The Gain and Optimal Length in the Erbium-Doped Fiber Amplifiers with 1480 nm Pumping

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Abstract—**The approximate analytic expressions for the gain and optimal length of the 1480 nm-pumped erbium-doped fiber amplifier with an arbitrary dopant distribution are obtained when the saturation by ASE is neglected. Numerical calculations show that the values predicted from the analytic expressions of the maximal gain and optimal fiber length as functions of input signal and pump powers are very accurate for narrow dopant distributions confined to the center of the fiber core.**

Introduction

The erbium-doped fiber can provide a broad-band, high-gain, and low-noise amplification for optical signals of wavelengths near 1530 nm. In [1], an analytic solution of the rate equations for an annual dopant distribution has been derived and the maximal gain at the optimal fiber length has been found, but the dependence of signal and pump powers on the fiber length are lost. In [2], the analytic expression for signal gain as an function of fiber length has been obtained. In this letter, we will find the approximate analytic expressions for gain and optimal length of the erbium-doped fiber amplifier with an arbitrary dopant distribution. Numerical calculations are carried out for Gaussian dopant distribution and are compared with the analytic results.

Theory

Regarding the erbium-ions doped in the single-mode fiber as homogeneously broadened two-level systems, we know that the population densities N_1 and N_2 of the lower and upper levels, respectively, are determined by the rates of stimulated absorption and emission and spontaneous emission between the lower and upper states. When the maximal gain is less than 20 dB, the amplified spontaneous emission can be neglected [3]. Thus, at stationary conditions we obtain [4]

$$N_2 = \tau\left[\left(\sigma_p^a N_1 - \sigma_p^e N_2\right)\frac{I_p}{h\nu_p} + \left(\sigma_s^a N_1 - \sigma_s^e N_2\right)\frac{I_s}{h\nu_s}\right] \quad (1a)$$

or

$$N_2 = \frac{I_p/I_{sp}^a + I_s/I_{ss}^a}{I_p/I_{sp}^a + I_p/I_{sp}^e + I_s/I_{ss}^a + I_s/I_{ss}^e + 1} N_t \quad (1b)$$

where $I_{sp}^a = h\nu_p/\tau\sigma_p^a$, $I_{sp}^e = h\nu_p/\tau\sigma_p^e$, $I_{ss}^a = h\nu_s/\tau\sigma_s^a$ and $I_{ss}^e = h\nu_s/\tau\sigma_s^e$. ν_s and ν_p are the signal and the pump frequencies, respectively; h is Planck's constant; τ is the spontaneous emission decay lifetime; σ_s^a and σ_s^e are the stimulated absorption and emission cross sections of the signal beam, while σ_p^a and σ_p^e are the stimulated absorption and emission cross sections of the pump beam, respectively; I_s and I_p are the optical intensities of the signal and pump beams, respectively; the total dopant density distribution $N_t(r) = N_1(r) + N_2(r)$ where N_t, N_1, and N_2 are all assumed to be radially symmetric. We define $I_s(z, r) = P_s(z)f_s(r)$ and $I_p(z, r) = P_p(z)f_p(r)$ where $P_s(z)$ and $P_p(z)$ are the signal and pump powers, and $f_s(r)$ and $f_p(r)$ are the normalized signal and pump transverse intensity profiles, respectively. If efficient long-wavelength pumping is used to avoid excited-state-absorption effects [5], we can assume $f_s(r) = f_p(r) = f(r)$ for the following derivations. The signal and pump powers will evolve according to the following propagation equations:

$$\frac{dP_s}{dz} = 2\pi\int_0^\infty I_s\left[\sigma_s^e N_2(r) - \sigma_s^a N_1(r)\right] r dr \quad (2a)$$

$$\frac{dP_p}{dz} = \mp 2\pi\int_0^\infty I_p\left[\sigma_p^a N_1(r) - \sigma_p^e N_2(r)\right] r dr. \quad (2b)$$

where the minus and plus signs in (2b) correspond to copropagating and counter-propagating pump waves, respectively.

A. Maximal Gain

Substituting (1b) into (2), we have

$$\frac{dP_s}{dz} = 2\pi\sigma_s^a P_s\int_0^\infty \cdot \frac{\left[\left(\sigma_s^e P_p/\sigma_s^a I_{sp}^a - P_p/I_{sp}^e\right)f - 1\right] N_t f}{P_p f/I_{sp}^a + P_p f/I_{sp}^e + P_s f/I_{ss}^a + P_s f/I_{ss}^e + 1} r dr \quad (3a)$$

$$\frac{dP_p}{dz} = \mp 2\pi\sigma_p^a P_p\int_0^\infty \cdot \frac{\left[\left(P_s/I_{ss}^e - \sigma_p^e P_s/\sigma_p^a I_{ss}^a\right)f + 1\right] N_t f}{P_p f/I_{sp}^a + P_p f/I_{sp}^e + P_s f/I_{ss}^a + P_s f/I_{ss}^e + 1} r dr \quad (3b)$$

If we neglect the constant "1" in the denominators of (3) and

Manuscript received November 12, 1991; revised January 10, 1992. This work was supported by National Science Council and Telecommunication Laboratories of R.O.C. under contract NSC-81-0417-E009-05.

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IEEE Log Number 9107076.

1041-1135/92$03.00

divide (3a) by (3b), we have

$$\frac{dP_s}{dP_p} = \mp \frac{\left[\left(\sigma_s^e/\sigma_s^a I_{sp}^a - 1/I_{sp}^e\right) - 1/RP_p\right]/\sigma_p^a}{\left[\left(1/I_{ss}^e - \sigma_p^e/\sigma_p^a I_{ss}^a\right) + 1/RP_s\right]/\sigma_s^a} \quad (4)$$

where $R = \int_0^\infty N_t(r)f(r)rdr / \int_0^\infty N_t(r)rdr$. The pumping efficiency $|dP_s/dP_p|$ increases with R for all P_s and P_p. For any length of the erbium-doped fiber, (4) can be integrated by parts to yield, for both copropagating and counterpropagating pumping

$$\left[\left(P_s^{out} - P_s^{in}\right)\left(\frac{1}{I_{ss}^e} - \frac{\sigma_p^e}{\sigma_p^a I_{ss}^a}\right) + \ln\left(\frac{P_s^{out}}{P_s^{in}}\right)\frac{1}{R}\right]\frac{\sigma_p^a}{\sigma_s^a}$$

$$= \left[\left(P_p^{in} - P_p^{out}\right)\left(\frac{\sigma_s^e}{\sigma_s^a I_{sp}^a} - \frac{1}{I_{sp}^e}\right) - \ln\left(\frac{P_p^{in}}{P_p^{out}}\right)\frac{1}{R}\right]. \quad (5)$$

From (5), we can find R by measuring P_s^{out}, P_p^{out} for given P_s^{in} and P_p^{in}. For the maximal gain, $dP_s/dP_p = 0$, we have the output pump power

$$P_{p,op}^{out} = \frac{1}{R\left(\sigma_s^e/\sigma_s^a I_{sp}^a - 1/I_{sp}^e\right)}. \quad (6)$$

If the maximal gain is defined as $G = P_s^{out}/P_s^{in}$, then it can be calculated from the following equation:

$$\frac{\nu_p}{\nu_s}\frac{P_s^{in}}{P_{p,op}^{out}}(G-1) + \frac{\sigma_p^a}{\sigma_s^a}\ln(G) = \frac{P_p^{in}}{P_{p,op}^{out}} - 1 - \ln\left(\frac{P_p^{in}}{P_{p,op}^{out}}\right) \quad (7)$$

B. Optimal Fiber Length for Maximal Gain

By combining (1a) and (2) and defining the confinement factor $\Gamma = A\int_0^\infty N_2(r)f(r)rdr / \int_0^\infty N_2(r)rdr$ where A is the effective doped area, we have the propagation equation for the signal beam:

$$\frac{dP_s}{dz} = -P_s\left[\alpha_s + \left(\frac{1}{h\nu_s}\frac{dP_s}{dz} \pm \frac{1}{h\nu_p}\frac{dP_p}{dz}\right)h\nu_s/P_s^{IS}\right] \quad (8)$$

where $\alpha_s = 2\pi\sigma_s^a\int_0^\infty N_t(r)f(r)rdr$ is the absorption constant of the signal beam, and $P_s^{IS} = Ah\nu_s/\tau\Gamma(\sigma_s^a + \sigma_s^e)$ is the intrinsic saturation power of the signal beam. Solving the differential equation (8), we obtain the output signal power at $z = L$

$$P_s^{out} = P_s^{in} \cdot \exp(-\alpha_s L)\exp\left[\left(\frac{P_p^{in} - P_p^{out}}{h\nu_p} + \frac{P_s^{in} - P_s^{out}}{h\nu_s}\right)\frac{h\nu_s}{P_s^{IS}}\right]. \quad (9)$$

Without pump beam, $P_s^{out} \simeq P_s^{in}\exp(-\alpha_s L)$ for small input signal power, and then $\alpha_s = \log(P_s^{in}/P_s^{out})/L$. When P_s^{in} approaches to P_s^{IS}, $P_s^{out} \simeq P_s^{IS}\exp(1 - \alpha_s L)$. It implies that both absorption constant and intrinsic saturation power for the signal beam can be obtained from the monochromatic absorption measurement [2]. The optimal fiber length L_{op} for the maximal gain can be obtained by reorganizing (9), and

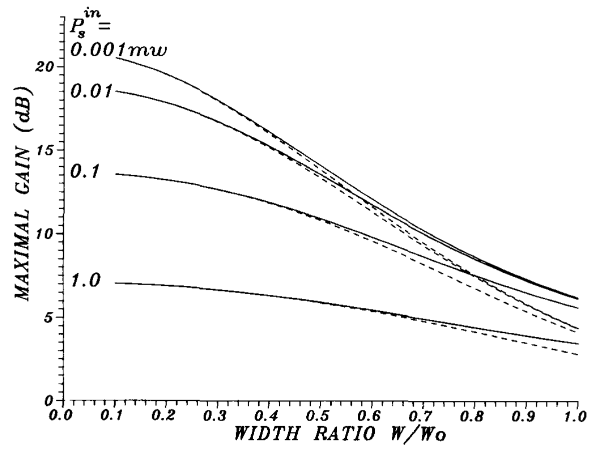

Fig. 1. The exact (solid lines) and the approximate (dashed lines) maximal gains as functions of the width-ratio (w/w_0) between Gaussian dopant distribution and transverse intensity profile for P_p^{in} = 15 mw.

$$L_{op} = -\frac{1}{\alpha_s} \cdot \left\{\ln(G) + \frac{h\nu_s}{P_s^{IS}}\left[\frac{P_{p,op}^{out} - P_p^{in}}{h\nu_p} + \frac{P_s^{in}}{h\nu_s}(G-1)\right]\right\}. \quad (10)$$

In summary, we can obtain the optimal fiber length for the maximal gain in the following way. At first, a piece of erbium-doped fiber with length L is chosen. From (5), we can obtain R by measuring P_s^{out} and P_p^{out} for given P_s^{in} and P_p^{in}. Subsequently we calculate $P_{p,op}^{out}$ from (6), and solve G from (7) for arbitrary P_s^{in} and P_p^{in}. Then we can obtain α_s and P_s^{IS} from the absorption measurement for the signal beam. Finally, the optimal fiber length can be calculated by substituting G into (10) for arbitrary P_s^{in} and P_p^{in}. The accuracy of the optimal fiber length calculated depends on that of the maximal gain calculated. The deviation of the optimal fiber length ΔL_{op} varies with the deviation of the maximum gain ΔG as $\Delta L_{op} = -\Delta G(1/G + P_s^{in}/P_s^{IS})/\alpha_s$.

Calculations and Discussion

To check the approximate expression of the maximal gain by numerical calculations, we take $f(r)$ in Gaussian form, $f(r) = \exp(-r^2/w_0^2)/\pi w_0^2$ where w_0 is the spot size, and the effective core area $\pi w_0^2 = 35\ \mu m^2$ [6]. $N_t(r)$ is also assumed to be in Gaussian form, $N_t(r) \simeq \exp(-r^2/w^2)/\pi w^2$. For the Al/P-silica erbium-doped fibers, typically we have parameters $\tau = 10.8$ ms, $\sigma_s^e = 5.7 \times 10^{-21}$ cm^2, and $\sigma_s^a = 6.6 \times 10^{-21}$ cm^2 for $\lambda_s = 1530$ nm; $\sigma_p^e = 0.87 \times 10^{-21}$ cm^2 and $\sigma_p^a = 2.44 \times 10^{-21}$ cm^2 for $\lambda_p = 1480$ nm [7]. α_s is arbitrarily chosen to be 0.5 m^{-1} and then P_s^{IS} is larger than 0.342 mw. We can compare the exact maximal gain calculated from (3) with that approximated by (7). Fig. 1 shows the exact (solid lines) and the approximate (dashed lines) maximal gains as functions of width ratio w/w_0 when the input pump power is fixed at 15 mw. At $w/w_0 = 0.4$, the difference ratios between approxi-

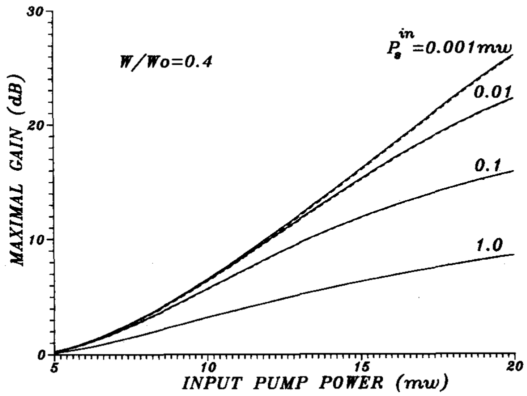

Fig. 2. The exact (solid lines) and the approximate (dashed lines) maximal gains as functions of the input pump power for Gaussian dopant distribution and transverse intensity profile with width-ratio $w/w_0 = 0.4$.

mate and exact maximal gains ($100\% \times \Delta G/G$) are -2.56%, -1.96%, -0.831%, and -0.221% for $P_s^{in} =$ 0.001, 0.01, 0.1, and 1.0 mw, respectively; and then the difference ratios between approximate and exact optimal fiber lengths ($100\% \times \Delta L_{op}/L_{op}$) are less than 0.125%, 0.173%, 0.226%, and 0.178% for $P_s^{in} = 0.001$, 0.01, 0.1, 1 mw, respectively. Fig. 2 shows that the maximal gain approximated by (7) matches the exact maximal gain calculated from (3) very well for $P_s^{in} > 0.001$ mw and $P_p^{in} > 5$ mw when $w/w_0 = 0.4$. For alumino-silicate glasses, it is easy to confine the erbium-ions to the central 25% of the fiber core [8]. Therefore, the approximate analytic expressions are useful in real cases.

Conclusion

We have obtained approximate analytic expressions for the maximal gain and optimal length of the erbium-doped fiber amplifier with an arbitrary dopant distribution. Numerical calculations show that the values predicted from the analytic expressions of the maximal gain and optimal fiber length as functions of input signal and pump powers are very accurate for narrow dopant distributions confined to the center of the fiber core.

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