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Modified propagation equations for soliton transmission in a polarization-division multiplexing system

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The modified coupled averaged propagation equations describing the orthogonally polarized soliton propagation in a random birefringent fibre are derived. These include the third-order dispersion and Raman shift terms. Using these equations, the effects of the third-order dispersion and Raman shift terms are found to reduce the soliton interactions in a polarization-division multiplexing transmission system.

1. Introduction

Recently polarization-division multiplexing (PDM) has been used to increase the capacity of the soliton-based transmission system [1–6]. It has been demonstrated that the interaction of orthogonally polarized solitons is weaker than that of parallely polarized solitons [1]. A good analytic description of PDM soliton interaction was made by the perturbation method [2–4]. The PDM system was considered by assuming the correlation length to be much shorter than the soliton period, so that the fluctuating local birefringence vector may be averaged over all polarization states. The soliton propagation of the PDM system can be described by the coupled averaged propagation equations (CAPE) [1, 7, 8], where the third-order dispersion and Raman shift terms are neglected. Using CAPE, De Angelis and Wabnitz [3] have numerically shown the interaction of the two orthogonal pulses. Initially, the two pulses attract each other. As the collision distance is approached, a complete polarization rotation by 90° for each pulse occurs in its own time slot. It has been shown that the collision distance of the PDM system is much larger than that of the parallely polarized system. However, the effects of third-order dispersion and Raman shift are not reported in a random birefringent PDM transmission system.

In this paper, we will derive the modified coupled averaged propagation equations (MCAPE) which describe the soliton propagation in a PDM transmission system including the effects of the third-order dispersion and Raman shift. Using these equations, we will show that the interactions of solitons are reduced by the third-order dispersion and Raman shift terms.

2. The modified propagation equations

The soliton propagation in a linearly birefringent fibre can be described by the modified coupled non-linear Schrödinger equations (MCNSE) [9]:

$$\mathrm{i}\frac{\partial U}{\partial Z}+\mathrm{i}\delta\frac{\partial U}{\partial T}+\frac{1}{2}\frac{\partial^2 U}{\partial T^2}-\mathrm{i}\frac{C_3}{6}\frac{\partial^3 U}{\partial T^3}+\left(|U|^2+\frac{2}{3}|V|^2\right)U$$
$$+\frac{1}{3}V^2U^*\exp(-\mathrm{i}R\delta Z)-\tau_{\mathrm{R}}\left[U\left(\frac{\partial|U|^2}{\partial T}+\frac{1}{3}\frac{\partial|V|^2}{\partial T}\right)+\frac{1}{3}V\frac{\partial V^*U}{\partial T}\right]=\mathrm{i}\gamma U \tag{1a}$$

$$\mathrm{i}\frac{\partial V}{\partial Z}-\mathrm{i}\delta\frac{\partial V}{\partial T}+\frac{1}{2}\frac{\partial^2 V}{\partial T^2}-\mathrm{i}\frac{C_3}{6}\frac{\partial^3 V}{\partial T^3}+\left(|V|^2+\frac{2}{3}|U|^2\right)V$$
$$+\frac{1}{3}U^2V^*\exp(\mathrm{i}R\delta Z)-\tau_{\mathrm{R}}\left[V\left(\frac{\partial|V|^2}{\partial T}+\frac{1}{3}\frac{\partial|U|^2}{\partial T}\right)+\frac{1}{3}U\frac{\partial U^*V}{\partial T}\right]=\mathrm{i}\gamma V \tag{1b}$$

where U and V are two polarization components of the electric field envelope normalized by the electric field scale Q. Z and T are normalized by dispersion length L_{D}, and time scale T_0, respectively. Q, L_{D} and T_0 are related by

$$Q=\left[\frac{\lambda|\beta_2|A_{\mathrm{eff}}}{2\pi n_2 T_0^2}\right]^{1/2},\quad L_{\mathrm{D}}=\frac{T_0^2}{|\beta_2|}$$

where λ is the wavelength, β_2 is the second-order dispersion, A_{eff} is the effective fibre cross-section area, and n_2 is the Kerr coefficient. $T_0=T_{\mathrm{W}}/1.763$ and T_{W} is the initial full pulsewidth at the half magnitude. The coefficients in Equations 1 are

$$\delta=\frac{\Delta\beta L_{\mathrm{D}}}{2T_0},\quad C_3=\frac{\beta_3 L_{\mathrm{D}}}{T_0^3},\quad \tau_{\mathrm{R}}=\frac{T_{\mathrm{R}}}{T_0},\quad \gamma=\alpha L_{\mathrm{D}}$$

where $\Delta\beta$ represents the inverse group velocity difference of two polarization components, β_3 is the third-order dispersion, α is the fibre loss, T_{R} is the slope of the Raman gain profile at the carrier frequency. $R=8\pi cT_0/\lambda$ is the normalized wave number and c is the velocity of light in a vacuum. However, in a real communication fibre, the orientation of fibre birefringence randomly varies with a correlation length which is typically of length 100 m or so [10]. When the soliton wavelength, λ, is at 1.55 μm, the index of refraction, Δn, between the two polarizations varies in the range 5×10^{-9} to 8×10^{-4}, $\Delta\beta$ is found in the range 1.7×10^{-2} to 2.7×10^{3} ps km^{-1}. If we take $T_{\mathrm{W}}=3$ ps and $\beta_2=-0.255$ ps^2 km^{-1} ($L_{\mathrm{D}}\cong 11.4$ km), we can find that $R\delta=(4\pi\Delta n/\lambda)\cdot L_{\mathrm{D}}\approx\Delta n\times10^{11}$ is much greater than unity over the entire range of Δn. Hence, the terms with the factor $\mathrm{e}^{\pm\mathrm{i}R\delta Z}$ are rapidly varying and can be neglected.

To derive modified coupled averaged propagation equations (MCAPE) for a PDM soliton system, we rewrite Equations 1 into a single-vector equation:

$$\mathrm{i}\frac{\partial\Psi}{\partial Z}+\mathrm{i}\sigma_1\frac{\partial\Psi}{\partial T}+\frac{1}{2}\frac{\partial^2\Psi}{\partial T^2}-\mathrm{i}\frac{C_3}{6}\frac{\partial^3\Psi}{\partial T^3}+\frac{5}{6}|\Psi|^2\Psi+\frac{1}{6}(\Psi^+\sigma_1\Psi)\sigma_1\Psi-\frac{1}{2}\tau_{\mathrm{R}}\left(\frac{\partial}{\partial T}|\Psi|^2\right)\Psi$$
$$-\frac{1}{6}\tau_{\mathrm{R}}\left[\frac{\partial}{\partial T}(\Psi^+\sigma_1\Psi)\right]\sigma_1\Psi-\frac{1}{3}\tau_{\mathrm{R}}\left\{\frac{\partial}{\partial T}[\Psi(\sigma_2\Psi)^+]\right\}\sigma_2\Psi=\mathrm{i}\gamma\Psi \tag{2}$$

where $\Psi = (U, V)^{\mathrm{t}}$ is the polarization state envelope vector, $\Psi^{+} = (\Psi^{*})^{\mathrm{t}}$, $\sigma_1 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ and $\sigma_2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$. The last two terms on the left-hand side of Equation 2 are transformed from Raman shift terms.

We assume that the polarization axes of the fibre periodically undergo a sudden rotation θ, which can take any value from 0 to 2π, and can be represented by an arbitrary rotation on the Poincaré sphere [7, 8]. When the rotation θ occurs, we also assume that a random phase ϕ is added to the phase difference between the polarization state envelopes U and V. The total transformation is given by

$$\begin{bmatrix} U' \\ V' \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta\, \mathrm{e}^{\mathrm{i}\phi} \\ -\sin\theta\, \mathrm{e}^{-\mathrm{i}\phi} & \cos\theta \end{bmatrix} \begin{bmatrix} U \\ V \end{bmatrix} \tag{3}$$

where the angles θ and ϕ are assumed to be uniformly distributed random variables on the Poincaré sphere. We have

$$\begin{aligned} &\mathrm{i}\frac{\partial\Psi}{\partial Z} + \mathrm{i}\delta\sigma\frac{\partial\Psi}{\partial T} + \frac{1}{2}\frac{\partial^2\Psi}{\partial T^2} - \mathrm{i}\frac{C_3}{6}\frac{\partial^3\Psi}{\partial T^3} + \frac{5}{6}|\Psi|^2\Psi + \frac{1}{6}(\Psi^{+}\sigma\Psi)\sigma\Psi - \frac{1}{2}\tau_{\mathrm{R}}\left(\frac{\partial}{\partial T}|\Psi|^2\right)\Psi \\ &\quad - \frac{1}{6}\tau_{\mathrm{R}}\left[\frac{\partial}{\partial T}(\Psi^{+}\sigma\Psi)\right]\sigma\Psi - \frac{1}{3}\tau_{\mathrm{R}}R^{-1}\left\{\frac{\partial}{\partial T}[R\Psi\Psi^{+}R^{+}\sigma_2^{+}]\right\}\sigma_2 R\Psi = \mathrm{i}\gamma\Psi \end{aligned} \tag{4}$$

where

$$R = \begin{bmatrix} \cos\theta & \sin\theta\, \mathrm{e}^{\mathrm{i}\phi} \\ -\sin\theta\, \mathrm{e}^{-\mathrm{i}\phi} & \cos\theta \end{bmatrix}$$

and

$$\sigma = R^{-1}\sigma_1 R = \begin{bmatrix} \cos 2\theta & -\sin 2\theta\, \mathrm{e}^{-\mathrm{i}\phi} \\ -\sin 2\theta\, \mathrm{e}^{-\mathrm{i}\phi} & -\cos 2\theta \end{bmatrix}$$

The second term on the left-hand side of Equation 4 can be ignored because δ varies randomly between positive and negative. Averaging Equation 4 over θ and ϕ on the Poincaré sphere, we obtain the modified coupled averaged propagation equations (MCAPE) for a PDM soliton system:

$$\begin{aligned} &\mathrm{i}\frac{\partial u}{\partial Z} + \frac{1}{2}\frac{\partial^2 u}{\partial T^2} - \mathrm{i}\frac{C_3}{6}\frac{\partial^3 u}{\partial T^3} + \left(|u|^2 + |v|^2\right)u \\ &\quad - \tau_{\mathrm{R}} u\frac{\partial}{\partial T}|u|^2 - \frac{1}{2}\tau_{\mathrm{R}} u\frac{\partial}{\partial T}|v|^2 - \frac{1}{2}\tau_{\mathrm{R}} v\frac{\partial}{\partial T}(v^{*}u) = \mathrm{i}\gamma u \end{aligned} \tag{5a}$$

$$\begin{aligned} &\mathrm{i}\frac{\partial v}{\partial Z} + \frac{1}{2}\frac{\partial^2 v}{\partial T^2} - \mathrm{i}\frac{C_3}{6}\frac{\partial^3 v}{\partial T^3} + \left(|u|^2 + |v|^2\right)v \\ &\quad - \tau_{\mathrm{R}} v\frac{\partial}{\partial T}|v|^2 - \frac{1}{2}\tau_{\mathrm{R}} v\frac{\partial}{\partial T}|u|^2 - \frac{1}{2}\tau_{\mathrm{R}} u\frac{\partial}{\partial T}(u^{*}v) = \mathrm{i}\gamma v \end{aligned} \tag{5b}$$

where $u = \sqrt{9/8}U$, $v = \sqrt{9/8}V$. The last three terms on the left-hand side of Equations 5 describe the averaged Raman effect, in which the first term is the self-frequency shift (SFS) term, and the other two are the cross-frequency shift (XFS) terms.

3. Numerical results

The typical fibre parameters used to solve Equations 5 numerically are: soliton wavelength $\lambda = 1.55\,\mu\text{m}$, $\beta_2 = -0.255\,\text{ps}^2\,\text{km}^{-1}$ $(D = 0.2\,\text{ps}\,\text{km}^{-1}\,\text{nm}^{-1})$, $\beta_3 = 0.14\,\text{ps}^3\,\text{km}^{-1}$, $\alpha = 0.22\,\text{dB}\,\text{km}^{-1}$, $n_2 = 3.2 \times 10^{-20}\,\text{m}^2\,\text{W}^{-1}$, and $T_R = 5\,\text{fs}$. The effective fibre cross-section is $35\,\mu\text{m}^2$. The fibre loss is periodically compensated by the lumped amplifiers and the amplification period is assumed to be $0.25L_D$. To show the effects of the third-order dispersion and Raman shift terms, we consider the soliton pulsewidth $T_W = 3$ ps. Equations 5 are solved by the split-step Fourier method with the initial condition $u(Z = 0, T) = \text{sech}(T + \Delta_0/2T_0)$ and $v(Z = 0, T) = \text{sech}(T - \Delta_0/2T_0)$ with $\Delta_0 = 3.5T_W$. In the absence of the third-order dispersion and Raman shift terms, Figs. 1a and 1b show the envelopes of $|u|$ and $|v|$ in the PDM soliton transmission system, respectively. The two pulses attract each other in the beginning and then repel to their own time slot after the collision distance $Z_c \cong 112L_D$. At the collision point the interaction leads to a complete polarization rotation by 90° for each pulse. The polarization-state each of the two pulses can exactly recover its own orientation at a distance which is a multiple of $2Z_c$. In Figs. 2a and 2b, we show the envelopes of $|u|$ and $|v|$, respectively, in the presence of the third-order dispersion and Raman shift terms. It is seen that the interaction leads to some polarization rotation which is much smaller than 90° at the collision distance. The polarization rotation at the second collision point is larger than that at the first collision point. Moreover, the degree of polarization rotation of the u polarization component is smaller than that of the v polarization component. In Fig. 2, the first collision distance Z_c is found at about $93L_D$ and the second collision distance is found at about $325L_D$. Its period is no longer $2Z_c$. In addition, we can see the obvious time delay of the two pulses, which is mainly due to the Raman shift terms. Figure 3 shows the variation of separation of two solitons as a function of normalized distance, curve (a) is obtained in the absence of the third-order dispersion and Raman shift terms and curve (b) is obtained in the presence of the third-order dispersion and Raman shift terms. For curve (a), the separation gradually reduces until the collision distance where the separation is minimum at about 7.2 ps. The oscillation period is $224L_D$. For curve (b), the first minimum separation is at about 8.9 ps at a

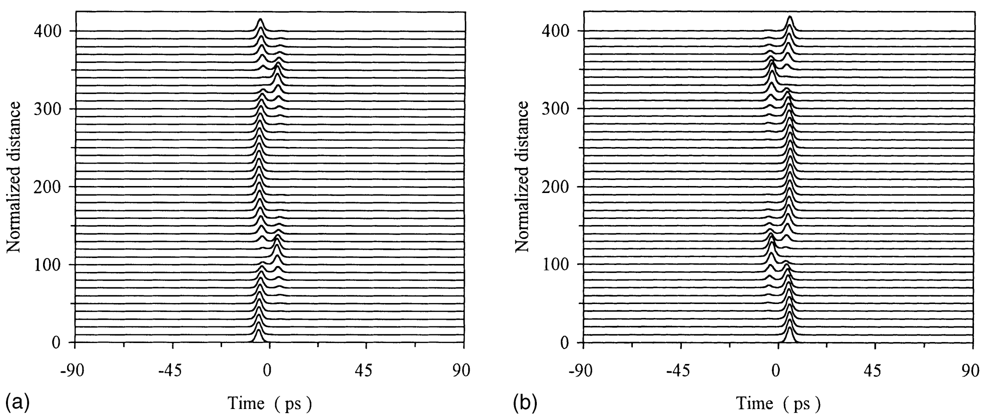

Figure 1 The interaction of two solitons in a PDM transmission system without the third-order dispersion and Raman shift terms: (a) the envelope of $|u|$, and (b) the envelope of $|v|$.

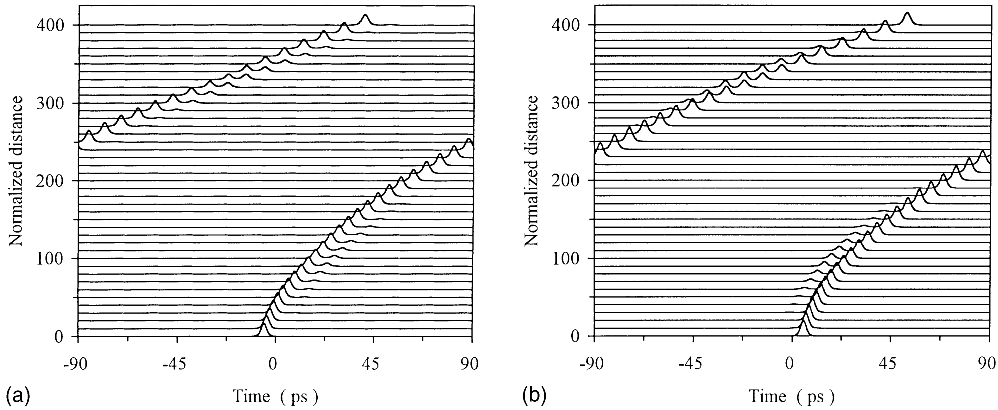

Figure 2 The interaction of two solitons in a PDM transmission system with the third-order dispersion and Raman shift terms: (a) the envelope of $|u|$, and (b) the envelope of $|v|$.

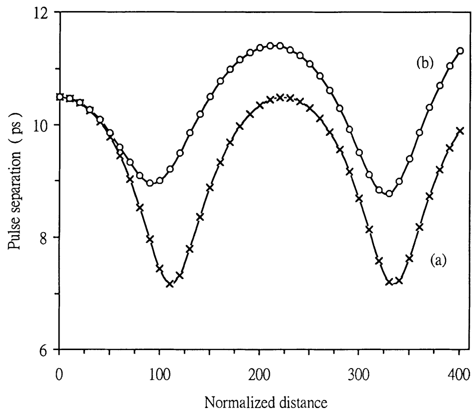

Figure 3 The variation of separation of two solitons as a function of normalized distance: curve (a) without the third-order dispersion and Raman shift terms, and curve (b) with the third-order dispersion and Raman shift terms.

distance $93L_D$, the next minimum separation is at about 8.8 ps at a distance $325L_D$ and the maximum separation is at about 11.4 ps at a distance $215L_D$ where the polarization rotation is almost equal to zero. Comparing curves (a) and (b), it can be seen that the third-order dispersion and Raman shift terms reduce the variation of pulse separation at the collision distance. The first collision distance will be shorter when the third-order dispersion and Raman shift terms are present. According to the degree of polarization rotation and the variation of pulse separation, we know that the third-order dispersion and Raman shift terms reduce soliton interaction.

4. Conclusion

In conclusion, we have derived the modified coupled averaged propagation equations of solitons with third-order dispersion and Raman shift terms in a random birefringent PDM soliton transmission system. It is found that the polarization rotation and the variation of pulse separation will be reduced when the third-order dispersion and Raman shift terms are present; i.e., the soliton transmission is reduced in the PDM transmission system.

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