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Exciton binding energy in a GaAs/$Al_xGa_{1-x}As$ quantum well with uniform electric field

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The effects of a uniform electric field on the binding energies of excitons and the subband energies in a GaAs/$Al_xGa_{1-x}As$ quantum well are studied by a perturbative variational approach. Our calculation is based on an effective-width infinite-well model. The effective-mass mismatch is also taken into account. Our results show that the electric field causes a large shift of the subband energy and exciton peak position. The calculated results are compared with the data observed from an optical-absorption experiment. Satisfactory agreement is obtained.

I. INTRODUCTION

Recently, growth of alternately thin-layered semiconductor heterostructures with controlled thicknesses and relatively sharp interfaces has become possible. More recently, much effort has been devoted to the study of the exciton states in these one-dimensional periodic structures and quantum wells.[1–17] A knowledge of the exciton binding energy is crucial to the interpretation of the photoluminescence spectra and photoluminescence excitation spectra, which are used to determine the electronic properties of heterostructures. The exciton binding energy is also important because it is now possible to make the layers so thin that some quantum-confinement effects of electrons and holes can readily be seen. For example, in bulk GaAs, the exciton resonances can be observed only at low temperature. At room temperature, excitonic resonances are very weak and thus nonresolvable. However, in multiple quantum wells, one remarkable phenomenon happens—the exciton resonances can be clearly observed at room temperature. When the electric field was applied in the direction perpendicular to the layers of molecular-beam-epitaxy- (MBE-) grown multiple-quantum wells in a *p-i-n* doped structure, the field-induced absorption was observed to have a large shift to lower energies accompanied by some broadening.[15–36] This exceptional quantum-confinement Stark effect has received a lot of attention for devices because it is a very large electroabsorption effect even at room temperature. This effect makes possible the fabrication of low-energy optical modulators and switches. Many authors[21–43] have devoted their efforts to studying the mechanism of the quantum-confinement Stark effect. Most of the theoretical works are usually based on the application of the conventional perturbation method, the traditional variation principle, Monte Carlo techniques, and exact numerical approaches. A method based on the Monte Carlo technique and variation principle was developed by Singh[31] and Singh and Hong[32] to study the ground-state problem in an arbitrary quantum well. An extensive calculation on several variants of the single-subband model of the infinite and finite wells with interwell Coulomb effects were reported recently by Mo and Sung.[40] Galbraith and Duggan[41] reported a variational calculation of the external-field effect on the exciton binding energy in a double-quantum-well model. Zhu[43] investigated the effects of the heavy- and light-hole mixing on the exciton spectra of a quantum well in the presence of the electric field by taking account the different exciton spinor components. The agreements with the experimental data of these theoretical works are, in general, only qualitative. It is, therefore, interesting to employ different theoretical treatments to study the exciton binding energy of a GaAs quantum well with uniform electric field.

In the present work, we propose a simple but much more efficient approximation method[44] to study the binding energy of the exciton in a type-*I* heterostructure with an electric field. For the purpose of illustration, numerical calculation will be performed only for the GaAs quantum well. This is because the experimental data for the GaAs/$Al_xGa_{1-x}As$ quantum well is abundant.

II. THEORY

We consider a Wannier exciton in a GaAs quantum well sandwiched between two semi-infinite $Al_xGa_{1-x}As$ slabs in the presence of a uniform electric field F along the positive z direction (i.e., perpendicular to the material layers). The origins of distance and of electrostatic potential are chosen at the center of the well. To write down the Schrödinger equation, it is convenient to retain the z-direction-motion Hamiltonian but transform the in-plane motion Hamiltonian into plane-relative coordinates: $x=x_e-x_h$ and $y=y_e-y_h$. This is because the exciton in-plane (x,y) motion is free except for the Coulomb potential. Then under the effective-mass approximation and by dropping the plane center-of-mass-coordinates part, the Schrödinger equation can be expressed as[25]

$$H\Psi_{\rm exc}(r_e,r_h)=E_{\rm exc}\Psi_{\rm exc}(r_e,r_h)\ , \tag{1}$$

where the exciton Hamiltonian H is

$$H=-\frac{\hbar^2}{2\mu_\perp}\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right]-\frac{\partial}{\partial z_e}\frac{\hbar^2}{2m^*_{e,z}}\frac{\partial}{\partial z_e}+eFz_e$$
$$+V_e\Theta(z_e^2-L^2/4)-\frac{\partial}{\partial z_h}\frac{\hbar^2}{2m^*_{h,z}}\frac{\partial}{\partial z_h}-eFz_h$$
$$+V_h\Theta(z_h^2-L^2/4)+E_{\rm gap}$$
$$-\frac{e^2}{k[x^2+y^2+(z_e-z_h)^2]^{1/2}}\ , \tag{2}$$

where $\Psi_{\rm exc}$ is the exciton envelope wave function, $E_{\rm exc}$ is the exciton energy and $E_{\rm gap}$ is the band-gap energy of the material that forms the well, and

$$\mu_\perp=\frac{m_{e\perp}m_{h\perp}}{m_{e\perp}+m_{h\perp}}$$

is the in-plane reduced mass. $E_{\rm gap}$ will be assumed to remain constant under the electric field. The variable r_e (r_h) is the electron (hole) position vector, $m^*_{e,z}$ ($m^*_{h,z}$) is the electron (hole) effective mass along the z axis, $m^*_{e\perp}$ ($m^*_{h\perp}$) is the electron (hole) x,y plane effective mass. These masses, are, in general, different in the wells and in the barriers, hence they are z dependent. The kinetic energy has been written, for z-dependent masses, in a way which restores the Hermitian character of the Hamiltonian.[45] V_e (V_h) is the barrier height confining the electron (hole). $\Theta(x)$ is the unit step function: $\Theta(x)=0$ for $x<0$, $\Theta(x)=1$ for $x>0$, and L is the well width. The dielectric constant k contained in the electron-hole Coulomb interaction term is assumed to be z independent. We shall neglect image effects to simplify the problem.

Equation (2) cannot be solved exactly; the usual method for solving this problem is the variational approach.[37–43,46] Here, we shall employ a different approximation approach, namely, the perturbative variational method (denoted as PVM) to study this problem. This method has been successfully applied to calculate the impurity level of both isotropic[44] and anisotropic[47–49] semiconductors. Following the treatment of the PVM, a term of $-\lambda e^2/k(x^2+y^2)^{1/2}$ is now added and subtracted from H. H is then divided into three terms:

$$H=H_e+H_h+H_{xy}+H'(\lambda)+E_{\rm gap}$$
$$=H_0(\lambda)+H'(\lambda)+E_{\rm gap}\ , \tag{3}$$

where $H_0=H_e+H_h+H_{xy}$, and

$$H_e=-\frac{\partial}{\partial z_e}\frac{\hbar^2}{2m^*_{e,z}}\frac{\partial}{\partial z_e}+eFz_e+V_e\Theta(z_e^2-L^2/4)\ , \tag{4a}$$

$$H_h=-\frac{\partial}{\partial z_h}\frac{\hbar^2}{2m^*_{h,z}}\frac{\partial}{\partial z_h}-eFz_h+V_h\Theta(z_h^2-L^2/4)\ , \tag{4b}$$

$$H_{xy}=\frac{\hbar^2}{2\mu_\perp}\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right]-\frac{\lambda e^2}{k(x^2+y^2)^{1/2}}\ , \tag{4c}$$

$$H'(\lambda)=\frac{\lambda e^2}{k(x^2+y^2)^{1/2}}-\frac{e^2}{k[x^2+y^2+(z_e-z_h)^2]^{1/2}}\ . \tag{4d}$$

The H_e or H_h are Hamiltonians for an electron or a hole in a one-dimensional confinement potential well acting under a uniform electric field. The H_{xy} is the Hamiltonian for a two-dimensional hydrogen atom. The Schrödinger equations for H_e, H_h, and H_{xy} can be solved separately.

For H_{xy}, the eigenfunctions can be solved exactly as

$$\Phi_{n,m}(\rho,\vartheta)=N_{n,m}\frac{e^{im\vartheta}}{\sqrt{2\pi}}\left[\frac{2\rho\lambda}{a_0(n+\frac{1}{2})}\right]^{|m|}$$
$$\times\left[\exp\left[-\frac{\rho\lambda}{a_0(n+\frac{1}{2})}\right]\right]L^{2|m|}_{n+|m|}$$
$$\times\left[\frac{2\rho\lambda}{a_0(n+\frac{1}{2})}\right]\ , \tag{5a}$$

with $n=0,1,2,3,\ldots$ and $m=0,\pm1,\pm2,\pm3,\ldots$ and $N_{n,m}$ is the normalization constant

$$N_{n,m}=\left[\frac{2}{a_0(n+\frac{1}{2})}\right]\left[\frac{(n-|m|)!}{(2n+1)[(n+|m|)!]^3}\right]^{1/2}\ . \tag{5b}$$

The eigenenergies for H_{xy} are

$$E_{xy,n}=-\frac{\mu_\perp\lambda^2e^4}{2k^2\hbar^2(n+\frac{1}{2})^2}=-\frac{\lambda^2R^*_y}{(n+\frac{1}{2})^2}\ , \tag{5c}$$

where $R^*_y=\mu_\perp e^4/2k^2\hbar^2$ is the effective Rydberg calculated with transverse effective reduced mass $\mu_\perp$.

For the Hamiltonians H_e and H_h, one can see that the potential-energy term in Eqs. (4a) and (4b) tends to $+\infty$ as z goes to $+\infty$, so that the solutions of the Schrödinger equation are restricted by a boundary condition that requires the wave function to vanish at infinity. On the other hand, the potential tends to $-\infty$ as z goes to $-\infty$, so that the system does not, strictly speaking, have true bound states.[41,46] In other words, the carrier initially confined in a well can always lower its potential energy by tunneling out the well when the field is not zero. However, provided the electric field is not too strong, we still expect to see resonances associated with the so-called "quasiband" states.[41,46] Although the actual state of an isolated quantum well with an electric field is quasibound, it may be approximated by a bound state if the applied field is not too strong so that the lifetime τ of the carrier in the well is long enough. Thus one can neglect the escape of the carrier outside the quantum well.[41,46,50] We will use the bound-state approximation to study the subband energies in a quantum well. For the GaAs/$Ga_{1-x}Al_xAs$ superlattice, the conduction-band-edge offset V_0 is about 85% of the band-gap difference between GaAs and $Ga_{1-x}Al_xAs$, thus electrons in the conduction band of GaAs are located, in general, in a quantum well with finite barrier height V_0.

Therefore, for our problem the Schrödinger equation for subband energies of the electric (or the hole) in a finite-barrier-height quantum well with a uniform electric field can be expressed as

$$H_z\phi_F(z)=-\frac{d}{dz}\frac{\hbar^2}{2m_z^*}\frac{d}{dz}\phi_F(z)-qFz\phi_F(z)+V_z\phi_F(z)$$
$$=E_Z\phi_F(z)\ , \tag{6a}$$

where

$$m_z^*(z_e)=\begin{cases}M_z, & |z|>L/2\\ m_z, & |z|<L/2\end{cases} \tag{6b}$$

and the confining potential V_z is defined as

$$V_z=\begin{cases}0 & \text{for}|z|>L/2\\ V_0 & \text{for}|z|\le L/2\ .\end{cases} \tag{6c}$$

We shall employ a simple perturbation approach proposed by Lee and Mei[51] to solve Eq. (6). The main idea of this approach is to treat the finite-height-barrier potential-well problem as the approximation of the corresponding infinite-height-barrier potential-well problem with a broadened well width $L+2\delta$, where δ is the penetration depth.

To find out the penetration depth, let us first assume the electric field is so weak that the bound-state approximation holds. Then the wave function inside the quantum well can be approximated as

$$\phi_F(z)=c\exp\left[\int_0^z K(z)dz\right]\ , \tag{7a}$$

where

$$K(z)=\left[\frac{2m_z^*}{\hbar^2}(V_0-qFz-E_Z)\right]^{1/2}\ . \tag{7b}$$

The integral contained in Eq. (7a) can be evaluated easily. One gets

$$\phi_F(z)=\left\{c\Big/\exp\left[\frac{2}{3qF}\left[\frac{2m_z^*}{\hbar^2}\right]^{1/2}(V_0-E_Z)^{3/2}\right]\right\}$$
$$\times\left[\frac{2}{3qF}\left[\frac{2m_z^*}{\hbar^2}\right]^{1/2}(V_0-qFz-E_Z)^{3/2}\right]\ . \tag{7c}$$

The first derivative of $\phi_F(z)$ can be expressed as

$$\frac{d\phi_F(z)}{dz}=\phi_F(z)\left[\frac{2m_z^*}{\hbar^2}(V_0-qFz-E_Z)\right]^{1/2}\ . \tag{8}$$

Therefore, the penetration depth at the boundary $z=-L/2$ is

$$\delta_L=\left[\frac{d\phi_F(z)}{dz}\Big/\phi_F(z)\right]_{Z=-L/2}$$
$$=\left[\frac{2m_z^*}{\hbar^2}(V_0-qFL/2-E_Z)\right]^{-1/2}\ . \tag{9a}$$

In a similar procedure, the penetration depth at the boundary $z=L/2$ can be obtained as

$$\delta_R=\left[\frac{d\phi_F(z)}{dz}\Big/\phi_F(z)\right]_{Z=L/2}$$
$$=\left[\frac{2m_z^*}{\hbar^2}(V_0+qFL/2-E_Z)\right]^{-1/2}\ . \tag{9b}$$

If we now move the well walls at $-L/2$ and $L/2$ to $-L/2-\delta_L$ and $L/2+\delta_R$, then outside the new well walls the probability for finding the electrons are rather small. This is equivalent to saying that for the first-order approximation one can replace the finite-well problem with well walls located at $-L/2$ and $L/2$ by an infinite-well problem with well walls located at $-L/2-\delta_L$ and $L/2+\delta_R$, as shown in Fig. 1. Thus, our problem corresponds to solving Eq. (6a) with V_Z defined as

$$V_Z=\begin{cases}0 & \text{for}-L/2-\delta_L\le z\le L/2+\delta_R\\ \infty & \text{otherwise}\ .\end{cases} \tag{9c}$$

The solution of Eqs. (6a) and (9c) can be expressed as

$$\phi_n(z)=\alpha_n\text{Ai}(\eta_n)+\beta_n\text{Bi}(\eta_n) \tag{10a}$$

with the boundary conditions

$$\phi_n(z)|_{z=L/2+\delta_R}=\phi_n(z)|_{z=-L/2-\delta_L}=0\ , \tag{10b}$$

where $\text{Ai}(\eta_n)$ and $\text{Bi}(\eta_n)$ are Airy functions;[52] α_n,β_n are normalization constants and

$$\eta_n=-\left[\frac{2m_z}{(e\hbar F)^2}\right]^{1/3}(E_{nZ}+qFz)\ . \tag{10c}$$

Equation (10b) yields

$$\text{Ai}(\eta_n^+)\text{Bi}(\eta_n^-)-\text{Ai}(\eta_n^-)\text{Bi}(\eta_n^+)=0\ , \tag{10d}$$

where η_n^+ or η_n^- is given by Eq. (10c) with $z=L/2+\delta_R$ or $-L/2-\delta_L$ and the eigenvalues E_n can be obtained by solving Eq. (10d) numerically.

The E_Z contained in Eqs. (9a) and (9b) is the total eigenenergy for the finite-well problem [Eqs. (6a) and (6c)] and is actually unknown. In the first-order-

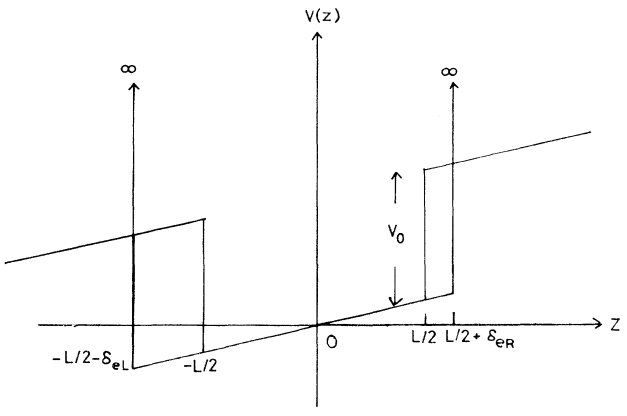

FIG. 1. A finite-height quantum well acted on by an electric field and its equivalent case of an infinite-height potential well.

approximation calculation, the E_Z is approximated by the eigenenergy $E_Z^{(0)}$ of the corresponding infinite-barrier-height quantum well. The value of q contained in Eqs. (9a) and (9b) is $-e$ for the electron, and e for the hole. One can define the effective well width for the electron ($\nu=e$) or hole ($\nu=h$) as

$$L_{\text{eff}\nu}=L+\delta_{\nu L}+\delta_{\nu R} \ . \tag{11}$$

Now from the above discussion, the solution for total unperturbed Hamiltonian H_0 can be expressed as

$$\Psi_{\text{exc}}^{(0)}(z_e,z_h,\rho,\vartheta,\lambda)=\Phi_{n,m}(\rho,\vartheta,\lambda)\phi_{en}(z)\phi_{hn}(z) \ , \tag{12}$$

where $\Phi_{n,m}(\rho,\vartheta,\lambda)$ and $\phi_{\nu n}(z)$ ($\nu=e$ for the electron and $\nu=h$ for the hole) are eigenfunctions obtained in Eqs. (5a) and (10a), respectively. The first-order energy correction is defined as

$$\Delta E_{\text{exc}}^{(1)}(\lambda)=\langle\Psi^{(0)}(\lambda)|H'(\lambda)|\Psi^{(0)}(\lambda)\rangle \ . \tag{13}$$

Since the second-order perturbation correction is in general much smaller than the first-order correction, the expression in Eq. (13) can be treated as the total-energy correction to the unperturbed energy

$$E_{\text{exc}}^{(0)}(\lambda)=E_{xy,n}(\lambda)+E_{e,z}+E_{h,z}+E_{\text{gap}},$$

where

$$E_{xy,n}(\lambda)=-\frac{\lambda^2 R^*}{(n+\frac{1}{2})^2}$$

is the eigenenergy for the two-dimensional hydrogen atom; $E_{e,z}$ ($E_{h,z}$) is the eigenenergy for the electron (hole) in a finite quantum well with a uniform electric field applied in the direction perpendicular to the well walls; and E_{gap} is the band-gap energy of the well-acting material. Therefore, the total eigenenergy for the total Hamiltonian H is

$$E_{\text{exc}}(\lambda)=E_{xy,n}(\lambda)+E_{e,z}+E_{h,z}+E_{\text{gap}}+\Delta E_{\text{exc}}^{(1)}(\lambda) \ . \tag{14}$$

For the ground state, the energy correction $\Delta E_{\text{exc}}^{(1)}(\lambda)$ can be evaluated as

$$\Delta E_{\text{exc},g}^{(1)}(\lambda)=\left[\frac{4\lambda}{a_0}\right]^2\frac{e^2}{k}\int_{-L/2-\delta_{eL}}^{L/2+\delta_{eR}}dz_e|\phi_{eF}(z)|^2 \times\int_{-L/2-\delta_{hL}}^{L/2+\delta_{hR}}dz_h|\phi_{hF}(z)|^2\left\{\frac{a_0}{4}+|z_e-z_h|-\frac{\pi}{2}|z_e-z_h|\left[H_1\left[\frac{4\lambda|z_e-z_h|}{a_0}\right]-N_1\left[\frac{4\lambda|z_e-z_h|}{a_0}\right]\right]\right\}, \tag{15}$$

where $H_1(x)$ and $N_1(x)$ are the Struve and Neumann functions[52] of order 1. The effective Bohr radius $a_0=k\hbar^2/\mu_\perp e^2$ and effective Rydberg $R^*=\mu_\perp e^4/(2k^2\hbar^2)$ are related with the in-plane reduced mass: $\mu_\perp=m_{e\perp}m_{h\perp}/(m_{e\perp}+m_{h\perp})$, where the equivalent in-plane effective masses $m_{e\perp}$ and $m_{h\perp}$ are defined as follows:

$$m_{e\perp}=\langle\Psi_{\text{exc}}|m_{e\perp}^*(z_e)|\Psi_{\text{exc}}\rangle$$

and

$$m_{h\perp}=\langle\Psi_{\text{exc}}|m_{h\perp}^*(z_h)|\Psi_{\text{exc}}\rangle \ .$$

In the practical calculation, the wave function Ψ_{exc} contained in $m_{e\perp}$ or $m_{h\perp}$ is replaced by the unperturbed wave function $\Psi_{\text{exc}}^{(0)}(z_e)$ or $\Psi_{\text{exc}}^{(0)}(z_h)$, respectively. The parameter λ contained in Eq. (14) can be determined by requiring the fast convergence of the perturbation series, and is equivalent to setting

$$\Delta E_{\text{exc}}^{(1)}(\lambda)=0 \ . \tag{16}$$

After obtaining λ from the above equation, one can substitute this λ into Eq. (14) to get the total eigenenergy of exciton $E_{\text{exc}}(\lambda)$ and the binding energy of the exciton: $E_B=E_{el}+E_{hl}+E_{\text{gap}}-E_{\text{exc}}$, where E_{el} (E_{hl}) is the confinement energy of the first electron (hole) subband.

III. RESULTS AND DISCUSSIONS

The values of the physical parameters used in this work are taken from some previous works.[25,53,54] The effective masses for the electron, light hole, and heavy hole inside the well are $m_{e,z}=0.0665m_0$, $m_{\text{lh},z}=0.088m_0$, and $m_{\text{hh},z}=0.45m_0$, where m_0 is the electron rest mass. Those in the barrier are $M_{e,z}=(0.0665+0.0835x)m_0$, $M_{\text{lh},z}=(0.088+0.049x)m_0$, and $M_{\text{hh},z}=(0.45+0.31x)m_0$, where x is the Al composition which is taken as $x=0.32$. The effective mass of the holes are anisotropic. In the plane motion, they are

$$m_{\text{lh}\perp}=\left[\frac{1}{4m_{\text{lh},z}}+\frac{3}{4m_{\text{hh},z}}\right]^{-1}, \quad m_{\text{hh},\perp}=\left[\frac{1}{4m_{\text{hh},z}}+\frac{3}{4m_{\text{lh},z}}\right]^{-1}, \tag{17a}$$

$$M_{\text{lh},\perp}=\left[\frac{1}{4M_{\text{lh},z}}+\frac{3}{4M_{\text{hh},z}}\right]^{-1}, \quad M_{\text{hh},\perp}=\left[\frac{1}{4M_{\text{hh},z}}+\frac{3}{4M_{\text{lh},z}}\right]^{-1} . \tag{17b}$$

The E_{gap} for GaAs and $Al_xGa_{1-x}As$ are assumed to be 1.424 eV and $1.424+1.247x$ eV, respectively. If one uses the 65%—35% rule[55,56] to share the difference between GaAs and $Al_xGa_{1-x}As$ band gaps, then one can obtain

$V_e = 0.26$ eV and $V_h = 0.14$ eV. The dielectric constant is $k = \sqrt{k_w k_b}$, where $k_w = 13.18$ (for GaAs) and $k_b = 13.18 - 3.12x$ (for $Al_xGa_{1-x}As$).

The electron ground-state energy was calculated as a function of the electric-field strength by using the same physical parameters as suggested by a previous exact calculation.[16] Our result shows the electron energy decreases monotonically as the electric field increases. Our result agrees with that obtained by Ref. 16 even when the electric field is as strong as 200 kV/cm. This shows that our approximation is reliable even for a stronger electric field. We also calculated the first three normalized subband energies ($n = 1$, 2, and 3) of a carrier in an infinite quantum well applied by a uniform electric field perpendicular to the well wall. Our ground-state wave function manifests a larger amplitude in the positive region of z for holes or in the negative region of z for electrons, while the first excited subband wave function still has a large amplitude in the $z > 0$ region for electrons or in the $z < 0$ region for holes even at $\tilde{F} = 20$, so that they have a positive-energy shift. For a much stronger electric field, the second-state wave function has a larger amplitude in the $z < 0$ region for electrons or in the $z > 0$ region for holes, hence the negative-energy shift appeared. Although the electric field is very large ($F > 20$), the second excited state still has a large amplitude in the $z > 0$ region for the electron or the $z < 0$ region for the hole, and thus the energy shift for the third state is still positive in the stronger field region.

The variations of the effective well widths of the electron ($L_{\text{eff},e}$) and the heavy hole ($L_{\text{eff},h}$) were calculated as a function of the electric-field strength for $L = 95$ Å. The results show that the effective well widths are slightly smaller than those of Miller *et al.*[25] ($L_{\text{eff},e} = 133$ Å, $L_{\text{eff},h} = 114$ Å). Our result shows that the effective width of the electrons increases slightly with the electric field while that of heavy holes decreases with the electric field. This is because the left-hand-side penetration depth δ_{eL} of the electron increases more rapidly than the decreases of the right-hand-side penetration depth δ_{eR}. In the case of the heavy hole, the situation is the opposite of the case of the electron. Therefore, the effective well widths are no longer symmetrical with respect to the $z = 0$ plane but have a position shift in the negative-z direction for the electron or shift in the positive-z direction for the hole. These position shifts of L_{eff} will affect the carrier subband energies.

The calculated subband energies of the electron, light hole, and heavy hole show that these energies decrease rapidly as the electric field increases. Since the effective mass of the heavy hole is larger than those of the electron and of the light hole, the heavy hole has a lower subband energy than either the electron or the light hole. When the field is large, the energies of the electron and light hole are positive while that of heavy hole becomes negative. These large shifts of energy affect the absorption spectra of exciton, so we can expect that the exciton peak has a large shift to lower energy as the electric field increases.

Figure 2 shows the electric-field dependence of the heavy-hole exciton binding energy in a GaAs quantum well for four well widths: $L = 95$, 150, 200 and 250 Å. One can note from the figure that the binding energy decreases for increasing the electric-field strength and finally approaches a constant value as the electric field becomes very large. The decrease is much more significant for larger L. The calculated heavy-hole-exciton binding energies were found to decrease rapidly as the well width increases and this means the excitons have a larger extension in a larger quantum well.

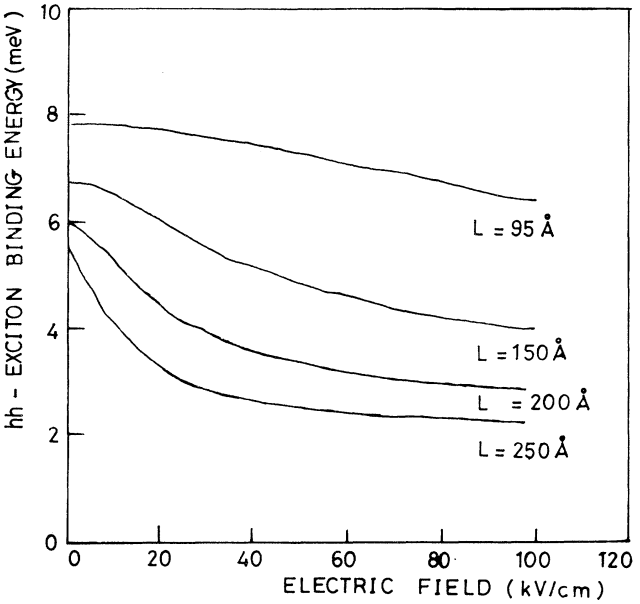

FIG. 2. Electric-field (in kV/cm) dependence of the heavy-hole-exciton binding energy (in meV) for four well widths: $L = 95$, 150, 200, and 250 Å (Al composition $x = 0.32$).

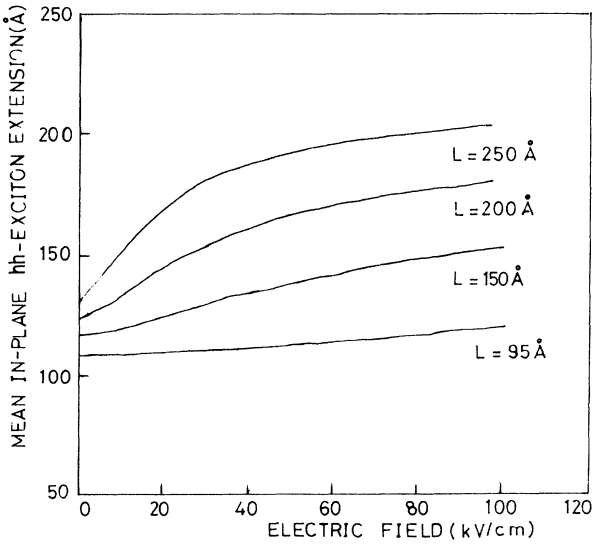

FIG. 3. Electric-field (in kV/cm) dependence of the mean in-plane heavy-hole-exciton extension (in Å) for four well widths: $L = 95$, 150, 200, and 250 Å (Al composition $x = 0.32$).

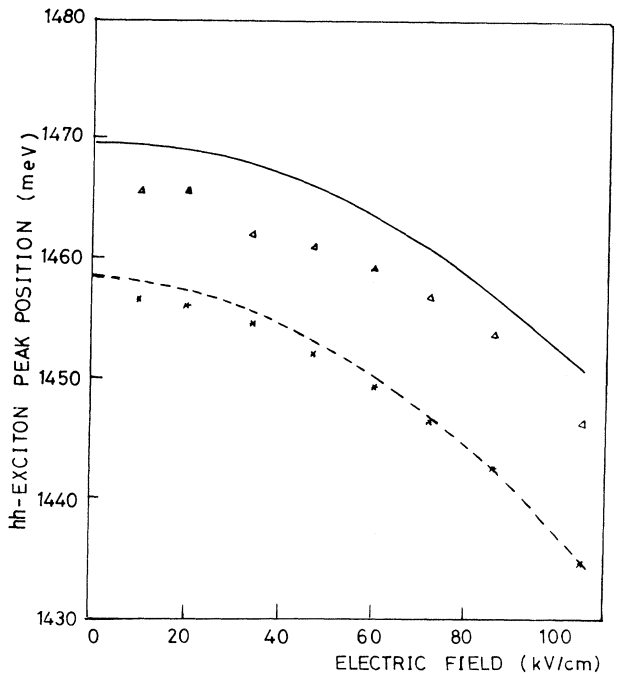

FIG. 4. Variations of heavy-hole- (dotted line) and light-hole- (solid line) exciton peak positions (in meV) with electric field (in kV/cm) for temperature $T=300$ K, well width $L=95$ Å, and Al composition $x=0.32$). The experimental data are taken from Ref. 37).

The electric-field dependence of the exciton binding energy can also be understood by the variation of the mean in-plane exciton extension $\langle\rho\rangle=\langle\Psi_{exc}|\rho|\Psi_{exc}\rangle$. Figure 3 presents the well width and the electric-field dependence of the mean in-plane heavy-hole-exciton extension in a GaAs quantum well for several values of L and F. The electric-field-induced increase of the exciton size in the layer plane will make the exciton resonances weaken and broaden. This is consistent with the observation of Wood *et al.*[15] Figure 4 presents a comparison between our calculated values and the measurements of the Stark shift of the heavy-hole-exciton and light-hole-exciton peak with the electric field applied in the GaAs-$Al_xGa_{1-x}As$ quantum well for $x=0.32$ and $L=95$ Å. The agreement between theory and experiment is better for the heavy-hole-exciton case. The discrepancy in the light-hole-exciton case might be due to the overestimation of light-hole subband energy. Since the light-hole effective mass is small, it has a larger ground-state energy; thus, the assumption that the barrier height is much larger than the carrier energy might not be very suitable for the case of the light-hole exciton.

IV. SUMMARY

The subband states and the exciton binding energies have been studied by an approximation method. The variations of the exciton binding energies with the electric-field strength and the well width are obtained. The mean in-plane exciton extension is found to increase with the electric-field strength. The excellent agreement of our calculated heavy-hole-exciton peak position with the experimental data shows that our bound-state assumption is suitable for heavy holes.

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