

Structure of even Ge isotopes by means of interacting boson model with a fermion pair model

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(Received 4 December 1991)

The energy levels of the even-even Ge isotopes with mass number between 64 and 78 are studied in the model of the traditional interacting boson approximation. To account for the multiple band structure of these isotopes, one boson is allowed to break and form a fermion pair. The two fermions are allowed to excite to $f_{5/2}$ and $g_{9/2}$ single-particle orbitals. It was found that the energy levels of the $^{64-78}\text{Ge}$ isotopes can be reproduced reasonably.

PACS number(s): 21.10.Re, 21.60.Ev, 23.20.Lv, 27.70.+q

I. INTRODUCTION

The nuclear properties of nuclei around the $N=40$ region and, more particularly, of the even-mass Ge isotopes have been investigated by a number of experimental and theoretical works [1–23]. The Ge nuclei are characterized by a complex nuclear system subjected to a variety of nuclear interactions which make these nuclei very unstable in shape. Hence both the coexistence of a shape transition from spherical to weakly deformed and a coexistence of different types of deformation occur in these isotopes. Qualitatively, these features can be explained with the help of the Nilsson model [24]. For the proton and neutron numbers in this mass region, the Nilsson single-particle energy diagrams display various rather large gaps at different deformations. Thus a competition and coexistence of several kinds of configurations corresponding to various shapes at the low spin region is expected. Guilbaut *et al.* [12] have presented shell model calculations for ^{68}Ge ; however, a satisfactory reproduction of the experimental data was not obtained. Ardouin *et al.* [7] successfully performed constrained Hartree-Fock calculations using Skyrme's effective interaction to analyze the different structures of $^{68-76}\text{Ge}$ isotopes in terms of an oblate-to-prolate transition. Petrovici *et al.* [8] studied in detail the shape coexistence phenomena dominating the structure of the nucleus ^{68}Ge by taking into account the dominant correlations on top of the symmetry projected quasiparticle mean-field solutions. de Lima *et al.* [9] performed two-quasiparticle-plus-rotor calculations [25] and an interacting boson approximation (IBA) model [26] calculation for the ^{68}Ge nucleus. The results obtained can well describe the yrast features of the level scheme. Barclay *et al.* [10] performed a two-quasiparticle-plus-IBA model proposed by Gelberg and Zemel [27] Morrison, Fassler, and Lima [28], and Yoshida, Arima, and Otsuka [29,30] to study the $B(E2)$ and g factors for the high spin states of the ^{68}Ge nucleus. Reasonable agreement between calculated and measured values was obtained.

The purpose of this work is twofold. First, we want to present a systematic study of the even-mass Ge isotopes.

Second, and most important, we desire to investigate to what extent the observed shape coexistence or multiple band structure of these nuclei can be interpreted in terms of the interacting-boson-plus-a-fermion pair model. This model has been successfully applied to study the positive and negative parity states and band crossing behavior of even-mass deformed nuclei [31–34].

II. MODEL

The even-mass Ge isotopes with $Z=32$ and $32 \leq N \leq 44$ will be studied systematically. Taking the ^{40}Ca nucleus as the core, the boson numbers for the isotopes ^{64}Ge and ^{66}Ge are $N=12$ and 13, respectively. For the other isotopes which pass the neutron midshell, the neutron boson numbers are counted as one-half of the number of neutron holes. Thus the IBA model assumes valence boson numbers 13, 12, 11, 10, 9, and 8 for the nuclei ^{68}Ge , ^{70}Ge , ^{72}Ge , ^{74}Ge , ^{76}Ge , and ^{78}Ge , respectively. In this work it is assumed that one of the bosons can be broken to form a fermion pair which may occupy the $f_{5/2}$ or $g_{9/2}$ orbitals.

Our model space includes the IBA space with N bosons and space with $N-1$ bosons plus two fermions. The model Hamiltonian can be expressed as [32]

$$H = H_B + H_F + V_{BF} ,$$

where H_B is the IBA boson Hamiltonian

$$H_B = a_0 \epsilon_d + a_1 p^\dagger p + a_2 L \cdot L + a_3 Q \cdot Q .$$

The octupole term $T_3 \cdot T_3$ and hexadecapole term $T_4 \cdot T_4$ have been omitted in H_B since they are generally believed to be less important. The fermion Hamiltonian H_F is

$$H_F = \sum_j \epsilon_j \sqrt{2j+1} [a_j^\dagger \times \bar{a}_j]^{(0)} + \frac{1}{2} \sum_{J,j} V^J \sqrt{2J+1} [(a_j^\dagger \times a_j^\dagger)^J \times (\bar{a}_j \times \bar{a}_j)^J]^{(0)} ,$$

with a_j^\dagger being the nucleon creation operator. The mixing Hamiltonian V_{BF} is assumed to be

$$V_{BF} = Q^B \cdot Q - Q^B \cdot Q^B,$$

where

$$Q^B = (d^\dagger \times \bar{s} + s^\dagger \times \bar{d})^{(2)} - \frac{\sqrt{7}}{2} (d^\dagger \times \bar{d})^{(2)},$$

$$Q = Q^B + \alpha (a_j^\dagger \times \bar{a}_j)^{(2)}$$

$$+ \beta [(a_j^\dagger \times a_j)^{(4)} \times \bar{d} - d^\dagger \times (\bar{a}_j \times \bar{a}_j)^{(4)}]^{(2)}.$$

In the calculation the fermion potential is taken as the Yukawa type with the Rosenfeld mixture. The oscillator constant $\nu = 0.96 A^{-1/3} \text{ fm}^{-2}$ with $A = 70$ is assumed. The single-particle energies and interaction strength parameters contained in the boson Hamiltonian H_B and V_{BF} were chosen to reproduce the energy level spectra of even Ge isotopes with mass number between 64 and 78. In our calculation the interaction parameters contained in H_B for each nucleus are unified for both the N pure boson configuration and $N - 1$ -boson-plus-one-fermion pair configuration. The energy bands with these two kinds of configurations are mixed through the diagonalization of the energy matrix in the whole model space.

III. RESULT AND DISCUSSION

The interaction strengths and single-particle energies for Ge isotopes are allowed to be mass number dependent. Table I lists the best fitted interaction strengths and single-particle energies for all isotopes. The mixing parameter β can be unified as $\beta = -0.02 \text{ MeV}$, while the parameter α has a significant change from nucleus ^{66}Ge to nucleus ^{68}Ge . Since here we have particle-particle to particle-hole transitions, it is not surprising we have a significant change of α at this point. It is well known [35] that the four terms of H_B relate to the pure symmetries in the following way: In the U(5) symmetry, only ϵ_d and $L \cdot L$ terms appear; in the SU(3) limit, only $L \cdot L$ and $Q \cdot Q$ terms appear; and in the O(6) limit, only $p^\dagger p$ and $L \cdot L$ terms appear. To correlate the variation of the interaction parameters to the limiting symmetries, the resulting interaction parameters contained in the pure boson Hamiltonian of Ge isotopes as a function of mass numbers are plotted in Fig. 1. To the far right, we listed the symmetries to which each of the terms belongs. At the bot-

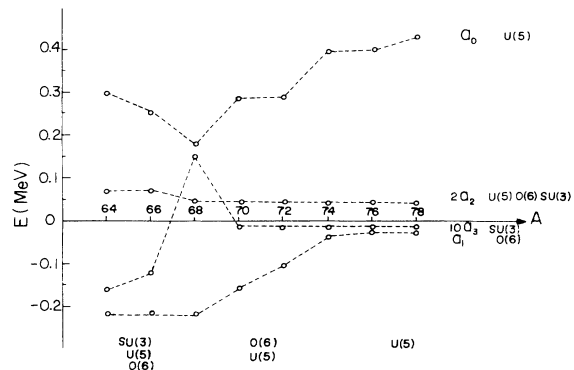


FIG. 1. Interaction parameters of H_B vs mass number A of Ge isotopes. Indicated on the right-hand side are the symmetries involved in each term. Indicated in the bottom are the symmetry regions for different Ge isotopes.

tom we indicated the possible relevant symmetries along the boson number axis. From Fig. 1 one can see that there are possibly symmetry changes from $A = 70$ to 72 , 72 to 74 , and 76 to 78 . The abrupt changes of the single-particle energies $\epsilon_{5/2}$ and $\epsilon_{9/2}$ for the nuclei ^{68}Ge , ^{70}Ge , and ^{72}Ge reflect the fact that there are structure changes in these nuclei. This is consistent with the result obtained by Lecomte *et al.* [20].

The calculated and observed energy spectra for the Ge isotopes are shown in Figs. 2–8. The levels marked with asterisks are not included in the least-squares fitting. Figure 2 shows the calculated and observed energy levels of the $N = Z$ nucleus ^{64}Ge . The structure of this very neutron deficient Ge isotope has been investigated recently with the use of particle- γ coincidence techniques in weak fusion-evaporation channels [1] and the evaporation code CASCADE [3] with the reaction $^{12}\text{C}(^{54}\text{Fe}, 2n)^{64}\text{Ge}$ at 150 MeV. Lister *et al.* [4] investigated the shape changes of ^{64}Ge experimentally and thus provide a direct test of a variety of nuclear models. Figure 3 shows the energy levels of the ^{66}Ge nucleus. From Figs. 2 and 3, it can be noted that our calculated energy levels of the nuclei ^{64}Ge and ^{66}Ge are all in good agreement with their experimental counterparts. The complex multiple band structures and shape coexistence of the nucleus ^{68}Ge have attracted

TABLE I. Interaction parameters (in MeV) adopted in this work.

Nucleus	Parameter (MeV)							
	a_0	a_1	a_2	a_3	α	β	$\epsilon_{5/2}$	$\epsilon_{9/2}$
	particle-particle							
^{64}Ge	0.2978	-0.22	0.035	-0.016	0.03	-0.02	0.249	1.534
^{66}Ge	0.2535	-0.22	0.035	-0.008	0.03	-0.02	0.186	1.488
	particle-hole							
^{68}Ge	0.1558	-0.22	0.023	0.015	-0.27	-0.02	0.111	1.080
^{70}Ge	0.2890	-0.155	0.023	-0.001	-0.27	-0.02	0.830	1.575
^{72}Ge	0.2890	-0.102	0.023	-0.001	-0.27	-0.02	1.200	1.687
^{74}Ge	0.3964	-0.035	0.023	-0.001	-0.27	-0.02	1.200	1.687
^{76}Ge	0.4000	-0.025	0.023	-0.001	-0.27	-0.02	1.200	1.687
^{78}Ge	0.4300	-0.025	0.023	-0.001	-0.27	-0.02	1.200	1.687

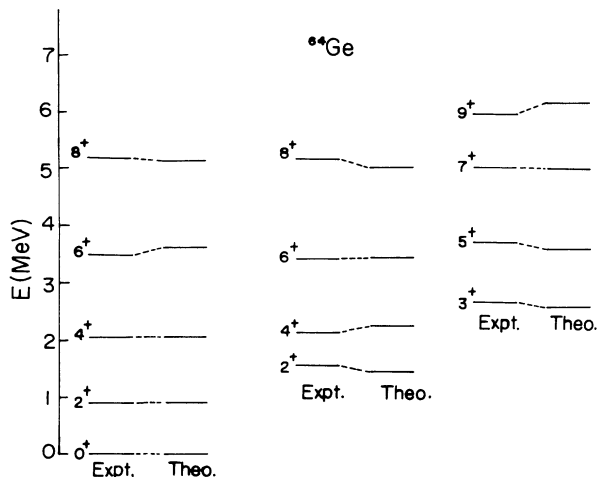


FIG. 2. Calculated and observed energy spectra for the nucleus ^{64}Ge . The experimental data are taken from Refs. [1–5].

much interest recently [7–11]. Petrovici *et al.* [8] investigated the shape coexistence phenomena which dominates the structure of the nucleus ^{68}Ge by using an approach of the excited variation after mean-field projection in a realistic model space. Chaturvedi *et al.* [11] employed the same approach to study the complex band structure of the ^{68}Ge nucleus and obtained good agreement between the theoretical and observed levels. de Lima *et al.* [9] studied the low and high spin states of ^{68}Ge through in-beam γ -ray spectroscopy via the $^{58}\text{Ni}(^{12}\text{C},2p)^{68}\text{Ge}$, $^{63}\text{Cu}(^7\text{Li},2n)^{68}\text{Ge}$, and $^{52}\text{Cr}(^{19}\text{F},p2n)^{68}\text{Ge}$ reactions. They observed three even parity collective bands which can be interpreted fairly well in terms of the rotation-aligned and interacting boson models. Our calculated results of the nucleus ^{68}Ge are shown in Fig. 4. The different bands are displayed in different columns for clear comparison. It can be seen from the figure that the complex multiple bands can be reproduced quite well. The calculated and observed energy levels of the isotopes $^{70-78}\text{Ge}$ are shown in Figs. 5–8. Ardouin *et al.* [14] in-

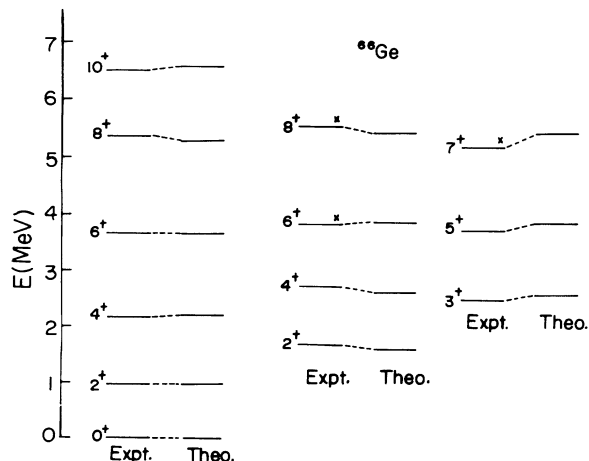


FIG. 3. Calculated and observed energy spectra for the nucleus ^{66}Ge . The experimental data are taken from Ref. [6].

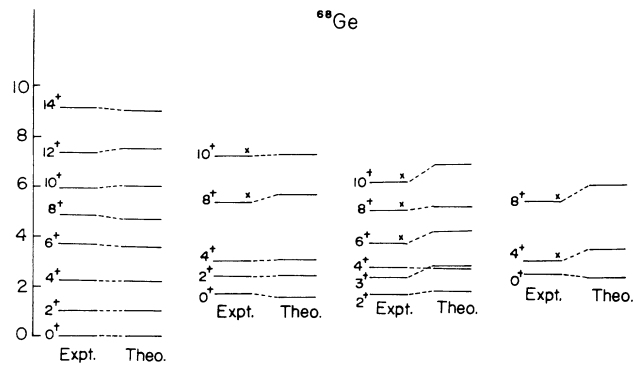


FIG. 4. Calculated and observed energy spectra for the nucleus ^{68}Ge . The experimental data are taken from Ref. [12].

vestigated the Ge nuclear structure with dynamic deformation theory, which is an improvement of the pairing-plus-quadrupole model. Satisfactory results were obtained. The spectroscopy of the nucleus ^{72}Ge is especially interesting because this $N=40$ semiclosed shell nucleus is one of the few even-even nuclei to have a 0^+ state for the first excited state [14,17,18]. Kotlinski *et al.* [17] studied the Coulomb excitation of ^{72}Ge using ^{16}O , ^{58}Ni , and ^{208}Pb targets. They proposed that 0_2^+ state is an intruder state. Our calculated 0_2^+ state has a discrepancy of 0.39 MeV above the observed value and is in a reversed order with the calculated 2_1^+ state. However, the calculated results in the other energy levels in general agree reasonably with the observed values. The calculated and observed energy levels of the nucleus ^{74}Ge are shown in Fig. 7. For this nucleus only a few levels have been identified experimentally. One can see from the figure that the agreement between the calculated and observed levels is satisfactory especially for those levels which were included in the least-squares fitting. The energy levels of the nuclei ^{76}Ge and ^{78}Ge are shown in Fig. 8. One can see that the agreement between the theoretical energy levels and experimental counterparts is quite reasonable.

The analysis of the relative wave-function intensities

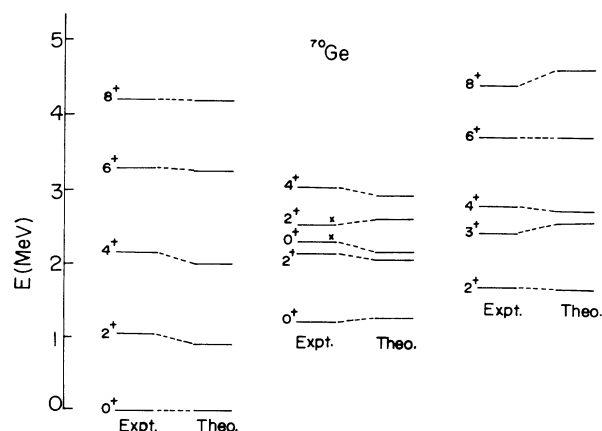


FIG. 5. Calculated and observed energy spectra for the nucleus ^{70}Ge . The experimental data are taken from Ref. [13].

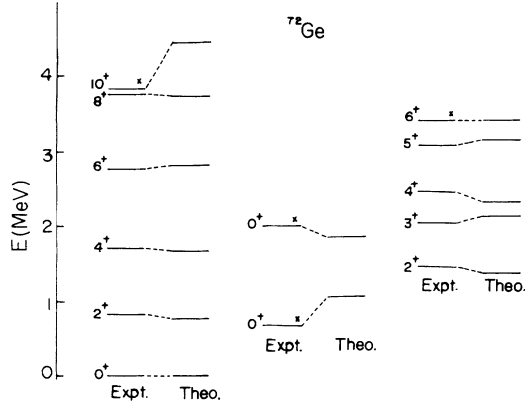


FIG. 6. Calculated and observed energy spectra for the nucleus ^{72}Ge . The experimental data are taken from Ref. [15].

for the energy levels of ^{64}Ge shows that most of the levels are dominated by the pure boson configuration except for the $J^\pi=5_1^+$, 6_2^+ , and 7_1^+ states, which are dominated by the configuration of $N-1$ boson plus two $f_{5/2}$ fermions, and the states $J^\pi=8_1^+$ and 9_1^+ , which are dominated by the configuration of $N-1$ boson plus two $g_{9/2}$ fermions. For the nucleus ^{66}Ge , most states are dominated by the pure boson configuration except for the states $J^\pi=4_2^+$, 5_1^+ , 6_2^+ , 7_1^+ , and 8_2^+ , which are dominated by the $N-1$ boson plus two $f_{5/2}$ fermion configuration, and the states $J^\pi=8_1^+$ and 10_1^+ , which are dominated by the configuration of $N-1$ boson plus two $g_{9/2}$ fermions. In our results it was found that the overlapping between different subspaces is very small. Table II shows the relative intensities of wave functions corresponding to N boson and $N-1$ -boson-plus-two- $f_{5/2}$ -or- $g_{9/2}$ -fermions configurations for each state of the nuclei ^{68}Ge , ^{70}Ge , and ^{72}Ge . The total intensity of N boson, $N-1$ -boson-plus-two- $f_{5/2}$ -fermions, and $N-1$ -boson-plus-two- $g_{9/2}$ -fermions configurations for each state is normalized to 1000. One can see that, in general, the energy levels of

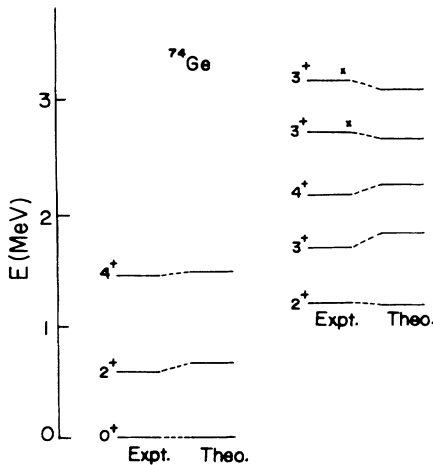


FIG. 7. Calculated and observed energy spectra for the nucleus ^{74}Ge . The experimental data are taken from Ref. [21].

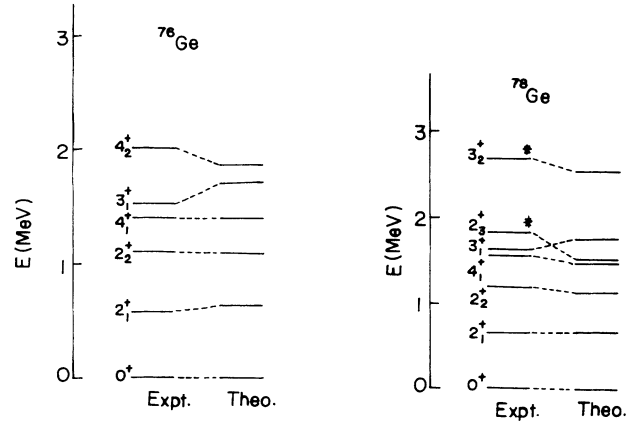


FIG. 8. Calculated and observed energy spectra for the nuclei ^{76}Ge and ^{78}Ge . The experimental data are taken from Refs. [22,23].

these three nuclei are dominated by the pure boson configurations. The $N-1$ -boson-plus-two- $f_{5/2}$ -fermions configuration is important only in the states $J^\pi=4_2^+$, 8_3^+ , 10_3^+ , and 12_2^+ of ^{68}Ge and 4_2^+ , 6_2^+ , 7_1^+ , and 8_2^+ of ^{70}Ge , while the $N-1$ -boson-plus-two- $g_{9/2}$ -fermions configuration is only dominant in the states of $j^\pi=8_1^+$, 12_1^+ , and 14_1^+ , of ^{68}Ge , 7_1^+ of ^{70}Ge , and 8_1^+ and 10_1^+ levels of ^{72}Ge . If we increase the $f_{5/2}$ or $g_{9/2}$ single-particle energy so that this orbit becomes effectively irrelevant, then the agreement between the calculated and observed levels will become worse. One can also find that the mixing between different configurations is very small in general. There are only four states ($J^\pi=4_2^+$, 7_1^+ , 8_2^+ , 8_2^+) which possess more than 10% mixing between different kinds of configurations. For the nuclei ^{74}Ge , ^{76}Ge , and ^{78}Ge , the pure boson configurations are dominant in nearly all states. Only the states $J^\pi=3_3^+$ and 4_3^+ of ^{74}Ge , 4_4^+ of ^{76}Ge , and 4_3^+ of the nucleus ^{78}Ge are dominated by the $N-1$ -boson-plus-two- $f_{5/2}$ -fermions configuration.

There are some experimental $B(E2)$ values for Ge isotopes [6,12,13,15,17,21,22]. The study of these values will give us a good test of the model wave functions. The electric quadrupole operator can be written as

$$T(E2) = e^B Q + e^F \alpha (a_j^\dagger \bar{a}_j)^{(2)} + \beta e^B [(a_j^\dagger a_j)^{(4)} \bar{d} - d^\dagger (\bar{a}_j \bar{a}_j)^{(4)}]^{(2)},$$

where Q is taken as

$$Q = (d^\dagger \bar{s} + s^\dagger d)^{(2)} - \kappa (d^\dagger \bar{d})^{(2)}.$$

In our calculation the fermion effective charge e^F is assumed to be 0.5. It was found that different values of the fermion effective charge cannot yield significant change in $B(E2)$ values. The boson effective charge in the $T(E2)$ operator has been determined by normalizing the calculated $B(E2)$ value to the corresponding observed data for the transition $2_1 \rightarrow 0_1$. The parameters α and β are assumed to have the same values as used in the mix-

TABLE II. Relative intensities of the N boson configuration (denoted as 0) and the $N-1$ bosons plus a fermion pair occupied in single fermion orbits $f_{5/2}$ (denoted as $f_{5/2}^2$) or $g_{9/2}$ (denoted as $g_{9/2}^2$) configurations for ^{68}Ge , ^{70}Ge , and ^{72}Ge isotopes. The total intensity of configurations with and without fermion-pair excitation for each state is normalized to 1000.

States	Nucleus								
	^{68}Ge			^{70}Ge			^{72}Ge		
	0	$f_{5/2}^2$	$g_{9/2}^2$	0	$f_{5/2}^2$	$g_{9/2}^2$	0	$f_{5/2}^2$	$g_{9/2}^2$
0 ₁	1.000	0.000	0.000	1.000	0.000	0.000	1.000	0.000	0.000
0 ₂	1.000	0.000	0.000	1.000	0.000	0.000	1.000	0.000	0.000
0 ₃	1.000	0.000	0.000	0.999	0.001	0.000	0.999	0.001	0.000
0 ₄	1.000	0.000	0.000	1.000	0.000	0.000	0.998	0.002	0.000
2 ₁	1.000	0.000	0.000	1.000	0.000	0.000	0.999	0.001	0.000
2 ₂	1.000	0.000	0.000	1.000	0.000	0.000	0.999	0.001	0.000
2 ₃	1.000	0.000	0.000	0.998	0.002	0.000	0.998	0.002	0.000
2 ₄	1.000	0.000	0.000	0.999	0.001	0.000	0.997	0.002	0.001
3 ₁	0.999	0.001	0.000	0.999	0.001	0.000	0.998	0.001	0.001
4 ₁	0.994	0.006	0.000	0.993	0.006	0.001	0.996	0.004	0.000
4 ₂	0.009	0.972	0.019	0.746	0.246	0.008	0.995	0.004	0.001
4 ₃	0.998	0.002	0.000	0.958	0.038	0.004	0.990	0.009	0.001
6 ₁	0.989	0.010	0.001	0.969	0.030	0.001	0.981	0.016	0.003
8 ₁	0.000	0.000	1.000	0.000	0.000	1.000	0.001	0.000	0.999

TABLE III. Calculated and the experimental $B(E2)$ values (in Weisskopf units) for ^{68}Ge and ^{70}Ge . The experimental data are adopted from Refs. [12] and [13].

Nucleus	$J_i \rightarrow J_f$	Expt.	This work	Other work ^a
^{68}Ge	2 ₁ →0 ₁	17.6	17.6	17.6
	2 ₂ →0 ₁	0.17	16.34	
	4 ₁ →2 ₁	13.9	29.2	29.09
	4 ₂ →2 ₁	0.5	0.88	
	4 ₂ →2 ₂	0.41	0.11	
	6 ₁ →4 ₁	12.0	25.28	33.67
	8 ₁ →6 ₁	15.0	0.06	16.07
	8 ₂ →6 ₁	12.0	8.35	19.13
	8 ₂ →6 ₂	23.0	5.83	
	8 ₃ →6 ₁	15.0	0.39	9.95
	8 ₄ →6 ₁	3.3	23.90	
	10 ₁ →8 ₁	24.0	54.45	17.60
	10 ₂ →8 ₃	> 23.0	4.2	
	12 ₂ →10 ₂	10.0	11.02	
14 ₁ →12 ₁	4.5	13.45		
^{70}Ge	2 ₁ →0 ₁	21.0	21.0	
	0 ₂ →2 ₁	48.0	50.2	
	2 ₂ →0 ₁	1.0	7.94	
	2 ₂ →0 ₂	25.0	22.02	
	2 ₂ →2 ₁	111.0	35.09	
	4 ₁ →2 ₁	24.0	35.04	
	0 ₃ →2 ₂	> 4.8	0.72	
	0 ₃ →2 ₁	> 0.14	0.16	
	4 ₂ →2 ₂	29.0	15.11	
	4 ₃ →2 ₁	2.0	0.69	
	6 ₁ →4 ₁	34.0	42.04	
6 ₂ →4 ₂	27.0	7.43		
8 ₁ →6 ₁	6.5	2.5		
8 ₂ →6 ₁	43.0	44.0		

^aReference [10].

ing Hamiltonian. The value of κ is chosen to be $-\sqrt{7}/2$, which is the generator of the SU(3) group. For illustration only we list the calculated and experimental $B(E2)$ values for ^{68}Ge and ^{70}Ge isotopes in Table III. Other theoretical work [10] is also presented for comparison. One can note from Table III that our calculated values agree reasonably with observed data and other theoretical values.

IV. SUMMARY

In summary, we have investigated the structure of the energy spectra of the isotope string of Ge with mass number between 64 and 78. We extended the IBA model to allow a boson to be broken to form a fermion pair which can occupy the $f_{5/2}$ or $g_{9/2}$ orbitals. The calculated energy levels are in satisfactory agreement with the observed values for the whole string of Ge isotopes.

The plot of the interaction strength versus mass number reveals a transition from the mixture of SU(3), O(6), and U(5) symmetry to O(6) and U(5) mixture and then finally U(5) symmetry as the mass number increases from 64 to 78. This structure change is apparently manifested in the steep change of the Hamiltonian between these two nuclei. We also analyze the relative intensities for configurations of pure N bosons and of $N-1$ -bosons-plus-two-fermions excitation. Our analysis shows that, in general, the mixings between these two kinds of configurations are small.

This work was supported by the National Science Council of ROC with Grant No. NSC81-0208-M009-04.

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