Incremental Maneuver Estimation Model for Target Tracking

WEN-THONG CHANG
SHAO-AN LIN
National Chiao Tung University
Taiwan

Target maneuver has long been a problem of tracking small targets with the Kalman filter. We describe an incremental model for maneuver detection and estimation for use in target tracking with the Kalman filter. The approach is similar to the multiple Kalman filter bank, but with a memory for the maneuver status for the track under consideration. The advantage of this approach is that the target acceleration can be more accurately estimated.

Manuscript received March 4, 1991; revised May 30, 1991.

IEEE Log No. 9104945.

This work was supported by the National Science Council under Contract D79015.

Authors' address: Dept. of Communication Engineering and Center for Telecommunication Research, National Chiao Tung University, Hsinchu, Taiwan, R.O.C.

0018-9251/92/0900-0439 $1.00 © 1992 IEEE

I. INTRODUCTION

Tracking of small targets with the Kalman filter [16] has been an active research subject during the past twenty years. By modeling the position and the velocity as a state vector, a dynamic equation is usually used to describe the movement of targets. By recursive calculation, the next state of a target can be easily estimated and a trajectory describing the movement history can be maintained.

Usually, there are three phases in the life of a trajectory; the initialization stage, the maintenance stage, and the deletion stage. The initialization stage is the creation of a new track. Hypothesis testing of those measurements that are not used to update the existing tracks is used to decide whether or not a new track is to be created. The deletion of a track is done by testing the track quality indicators to justify whether or not a track is to be maintained. We mainly discuss the maintenance stage. In this stage, the main concern is the maintenance of the trajectory. When there is more than one return at a measurement time, the system has to decide a best return measurement according to its track estimations. Besides the target-originated measurements, there exist other types of measurements, such as clutter caused by background noise, man-made jamming and thermal noise of the internal system, etc.

In the maintenance of a track, a difficult problem is the detection of the target maneuver and the estimation of its amount. Several approaches have been proposed to solve this problem, such as nonlinear estimation [17, 8], input estimation [7], and the extended Kalman filtering [15]. Thorp [20] described a technique that switches between two Kalman filters when maneuver is detected. Augmented state filter is another popular approach [3, 14]. In this method, the state vector of the Kalman filter is augmented while maneuver is detected and is restored to the original state dimension while the maneuver vanishes. The drawback of such approach is that there is a delay between the time when the maneuver actually occurs and when it is detected.

In the above approaches, a set of known maneuvers is usually used [18, 12]. With this set of distinct maneuver events, a bank of Kalman filters is used simultaneously to predict a set of next state estimations. Then, Bayes' rule is used to calculate the posterior probability of an acceleration given the measurements. This approach is not flexible since the maneuvers are usually random and unknown in the real world. Thus, an improved algorithm for maneuver estimation is introduced here.

In our method, the tracking filter is still based on the Kalman filter. However, a separate vector to record the acceleration is incorporated into the system equation. If, there is no acceleration, the vector assumes a value of zero. Owing to the steady state characteristics of the Kalman filter, there must be a perturbation to cause a large deviation in estimation. So, if there is a large deviation between the estimation and the measurement, we assume that there is a change in the target acceleration. Using the known maneuver value of the previous time as the reference, a set of incremental maneuver is assumed. This set of incremental values together with the reference is used to estimate the next states and Bayes' rule is used to find the best possible one. Thus, for each track under consideration, besides the steady state estimation, N more possible different positions according to N assumed incremental maneuver values are estimated. According to the probability of the acceleration given the measurement the reference value is updated. Since the incremental acceleration is a vector form, we can describe it by a scale value and a direction unit. The amount of the incremental maneuver value is derived according to the sampling time interval, the noise covariance, and the object dynamic characteristics.

II. PROBLEM FORMATION

Let us consider a dynamic system in the multitarget environment with T parallel Kalman filters, where T is the total number of existing tracks currently under consideration. These state equations can be described as follows,

$$\mathbf{x}^t(k+1) = \mathbf{F}(k)\mathbf{x}^t(k) + \mathbf{G}(k)\mathbf{u}^t(k) + \mathbf{v}(k), \quad t = 1,\ldots,T \tag{1}$$

where k is the scan index at time Δk, Δ is the sampling time interval, and the superscript t is the track number. The state vector is a four-dimensional vector of position and velocity,

$$\mathbf{x}^t(k) = (x^t(k) \quad y^t(k) \quad \dot{x}^t(k) \quad \dot{y}^t(k))' \tag{2}$$

and $\mathbf{u}^t(k)$ is the maneuvering vector, corresponding to the acceleration generated by the pilot at scan k for the tth track in both the x and y direction,

$$\mathbf{u}^t(k) = (\ddot{x}^t(k) \quad \ddot{y}^t(k))'. \tag{3}$$

$\mathbf{F}(k)$ is the transition matrix that transits the state vector from scan k to the next scan $k+1$, and is defined as

$$\mathbf{F}(k) = \begin{pmatrix} 1 & 0 & \Delta & 0 \\ 0 & 1 & 0 & \Delta \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \mathbf{I}_2 & \Delta\mathbf{I}_2 \\ \mathbf{0} & \mathbf{I}_2 \end{pmatrix} \tag{4}$$

where $\mathbf{I}_2$ is a 2×2 identity matrix. $\mathbf{G}(k)$ is also a transition matrix that transits the state vector according to the occurrence of any non-zero external force $\mathbf{u}^t(k)$ and is defined as

$$\mathbf{G}(k) = \begin{pmatrix} \frac{\Delta^2}{2} & 0 \\ 0 & \frac{\Delta^2}{2} \\ \Delta & 0 \\ 0 & \Delta \end{pmatrix} = \begin{pmatrix} \frac{\Delta^2}{2}\mathbf{I}_2 \\ \Delta\mathbf{I}_2 \end{pmatrix}. \tag{5}$$

Both $\mathbf{F}(k)$ and $\mathbf{G}(k)$ are constant matrices since the sampling interval Δ is assumed to be a constant for all k. The noise term $\mathbf{v}(k)$ is a zero-mean white Gaussian random process with known variance,

$$\mathbf{E}[\mathbf{v}(k)\mathbf{v}(j)'] = \mathbf{Q}(k)\delta_{kj} \tag{6}$$

where

$$\mathbf{Q}(k) = \begin{pmatrix} \frac{\Delta^4}{4}\mathbf{I}_2 & \frac{\Delta^3}{2}\mathbf{I}_2 \\ \frac{\Delta^3}{2}\mathbf{I}_2 & \Delta^2\mathbf{I}_2 \end{pmatrix} q \tag{7}$$

and q is the variance of the noise process and δ_{kj} is the delta function.

The target-originated measurement, which occurs with the probability of detection (P_D), can be modeled as

$$\mathbf{z}^t(k) = \mathbf{H}(k)\mathbf{x}^t(k) + \mathbf{w}(k) \tag{8}$$

where $\mathbf{H}(k)$ (usually independent of k), is a transition matrix and is defined as

$$\mathbf{H}(k) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} = (\mathbf{I}_2 \quad \mathbf{0}) \tag{9}$$

and $\mathbf{w}(k)$ is a zero-mean white Gaussian random process with diagonalized non-zero covariance matrix,

$$\mathbf{E}[\mathbf{w}(k)\mathbf{w}(j)'] = \mathbf{R}(k)\delta_{kj} \tag{10}$$

and

$$\mathbf{R}(k) = \begin{pmatrix} R_{11} & 0 \\ 0 & R_{22} \end{pmatrix} \tag{11}$$

where R_{11} and R_{22} are predefined variance in x and y direction, respectively. For convenience, the scan index k of $\mathbf{F}(k)$, $\mathbf{G}(k)$, $\mathbf{H}(k)$, $\mathbf{Q}(k)$, and $\mathbf{R}(k)$ can be omitted.

The main idea that we estimate an incremental maneuvering change between any two time intervals k and $k+1$, is that, for sufficiently small sampling time interval Δ, the amount of the abrupt change in maneuver is usually small. The new maneuver value after change is assumed to be the sum of the maneuver before change and a small increment. For detection, we partition the entire domain into several regions, and associate each region with an incremental acceleration vector. These regions represent the search area for a new target position if the target is accelerated by the corresponding acceleration. The number of partition is a tradeoff between the computation speed and the estimation accuracy. In

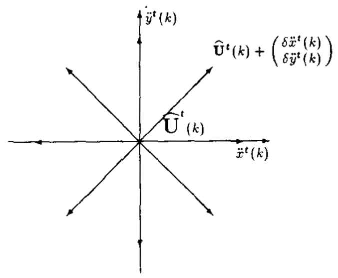


Fig. 1. Example choice of $\mathbf{U}_m^t(k)$.

our experiment, we take eight incremental maneuver modes evenly around the entire plane as an example.

For each track under consideration, eight additional new states are generated with these eight different maneuver deviation with respect to the reference maneuver. These eight incremental maneuver vectors distributed isotropically around the previous maneuver is shown in Fig. 1.

The state prediction equations become

$$\hat{\mathbf{X}}_m^t(k+1 \mid k) = \mathbf{F}\hat{\mathbf{X}}^t(k \mid k) + \mathbf{G}\mathbf{U}_m^t(k), \quad t = 1,\ldots,T \quad m = 0,\ldots,\mathcal{M}-1 \tag{12}$$

where T is the total number of existing tracks at scan k and $\mathcal{M}$ is nine in our case.

The variables $\mathbf{U}_m^t(k)$ indicate the nine maneuver values labelled from 0 to 8, defined as

$$\mathbf{U}_m^t(k) = \begin{cases} \hat{\mathbf{U}}^t(k) + \mathbf{C}_m^t(k)\mathbf{A}_m^t(k), & m = 0,\ldots,7 \\ \hat{\mathbf{U}}^t(k), & m = 8 \end{cases} \tag{13}$$

where $\hat{\mathbf{U}}^t(k)$ is the previous maneuver value, $\mathbf{C}_m^t(k)\mathbf{A}_m^t(k)$ is the incremental value $\mathbf{C}_m^t(k)$ is a scale matrix, and $\mathbf{A}_m^t(k)$ is a unit directional vector of the mth maneuver for the tth track at scan k. They are used to denote the amount of acceleration and the direction of acceleration relative to $\hat{\mathbf{U}}^t(k)$, respectively.

As shown in Fig. 1, the reference acceleration $\hat{\mathbf{U}}^t(k)$ is the previous maneuver value. The unit directional vector of the mth maneuver value is defined as

$$\mathbf{A}_m^t(k) = \begin{cases} \begin{pmatrix} \cos(m\pi/4) \\ \sin(m\pi/4) \end{pmatrix}, & m = 0,\ldots,7 \\ \begin{pmatrix} 0 \\ 0 \end{pmatrix}, & m = 8. \end{cases} \tag{14}$$

Due to the variance of the measurement $\mathbf{z}(k+1)$, a validation region for each mode is set to take into account this measurement deviation. For this validation region, two conditions have to be considered. The first is when the validation gate is too large. In this case, ambiguity may arise as to which region a target belongs. The second is when the validation gate is too small. In this case, it is possible that a return signal will fall outside any of the validation regions. Therefore, the validation region and the incremental amount should be considered together. If one is kept fixed, the other has to be adjusted accordingly. In our approach, we choose a fixed gate size. The scale matrix is thus a function of the sampling time interval Δ, the gate size γ^2, and the innovation covariance $\mathbf{S}^t(k+1)$. It is derived in the Appendix as

$$\mathbf{C}_m^t[i,j](k+1) = \sqrt{2\gamma^2 \frac{4}{\Delta^4} |\mathbf{S}^t[i,j](k+1)|} \tag{15}$$

where the indices i and j represent the row and column, respectively.

With the nine estimated states in (12), the predictions of the measurement for the tth track are as follows.

$$\begin{aligned} \hat{\mathbf{Z}}_m^t(k+1 \mid k) &= \mathbf{H}\hat{\mathbf{X}}_m^t(k+1 \mid k) \\ &= \mathbf{H}(\mathbf{F}\hat{\mathbf{X}}^t(k \mid k) + \mathbf{G}\mathbf{U}_m^t(k)) \\ &= \mathbf{H}(\mathbf{F}\hat{\mathbf{X}}^t(k \mid k) + \mathbf{G}\hat{\mathbf{U}}^t(k) \\ &\quad + \mathbf{G}\mathbf{C}_m^t(k)\mathbf{A}_m^t(k)) \\ &= \hat{\mathbf{Z}}^t(k+1 \mid k) + \mathbf{H}\mathbf{G}\mathbf{C}_m^t(k)\mathbf{A}_m^t(k). \end{aligned} \tag{16}$$

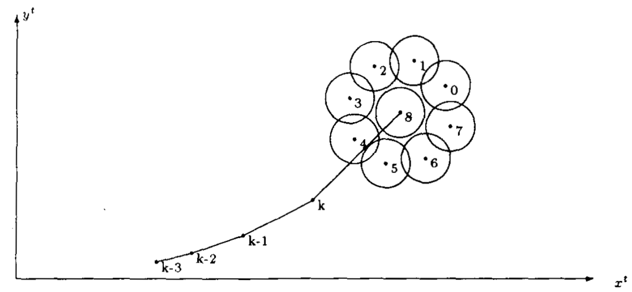


Fig. 2. Search area of $\hat{\mathbf{Z}}_m^t(k+1 \mid k)$ for incremental maneuver in Fig. 1.

The innovation vectors between these estimation and the rth true measurement at scan $k+1$ are

$$\tilde{\mathbf{Z}}^t_{mr}(k+1) = \mathbf{z}_r(k+1) - \hat{\mathbf{Z}}^t_m(k+1 \mid k), \quad r = 1,\ldots,R(k+1) \tag{17}$$

where $R(k+1)$ is the total number of measurements at scan $k+1$. An illustration of possible $\hat{\mathbf{Z}}^t_m(k+1 \mid k)$ is shown in Fig. 2, where the points labelled from 0 to 8 indicate the estimated positions for each of the assumed incremental modes.

The validation region is defined with $\hat{\mathbf{Z}}^t_m(k+1 \mid k)$ as the center. The distance between the measurement $\mathbf{z}_r(k+1)$ and the estimated position is normalized with the measurement covariance and is defined as

$$d^t_{mr}(k+1)^2 \triangleq \tilde{\mathbf{Z}}^t_{mr}(k+1)'\mathbf{S}^t(k+1)^{-1}\tilde{\mathbf{Z}}^t_{mr}(k) \tag{18}$$

where $\mathbf{S}^t(k+1) = \mathbf{E}[\tilde{\mathbf{Z}}^t_{mr}(k+1 \mid k)\tilde{\mathbf{Z}}^t_{mr}(k+1 \mid k)' \mid \mathcal{Z}(k+1)]$ is the innovation covariance matrix. Since the return is a Gaussian random process with mean equal to $\hat{\mathbf{Z}}^t_m(k+1 \mid k)$ and covariance equal to $\mathbf{S}^t(k+1)$, the term $d^t_{mr}(k+1)^2$ is the sum of M independent Gaussian random variables. Thus it will be a central χ^2-*distribution* with M degrees of freedom; M being the order of the vector $\mathbf{z}_r(k+1)$.

III. ASSOCIATION PROBLEM

The purpose of the data assignment is to make a best pair between the return signals and the existing tracks under consideration. There exist several well-known algorithms to solve this kind of problem. They are the nearest neighborhood standard filter (NNSF) [4, 6], the probabilistic data association filter (PDAF) [4, 1], the joint probability data association filter (JPDAF) [4, 10, 9], the track splitting filter [4, 6], the multiple hypothesis tracking [11], and the traveling salesman problem method [19], etc.

After the estimation, the next thing is the verification of the goodness of these estimations. The verification is measured by the closeness of the estimation with the correct measurement. So, the problem with this is how to locate the correct measurement in the presence of noises. In the NNSF, the measurement that is nearest to the predicted estimation is used. The distance measure used is the weighted norm of the innovation defined as

$$d^t_{mr}(k+1)^2 = \tilde{\mathbf{Z}}^t_{mr}(k+1)'\mathbf{S}^t(k+1)^{-1}\tilde{\mathbf{Z}}^t_{mr}(k) \le \gamma^2 \tag{19}$$

where $\tilde{\mathbf{Z}}^t_{mr}(k+1) = \mathbf{z}_r(k+1) - \hat{\mathbf{Z}}^t_{mr}(k+1)$ is the innovation corresponding to the measurement $\mathbf{z}_r(k+1)$ with the estimation $\hat{\mathbf{Z}}^t_{mr}(k+1)$.

From (19), measurements outside validation gate γ^2 are ignored. Only these within the gate are considered and the one closest to the estimation is chosen.

This is the simplest method for tracking a target in clutter environment. The problem with choosing the nearest neighbour is its high sensitivity to the false alarms. Thus its performance is not satisfactory in the multitarget environment.

The PDAF is a suboptimal Bayesian algorithm. A track is updated with all the observations within its gate. Since only one of the validated returns falling within the gate is target originated, given N observations within the gate of the track t, there will be $N+1$ hypotheses to be formed. The extra one is the case when none of the observations is valid. Using the results of [6, p. 300], the probabilities $(\beta^t_r(k))$ associated with the $N+1$ hypotheses for the tth track and the rth return are

$$\beta^t_r(k) = \begin{cases} \dfrac{e_r}{b + \sum_{j=1}^N e_j}, & r = 1,\ldots,N \\ \dfrac{b}{b + \sum_{j=1}^N e_j}, & r = 0 \text{ (no valid return)} \end{cases} \tag{20}$$

where

$$e_j \triangleq P_D \exp\left(-\tfrac{1}{2} d^t_j(k)^2\right) \tag{21}$$

$$b \triangleq (1 - P_D)\lambda(2\pi)^{M/2}\sqrt{|\mathbf{S}^t(k)|}. \tag{22}$$

The assignment is based on these different hypotheses [2]. Results for a single target in clutter [5] have shown a significant decrease in the number of lost tracks when the probabilistic data association (PDA) method is compared with the standard nearest-neighbor correlation method. However, in multiple target environment, it is still not satisfactory. Data association in multitarget environment has to be considered as a global problem. JPDA considers the temporal relationship of the measurements. The main difference between this method and the previous one is that all combinations of measurements from the initial to the present time are used, rather than just in terms of the latest set of measurements [4].

The JPDA method is identical to the PDA except that the association probabilities are computed with all observations for all tracks. The state estimation gain and covariance are the same as that of PDA. The probability computation of (20) is extended to include multiple tracks. Thus, in the calculation of the probability $\beta^t_r(k)$, there are problems of increasing memory and computation requirements. A suboptimal *ad hoc* formula for the $\beta^t_r(k)$s is proposed by Fitzgerald [13] as the following

$$\beta^t_r(k) = \frac{1}{c}\frac{p^t_r(k)}{\sum_r p^t_r(k) + \sum_t p^t_r(k) - p^t_r(k) + p_0} \tag{23}$$

where

$$p^t_0(k) = \lambda^N(1 - P_D). \tag{24}$$

and

$$p_r^t(k) = \frac{\lambda^{N-1} p_D}{(2\pi)^{M/2}\sqrt{|\mathbf{S}^t(k)|}} \exp\left(-\frac{1}{2} d_r^t(k)^2\right), \quad r = 1,\ldots,N \tag{25}$$

and the constant p_0 is expected to improve the performance in the presence of clutter, and c is a normalization constant such that $\sum_{r=0}^{R(k)} \beta_r^t(k) = 1$ is satisfied. The probabilities $p_r^t(k)$ are the likelihood ratios which are related to the normalized distance in (18).

IV. UPDATING PROBLEM

In this section, the actual implementation is described. The system includes two parts, the maneuver updating and the state updating. In the maneuver updating, the final increment is considered to be the weighted average of all the increments. The probability densities associated with each mode are considered to be a Gaussian distribution with mean $\hat{\mathbf{Z}}_m^t(k+1 \mid k)$ and variance $\mathbf{S}^t(k+1)$ as in (16). The original recursive technique for computing $\hat{\mathbf{U}}^t(k+1)$ is developed in detail by Moose [14]. It is developed for the model of multiple Kalman filters with fixed maneuver mean. For uniform sampling time, it has the following form

$$\hat{\mathbf{U}}^t(k+1) = \sum_{m=0}^{\mathcal{M}-1} \mathbf{U}_m^t(k) P\{\mathbf{U}^t(k+1) = \mathbf{U}_m^t(k) \mid \mathcal{Z}^t(k+1)\} \tag{26}$$

$$\begin{aligned} P\{\mathbf{U}^t(k+1) = \mathbf{U}_m^t(k) \mid \mathcal{Z}^t(k+1)\} &= \frac{1}{c} P\{\mathcal{Z}^t(k+1) \mid \mathbf{U}^t(k) = \mathbf{U}_m^t(k)\} \\ &\quad \times \sum_{i=0}^{\mathcal{M}-1} P\{\hat{\mathbf{U}}^t(k) = \mathbf{U}_m^t(k) \mid \mathcal{Z}^t(k)\}\Theta_{mi}. \end{aligned} \tag{27}$$

The meaning of these terms are as follows.

1) The variable c is a normalization constant that satisfies

$$\sum_{m=0}^{\mathcal{M}-1} P\{\mathbf{U}^t(k+1) = \mathbf{U}_m^t(k) \mid Z^t(k+1)\} = 1. \tag{28}$$

2) The term $P\{\mathcal{Z}^t(k+1) \mid \mathbf{U}^t(k) = \mathbf{U}_m^t(k)\}$ on the left-hand side of (27) are Gaussian distribution functions with mean $\hat{\mathbf{Z}}_m^t(k+1 \mid k)$ as in (16) and variance $\mathbf{S}^t(k+1)$.

3) The probability $\Theta_{mi} = P\{\hat{\mathbf{U}}^t(k) = \mathbf{U}_m^t(k) \mid \hat{\mathbf{U}}^t(k-1) = \mathbf{U}_i^t(k)\}$ is obtained from semi-Markov considerations [14]. For many tracking situations, it is resonable to assume that the consistency of the center mode $\hat{\mathbf{U}}^t(k)$ with the previous one $\hat{\mathbf{U}}^t(k-1)$ is of higher probability. Unless the case abruptly changes, Θ can be approximated by a value p near unity for $i = m$ and $(1-p)/(\mathcal{M}-1)$ for $i \neq m$.

The recursive update equations for the case of a single target are given in [4]. In the case of multiple targets and multiple maneuver modes, the probabilities of data association $\beta_{mr}^t(k)$ are taken into consideration.

The state equation estimated at scan $k+1$ for the tth track using the mth maneuver mode is described in (12) as

$$\hat{\mathbf{X}}_m^t(k+1 \mid k) = \mathbf{F}\hat{\mathbf{X}}^t(k \mid k) + \mathbf{G}\mathbf{U}_m^t(k), \quad t = 1,\ldots,T \quad m = 0,\ldots,\mathcal{M}-1. \tag{29}$$

For each measurement $\mathbf{z}_r(k+1)$, under the hypothesis that the rth validated return is from the tth track with the mth maneuver mode, the state is updated by

$$\hat{\mathbf{X}}_{mr}^t(k+1 \mid k+1) = \hat{\mathbf{X}}_m^t(k+1 \mid k+1) + \mathbf{W}^t(k+1)\tilde{\mathbf{Z}}_{mr}^t(k+1) \qquad r = 1,\ldots,R(k) \tag{30}$$

where $\tilde{\mathbf{Z}}_{mr}^t(k+1)$ is the innovation defined in (17), $\mathbf{W}^t(k+1) = \mathbf{P}^t(k+1 \mid k)\mathbf{H}'\mathbf{S}^t(k+1)^{-1}$ is the filter gain and $\mathbf{S}^t(k+1)$ is the innovation covariance and is updated using the state prediction covariance as the following

$$\mathbf{S}^t(k+1) = \mathbf{H}\mathbf{P}^t(k+1 \mid k)\mathbf{H}' + \mathbf{R}. \tag{31}$$

Using the total probability theorem, the conditional mean of the overall state of the tth track at time $k+1$ can be written as

$$\begin{aligned} \hat{\mathbf{X}}^t(k+1 \mid k+1) &= \mathbf{E}[\mathbf{X}^t(k+1) \mid \mathcal{Z}(k+1)] \\ &= \mathbf{E}[\mathbf{X}^t(k+1) \mid \theta_0^t, \mathcal{Z}(k+1)] P\{\theta_0^t \mid \mathcal{Z}(k+1)\} \\ &\quad + \sum_{r=1}^{R(k+1)} \sum_{m=0}^{\mathcal{M}-1} \mathbf{E}[\mathbf{X}^t(k+1) \mid \theta_{mr}^t, \mathcal{Z}(k+1)] \\ &\quad \times P\{\theta_{mr}^t \mid \mathcal{Z}^t(k+1)\} \\ &= \hat{\mathbf{X}}_0^t(k+1 \mid k+1)\beta_0^t(k+1) \\ &\quad + \sum_{r=1}^{R(k+1)} \sum_{m=0}^{\mathcal{M}-1} \hat{\mathbf{X}}_{mr}^t(k+1 \mid k+1)\beta_{mr}^t(k+1) \end{aligned} \tag{32}$$

where $\hat{\mathbf{X}}_0^t(k+1 \mid k+1) = \hat{\mathbf{X}}^t(k+1 \mid k)$ and $\hat{\mathbf{X}}_{mr}^t(k+1 \mid k+1)$ is the updated state estimation conditioned on the event θ_{mr}^t that the rth validated measurement is correct. Using the fact that

$$\beta_0^t(k+1) + \sum_{r=1}^{R(k+1)} \sum_{m=0}^{\mathcal{M}-1} \beta_{mr}^t(k+1) = 1 \tag{33}$$

and substituting (30) into (32), the state update equation is as follows.

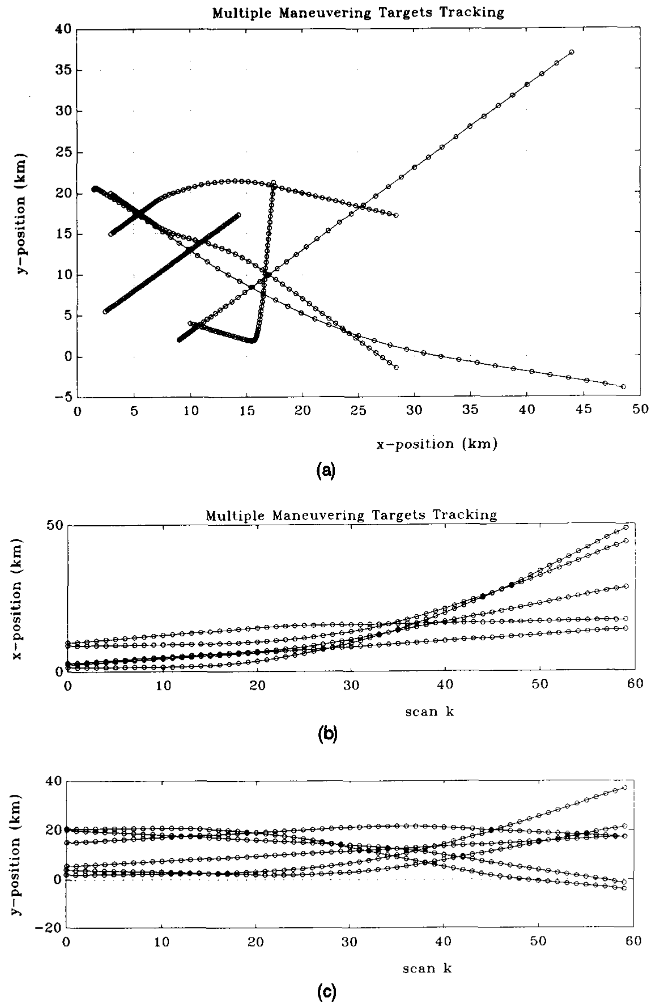


Fig. 3. Actual and estimated positions of example. (a) Trajectory in X-Y plane. (b) X position versus k. (c) Y position versus k.

$$\hat{\mathbf{X}}^t(k+1 \mid k+1) = \hat{\mathbf{X}}^t(k+1 \mid k) + \mathbf{W}^t(k+1)\tilde{\mathbf{Z}}^t(k+1) \tag{34}$$

where

$$\tilde{\mathbf{Z}}^t(k+1) = \sum_{r=1}^{R(k+1)} \sum_{m=0}^{\mathcal{M}-1} \tilde{\mathbf{Z}}^t_{mr}(k+1)\beta^t_{mr}(k+1) \tag{35}$$

is known as the combined innovation.

The overall error covariance associated with the updated state estimator is derived in [4]. It is modified to our model as

$$\mathbf{P}^t(k+1 \mid k+1) = \beta^t_0(k+1)\mathbf{P}^t(k+1 \mid k) + (1-\beta^t_0(k+1))\mathbf{P}^t_c(k+1 \mid k+1) + \tilde{\mathbf{P}}^t(k+1) \tag{36}$$

where

$$\tilde{\mathbf{P}}^t(k+1) = \mathbf{W}^t(k+1)\left[\sum_{r=1}^{R(k+1)} \sum_{m=0}^{\mathcal{M}-1} \beta^t_{mr}(k+1) \times \tilde{\mathbf{Z}}^t_{mr}(k+1)\tilde{\mathbf{Z}}^t_{mr}(k+1)' - \tilde{\mathbf{Z}}^t(k+1)\tilde{\mathbf{Z}}^t(k+1)'\right]\mathbf{W}^t(k+1)' \tag{37}$$

and

$$\mathbf{P}^t_c(k+1 \mid k+1) = [\mathbf{I} - \mathbf{W}^t(k+1)\mathbf{H}]\mathbf{P}^t(k+1 \mid k) \tag{38}$$

is the covariance of the state updated with the correct measurement, i.e., in the absence of measurement origin uncertainty.

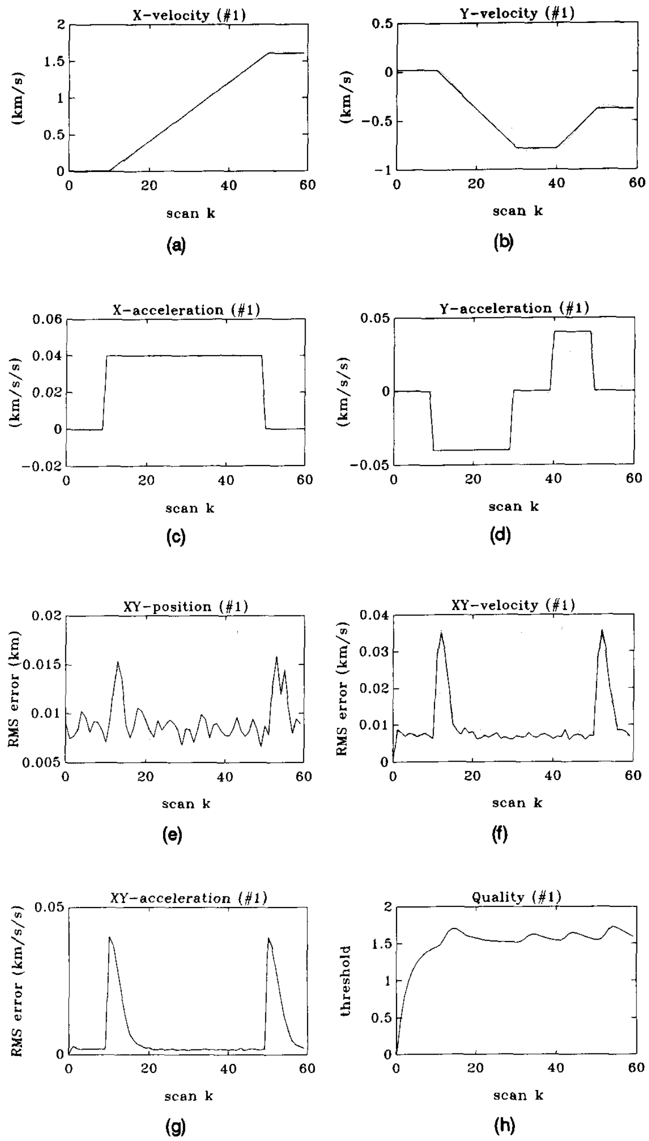


Fig. 4. Statistics of track 1. (a) X velocity versus k. (b) Y velocity versus k. (c) X acceleration versus k. (d) Y acceleration versus k. (e) rms position error (x in solid line, y in dotted line). (f) rms velocity error (x in solid line, y in dotted line). (g) rms acceleration error (x in solid line, y in dotted line). (h) Quality index.

V. SIMULATION AND RESULTS

Computer simulations are used to evaluate the proposed incremental maneuver estimation model. A Monte Carlo simulation of 50 runs was obtained and the rms values of the estimation error were computed. The dynamic models for the targets have been digitized using the sampling period Δ normalized to $1s$ and the state vectors have been represented in a 2-dimension Cartesian coordinates. Furthermore, only position measurements are assumed so that the measurement transition matrix $\mathbf{H}(k) = (\mathbf{I}_2 \quad \mathbf{0})$ is used in (9) for all k. The initial conditions of the filter are assumed to be the starting positions of the targets which are usually obtained from another search radar. During the simulation, false targets are generated by a normal distribution with the true target location as the mean. This is used to test the effectiveness of the association model.

The covariance matrix $\mathbf{Q}(k)$ of the plant noise $\mathbf{w}(k)$ is defined in (7), with the associated variance q equal to $0.01\text{km}^2\text{s}^{-4}$. The measurement noise covariance matrix $\mathbf{R}(k)$ for $\mathbf{v}(k)$ in (11) is defined with $R_{11} = R_{22} = 0.01\text{km}^2$, the off-diagonal terms are zero assuming that all the measurement noises are uncorrelated. The probability of validation was

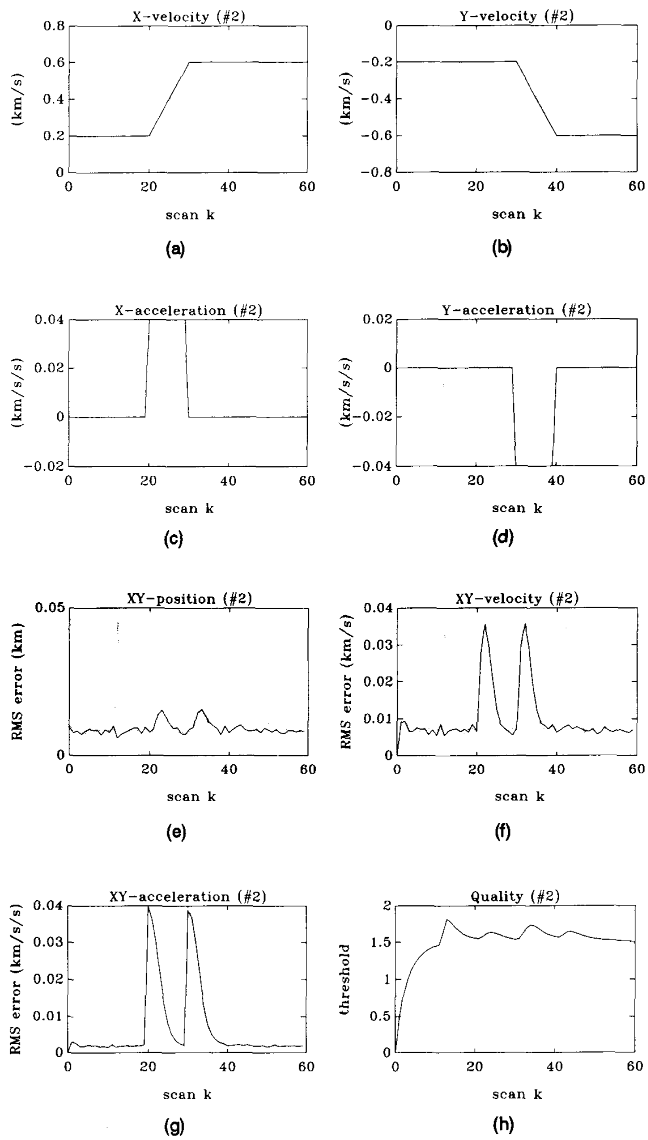


Fig. 5. Statistics of track 2. (a) X velocity versus k. (b) Y velocity versus k. (c) X acceleration versus k. (d) Y acceleration versus k. (e) rms position error (x in solid line, y in dotted line). (f) rms velocity error (x in solid line, y in dotted line). (g) rms acceleration error (x in solid line, y in dotted line). (h) Quality index.

chosen to be $P_G = 0.95$ with $\lambda = 0.2$. The probability of detection was chosen also to be $P_D = 0.95$. The corresponding threshold of the validation gate, as obtained from the table of χ_2^2 distribution is $\gamma^2 = 6.0$. In order to investigate the performance of the incremental maneuver estimation model for tracking maneuvering and nonmaneuvering targets in clutter environment, the *ad hoc* formula in **JPDAF** is used.

The initial conditions of the trajectories are listed in Table I. The maneuver events are listed in Table II. The true trajectory and the estimated position (indicated with a circle) are plotted in Fig. 3. These targets pass through the same location at different times. For illustration, the details of track 1 in this example are described below. This track starts at the position (1.5, 20.5). Due to its small initial velocity, the movement of this target is very slow. As can be seen in Fig. 3, most of the initial trajectories are quite near the initial point. The target moves first toward the positive Y direction, then turns to the negative Y direction due to the negative acceleration applied at time scan 10. In Fig. 4(a) and (b), we show the true (solid line) and the estimated (dotted line) velocities. To investigate the maneuver following capability of the proposed model, a staircase-like maneuver command is applied to both the x and y directions.

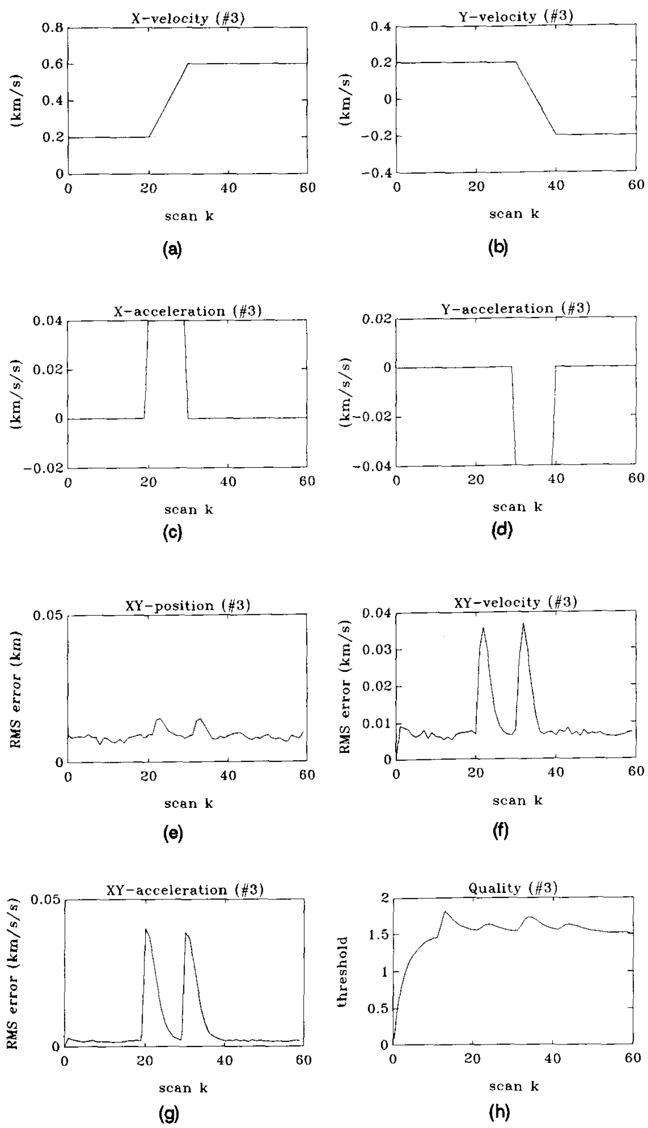


Fig. 6. Statistics of track 3. (a) X velocity versus k. (b) Y velocity versus k. (c) X acceleration versus k. (d) Y acceleration versus k. (e) rms position error (x in solid line, y in dotted line). (f) rms velocity error (x in solid line, y in dotted line). (g) rms acceleration error (x in solid line, y in dotted line). (h) Quality index.

The result shown in Fig. 4(c) and (d) indicates that the performance of the maneuver following capability is quite satisfactory. The dotted line is the estimated maneuver value. The delay between the estimated value and the true maneuver is the set up time that is common for all steady state systems. Since, in our model, the all-neighbors association method is used, abrupt maneuver change can still be detected. The estimated maneuver value will approach the true state as long as the maneuver is applied long enough.

Fig. 4(e) shows the rms errors of the position estimation in both the x (solid line) and y (dotted line) directions versus scan index k. Fig. 4(f) shows the rms errors of the velocity estimation $\dot{x}$ (solid line) and $\dot{y}$ (dotted line) versus scan index k. Fig. 4(g) shows the rms errors of the acceleration estimation $\ddot{x}$ (solid line) and $\ddot{y}$ (dotted line) versus scan index k. These errors

TABLE I
Initial Positions and Velocities of Example

target	x km	y km	$\dot{x}$ km/s	$\dot{y}$ km/s
#1	1.50	20.5	0.01	0.02
#2	3.00	20.0	0.20	−0.2
#3	3.00	15.0	0.20	0.20
#4	2.50	5.50	0.20	0.20
#5	10.0	4.00	0.25	−0.1
#6	9.00	2.00	0.03	0.03

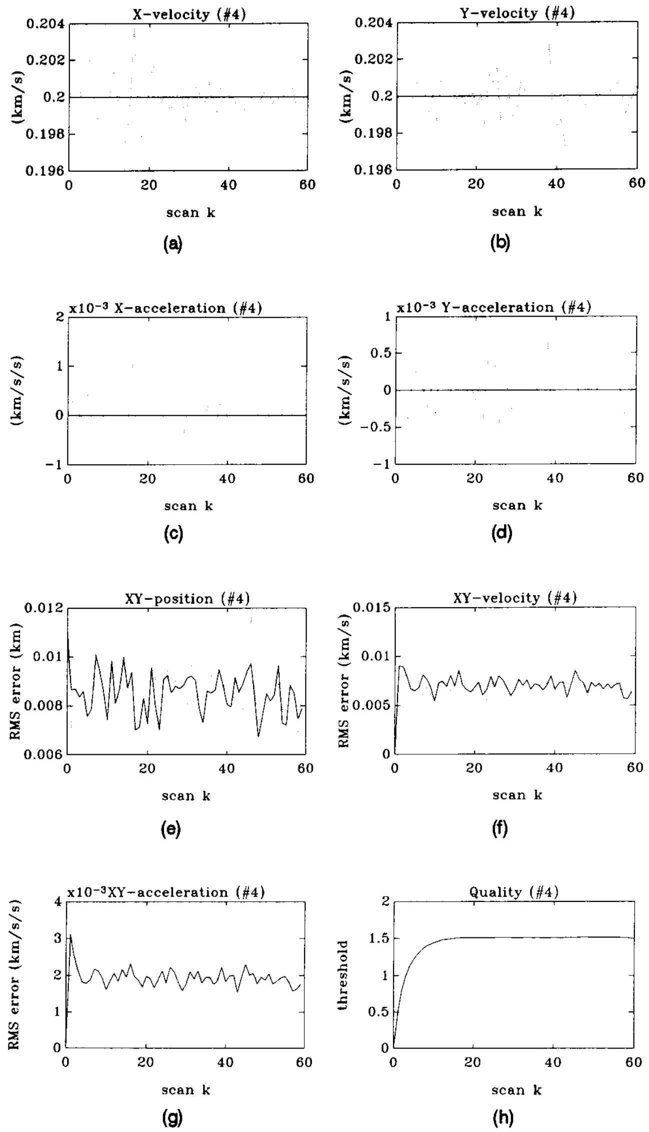


Fig. 7. Statistics of track 4. (a) X velocity versus k. (b) Y velocity versus k. (c) X acceleration versus k. (d) Y acceleration versus k. (e) rms position error (x in solid line, y in dotted line). (f) rms velocity error (x in solid line, y in dotted line). (g) rms acceleration error (x in solid line, y in dotted line). (h) Quality index.

TABLE II
Maneuver Events of Example

target	#1		#2		#3		#4		#5		#6	
scan k	$\ddot{x}$	$\ddot{y}$	$\ddot{x}$	$\ddot{y}$	$\ddot{x}$	$\ddot{y}$	$\ddot{x}$	$\ddot{y}$	$\ddot{x}$	$\ddot{y}$	$\ddot{x}$	$\ddot{y}$
0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
10	0.04	−0.04	—	—	—	—	—	—	—	—	0.01	0.01
15	—	—	—	—	—	—	—	—	—	—	0.02	0.02
20	—	—	0.04	0.00	0.04	0.00	—	—	−0.04	0.04	0.03	0.03
25	—	—	—	—	—	—	—	—	0.00	0.04	0.04	0.04
30	0.04	0.00	0.00	−0.04	0.00	−0.04	—	—	—	—	0.05	0.05
35	—	—	—	—	—	—	—	—	—	—	0.04	0.04
40	0.04	0.04	0.00	0.00	0.00	0.00	—	—	0.00	0.00	0.03	0.03
45	—	—	—	—	—	—	—	—	—	—	0.02	0.02
50	0.00	0.00	—	—	—	—	—	—	—	—	0.01	0.01

Note: Unit of maneuver is km/s^2. Scan k indicates time when maneuver is applied.

are the consequence of the state noise sequence $\mathbf{v}(k)$ and the measurement noise sequence $\mathbf{w}(k)$.

To compare with other approaches, such as the fading memory average method, the quality indicator of the average of the innovations is also calculated. Let the effective window length be $n = 4$, the fading factor resulted from [8] is equal to 0.75. The quality indicator as shown in Fig. 4(h) appears to be smooth when the system is stable; it is an apparent indication that the maneuver is detected and estimated almost simultaneously. The subsequent figures show the details of tracks 2–6.

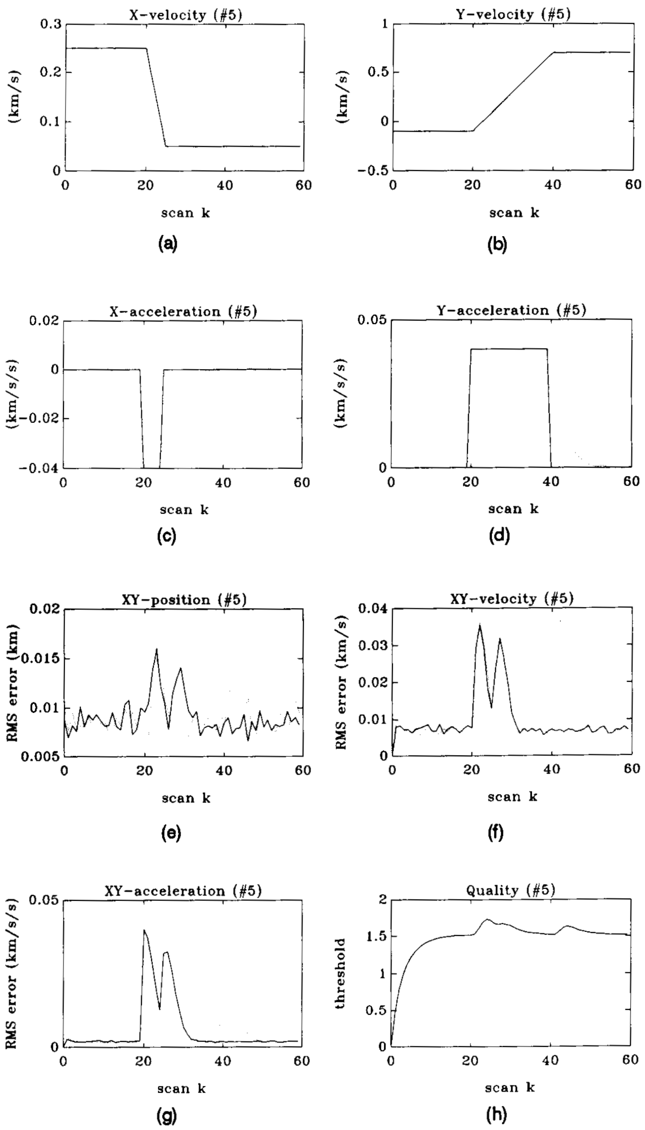


Fig. 8. Statistics of track 5. (a) X velocity versus k. (b) Y velocity versus k. (c) X acceleration versus k. (d) Y acceleration versus k. (e) rms position error (x in solid line, y in dotted line). (f) rms velocity error (x in solid line, y in dotted line). (g) rms acceleration error (x in solid line, y in dotted line). (h) Quality index.

VI. CONCLUSION

The conventional approaches for tracking in the environment of multiple targets with maneuver are not all satisfactory, such as the Kalman filter bank with known external force or the sliding window method. These methods suffer from the problem of computation load or time-lag.

In our experiment, the proposed incremental maneuver detection model has shown good maneuver following capability. Moreover, it needs only a finite number of Kalman filters to handle all possible maneuver values. And it responds quickly as maneuver occurs. Also when there is an abrupt maneuver change, the model can still track the targets in short time.

APPENDIX. DERIVATION OF SCALE MATRIX

For a return falling within these regions ($m = 0,\ldots,7$) associated with maneuver, the innovation is first normalized with the covariance matrix $\mathbf{S}(k+1)$. That is

$$d_m(k+1)^2 = \tilde{\mathbf{Z}}_m(k+1)'\mathbf{S}(k+1)^{-1}\tilde{\mathbf{Z}}_m(k+1) \quad (39)$$

where $\tilde{\mathbf{Z}}_m(k+1)$ is $\hat{\mathbf{Z}}_m(k+1 \mid k) - \hat{\mathbf{Z}}(k+1 \mid k)$ and is equal to $\mathbf{HGC}_m\mathbf{A}_m$.

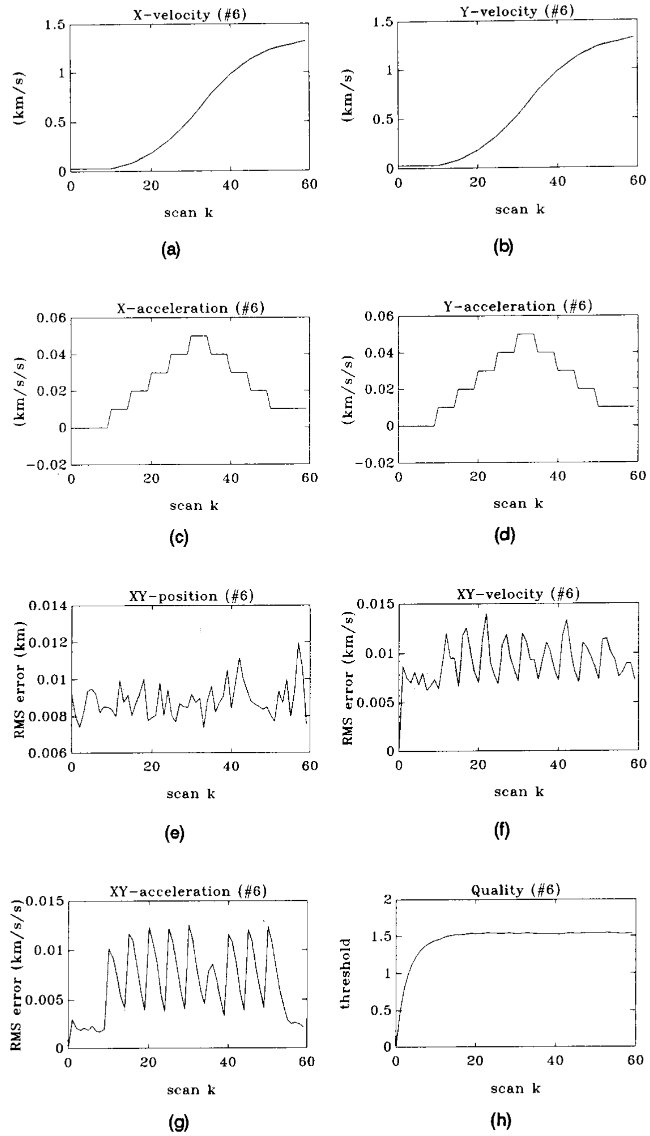


Fig. 9. Statistics of track 6. (a) X velocity versus k. (b) Y velocity versus k. (c) X acceleration versus k. (d) Y acceleration versus k. (e) rms position error (x in solid line, y in dotted line). (f) rms velocity error (x in solid line, y in dotted line). (g) rms acceleration error (x in solid line, y in dotted line). (h) Quality index.

In our case, we set this distance to be at least twice larger than the gate size γ^2. With $2\gamma^2 = d_m(k+1)^2$, we have the following equation

$$\begin{aligned}2\gamma^2 &= (\mathbf{HGC}_m\mathbf{A}_m)'\mathbf{S}^{-1}(\mathbf{HGC}_m\mathbf{A}_m)\\ &= \frac{\Delta^4}{4}(\mathbf{C}_m\mathbf{A}_m)'\mathbf{S}^{-1}(\mathbf{C}_m\mathbf{A}_m)\\ &= \frac{\Delta^4}{4}\mathbf{A}_m'\mathbf{C}_m'\mathbf{S}^{-1}\mathbf{C}_m\mathbf{A}_m. \qquad (40)\end{aligned}$$

With the fact that $\mathbf{A}_m'\mathbf{A}_m = 1$ and assuming that this scale matrix $\mathbf{C}_m$ is diagonal, the following result can be obtained

$$\mathbf{C}_m[i,j](k+1) = \sqrt{2\gamma^2\frac{4}{\Delta^4}|\mathbf{S}[i,j](k+1)|} \qquad (41)$$

where the indices i and j represent the row and column, respectively.

REFERENCES

[1] Zhou, B., Mahalanabis, A. K., and Bose, N. K. (1990) Improved multi-target tracking clutter by PDA smoothing. *IEEE Transactions on Aerospace and Electronic Systems*, **26** (Jan. 1990), 113–121.

[2] Bar-Shalom, Y. (1978)
Tracking methods in a multitarget environment.
IEEE Transactions on Automatic Control, **AC-23** (Aug. 1978), 618–626.

[3] Bar-Shalom, Y., and Birmiwal, K. (1982)
Variable dimension filter for maneuvering target tracking.
IEEE Transactions on Aerospace and Electronic Systems, **AES-18** (Sept. 1982), 621–629.

[4] Bar-Shalom, Y., and Fortmann, T. E. (1988)
Tracking and Data Association.
New York: Academic Press, 1988.

[5] Bar-Shalom, Y., and Tse, E. (1975)
Tracking in a cluttered environment with probabilistic data association.
IEEE Transactions on Automatic Control, **AC-11** (Sept. 1975), 451–460.

[6] Blackman, S. S. (1986)
Multiple Target Tracking with Radar Applications.
Dedham, MA: Artech House, 1986.

[7] Bogler, P. L. (1987)
Tracking a maneuvering target using input estimation.
IEEE Transactions on Aerospace and Electronic Systems, **AES-23** (May 1987), 298–310.

[8] Bullock, T. E., and Sangsuk-Iam, S. (1984)
Maneuver detection and tracking with a nonlinear target model.
In *Proceedings of 23rd Conference on Decision and Control*, Las Vegas, NV, Dec. 1984, 1122–1126.

[9] Chang, K.-C., and Bar-Shalom, Y. (1986)
A simplification of the JPDAM algorithm.
IEEE Transactions on Automatic Control, **AC-31** (Oct. 1986), 989–991.

[10] Chang, K.-C., and Bar-Shalom, Y. (1984)
Joint probabilistic data association for multitarget tracking with possibly unresolved measurements and maneuvers.
IEEE Transactions on Automatic Control, **AC-29** (July 1984), 585–594.

[11] Demirbas, K. (1987)
Maneuvering target tracking with hypothesis testing.
IEEE Transactions on Aerospace and Electronic Systems, **AES-23** (Nov. 1987), 757–766.

[12] Emre, E., and Seo, J. (1989)
A unifying approach to multitarget tracking.
IEEE Transactions on Aerospace and Electronic Systems, **25** (July 1989), 520–528.

[13] Fitzgerald, R. J. (1986)
Development of practical PDA logic for multitarget tracking by microprocessor.
In *Proceedings of the American Controls Conference*, Seattle, WA, 1986, 889–898.

[14] Gholson, N. H., and Moose, R. L. (1977)
Maneuvering target tracking using adaptive state estimation.
IEEE Transactions on Aerospace and Electronic Systems, **AES-13** (May 1977), 310–317.

[15] Greene, M., and Stensby, J. (1987)
Radar target pointing error reduction using extended Kalman filtering.
IEEE Transactions on Aerospace and Electronic Systems, **AES-23** (Mar. 1987), 273–412.

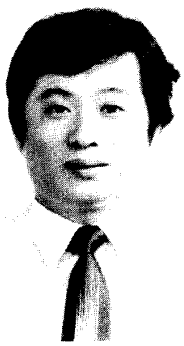
[16] Pearson, J. B., III, and Stear, E. B. (1974)
Kalman filter applications in airborne radar tracking.
IEEE Transactions on Aerospace and Electronic Systems, **AES-10** (May 1974), 319–329.

[17] Miller, K. S., and Leskiw, D. M. (1982)
Nonlinear estimation with radar observations.
IEEE Transactions on Aerospace and Electronic Systems, **AES-18** (Mar. 1982), 192–200.

[18] Roecker, J. A., and McGillem, C. D. (1989)
Target tracking in maneuver-centered coordinates.
IEEE Transactions on Aerospace and Electronic Systems, **25** (Nov. 1989), 836–843.

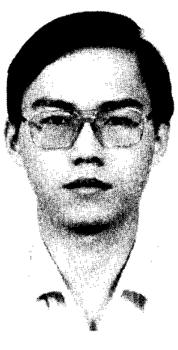
[19] Sengupta, D., and Iltis, R. A. (1989)
Neural solution to the multitarget tracking data association problem.
IEEE Transactions on Aerospace and Electronic Systems, **25** (Jan. 1989), 96–108.

[20] Thorp, J. S. (1973)
Optimal tracking of maneuvering targets.
IEEE Transactions on Aerospace and Electronic Systems, **AES-9** (July 1973), 512–519.



Wen-Thong Chang was born on November 15, 1956. He received the B.S. degree from the Department of Communication Engineering, National Chiao-Tung University, Hsinchu, Taiwan, in 1979, and the Ph.D. degree from the Department of Electrical Engineering. Carnegie-Mellon University, Pittsburgh, PA in 1985.

Currently, he is an Associate Professor in the Department of Communication Engineering, National Chiao-Tung University. His research interests include radar signal processing, digital image processing, digital signal processing, and computer vision.



Shao-An Lin was born on March 22, 1962. He received both the B.S. and the M.S. degree in electronics engineering from the National Chiao-Tung University, Hsinchu, Taiwan, in 1984 and 1990, respectively.

Since 1984, he has been an Assistant Researcher with the Chung Shan Institute of Science and Technology, Lung-Tan, Taiwan. His research interests include multiple maneuvering targets tracking, digital signal processing, and computer architecture.