Integral variable structure control approach for robot manipulators

T.-L. Chern
Y.-C. Wu

Indexing term: Control theory

Abstract: An integral variable structure control (IVSC) approach for robot manipulators is presented to achieve accurate servo-tracking in the presence of load variations, parameter variations and nonlinear dynamic interactions. A procedure is proposed for choosing the control function so that it guarantees the existence of the sliding mode and for determining the coefficients of the switching plane and the integral control gain such that the IVSC approach has the desired properties. Furthermore, a modified proper continuous function is introduced to overcome the chattering problem. The proposed IVSC approach has been simulated for the first three links of a PUMA 560 robot arm as an illustration. The simulation results demonstrate the potential of the proposed scheme.

1 Introduction

Most industrial robots are composed of multilinks. Such a robot arm is a highly nonlinear system with complicated coupled dynamics and uncertainty (various loads, inertia, gravitational forces etc.). With regard to such a complicated system, various controllers have been developed, such as adaptive controllers [1–3], robust controllers [4–6] and controllers based on the theory of variable structure [7–10].

The integral variable structure control (IVSC) approach previously proposed in Reference 11 considered the single-input single-output (SISO) system and has been successfully applied to electrohydraulic servo control systems. The IVSC approach comprises an integral controller for achieving a zero steady-state error under step input and a variable structure controller (VSC) [12–14] for enhancing the robustness. With this special scheme, two control loops are obtained, and it yields improved performance when compared to conventional VSC and linear approaches [11]. This paper extends previous results to the multi-input multi-output (MIMO) case, with an application to robot manipulators. The control of the first three links of a PUMA 560 robot arm has been simulated for illustrating the design procedure and demonstrating the robustness property.

Paper 8569D (C9, C15), first received 21st January and in revised form 13th November 1991

T.-L. Chern is with the Institute of Electronics, National Chiao Tung University, Hsinchu, Taiwan, Republic of China

Y.-C. Wu is with the Institute of Control Engineering, National Chiao Tung University, Hsinchu, Taiwan, Republic of China

2 Description of methodology

The IVSC approach presented here is derived for the class of second-order dynamic equations with a positive-definite symmetric inertia matrix. Since the dynamics of most mechanical systems can be modelled in this form, this approach will have wide applications.

Consider the dynamic equation [9]

$$M\ddot{\theta} + B\dot{\theta} + D\theta = W + U \tag{1a}$$

where θ, $\dot{\theta}$, $\ddot{\theta}$ are $n \times 1$ position, velocity and acceleration vectors, respectively; $M = M(\theta, \dot{\theta})$ is an $n \times n$ symmetric positive-definite inertia matrix; $B = B(\theta, \dot{\theta})$ is an $n \times n$ matrix; $D = D(\theta, \dot{\theta})$ is an $n \times n$ matrix; $W = W(\theta, \dot{\theta})$ is an $n \times 1$ vector representing the gravity term; and U is an $n \times 1$ control vector.

The corresponding state-space model can be written as

$$\begin{bmatrix} \dot{\theta} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & I \\ -M^{-1}D & -M^{-1}B \end{bmatrix} \begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ M^{-1} \end{bmatrix} U + \begin{bmatrix} 0 \\ M^{-1} \end{bmatrix} W \tag{1b}$$

The proposed configuration of the IVSC approach is shown in Fig. 1. It combines an integral controller, a

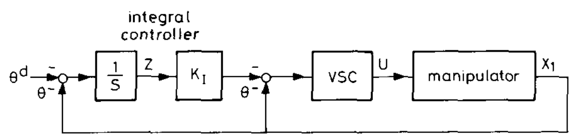

Fig. 1 *Block diagram of an integral-variable-structure-controlled manipulator control system*

VSC and the plant (eqn. 1), and is described as follows:

$$\begin{bmatrix} \dot{\theta} \\ \ddot{\theta} \\ \dot{Z} \end{bmatrix} = \begin{bmatrix} 0 & I & 0 \\ -M^{-1}D & -M^{-1}B & 0 \\ -I & 0 & 0 \end{bmatrix} \begin{bmatrix} \theta \\ \dot{\theta} \\ Z \end{bmatrix} + \begin{bmatrix} 0 \\ M^{-1} \\ 0 \end{bmatrix} U + \begin{bmatrix} 0 \\ M^{-1} \\ 0 \end{bmatrix} W + \begin{bmatrix} 0 \\ 0 \\ I \end{bmatrix} \theta^d \tag{2}$$

where $\theta^d = [\theta_1^d \theta_2^d \cdots \theta_n^d]^T$ represent the desired position; $Z = [z_1 z_2 \cdots z_n]^T$ is an $n \times 1$ vector; I is the $n \times n$ identity matrix; $K_I = \text{diag}\,[k_1 k_2 \cdots k_n]$ is the gain matrix of the integral controller; and the control function $U = [U_1 U_2 \cdots U_n]^T$ is piecewise linear of the form

$$U_i = \begin{cases} U_i^+(x, t) & \text{if } \sigma_i > 0 \\ U_i^-(x, t) & \text{if } \sigma_i > 0 \end{cases} \quad i = 1, \ldots, n \tag{3}$$

where σ_i is the ith component of the n-dimensional switching plane $\sigma = 0$ and is chosen as

$$\sigma_i = c_i(\theta_i - k_i z_i) + \dot{\theta}_i \quad i = 1, \ldots, n \tag{4a}$$

or, in matrix form,

$$\sigma = C(\theta - K_I Z) + \dot{\theta} \tag{4b}$$

where

$$\sigma = [\sigma_1 \sigma_2 \quad \cdots \quad \sigma_n]^T$$

$$C = \mathrm{diag}\,[c_1 c_2 \quad \cdots \quad c_n] \quad c_i > 0$$

Design of such a system involves

(a) the choice the functions U^+ and U^- to guarantee the existence of a sliding mode

(b) the determination of the switching function σ and the integral control gain K_I, such that the system has the desired eigenvalues

(c) the elimination of chattering of the control input.

2.1 Control function

From eqns. 2 and 4, one has

$$\dot{\sigma} = -M^{-1}D\theta - M^{-1}B\dot{\theta} + C\dot{\theta} - CK_I(\theta^d - \theta) + M^{-1}W + M^{-1}U \tag{5}$$

Let

$$M = M^0 + \Delta M$$

$$B = B^0 + \Delta B$$

$$D = D^0 + \Delta D$$

$$W = W^0 + \Delta W$$

where M^0, B^0, D^0 and W^0 are nominal values of M, B, D and W, and ΔM, ΔB, ΔD and ΔW are the deviations.

Let the control function U be decomposed as

$$U = U_{eq} + \Delta U \tag{6a}$$

where U_{eq}, called equivalent control, is defined as the solution of the problem $\dot{\sigma} = 0$ under $M = M^0$, $B = B^0$, $D = D^0$ and $W = W^0$. That is,

$$U_{eq} = D^0\theta + B^0\dot{\theta} - M^0 C\dot{\theta} + M^0 CK_I(\theta^d - \theta) - W^0 \tag{6b}$$

The function ΔU is used to eliminate the influence due to the plant parameter variations in ΔM, ΔB, ΔD and ΔW so as to guarantee the existence of a sliding mode. It is constructed as follows:

$$\Delta U = M^0 \Delta\tau \tag{6c}$$

where

$$\Delta\tau = \Psi(\theta - K_I Z) + \Phi\dot{\theta} + \varphi \tag{6d}$$

$$\Psi = \mathrm{diag}\,[\Psi_1 \Psi_2 \quad \cdots \quad \Psi_n]$$

$$\Phi = \mathrm{diag}\,[\Phi_1 \Phi_2 \quad \cdots \quad \Phi_n]$$

$$\varphi = [\varphi_1 \varphi_2 \quad \cdots \quad \varphi_n]^T$$

$$\Psi_i = \begin{cases} \Psi_i^+ & \text{if } (\theta_i - k_i z_i)\sigma_i > 0 \\ \Psi_i^- & \text{if } (\theta_i - k_i z_i)\sigma_i < 0 \end{cases} \quad i = 1, \ldots, n$$

$$\Phi_i = \begin{cases} \Phi_i^+ & \text{if } \dot{\theta}_i \sigma_i > 0 \\ \Phi_i^- & \text{if } \dot{\theta}_i \sigma_i < 0 \end{cases} \quad i = 1, \ldots, n$$

$$\varphi_i = \begin{cases} \varphi_i^+ & \text{if } \sigma_i > 0 \\ \varphi_i^- & \text{if } \sigma_i < 0 \end{cases} \quad i = 1, \ldots, n$$

For a mechanical system such as a robot arm, each diagonal component of $M^{-1}M^0$ is larger than the absolute value of the sum of other components in the same row [10]. Thus, the following equation is obtained:

$$M^{-1}M^0 = I + \Delta I \tag{7}$$

where $\Delta I = [\Delta i_{ij}]$ $(i = 1, \ldots, n,\ j = 1, \ldots, n)$ and each entry $\Delta i_{ij} \ll 1$.

The condition for the existence of a sliding motion on the ith switch plane is [12–14]

$$\lim_{\sigma_i \to 0} \sigma_i \dot{\sigma}_i < 0 \tag{8}$$

Substituting eqn. 6 into eqn. 5 yields

$$\dot{\sigma}_i = -M^{-1}\,\Delta D\theta - M^{-1}\,\Delta B\dot{\theta} - \Delta IC\theta + \Delta ICK(\theta^d - \theta) + M^{-1}\,\Delta W + \Delta I\,\Delta\tau + \Delta r$$

Let

$$\Delta W = [\Delta w_1, \ldots, \Delta w_n]^T$$

$$M^{-1} = [m_{ij}^{-1}] \quad (i = 1, \ldots, n, j = 1, \ldots, n)$$

$$M^{-1}\,\Delta D = [\Delta d_{ij}] \quad (i = 1, \ldots, n, j = 1, \ldots, n)$$

$$M^{-1}\,\Delta B = [\Delta b_{ij}] \quad (i = 1, \ldots, n, j = 1, \ldots, n)$$

Each component of $\dot{\sigma}$ is represented as

$$\begin{aligned}\dot{\sigma}_i &= (-\Delta d_{ii} - \Delta i_{ii} c_i - \Delta i_{ii} c_i k_i)(\theta_i - k_i z_i) - \Delta b_{ii}\dot{\theta} + g_i + \Delta\tau_i \\ &= (-\Delta d_{ii} - \Delta i_{ii} c_i - \Delta i_{ii} c_i k_i + \Psi_i)(\theta_i - k_i z_i) + (-\Delta b_{ii} + \Phi_i)\dot{\theta} + (g_i + \varphi_i)\end{aligned} \tag{9a}$$

where

$$g_i = -\sum_{j \neq i}^{n}(\Delta d_{ij} + \Delta i_{ij} c_j + \Delta i_{ij} c_j K_j)\theta_j - \sum_{j \neq i}^{n}(\Delta b_{ij}\dot{\theta}_j) + \sum_{j=1}^{n}(\Delta i_{ij} c_j K_j \theta_j^d + m_{ij}^{-1}\,\Delta w_j) + \sum_{j=1}^{n}(\Delta i_{ij}\,\Delta\tau_j) - (\Delta d_{ii} + \Delta i_{ii} c_i + \Delta i_{ii} c_i K_i)K_i Z_i \tag{9b}$$

Then

$$\lim_{\sigma_i \to 0} \dot{\sigma}_i \sigma_i = (-\Delta d_{ii} - \Delta i_{ii} c_i - \Delta i_{ii} c_i k_i + \Psi_i)(\theta_i - k_i z_i)\sigma_i + (-\Delta b_{ii} + \Phi_i)\dot{\theta}_i \sigma_i + (g_i + \varphi_i)\sigma_i \tag{10}$$

and the conditions for satisfying the inequality eqn. 8 are

$$\Psi_i = \begin{cases} \Psi_i^+ < \inf(\Delta d_{ii} + \Delta i_{ii} c_i + \Delta i_{ii} c_i k_i) \\ \quad \text{if } (\theta_i - k_i z_i)\sigma_i > 0 \\ \Psi_i^- > \sup(\Delta d_{ii} + \Delta i_{ii} c_i + \Delta i_{ii} c_i k_i) \\ \quad \text{if } (\theta_i - k_i z_i)\sigma_i < 0 \end{cases} \quad i = 1, \ldots, n \tag{11a}$$

$$\Phi_i = \begin{cases} \Phi_i^+ < \inf(\Delta b_{ii}) & \text{if } \dot{\theta}_i \sigma_i > 0 \\ \Phi_i^- > \sup(\Delta b_{ii}) & \text{if } \dot{\theta}_i \sigma_i < 0 \end{cases} \quad i = 1, \ldots, n \tag{11b}$$

$$\varphi_i = \begin{cases} \varphi_i^+ < \inf|g_i| & \text{if } \sigma_i > 0 \\ \varphi_i^- > \sup|g_i| & \text{if } \sigma_i < 0 \end{cases} \quad i = 1, \ldots, n \tag{11c}$$

Note that g_i in eqn. 9b is dependent not only on parameter variations, load variation and coupling effects but also on the control parameters c_j, K_j, $\Delta\tau_j$ $(j = 1, \ldots, n)$. Since the plant parameter variations Δd_{ij}, Δb_{ij}, Δw_j $(i = 1, \ldots, n, j = 1, \ldots, n)$ are bounded and the term Δi_{ij} $(i = 1, \ldots, n, j = 1, \ldots, n) \ll 1$ as described in eqn. 7, one can guarantee the existence of the gain φ_i such that the inequality eqn. 11c is held.

2.2 Determination of switching plane and integral control gain

While in the sliding motion, the system described by eqn. 2 can be reduced to the following linear equations [12–14]:

$$\dot{\theta} = -C(\theta - K_I Z) \quad (12a)$$

$$\dot{Z} = \theta^d - \theta \quad (12b)$$

Since C and K_I are diagonal matrices, the MIMO system can be decomposed into n sets of SISO systems, as follows:

$$\begin{bmatrix} \dot{\theta}_i \\ \dot{z}_i \end{bmatrix} = \begin{bmatrix} -c_i & c_i k_i \\ -1 & 0 \end{bmatrix} \begin{bmatrix} \theta_i \\ z_i \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \theta_i^d \quad i = 1, \ldots, n \quad (13)$$

The closed-loop transfer function of the system described by eqn. 13 is

$$H_i(S) = \frac{\theta_i(S)}{\theta_i^d} = \frac{c_i k_i}{S^2 + c_i S + c_i k_i} \quad i = 1, \ldots, n \quad (14)$$

where $\theta_i(S)$ and $\theta_i^d(S)$ are the Laplace transforms of θ_i and θ_i^d, respectively. The characteristic equations of the systems are

$$S^2 + c_i S + c_i k_i = 0 \quad i = 1, \ldots, n \quad (15)$$

Since these characteristic equations are independent of the plant parameters, the IVSC approach is robust to the plant parameter variations. It can achieve a zero steady-state error, and its eigenvalues can be set arbitrarily. Let the desired eigenvalues of the systems be λ_{1i}, $\lambda_{2i}(i = 1, \ldots, n)$, or the equivalent desired characteristic equations

$$S^2 + \alpha_{1i} S^1 + \alpha_{2i} = 0 \quad i = 1, \ldots, n \quad (16)$$

Then the switching plane coefficients c_i $(i = 1, \ldots, n-1)$ and the integral control gains k_i $(i = 1, \ldots, n)$ can be chosen as follows:

$$c_i = \alpha_{1i}$$

$$k_i = \alpha_{2i}/\alpha_{1i}$$

2.3 Chattering considerations

For the control law given by eqn. 6d, if Ψ_i, Φ_i and φ_i $(i = 1, \ldots, n)$ are chosen as

$$\Psi_i = \Psi_i^+ = -\Psi_i^-$$

$$\Phi_i = \Phi_i^+ = -\Phi_i^-$$

and

$$\varphi_i = \varphi_i^+ = -\varphi_i^-$$

then the control function $\Delta\tau_i$ $(i = 1, \ldots, n)$ can be represented as

$$\Delta\tau_i = (\Psi_i |\theta_i - k_i z_i| + \Phi_i |\dot{\theta}| + \varphi_i) \operatorname{sign}(\sigma_i) \quad (17)$$

Since the control $\Delta\tau_i$ contains the sign function sign (σ_i), direct application of such control signals to the plant may give rise to chatterings. To obtain continuous control signals, the sign function sign (σ_i) in eqn. 17 can be replaced by a modified proper continuous function as [11]

$$P_i(\sigma_i) = \frac{\sigma_i}{|\sigma_i| + \delta_i} \quad (18)$$

where δ_i is chosen as a function of $|\theta_i - \theta_i^d|$ as

$$\delta_i = \delta_{1i} + \delta_{2i} |\theta_i - \theta_i^d| \quad i = 1, \ldots, n \quad (19)$$

where δ_{1i} and δ_{2i} are positive constants.

3 Control of first three links of PUMA 560 robot arm

The PUMA 560 robot arm has six links and six rotational joints. However, for simplicity, it is assumed that the wrist joints are not active. The mathematical model of the first three links of the PUMA 560 robot is given by [16]

$$M(\theta)\ddot{\theta} + B(\theta, \dot{\theta})\dot{\theta} = W(\theta, \dot{\theta}) + U \quad (20)$$

where $\theta \in R^3$, $U \in R^3$, $W(\theta, \dot{\theta}) \in R^3$, $M(\theta) \in R^{3 \times 3}$ and $B(\theta, \dot{\theta}) \in R^{3 \times 3}$.

Based on the block diagram shown in Fig. 1, by combining eqn. 20 and the IVSC, one obtains a set of state equations of the integral-variable structure-controlled three-link manipulator control system, as follows:

$$\begin{bmatrix} \dot{\theta} \\ \ddot{\theta} \\ \dot{Z} \end{bmatrix} = \begin{bmatrix} 0 & I & 0 \\ 0 & -M^{-1}B & 0 \\ -I & 0 & 0 \end{bmatrix} \begin{bmatrix} \theta \\ \dot{\theta} \\ Z \end{bmatrix} + \begin{bmatrix} 0 \\ M^{-1} \\ 0 \end{bmatrix} U + \begin{bmatrix} 0 \\ M^{-1} \\ 0 \end{bmatrix} W + \begin{bmatrix} 0 \\ 0 \\ I \end{bmatrix} \theta^d \quad (21)$$

where $\theta^d = [\theta_1^d \theta_2^d \theta_3^d]^T$ represents the desired position, $Z = [z_1 z_2 z_3]^T$ is a 3×1 vector, I is 3×3 identity matrix, and $K_I = \operatorname{diag}[k_1 k_2 k_3]$ is the gain matrix of the integral controller. When following the design procedure described in Section 2, one obtains

$$U = U_{eq} + \Delta U$$
$$= B^0\dot{\theta} - M^0 C\dot{\theta} + M^0 C K_I(\theta^d - \theta) - W^0 + M^0 \Delta\tau \quad (22)$$

where

$$C = \operatorname{diag}[c_1 c_2 c_3]$$

$$\Delta\tau = (\Psi |\theta - K_I Z| + \Phi |\dot{\theta}| + \varphi) P$$

in which

$$\Psi = \operatorname{diag}[\Psi_1 \Psi_2 \Psi_3]$$

$$\Phi = \operatorname{diag}[\Phi_1 \Phi_2 \Phi_3]$$

$$\varphi = [\varphi_1 \varphi_2 \varphi_3]^T$$

and

$$p = [p_1(\sigma) p_2(\sigma) p_3(\sigma)]^T$$

$$P_i(\sigma_i) = \frac{\sigma_i}{|\sigma_i| + \delta_{1i} + \delta_{2i} |\theta_i - \theta_i^d|}$$

The control gains are chosen, according to eqn. 11, as

$$\Psi_i = \Psi_i^+ = -\Psi_i^- < -\sup |\Delta i_{ii} c_i + \Delta i_{ii} c_i k_i| \quad i = 1, 2, 3 \quad (23a)$$

$$\Phi_i = \Phi_i^+ = -\Phi_i^- < -\sup |\Delta b_{ii}| \quad i = 1, 2, 3 \quad (23b)$$

and

$$\varphi_i = \varphi_i^+ = -\varphi_i^- < -\sup |g_i| \quad i = 1, 2, 3 \quad (23c)$$

The σ function is obtained from eqn. 4 as

$$\sigma = C(\theta - K_I Z) + \dot{\theta} \quad (24)$$

In the sliding motion, the system described by eqn. 21 can be reduced to the following simple linear form:

$$\dot{\theta} = -C(\theta - K_I Z) \quad (25a)$$

$$\dot{Z} = \theta^d - \theta \quad (25b)$$

Since C and K_I are diagonal matrices, the MIMO system can be decomposed into three SISO systems, as follows:

$$\begin{bmatrix} \dot{\theta}_i \\ \dot{z}_i \end{bmatrix} = \begin{bmatrix} -c_i & c_i k_i \\ -1 & 0 \end{bmatrix} \begin{bmatrix} \theta_i \\ z_i \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \theta_1^d \quad i = 1, 2, 3 \quad (26)$$

The characteristic equations of the system are

$$S^2 + c_i S + c_i k_i = 0 \quad i = 1, 2, 3 \quad (27)$$

It is clear that the dynamic performance of the system can now be determined by simply choosing the coefficients c_1, c_2, c_3 and the gains k_1, k_2, k_3. Let the characteristic equation of the system with desired eigenvalues λ_{1i} and λ_{2i} be

$$S^2 - (\lambda_{1i} + \lambda_{2i})S + \lambda_{1i}\lambda_{2i} = 0 \quad (28)$$

Then, c_i and k_i can be chosen as

$$c_i = -(\lambda_{1i} + \lambda_{2i}) \quad (29a)$$

$$k_i = \frac{-\lambda_{1i}\lambda_{2i}}{(\lambda_{1i} + \lambda_{2i})} \quad (29b)$$

4 Simulation results and discussions

The robustness of the proposed IVSC approach against large variations of plant parameters and load have been simulated for demonstration. The nominal values of the PUMA 560 robot are taken from Reference 16 and given in Table 1, in Appendix 7.

Choosing the eigenvalues of the systems of eqn. 26 as

$$\lambda_{1i} = -20 + j15 \quad \lambda_{2i} = -20 - j15 \quad i = 1, 2, 3$$

one obtains the coefficients of the switching plane and the integral control gain given by eqn. 29 as

$$K = \text{diag}\,[15.625 \quad 15.625 \quad 15.625] \quad (30a)$$

$$C = \text{diag}\,[40 \quad 40 \quad 40] \quad (30b)$$

The gains Ψ_i, Φ_i and φ_i must be chosen to satisfy eqn. 23, and, based on simulations, one possible set of the switching gains is chosen as follows:

$$\Psi = \text{diag}\,[-500 - 500 - 500] \quad (31a)$$

$$\Phi = \text{diag}\,[-10 - 10 - 10] \quad (31b)$$

and

$$\varphi = [-1 - 1 - 1]^T \quad (31c)$$

Thus, the IVSC design gives a control function

$$U = U_{eq} + \Delta U$$
$$= -M^0 C\dot{\theta} + M^0 C K_I(\theta^d - \theta) - W^0 + M^0 \, \Delta\tau$$

where C, K_I, Ψ, Φ and φ are given in eqns. 30–31, and

$$\Delta\tau = (\Psi\,|\theta - K_I Z| + \Phi\,|\dot{\theta}| + \varphi)P$$

where

$$p = [p_1(\sigma)p_2(\sigma)p_3(\sigma)]^T$$

in which

$$P_i(\sigma_i) = \frac{\sigma_i}{|\sigma_i| + 0.1 + 10\,|\theta_i - \theta_i^d|}$$

and

$$\sigma = [\sigma_1 \sigma_2 \sigma_3]^T = C(\theta - K_I Z) + \dot{\theta}$$

The simulation results of the dynamic responses are plotted in Figs. 2–6. To examine the robustness property

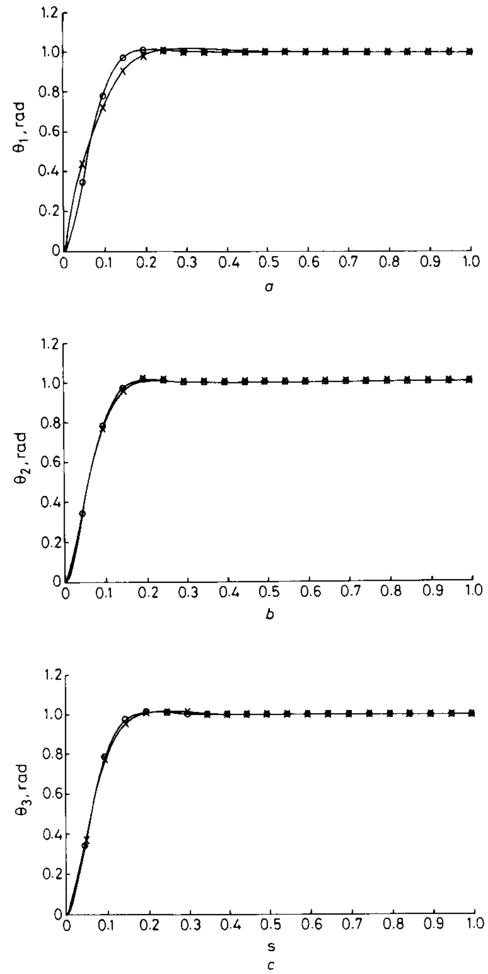

Fig. 2 *Angular responses of IVSC approach with function $p_i(\sigma_i)$, $\delta_i = 0.1 + 10\,|X_i - \theta_i^d|$ and input command $\theta^d = [1 \; 1 \; 1]$ rad*
—×— zero load
—○— with load (200% change in m_3 and 500% change in gyration radius)
a Response θ_1
b Response θ_2
c Response θ_3

of the control system, assume that there is a load attached to the end effector and the total mass and gyration radius of the arm are $m_3 = 15.03$, $k_{3xx} = 0.0906$, $k_{3yy} = 0.093$ and $k_{3zz} = 0.0126$. This corresponds to an increase of 200% in mass and 500% in the gyration radius of the third link. The three links angle responses are shown in Figs. 2 and 3. It is clear that the response can almost be maintained under severe variations of the plant parameters and load.

Figs. 4–6 show the waveforms of the control function of three links with zero load. It is clear that the chattering phenomena can be eliminated by using a modified proper continuous function. Thus, the IVSC approach seems amenable for practical implementation.

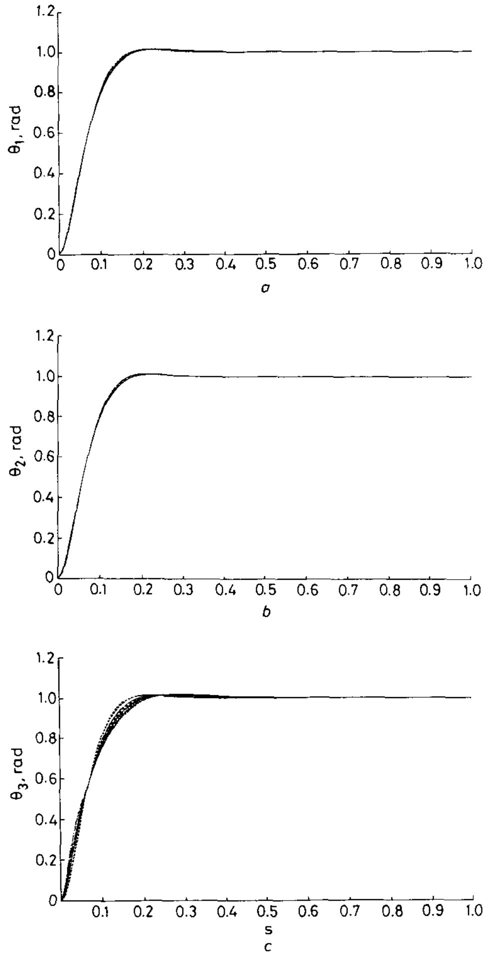

Fig. 3 *Angular responses of IVSC approach under random deviations of m_3 from 0 to 200%, gyration radius from 0 to 500%, with $p_i(\sigma_i)$, $\delta_i = 0.1 + 10|X_i - \theta_i^d|$ and input command $\theta^d = [1 \;\; 1 \;\; 1]$ rad*

a Response θ_1
b Response θ_2
c Response θ_3

5 Conclusions

An IVSC design methodology for MIMO system is presented and applied to the control of the first three links of a PUMA 560 robot arm. It has been shown that the IVSC approach is robust to the plant parameter variations. It can achieve a zero steady-state error for step input and is possible for arbitrary eigenvalue assignment. The control of the three links of a PUMA 560 robot arm is considered for demonstrating the design procedure and the potential of the IVSC approach. Simulations show that the proposed approach can give an almost accurate servo-tracking response in the face of large plant parameter variations, load variations and nonlinear dynamic interactions. It is a robust and practical control law for robot manipulators.

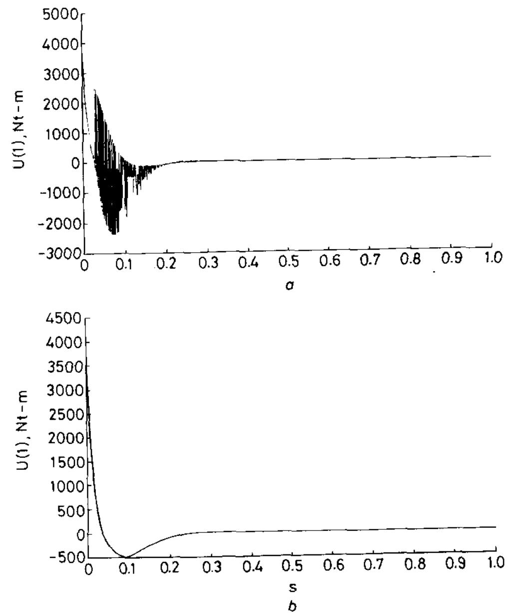

Fig. 4 *Control signal of link-1 in IVSC approach, with zero load and $\theta^d = [1 \;\; 1 \;\; 1]^T$ rad*

a Without $p_1(\sigma_1)$, $\delta_1 = 0$
b With $p_1(\sigma_1)$, $\delta_1 = 0.1 + 10|X_1 - \theta_1^d|$

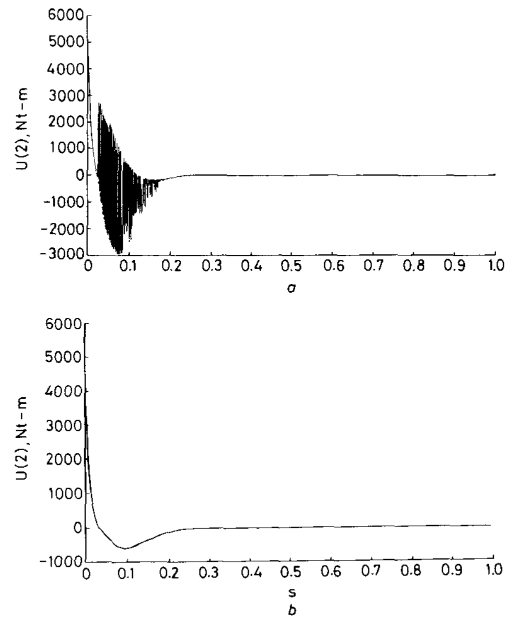

Fig. 5 *Control signal of link-2 in IVSC approach, with zero load and $\theta^d = [1 \;\; 1 \;\; 1]^T$ rad*

a Without $p_2(\sigma_2)$, $\delta_2 = 0$
b With $p_2(\sigma_2)$, $\delta_2 = 0.1 + 10|X_2 - \theta_2^d|$

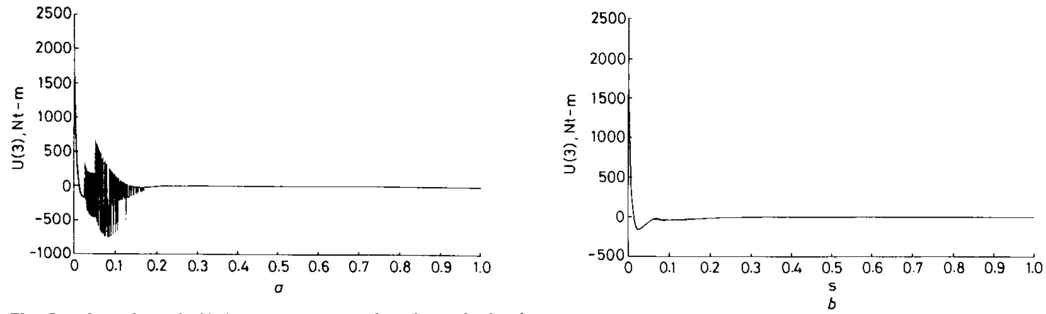

Fig. 6 *Control signal of link-3 in IVSC approach, with zero load and* $\theta^d = [1 \;\; 1 \;\; 1]^T$ *rad*
a Without $p_3(\sigma_3)$, $\delta_3 = 0$
b With $p_3(\sigma_3)$, $\delta_3 = 0.1 + 10|X_3 - \theta_3^d|$

6 References

1 KOIVO, A.J., and GUO, T.H.: 'Adaptive linear controller for robotic manipulator', *IEEE Trans.*, 1984, **AC-28**, pp. 162–171
2 SINGH, S.N.: 'Control and stabilization of a nonlinear uncertain elastic robotic arm', *IEEE Trans.*, 1988, **AES-24**, pp. 114–123
3 KIM, G.K., and SHIN, K.G.: 'An adaptive model following control of industrial manipulator', *IEEE Trans.*, 1983, **AES-19**, pp. 805–814
4 SINGH, S.N., and SCHY, A.A.: 'Robust trajectory following control of robotic system', *J. Dynamic Systems, Measurement and Control*, 1985, **107**, pp. 308–315
5 HA, I.J., and GILBERT, E.G.: 'Robust tracking in nonlinear systems', *IEEE Trans.*, 1987, **AC-32**, pp. 763–771
6 SINGH, S.N., and SCHY, A.A.: 'Control of elastic robotic system by nonlinear inversion and modal damping', *J. Dynamic Systems, Measurement, and Control*, 1986, **108**, pp. 180–189
7 YOUNG, K.D.: 'A variable structure model following control design for robotics applications', *IEEE Trans.*, 1988, **RA-4**, pp. 556–561
8 SINGH, S.N., and ZAKHARIA, Y.N.: 'Variable structure control of a robotic arm in the presence of uncertainty', *J. Robotic Syst.*, 1989, **6**, pp. 111–132
9 YEUNG, K.S., and CHEN, Y.P.: 'A new controller design for manipulators using the theory of variable structure systems', *IEEE Trans.*, 1988, **AC-33**, pp. 200–206
10 HASHIMOTO, H., MARUYAMA, K., and HARASHIMA, F.: 'A microprocessor-based robot manipulator control with sliding mode', *IEEE Trans.*, 1987, **IE-34**, pp. 11–18
11 CHERN, T.L., and WU, Y.C.: 'Design of integral variable controller and application to electrohydraulic velocity servo systems', *IEE Proc. D, Control Theory & Appl.*, Sept. 1991, pp. 439–444
12 UTKIN, V.I.: 'Variable structure systems with sliding modes', *IEEE Trans.*, 1977, **AC-22**, pp. 212–222
13 UTKIN, V.I.: 'Sliding modes and their application in variable structure system' (Mir, Moscow, 1978)
14 ITKIS, U.: 'Control systems of variable structure' (John Wiley, New York, 1976)
15 AMBROSINO, G., CELENTANO, G., and GAROFALO, G.: 'Variable structure model reference adaptive control system', *Int. J. Contr*, 1984, **39**, pp. 1339–1349
16 TARN, T.J., BEJCZY, A.K., YUN, X., and DING, X.: 'Dynamic equation for PUMA robot arm'. Dept. of Systems Science & Mathematics, Washington University, St. Louis, Missouri 63130, USA, Robotics Lab. Report SSM-RL-85-02, 1985

7 Appendix

Table 1: PUMA 560 robot arm parameters with zero load

Link	Mass	Centre of mass			Radius of gyration			Reflected motor inertia
	m_i(kg)	$\bar{x}_i$(m)	$\bar{y}_i$(m)	$\bar{z}_i$(m)	k^2_{ixx}(m^2)	k^2_{iyy}(m^2)	k^2_{izz}(m^2)	I_{ai}(kg/m^2)
1	12.96	0.0000	0.3088	0.0389	0.1816	0.0152	0.1811	0.7766
2	22.37	−0.3289	0.0050	0.2038	0.0596	0.1930	0.1514	2.3616
3	5.01	0.0204	0.0137	0.0037	0.0151	0.0155	0.0021	0.5827

$a_2 = 0.4318$ m; $a_3 = -0.0191$ m; $d_3 = 0.1505$ m; $d_4 = 0.4331$ m