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AN ACCURATE COMPUTATION FOR RAPIDLY VARIED FLOW IN AN OPEN CHANNEL

JINN-CHUANG YANG AND KUN-NAN CHEN

Department of Civil Engineering, National Chiao Tung University, Hsinchu, Taiwan 30050, R.O.C.

AND

HONG-YUAN LEE

Department of Civil Engineering, National Taiwan University, Taipei, Taiwan, R.O.C.

SUMMARY

A new method combining the Preissmann four-point scheme and the Holly–Preissmann reach-back scheme is introduced to solve the rapidly varied flow problem in an open channel. The Preissmann four-point scheme is well known for the computation of one-dimensional unsteady flow. The Holly-Preissmann reach-back scheme integrates the Holly–Preissmann two-point scheme with the concept of reach-back characteristics, which allows the characteristics to project several time steps beyond the current time level. A spontaneous surge formation case is used to demonstrate and evaluate the applicability of the new method. It has been found that the results from this method are quite compatible with those of Preissmann four-point scheme. In addition, with the appropriate choice of the number of reach-back time steps, this new method can always avoid the numerical oscillation which usually exists when one uses the Preissmann four-point scheme for the condition of Courant number not close to unity.

KEY WORDS Rapidly varied flow Open channel Numerical simulation

INTRODUCTION

The accurate numerical simulation of a transient flow such as a surge is strongly hampered by the difficulties in treatment of the non-linear advection terms in the de St. Venant equations. Many numerical methods have been proposed to tackle the advection non-linearity problem. However, most schemes still cannot simulate the surge problem without giving numerical oscillation and damping. The method of characteristics with a characteristic grid system can generate very accurate solutions, but the grid system is awkward. The difficulty arises because the hydrograph at a specific point or the water surface profile at a specific time is required. When the method of characteristics with a fixed grid system is employed, the commonly used linear interpolation can always lead to an inevitable smoothing of the solution. It is known that the shock-fitting method used for rapidly varied flow gives less improvement in the solution. The explicit form of the finite difference method requires the Courant condition[1] for stability. Cooley and Moin[2] and Keuning[3] indicated that the existing finite difference technique was superior to the finite element technique in the case of gradually varied flow. For rapidly varied flow, Katapodes[4] introduced pseudoviscosity in a Galerkin formulation and succeeded in improving the accuracy of simulation drastically. However, as far as the brevity of the algorithm and the computational time are

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concerned, it is hard to judge whether Katapodes' proposed method is better than the Preissmann four-point scheme or not.

Cunge[5] has executed detailed numerical experiments for a spontaneous surge case using the Preissmann four-point method. A review of previous numerical methods suggests that the Preissmann four-point method is considered to be one of the best for simulation of a rapidly varied flow such as a surge, but it has the disadvantage of smearing a discontinuous profile. Toda[6] has proposed a hybrid method based on the concepts of the Holly–Preissmann two-point scheme (H–P method) and the Preissmann four-point scheme to solve the mass and momentum conservation equations for rapidly varied flow. The H–P method is based on the fact that the higher-order interpolating polynomials are constructed by the use of the dependent variables and their derivatives at two adjacent points on the spatial axis. Application of the H–P method has proven it to be a powerful technique which reduces the numerical oscillation and damping to the greatest extent in the dispersion equation.[7–9] However, Toda indicated that the hybrid method gives insignificant improvement for the computation of rapidly varied flow and places very rigid restrictions on the control parameters such as spatial and temporal weighting factors for the solution stability. In this paper a method is developed on the basis of Toda's idea, but allowing the characteristics to project several time steps beyond the current time level to fall on the spatial axis, and in which the characteristic foot is solved by the H–P method. This method is denoted hereafter as the hybrid Holly–Preissman reach-back method (hybrid HPRB method). In fact, the Holly–Preissmann two-point scheme with the reach-back characteristics technique (i.e. HPRB method) has also been successfully applied to solve the 1D and 2D dispersion equations.[10,11] This newly introduced hybrid HPRB method is examined through its application to a simple rapidly varied flow case for which an analytical solution exists. The computed results are also compared with those obtained from the use of the Preissmann four-point scheme.

GOVERNING EQUATIONS

The channel is assumed to have a uniform rectangular cross-section and to be frictionless and horizontal. The de St. Venant equations can then be written as

$$\frac{\partial h}{\partial t}+\frac{\partial q}{\partial x}=0, \tag{1}$$

$$\frac{\partial q}{\partial t}+\frac{\partial}{\partial x}\left(\frac{q^2}{h}\right)+\frac{\partial}{\partial x}\left(\frac{g}{2}h^2\right)=0, \tag{2}$$

where q denotes the discharge per unit width, h is the flow depth, t is the time, x is the distance along the flow direction and g is the gravitational constant. If one rewrites the two equations in conservation form, they become

$$\frac{\partial \mathbf{f}}{\partial t}+\frac{\partial}{\partial x}[\mathbf{g}(\mathbf{f})]=0, \tag{3}$$

in which

$$\mathbf{f}=\begin{bmatrix} h \\ q \end{bmatrix}, \qquad \mathbf{g}(\mathbf{f})=\begin{bmatrix} q \\ q^2/h+gh^2/2 \end{bmatrix}.$$

This conservation form admits a weak solution to the hydraulic transient or rapidly varied flow problem.[5] The method introduced herein is that the advection term is evaluated by the Holly–Preissmann two-point scheme with the reach-back technique (HPRB), while the other terms are still computed by the Preissmann four-point scheme. It is noted that this hybrid HPRB method is identical to Toda's hybrid method when the reach-back number $m=1$.

To apply the HPRB method to the evaluation of the momentum advection terms of equation (2), one rewrites the momentum equation in the total derivative form

$$\frac{\mathrm{D}q}{\mathrm{D}t}+\left(gh-\frac{q^2}{h^2}\right)\frac{\partial h}{\partial x}=0 \tag{4}$$

along

$$\frac{\mathrm{d}x}{\mathrm{d}t}=2\frac{q}{h}, \tag{5}$$

where $\mathrm{D}q/\mathrm{D}t$ denotes the total derivative of q along the trajectory of equation (5). Integration of equations (4) and (5) leads to

$$q_\eta-q_\varepsilon+\int_{t_\varepsilon}^{t_\eta}\left(gh-\frac{q^2}{h^2}\right)\frac{\partial h}{\partial x}\,\mathrm{d}t=0, \tag{6}$$

$$x_\eta-x_\varepsilon=\int_{t_\varepsilon}^{t_\eta}2\frac{q}{h}\,\mathrm{d}t, \tag{7}$$

where η and ε denote the head and foot of the trajectory respectively. The value of q_ε is to be evaluated by the HPRB method, which requires q and qx (i.e. spatial derivative of q) as dependent variables on the computational grid. The grid graph is shown in Figure 1. Since qx is introduced in the above formulations, one more equation is required to evaluate it. Toda[6] used a simple method to determine qx, which is expressed as

$$qx=\frac{\partial q}{\partial x}, \tag{8}$$

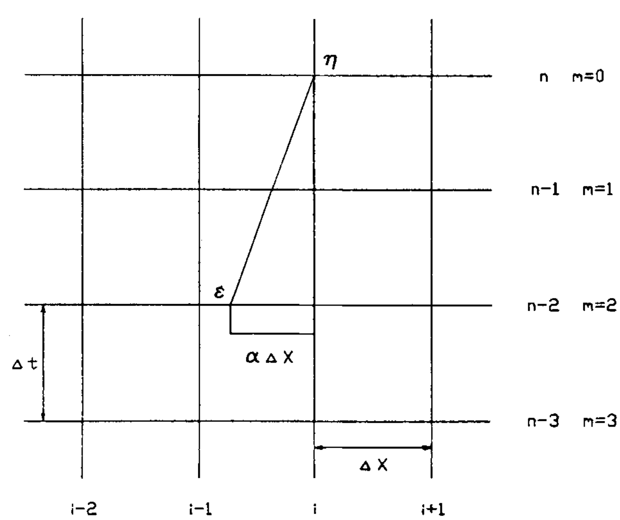

Figure 1. HPRB method grid diagram

where qx is independent of q as a dependent variable. Now, instead of solving equations (1) and (2), one has to solve equations (1) and (6)–(8) simultaneously.

SOLUTION ALGORITHM

With the use of the Preissmann four-point scheme, equations (1) and (8) can be transformed into the discrete forms

$$\frac{h_i^n - h_i^{n-1}}{2\Delta t} + \frac{h_{i-1}^n - h_{i-1}^{n-1}}{2\Delta t} + \theta \frac{q_i^n - q_{i-1}^n}{\Delta x} + (1-\theta) \frac{q_i^{n-1} - q_{i-1}^{n-1}}{\Delta x} = 0, \tag{9}$$

$$\theta \frac{qx_{i-1}^n - qx_i^n}{2} + (1-\theta) \frac{qx_{i-1}^{n-1} - qx_i^{n-1}}{2} = \theta \frac{q_i^n - q_{i-1}^n}{\Delta x} + (1+\theta) \frac{q_i^{n-1} - q_{i-1}^{n-1}}{\Delta x}. \tag{10}$$

Equation (6) can be simply approximated as

$$q_\eta - q_\varepsilon = -[\theta \Gamma_\eta + (1-\theta)\Gamma_\varepsilon] m\,\Delta t, \tag{11}$$

where m is the reach-back number and Γ denotes an approximation of $(gh - q^2/h^2)\,\partial h/\partial x$ at either the foot ε or head η of the trajectory. Point η is just point i at time level n, and Γ_ε can be approximated as the average value at points $i-1$ and i at time level $n-m$. The discretized form of equation (11) can be expressed as

$$q_i^n - q_\varepsilon + \left[\left(g h_i^n - \frac{(q_i^n)^2}{(h_i^n)^2} \right) \frac{h_i^n - h_{i-1}^n}{\Delta x} \theta \right] m\Delta t$$
$$+ \left[g \frac{h_i^{n-m} - h_{i-1}^{n-m}}{2} - \frac{1}{4} \left(\frac{q_i^{n-m}}{h_i^{n-m}} + \frac{q_{i-1}^{n-m}}{h_{i-1}^{n-m}} \right)^2 \right] \frac{h_i^{n-m} - h_{i-1}^{n-m}}{\Delta x} (1-\theta) m\,\Delta t = 0. \tag{12}$$

Equation (7) is approximated as

$$x_\eta - x_\varepsilon = \alpha \Delta x = 2 \left[\phi \left(\frac{q}{h} \right)_\eta + (1-\phi) \left(\frac{q}{h} \right)_\varepsilon \right] m\,\Delta t. \tag{13}$$

θ and ϕ appearing in equations (12) and (13) are weighting factors. q_ε appearing in the previous formulations can be evaluated by using a cubic interpolating polynomial based on the known quantities q_{i-1}^{n-m}, q_i^{n-m}, qx_{i-1}^{n-m} and qx_i^{n-m}:

$$q_\varepsilon = A_0 + A_1 \alpha + A_2 \alpha^2 + A_3 \alpha^3, \tag{14}$$

where $\alpha = (x_i - x_\varepsilon)/\Delta x$ and the coefficients A_0–A_3, listed in Appendix I, are determined such that for $\alpha = 0$, $q_\varepsilon = q_i^{n-m}$ and $qx_\varepsilon = qx_i^{n-m}$, and for $\alpha = 1$, $q_\varepsilon = q_{i-1}^{n-m}$ and $qz_\varepsilon = qx_{i-1}^{n-m}$.

However, in order to compute q_η, one has to know the value of α. First of all, one approximates q_η from equation (12) with a purely explicit form of the source terms,

$$q_\eta = q_\varepsilon - m\Delta t \left[g \frac{h_i^{n-1} + h_{i-1}^{n-1}}{2} - \frac{1}{4} \left(\frac{q_i^{n-1}}{h_i^{n-1}} + \frac{q_{i-1}^{n-1}}{h_{i-1}^{n-1}} \right)^2 \right] \frac{h_i^{n-1} - h_{i-1}^{n-1}}{\Delta x}; \tag{15}$$

h_ε can be evaluated by linear interpolation with respect to x as

$$h_\varepsilon = \alpha h_{i-1}^{n-m} + (1-\alpha) h_i^{n-m}. \tag{16}$$

Assuming $h_\eta = h_\varepsilon$ and substituting equations (14)–(16) into equation (13), the latter becomes

$$A_3\alpha^3+\left(A_2-\frac{1}{2m}\frac{\Delta x}{\Delta t}(h_{i-1}^{n-m}-h_i^{n-m})\right)\alpha^2+\left(A_1-\frac{1}{2m}\frac{\Delta x}{\Delta t}h_i^{n-m}\right)\alpha$$
$$+A_0-\phi\left[g\frac{h_i^{n-1}+h_{i-1}^{n-1}}{2}-\frac{1}{4}\left(\frac{q_i^{n-1}}{h_i^{n-1}}+\frac{q_{i-1}^{n-1}}{h_{i-1}^{n-1}}\right)^2\right]\frac{h_i^{n-1}-h_{i-1}^{n-1}}{\Delta x}m\Delta t=0 \quad (17)$$

In the vicinity of the surge front, owing to the rapid changes of depth and discharge, the approximations of q_η, h_ε and h_η by equations (15) and (16) and the assumption $h_\eta=h_\varepsilon$, which yield a direct and simple trajectory evaluation by equation (17), may have some effects on the solution stability and accuracy. However, from the later demonstration case one can observe that the solution stability and accuracy have been successfully improved with the use of this newly developed method by careful selection of the reach-back number. Therefore the error which may exist due to the approximations of q_η, h_ε and h_η will not be considered to be a major one and the approximation relations should be acceptable.

Solution of equation (17), which is a cubic polynomial in α, yields a value for α as a real root between zero and unity. Once α has been determined, q_ε is obtained from equation (14). Therefore, for N computational points, a system of $3(N-1)$ equations will be constructed on the basis of equations (9), (10) and (12) for three unknowns. With the addition of three boundary conditions the system of equations can be solved by a Newton–Raphson method. Through linearization,

$$h_i^n=h_i^{n-1}+\Delta h_i,\qquad q_i^n=q_i^{n-1}+\Delta q_i,\qquad qx_i^n=qx_i^{n-1}+\Delta q_i. \quad (18)$$

The increments of the unknowns during a time interval Δt become the dependent variables. Substitution of equations (18) into equations (9), (10) and (12) leads to the linear system

$$A_i\Delta h_i+B_i\Delta q_i=D_i\Delta h_{i-1}+E_i\Delta q_{i-1}+G_i, \quad (19)$$

$$A_i'\Delta h_i+B_i'\Delta q_i=D_i'\Delta h_{i-1}+E_i'\Delta q_{i-1}+G_i', \quad (20)$$

$$B_i''\Delta q_i+C_i''\Delta qx_i=E_i''\Delta q_{i-1}+F_i''\Delta qx_{i-1}+G_i'', \quad (21)$$

where the coefficients A_i–G_i'' are as given in Appendix I. In the system of equations (19)–(21), Δqx_i appears only in equation (21). The boundary values for Δqx_i can be obtained by using the updated boundary values of Δh_i and Δq_i. Therefore it will be convenient to solve for Δh_i and Δq_i from equations (19) and (20), which are not coupled with equation (21). The double-sweep method with two boundary conditions results in a solution for Δh_i and Δq_i ($i=1, 2, \ldots, N$). Then Δqx_i can be determined explicitly with one boundary condition through equation (21). After Δh_i, Δq_i and Δqx_i have been determined in this manner, h_i^n, q_i^n and qx_i^n can be obtained from equations (18) and the computation proceeds to the next time step.

DEMONSTRATION AND EVALUATION

A test case is adopted for the evaluation of the method proposed in the previous section. The test case is a spontaneous surge formation. A uniform subcritical flow is assumed to take place in a frictionless, horizontal, rectangular prismatic channel. The depth is 2 m and the discharge per unit width is 2 m^2 s^{-1}. Suddenly a sluice gate is closed at the downstream end, resulting in spontaneous development of a surge transmitting upstream with constant speed. For this simple case the analytical solutions for surge celerity and surge depth exist.

The surge celerity V_c and the rapidly varied depth h can be obtained by simultaneously solving the following two equations for mass and momentum conservation:[5]

$$V_c = \frac{q_1 - q_2}{h_2 - h_1}, \tag{22}$$

$$V_c = \frac{q_2^2/h_2 + gh_2^2/2 - (q_1^2/h_1 + gh_1^2/2)}{q_1 - q_2}, \tag{23}$$

where subscripts 1 and 2 denote the upstream and downstream sides of the surge respectively. By substituting the given data for this test case into equations (22) and (23), one obtains $V_c = 4{\cdot}21\ \mathrm{m\,s^{-1}}$ and $h_2 = 2{\cdot}475$ m. The travelling distance can be computed by multiplying V_c by time. These results are used for comparison with the numerical solutions. The newly proposed scheme (i.e. hybrid HPRB method) may require one to assume/calculate the initial conditions when one applies the reach-back technique for computing the advection term. In addition, the results from the use of the Preissmann four-point scheme, which is one of the best schemes for rapidly varied flow computation,[5] are also used for the comparison study.

In fact, it is clear that the newly proposed technique (hybrid HPRB method) is identical to Toda's hybrid method[6] when the reach-back number $m = 1$. Toda indicated that many difficulties exist with the use of the hybrid method for rapidly varied flow computation. The main disadvantage of Toda's hybrid method (i.e. hybrid HPRB method with $m = 1$) is that it is very sensitive to the Courant number. On the basis of Toda's study, it is known that the solution becomes unstable when the Courant number is smaller than 0·8. In addition, the stability of the simulation from Toda's hybrid method is also very sensitive to the values of the weighting factors θ and ϕ. When $\theta < 0{\cdot}7$, the computation becomes unstable even for the condition of Courant number approaching unity. Increasing θ can reduce numerical oscillation, but smearing of the wave front occurs. As for the ϕ-value, Toda concluded that for all cases $\phi = 0$ almost leads to the exact celerity without the existence of phase error.

In this study, in order to show the merits of the newly proposed method which integrates Toda's hybrid scheme with the reach-back characteristics technique (i.e. hybrid HPRB method), the analyses are performed through the examination of some key parameters, namely the spatial weighting factor ϕ, the temporal weighting factor θ and the Courant number *Cr*.

EXAMINATION OF θ-EFFECT

From References 5 and 6 it is known that the best simulation of surge wave propagation using the Preissmann four-point scheme or Toda's hybrid method can be obtained when the Courant number approaches unity. The best ϕ-value for the Preissmann four-point scheme is 0·5 and for Toda's hybrid scheme is zero. In order to verify the θ-effect, the best choice of ϕ-value and Courant number *Cr* is fixed and assigned to the respective method. Therefore it is expected that the effect induced by the θ-value can be clearly verified. Figure 2 shows the simulation results obtained by the use of the Preissmann four-point scheme for various values of θ when $Cr = 0{\cdot}985$. It is evident that $\theta = 2/3$ seems to be the best choice, which was also pointed out by Cunge.[5] When $\theta = \frac{1}{2}$, the value of the wave front approaches the exact solution, but severe numerical oscillation occurs. Increasing the θ-value can reduce numerical oscillation, but the wave front will be smoothed.

Figure 3 shows the results simulated by Toda's hybrid scheme (i.e. HPRB method with $m = 1$) for the same case mentioned above with various values of θ. As pointed out by Toda,[6] the solution becomes unstable when $\theta \leqslant 0{\cdot}7$. In order to have a stable solution, one may need to use a larger value of θ; this, however, smears the discontinuous wave front as shown in Figure 3.

When the hybrid HPRB method with $m > 1$ is used, it has been found that the low bound of the θ-value mentioned previously for Toda's hybrid method (i.e. hybrid HPRB method with $m = 1$) can

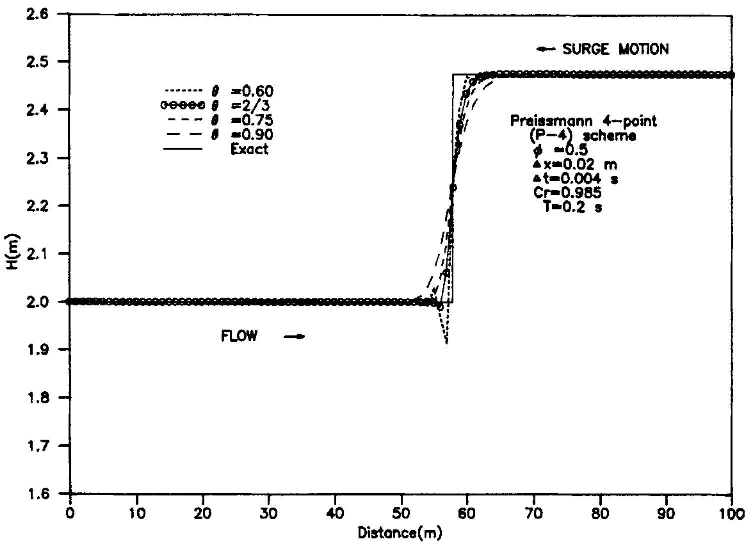

Figure 2. Comparison of numerical solution from Preissmann four-point scheme and analytical solution for $Cr = 0{\cdot}985$

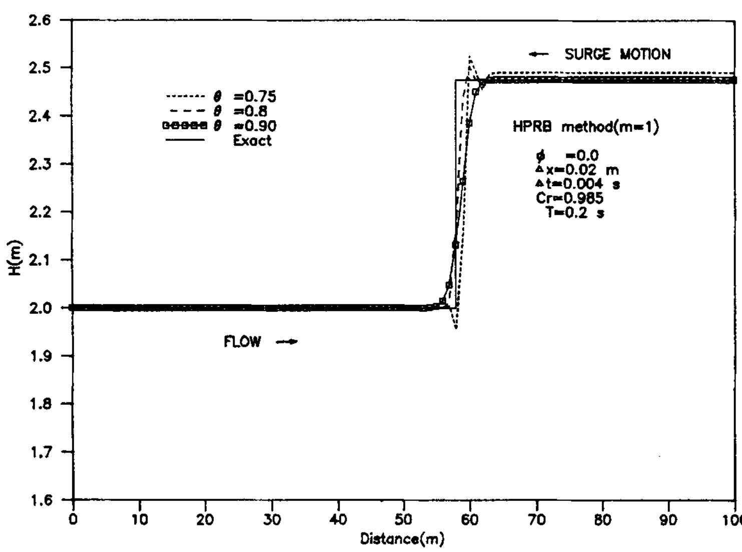

Figure 3. Comparison of numerical solution from hybrid HPRB method with $m = 1$ and analytical solution for $Cr = 0{\cdot}985$

be further decreased by an increase of the reach-back number m. Figure 4 shows the results simulated by the hybrid HPRB method with $m = 2$ for various θ-values. However, according to several test simulations, when the Courant number approaches unity, the use of a larger m-value will further smooth the wave front. From a comparison of Figures 3 and 4 it is evident that the case with $m = 2$ gives a smoother wave front than that with $m = 1$. Therefore no further results with larger m-value are shown here.

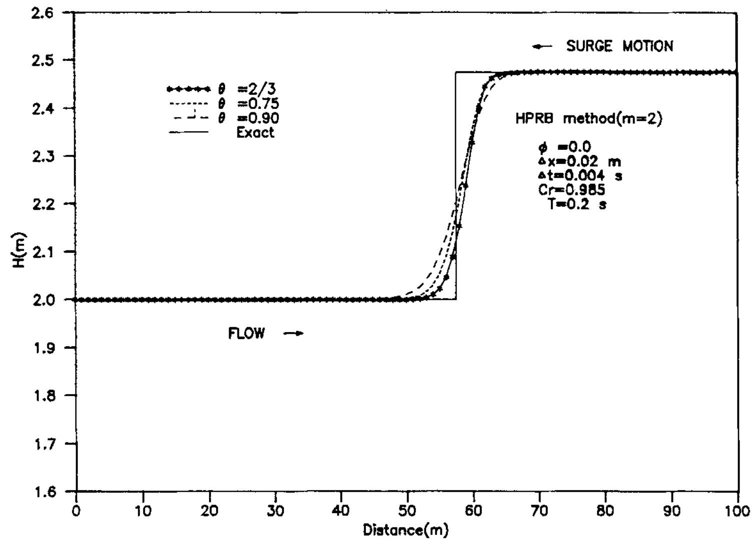

Figure 4. Comparison of numerical solution from hybrid HPRB method with $m=2$ and analytical solution for $Cr=0.985$

The best results obtained from each method discussed above were picked out and a comparison is shown in Figure 5. Figure 5 consists of the results from the Preissmann four-point scheme with $\theta=\frac{2}{3}$, Toda's hybrid method (i.e. hybrid HPRB method with $m=1$) and the hybrid HPRB method with $m=2$ and $\theta=\frac{2}{3}$. It is obvious that all of these results are quite compatible. The result from the hybrid HPRB method with $m=1$ and $\theta=0.9$ is very close to that from the Preissmann four-point method. It is very difficult to distinguish their differences, since the former has the wave front smoothed and the latter has numerical oscillation occurring at the foot of the wave front. Again from Figure 5 it can be observed that the result from the hybrid HPRB method with $m=2$ has the wae front smoothed a little bit more. Thus at this stage, as far as solution accuracy is concerned, one may conclude that for the condition of Courant number approaching unity, in which the simulation is supposed to have no instability problem, one has no need to increase the m-value. However, a larger value of θ may be needed. From the above discussion one can see that for both the Preissmann four-point and hybrid HPRB methods the weighting factor θ plays quite an important role in the solution stability and accuracy. An increase of θ-value will have a positive effect on the stability, but one may have to sacrifice the accuracy of the wave front shape.

EXAMINATION OF ϕ-EFFECT

Figure 6 shows the results computed by the use of the hybrid HPRB method with $m=2$ for various values of ϕ. For this case the best choice of θ-value, $\theta=\frac{2}{3}$, which was found previously is used. Our findings are similar to what Toda concluded, i.e. that $\phi=0$ gives the least phase error. Therefore one may conclude that an increase of m-value has no effect at all on the choice of ϕ-value.

EXAMINATION OF Cr-EFFECT

Several cases with various values of Δx and Δt, listed in Table I, are studied here to investigate the effect of the Courant number Cr on the simulation.

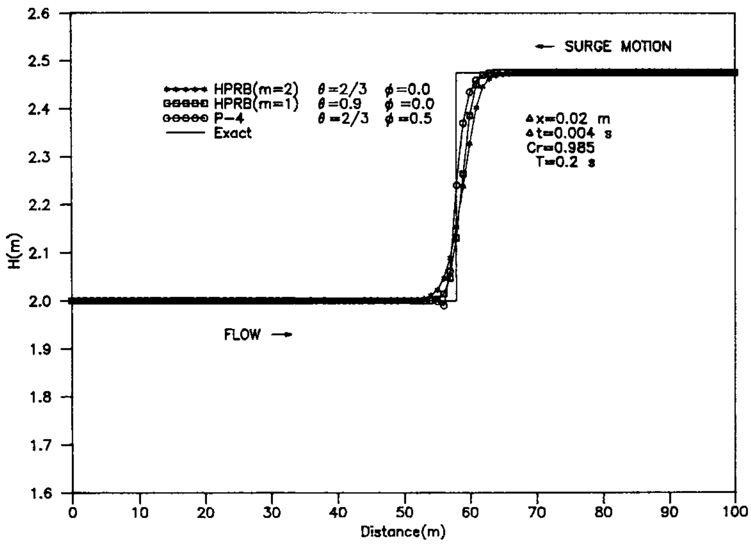

Figure 5. Comparison of numerical solutions from hybrid HPRB method and Preissmann four-point scheme and analytical solution for $Cr = 0{\cdot}985$

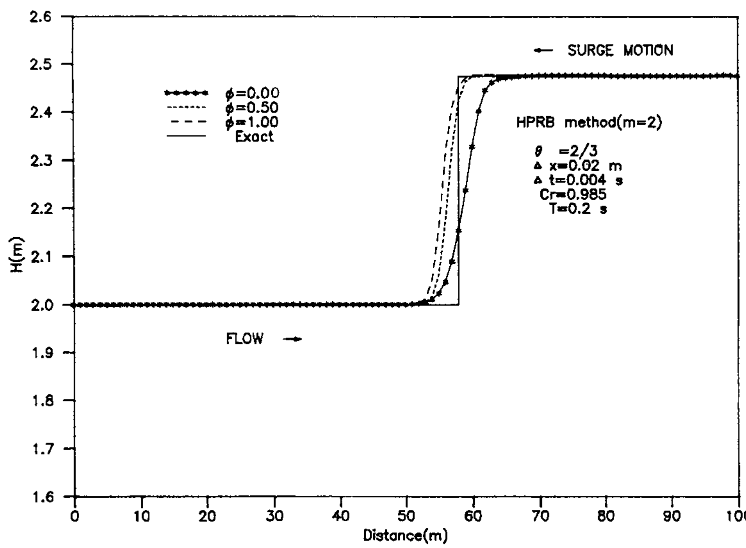

Figure 6. Examination of ϕ-value for hybrid HPRB method

Figure 7 shows the results computed by the use of the Preissmann four-point scheme with $\theta = 0{\cdot}9$ and $\phi = 0{\cdot}5$ for various Courant numbers, $Cr = 0{\cdot}8$, 0·616, 0·493 and 0·37. For the case with smaller Courant number, numerical oscillaton appears. The results computed using the hybrid HPRB method with $\theta = 0{\cdot}9$ and $\phi = 0$ are shown in Figure 8. In Figure 8, for cases with $Cr = 0{\cdot}8$, 0·616 and 0·493, the reach-back number $m = 2$ is used. When the Courant number is further

Table I

Δt	Δx	$Cr\ (=\sqrt{(gh)}\Delta t/\Delta x)$	Time steps
0·00325	0·02	0·0	62
0·0025	0·02	0·616	80
0·002	0·02	0·493	110
0·0015	0·02	0·37	133

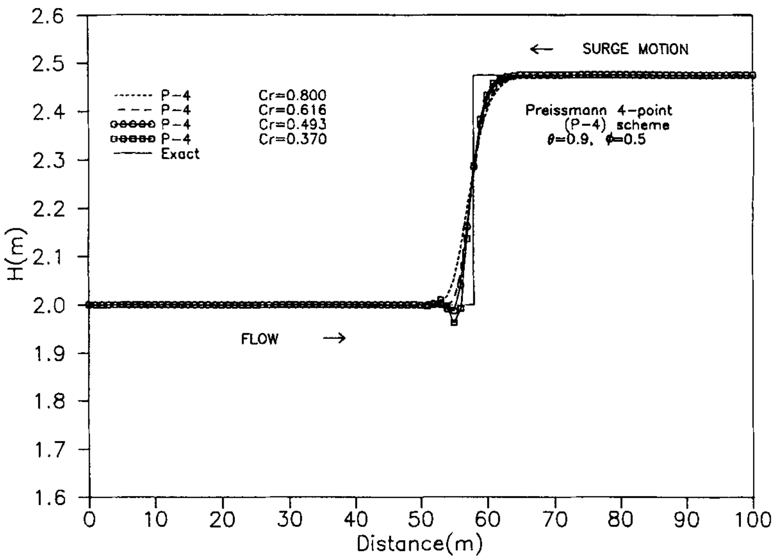

Figure 7. Results from Preissmann four-point scheme for various Courant numbers

reduced to 0·37, the reach-back number $m=4$ is needed to avoid numerical oscillation. However, it is obvious that the wave front has been further diffused owing to the increase of m-value.

From Figures 7 and 8 one can tell that the results for $Cr=0{\cdot}8$ and 0·616 computed by the hybrid HPRB method are clearly compatible with those by the Preissmann four-point method. It is obvious that the simulation results from the Preissmann four-point scheme have the numerical oscillation phenomenon at the wave front. The oscillation problem is getting more severe as the Courant number becomes smaller. It seems that the hybrid HPRB method with careful selection of m-value gives less oscillation and has the better wave front shape close to the exact solution.

When the Courant number is further reduced, it is not unexpected that the result from the Preissmann four-point scheme becomes worse. Figures 9 and 10 show the comparison results for the cases with $Cr=0{\cdot}493$ and 0·37. One can observe that the results computed by the Preissmann four-point scheme are almost unacceptable. On the other hand, the result computed by the hybrid HPRB method is still quite close to the exact solution. For $Cr=0{\cdot}37$ it is necessary to use a larger value of m in order to get a stable solution. From Figure 10, apparently, the result from the hybrid HPRB method is quite convincing for the condition $Cr=0{\cdot}37$. Although the wave front has been smoothed out a little bit, the solution is still stable and has no numerical oscillation at all.

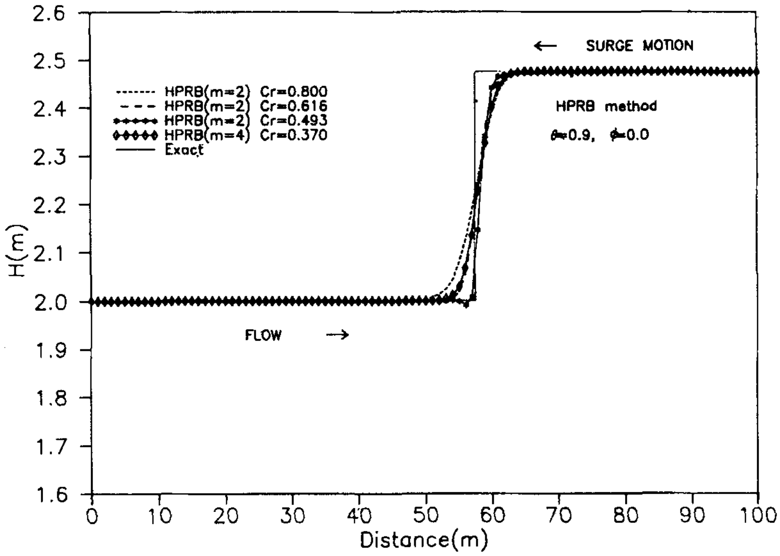

Figure 8. Results from hybrid HPRB method for various Courant numbers

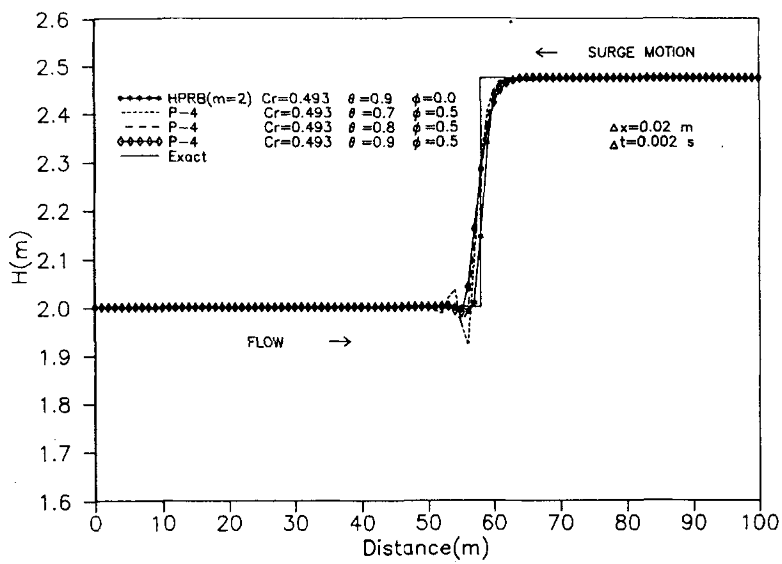

Figure 9. Comparison of hybrid HPRB method and Preissmann four-point scheme for $Cr = 0{\cdot}493$

CONCLUSIONS

A new hybrid method combining the Holly–Preissmann two-point scheme, the reach-back characteristics technique and the Preissmann four-point scheme has been introduced in this paper and denoted the hybrid HPRB method. The newly introduced hybrid HPRB method has

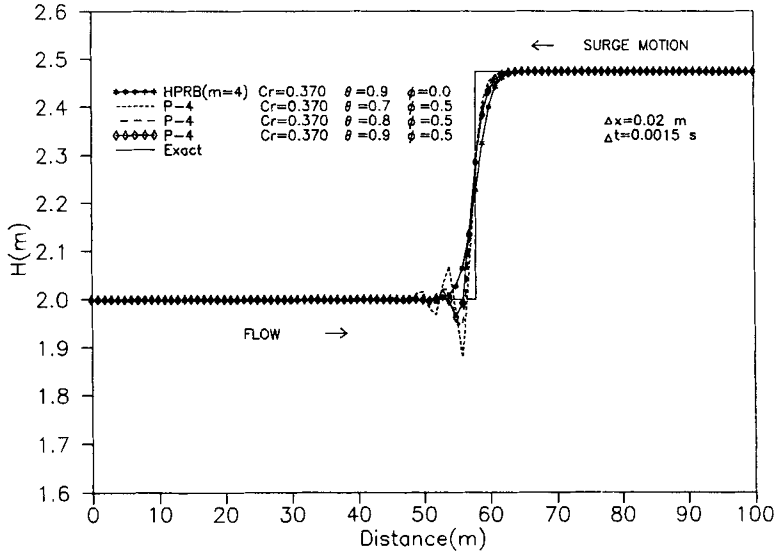

Figure 10. Comparison of hybrid HPRB method and Priessmann four-point scheme for $Cr = 0{\cdot}37$

been evaluated by comparison with the Preissman four-point scheme and the exact solution through a spontaneous surge formation case study. The conclusions can be stated as follows.

(1) For a Courant number approaching unity, the hybrid HPRB method with reach-back number $m = 1$ can give results quite close to those from the Preissmann four-point scheme. When unit Courant number is retained, a larger value of reach-back number may be required.
(2) From the test simulation it can be concluded that $\phi = 0$ will be the best choice for the hybrid HPRB method. An increase of m-value has no effect at all on the choice of ϕ-value.
(3) With the use of the Preissmann four-point scheme for the computation of rapidly varied flow, the solution stability is severely endangered when the Courant number becomes much smaller than unity. The hybrid HPRB method can avoid this instability problem as long as the proper reach-back number m can be chosen.

For practical problems such as surge propagation in a natural irregular channel, it will be very difficult if not impossible to keep the Courant number approaching unity along the river for all the computational points. Therefore it is certain that the hybrid HPRB method will be more suitable for practical applications as far as the solution stability is concerned.

APPENDIX I

Coefficients A_0–A_3

$$A_0 = q_i^{n-m}, \qquad A_1 = -qx_i^{n-m}\Delta x,$$
$$A_2 = -3q_i^{n-m} + 3q_{i-1}^{n-m} + 2qx_i^{n-m}\Delta x + qx_{i-1}^{n-m}\Delta x,$$
$$A_3 = 2q_i^{n-m} - 2q_{i-1}^{n-m} - qx_i^{n-m}\Delta x - qx_{i-1}^{n-m}\Delta x.$$

Coefficients used in equations (19)–(21)

$$A_i=\frac{1}{2\Delta t},\qquad B_i=\frac{\theta}{\Delta x},\qquad D_i=-\frac{1}{2\Delta t},$$

$$E_i=\frac{\theta}{\Delta x},\qquad G_i=\frac{q_{i-1}^{n-1}-q_i^{n-1}}{\Delta x},$$

$$A_i'=\frac{\theta}{\Delta x}\left(g(2h_i^{n-1}-h_{i-1}^{n-1})+\frac{(q_i^{n-1})^2}{(h_i^{n-1})^3}(h_i^{n-1}-h_{i-1}^{n-1})\right),$$

$$B_i'=\left(\frac{1}{m\Delta t}-2\frac{q_i^{n-1}}{(h_i^{n-1})^2}\frac{h_i^{n-1}-h_{i-1}^{n-1}}{\Delta x}\right),$$

$$D_i'=\frac{\theta}{\Delta x}\left(gh_i^{n-1}-\frac{(q_i^{n-1})^2}{(h_i^{n-1})^2}\right),\qquad E_i'=0,$$

$$G_i'=-\frac{q_i^{n-1}-q_\varepsilon}{m\Delta t}$$
$$-\left\{\theta\left(gh_i^{n-1}-\frac{(q_i^{n-1})^2}{(h_i^{n-1})^2}\right)+(1-\theta)\left[g\frac{h_{i-1}^{n-1}+h_i^{n-1}}{2}-\frac{1}{4}\left(\frac{q_i^{n-1}}{h_i^{n-1}}+\frac{q_{i-1}^{n-1}}{h_{i-1}^{n-1}}\right)^2\right]\right\}\frac{h_i^{n-1}-h_{i-1}^{n-1}}{\Delta x},$$

$$B_i''=\frac{\theta}{\Delta x},\qquad C_i''=-\frac{\theta}{2},\qquad E_i''=\frac{\theta}{\Delta x},\qquad F_i''=\frac{\theta}{2},$$

$$G_i''=\frac{qx_{i-1}^{n-1}+qx_i^{n-1}}{\Delta x}-\frac{q_i^{n-1}-q_{i-1}^{n-1}}{\Delta x}.$$

APPENDIX II: NOTATION

A_0–A_3	coefficients used in equation (14)
A_i–G_i''	coefficients used in equations (19)–(21)
Cr	Courant number
g	gravitational constant
h	flow depth
hx	spatial derivative of h
i	computational point index
m	reach-back number
n	time level
q	flow discharge per unit width
t	time
V_c	front propagation velocity
x	distance along flow direction
Δh	increment of depth
Δq	increment of discharge
Δqx	increment of discharge derivative
Δt	time interval
Δx	distance between two neighbouring computational points
α	root of cubic interpolating polynomial
ε	foot of characteristics

η head of characteristics
θ temporal weighting factor
ϕ spatial weighting factor

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