Actually, to illustrate how conjugate coefficients can improve the arithmetic complexity the authors consider only the sum of two complex multiplications:

$$(a + jb)(c + jd) + (a - jb)(e + jf) = a[(c + jd) + (e + jf)] + jb[(c + jd) - (e + jf)] \tag{1}$$

ignoring the difference between two complex multiplications.

At the same time, the 'butterfly' operation for a length-2^n FFT requires the difference of these multiplications to be considered.

$$(a + jb)(c + jd) - (a - jb)(e + jf) = a[(c + jd) - (e + jf)] + jb[(c + jd) + (e + jf)] \tag{2}$$

The combined calculation of eqns. 1 and 2 corresponds to the two-pointed cyclic convolution calculation:

$$\begin{bmatrix} p + jq \\ r + js \end{bmatrix} = \begin{bmatrix} a & jb \\ jb & a \end{bmatrix} \cdot \begin{bmatrix} (c + e) + j(d + f) \\ (c - e) + j(d - f) \end{bmatrix} \tag{3}$$

The cyclic convolution of eqn. 3 is transformed to the next skew-cyclic convolution:

$$\begin{bmatrix} p + jq \\ -s + jr \end{bmatrix} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix} \cdot \begin{bmatrix} (c + e) + j(d + f) \\ (d - f) + j(e - c) \end{bmatrix} \tag{4}$$

which corresponds to two real skew-cyclic convolutions:

$$\begin{bmatrix} p \\ -s \end{bmatrix} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix} \cdot \begin{bmatrix} c + e \\ d - f \end{bmatrix} \tag{5a}$$

$$\begin{bmatrix} q \\ r \end{bmatrix} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix} \cdot \begin{bmatrix} d + f \\ e - c \end{bmatrix} \tag{5b}$$

The calculation of both eqn. 5*a* and *b* ratios and, consequently, both eqn. 1 and 2 ratios, requires six real multiplications and six real additions, but not four real multiplications, as asserted in the article [1]. This means that the CPFFT algorithm has analogous arithmetic complexity to the split-radix FFT algorithm [2–5].

Note that if eqn. 3 has the appearance of the two-pointed skew-cyclic convolution

$$\begin{bmatrix} p + jq \\ r + js \end{bmatrix} = \begin{bmatrix} a & jb \\ -jb & a \end{bmatrix} \cdot \begin{bmatrix} c + jd \\ e + jf \end{bmatrix}$$

it can be transformed to the following two-pointed cyclic convolution:

$$\begin{bmatrix} p + jq \\ -s + jr \end{bmatrix} = \begin{bmatrix} a & b \\ b & a \end{bmatrix} \cdot \begin{bmatrix} c + jd \\ -f + jc \end{bmatrix}$$

the calculation of which requires four real multiplications. This point is noted in a monograph by Blahut [6].

10th March 1992

A. M. Krot and H. B. Minervina (*Institute of Engineering Cybernetics of the Byelorussian Academy of Sciences, Surganov Str. 6, 220012 Minsk, Byelorussia*)

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POLY-OXIDE/POLY-Si/SiO_2/Si STRUCTURE FOR ELLIPSOMETRY MEASUREMENT

T. S. Chao, C. L. Lee, T. F. Lei and Y. T. Yen

Indexing terms: Measurement, Thin films, Elliposometry

A multiple poly-oxide/poly-Si/SiO_2/Si sandwiched structure is proposed for the conventional single incident angle and single wavelength ellipsometry measurement of the thicknesses and refractive indices of the poly-oxide and the poly-Si at the same time. The structure is simple and gives accurate results.

Ellipsometry is a powerful technique for measuring the thickness and refractive index of a thin film owing to its nondestructiveness, ease of use, and high sensitivity to a monolayer thickness [1–2]. However, for a thin film, such as a poly-Si film which is optically-absorptive, ellipsometry cannot be readily applied due to the existence of the imaginary part in the refractive index. Generally, more than two measurements are needed to solve for the thickness *T*, and complex refractive index $N\text{–}iK$. Various alternative approaches have been adopted [3]; they are

(1) multiple film thickness (MFT) measurement [4]

(2) multiple incident medium (MIM) measurement [5]

(3) multiple wavelength ellipsometry measurement [6, 7]

(4) multiple angle incident (MAI) ellipsometry measurement [8, 9].

However, all these methods suffer drawbacks, such as the need for more than two samples to be prepared whose optical characteristics may not be the same, or the need for a complicated instrument setup, so that the techniques are difficult to apply to in-line measurement in the manufacturing process.

In this Letter, a sandwiched poly-poly/poly-Si/SiO_2/Si structure is proposed for conventional ellipsometry measurement with a single incident angle and a single wavelength to measure the thicknesses and the refractive indices of the poly-oxide and poly-Si. The structure is compatible with that of MOS ICs. This makes possible the in-situ ellipsometry measurement of the poly-oxide and poly-Si of MOS ICs during manufacture.

Fig. 1 shows the sandwiched poly-oxide/poly-Si/SiO_2/Si structure. In this structure, the bottom oxide has two different thicknesses at two neighbouring regions. The thickness and refractive index of this oxide at two regions can be first measured with an ellipsometer. After depositing the poly-Si using

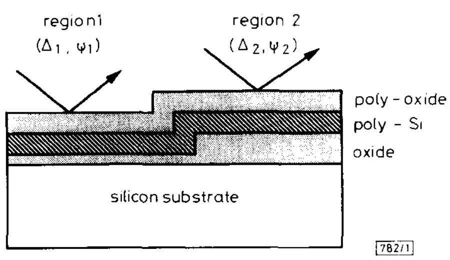

Fig. 1 *Multiple sandwiched structure of poly-oxide/poly-Si/SiO_2/Si structure for conventional ellipsometry measurement*

low-pressure chemical vapour deposition (LPCVD) and oxidising the deposited poly-Si, the laser beam of the ellipsometer can be applied to regions 1 and 2. Two sets of (Δ, ψ) data can be obtained. The ellipsometric equations of this system can be written as [3]

$$f_1(\Delta_1, \psi_1, \phi, N_s, K_s, N_{ox}, T_{ox1}, N_{poly}, K_{poly}, T_{poly}, \lambda) = T_{poly\text{-}oxide} \quad (1a)$$

$$g_1(\Delta_1, \psi_1, \phi, N_s, K_s, N_{ox}, T_{ox1}, N_{poly}, K_{poly}, T_{poly}, \lambda) = 0 \quad (1b)$$

$$f_2(\Delta_2, \psi_2, \phi, N_s, K_S, N_{ox}, T_{ox2}, N_{poly}, K_{poly}, T_{poly}, \lambda) = T_{poly\text{-}oxide} \quad (2a)$$

$$g_2(\Delta_2, \psi_2, \phi, N_s, K_s, N_{ox}, T_{ox2}, N_{poly}, K_{poly}, T_{poly}, \lambda) = 0 \quad (2b)$$

where Δ_1, Δ_2, ψ_1 and ψ_2 are the measured ellipsometric angles, ϕ is the incident angle, N_s and K_s are the silicon substrate index (3·858–i0·018, at 632.8 nm), N_{ox} and N_{poly}-iK_{poly} are the refractive indices of the oxide and poly-Si, respectively, and T_{ox1}, T_{ox2}, T_{poly} and $T_{poly\text{-}oxide}$ are the thicknesses of the bottom oxides, the poly-Si, and the poly-oxide, respectively. In eqns. 1 and 2, there are four parameters, i.e. N_{poly}, K_{poly}, T_{poly} and $T_{poly\text{-}oxide}$ whose values are to be determined. In principle, the values of these four parameters can be determined. Eqn. 1*a* and *b* represent a curve in a 4-dimensional space of the variables N_{poly}, K_{poly}, T_{poly} and $T_{poly\text{-}oxide}$. Eqn. 2*a* and *b* represent another curve in the same 4-dimensional space. To determine the values for N_{poly}, K_{poly}, T_{poly} and $T_{poly\text{-}oxide}$, the intersection of these two curves needs to be found. However, owing to the measurement and instrumental errors [10], these two curves do not intersect each other; instead they pass close to each other, with minimum separation at the point of the solution in 4-dimensional space. Hence, the solution can be determined by finding the minimum of the distance between these two curves.

The above process for finding the solution is as follows. First, T_{poly} and $T_{poly\text{-}oxide}$ are given reasonable values and the values of N_{poly} and K_{poly} are computed using eqns. 1*a* and *b* and 2*a* and *b* for (Δ_1, ψ_1) and (Δ_2, ψ_2), respectively. The values of T_{poly} and $T_{poly\text{-}oxide}$ are then varied in a range and N_{poly} and K_{poly} are computed for each specified T_{poly} and $T_{poly\text{-}oxide}$. This forms the two curves in 4-dimensional space. The minimum distance between these two curves is then determined by comparison. After the minimum is found, the centre point of these minimum distances is taken as the solution. The above computing process is very fast. It takes less than 1 second on a IBM compatible 486 personal computer.

Experiments were carried out to demonstrate the above methodology. In experiments, oxides of two different thicknesses (262 Å, 1074 Å) were grown on a silicon substrate. An LPCVD amorphous silicon film, approximately 1000 Å thick, was deposited on the wafer at 550°C. The wafer was then annealed at 1000°C to become a poly-Si film and the poly-Si film was oxidised in a dry oxygen ambient at 1000°C for 25 min. Applying the ellipsometry measurement, two sets of (Δ, ψ) values were obtained. The measured values obtained by using the above procedures for T_{poly} and $T_{poly\text{-}oxide}$, N_{poly} and K_{poly} are listed in Table 1. These results are compared

Table 1 THICKNESSES AND REFRACTIVE INDICES OF POLY-OXIDE, AND POLY-Si MEASURED BY MULTIPLE SANDWICHED STRUCTURE (MSS), MULTIPLE-ANGLE INCIDENT (MAI) ELLIPSOMETRY AND CROSS-SECTION TEM METHODS

	MSS	MAI	TEM
Poly-oxide N	1·46	1·46	—
Poly-oxide T (Å)	451	455	455 ± 30
Poly-Si T (Å)	807	811	850 ± 30
Poly-Si N	4·148	4·020	—
Poly-Si K	0·026	0·024	—

with those obtained for the same wafer by using MAI ellipsometry [9] and by using cross-section TEM measurement. Those data are also shown in Table 1. For the values obtained by MAI ellipsometry, the fitting error was less than 1·0°. For cross-section TEM, the structure is shown in Fig. 2. It can be seen that the results obtained using the poly-oxide/poly-Si/SiO_2/Si structure (MSS) and by MAI ellipsometry are very close both in values of thickness and in optical constants. The values of T_{poly} and $T_{poly\text{-}oxide}$ obtained by the TEM picture are estimated to be 850 ± 30 Å and 445 ± 30 Å, respectively, where the estimation tolerances are rather large.

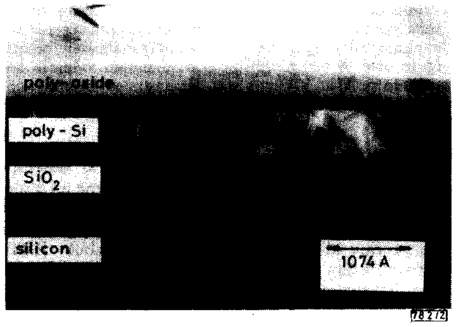

Fig. 2 *Cross-section TEM photograph of poly-oxide/poly-Si/SiO_2/Si structure*

Hence, it can be concluded that it is very simple to use the multiple sandwiched poly-oxide/poly-Si/SiO_2/Si structure proposed for conventional single incident angle and single incident wavelength ellipsometry measurement for measuring the thickness and the refractive index of the poly-oxide and poly-Si simultaneously. The structure can easily be incorporated into MOS ICs for in-line measurement during manufacture. The measurement also gives precise values for the thickness and refractive indices.

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T. S. Chao, C. L. Lee, T. F. Lei and Y. T. Yen (*Department of Electronics and Engineering and Institute of Electronics, National Chiao Tung University, Hsinchu, Taiwan, Republic of China*)

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