Charge-coupled device polarimetry and its measurement of the Stokes vector of light transmitted by a polymer plate

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The Stokes vector of light transmitted by a polycarbonate plate is studied by a modified charge-coupled device polarimeter. A sheet polarizer is measured for comparison.

By use of photographic techniques Walravan[1] was able to identify some important features of the target from the ground through the atmosphere by different polarization parameters. According to the same principle, Prosch *et al.*[2] neglected circular polarization and developed a video polarimeter to measure and map out the polarization properties of a lake and its environment. They deduced p, the degree of polarization, α, the azimuth of polarization, and I, the radiance, of any target by measuring three radiances through three linear polarizers spaced at a 60° angle. Besides measuring the original radiances we modified Prosch's video polarimeter by introducing an extra quarter-wave plate in front of an analyzer and measured another three radiances at the same angles. In addition to p, α, and I we can also obtain the ellipticity χ. This technique permits us to measure all the Stokes parameters of any sample in air. For comparison we measured the azimuth angle of a Polaroid relative to an arbitrary axis and found that this technique can accurately measure the azimuth angle of the Polaroid as well as the azimuth angle of the quarterwave plate. Prior knowledge of the direction of the quarterwave plate is not necessary with this technique. By analyzing the polarization properties of linearly polarized light through a polycarbonate (PC) plate, we clearly observed a birefringence effect[3] in its two-dimensional distribution of polarization and azimuth angles. Most plastics under stress exhibit birefringence.[4] Since our sample is molded by injection, flow-induced birefringence is expected. We are interested in exploring the possibility of using a charge-coupled device polarimeter to study this effect quantitatively.

By use of the Stokes vector[5] **S** to represent the electromagnetic wave, we can write

$$\mathbf{S} = \begin{bmatrix} I \\ Q \\ U \\ V \end{bmatrix} = I \begin{bmatrix} 1 \\ p\cos 2\chi \cos 2\alpha \\ p \cos 2\chi \sin 2\alpha \\ p \sin 2\chi \end{bmatrix}.$$

All the notations is the same as in Ref. 5. The Stokes vector $\mathbf{S}_s$ for light transmitted by any sample and its relation to the measured Stokes vector $\mathbf{S}_f$ through a quarter-wave plate and a linear analyzer can be written as

$$S_f = \mathbf{M}_p \mathbf{M}_q \mathbf{S}_s,$$

where $\mathbf{M}_p$ and $\mathbf{M}_q$ are the Mueller matrices of the linear analyzer and the quarter-wave plate, respectively. If α_p and α_q denote their directions of optical axes with respect to the reference axis,

$$\mathbf{M}_q = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos^2 2\alpha_q & \cos 2\alpha_q \sin 2\alpha_q & -\sin 2\alpha_q \\ 0 & \cos 2\alpha_q \sin 2\alpha_q & \sin^2 2\alpha_q & \cos 2\alpha_q \\ 0 & \sin 2\alpha_q & -\cos 2\alpha_q & 0 \end{bmatrix},$$

$$\mathbf{M}_p = \frac{1}{2}\begin{bmatrix} 1 & \cos 2\alpha_p & \sin 2\alpha_p & 0 \\ \cos 2\alpha_p & \cos^2 2\alpha_p & \cos 2\alpha_p \sin 2\alpha_p & 0 \\ \sin 2\alpha_p & \cos 2\alpha_p \sin 2\alpha_p & \sin^2 2\alpha_p & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

The transmitted radiance of S_f at α_p is

$$\bar{I}_p(\alpha_p) = \tfrac{1}{2} I[1 + p\cos 2\chi \cos 2(\alpha - \alpha_q)\cos 2(\alpha_p - \alpha_q) + p \sin 2\chi \sin 2(\alpha_p - \alpha_q)],$$

while the transmitted radiance at α_p without a quarter-wave plate is

$$I_p(\alpha_p) = \tfrac{1}{2} I[1 + p \cos 2\chi \cos 2(\alpha - \alpha_p)].$$

All polarization parameters I, χ, α, p, and α_q can be obtained by taking three I_p's and three $\bar{I}_p$'s at 0°, 60°, and 120°, respectively. The extra measurement of $\bar{I}_p$ is used for normalization to remove the absorption effect by the quarter-wave plate. It is easy to prove that

$$I = \tfrac{2}{3}[I_p(0) + I_p(60) + I_p(120)];$$

$$\bar{I} = \tfrac{2}{3}[\bar{I}_p(0) + \bar{I}_p(60) + \bar{I}_p(120)],$$

$$\tan 2\alpha = \frac{I_b}{I_a}; \quad \tan 2\alpha_q = -\frac{(I_a - \bar{I}_a)}{(I_b - \bar{I}_b)}, \tag{1}$$

$$p \sin 2\chi = -\frac{(\bar{I}_a \tan 2\alpha_q - \bar{I}_b)}{(1 + \tan^2 2\alpha_q)^{1/2}}; \quad p\cos 2\chi = (I_a{}^2 + I_b{}^2)^{1/2}, \tag{2}$$

where

$$I_a = [2I_p(0) - I_p(60) - I_p(120)]/I;$$

$$\bar{I}_a = [2\bar{I}_p(0) - \bar{I}_p(60) - \bar{I}_p(120)]/\bar{I},$$

$$I_b = \sqrt{3}[I_p(60) - I_p(120)]/I;$$

$$\bar{I}_b = \sqrt{3}[\bar{I}_p(60) - \bar{I}_p(120)]/\bar{I}.$$

The modified polarimeter (Fig. 1) is basically constructed of the following parts: (a) light source L_s that contains a tungsten lamp followed by a color filter for matching the quarter-wave plate in the analyzing system and illuminates the tested sample; (b) the analyzing system that consists of a quarter-wave plate and a linear polarizer, which should be as close to the charge-coupled device camera (Photometrics CC200 without cooling) as possible for obtaining a wider view. All the necessary radiances (three I_p's and three $\bar{I}_p$'s) are taken by rotating the polarizer with respect to a reference axis and stored in a computer. All polarization parameters can be obtained by Eqs. (1) and (2). A PC plate is illuminated by a linearly polarized monochromatic light,

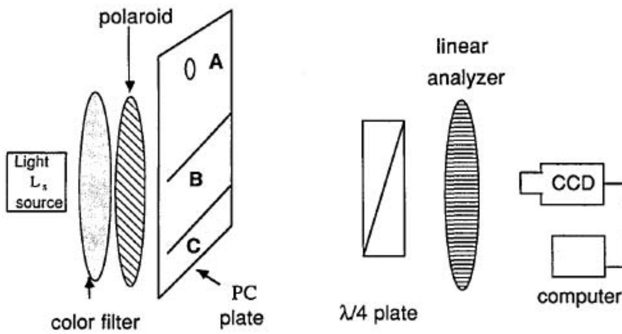

Fig. 1. Schematic of the polarimeter.

and its polarization properties are studied. By use of the inlet of injection A as a reference position, we analyze two separated lines across the PC plate at a distance of 1.5 cm (B) and 2.1 cm (C) away from inlet (A). For comparison, an illuminated Polaroid is also used as a sample.

After measuring all the polarization parameters of light transmitted by a Polaroid, we find that the degree of polarization p [(top) showing Fig. 2, 0.937 ± 0.002 without and 0.879 ± 0.005 with the quarter-wave plate] is less than that which is given (0.975). This may be caused by the scattering of optical surfaces, while α_q (-0.019 ± 0.062 in degrees) of the quarterwave plate and α (3.782 ± 0.285 in degrees) of the Polaroid can be measured accurately [Fig. 2 (bottom)]. Although there is a small deviation in ellipticity (6.047 ± 0.529 in degrees), it is uniformly distributed. By use of the same technique to analyze qualitatively the polarization properties of a linearly polarized light through a PC plate, we clearly observed a flow-induced birefringence effect in its azimuth angle distribution. The polarization properties of the PC plate are analyzed at two horizontal lines separated by 0.6 cm, as shown in Fig. 3. The distribution of polarization indicates how the long-chain molecules are aligned. From the distribution one can tell that the molecules closer to the inlet of injection are better aligned. The distribution of the azimuth angle also behaves similarly, which is a result of the birefringence of the polymer. This effect is induced by the injection flow.[5] The birefringence property of polymers has been of great interest in polymer science. With this technique we are doing more work on the stress-related birefringence of polymer.

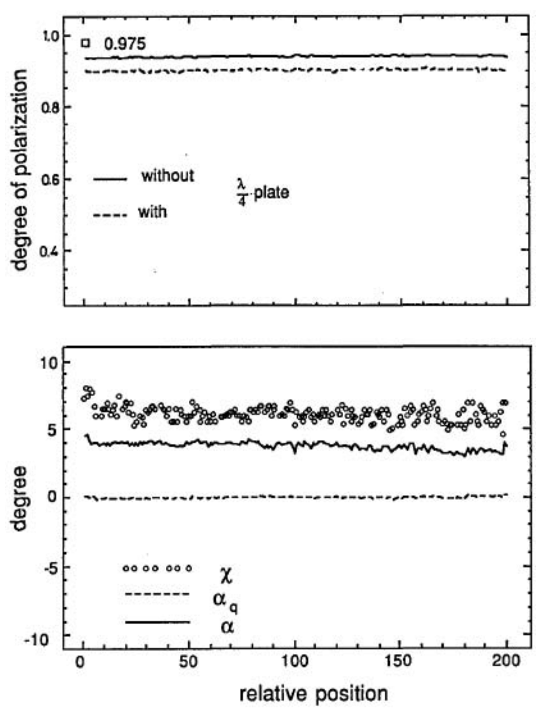

Fig. 2. Polarization properties of the Polaroid: The degree of polarization of light transmitted through a Polaroid is measured by a linear analyzer with/without a quarter-wave plate (top). The measured azimuth angle of the quarter-wave plate α_q, Polaroid α, and the ellipticity χ of the Polaroid (bottom) are shown.

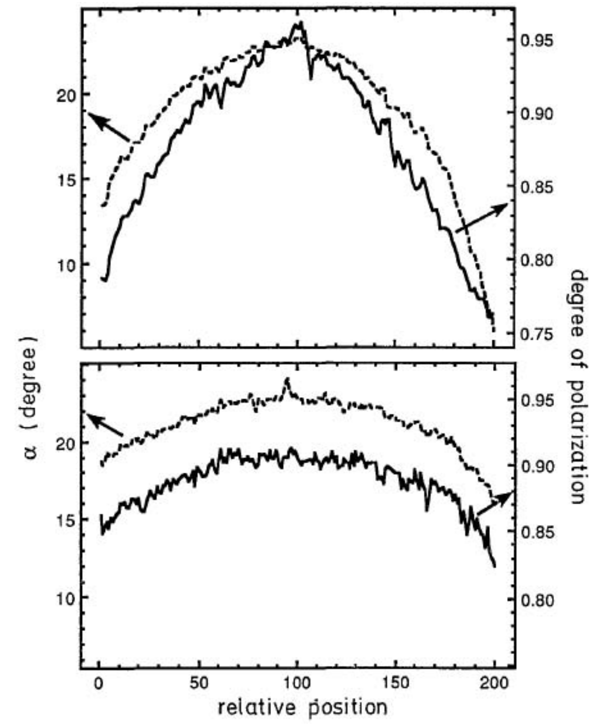

Fig. 3. Polarization properties of light transmitted through a PC plate. The degree of polarization (solid curve) and the azimuth angle (dashed curve) of the PC plate at B, 1.5 cm from the inlet A (top) are shown. The same parameters are measured at C, 2.1 cm from inlet A (bottom).

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