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# Two new generalized equations for minority-carrier transport in bipolar transistors with heavily doped base and non-uniform band structure

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Received 12 June 1991, in final form 21 January 1992, accepted for publication 11 March 1992

**Abstract.** Two new, simple, general and rigorous closed-form integral solutions to the base current density and to the base transit time for electron transport in NPN bipolar transistors with heavily doped base and non-uniform band structure are presented. The expressions include the effects of non-uniform band structure, the influence of Fermi–Dirac statistics, the position-dependent mobility and the velocity saturation effect at the base–collector depletion-layer edge on the base side. An application of this new simplified base current density of equation to Early voltage in a narrow-base bipolar transistor is illustrated. A simple example for the effects of the minority-carrier density and of the base built-in field on the minority-carrier base transit time is also qualitatively discussed.

## 1. Introduction

Two important transport quantities of minority carriers across the quasi-neutral base of a transistor have received considerable attention in the physics of transistor action. One is the current density and is described by the familiar integral relationship between  $J_n$  and  $V_{BE}$ , which are connected by a so-called Gummel number (denominator of equation (1)) as

$$J_n = - \frac{qD_n n_i^2 \exp(qV_{BE}/kT)}{\int_0^W N_A dx} \quad (1)$$

This relation for the electron current flow through the base region of an NPN transistor was first derived in a closely analogous expression (for PNP) by Moll and Ross [1] and is widely used in transistor modelling analysis.

Another significant parameter is the minority-carrier transit time ( $\tau$ ) through the base region. Again, a similar result (for NPN) to Moll and Ross's [1] original formulation (for PNP) of the integral relation for  $\tau$  which depends on the base doping is expressed as

$$\tau = \int_0^W \left( \frac{1}{D_n N_A(x)} \int_x^W N_A(x) dx \right) dx. \quad (2)$$

It is noted that equations (1) and (2) are good only for the simplest case with some approximations, such as non-degenerate carrier densities, low-level injection, position-

independent diffusion coefficients and uniform base structure.

Although there is a large body of papers [2–7] published on these subjects in the last fifteen years, all of the previous results based on the conventional drift-diffusion transport equation were expressed in terms of the carrier diffusion coefficient instead of the carrier mobility as given by equations (11) and (12) below. It is noted that a number of these are on minority carriers in transparent emitters [2–6] rather than on base transport [7, 8]. In 1978, Abram *et al* [9] generalized the use of mobility (or average mobility) rather than diffusion coefficient in their work. However, a consistent and systematic study (including the heavy-doping and non-uniform band structure effects) of the base transport equation does not seem to exist in the literature.

In this paper, two new, simple, closed-form integral relations are generalized to be valid for bipolar transistors with heavily doped base and non-uniform band structure under the assumption of one-dimensional analysis. The results show the complex heavy-doping and non-uniform band structure effects. These effects may include actual bandgap narrowing, carrier degeneracy, influence of the impurity band, position-dependent mobility, and the built-in electric field due to a graded doping density or a graded band structure. In addition, the velocity saturation effect at the base–collector depletion-layer edge on the base side is also accounted for.

## 2. Theory and discussion

As shown in figure 1, we consider the base of a one-dimensional NPN junction transistor with quasi-neutral base width  $W_p$  and assume that the edges of the base-emitter and base-collector depletion layers are located at  $x = 0$  and  $x = W_p$ , respectively. The starting equations, for generalizing Moll-Ross integral relations, are the current density flow equations, expressed in their most general consistent form, involving the gradients of the electrostatic potential, the two carrier concentrations and the activity coefficients  $\gamma_n$  and  $\gamma_p$  for both the minority (electron) and majority (hole) carriers [10]

$$J_n = kT\mu_n \frac{dn}{dx} - qn\mu_n \frac{dV}{dx} + kTn\mu_n \frac{d \ln \gamma_n}{dx} \quad (3)$$

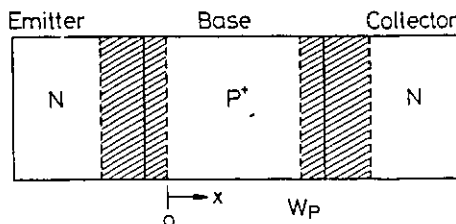
$$J_p = -kT\mu_p \frac{dp}{dx} - qp\mu_p \frac{dV}{dx} - kTp\mu_p \frac{d \ln \gamma_p}{dx} \quad (4)$$

and

$$\gamma_n(x)\gamma_p(x) = \frac{n_i^2(0)}{n_{ie}^2(x)} = \exp\left(\frac{-\Delta E_g(x)}{kT}\right) \quad (5)$$

where the symbols in the above equations have their usual meaning except that  $\Delta E_g$  represents the effective bandgap shrinkage which accounts for the actual bandgap narrowing effect ( $\Delta_{BGN}$ ), the density-of-state effects ( $\Delta_{DOS}$ ), and the influence of Fermi-Dirac statistics ( $\Delta_{FD}$ ). Parameters  $\Delta E_g$ ,  $\Delta_{BGN}$ ,  $\Delta_{DOS}$  and  $\Delta_{FD}$  are defined in [10]. It should be mentioned here that the Fermi-Dirac integral of order one-half ( $F_{1/2}$ ) used in [10] for  $\Delta_{FD}$  implies that the density of states is proportional to the square root (parabolic) of the kinetic energy (see appendix 1). This is not the general situation for the case of high doping [9, 11, 12].

It is important to note that equations (3) and (4) have moved the influence of Fermi-Dirac statistics in the diffusion terms to the activity coefficient terms and replaced the carrier diffusivities in the diffusion terms by the carrier mobilities. The activity coefficients here account for the non-ideal behaviour of the carriers due to heavy-doping effects and non-uniform band structure. These coefficients are functions of electron affinity, bandgap, doping density and the density of states that vary with position. It is also noted that equations (3) and (4) are similar in form to those derived by Van Overstraeten *et al* [12]. The latter, however, retained the effects of degeneracy in the diffusion terms and regarded the



**Figure 1.** One-dimensional NPN bipolar junction transistor showing base-emitter and base-collector junction layers, and quasi-neutral base width  $W_p$ .

intrinsic Fermi level  $E_i$  as a measure of the electrostatic potential, which is not suitable for the case of non-uniform band structure [13]. The use of equation (5) implies low injection as discussed in [10].

It is a necessary condition for a good transistor that  $\beta \gg 1$ . This implies that the hole current is small and may be neglected, i.e.  $J_p \approx 0$ . From this, the electric field ( $E = -dV/dx$ ) can be evaluated by equating (4) to zero as

$$\frac{dV}{dx} = -\frac{kT dp}{qp dx} - \frac{kT d \ln \gamma_p}{q dx} \quad (6)$$

Eliminating the electric field by substituting (6) into (3) one obtains, after some manipulation, the following equation for the minority-carrier current density  $J_n$

$$\frac{J_n p \gamma_n \gamma_p}{kT \mu_n} = \frac{d(pn \gamma_n \gamma_p)}{dx} \quad (7)$$

Integrating both sides of the above equation between  $x = 0$  and  $x = W_p$  and treating recombination as negligible so that  $J_n$  can be removed from the integral gives

$$\begin{aligned} \frac{J_n}{kT} \int_0^{W_p} \frac{p \gamma_n \gamma_p}{\mu_n} dx &= \int_0^{W_p} d(pn \gamma_n \gamma_p) \\ &= p(W_p)n(W_p)\gamma_n(W_p)\gamma_p(W_p) - p(0)n(0)\gamma_n(0)\gamma_p(0). \end{aligned} \quad (8)$$

The above equation is the general equation from which the Moll-Ross current relation, or its extension, is obtained.

To account for the velocity saturation effect, we simply consider the collector current density  $J_c$  as [14]

$$J_c (= -J_n) = qV_d n(W_p) \quad (9)$$

where  $V_d$  is the electron saturation velocity and  $n(W_p)$  is the electron concentration within the space-charge layer required to support the drift current. Also, at the emitter end of the base,  $x = 0$ , we use the boundary condition [10]

$$\begin{aligned} p(0)n(0)\gamma_n(0)\gamma_p(0) &= n_i^2(0) \exp\left(\frac{E_{FN} - E_{FP}}{kT}\right) \Big|_{x=0} \\ &\approx n_i^2(0) \exp\left(\frac{qV_{BE}}{kT}\right). \end{aligned} \quad (10)$$

Combining equations (5), (7), (8) and (9), by purely algebraic operations, we obtain

$$J_c = \frac{q \exp(qV_{BE}/kT)}{\frac{p(W_p)}{V_d n_{ie}^2(W_p)} + \frac{q}{kT} \int_0^{W_p} \frac{p(x)}{\mu_n(x) n_{ie}^2(x)} dx} \quad (11)$$

which can be written as

$$\begin{aligned} J_c &= \frac{qn_i^2(0) \exp(qV_{BE}/kT)}{\frac{p(W_p)}{V_d} \exp\left(\frac{-\Delta E_g(W_p)}{kT}\right)} \\ &\quad + \frac{q}{kT} \int_0^{W_p} \frac{p(x)}{\mu_n(x)} \exp\left(\frac{-\Delta E_g(x)}{kT}\right) dx. \end{aligned} \quad (12)$$

Equations (11) and (12) are the generalization of the Moll-Ross current relation. The denominator in the above relation can be considered as the effective base Gummel number. An application of this result to Early voltage in a narrow base bipolar transistor is given in appendix 2.

It is clear from equations (11) and (12) that the velocity saturation effect, the shape of the base doping profile, position-dependent mobility and position-dependent complex heavy-doping effects are included. The velocity saturation effect has usually been neglected in previous studies [7-9, 12, 15, 16]. Also, the previous results are mainly based on the conventional drift-diffusion transport equation which will lead equation (11) or (12) to be expressed in terms of the carrier diffusion coefficient [8, 12, 15] or average (or effective) diffusion coefficient [7, 15] instead of the carrier mobility. It must be emphasized that the statement expressed mathematically by the conventional diffusion model is in general consistent only for a uniform medium, whose structure and diffusion properties in the neighbourhood of any point are the same in all directions. Thus, it is not strictly physically correct for the case studied here. It is also extremely important to note that the values of the diffusion coefficients must be calculated through the complex degenerate Einstein relations, while the mobility can be directly obtained from experiments. The results presented here are more general and more physically significant.

In order to solve the transit time of minority carriers across the quasi-neutral base of a transistor, we return to (7) for arbitrary  $x$ . After some algebraic manipulation, we see using also equation (5) that

$$n(x) = -J_n \left( \frac{n_{ie}^2(x)}{p(x)} \right) \left( \frac{p(W_p)}{qV_d n_{ie}^2(W_p)} + \frac{1}{kT} \int_x^{W_p} \frac{p(y) dy}{\mu_n(y) n_{ie}^2(y)} \right). \quad (13)$$

Integrating both sides of the above equation between  $x = 0$  and  $x = W_p$  and dividing by  $-J_n/q$  gives the average base transit time for the transfer of the minority carriers (electrons) across the base.

$$\begin{aligned} - \left( q \int_0^{W_p} n(x) dx \right) / J_n &= \tau_{in} \\ &= \int_0^{W_p} \left[ \left( \frac{n_{ie}^2(x)}{p(x)} \right) \left( \frac{p(W_p)}{V_d n_{ie}^2(W_p)} + \frac{q}{kT} \int_x^{W_p} \frac{p(y) dy}{\mu_n(y) n_{ie}^2(y)} \right) \right] dx. \end{aligned} \quad (14)$$

Equation (14) is the generalized form of the base transit-time relation (2). In the case of low current level, non-degeneracy and uniform base, the new relations (11) or (12) and (14) reduce to the classical conventional limits (1) and (2), respectively.

Now, a simple example for the effects of the minority-carrier density and of the base built-in field on the minority-carrier base transit time is illustrated. For simplicity, we consider the electron mobility as constant and assume that the velocity saturation can be neglected.

Equation (11) then becomes

$$J_n = - \frac{kT \mu_n p(0) n(0) / n_{ie}^2(0)}{\int_0^{W_p} (p(x) / n_{ie}^2(x)) dx}. \quad (15)$$

Combining (5) and (15) yields

$$J_n = - \frac{kT \mu_n p(0) n(0) \exp(-\Delta E_g(0) / kT)}{\int_0^{W_p} p(x) \exp(\Delta E_g(x) / kT) dx}. \quad (16)$$

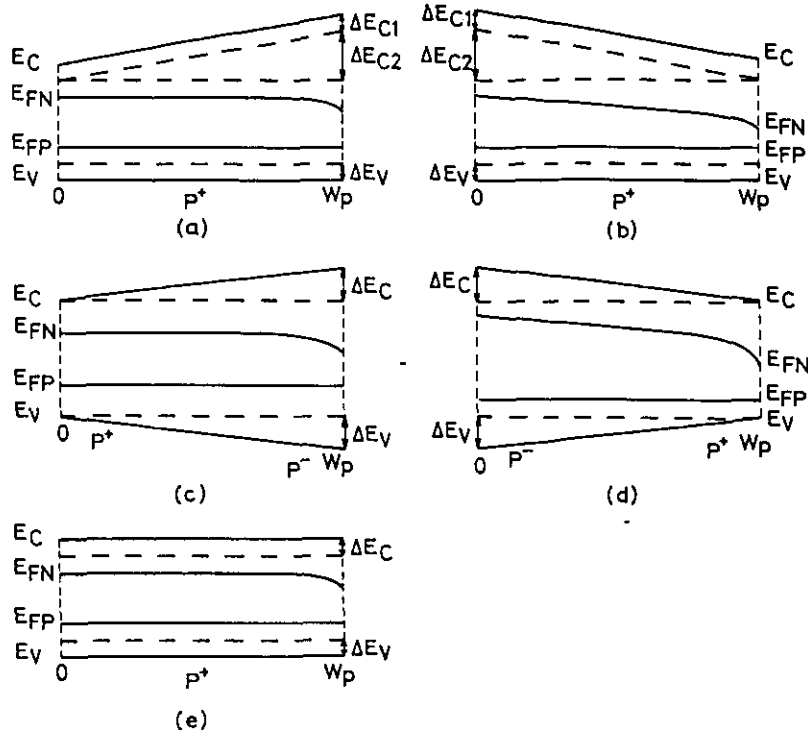
Substituting (16) into the first equation (14), the electron base transit time can be written as

$$\begin{aligned} \tau_{in} &= \left( \frac{1}{V_T \mu_n} \right) \left( \frac{\int_0^{W_p} p(x) dx}{n(0)} \right) \\ &\quad \times \left( \frac{\int_0^{W_p} p(x) \exp(-\Delta E_g(x) / kT) dx}{p(0) \exp(-\Delta E_g(0) / kT)} \right) \end{aligned} \quad (17)$$

where  $V_T (= kT/q)$  is the thermal voltage. Here we have separated the base transit time into three individual effects. The first term on the right-hand side is from the electron mobility contribution. Clearly, the higher mobility will reduce the transit time. The second term describes the minority-carrier effect, while the last term includes the shape of the base doping profile (or majority-carrier density) and the overall bandgap narrowing (or built-in field) effects.

As shown in figure 2(a) and (b) for the compositional graded band structures, it is important to note that the effects of minority carrier density and the compositional graded bandgap narrowing occur simultaneously but in the opposite contribution because the minority carrier density increases as the energy gap of the semiconductor decreases or vice versa. For the case of figure 2(a), the bandgap narrowing effect (energy band graded up) causes a retarding built-in field which increases the base transit time, while the second term (minority-carrier density effect) simultaneously tends to reduce it because the minority-carrier density decreases as the bandgap increases. On the other hand, for the case of figure 2(b), the bandgap narrowing effect (energy band graded down) leads to an 'assisting' (or accelerating) built-in field which reduces the base transit time, while the minority-carrier effect simultaneously tends to increase it because the minority-carrier density increases as the bandgap is getting smaller. In the case of figure 2(b), the value of the second term can be greater than  $W_p$ . Basically, both effects are in opposite directions. However, the bandgap narrowing effect on  $\tau_{in}$  is generally larger than that of the minority-carrier density effect because it affects the exponential term.

Also shown in figure 2(c) and (d) are bases with the the same host semiconductor but with graded doping densities in the opposite directions. The built-in field behaviours are similar to the cases (a) and (b) as discussed above. However, the value of the second term for these cases is typically smaller than  $W_p$  in the case of low injection. Figure 2(e) shows the uniformly heavily doped base. In this case,  $\Delta E_g$  is constant and the third term reduces to  $W_p$ . No built-in field effect exists in this case, while the minority-carrier density is greater than that of the lightly doped case. Quasi-Fermi levels  $E_{FN}$  and  $E_{FP}$



**Figure 2.** Base band structures: (a) and (b) are for the uniformly compositionally graded heavily doped base, (c) and (d) are for the non-uniformly heavily doped base, and (e) is for the uniformly heavily doped base.  $\Delta E_{C1}$  in (a) and (b),  $\Delta E_C$  in (c) and (d) and  $\Delta E_V$  in (a), (b) and (e) are for the actual bandgap narrowing due to uniformly heavily doped effects (high doping concentration),  $\Delta E_{C2}$  is due to compositional graded band structure, e.g.  $\text{Al}_x\text{Ga}_{1-x}\text{As}$  ( $x: 0 \rightarrow 1$  or  $1 \rightarrow 0$ ), and  $\Delta E_C$  and  $\Delta E_V$  in (c) and (d) are due to doping graded band structure. (a) and (c) cause the retarding fields (graded-up), while (b) and (d) lead to 'assisting' (or accelerating) fields (graded-down).

are also shown in figure 2 for the normal active mode operation under low-level injection conditions. The situation discussed above will become more complicated for the case of high-level injection.

Finally, we would like to mention that in our general results derived above we have included the velocity saturation effect which is important for operation in the normal active region at moderate currents. Also, the results presented here are valid for either homojunction or graded heterojunction bipolar devices including the complex heavy-doping and non-uniform band structure effects. The results, however, cannot be applied to abrupt heterojunction bipolar transistors if the thermionic emission mechanism for electron transport is important at the junction interface. The reason is quite simple—because we assume  $qV_{BE} = E_{FN}(0) - E_{FP}(0)$ , which is not true for the thermionic emission case. It has also to be noted that the similar derived expressions can be obtained for minority-carrier transport in the heavily doped emitter regions with non-uniform band structure.

## Appendix 1

Derivations of the carrier concentrations in terms of the function  $F_{1/2}$  for heavily doped semiconductors are pre-

sented in this appendix. Only the electron concentration is considered here because the derivation of the hole concentration is similar. The concentration or number of electrons per unit volume in a heavily position-dependent band structure is generally represented as

$$n = \int_{E_c}^{\infty} f(E, E_{FN}, T) g_c(E, x) dE \quad (\text{A1})$$

where  $g_c$  is the density of states for the conduction band and the Fermi function is given by

$$f = \frac{1}{1 + \exp[(E - E_{FN})/kT]}. \quad (\text{A2})$$

For mathematical tractability and simplicity, if we consider the rigid band model [17] and assume the standard parabolic structure under isothermal conditions, then the density of states can be written as

$$g_c = \frac{1}{2\pi^2} \left( \frac{2m_n^*}{\hbar^2} \right)^{3/2} (E - E_c)^{1/2}. \quad (\text{A3})$$

It is extremely important to note that the density-of-states effective mass  $m_n^*$  can be changed due to band distortion or tailing for the case of heavily doped semiconductors.

When we introduce the dimensionless variables

$$y = \frac{E - E_c}{kT} \quad \text{and} \quad \eta_c = \frac{E_{FN} - E_c}{kT} \quad (\text{A4})$$

and assume  $m_n^*$  and  $T$  to be constant, equations (A1)–(A4), by purely algebraic operations, give

$$n = 4\pi \left( \frac{2kTm_n^*}{h^2} \right)^{3/2} \int_0^\infty \frac{y^{1/2} dy}{1 + \exp(y - \eta_c)}. \quad (\text{A5})$$

The Fermi–Dirac integral of order  $n$  is defined as

$$F_n(\eta_c) \equiv \frac{1}{\Gamma(n+1)} \int_0^\infty \frac{y^n dy}{1 + \exp(y - \eta_c)}. \quad (\text{A6})$$

With this expression the electron concentration in the conduction band is

$$n = 2 \left( \frac{2\pi kTm_n^*}{h^2} \right)^{3/2} F_{1/2}(\eta_c) \quad (\text{A7})$$

where the factor 1/2 is due to assuming  $g_c \propto (E - E_c)^{1/2}$ .

As an example, if the density of states is given by

$$g_c \propto [k_1(E - E_c)^{1/2} + k_2(E - E_c) + k_3(E - E_c)^{3/2} + \dots] \quad (\text{A8})$$

then we have the form

$$n = \sum_n C_n F_n \quad n = \frac{1}{2}, 1, \frac{3}{2}, \dots \quad (\text{A9})$$

## Appendix 2

In this appendix, we will illustrate an application of equation (12) to Early voltage in narrow-base bipolar transistors. For simplicity, we consider a simple case of devices with uniformly heavily doped base, i.e.  $p \approx N_A^+ \approx N_A = \text{constant}$  (completely ionized) in the quasi-neutral base region. This implies that the bandgap narrowing is independent of the position. We also assume that the mobility is constant. Under these conditions, equation (12) can be rewritten as

$$J_c = \frac{qn_i^2(0)\exp(qV_{BE}/kT)}{\frac{p}{V_d} \exp\left(\frac{-\Delta E_g}{kT}\right) + \frac{q}{kT} \frac{pW_p}{\mu_n} \exp\left(\frac{-\Delta E_g}{kT}\right)}. \quad (\text{A10})$$

The Early voltage  $V_A$  is defined by convention from figure 3

$$\left. \frac{dJ_c}{dV_{CE}} \right|_{V_{BE}=\text{constant}} = \frac{J_c}{V_A + V_{CE}} = \frac{dJ_c}{dW_p} \frac{dW_p}{dV_{CE}} \quad (\text{A11})$$

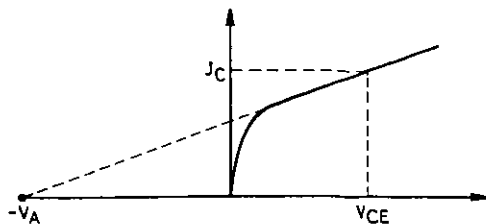


Figure 3.  $J_c$  versus  $V_{CE}$  characteristics including the Early voltage  $V_A$ .

where  $V_A$  is the Early voltage [18]. To determine this quantity, we have to find  $dJ_c/dW_p$  from (A10) as

$$\frac{dJ_c}{dW_p} = \frac{-J_c}{W_p + \frac{kT}{q} \frac{\mu_n}{V_d}} = \frac{-J_c}{W_p + \frac{D_n F_{-1/2}(\eta_c)}{V_d F_{1/2}(\eta_c)}}. \quad (\text{A12})$$

Substituting equations (A10) and (A12) into (A11), we obtain

$$V_A + V_{CE} = -\frac{W_p + \frac{kT}{q} \frac{\mu_n}{V_d}}{dW_p/dV_{CE}}. \quad (\text{A13})$$

This is the general result of the Early voltage for NPN transistors with uniformly heavily doped base including the electron saturation velocity effect. It is important to note that the conventional expression for  $V_A$  is similar to the result given by equation (A13) excluding the electron saturation velocity term. However, the value of  $W_p$  for the heavy-doping case is greater than for the lightly doped one, i.e.  $W_p^H > W_p^L$ .

## Acknowledgments

Support of this work by the ROC National Science Council, Engineering Division under Grant NSC 80-0404-E009-54 is gratefully acknowledged. The authors are indebted to anonymous reviewers for a careful reading and criticism of the manuscript.

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