[3] I. P. Kaminow, "FSK with direct detection in optical multiple-access FDM networks," *IEEE J. Select. Areas Commun.*, vol. 8, pp. 1005-1114, 1990.
[4] A. E. Willner, E. Desurvire, H. M. Presby, C. A. Edwards, and J. R. Simpson, "Use of LD-Pumped erbium-doped fiber preamplifiers with optimal noise filtering in a FDMA-FSK 1 Gb/s star network," *IEEE Photon. Technol. Lett.*, vol. 2, no. 9, 1990.
[5] N. A. Olsson, "Lightwave systems with optical amplifiers," *J. Lightwave Technol.*, vol. 7, pp. 1071-1082, 1989.
[6] R. Ramaswami and P. A. Humblet, "Amplifier induced crosstalk in multichannel optical networks," *J. Lightwave Technol.*, vol. 8, pp. 1882-1896, 1990.
[7] A. E. Willner, "Simplified model of an FSK-to-ASK direct-detection system using a Fabry-Perot demodulator," *IEEE Photon. Technol. Lett.*, vol. 2, pp. 363-366, 1990.
[8] O. K. Tonguz and L. G. Kazovsky, "Theory of direct detection lightwave receivers using optical amplifiers," *J. Lightwave Technol.*, vol. 9, pp. 174-181, 1991.
[9] G. J. Foschini, L. J. Greenstein, and G. Vannucci, "Noncoherent detection of coherent lightwave signals corrupted by phase noise," *IEEE Trans. Commun.*, vol. 36, pp. 306-314, 1988.
[10] A. Elrefaie, M. W. Maeda, and R. Guru, "Impact of laser linewidth of optical channel spacing requirements for multichannel FSK and ASK systems," *IEEE Photon. Technol. Lett.*, vol. 1, pp. 88-90, 1989.
[11] J. Gowar, *Optical Communications Systems.* Englewood Cliffs, NJ: Prentice-Hall, 1984.
[12] K. Inoue, H. Toba, and K. Nosu, "Multichannel amplification utilizing an Er^{3+}-doped fiber amplifier," *J. Lightwave Technol.*, vol. 9, pp. 368-374, 1991.

Distributed Erbium-Doped Fiber Amplifiers with Stimulated Raman Scattering

Senfar Wen and Sien Chi

Abstract—**The effect of stimulated Raman scattering (SRS) on the distributed erbium-doped fiber amplifier with 1.48 μm pump wavelength and 1.55 μm signal wavelength is numerically studied. The contributions of the gains from SRS and the pumped erbium ions doped in the fiber core are shown. It is found that SRS enhances both the gain and signal-to-noise ratio of the amplifier except the signal-to-noise ratio may be decreased by SRS for high initial signal power.**

THE erbium-doped fiber amplifier (EDFA) has been considered as an effective optical amplifier for the fiber communication system because it has high gain and low noise, and its gain band coincides the ultralow loss regime of the wavelength near 1.55 μm [1]. The EDFA can be used as lumped amplifier or distributed amplifier. The distributed EDFA (DEDFA) has the advantage of lower amplified-spontaneous-emission noise (ASEN) power than the lumped EDFA (LEDFA) [2]. In the literature, the Raman effect [3] in the fiber was usually neglected in the theoretical studies of the EDFA. The neglection is valid for LEDFA but is invalid for the DEDFA with the length of several tens of kilometers and high pump power. In this letter, we will study the effects of SRS on the gain and signal-to-noise ratio of the DEDFA.

For the pump wavelength $\lambda_p = 1.48$ μm and the signal wavelength $\lambda_s = 1.55$ μm, the EDFA can be modelled as a two-level system [4]. The spectra of the absorption cross-section (σ_a) and emission cross-section (σ_e) of the doped Er^{3+} vary with the compositions in glass [5]. The spectra used by [6] are shown in Fig. 1 where the Raman gain (g_r) are also shown. From the figure, it is seen that the gain bandwidth is large and the spectrum of the ASEN is broadband. We consider ASEN as a number of optical beams of frequency bandwidth $\Delta\nu_k$ centered at the wavelength $\lambda_k = c/\nu_k$ to resolve the ASEN spectrum [4]. The equations to describe the spatial development of the signal power (P_s), pump power (P_p) and ASEN power (P_k, $k = 1, N$) in the erbium-doped fiber with SRS can be derived by assuming the steady-state condition for the rate equations and are given by

$$\frac{dP_s}{dz} = \left[(\alpha_{gs} + \alpha_{ls})\frac{n_2}{n_t} - \alpha_{ls} + \gamma_{rs}(P_p^+ + P_p^-) - \alpha_{is}\right]P_s, \tag{1}$$

$$u^{\pm}\frac{dP_p^{\pm}}{dz} = \left[(\alpha_{gp} + \alpha_{lp})\frac{n_2}{n_t} - \alpha_{lp} - \frac{\lambda_s}{\lambda_p}\gamma_{rs}P_s - \sum_{k=1}^{N}\frac{\lambda_k}{\lambda_p}\gamma_{rk}(P_k^+ + P_k^-) - \alpha_{ip}\right]P_p^{\pm} \tag{2}$$

$$u^{\pm}\frac{dP_k^{\pm}}{dz} = \left[(\alpha_{gk} + \alpha_{lk})\frac{n_2}{n_t} - \alpha_{lk} + \gamma_{rk}(P_p^+ + P_p^-) - \alpha_{ik}\right]P_k^{\pm} + \left[\alpha_{gk}\frac{n_2}{n_t} + \gamma_{rk}(P_p^+ + P_p^-)\right]mh\nu_k\Delta\nu_k \tag{3}$$

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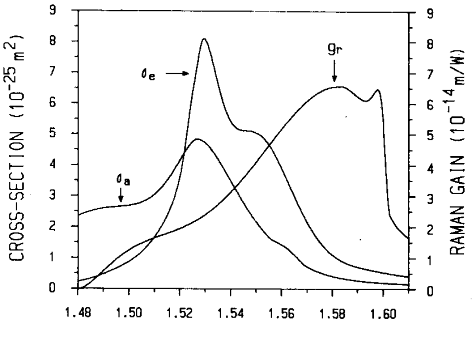

Fig. 1. Spectra of the absorption cross-section σ_a and emission cross-section σ_e of Er^{3+} in the erbium-doped fiber and the spectrum of the Raman gain g_r with the pump wavelength at 1.48 μm.

where

$$\frac{n_2}{n_t} = \left[\frac{P_2}{p_s^{th}} + \frac{P_p^+ + P_p^-}{P_p^{th}} + \sum_{k=1}^{N} \frac{P_k^+ + P_k^-}{P_k^{th}}\right] \Bigg/ \left[1 + \frac{P_s}{P_s^{is}} + \frac{P_p^+ + P_p^-}{P_p^{is}} + \sum_{k=1}^{N} \frac{P_k^+ + P_k^-}{P_k^{is}}\right] \quad (4)$$

is the ratio of the population density of the upper state n_2 and the Er^{3+} density n_t. For the two-level system, $n_t = n_1 + n_2$, where n_1 is the population density of the lower state. The superscript + (−) designates the optical beam propagating along + (−) z direction and $u^+ = 1$, $u^- = -1$. In (3), $m = 2$ represents the two polarization modes of the ASEN. The coefficients in (1)–(4) are $\alpha_{gk} = \sigma_{ek}\Gamma_k n_t$, $\alpha_{lk} = \sigma_{ak}\Gamma_k n_t$, $P_k^{th} = A_e h\nu_k / \sigma_{ak}\Gamma_k\tau$, $P_k^{is} = A_e h\nu_k / (\sigma_{ak} + \sigma_{ek})\Gamma_k\tau$, and $\gamma_{rk} = g_{rk}/A_{ok}$ where $k = s$, p correspond to the signal and pump waves, respectively, and $k = 1$, N correspond to the ASEN with center of wavelength λ_k; α_{ik} is the intrinsic loss of the silica based host glass; $h\nu_k$ is the photon energy; τ is the spontaneous lifetime of the upper level; A_e is the effective area of the Er^{3+} in the fiber core; A_{ok} is the effective area of the optical mode with wavelength λ_k; Γ_k is the overlap integral between the dopant and optical mode. We assume a step-like Er^{3+} density profile where Er^{3+} is uniformly doped over a radius of b in the fiber core and assume Gaussian approximation for the optical mode with power spot size a_k. If the radius b is small enough that the doped Er^{3+} is uniformly pumped, then we have $A_e = \pi b^2$, $A_{ok} = \pi a_k^2$, and $\Gamma_k = 1 - \exp(-b^2/a_k^2)$.

The coupled equations (1)–(3) are numerically solved with the absorption cross-section σ_{ak} and emission cross-section σ_{ek} and Raman gain g_{rk} given in Fig. 1 and the following parameters: $\tau = 10$ ms, fiber length $L = 100$ km, $b = 1$ μm, and $\alpha_{ik} = 0.26$ dB/km, $a_k = 3.34$ μm for all k for simplicity. The initial signal power P_{s0} is launched into the fiber at $z = 0$ and the initial pump powers P_{p0} are launched into the fiber at $z = 0$ and L bidirectionally. We use 130 points to sample the ASEN spectrum, which corresponds to spacing of the $\Delta\lambda = 1$ nm.

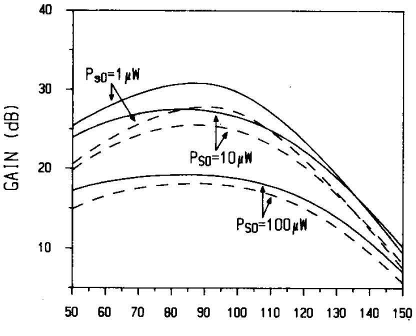

Fig. 2. Variation of the gain G of the DEDFA with respect to the Er^{3+} density n_t for the fiber length $L = 100$ km, the pump power $P_{p0} = 50$ mW, and the initial signal power $P_{s0} = 1$, 10, 100 μW. The gains of the DEDFA with and without SRS are shown by the solid and dashed lines, respectively.

Fig. 2 shows the relations of the gains G of the DEFFA and n_t for $P_{p0} = 50$ mW and $P_{s0} = 1$, 10, 100 μW. The gain G is defined as $G = 10 \times \log(P_s(L)/P_s(0))$ by convention, which is the net gain of the amplifier having the intrinsic loss and the gains from the pumped erbium ions and SRS. It is seen that G decreases as P_{s0} increases and there exists an optimum n_t to obtain the maximum gain for each case. In the figures of this paper, the corresponding data of the DEDFA without SRS ($g_r = 0$) are also shown by the dashed lines for comparison. It is found that the gain of the DEDFA with SRS is always larger than the gain without SRS for the same P_{s0}. For $P_{s0} = 1$ μW, the optimum n_t for the DEDFA without SRS is 90.3×10^{20} m^{-3} and the maximum gain is 27.8 dB; while the optimum n_t of the DEDFA with SRS is decreased to 86.9×10^{20} m^{-3} and the maximum gain is increased to 30.8 dB. There is 3 dB improvement of the optimum gain for the DEDFA with SRS to compare with the optimum gain of the DEDFA without SRS in this case. The gain enhancement due to SRS can be reasoned by studing the variations of the gain coefficients in (1). The gain coefficient from the pumped Er^{3+} is $\alpha_{es} = (\alpha_{gs} + \alpha_{ls})n_2/n_t - \alpha_{ls}$ and the gain coefficient from SRS is $\alpha_{rs} = \gamma_{rs}(P_p^+ + P_p^-)$, and the total gain coefficient is $\alpha_{ts} = \alpha_{es} + \alpha_{rs}$. For the DEDFA without SRS, $\alpha_{rs} = 0$. The gains from the two effects are the integrations of the gain coefficients over the fiber length. Fig. 3 shows the variations of the gain coefficients along the fiber with $P_{s0} = 1$ μW and $n_t = 90 \times 10^{20}$ m^{-3}. The gain coefficient α_{es} of the DEDFA without SRS is shown by the dashed line and the intrinsic loss coefficient α_{is} is also shown for reference. It is seen that the gain coefficient from SRS is significant at the beginning but decays quickly due to not enough pump power. The gain coefficient from the pumped Er^{3+} is lower than the case without SRS due to part of the pump power is depleted by SRS. The total gain coefficient of the DEDFA with SRS is larger than the gain coefficient of the DEDFA without SRS except in the middle of the transmission length. Although the gain from the pumped Er^{3+} is reduced due to part of the pump powers is depleted by SRS, the gain from SRS is larger than the reduction of the gain from the pumped Er^{3+}. In this case, there is 2.95 dB

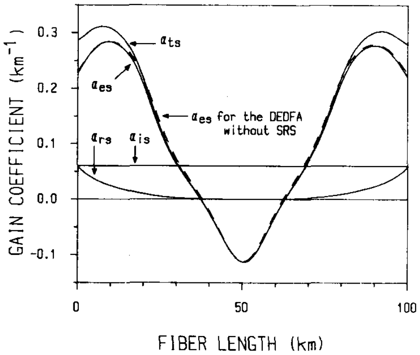

Fig. 3. Variations of the gain coefficients α_{ts}, α_{es}, and α_{rs} of the DEDFA along the fiber with the fiber length L = 100 km, Er^{3+} density $n_t = 90 \times 10^{20}$ m^{-3}, the pump power P_{p0} = 50 mW and initial signal power P_{s0} = 1 μW. The gain coefficients of the DEDFA with and without SRS are shown by the solid and dashed lines, respectively. The intrinsic loss coefficient α_{is} is shown for reference.

gain enhancement. It is found that when the effect of SRS increases, e.g., increasing the pump power or reducing the effective optical mode area A_{ok} in (4), the gain enhancement is improved.

Fig. 4 shows the relations of G and P_{p0} for $n_t = 90 \times 10^{20}$ m^{-3} and P_{s0} = 1, 10, and 100 μW. The gain decreases as P_{s0} increases. It is seen that the gain of the DEDFA with SRS is larger than the gain of the DEDFA without SRS for the same P_{s0} and the gain enhancement increases as the pump power. It is noticed that the gains are saturated for the pump power larger than about 50 mW for the both cases, but the slope of the gain of the DEDFA with SRS is larger than the DEDFA without SRS. This indicates that the gain enhancement due to SRS is more significant for higher pump power. Although the gain is enhanced by SRS, the power of the ASEN is also enhanced. Using the bandpass optical filter with center of wavelength at $\lambda = \lambda_s$ and bandwidth $\Delta\lambda_f$ = 0.2 nm at the end of the fiber, we define the signal-to-noise ratio (SNR) as the ratio of the output signal power and the output ASEN power and SNR $= 10 \times \log(\Delta\lambda P_s(L)/\Delta\lambda_f P_n^+(L))$ where $P_n^+(L)$ is the power of the forward ASEN with bandwidth $\Delta\lambda$ centered at λ_s. Fig. 5 shows the SNR in Fig. 4. It is seen that SNR is enhanced for the DEDFA with SRS to compare with the DEDFA without SRS for P_{s0} = 1 and 10 μW. For the case with P_{s0} = 1 μW and P_{p0} = 50 mW, SNR is enhanced 0.57 dB. From the figure, SNR increases as the pump power and is saturated for the pump power larger than about 40 mW. When the pump power is high, the signal is amplified in about the same rate as the ASEN and the improvement of the SNR is little. As P_{s0} increases, SNR increases but the enhancement of SNR by SRS decreases. In Fig. 5, for the case with P_{s0} = 100 μW, SNR is not only enhanced less than the other two cases, but decreased by SRS for 46 mW $< P_{p0} <$ 85 mW. However, the decrement of SNR is small and is not more than 0.1 dB. The gain of a distributed amplifier is usually designed to be zero where the fiber loss is just compensated. From Fig. 4, the required pump powers for the zero gain of the DEDFA for different P_{s0} are around 25 mW and the corresponding gain enhancements due to SRS are about 2 dB. However, to obtain high SNR for low initial signal power, higher pump power is required as indicated in Fig. 5.

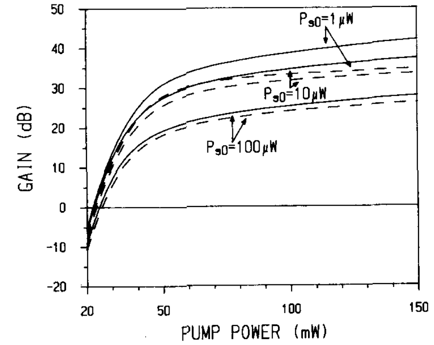

Fig. 4. Variation of the gain G of the DEDFA with respect to the pump power P_{p0} for the Er^{3+} density $n_t = 90 \times 10^{20}$ m^{-3}, the fiber length L = 100 km, and the initial signal power P_{s0} = 1, 10, 100 μW. The gains of the DEDFA with and without SRS are shown by the solid and dashed lines, respectively.

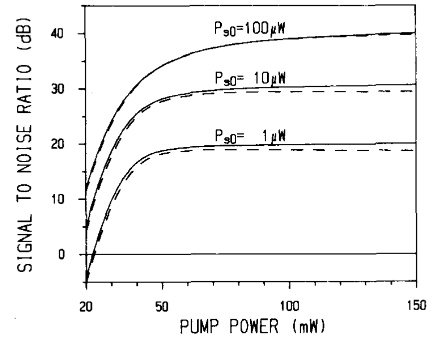

Fig. 5. Variation of the signal-to-noise ratio SNR of the DEDFA with respect to the pump power P_{p0} for the Er^{3+} density $n_t = 90 \times 10^{20}$ m^{-3}, the fiber length L = 100 km, and the initial signal power P_{s0} = 1, 10, 100 μW. the SNR's of the DEDFA with and without SRS are shown by the solid and dashed lines, respectively.

In conclusion, we have considered the distributed erbium-doped fiber amplifier with stimulated Raman scattering. The pump wavelength is 1.48 μm and the signal wavelength is 1.55 μm for the amplifier. The contributions of the gains from SRS and the pumped Er^{3+} in the fiber core are shown. We have found that both the gain and SNR of the amplifier is enhanced by SRS except SNR may be decreased by SRS for high initial signal power.

REFERENCES

[1] R. J. Mears, L. Reekie, I. M. Jauncey, and D. N. Payne, "Low-noise erbium-doped fiber amplifier operating at 1.54 μm," *Electron. Lett.*, vol. 23, pp. 1026-1028, 1987.

[2] C. R. Giles and E. Desurvire, "Propagation of signal and noise in concatenated erbium-doped fiber optical amplifiers," *J. Lightwave Technol.*, vol. 9, pp. 147-154, 1991.

[3] R. H. Stolen and E. P. Ippen, "Raman gain in glass optical waveguides," *Appl. Phys. Lett.*, vol. 22, pp. 276-278, 1972.

[4] C. R. Giles and E. Desurvire, "Modeling erbium-doped fiber amplifiers," *J. Lightwave Technol.*, vol. 9, pp. 271-283, 1991.

[5] W. J. Miniscalco, "Erbium-doped glasses for Fiber amplifiers at 1500 nm," *J. Lightwave Technol.*, vol. 9, pp. 234-250, 1991.

[6] E. Desurvire and J. R. Simpson, "Amplification of spontaneous emission in erbium-doped single-mode fibers," *J. Lightwave Technol.*, vol. 7, pp. 835-845, 1989.

Bending Loss Effect on Signal Gain in an Er^{3+}-Doped Fiber Amplifier

Masaharu Ohashi and Kazuyuki Shiraki

***Abstract*—This letter gives a theoretical description of the bending loss effect on signal gain in an optical fiber amplifier when Er^{3+}-doped fiber is wound on a bobbin. The dependence of signal gain on bending radius for an Er^{3+}-doped fiber amplifier pumped by a 1.48 μm laser diode is determined using the rate equation for a three-level laser system, taking the bending property into account. The fiber parameters necessary to achieve maximum signal gain are shown to be dependent on the bending loss.**

I. INTRODUCTION

Er^{3+}-doped fiber is an attractive medium for optical signal amplification in the 1.5 μm wavelength region [1], [2]. A lot of effort has been concentrated on demonstrating optical transmission systems using Er^{3+}-doped fiber amplifiers [3]-[5]. As the Er^{3+}-doped fiber amplifier has many advantages, such as high-gain, wide bandwidth, low-noise, and polarization independence, it is widely used as if it were a required optical component. Therefore, it is necessary to design and fabricate an Er^{3+}-doped fiber amplifier module for practical use. The design and fabrication of such a module has been reported [6]. There have been few reports, however, on how the bending loss influences signal gain in Er^{3+}-doped fiber amplifiers [7].

This letter investigates theoretically the relationship between the fiber parameters of an Er^{3+}-doped fiber with a step-index profile and the signal gain for an Er^{3+}-doped fiber amplifier pumped by a 1.48 μm laser diode, using the rate equation for a three-level laser system when the Er^{3+}-doped fiber is wound on a bobbin. The bending loss effect on the signal gain is shown in relation to the fiber parameters.

II. BASIC BACKGROUND

The three-level laser system approximation is justified for Er^{3+}: glass fibers as shown by Desurvire *et al.* [8], [9]. Fiber amplifiers are analyzed in [8]-[10].

Let $P_s^+(z, \nu)$ and $P_s^-(z, \nu)$ be the forward and backward optical powers at frequency ν in a frequency interval $\Delta\nu$, and at longitudinal fiber coordinate z. They correspond to the signal powers that propagate in opposite directions in the fiber. When Er^{3+}-doped fiber is wound on a bobbin with a radius of R, the steady-state variation of $P_s^\pm(z, \nu)$ along the fiber follows the equation [9], [10]:

$$\frac{dP_s^\pm(z,\nu)}{dz} = \pm\left\{\gamma_e(z,\nu)\left[P_s^\pm(z,\nu) + P_0\right] - \left[\gamma_a(z,\nu) + \alpha_b(\nu)\right]P_s^\pm(z,\nu)\right\} \quad (1)$$

where $\gamma_e(z, \nu)$ and $\gamma_a(z, \nu)$ are the spectral emission and absorption coefficients, respectively. α_b denotes the bending loss coefficient with a bending radius of R. P_0 is the power of two photons per unit frequency in bandwidth $\Delta\nu$, and corresponds to the spontaneous emission in the two polarization modes of the fiber [9] as follows:

$$P_0 = 2h\nu_s\Delta\nu. \quad (2)$$

The length dependence of the pump power is expressed as

$$\frac{dP_p(z)}{dz} = -\left[\gamma_p(z) + \alpha_b\right]P_p(z) \quad (3)$$

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