Design of integral variable structure controller and application to electrohydraulic velocity servosystems

T.-L. Chern
Y.-C. Wu

Abstract: An integral variable structure controller (IVSC) for robust servotracking is proposed. It comprises an integral controller, which is designed for achieving zero steady-state error under step input, and a variable structure controller (VSC) which is designed for enhancing robustness. A procedure is developed for determining the coefficients of the switching plane and the integral control gain such that the overall closed-loop system has the desired eigenvalues. Furthermore, a modifed proper continuous function is introduced to overcome the chattering problem. An electrohydraulic velocity servocontrol system using the proposed IVSC approach is illustrated. Simulation results show that the proposed IVSC approach can achieve accurate servotracking and is fairly robust to plant parameter variations and external load disturbances.

1 Introduction

The most distinct feature of the variable structure controller (VSC) is the existence of a sliding mode which occurs on a predetermined switching plane [1–3]. Once in sliding mode, the system will be forced to slide along, or at the vicinity of, the switching plane and hence is theoretically robust to plant parameter variations. However, a controller based on variable structure configuration may fail to meet the desired performance when the system is commanded to track an arbitrary or a step input, or is subjected to external disturbances. To solve this problem, we propose an IVSC approach which comprises an integral controller followed by a variable structure controller. The design of such a system involves the determination of the switching plane, the integral control gain and the control function to guarantee the existence of a sliding mode.

The performances of the proposed IVSC approach, the conventional VSC and the linear approach have been compared by computer simulation, with an electrohydraulic velocity control system as an illustration. The dynamics of this type of system are usually very complex and highly nonlinear due to the flow-pressure relation-

ship [4, 5]. The simulated results show that the proposed IVSC approach can almost maintain an identical response in the face of large plant parameter variations and external disturbances. It yields improved performance when compared to the conventional VSC and the linear approaches.

2 Integral variable structure system (IVSS)

The proposed configuration of the IVSC combines an integral controller followed by a VSC, as shown in Fig. 1, and is described as follows:

$$\dot{X}_i = X_{i+1} \quad i = 1, \ldots, n - 1 \tag{1a}$$

$$\dot{X}_n = - \sum_{i=1}^{n} a_i X_i + bU - f(t) \tag{1b}$$

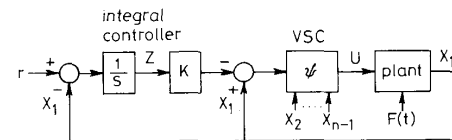$$\dot{Z} = r - X_1 \tag{1c}$$

Fig. 1   Block diagram of an IVSS

where X_1 is the output signal, r is the input command, K is the gain of the integral controller, a_i and b are the plant parameters, $f(t)$ are disturbances, and the control function U is piecewise linear of the form

$$U = \begin{cases} U^+(x, t) & \text{if } \sigma > 0 \\ U^-(x, t) & \text{if } \sigma < 0 \end{cases} \tag{2}$$

where σ is the switching function given by

$$\sigma = c_1(X_1 - KZ) + \sum_{i=2}^{n} c_i X_i$$

$$c_i = \text{constant}$$

$$c_n = 1 \tag{3}$$

The design of such a system involves

(a) the choice of the control function U so that it gives rise to the existence of a sliding mode, and

(b) the determination of the switching function σ and the integral control gain K such that the system has the desired eigenvalues.

2.1 Choice of control function

From eqns. 1 and 3, we have

$$\dot{\sigma} = -c_1 K(r - X_1) + \sum_{i=2}^{n} c_{i-1} X_i$$

$$- \sum_{i=1}^{n} a_i X_i + bU - f(t) \qquad (4)$$

Let

$$a_i = a_i^0 + \Delta a_i \quad i = 1, \ldots, n$$

$$b = b^0 + \Delta b$$

where a_i^0 and b^0 are nominal values of a_i and b, and Δa_i and Δb are the variations of a_i and b, respectively.

Let the control function U be decomposed as

$$U = U_{eq} + \Delta U \qquad (5a)$$

where U_{eq}, called equivalent control, is defined as the solution of the problem $\dot{\sigma} = 0$ under $f(t) = 0$, $a_i = a_i^0$ and $b = b^0$. That is

$$U_{eq} = \left[c_1 K(r - X_1) - \sum_{i=2}^{n} c_{i-1} X_i + \sum_{i=1}^{n} a_i^0 X_i \right] \Big/ b^0 \qquad (5b)$$

ΔU is used to eliminate the influence due to the plant parameter variations in Δa_i, Δb and the disturbances $f(t)$ to guarantee the existence of a sliding mode. This function is constructed as follows:

$$\Delta U = \Psi_1 (X_1 - KZ) + \sum_{i=2}^{n} \Psi_i X_i \qquad (5c)$$

where

$$\Psi_1 = \begin{cases} \alpha_1 & \text{if } (X_1 - KZ)\sigma > 0 \\ \beta_1 & \text{if } (X_1 - KZ)\sigma < 0 \end{cases}$$

$$\Psi_i = \begin{cases} \alpha_i & \text{if } X_i \sigma > 0 \\ \beta_i & \text{if } X_i \sigma < 0 \end{cases} \quad i = 2, \ldots, n \qquad (5d)$$

It is known that the condition for the existence of a sliding motion is [1]

$$\lim_{\sigma \to 0} \sigma \dot{\sigma} < 0 \qquad (6)$$

Substitution of eqn. 5 into eqn. 4 yields

$$\dot{\sigma} = -\sum_{i=1}^{n} \Delta a_i X_i - f(t) + \Delta b U_{eq} + b \, \Delta U$$

$$= -\sum_{i=1}^{n} \Delta a_i X_i - f(t)$$

$$+ \Delta b/b^0 \left[c_1 K(r - X_1) - \sum_{i=2}^{n} c_{i-1} X_i + \sum_{i=1}^{n} a_i^0 X_i \right]$$

$$+ b \left[\Psi_1 (X_1 - KZ) + \sum_{i=2}^{n} \Psi_i X_i \right] \qquad (7)$$

Then

$$\lim_{\sigma \to 0} \dot{\sigma} \sigma = \left(-\Delta a_1 + \frac{a_1^0 \Delta b}{b^0} + b\Psi_1 \right)(X_1 - KZ)\sigma$$

$$+ \sum_{i=2}^{n} \left[\left(-\Delta a_i + \frac{a_i^0 \Delta b}{b^0} \right. \right.$$

$$\left. \left. - \frac{c_{i-1} \Delta b}{b^0} + b\Psi_i \right) X_i \sigma \right]$$

$$+ \left\{ -KZ\left(\Delta a_1 - \frac{a_1^0 \Delta b}{b^0} \right) \right.$$

$$+ \frac{\Delta b}{b^0} [c_1 K(r - X_1)] - f(t) \bigg\} \sigma \qquad (8)$$

If we neglect the term $N(t) \equiv \{ -KZ(\Delta a_1 - a_1^0 \Delta b/b^0) + \Delta b/b^0 [c_1 K(r - X_1)] - f(t)\}\sigma$ in eqn. 8, then the conditions for satisfying the inequality (eqn. 6) are

$$\Psi_i = \begin{cases} \alpha_i < (\Delta a_i - a_i^0 \Delta b/b^0 + c_{i-1} \Delta b/b^0)/b \\ \beta_i > (\Delta a_i - a_i^0 \Delta b/b^0 + c_{i-1} \Delta b/b^0)/b \end{cases}$$

$$i = 1, \ldots, n \quad c_0 = 0 \qquad (9)$$

However, the term $N(t)$ may not be neglected in the presence of an input command, the plant parameter variations and/or the external disturbances. Hence, once the effect of the term $N(t)$ exceeds the other two terms in eqn. 8 so that the inequality of eqn. 6 is violated, then the sliding mode breaks down and the system gives rise to a limit cycle. Fortunately, by increasing the control gain Ψ_i the effect due to the term $N(t)$ can be arbitrarily suppressed so that the magnitude of the limit cycle can be reduced to within a tolerable range, the validity of the assumption can be shown by the simulation, and hence a quasi-ideal sliding motion can be obtained.

2.2 Determination of switching plane and integral control gain

While in the sliding motion, the system described by eqn. 1 can be reduced to the following linear equations [1–3]

$$\dot{X}_i = X_{i+1} \quad i = 1, \ldots, n-2 \qquad (10a)$$

$$\dot{X}_{n-1} = -\sum_{i=1}^{n-1} c_i X_i + c_1 KZ \qquad (10b)$$

$$\dot{Z} = r - X_1 \qquad (10c)$$

or, in matrix form

$$\dot{X} = AX + Br \qquad (11)$$

where

$$X = \begin{bmatrix} Z \\ -- \\ X_1 \\ \vdots \\ X_{n-1} \end{bmatrix}_{n \times 1}$$

$$A = \begin{bmatrix} 0 & -1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ c_1 K & -c_1 & -c_2 & \cdots & -c_{n-1} \end{bmatrix}_{n \times n}$$

$$B = \begin{bmatrix} 1 \\ -- \\ 0 \\ \vdots \\ 0 \end{bmatrix}_{n \times 1}$$

The closed-loop transfer function of the system described by eqn. 11 is

$$H(S) = \frac{X_1(S)}{R(S)}$$

$$= \frac{c_1 K}{S^n + c_{n-1} S^{n-1} + \cdots + c_1 S + c_1 K} \qquad (12)$$

where $R(S)$ and $X_1(S)$ are the Laplace transforms of r and X_1, respectively. The characteristic equation of the

system is

$$S^n + c_{n-1}S^{n-1} + \cdots + c_2 S^2 + c_1 S + c_1 K = 0 \qquad (13)$$

Because this characteristic equation is independent of the plant parameters, the IVSC approach is robust to the plant parameter variations. It can achieve a zero steady-state error and its eigenvalues can be set arbitrarily. Let the desired eigenvalues of the system be $\lambda_1, \ldots, \lambda_n$, or an equivalent desired characteristic equation

$$S^n + \alpha_1 S^{n-1} + \cdots + \alpha_n = 0$$

Then the switching plane coefficients c_i for $i = 1, \ldots, n - 1$ and the integral control gain K can be chosen as follows

$$c_{n-1} = \alpha_1$$
$$\vdots$$
$$c_1 = \alpha_{n-1}$$
$$K = \frac{\alpha_n}{\alpha_{n-1}}$$

3 Chattering

For the control law as given by eqn. 5, if Ψ_i $(i = 1, \ldots, n)$ are chosen as

$$\Psi_i = \alpha_i = -\beta_i$$

then the control function U can be represented as

$$U = \left[c_1 K(r - X_1) + \sum_{i=2}^{n} c_{i-1} X_i - \sum_{i=1}^{n} a_i^0 X_i \right] \Big/ b^0$$
$$+ \left(\Psi_1 | X_1 - KZ | + \sum_{i=2}^{n} \Psi_i | X_i | \right) \text{sign} \, (\sigma) \qquad (14)$$

Because the control U gives rise to chattering due to the sign function sign (σ), direct application of such a control signal to the plant may be impractical. To obtain a continuous control signal, the discontinuous sign function sign (σ) in eqn. 14 can be replaced by a proper continuous function [6–8] as

$$S_\delta(\sigma) = \frac{\sigma}{|\sigma| + \delta} \qquad (15)$$

where δ is a positive constant. However, under different operating conditions, the proper continuous function with a constant δ may not effectively eliminate the chattering phenomena. For improving the result, δ is chosen alternatively as a function of $| X_1 - r |$ as

$$\delta = \delta_0 + \delta_1 | X_1 - r |$$

where δ_0 and δ_1 are positive constants. Then the modified proper continuous function is given by

$$M_\delta(\sigma) = \frac{\sigma}{|\sigma| + \delta_0 + \delta_1 | X_1 - r |} \qquad (16)$$

4 Electrohydraulic velocity control problem

The block diagram of the electrohydraulic velocity control system to be studied is shown in Fig. 2. The objective of the control is to keep the velocity ω_c of the hydraulic system following the desired trajectory as closely as possible, regardless of the operating points.

The relation between the valve displacement X_v and the load flow rate Q_L is governed by the well known orifice law given by [4]

$$Q_L = X_v K_{j\sqrt{}}[P_s - \text{sign} \, (X_v)P_L] = X_v K_s \qquad (17)$$

where K_j is a constant for a specific hydraulic motor, P_s is the supply pressure, P_L is the load pressure and K_s is the valve flow gain which varies at different operating
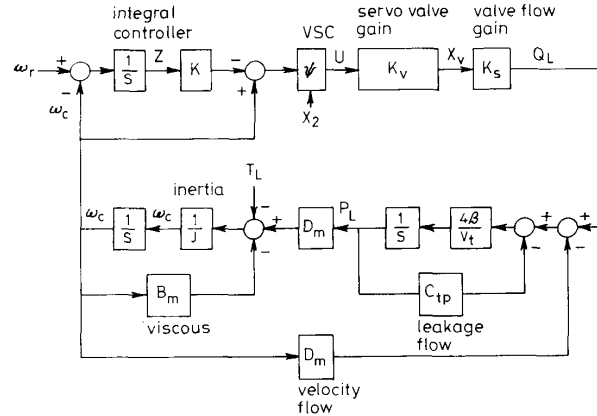

Fig. 2 *Block diagram of a hydraulic velocity control system with IVSC*

points. The flow continuity property of the servo valve and motor chamber yields

$$Q_L = D_m \omega_c + C_{tp} P_L + (V_t - 4\beta)\dot{P}_L \qquad (18)$$

where D_m is the volumetric displacement, C_{tp} is the total leakage coefficient, V_t is the total volume of the oil, β is the bulk modulus of the oil and ω_c is the velocity of the motor shaft. The torque balance equation for the motor is given by

$$D_m P_L = J\dot{\omega}_c + B_m \omega_c + T_L \qquad (19)$$

where B_m is the viscous damping coefficient and T_L is the external load disturbance which is assumed to be dependent upon the velocity of the shaft or slowly time varying as described by

$$T_L = f(X_1) \qquad (20)$$

The above form of the external load disturbance can frequently be found in industrial processes [5] involving hydraulic servo systems.

By combining eqns. 17–19, the servo valve gain K_v and the IVSC, we obtain a set of state equations of the integral-variable-structure-controlled electrohydraulic servo system as follows:

$$\dot{X}_1 = X_2 \qquad (21a)$$
$$\dot{X}_2 = -a_1 X_1 - a_2 X_2 + bU - f(t) \qquad (21b)$$
$$\dot{Z} = r - X_1 \qquad (21c)$$

where

$$a_1 = \frac{4\beta}{V_t} \frac{D_m^2}{J} + \frac{4\beta}{V_t} \frac{B_m}{J} C_{tp}$$

$$a_2 = \frac{B_m}{J} + \frac{4\beta}{V_t} C_{tp}$$

$$b = K_v K_s \frac{4\beta}{V_t} \frac{D_m}{J}$$

$$f(t) = \frac{4\beta}{V_t} \frac{C_{tp}}{J} T_L + \frac{1}{J} \dot{T}_L$$

$X_1 = \omega_c$ is the velocity of the motor shaft
$r = \omega_r$ is the reference input
$K =$ the gain of the integral controller.

Table 1: System parameters for simulation

Parameter	Value	Dimension
K_s	$2.3 \times 10^{-7} \times \sqrt{[P_s - \text{sign}\ (X_v)P_L]}$	m^2/s
P_s	1.4×10^7	Nt/m^2
β	3.5×10^7	Nt/m^2
V_t	3.3×10^{-5}	m^3/rad
C_{tp}	2.3×10^{-11}	$m^5/s/Nt$
D_m	1.6×10^{-5}	m^3/rad
J	5.8×10^{-3}	$kg\text{-}m\text{-}s^2$
B_m	0.864	$kg\text{-}m\text{-}s/rad$
K_v	0.5	m/V

Following the design procedure as described in the previous Section, we obtain

$$U = [c_1 K(r - X_1) + a_1^0 X_1 + a_2^0 X_2]/b^0$$
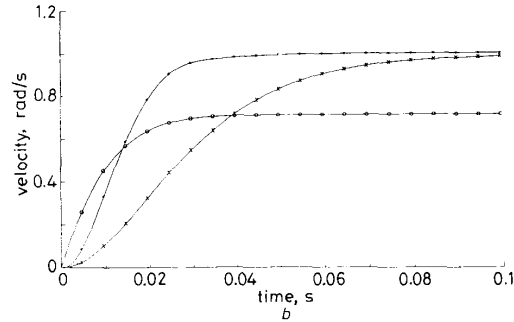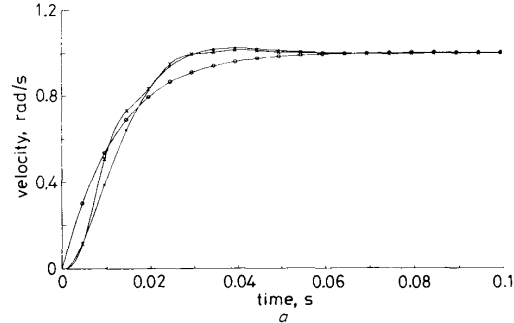$$+ (\Psi_1 |X_1 - KZ| + \Psi_2 |X_2|)M_\delta(\sigma) \qquad (22a)$$

Fig. 3 *Velocity responses of IVSC, VSC and linear approaches*

For $M_\delta(\sigma)$, $\delta = 20 + 100|X_1 - \omega_r|$
—●— IVSC
—○— VSC
—×— linear
$a\ T_L = 0$ kg-m
$b\ T_L = 20|\omega_c|$ kg-m

where a_1^0, a_2^0 and b^0 are nominal values of a_1, a_2 and b; Δa_1, Δa_2 and Δb are the deviation from a_1^0, a_2^0 and b^0, respectively, and

$$\Psi_i < -|\Delta a_i - a_i^0 \Delta b/b^0 + c_{i-1} \Delta b/b^0|/b$$
$$i = 1, 2 \quad c_0 = 0 \quad (22b)$$

The σ function is obtained from eqn. 3 as

$$\sigma = c_1(X_1 - KZ) + X_2 \qquad (23)$$

In the sliding motion, the system described by eqn. 21 can be reduced to the following simple linear form

$$\dot{X}_1 = -c_1(X_1 - KZ)$$
$$\dot{Z} = r - X_1 \qquad (24)$$

The characteristic equation of this reduced system is

$$S^2 + c_1 S + c_1 K = 0 \qquad (25)$$

It is clear that the dynamic performance of the system can now be determined by simply choosing the coefficient c_1 and the gain K. Let the characteristic equation of the system with desired eigenvalues λ_1 and λ_2 be

$$S^2 - (\lambda_1 + \lambda_2)S + \lambda_1 \lambda_2 = 0 \qquad (26)$$

Then c_1 and K can be chosen as

$$c_1 = -(\lambda_1 + \lambda_2) \qquad (27a)$$

$$K = \frac{-\lambda_1 \lambda_2}{(\lambda_1 + \lambda_2)} \qquad (27b)$$

5 Simulation results and discussion

The robustness of the proposed IVSC approach against large variations of plant parameters and external load disturbances have been simulated for demonstration. The results were compared to those obtained by a conventional VSC and a conventional linear controller. The nominal values of the hydraulic system parameters are listed in Table 1. By considering different operating points, we assume the range of the plant parameter variations to be

$$|\Delta a_1| < a_1^0 \times 500\%$$

$$|\Delta a_2| < a_2^0 \times 500\%$$

$$|\Delta b| < b^0 \times 500\%$$

Choosing the poles of the system as described by eqn. 24 at $-100 \pm j80$, we obtain the coefficients of the switching plane and the integral control gain given by eqn. 27 as $c_1 = 200$ and $K = 82$.

Thus, from eqn. 22b, the gains Ψ_1 and Ψ_2 must be chosen to satisfy the following inequalities:

$$\Psi_1 < -0.48 \qquad \Psi_2 < -0.00057$$

Based on simulations, one possible set of switching gains is chosen as follows:

$$\Psi_1 = -0.7 \qquad \Psi_2 = -0.002$$

This IVSC design gives a control function

$$U = [c_1 K(r - X_1) + a_1^0 X_1 + a_2^0 X_2]/b^0$$
$$+ (\Psi_1 |X_1 - KZ| + \Psi_2 |X_2|)M_\delta(\sigma)$$

where $M_\delta(\sigma)$ is given by eqn. 16 and $\sigma = 200(X_1 - KZ) + X_2$.

A conventional VSC approach is presented for performance comparison. In this approach, let the control function U be

$$U = [80(X_1 - r)(80 - a_2^0) + X_1 a_1^0]/b^0$$
$$+ (-0.5|X_1 - r| - 0.001|X_2|)M_\delta(\sigma)$$

where $\sigma = 80(X_1 - r) + X_2$. Also a conventional linear controller with the transfer function

$$\left(\frac{2.06}{S} \times \frac{S + 200}{S + 100} \times \frac{S + 48.3}{S + 56.8} \right)$$

has been chosen for comparison. It was designed with the aims of matching and dynamic response of the IVSC approach by means of a computer-aided technique [9].

The simulation results of the dynamic responses under various operating conditions are plotted in Figs. 3–7. Fig. 3 shows the dynamic responses of the three approaches when a shaft-velocity-dependent external load dis-

turbance T_L is present. It is clear that the response can almost be maintained for the IVSC approach, whereas it varies significantly for both the linear and the VSC approaches. Fig. 4 shows the dynamic responses of the three approaches under a constant external load dis-
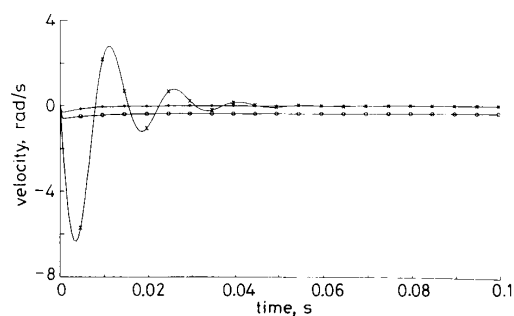

Fig. 4 *Velocity responses of the IVSC, VSC and linear approaches*

$T_L = 20$ kg-m, input command $\omega_r = 0$ rad/s and function $M_\delta(\sigma)$, $\delta = 20 + 100 |X_1 - \omega_r|$
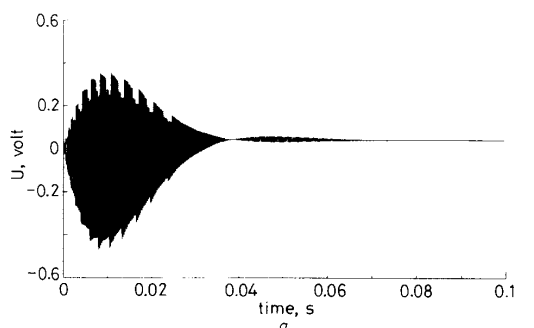—●— IVSC
—○— VSC
—×— linear
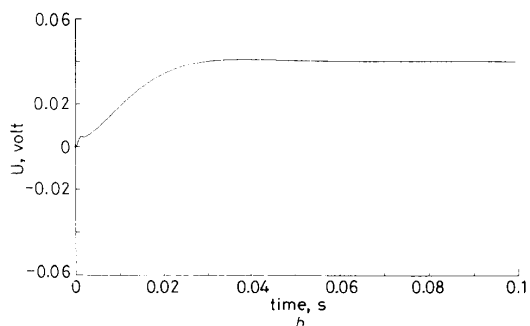
Fig. 5 *Control signal in the IVSC approach with $\omega_r = 1$ rad/sec*

a without $M_\delta(\sigma)$, $\delta = 0$
b with $M_\delta(\sigma)$, $\delta = 20 + 100 |X_1 - \omega_r|$

turbance. Results show that the IVSC approach results in fast convergence to the zero steady-state, but the other two approaches give rise to significant deviations and/or steady-state errors. Thus, we conclude that the proposed IVSC approach is robust to the external load disturbances.

Fig. 5 shows the waveforms of the control function U. It is clear that by using a modified proper continuous function, chattering phenomena can be eliminated and the strength of the control signal can also be significantly reduced. Thus, the IVSC approach seems amenable for practical implementation. Fig. 6 shows the dynamic responses of the IVSC approach under different values of

δ. It is obvious that the proposed approach with the value of δ dependent on $|X_1 - r|$ is robust to the different input command levels.
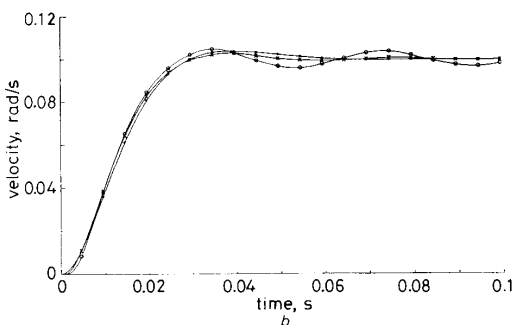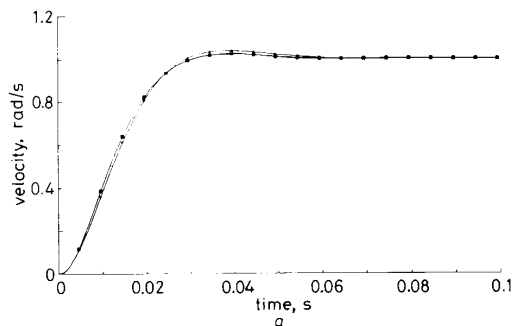

Fig. 6 *Velocity responses of the IVSC approach under different values of δ*

—●— $\delta = 0$
—○— $\delta = 100$
—×— $\delta = 20 + 100 |X_1 - \omega_r|$
a $\omega_r = 1$ rad/s
b $\omega_r = 0.1$ rad/s

Fig. 7 shows the dynamic responses of the three approaches under changes of the servo valve gain K_v, inertia J and total leakage coefficient C_{tp}, respectively. From the observations, we conclude that the IVSC approach is also insensitive to the variations of the parameters K_v, J and C_{tp}. This conclusion agrees with that predicted by the characteristic equation as given by eqn. 25 which should be independent of the plant parameters.

6 Conclusion

Most practical control systems are usually required to track an input signal in the presence of external disturbances. In this paper we have presented an IVSC configuration and developed a procedure for determining the coefficients of the switching plane and the integral control gain. It was shown in Section 2 that the IVSC approach is theoretically robust to the plant parameter variations. It can achieve a zero steady-state error for step input and its eigenvalues can be set arbitrarily. To solve the chattering problem, we also introduced a modified proper continuous function which can eliminate chattering under different operating conditions. An electrohydraulic velocity control problem was used to demonstrate the design procedure of the IVSC approach. Simulations showed that the proposed approach can give accurate servotracking response in the face of large plant parameter variations and external disturbances. It is a robust and practical control law for servomechanism systems.
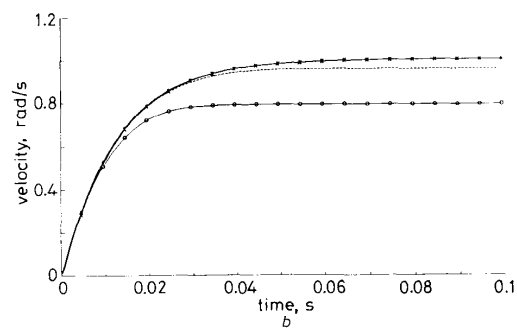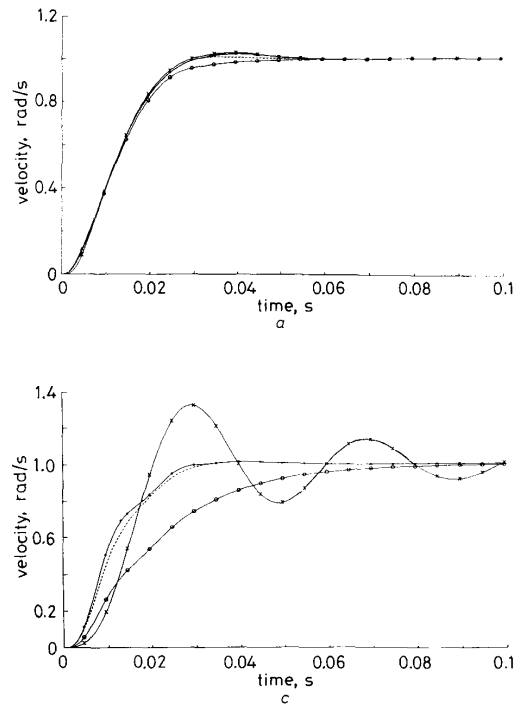
Fig. 7 *Velocity responses of the IVSC, VSC and linear approaches under changes of the servo valve gain K_v, inertia J and leakage coefficient C_{tp}*

a IVSC approach with function $M_\delta(\sigma)$, $\delta = 20 + 100|X_1 - \omega_r|$
—●— normal
—○— -50% changes in K_v
—×— 500% changes in J
– – – – 100% changes in C_{tp}
b VSC approach with function $M_\delta(\sigma)$, $\delta = 20 + 100|X_1 - \omega_r|$
—●— normal
—○— -50% changes in K_v
–×– 500% changes in J
– – – – 100% changes in C_{tp}
c Linear approach
—●— normal
—○— -50% changes in K_v
—×— 500% changes in J
– – – – 100% changes in C_{tp}

7   References

1 UTKIN, V.I.: 'Variable structure systems with sliding modes', *IEEE Trans.*, 1977, **AC-22**, pp. 212–222
2 UTKIN, V.I.: 'Sliding modes and their application in variable structure systems' (Mir: Moscow, 1978)
3 ITKIS, U.: 'Control systems of variable structure' (John Wiley, 1976)
4 MERRIT, H.E.: 'Hydraulic control system' (John Wiley, 1967)
5 Yun, J.S., and CHO, H.S.: 'Adaptive model following control of electrohydraulic velocity control systems subjected to unknown disturbances', *IEE Proc. D, Control Theory & Appl.*, 1988, **135**, pp. 149–156

6 AMBROSINO, G., GELENTANO, G., and GAROFALO, F.: 'Variable structure model reference adaptive control system', *Int. J. Contr.*, 1984, **39**, pp. 1339–1349
7 HASHIMOTO, H., MARUYAMA, K., and HARASHIMA, F.: 'A microprocessor-based robot manipulator control with sliding mode', *IEEE Trans.*, 1987, **IE-34**, pp. 11–18
8 YUNG, K.D.: 'A variable structure model following control design for robotics applications', *IEEE Trans., Robot. Autom.*, 1988, **4**, pp. 556–561
9 ASHWORTH, LT. CDR. M.J., and TOWILL, D.R.: 'Computer-aided design of tracking systems', *Radio & Electron. Eng.*, 1978, **48**, pp. 479–492