Throughput efficiency of a class of continuous ARQ schemes under Markovian error patterns

Tsern-Huei Lee

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Abstract: The independent error model is often adopted to simplify the analysis of ARQ strategies. This assumption, however, is not true for channels where transmission errors occur in bursts. The limiting throughput efficiency of a class of continuous ARQ schemes with repeated transmissions is analysed under Markovian error patterns. The scheme without repeated transmissions, i.e. the classic go-back-N scheme, has been analysed in a previous paper. The results show that the throughput efficiency can be significantly increased by transmitting multiple copies of each data block contiguously to the receiver. The results obtained are also compared under Markovian error patterns with those obtained under the independent error model.

1 Introduction

Automatic repeat request (ARQ) strategies are widely used to handle transmission errors in a two-way communication system. The basic concept of ARQ is to detect data blocks with errors at the receiver and then to request the transmitter to retransmit the information in those erroneous data blocks. Stop-and-wait, go-back-N and selective-repeat are the three most popular ARQ schemes in use and have been extensively studied. Many modifications of these basic techniques [2–5, 9] were proposed to achieve a better performance for communications over channels having high error rates. However, most of the results regarding the performance of ARQ schemes were derived based on the assumption of independent errors. This assumption, which greatly simplifies the analysis, is not true for situations where transmission errors occur in bursts, Markov error models are usually used to describe such channels.

Leung, Kikumoto and Sorenson [12] analysed the limiting throughput efficiency of the go-back-N ARQ scheme under Markovian error patterns. By Markovian error pattern it is meant that the probability that a particular data block arrives at the receiver with error depends on the outcome (success or failure) of the immediately preceding block. Let X_i denote the outcome of the transmission in the ith slot so that $X_i = 0$ means the transmission was a success and $X_i = 1$ means the transmission was a failure. Then the error model studied in Reference 12 can be described by prob $[X_{i+1} = 0 | X_i = 0] = p$ and prob $[X_{i+1} = 1 | X_i = 1 = q$. In other words, the channel can be in one of two states, say state 0 and state 1. A transmission will be received without (with) error with probability 1 if the channel is in state 0 (state 1). The state transition probabilities are controlled by the two parameters p and q. In this model, the average length of an error burst is equal to $1/(1-q)$. The quantity $c = p + q$ was defined in Reference 12 as the clustering coefficient of the Markov system. It was found that the go-back-N ARQ scheme is more efficient for a Markov system than an independent error system if and only if (iff) $p + q \geqslant 1$.

Lee [13] studied the throughput efficiencies of the Sastry's modified scheme [2] and the scheme proposed by Moeneclaey and Bruneel [9], under the same Markov error model. It was found that the Sastry's modification is outperformed by either the go-back-N scheme or the scheme proposed in Reference 9. Moreover, the scheme proposed by Moeneclaey and Bruneel was shown to be more efficient for a Markov system than an independent error system iff $p + q \leqslant 1$.

2 Investigated ARQ schemes

The operation of the investigated ARQ schemes can be described as follows. A chunk of n ($n \geqslant 1$) or less copies of each data block are transmitted contiguously to the receiver. An error detection procedure is performed at the receiver on each received copy. A positive (ACK) or a negative acknowledgement (NAK) is sent to the transmitter according to whether the copy is received successfully or erroneously. The data block is considered to be successfully delivered as long as at least one of the copies is correctly received. If all the n copies of a data block are negatively acknowledged, then, just as in the classic go-back-N ARQ strategy, the transmitter goes back to that data block and sends another chunk of n or less copies of the same data block. $n = 1$ corresponds to the go-back-N scheme and $n = \infty$ corresponds to the scheme proposed in Reference 9.

Assume all the data blocks are of fixed length and time is divided into slots so that the duration of each slot is equal to the transmission time of a copy of any data block. Transmissions and retransmissions can only start from slot boundaries. The round-trip delay r is assumed to be fixed and is equal to an integral number of slots. The round-trip delay is defined to be the time interval between the end of the transmission of a copy and the receipt of its response. The feedback channel is assumed to be noiseless for simplicity.

Consider a particular (re)transmission of a data block. Clearly, if the round-trip delay is larger than or equal to $n - 1$, then all the n copies have to be sent before any response of the data block arrives at the transmitter.

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The author is with the Department of Communications Engineering and Centre for Telecommunications Research, National Chiao Tung University, 1001 Ta Hsueh Road, Hsinchu, Taiwan 30050, Republic of China

However, if $r < n - 1$, then an ACK for the data block may return to the transmitter before all the n copies are transmitted. When this occurs, the transmitter will start transmitting the next data block rather than continuing transmitting the rest copies. Therefore, the phrase 'or less' was used in the description at the beginning of this Section. Fig. 1 illustrates the operation of one of the investigated ARQ schemes for $n = 6$ and $r = 3$. Notice that all the six copies of a data block have to be transmitted if the first two copies arrive at the receiver erroneously (for example, see the transmission of data block i). However, it is also possible that less than the maximal allowed number of copies (six, in this example) are actually (re)transmitted. For example, since the first copy of data block $i + 1$ is successfully received, there are only four copies transmitted.

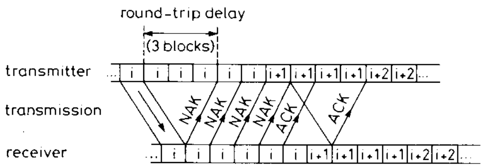

Fig. 1 *Operation of investigated ARQ scheme*
$n = 6; r = 3$

3 Analysis of throughput efficiency

Remember that the random variable X_i is used to denote the outcome of the transmission (or equivalently, state of the channel) in the ith slot so that $X_i = 0$ means the transmission in the ith slot is a success and $X_i = 1$ means the transmission is a failure. For convenience, let

$$T = \begin{bmatrix} p & 1-p \\ 1-q & q \end{bmatrix}$$

denote the channel state transition matrix. It can be shown that the matrix T can be decomposed into

$$T = Q\Lambda Q^{-1}$$

where

$$Q = \begin{bmatrix} 1 & 1-p \\ 1 & q-1 \end{bmatrix}$$

and

$$\Lambda = \begin{bmatrix} 1 & 0 \\ 0 & p+q-1 \end{bmatrix}$$

Therefore, the k-step transition matrix denoted by T^k is equal to $Q\Lambda^k Q^{-1}$. After some algebraic manipulation

$$T^k = \frac{1}{2-(p+q)}\begin{bmatrix} 1-q & 1-p \\ 1-q & 1-p \end{bmatrix} + \frac{(p+q-1)^k}{2-(p+q)}\begin{bmatrix} 1-p & p-1 \\ q-1 & 1-q \end{bmatrix}$$

Let $t_{ij}^k (i, j = 0.\ 1)$ denote the (i,j)th entry of T^k. For convenience, allow k to be zero and define $t_{00}^0 = t_{11}^0 = 1$ and $t_{01}^0 = t_{10}^0 = 0$.

Consider a particular value of n. The transmission sequence can be divided into cycles so that each cycle starts with n consecutive 1s, i.e. an unsuccessful (re)transmission of a data block. Fig. 2 shows a typical transmission sequence for $n = 3$ and $r = 4$. For clarity, spaces are inserted in Fig. 2.

Define the throughput efficiency $\eta(n)$ to be

$$\eta(n) = \lim_{l \to \infty} \frac{\text{number of data blocks successfully received by slot } l}{l}$$

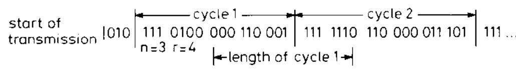

Fig. 2 *Typical transmission sequence for n = 3 and r = 4*

where l is an integer. This definition is independent of the code used for error detection. The actual throughput efficiency is equal to $\eta_0\,\eta(n)$ where $\eta_0 = k/m$ if a (m, k) code is used. Let $\alpha(n)$ and $L(n)$ denote the average number of data blocks successfully transmitted in a cycle and the average length of a cycle, respectively. By the regenerative theorem [15], $\eta(n) = \alpha(n)/L(n)$. Therefore, the remaining work is to compute the values of $\alpha(n)$ and $L(n)$.

Since each cycle starts with n consecutive 1s, it is not hard to see that the value of $L(n)$ is equal to $t_{10}^r L_S(n) + t_{11}^r L_F(n)$, where $L_S(n)$ and $L_F(n)$ represent the expected cycle lengths conditioning on the transmission in the $(n + r)$th slot of a cycle is a success or a failure, respectively. Similarly, $\alpha(n) = t_{10}^r \alpha_S(n) + t_{11}^r \alpha_F(n)$, where $\alpha_S(n)$ and $\alpha_F(n)$ denote the expected number of data blocks successfully received in a cycle conditioning on the same events stated for $L_S(n)$ and $L_F(n)$, respectively. The following two cases should be considered separately.

Case 1, $n \leqslant r + 1$: When $n \leqslant r + 1$, all the n copies of each (re)transmission have to be sent. Let S_{n+r} denote the event that the transmission in the $(n + r)$th slot is a success. Let $A_i, i = 1, 2, \ldots, n$, denote the event that the first successful retransmission of the negatively acknowledged data block occurs at the $(n + r + i)$th slot. Clearly, $(\bigcup_{i=1}^{n} A_i)^c$ represents the event that there is no data block successfully received in a cycle.

The values of $L_S(n)$ and $L_F(n)$ can now be computed. Notice that, as far as the length of a cycle is concerned, there is no difference if the leading $(n + r)$ slots of the cycle are deleted and the following $(n + r)$ slots added (see Fig. 2). A recursive formula for $L_S(n)$ can therefore be expressed as

$$L_S(n) = \sum_{i=1}^{n} \text{prob}\,(A_i \mid S_{n+r})[n + t_{00}^{n-i} L_S(n) + t_{01}^{n-i} L_F(n)] + (1-p)q^{n-1}(n+r)$$

Since

$$\text{prob}\,(A_i \mid S_{n+r}) = \begin{cases} p & i = 1 \\ (1-p)q^{i-2}(1-q) & 2 \leqslant i \leqslant n \end{cases}$$

then

$$\begin{aligned} L_S(n) = {} & p[n + t_{00}^{n-1} L_S(n) + t_{01}^{n-1} L_F(n)] + (1-p) \\ & \times (1-q)[n + t_{00}^{n-2} L_S(n) + t_{01}^{n-2} L_F(n)] \\ & + (1-p)q(1-q)[n + t_{00}^{n-3} L_S(n) + t_{01}^{n-3} L_F(n) \\ & + \cdots + (1-p)q^{n-3}(1-q) \\ & \times [n + t_{00} L_S(n) + t_{01} L_F(n)] + (1-p)q^{n-2} \\ & \times (1-q)[n + L_S(n)] + (1-p)q^{n-1}(n+r) \end{aligned} \quad (1)$$

After some algebraic manipulations, eqn. 1 reduces to

$$a_{11}L_S(n) + a_{12}L_F(n) = n + r(1-p)q^{n-1} \quad (2)$$

where

$$a_{11} = 1 - \left[pt_{00}^{n-1} + (1-p)(1-q)\sum_{i=0}^{n-2} q^i t_{00}^{n-2-i}\right] \quad (3)$$

and

$$a_{12} = -\left[pt_{01}^{n-1} + (1-p)(1-q)\sum_{i=0}^{n-3} q^i t_{01}^{n-2-i}\right] \quad (4)$$

Similarly, a recursive formula for $L_F(n)$ can be derived and another equation of $L_S(n)$ and $L_F(n)$ can be obtained:

$$a_{21}L_S(n) + a_{22}L_F(n) = n + rq^n \quad (5)$$

where

$$a_{21} = -[(1-q)t_{00}^{n-1} + q(1-q)\sum_{i=0}^{n-2} q^i t_{00}^{n-2-i}] \quad (6)$$

and

$$a_{22} = 1 - \left[(1-q)t_{01}^{n-1} + q(1-q)\sum_{i=0}^{n-3} q^i t_{01}^{n-2-i}\right] \quad (7)$$

As for the computation of $\alpha(n)$, since $\alpha(n) = t_{10}^r \alpha_S(n) + t_{11}^r \alpha_F(n)$, only the values of $\alpha_S(n)$ and $\alpha_F(n)$ are required. By similar derivations

$$a_{11}\alpha_S(n) + a_{12}\alpha_F(n) = 1 - (1-p)q^{n-1} \quad (8)$$

and

$$a_{21}\alpha_S(n) + a_{22}\alpha_F(n) = 1 - q^n \quad (9)$$

It can be verified that $\alpha_S(n) \geqslant \alpha_F(n)$ iff $[1-(1-p)q^{n-1}](a_{21}+a_{22}) - (1-q^n)(a_{11}+a_{12}) \geqslant 0$. Since $a_{21} + a_{22} = q^n$ and $a_{11} + a_{12} = (1-p)q^{n-1}$, then $\alpha_S(n) \geqslant \alpha_F(n)$ iff $p + q \geqslant 1$. Similarly, it can be verified that $L_S(n) \geqslant L_F(n)$ iff $p + q \geqslant 1$. Let $\eta_1(n)$ represent the throughput efficiency for values of n belonging to case 1. Then

$$\eta_1(n) = \frac{t_{10}^r \alpha_S(n) + t_{11}^r \alpha_F(n)}{t_{10}^r L_S(n) + t_{11}^r L_F(n)}$$

where $L_S(n)$ and $L_F(n)$ can be computed by solving eqns. 2 and 5 and $\alpha_S(n)$ and $\alpha_F(n)$ can be computed by solving eqns. 8 and 9.

Case 2, $n \geqslant r+1$: When $n > r + 1$, the transmitter may receive an ACK before all the n copies of a data block are transmitted. Therefore, the recursive formula for $K_S(n)$ can be expressed as

$$\begin{aligned} L_S(n) = {} & p[r + 1 + t_{00}^r L_S(n) + t_{01}^r L_F(n)] + (1-p) \\ & \times (1-q)[r + 2 + t_{00}^r L_S(n) + t_{01}^r L_F(n)] \\ & + (1-p)q(1-q)[r + 3 + t_{00}^r L_S(n) + t_{01}^r L_F(n)] \\ & + \cdots + (1-p)q^{n-r-2}(1-q) \\ & \times [n + t_{00}^r L_S(n) + t_{01}^r L_F(n)] + (1-p)q^{n-r-1} \\ & \times (1-q)[n + t_{00}^{r-1} L_S(n) + t_{01}^{r-1} L_F(n)] \\ & + \cdots + (1-p)q^{n-2}(1-q)[n + L_S(n)] \\ & + (1-p)q^{n-1}(n+r) \end{aligned} \quad (10)$$

After some manipulations, eqn. 10 can be reduced to

$$b_{11}L_S(n) + b_{12}L_F(n) = X \quad (11)$$

where

$$\begin{aligned} b_{11} = 1 - \Bigg\{ & \left[p + (1-p)(1-q)\sum_{i=0}^{n-r-2} q^i\right] t_{00}^r \\ & + (1-p)(1-q)\sum_{i=n-r-1}^{n-2} q^i t_{00}^{n-2-i} \Bigg\} \end{aligned} \quad (12)$$

$$\begin{aligned} b_{12} = -\Bigg\{ & \left[p + (1-p)(1-q)\sum_{i=0}^{n-r-2} q^i\right] t_{01}^r \\ & + (1-p)(1-q)\sum_{i=n-r-1}^{n-3} q^i t_{01}^{n-2-i} \Bigg\} \end{aligned} \quad (13)$$

and

$$\begin{aligned} X = {} & p(r+1) + (1-p)(1-q)\sum_{i=0}^{n-r-2} q^i(r+2+i) \\ & + (1-p)(1-q)\sum_{i=n-r-1}^{n-2} nq^i + (1-p)q^{n-1}(n+r) \end{aligned} \quad (14)$$

A recursive expression for $L_F(n)$ can be similarly derived and another equation of $L_S(n)$ and $L_F(n)$ can be obtained. The result is shown in the following equation:

$$b_{21}L_S(n) + b_{22}L_F(n) = Y \quad (15)$$

where

$$\begin{aligned} b_{21} = -\Bigg\{ & \left[(1-q) + q(1-q)\sum_{i=0}^{n-r-2} q^i\right] t_{00}^r \\ & + q(1-q)\sum_{i=n-r-1}^{n-2} q^i t_{00}^{n-2-i} \Bigg\} \end{aligned} \quad (16)$$

$$\begin{aligned} b_{22} = 1 - \Bigg\{ & \left[(1-q) + q(1-q)\sum_{i=0}^{n-r-2} q^i\right] t_{01}^r \\ & + q(1-q)\sum_{i=n-r-1}^{n-3} q^i t_{01}^{n-2-i} \Bigg\} \end{aligned} \quad (17)$$

and

$$\begin{aligned} Y = {} & (1-q)(r+1) + q(1-q)\sum_{i=0}^{n-r-2} q^i(r+2+i) \\ & + q(1-q)\sum_{i=n-r-1}^{n-2} nq^i + q^n(n+r) \end{aligned} \quad (18)$$

The two simultaneous equations of $\alpha_S(n)$ and $\alpha_F(n)$ which are obtained from their recursive expressions can be similarly derived. The results are

$$b_{11}\alpha_S(n) + b_{12}\alpha_F(n) = 1 - (1-p)q^{n-1} \quad (19)$$

and

$$b_{21}\alpha_S(n) + b_{22}\alpha_F(n) = 1 - q^n \quad (20)$$

Again, it can be verified that $\alpha_S(n) \geqslant \alpha_F(n)$ and $L_S(n) \geqslant L_F(n)$ iff $p + q \geqslant 1$. Moreover, the throughput efficiency for values of n belonging to case 2, denoted by $\eta_2(n)$, can be computed by

$$\eta_2(n) = \frac{t_{10}^r \alpha_S(n) + t_{11}^r \alpha_F(n)}{t_{10}^r L_S(n) + t_{11}^r L_F(n)}$$

where $L_S(n)$ and $L_F(n)$ should be computed by solving eqns. 11 and 15 and $\alpha_S(n)$ and $\alpha_F(n)$ should be computed by solving eqns. 19 and 20.

It can be easily verified that $a_{ij} = b_{ij}$ $(i, j = 1, 2)$, $X = n + r(1-p)q^n$, and $Y = n + rq^n$ when $n = r + 1$. Therefore, $\eta_1(r+1) = \eta_2(r+1)$, which is a verification of the derivations. Besides, when $n = 1$, which belongs to case 1, the throughput efficiency $\eta_1(1)$ is given by

$$\eta_1(1) = \frac{(1-q)[1-(p+q-1)^{r+1}]}{(r+1)[2-(p+q)](1-p) + (1-q)[p+q-1)^{r+1}]} \tag{21}$$

the same as eqn. 2.10 obtained in Reference 12. Finally, when $p + q = 1$, which corresponds to the independent error model, the expressions of $\eta_1(n)$ and $\eta_2(n)$ can be reduced to

$$\eta_1(n) = \frac{1-q^n}{n+rq^n} \tag{22}$$

and

$$\eta_2(n) = \frac{(1-q)(1-q^n)}{r(1-q)+r(1-q)q^n+1-q^{n-r}} \tag{23}$$

which were also obtained in Reference 11.

The above derivations for case 2 cannot be used to compute the throughput efficiency for the case when $n = \infty$ because all the values of $L_S(n)$, $L_F(n)$, $\alpha_S(n)$, and $\alpha_F(n)$ become infinity for this case. A different approach is used here to compute $\eta_2(\infty)$. When $n = \infty$, a new data block will not be transmitted until an ACK for the immediately preceeding data block arrives at the transmitter. The transmission sequence can therefore be divided into cycles so that each cycle ends with a 0, i.e. a successful transmission. Fig. 3 illustrates a typical transmission

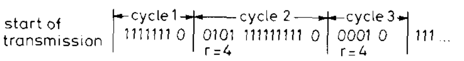

Fig. 3 *Typical transmission sequence for $n = \infty$ and $r = 4$*

sequence for this particular ARQ strategy. Let β_S and β_F denote the average cycle lengths conditioning on the transmission in the rth slot of a cycle is a success or a failure, respectively. Then

$$\beta_S = r + p + 2(1-p)(1-q) + 3(1-p) \times q(1-q) + 4(1-p)q^2(1-q) + \cdots \tag{24}$$

After some manipulation

$$\beta_S = r + 1 + \frac{1-p}{1-q} \tag{25}$$

The value of β_F can be similarly computed and the result is

$$\beta_F = r + 1 + \frac{q}{1-q} \tag{26}$$

It can be verified that $\beta_S \leqslant \beta_F$ iff $p + q \geqslant 1$. Since $\eta_2(\infty) = 1/(t^r_{00}\beta_S + t^r_{01}\beta_F)$

$$\eta_2(\infty) = \frac{(1-q)[2-(p-q)]}{(r+1)[2-(p-q)](1-q) + (1-p)[1-(p+q-1)^{r+1}]} \tag{27}$$

The same result was given in Reference 13 without derivation.

4 Numerical results

In this Section some examples are studied and the results for Markov systems are compared with those for independent error systems. For a Markov system with state transition matrix

$$T = \begin{bmatrix} p & 1-p \\ 1-q & q \end{bmatrix}$$

the stationary probability distribution is given by $[1-q \quad 1-p]/(2-p-q)$. The effective block error probability for the corresponding independent error system is therefore equal to $(1-p)/(2-p-q)$.

Fig. 4 shows the comparison of the throughput efficiencies of a Markov system and its corresponding inde-

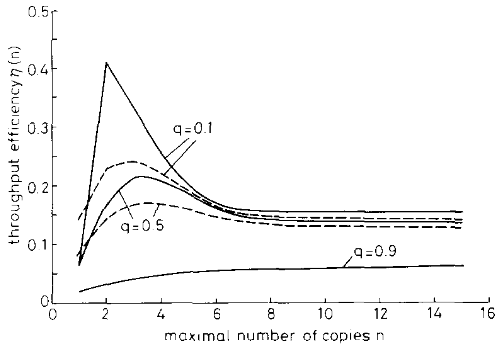

Fig. 4 *Throughput efficiency against maximal number of copies*
$r = 5$; $p = 0.1$
——— Markov error model
– – – independent error model

pendent error system for round-trip delay $r = 5$ with $p = 0.1$ and $q = 0.1$, 0.5, and 0.9. There is only one curve for $q = 0.9$ because a Markov system is identical to its corresponding independent error system if $p + q = 1$. The classic go-back-N ARQ scheme ($n = 1$) is less efficient for a Markov system than its corresponding independent error system when $p + q < 1$ [12]. The curve for $q = 0.9$ can be shown [11] to be monotonically increasing with $\eta_2(\infty) = 1/15$. Besides, the maximum throughput efficiency of a Markov system is larger than that of its corresponding independent error system under the considered values of p and q. For example, the maximum throughput efficiency is equal to 0.41 (which occurs at $n = 2$) for the Markov system and is equal to 0.241 (which occurs at $n = 3$) for its corresponding independent error system when $p = q = 0.1$.

Fig. 5 shows another comparison for $p = 0.5$. The same properties as stated for the curves shown in Fig. 4

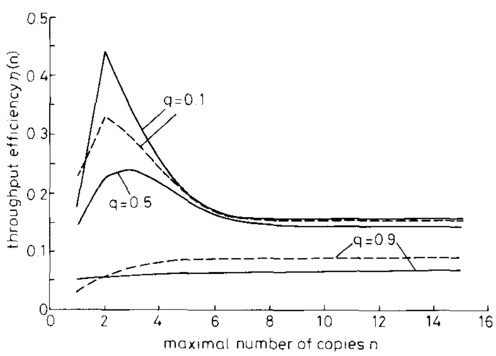

Fig. 5 *Throughput efficiency against maximal number of copies*
$r = 5$; $p = 0.5$
——— Markov error model
– – – independent error model

can be observed in this Figure. The only exception is that the maximum throughput efficiency of the Markov system is smaller than that of its corresponding independent error system when $p = 0.9$. Fig. 6 shows the throughput efficiencies for $p = 0.9$. It can be seen that, under such a large value of p, the go-back-N scheme is the optimal choice for the Markov system. However, the

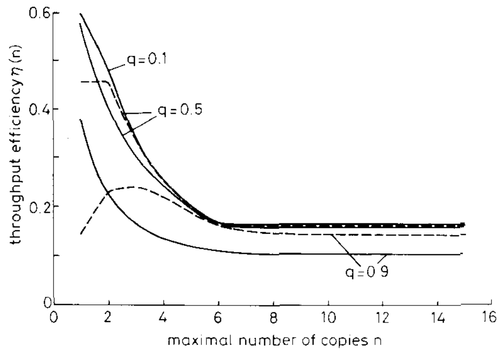

Fig. 6 *Throughput efficiency against maximal number of copies*
$r = 5$; $p = 0.9$
- · —— Markov error model
- - - - independent error model

optimal value of n could be greater than unity for the corresponding independent error system. Again, the maximum throughput efficiency of the Markov system is larger than that of its corresponding independent error system under the considered values of p and q.

Figs. 7–9 show similar results for round-trip delay $r = 10$. The maximum throughput efficiency of the

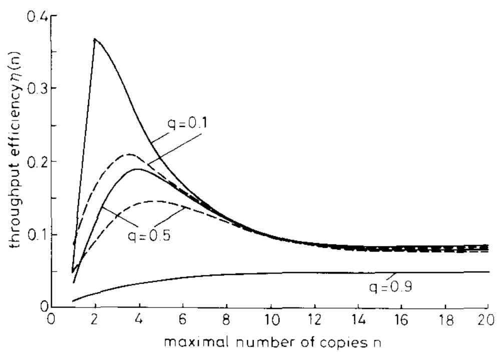

Fig. 7 *Throughput efficiency against maximal number of copies*
$r = 10$; $p = 0.1$
—— Markov error model
- - - - independent error model

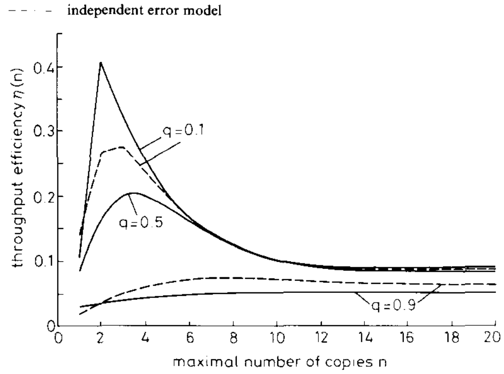

Fig. 8 *Throughput efficiency against maximal number of copies*
$r = 10$; $p = 0.5$
—— Markov error model
- - - - independent error model

Markov system is again smaller than that of its corresponding independent error system only when $p = 0.5$ and $q = 0.9$ (Fig. 8).

Figs. 4–9 show that the maximum throughput efficiency is in general greater than $\eta_1(1)$, the throughput efficiency of the classic go-back-N ARQ scheme. Moreover, the function $\eta_2(n)$ converges rapidly to $\eta_2(\infty)$. Consider the curve for $q = 0.1$ in Fig. 9. This curve is a monotonic decreasing function of n of $n \geqslant 10$ with $\eta_2(\infty) = 0.09$. Consider the values of $\eta_2(11)$ and $\eta_2(20)$. Intuitively, $\eta_2(11)$ can be approximated by $1/11 \cong 0.091$ because for $r = 10$ all the 11 copies have to be sent in each transmission and the probability that at least one of the 11 copies will be received successfully is equal to $1 - (0.1)^{11} \simeq 1$.

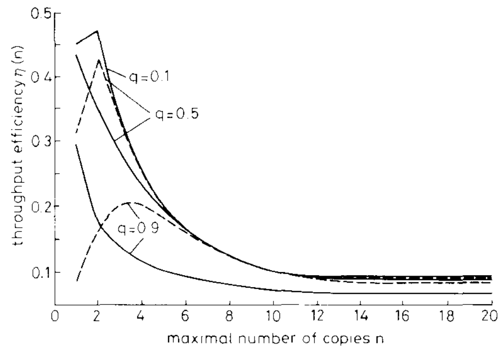

Fig. 9 *Throughput efficiency against maximal number of copies*
$r = 10$; $= 0.9$
—— Markov error model
- - - - independent error model

Similarly, $\eta_2(20)$ can be approximated by $1/[0.9 \sum_{i=1}^{10}(0.1)^{i-1}(10 + i) + 20x(0.1)^{10}] \simeq 0.090$. The values of $\eta_2(11)$ and $\eta_2(20)$ reveal that $\eta_2(n)$ does converge to $\eta_2(\infty)$ very fast for this example. Similar arguments can be applied to the curves in the other Figures. As a result, $\eta_2(\infty)$ can be used as a good approximation of $\eta_2(n)$ as long as n is moderately larger than r. This observation is very helpful in searching for the optimal value of n. A final point which is worth mentioning is that, given the state transition matrix $\boldsymbol{T}$, the optimal value of n tends to be larger as the round-trip delay increases. For example, when $p = q = 0.5$, the optimal value of n is equal to 3 for $r = 5$ and is equal to 4 for $r = 10$.

5 Conclusions

The limiting throughput efficiency of a class of continuous ARQ schemes with repeated transmissions has been analysed. It was shown that the throughput efficiency can be significantly increased by transmitting multiple copies of a data block contiguously to the receiver, especially for channels having large round-trip delays. The function $\eta_1(n)$ seems to be convex (if n is considered as a continuous variable) and the function $\eta_2(n)$ seems to be either monotonically increasing or decreasing. However, to formally prove these properties seems very difficult unless $p + q = 1$. The performance of investigated ARQ schemes under other related error patterns can be further studied.

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