Analysis of Beam Size Effect on the Laser Induced Fréedericksz Transition and the Dynamic Response in Nematie Liquid Crystal (5CB) Films with a Free Surface

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The non-local effect caused by the finite laser beam size in the laser induced Fréedericksz transition for the homeotropically aligned nematic liquid crystal film is derived analytically and calculated numerically. The influences on the transition threshold and the dynamic response are discussed. The coupling between backflow effect and the non-local effect is shown to be important for the films with a free surface by the calculated results and by comparing with the existing experimental results.

I. INTRODUCTION

Recently, the turn off times of molecular reorientation of nematic liquid crystal (NLC) films have been studied in several works.[1-3] The films with a free surface (FS) and the films sandwiched between two glasses, i.e., the so-called hard boundaries (HB) films, have been shown having different turn off times in both of the laser induced Freedericksz transition work' and the magnetic field induced Freedericksz work.' The backflow effect and the different flow boundary conditions are considered as the cause of the difference. A careful derivation has been given in Ref. 2 for a uniform magnetic field. However in a more detail measurement on the turn off time caused by varying the intensity of incidental Ar^+ laser light,[3] it is found that the difference can not be explained by the backflow effect only. Although several studies of the laser-induced molecular reorientation in nematic liquid crystal films have shown that the non-local effect is significant.[4,5] A pure non-local effect correction, where the angle variation on transverses direction caused by the finite laser beam size is considered, can not explain the difference satisfactorily, either. In this work, we start with the original equation of motion of molecular orientation under a laser field with a Gaussian distribution, then the contribution on

non-local effect, backflow effect,[6] which is caused by the coupling of orientational motion and translational motion, and particularly the coupling of these two effects are derived and discussed.

II. EQUATION OF MOTION

For comparing with experiments,[3] we consider homeotropical NLC films with thickness ranging from 150 to 300 μm and a linearly polarized Ar$^+$ laser beam being normally incidental upon the films. The laser beam intensity at the NLC sample is assumed having a symmetrical Gaussian profile, i.e., $I(\rho) = I_0 \exp(-\rho^2/w^2)$, with 2w equal to 500 μm. We set z-axis along the light propagation direction, and the polarization direction of the laser beam along the x-axis, the director $\hat{n}$ is $(n_x, 0, n_z) = (\sin\theta, 0, \cos\theta)$. Here the orientation angle of NLC director, θ, is a function of x, y and z to reflect the finite-size effect on the molecular reorientation.

Following the Ericksen and Leslie's continuum theory of NLC,[7] and using the one constant approximation, the coupled differential equations of motion for small angle θ are obtained as:

$$C^{-1}I\theta + K\left(\frac{\partial^2}{\partial z^2} + \nabla_t^2\right)\theta - \gamma_1\dot{\theta} - \alpha_2\frac{\partial}{\partial z}v_x = 0, \tag{1a}$$

$$\alpha_2\frac{\partial}{\partial z}\dot{\theta} + \eta_c\frac{\partial^2}{\partial z^2}v_x + \frac{1}{2}\alpha_4\nabla_t^2 v_x = 0, \tag{1b}$$

where K is the elastic constant, $C^{-1} \equiv n_o'[1-(n_o'/n_e')^2]/c$, c is the speed of light, n_o' and n_e' are respectively the ordinary and the extraordinary refractive index of NLC with respect to the pump beam wavelength, γ_1 is the viscosity coefficient for molecular rotation, $\eta_c \equiv 1/2(\alpha_4 + \alpha_5 - \alpha_2)$, $\gamma_1 \equiv (\alpha_3-\alpha_2)$ and $\alpha_1, \alpha_2..., \alpha_6$ are the various viscosity coefficients coupling the rotational and translational motion, and ∇_t^2 is the Laplacian operator in the transverse plane. The term $1/2\alpha_4\nabla_t^2 v_x$ in Eq. (1b) shows the coupling between the backflow effect and the non-local effect. The detailed derivations are given in the appendix. For neatness of equations, the partial derivatives with respect to spatial coordinates will be denoted by ∂_α or with ",α" in subscript. For example, $\partial_z^2 \equiv \partial^2/\partial z^2$, and $v_{x,z} \equiv \partial/\partial z\, v_x$.

III. THRESHOLD INTENSITY, I_{th}

Here, we derive the threshold laser intensity, I_{th}, of the laser induced Freedericksz transition for a film with thickness, d. Only when I_o is greater than I_{th}, the molecular angle $\theta(z)$ is not zero.

In the steady state, $\theta = v_x = 0$, Eq. (1) becomes:

$$C^{-1}I(\rho)\theta(\rho, \phi, z) + K(\partial_z^2 + \nabla_t^2)\theta(\rho, \phi, z) = 0, \tag{2}$$

where the cylindrical coordinates (ρ, ϕ, z) are used.

Due to the radial symmetry of the laser intensity and the one-constant approximation, θ is independent of ϕ, i.e., $\theta = \theta(z, \rho)$. Using $\nabla_t^2 = \partial^2/\partial\rho^2 + 1/\rho \, \partial/\partial\rho \equiv \nabla_\rho^2$, Eq. (2) becomes:

$$C^{-1}I(\rho)\theta(\rho, z) + K(\partial_z^2 + \nabla_\rho^2)\theta(\rho, z) = 0. \tag{3}$$

With the variable separation technique and setting $\theta(\rho, z) = R(\rho)Z(z)$, we get:

$$\partial_z^2 Z(z) = -q^2 Z(z), \tag{4a}$$

$$\nabla_\rho^2 R(\rho) + \left[\frac{1}{CK}I(\rho) - q^2\right]R(\rho) = 0, \tag{4b}$$

where q is a constant. For films with a strong homeotropical anchoring[1,2] on the free surface, the boundary conditions for both of the FS samples and the HB samples are $\theta(z = \pm d/2) = 0$, thus from Eq. (4a), we obtain

$$Z(z) = \cos(qz), \tag{5}$$

with $q = \pi/d$ for the lowest order solution, which corresponds to a small angle variation and thus a small energy increasing.

In general, Eq. (4b) can not be solved analytically for a Gaussian type of $I(\rho)$. However, L. Csillag et al.[5] have introduced an approximation for the Gaussian beam profile and derived the threshold analytically. We summarize their results at follows. In order to obtain the analytic solution, they replaced the Gaussian profile by an effective profile?

$$I(\rho) = \begin{cases} I_o, & \text{for } \rho \le w, \\ 0, & \text{for } \rho > w, \end{cases} \tag{6}$$

which keeps the power $P = I_o(\pi w^2)$ unchanged. In this situation, the non-trivial solution of Eq. (4b) with the boundary conditions $R(\rho = 0)$ = finite, $R'(\rho = 0)$ =0 and $R(\rho = \infty) = 0$ is:[5]

$$R(\rho) = \begin{cases} \theta_o J_o(\lambda\rho), & \rho < w, \\ \theta_o \beta K_o(q\rho), & \rho > w, \end{cases}$$

where J_o and K_o are the zeroth-order Bessel-function and modified Hankel-function, respectively, β is a constant, and $\lambda^2 = (I_o/CK)-(\pi/d)^2$. Solutions of physical meaning are obtained only with A $\ge$ 0, which implies the existence of a certain threshold. The threshold intensity is then expressed in an empirical form,

$$I_{th} = I_{tho}(1 + b^2 k^{2(m-1)}), \tag{7}$$

where $I_{tho} = CK(\pi/d)^2$, $k = \pi w/d$, and the fitted parameters are $b = 1.43$ and $m = 0.24$.'

Eq. (4b) with the original Gaussian laser beam profile can be solved easily by numerical method' for the same boundary conditions without the approximation of Eq. (6) for given w and

d. The numerical solution of the maximum molecular reorientation angle θ_o of Eq. (4b) as a function of the laser intensity I,, for w = 250 μm and d = 200 μm is shown in Fig. (la) as an example. The existence of threshold is obvious from the curve. In Fig. (lb) we show some of
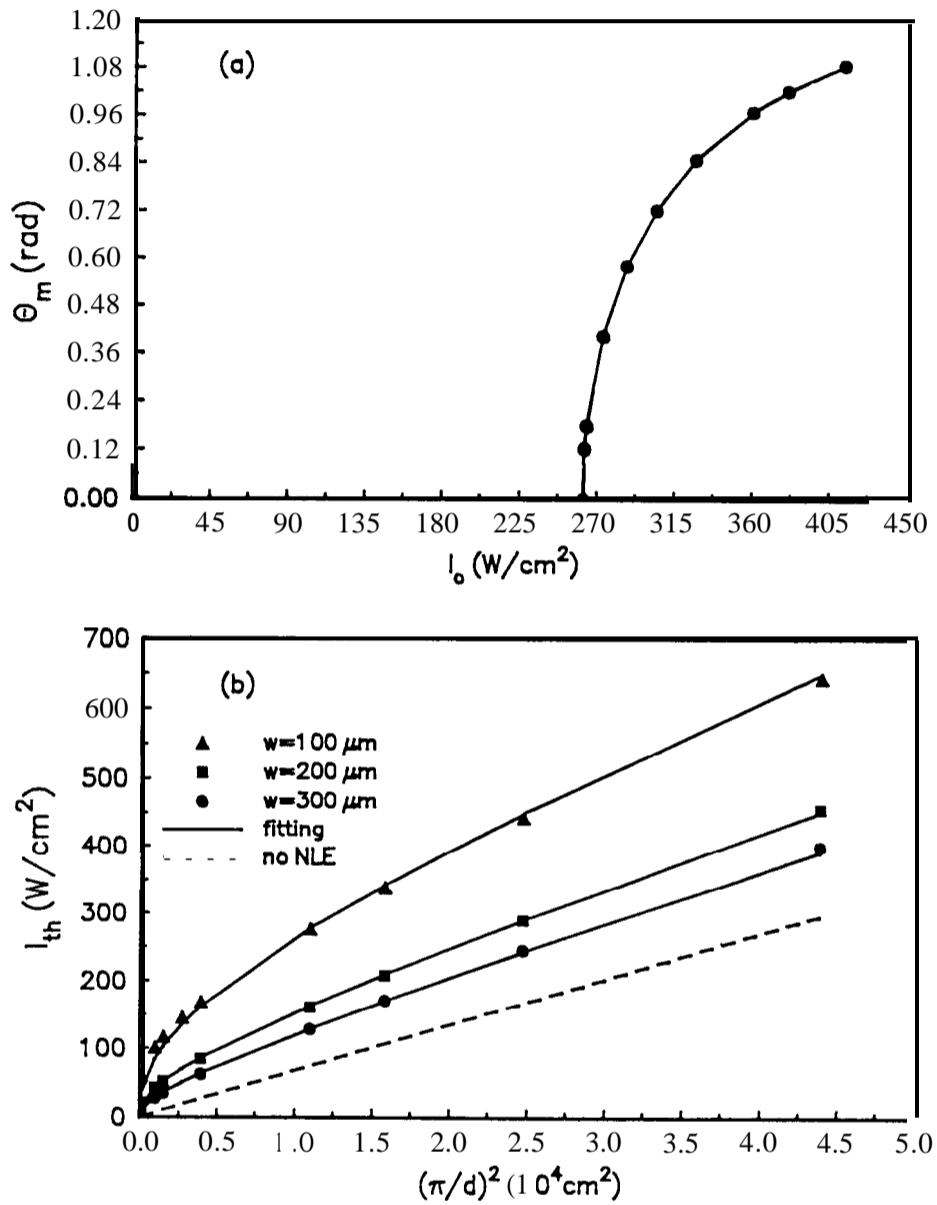

FIG. 1.   (a) The numerical solution of the maximum molecular reorientation angle θ_o as a function of the laser intensity&, for a Gaussian profile intensity with w = 250 μm and d = 200 μm; (b) the threshold intensity (I_{th}) versus inverse of square of film thickness $(1/d^2)$ for w = 100, 200, and 300 μm, respectively. Solution without non-local effect is shown with dashed line.

these I_{th} as a function of $(\pi/d)^2$, for $w = 100, 200$, and 300 μm, respectively. Here we have used $n_o' = 1.54$, $n_e' = 1.73$," and $K = 0.72 \times 10^{-6}$ dyne." Fitting these points of numerical results with same form as in Eq. (7), we found that b and m are 1.7 and 0.4, respectively. Thus the threshold intensity estimated with approximation in Eq. (6) is smaller than that with a Gaussian profile by a factor about 1.26, for the cases with $w \cong 250$ μm and d ranging from 150 to 300 μm. However, Eq. (7) is still a good expression for I_{th} only that the values for b and m are readjusted as mentioned.

Now, Eq. (7) can be rewritten as

$$I_{th} = CK^*(\pi/d)^2, \tag{8}$$

where $K^* = K[1 + b^2 k^{2(m}-1)]$, $b = 1.7$, and $m = 0.4$. This the same form for the threshold for a infinite large beam, I_{tho}, only that an effective elastic constant K^* is used instead of the true elastic constant K. Since K^* is greater than K, the non-local effect makes the threshold intensity increased.

IV. DYNAMIC RESPONSE TIME CONSTANTS, τ_{on} AND τ_{off}

If the laser intensity I_0 is changed abruptly from an initial intensity I_1 smaller than I_{th} to an intensity I_0 larger than I_{th}, the molecules will begin to rotate, we define the beginning exponential time constant as the turn-on time constant, τ_{on}. Similarly, if the laser intensity is changed from I_2 larger than I_{th} to I_0 smaller than I_{th}, the molecular orientation will relax to θ = ' 0, the turn-off time constant, τ_{off}, is defined as the exponential time constant at the end. In both cases, the time constant are evaluated at small angles, and $\theta \propto e^{t/\tau}$. However τ_{on} is positive while τ_{off} is negative. In this chapter, we derive these time constants as a function of laser intensity I_o.

Again, using variable separation:

$$\theta(t, \rho, z) = \theta_1(t, z)R_\theta(\rho), \tag{9a}$$

$$V_x(t, \rho, z) = v_1(t, z)R_v(\rho), \tag{9b}$$

and substituting into Eq. (1), one obtains:

$$C^{-1}I(\rho)\theta_1 R_\theta(\rho) + K[\partial_z^2(\theta_1)R_\theta(\rho) + \theta_1\nabla_\rho^2 R_\theta(\rho)]$$
$$-\gamma_1\dot{\theta}_1 R_\theta(\rho) - \alpha_2 v_{1,z}R_v(\rho) = 0, \tag{10a}$$

$$\alpha_2(\dot{\theta}_1)_{,z}R_\theta(\rho) + \eta_c\partial_z^2(v_1)R_v(\rho) + \frac{1}{2}\alpha_4 v_1\nabla_\rho^2 R_v(\rho) = 0. \tag{10b}$$

Here we still use the approximation as in Eq. (6) but with the modified values of b and m in I_{th}, then $R_\theta(\rho), \nabla_\rho^2 R_\theta(\rho), R_v(\rho)$ and $\nabla_\rho^2 R_v(\rho)$ are same functions of p except with different constant

factors. Here we also assume that $R_\theta(\rho)$ and $R_v(\rho)$ have the same form of $R(\rho)$ which is the steady state solution when I_o is just above I_{th}. Eq. (4b) becomes:

$$\nabla_\rho^2 R(\rho) + a^2 R(\rho) = 0, \tag{11}$$

where $a^2 \equiv (b/w)^2 (\pi w/d)^{2m}$. Eq. (10) is then simplified as:

$$C^{-1} I_o \theta_1 + K[\partial_z^2 - a^2]\theta_1 - \gamma_1 \dot{\theta}_1 - \alpha_2 v_{1,z} = 0, \tag{12a}$$

$$\partial_z [\alpha_2 \theta_1 + \eta_c v_{1,z}] - \frac{1}{2}\alpha_4 a^2 v_1 = 0. \tag{12b}$$

For HB case, the following trial solution is used to satisfy the boundary conditions $\theta(z = \pm d/2) = v_x(z = \pm d/2) = 0,$[12]

$$\theta_1(t, z) = \theta_o \cos(qz)e^{t/\tau}, \tag{13a}$$

$$v_1(t, z) = v_o[\sin(qz) - 2z/d]e^{t/\tau}. \tag{13b}$$

Substituting into Eq. (12), one obtains:

$$[C^{-1} I_o - K(q^2 + a^2) - \frac{\gamma_1}{\tau}]\theta_o \cos(qz) - \alpha_2[q\cos(qz) - \frac{2}{d}]v_o = 0, \tag{14a}$$

$$-\frac{\alpha_2}{\tau}q\theta_o \sin(qz) - \eta_c q^2 v_o \sin(qz) - \frac{1}{2}\alpha_4 a^2 v_o[\sin(qz) - \frac{2z}{d}] = 0. \tag{14b}$$

The variable z is eliminated by integrating from $z = -d/2$ to $z = +d/2$, after multiplying Eqs. (14a) and (14b) with $\cos(qz)$ and $\sin(qz)$, respectively, these equations then become

$$[C^{-1} I_o - K(q^2 + a^2) - \frac{\gamma_1}{\tau}]\theta_o - \alpha_2(q - \frac{8}{\pi d})v_o = 0, \tag{15a}$$

$$-\frac{\alpha_2}{\tau}q\theta_o - [\eta_c q^2 + \frac{1}{2}\alpha_4 a^2(1 - \frac{8}{\pi^2})]v_o = 0. \tag{15b}$$

Combining the last two equations, the time constant $\tau_{L,HB}(I_o)$ is obtained as:

$$\tau_{L,HB}^{-1}(I_o) = \frac{1}{C\gamma_{1L,HB}^*}(I_o - I_{th}), \tag{16a}$$

where

$$\gamma_{1L,HB}^* \cong \gamma_1\{1 - \frac{1}{6}(\alpha_2^2/\gamma_1\eta_c)[1 + \frac{1}{12}(\alpha_4/\eta_c)(a/q)^2]^{-1}\}. \tag{16b}$$

The subscript L denotes the finite " Laser" beam size effect to distinguish from a uniform field.
For FS sample, the boundary conditions are:[12]

$$\theta = v_x = 0, \text{ at } z = -d/2; \quad \theta = v_{x,z} = 0, \text{ at } z = +d/2. \tag{17}$$

To satisfy these boundary conditions, we choose the trial solutions for Eq. (12) as

$$\theta_1(t,z) = \theta_o \cos(qz) e^{t/\tau}, \tag{18a}$$

$$v_1(t,z) = v_o[1 + \sin(qz)] e^{t/\tau}. \tag{18b}$$

By similar work, the response time constant of FS samples, $\tau_{L,FS}(I_o)$, is obtained as

$$\tau_{L,FS}^{-1}(I_o) = \frac{1}{C\gamma_{1L,FS}^*}(I_o - I_{th}), \tag{19a}$$

where

$$\gamma_{1L,FS}^* \cong \gamma_1\{1 - (\alpha_2^2/\gamma_1\eta_c)[1 + \frac{1}{2}(\alpha_4/\eta_c)(a/q)^2]^{-1}\}. \tag{19b}$$

If the non-local effect is neglected, then a = 0, and $\gamma^*_{1L,HB}$ reduces to $\gamma^*_{1,HB} = \gamma_1[1 - 1/6(\alpha_2^2/\gamma_1\eta_c)]$ and $\gamma^*_{1L,FS}$ reduces to $\gamma^*_{1,FS} = \gamma_1(1 - \alpha_2^2/\gamma_1\eta_c)$, which are the effective viscosity coefficients in the uniform field. Thus the non-local effect make the effective viscosity coefficient increased.

According to our definitions, both τ_{on} and τ_{off} are with small molecular orientation. Therefore Eqs. (16a) and (19a) can be used for both cases.

If the applied laser intensity is reduced to zero from I_1, the zero intensity turn off rate $\tau_L^{-1}(0)$ from Eqs. (16a and 19a) is,

$$\tau_{L,1}^{-1}(0) = -(K^*/\gamma_{1L,i}^*)(\pi^2/d^2), \tag{20}$$

where $\gamma^*_{1L,i} = \gamma^*_{1L,HB}$ (or $\gamma^*_{1L,FS}$) for HB (or FS) samples. For the case of uniform field, $\tau^{-1}(0) = -(K/\gamma^*_1)(\pi^2/d^2)$, with $\gamma^*_1 = \gamma^*_{1,HB} = \gamma^*_{1,FS}$ for HB and FS samples, respectively.

V. RESULTS

Here we present our calculated results for various situations. Using the parameters for 5CB as in previous sections, the threshold intensity of the case without non-local effect, I_{tho}, as a function of $1/d^2$ is also shown in Fig. (1b) by a dashed line. The non-local effect on the threshold intensity is obvious from this figure.

For comparing the non-local effect and the backflow effect on the dynamic properties, the time constants are calculated with Eq. (16a) and (19a) for the following 5 situations, and the effective elastic constants and effective viscosities used are shown in the parenthesis:

(1) neglecting both of the two effects, (K, γ_1);

(2) including only non-local effect, (K^*, γ_1);

(3) including only backflow effect, (K, γ^*_1);

(4) including both the two effects, but neglecting the non-local effect on the effective viscosity, (K^*, γ^*_1);

and

(5) including both the two effects and the coupling of these two effects, which is the non-local effect on the effective viscosity, (K^*, γ^*_{1L}).

Beside the parameters mentioned earlier, the other parameters for 5CB used are: $\gamma_1 = 0.68$ P, $\alpha_2 = -0.723$ P, $\alpha_4 = 0.8$ P, $\eta_c = 1.53\, p$,[2] and $d = 200\,\mu m$.

The calculated results for $1/\tau$ versus intensity I_o are shown in Fig. 2(a) for FS case. The long dashed line is for situation (1), where neither non-local effect nor backflow effect is considered. The short dashed line is for situation (2), it showed that the pure non-local effect would shift line 1 to the right for a constant value leaving the slope unchanged. The dotted line is for situation (3), it shows a pure backflow effect makes the slope steeper but leaves the threshold unchanged. The alternate dashed line shows the combination of situation (2) and (3), the slope is same as for a pure backflow effect, but the threshold is shifted as a pure non-local effect. With the solid line, the coupling of non-local effect and the backflow effect is added and the line is less steeper than lines 3 and 4 but still steeper than lines 1 and 2.

For the HB case, the relations of these five situations are similar to FS as shown in Fig. 2(b). The shift of threshold is the same as for FS film. However, the influence of backflow is smaller, the coupling with non-local effect is even less significant. Line 4 is almost overlapping with line 5. The coupling effect can be neglected for HB case, and the equations of motion can be simplified once more by dropping the transverse Laplacian term in Eq. (1b).

VI. CONCLUSION

The response time of the free surface film is less than that for the hard boundaries film. When consider backflow only, the ratio of the response time for HB and FS surface (which is same as to comparing the ratio of zero field turn off times at same thickness) is the same as the ratio under a uniform magnetic field.[2] Due to the non-local effect caused by the finite size of laser beam and the elastic interaction of liquid crystals, the effective viscosity caused by backflow is modified. We call this the coupling effect between non-local effect and backflow effect. Due to this coupling, we predict the measured ratio as above mentioned should be more closer to 1 than that from the uniform magnetic field experiments. Our example shows that the ratio changes from 1.8 to values between 1.55 and 1.69 for the thickness between 150 and 300 μm. The ratio is slightly dependent on the thickness due to the thickness dependence of γ^*_{1L} as in Eqs. (16) and (19). This conclusion can explain the recent experimental study on the dynamics of laser induced Freedericksz transition.[3]
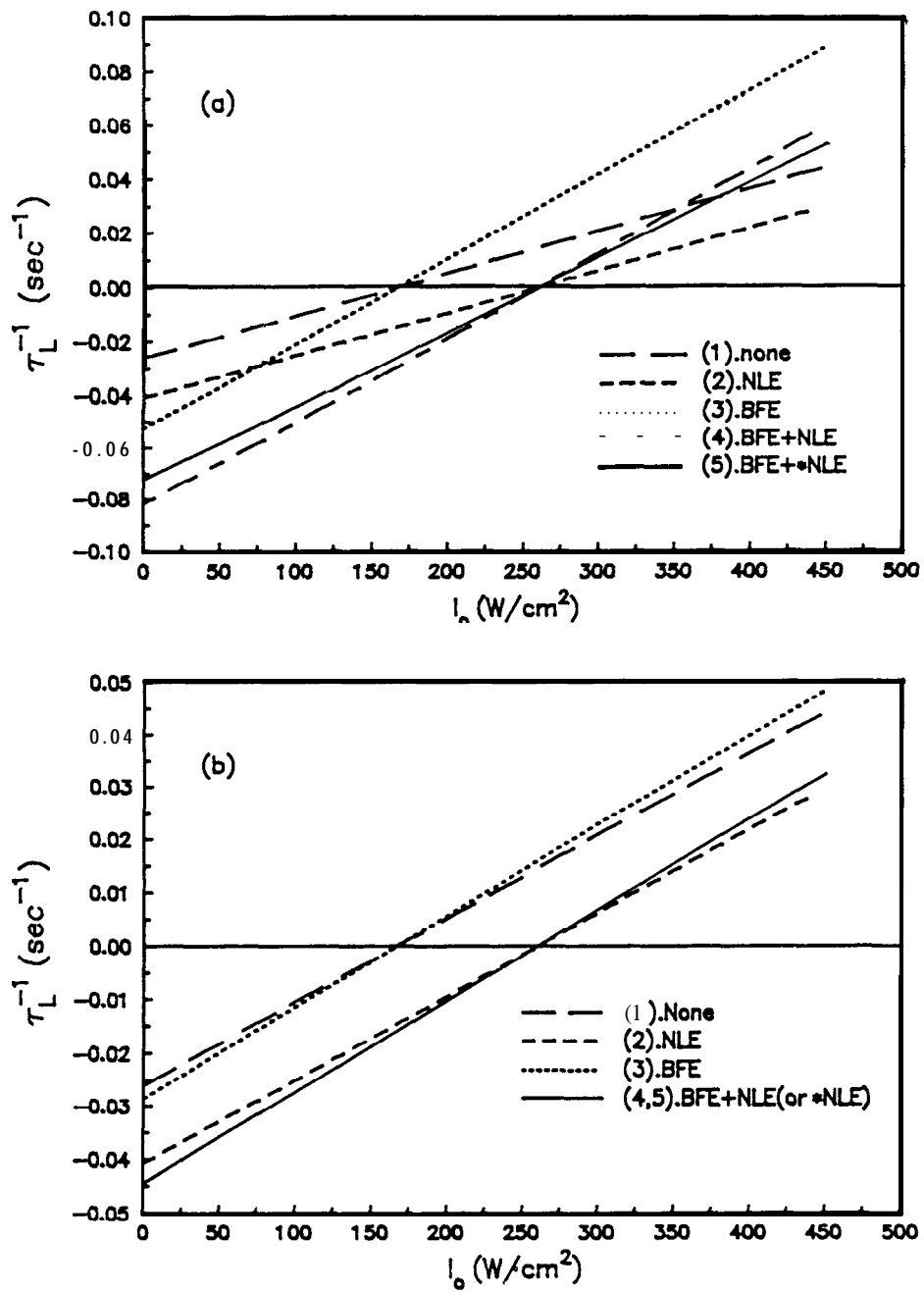
FIG. 2. Calculated dynamic response rate (l/t) as a function of the laser intensity (I_o) for 5 situations: (1) neglecting both the non-local effect (NLE) and the backflow effect (BFE); (2) including only NLE, neglecting the **BFE**; (3) including only BFE, neglecting the NLE; (4) including both the two effects, but neglecting the NLE on the effective viscosity; and (5) including both the two effects and the NLE coupling on the effective viscosity. (a) For FS case, and (b) for HB case.

ACKNOWLEDGMENTS

APPENDIX: DERIVATIONS OF THE EQUATIONS OF MOTION FOR MOLECULAR REORIENTATION INDUCED BY A LASER BEAM IN NEMATIC LIQUID CRYSTAL FILMS

We set z-axis along the light propagation direction, the polarization direction of the laser beam along x-axis, the director n then can be written as: $\hat{n} = (n_x, 0, n_z) = (\sin\theta, 0, \cos\theta)$, here θ is a function of x, y and z, to reflect the finite-size effect on the molecular reorientation. With the one constant approximation, the elastic free energy density, F, is given by:[7]

$$F = \frac{1}{2}K[n_{x,x}^2 + n_{z,z}^2 + 2n_{x,x}n_{z,z} \\ +(n_{x,y}^2 + n_{z,y}^2 + n_{z,x}^2 + n_{x,z}^2 - 2n_{x,z}n_{z,x})], \tag{A.1}$$

where K is the distortion elastic constant of NLC. The comma in subscript denotes partial differentiation w.r.t. spatial coordinates, e.g., $n_{x,z} \equiv \partial n_x/\partial z$.

The external applied body force[7] G on the director $\hat{n}$ due to the electric field E of the laser beam is: $G = \varepsilon_a/4\pi <(\hat{n}\cdot E)E>$, where ε_a is the anisotropy of the dielectric constant; $\varepsilon_a = (n_e')^2 - (n_o')^2$, n_o' and n_e' are the ordinary and the extraordinary refractive indices, respectively, and $E = (E_x, 0, E_z)$. E_x is determined by the continuity of the tangential component of E, $E_x = E_o$, where E_o is the electric vector of the incoming light beam; E_z is calculated from the condition of $z\cdot\varepsilon\cdot E = 0$, which leads to $E_z = -1/2E_o\varepsilon_a\sin2\theta/[(n_e')^2 - \varepsilon_a\sin^2\theta]$.[13] Thus,

$$\vec{G} \equiv (G_x, 0, G_z) = E_0^2 \frac{\varepsilon_a}{4\pi} \frac{(n_o')^2\sin\theta}{(n_e')^2 - \varepsilon_a\sin^2\theta}\left[1, 0, -\frac{\varepsilon_a\sin\theta\cos\theta}{(n_e')^2 - \varepsilon_a\sin^2\theta}\right]$$

Using the Ericksen and Leslie's continuum theory of NLC,[7] assuming the fluid-flow velocity $v = (v_x, v_y, v_z)$, with each component v_i, $i = x$, y, and z, being a function of x, y, and z, and neglecting the inertial effect, we can obtain six coupled partial differential equations as following:

$$G_x + y\sin\theta + K\{\cos\theta\nabla^2\theta - \sin\theta[(\theta_{,x})^2 + (\theta_{,y})^2 + (\theta_{,z})^2]\} \\ -\gamma_1\cos\theta - \alpha_2\cos\theta v_{x,z} - \alpha_3\cos\theta v_{z,x} - \gamma_2\sin\theta v_{x,x} = 0, \tag{A.2a}$$

$$\alpha_3\sin\theta v_{x,y} + \alpha_2\sin\theta v_{y,x} + \alpha_3\cos\theta v_{z,y} + \alpha_2\cos\theta v_{y,z} = 0, \tag{A.2b}$$

$$G_z + \gamma \cos \theta + K\{- \sin \theta \nabla^2 \theta - \cos \theta[(\theta_{,x})^2 + (\theta_{,y})^2 + (\theta_{,z})^2]\}$$

$$+\gamma_1 \sin \dot{\theta} - \alpha_3 \sin \theta v_{x,z} - \alpha_2 \sin \theta v_{z,x} - \gamma_2 \cos \theta v_{z,z} = 0, \tag{A.2c}$$

$$t_{xx,x} + t_{yx,y} + t_{zx,z} = 0, \tag{A.2d}$$

$$t_{xy,x} + t_{yy,y} + t_{zy,z} = 0, \tag{A.24}$$

$$t_{xz,x} + {}_{yz,y} + t_{zz,z} = 0, \tag{A.2f}$$

where

$$t_{xx} = -p - K(\theta_{,x})^2 + \frac{1}{2}\gamma_2 \sin(2\theta)\dot{\theta} + \frac{1}{4}(2\alpha_1 \sin^2 \theta - \alpha_2 - \alpha_3 + \alpha_5 + \alpha_6) \sin(2\theta)v_{x,z}$$

$$+\frac{1}{4}(2\alpha_1 \sin^2 \theta + \alpha_2 + \alpha_3 + \alpha_5 + \alpha_6) \sin(2\theta)v_{z,x}$$

$$+[\alpha_1 \sin^4 \theta + \alpha_4 + (\alpha_5 + \alpha_6) \sin^2 \theta]v_{x,x} + \frac{1}{4}\alpha_1 \sin^2(2\theta)v_{z,z}$$

$$t_{yx} = -K(\theta_{,x})(\theta_{,y}) + \frac{1}{2}[\alpha_4 + (\alpha_6 + \alpha_3) \sin^2 \theta]v_{x,y}$$

$$+\frac{1}{2}[\alpha_4 + (\alpha_6 - \alpha_3) \sin^2 \theta]v_{y,x}$$

$$+\frac{1}{4}(\alpha_6 - \alpha_3) \sin(2\theta)v_{y,z} + \frac{1}{4}(\alpha_6 + \alpha_3) \sin(2\theta)v_{z,y},$$

$$t_{zx} = -K(\theta_{,x})(\theta_{,z}) + (\alpha_2 \cos^2 \theta - \alpha_3 \sin^2 \theta)\dot{\theta}$$

$$+\frac{1}{2}(\frac{1}{2}\alpha_1 \sin^2 2\theta - \alpha_2 \cos^2 \theta + \alpha_3 \sin^2 \theta + \alpha_4 + \alpha_5 \cos^2 \theta + \alpha_6 \sin^2 \theta)v_{x,z}$$

$$+\frac{1}{2}(\frac{1}{2}\alpha_1 \sin^2 2\theta + \alpha_2 \cos^2 \theta - \alpha_3 \sin^2 \theta + \alpha_4 + \alpha_5 \cos^2 \theta + \alpha_6 \sin^2 \theta)v_{z,x}$$

$$+\frac{1}{2}(\alpha_1 \sin^2 \theta + \alpha_5) \sin(2\theta)v_{x,x} + \frac{1}{2}(\alpha_1 \cos^2 \theta + \alpha_6) \sin(2\theta)v_{z,z},$$

$$t_{xy} = -K(\theta_{,x})(\theta_{,y}) + \frac{1}{2}[\alpha_4 + (\alpha_5 + \alpha_2) \sin^2 \theta]v_{x,y}$$

$$+\frac{1}{2}[\alpha_4 + (\alpha_5 - \alpha_2) \sin^2 \theta]v_{y,x}$$

$$+\frac{1}{4}(\alpha_5 - \alpha_2) \sin(2\theta)v_{y,z} + \frac{1}{4}(\alpha_5 + \alpha_2) \sin(2\theta)v_{z,y},$$

$$t_{yy} = -p - K(\theta_{,y})^2 + \alpha_4 v_{y,y},$$

$$t_{zy} = -K(\theta_{,y})(\theta_{,z}) + \frac{1}{2}[\alpha_4 + (\alpha_5 + \alpha_2)\cos^2\theta]v_{z,y}$$

$$+\frac{1}{2}[\alpha_4 + (\alpha_5 - \alpha_2)\cos^2\theta]v_{y,z}$$

$$+\frac{1}{4}(\alpha_5 + \alpha_2)\sin(2\theta)v_{x,y} + \frac{1}{4}(\alpha_5 - \alpha_2)\sin(2\theta)v_{y,x},$$

$$t_{xz} = -K(\theta_{,x})(\theta_{,z}) + (-\alpha_2\sin^2\theta + \alpha_3\cos^2\theta)\dot{\theta}$$

$$+\frac{1}{2}(\frac{1}{2}\alpha_1\sin^2 2\theta + \alpha_2\sin^2\theta - \alpha_3\cos^2\theta + \alpha_4 + \alpha_5\sin^2\theta + \alpha_6\cos^2\theta)v_{x,z}$$

$$+\frac{1}{2}(\frac{1}{2}\alpha_1\sin^2 2\theta - \alpha_2\sin^2\theta + \alpha_3\cos^2\theta + \alpha_4 + \alpha_5\sin^2\theta + \alpha_6\cos^2\theta)v_{z,x}$$

$$+\frac{1}{2}(\alpha_1\sin^2\theta + \alpha_6)\sin(2\theta)v_{x,x} + \frac{1}{2}(\alpha_1\cos^2\theta + \alpha_5)\sin(2\theta)v_{z,z},$$

$$t_{yz} = -K(\theta_{,y})(\theta_{,z}) + \frac{1}{2}[\alpha_4 + (\alpha_6 + \alpha_3)\cos^2\theta]v_{z,y}$$

$$+\frac{1}{2}[\alpha_4 + (\alpha_6 - \alpha_3)\cos^2\theta]v_{y,z}$$

$$+\frac{1}{4}(\alpha_6 + \alpha_3)\sin(2\theta)v_{x,y} + \frac{1}{4}(\alpha_6 - \alpha_3)\sin(2\theta)v_{y,x},$$

$$t_{zz} = -p - K(\theta_{,z})^2 - \frac{1}{2}\gamma_2\sin(2\theta)\dot{\theta} + \frac{1}{4}(2\alpha_1\cos^2\theta + \alpha_2 + \alpha_3 + \alpha_5 + \alpha_6)\sin(2\theta)v_{x,z}$$

$$+\frac{1}{4}(2\alpha_1\cos^2\theta - \alpha_2 - \alpha_3 + \alpha_5 + \alpha_6)\sin(2\theta)v_{z,x}$$

$$+[\alpha_1\cos^4\theta + \alpha_4 + (\alpha_5 + \alpha_6)\cos^2\theta]v_{z,z} + \frac{1}{4}\alpha_1\sin^2(2\theta)v_{x,x}.$$

In above equations, γ and p are arbirary (indeterminate) constants, and $\gamma_1 \equiv \alpha_3 - \alpha_2, \gamma_2 \equiv \alpha_3 + \alpha_2$ and $\alpha_1,...,\alpha_6$ are the viscosity coefficients following the notations of Ref. (14).

Combining Eq. (A.2a) and Eq. (A.2c) by subtracting Eq. (A.2a) x $\cos\theta$ and Eq. (A.2c) x $\sin\theta$, one can obtain:

$$\frac{\frac{1}{2}\varepsilon_a n_o' n_e' I \sin(2\theta)}{c[(n_e')^2 - \varepsilon_a \sin^2\theta]^{3/2}} + K\nabla^2\theta - \gamma_1\dot{\theta} + \frac{1}{2}[\gamma_1 - \gamma_2\cos(2\theta)]v_{x,z}$$

$$\tag{A.3}$$

$$-\frac{1}{2}[\gamma_1 + \gamma_2\cos(2\theta)]v_{z,x} - \gamma_2\sin\theta\cos\theta(v_{x,x} - v_{z,z}) = 0.$$

where $I = (<E^2>/4\pi)(1/cn'_{eff})$, and $n'_{eff} = n_o'n_e'[(n_e')^2 - \varepsilon_a\sin^2\theta)^{-1/2}$ is the effective refractive index of the e-beam.

The equations are so complicate, it is impossible to solve these equations of motion analytically. Even using numerical approach, it would still be a terrible tedious work. To simplify these

equations, we consider the case with small angle θ, therefore we can set $\sin\theta \cong \theta$ and $\cos\theta \cong 1$, then linearize these equations by keeping only terms up to the first order in θ and v_i. The results **are**

$$C^{-1}I\theta + K\nabla^2\theta - \gamma_1\dot\theta - \alpha_2 v_{x,z} - \alpha_3 v_{z,x} = 0, \tag{A.4a}$$

$$\alpha_2(\dot\theta)_{,z} + \frac{1}{2}[(\alpha_5 - \alpha_2)v_{x,z,z} + (\alpha_5 + \alpha_2)v_{z,x,z}] + \frac{1}{2}\alpha_4\nabla^2 v_x = 0, \tag{A.4b}$$

$$\alpha_3(\dot\theta)_{,x} + (\alpha_5 + \alpha_1)v_{z,z,z} + \frac{1}{2}(\alpha_3 + \alpha_4 + \alpha_6)\nabla^2 v_z = 0, \tag{A.4c}$$

$$[\alpha_2 + \alpha_3 + \alpha_5(1 - \alpha_3/\alpha_2)]v_{z,y,z} + \alpha_4\nabla^2 v_y = 0, \tag{A.4d}$$

$$\alpha_3 v_{z,y} + \alpha_2 v_{y,z} = 0, \tag{A.4e}$$

where $C^{-1} \equiv n_o'[1-(n_o'/n_e')^2]/c$, c is the speed of light, and we have used the incompressible condition Of material, i.e., $\Sigma_i v_{i,i} = 0$.[7]

Physically, from Eq. (A.4a), we know that the external light field $< E^2 >$, which is a function of transverse coordinate, x and y, induces molecular reorientation, θ, and θ induces the fluid-flow (v_x, v_y, v_z) through Eqs. (A.4b)-(A.4e). Conversely, v_x and v_z also influence θ through Eq. (A.4a). The induced fluid-flow is the so-called backflow.[6]

Now comparing the force sources of the backflow, i.e., $\alpha_2(\dot\theta)_{,z}$ in Eq. (A.4b) and $\alpha_3(\dot\theta)_{,x}$ in Eq. (A.4c), because $|(\dot\theta)_{,x}| \leq |(\dot\theta)_{,z}|$ due to the non-local effect, and $|\alpha_3| << |\alpha_2|$ for 5CB **in** the nematic phase,[15] $|\alpha_3(\dot\theta)_{,x}|$ is much less $|\alpha_2(\dot\theta)_{,z}|$. Therefore $|v_z|$ is much smaller than $|v_x|$. The effect of v_z can be neglected in the Eqs. (A.4a) and (A.4b). Moreover, comparing the coefficients of $v_{x,z}$ and $v_{z,x}$ in these two equations: $|\alpha_3| << |\alpha_2|$ and $|4(\alpha_5 + \alpha_2)| \leq |(\alpha_5 - \alpha_2)|$, we can also conclude that the effect of v_z can be neglected. The effect of v_y can be also neglected by similar considerations through Eqs. (A.4d) and (A.4e). Thus the equations of motion are further simplified as following:

$$C^{-1}I\theta + K(\partial_z^2 + \nabla_t^2)\theta - \gamma_1\dot\theta - \alpha_2 v_{x,z} = 0, \tag{A.5a}$$

$$\alpha_2(\dot\theta)_{,z} + \eta_c\partial_z^2 v_x + \frac{1}{2}\alpha_4\nabla_t^2 v_x = 0, \tag{A.5b}$$

where $\eta_c \equiv 1/2(\alpha_5 + \alpha_4 - \alpha_2)$, $\partial_z^2 \equiv \partial^2/\partial z^2$, and ∇_t^2 is the Laplacian operator in the transverse coordinate.

When the external applied field $< E^2 >$ is a uniform field, $\nabla_t^2\theta = \nabla_t^2 v = 0$, and the Eqs. (A.5a) and (A.5b) are reduced to the simple forms as for the case of the uniform magnetic field."

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