Theory of phase-conjugate oscillators. II

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This paper is the second in a series describing propagation of electromagnetic radiation in phase-conjugate oscillators. We apply the formulation developed in the first paper to study the phase-conjugate resonator (PCR) and the Fabry–Perot cavity with an intracavity phase-conjugate mirror (PCM). Our results show that nondegenerate oscillation occurs in the PCR for large parametric gains and fixed separation between the conventional reflector and the PCM. The bandwidth is considerably reduced from that of a PCM alone. In the phase-conjugate oscillator, oscillation will occur at the pump frequency only if there is no separation between the conventional mirrors and the phase-conjugate element and if the nonlinear medium fills the entire Fabry–Perot cavity.

INTRODUCTION

In Part I of this series[1] we formulated the power reflectivity and transmissivity as well as the oscillation condition in phase-conjugate oscillators. These oscillators consist of conventional Fabry–Perot cavities with an intracavity, transparent Kerr medium that is pumped externally by a pair of off-axis, counterpropagating pump beams. An external signal with a frequency different from that of the pump beams is injected into the cavity along its axis. Nearly degenerate four-wave mixing (NDFWM) generates a wave that is a phase conjugate of the signal beam. Additional waves arise owing to the presence of the conventional end mirrors of the cavity. Externally driven, intracavity four-wave mixing in Fabry–Perot resonators has been considered before by Agrawal[2] and Yaholam and Yariv[3] for saturable absorbers and photorefractive media, respectively, and for equal frequency of the interacting waves. In contrast to our study, Refs. 2 and 3 studied the external driving field that bounces back and forth between the two mirrors of the cavity as the counterpropagating pumps for the nonlinear medium, while another external, weak signal that is not part of the cavity probes such a device, thereby combining NDFWM with cavity operation.

We apply our general formulation of nondegenerate operation of phase-conjugate oscillators to the special case of a phase-conjugate resonator (PCR), a Fabry–Perot cavity with one conventional mirror, and a phase-conjugate mirror (PCM). This device has been studied extensively.[4–8] References 4–6 examined the transverse modes in a PCR. References 7 and 8 looked at the fields inside and outside a PCR without taking into account the nature of the nonlinear process responsible for phase-conjugate reflection. Reference 9 examined the effects of a noisy probe field incident upon a phase-conjugate Fabry–Perot resonator. Our theory describes nondegenerate oscillation in a PCR by taking into account the four-wave mixing process responsible for generating the phase-conjugate wave. Analytic expressions are obtained for power reflectivity and transmissivity as well as for the oscillation condition, showing the dependence of these parameters on the linear and parametric gain of the PCM, mirror reflectivities, and separation between mirrors and the PCM. Numerical plots are presented to illustrate this parameter dependence.

PHASE-CONJUGATE OSCILLATORS BOUNDED BY ONE CONVENTIONAL REFLECTOR AND ONE PHASE-CONJUGATE MIRROR

Figure 1 shows a linear optical resonator bounded by a PCM and one conventional mirror, r_2. The PCM is a transparent Kerr medium with linear gain (or loss), and it is pumped externally by a pair of off-axis, counterpropagating laser beams of frequency ω. A weak probe field at frequency $\omega + \delta$, where $\delta \ll \omega$, is incident upon the PCM from the left. The field E_2 at frequency $\omega - \delta$, which propagates backward to the left of the PCM, is generated by NDFWM at the PCM. The field E_4 at frequency $\omega + \delta$, which propagates backward to the left of the PCM, is generated by reflections off the conventional mirror of the beam transmitted by the PCM. Waves G_1 and G_3, at frequencies $\omega + \delta$ and $\omega - \delta$, respectively, propagate in the forward direction to the right of the conventional mirror and can be similarly interpreted. Following the formulation developed in Ref. 1, the complex phase-conjugate reflectivity r_p and transmissivity t_p and the coherent reflectivity r_s and transmissivity t_s are given by

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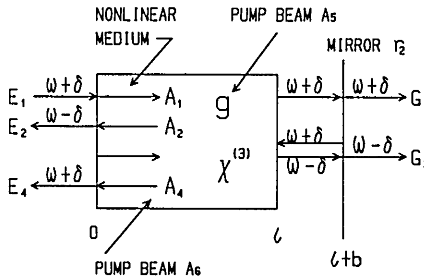

Fig. 1. Basic geometry of a linear phase-conjugate resonator by NDFWM. In this case the incident probe wave, whose frequency $\omega + \delta$ is slightly detuned from that of the pump waves (both at frequency ω), will result in a conjugate wave with an inverted frequency $\omega - \delta$. g is the linear nonsaturating background (intensity) net gain coefficient.

$$r_p = \frac{E_2}{E_1^*} = -i\kappa_2^*\beta\left[1 + \frac{|r_2|^2\alpha^2}{\exp(-2i\Delta kb) - \kappa_1\kappa_2^*\beta^2|r_2|^2}\right],$$

$$t_p = \frac{G_3}{E_1^*} = \frac{-i\kappa_2^*\beta\alpha r_2^*(1 - r_2^2)\exp(-i\Delta kl/2)\exp(-i\Delta kb + ik_1(l + b)]}{t_2[\exp(-2i\Delta kb) - \kappa_1\kappa_2^*\beta^2|r_2|^2]},$$

$$r_s = \frac{E_4^*}{E_1^*} = \frac{\alpha^2\exp[i(k_1 + k_2)(l + b)]}{\exp(-i\Delta kb) - \kappa_1\kappa_2^*\beta^2|r_2|^2\exp(i\Delta kb)},$$

$$t_s = \frac{G_1}{E_1} = \frac{(1 - r_2^2)\alpha^*\exp[-ik_1(l + b)]\exp(i\Delta kl/2)\exp(2i\Delta kb)}{t_2[\exp(2i\Delta kb) - \kappa_1^*\kappa_2\beta^{*2}|r_2|^2]}, \qquad (1)$$

where

$$\kappa_i^* = \frac{\omega_i}{2}(\mu/\epsilon)^{1/2}\chi^{(3)}\mathcal{A}_5\mathcal{A}_6\exp(gl/2) = \kappa_i^{*\prime}\exp(gl/2),$$

$$\alpha = \frac{1}{D}s, \qquad \Delta k = k_1 - k_2,$$

$$\beta = \frac{2}{D}\sinh\left(\frac{sl}{2}\right),$$

$$s = [-(\Delta k - ig)^2 - 4\kappa_1\kappa_2^*]^{1/2},$$

$$D = (-g - i\Delta k)\sinh(sl/2) + s\cosh(sl/2). \qquad (2)$$

The oscillation condition is

$$\kappa_1\kappa_2^*\beta^2|r_2|^2 = \exp(-2i\Delta kb). \qquad (3)$$

Note that the power reflectivities R_p and R_s, power transmissivities T_p and T_s, and the oscillation condition are independent of the phase introduced by reflection or transmission at mirror 2, where $R_p = |r_p|^2$, $R_s = |r_s|^2$, $T_p = |t_p|^2$, and $T_s = |t_s|^2$. Note that in the absence of mirror 2 ($r_2 = 0$) $r_s = t_p = 0$, while r_p and t_s reduce to the results of Ref. 1 for a PCM, with the exception that wave G_1 is now measured at $l + b$ instead of l.

We now discuss four different operation conditions: (1) $\Delta k = 0, g = 0, \kappa_1 = \kappa_2 = \kappa$; (2) $\Delta k = 0, g \neq 0, \kappa_1 = \kappa_2 = \kappa$; (3) $\Delta k \neq 0, g = 0, \kappa_1^*\kappa_2 > 0$; (4) $\Delta k \neq 0, g \neq 0, \kappa_1^*\kappa_2 > 0$. For each case, we discuss the complex phase-conjugate reflectivity and transmissivity, coherent reflectivity and transmissivity, and oscillation condition.

Condition (1). $\Delta k = 0, g = 0, \kappa_1 = \kappa_2 = \kappa$

Condition (1) is the case of degenerate four-wave mixing (DFWM) in the phase-conjugate element of the PCR.

By Eqs. (1) and (2) the complex phase-conjugate reflectivity r_p and transmissivity t_p and coherent reflectivity r_s and transmissivity t_s are given by

$$r_p = -i\frac{\kappa^*}{|\kappa|}\tan|\kappa|l\frac{1 + |r_2|^2}{1 - |r_2|^2\tan^2|\kappa|l},$$

$$t_p = -\frac{i\kappa^*}{|\kappa|}\tan|\kappa|l\sec|\kappa|l\exp[ik(l + b)]\frac{t_2 r_2^*}{1 - |r_2|^2\tan^2|\kappa|l},$$

$$r_s = \sec^2|\kappa|l\exp[2ik(l + b)]\frac{r_2^*}{1 - |r_2|^2\tan^2|\kappa|l},$$

$$t_s = \sec|\kappa|l\exp[-ik(l + b)]\frac{t_2}{1 - |r_2|^2\tan|\kappa|l}, \qquad (4)$$

where $k = k_1 = k_2$. Note that, if $r_2 = 0$, then we recover the results of Refs. 1 and 9 for DFWM in a PCM. Identifying $\tan^2|\kappa|l$ and $\sec^2|\kappa|l$ as the phase-conjugate power reflectivity and coherent transmissivity of a PCM,[1,9] we note that Eq. (5) below is similar to the results obtained in Ref. 8 for the fields in a PCR. The differences arise because Vesperinas[8] considers the external probe field to be incident upon the mirror instead of upon the PCM and does not account for the nonlinear process responsible for phase conjugation at the PCM.

By Eqs. (2) and (3) the oscillation condition is

$$|r_2|^2\tan^2|\kappa|l = 1. \qquad (5)$$

Equation (5) is identical to the result obtained in Ref. 9 and shows that, if mirror 2 is perfectly reflecting, oscillation occurs at $|\kappa|l = \pi/4$, which is lower by a factor of 2 than that for a PCM alone.

Note that for the PCR the oscillation condition [Eq. (5)] and the four power coefficients in Eq. (4) are independent of separation b between the PCM and the conventional mirror.

Condition (2). $\Delta k = 0, g \neq 0, \kappa_1 = \kappa_2 = \kappa$

Condition (2) is the case of DFWM in the nonlinear medium with linear gain or loss in addition to the parametric gain.

If one substitutes Eq. (2) into Eq. (3) and simplifies, the oscillation condition becomes

$$\tan\{[|\kappa|^2 - (g/2)^2]^{1/2}l\} = \frac{2[|\kappa|^2 - (g/2)^2]^{1/2}}{g + 2|\kappa r_2|}. \qquad (6)$$

Equation (6) shows how the presence of linear gain (or loss) will affect the values of parametric gain required for oscillation in a PCR.

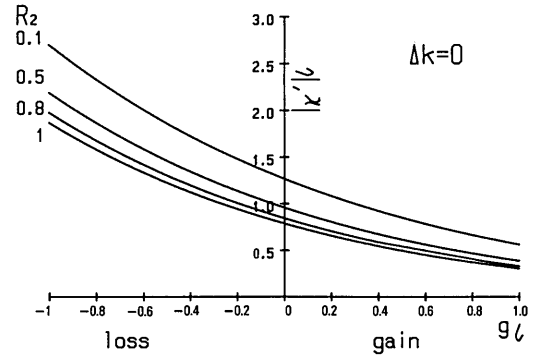

Fig. 2. Parametric gain $|\kappa'|l$ versus linear gain gl at the oscillation condition for $\Delta k = 0$ and several values of conventional mirror's reflectivity: $|r_2|^2 = 1$, 0.8, 0.5, and 0.1.

The resonator oscillation frequency ω is also independent of the separation between the two reflectors for DFWM in the PCM.

In Fig. 2 we plot the parametric gain $\kappa' l$ required for oscillation versus the linear gain or loss gl for several values of conventional mirror reflectivity: $|r_2|^2 = 1$, 0.8, 0.5, and 0.1. When there is no linear gain or loss ($g = 0$) and $|r_2|^2 = 1$, the coupling strength required for oscillation is $\pi/4$, as expected from Ref. 9 for a PCM in the presence of a perfectly reflecting mirror. Linear absorption losses in the medium substantially increase the threshold value of $|\kappa'|l$ at which oscillation will occur. If the medium were somehow to exhibit linear gain instead of absorption, then the threshold value of the coupling strength would be correspondingly lowered owing to the additional gain available from the medium. Also, for a given linear gain or loss in the nonlinear medium, the threshold value of $|\kappa'|l$ for oscillation increases as losses at the conventional mirror of the PCR increase.

Condition (3). $\Delta k \neq 0, g = 0, \kappa_2\kappa_2^* > 0$

Condition (3) is the case of NDFWM in the PCM of the PCR.

The oscillation condition can be separated into an amplitude part and a phase part. The equality of the amplitude part yields

$$4|r_2|^2\kappa_1\kappa_2^* \sin^2\frac{(\Delta k^2 + 4\kappa_1\kappa_2^*)^{1/2}}{2}l = \Delta k^2 + 4\kappa_1\kappa_2^* \cos^2\frac{(\Delta k^2 + 4\kappa_1\kappa_2^*)^{1/2}}{2}l, \quad (7)$$

and the equality phase part yields

$$\tan(b\Delta k) = -\frac{\Delta k}{(\Delta k^2 + 4\kappa_1\kappa_2^*)^{1/2}} \tan\frac{(\Delta k^2 + 4\kappa_1\kappa_2^*)^{1/2}}{2}l. \quad (8)$$

Owing to NDFWM in the PCM of the PCR, the oscillation condition is now a function of separation b between the PCM and the conventional mirror. For practical applications, if we choose $b = 0$ (i.e., the conventional mirror is placed exactly at the back of the PCM), and if we consider nondegenerate oscillation ($\Delta k \neq 0$), then Eq. (8) requires that $(\Delta k^2 + 4\kappa_1\kappa_2^*)^{1/2}l = 2p\pi$, where p is an integer. This however cannot satisfy Eq. (7) because $\kappa_1\kappa_2^*$ is a real and positive number. Hence, if $b = 0$, oscillation will occur only at the pump frequency when $\Delta k = 0$. As shown in Fig. 2, the minimum parametric gain required for oscillation will occur for a perfectly reflecting conventional mirror when $|r_2|^2 = 1$. Hence for this special case the oscillation condition of Eq. (7) can be simplified to

$$\cos(\Delta k^2 + 4\kappa_1\kappa_2^*)^{1/2}l = -\frac{\Delta k^2}{4\kappa_1\kappa_2^*}. \quad (9)$$

Since the magnitude of the cosine function cannot exceed unity, by Eq. (9), oscillation can be achieved only when the absolute value of the maximum wave number detuning $|\Delta k|$ is equal to or less than $2(\kappa_1\kappa_2^*)^{1/2}$. Let $\kappa_1\kappa_2^* = \kappa^2$; then Eq. (9) can be solved for κl as a function of $\Delta kl/2\pi$ by numerical methods and we can obtain b/l from Eq. (8).

We plot the phase-conjugate power reflectivity R_p versus normalized wavelength detuning $\Psi = \Delta kl/2\pi = (-\Delta\lambda/2)(2nl/\lambda^2)$ in Fig. 3. The dotted–dashed curve represents the case of no conventional mirror, i.e., an ordinary PCM based on NDFWM, with a parametric gain of $\pi/2$. The solid curve corresponds to a linear resonator with a PCM and a perfectly reflecting conventional mirror placed exactly at the back of the PCM. The parametric gain for this curve is $\pi/4$. Note that the introduction of the mirror not only reduces the parametric gain required for oscillation but also dramatically decreases the bandwidth of the filter. This is because feedback from the mirror in the form of reflected light along the axis of the resonator increases the internal gain available from the PCM, thus increasing the finesse of the resonator.[10] Oscillation occurs at pump frequency in both cases. A larger value for parametric gain above this threshold value ($\kappa l = 1.110\,65$, as in the dashed curve of the same figure) shows that nondegenerate oscillation is now possible at $\Delta kl/2\pi = \pm 0.353$, but for the fixed separation of $b/l = 0.707\,22$. The bandwidth of the dashed curve is less than that of the solid curve.

Condition (4). $\Delta k \neq 0, g \neq 0, \kappa_1^*\kappa_2 > 0$

Condition (4) is the case of NDFWM in the externally pumped nonlinear medium with linear gain or loss besides the usual parametric gain.

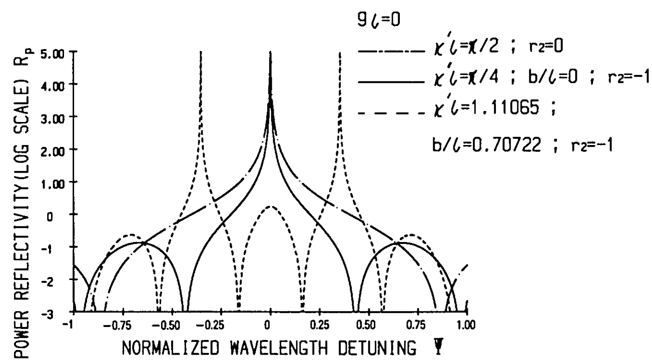

Fig. 3. Phase-conjugate power reflectivity R_p versus normalized wavelength detuning Ψ for linear gain $g = 0$. The dotted–dashed curve represents the case of no conventional mirrors and $\kappa l = \pi/2$. The solid and dashed curves correspond to a linear resonator with a PCM and a perfectly reflecting conventional mirror at its two ends with nonlinear gain $\kappa l = \pi/4$ and $\kappa l = 1.110\,65$, respectively. Note that when nonlinear gain is above $\pi/4$, nondegenerate oscillation is possible and the bandwidth of the dashed line is narrower than the other two lines.

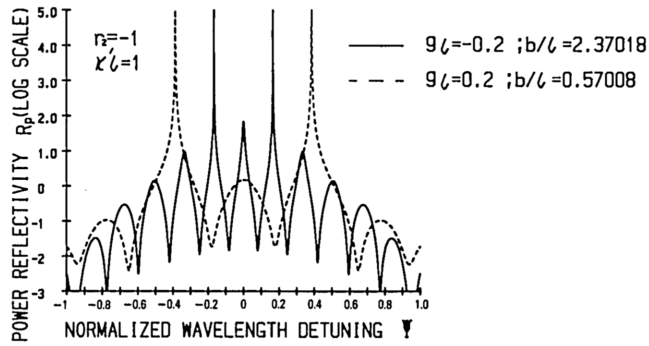

Fig. 4. Phase-conjugate and coherent power reflectivities R_p versus normalized wavelength detuning Ψ with $r_2 = -1$, $\kappa' l = 1$, for $gl = -0.2$, $b/l = 2{,}37018$ and $gl = 0.2$, $b/l = 0.57008$. These graphs show that for constant nonlinear gain $\kappa' l$, it is possible to detune the frequency of oscillation if we vary linear gain gl and choose a suitable b/l.

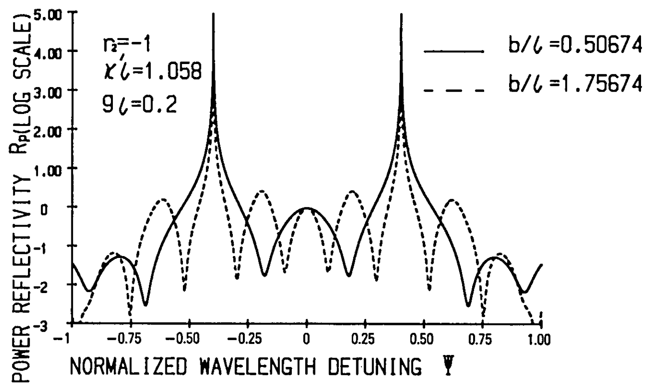

Fig. 5. Phase-conjugate power reflectivity versus normalized wavelength detuning at oscillation to show the effect of different b/l.

Numerical plots of the phase-conjugate reflectivity of the PCR are shown in Fig. 4 as a function of normalized wavelength detuning Ψ, respectively for $r_2 = -1$, $\kappa' l = 1$, and linear gain $gl = \pm 0.2$. For filter applications, linear gain ($gl > 0$) is better than linear loss ($gl < 0$) because the central peak (at $\Delta kl/2\pi = 0$) is smaller for linear gain than for linear loss, and the sidelobe structure is reduced.

We may rewrite Eq. (3) as

$$\kappa_1 \kappa_2^* \beta^2 |r_2|^2 = \exp[-i(2\Delta k b + 2\pi p)], \tag{10}$$

where p is an integer. The b/l has a multiple-valued solution for the phase part of Eq. (10) when $\kappa' l$, gl, and Ψ are fixed. We plot the phase-conjugate power reflectivity versus normalized wavelength detuning in Fig. 5 to show the effects of different values of b/l corresponding to $p = 0, 1$ on the mode spacing at the oscillation condition. This figure shows that, when we increase p (i.e., increase separation b/l), we will produce the detrimental effect of decreasing the mode spacing and increasing the power reflectivity in sidelobes other than that at the oscillation frequency.

PHASE-CONJUGATE OSCILLATOR

Figure 6 shows an optical resonator that consists of two partially reflecting mirrors, r_1 and r_2, and an intracavity PCM that provides linear gain (or loss) besides parametric gain through FWM.

Condition (1). $g = 0, \kappa_1 = \kappa_2 = 0, \Delta k = 0, R_1 R_2 \neq 0$
Condition (1) is the case of a Fabry–Perot cavity, with no intracavity PCM.

The coherent reflection and transmission are

$$r_s = \frac{r_2^* \exp(ikd) + r_1^* \exp(-ikd)}{r_1^* r_2^* \exp(ikd) + e(-ikd)},$$

$$t_s = \frac{1}{t_1 t_2}\left[\frac{1 + r_1^2 r_2^2 - r_1^2 - r_2^2}{r_1 r_2 \exp(-ikd) + \exp(ikd)}\right], \tag{11}$$

where $d = l + a + b$, and $k = k_1 = k_2$. These two equations can reduce to Airy's formula. Note that $t_s = 0$ if $r_2^2 = 1$, since there is no transmitted wave G_1 if mirror 2 is perfectly reflecting. Also note that $r_p = t_p = 0$, as expected in the absence of the intracavity PCM.

Condition (2). $g \neq 0, \kappa_1 = \kappa_2 = 0, \Delta k = 0, R_1 R_2 \neq 0$
Condition (2) is the case of a Fabry–Perot cavity with an intracavity linear gain medium.

By using $r_1 = \sqrt{R_1}$, $r_2 = \sqrt{R_2}$, we can write the oscillation condition as

$$1 + R_1 R_2 \exp(2gl) - 2\sqrt{R_1 R_2} \exp(gl) \cos(2kd) = 0, \tag{12}$$

where R_1 and R_2 are real numbers. Solving this equation, we obtain

$$\sqrt{R_1 R_2} \exp(gl) = 1, \tag{13a}$$

which means that laser round-trip gain must be equal to unity, and

$$2n \frac{\omega}{c}(l + a + b) = 2p\pi, \tag{13b}$$

where p is an integer. Hence in a Fabry–Perot cavity with an intracavity gain medium the net phase change acquired in one round trip must be an integral multiple of 2π.

Condition (3). $g = 0, \kappa_1 = \kappa_2 = \kappa, \Delta k = 0, R_1 R_2 \neq 0$
Condition (3) is the case of DFWM in the PCM bounded by two conventional mirrors (see Ref. 7).

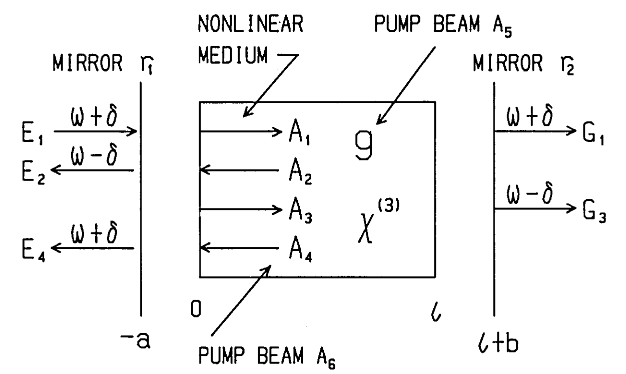

Fig. 6. Basic geometry of linear phase-conjugate oscillator by NDFWM. In this case, the incident probe wave, whose frequency $\omega \pm \delta$ is slightly detuned from that of the pump waves (both at frequency ω), will result in a conjugate wave with an inverted frequency $\omega \mp \delta$. g is the linear nonsaturating background (intensity) net gain coefficient.

By using the formulation developed in Ref. 1, we obtain the phase-conjugate reflection coefficient

$$r_p = \frac{-t_1(1 - r_1^{*2})(1 + R_2)i\frac{\kappa^*}{|\kappa|}\tan|\kappa|l}{t_1^*\{1 + R_1R_2 - \tan^2|\kappa|l(R_1 + R_2) + (1 + \tan^2|\kappa l|)[r_1r_2\exp(-2ikd) + r_1^*r_2^*\exp(2ikd)]\}}. \quad (14)$$

Equation (14) shows that when $\kappa = 0$, then $r_p = 0$, which means that, when there is no four-wave mixing, the phase-conjugate wave cannot be generated. Note also that, if $R_1 = 0$, we recover the results for r_p [Eq. (4)] for the PCR. If $R_1 = 1$, then $r_p = 0$ because in that case phase-conjugate light cannot be transmitted to the left by mirror 1.

Similarly, we can obtain the coherent reflection coefficient

$$r_s = \frac{(1 - \tan^2|\kappa|l)[r_2^*\exp(2ikd) + R_1r_2\exp(-2ikd)] + r_1\tan^2|\kappa|l + r_1^* + R_2(r_1^*\tan^2|\kappa|l + r_1)}{1 + R_1R_2 - \tan^2|\kappa|l(R_1 + R_2) + (1 + \tan^2|\kappa l|)[r_1r_2\exp(-2ikd) + r_1^*r_2^*\exp(2ikd)]}, \quad (15)$$

where $R_i = |r_i|^2$ and $i = 1, 2$. Note that this equation reduces to Airy's formula as given by Eq. (11) when $|\kappa|l = 0$. For oscillation to occur the denominator of Eq. (15) must be zero. The resulting equation can reduce to the result of Yeh.[11]

Condition (4). $g \neq 0, \kappa_1 = \kappa_2 = \kappa, \Delta k = 0, R_1R_2 \neq 0.$ Condition (4) is the case of DFWM in a transparent medium with linear gain (or loss) and parametric gain bounded by two conventional mirrors.

In Fig. 7 we plot the threshold parametric gain $|\kappa'|l$ as a function of the normalized cavity length $\phi = k(l + a + b) = kd$ with $r_2 = -1$, $r_1 = \sqrt{0.9}$, and linear gain (or loss) $gl = \pm 0.052\,68$. Note that for cavity lengths in which $\phi = \gamma\pi$, oscillation can still occur for γ not an integer, and the minimum threshold occurs when γ is an integer. In Fig. 8 we plot the parametric gain $\kappa'l$ versus the linear gain gl at threshold oscillation conditions for varius γ when $|r_2| = -1$, $|r_1| = \sqrt{0.9}$. The effects of gain (or loss) on oscillation are evident. When γ is an integer (minimum threshold) and the linear gain of the medium reaches its threshold value $gl = 1/2\ \ln(1/R_1R_2)$, then $\kappa'l = 0$ for oscillation, as given by Eq. (13a).

Condition (5). $g = 0, \Delta k \neq 0, |\kappa_1\kappa_2| \neq 0, R_1R_2 \neq 0$ Condition (5) is the case of NDFWM in the transparent nonlinear medium bounded by two conventional mirrors.

Consider condition (1) of this section, with $a = b = 0$, $\kappa_1\kappa_2^* = \kappa^2$, and adjust the phase of r_1 and r_2 such that

$$r_1r_2\exp[-i(k_1 + k_2)l] + r_1^*r_2^*\exp[i(k_1 + k_2)l] = 2|r_1r_2|\cos[(k_1 + k_2)l + \xi], \quad (16)$$

where ξ is the total phase of reflection from mirrors r_1 and r_2. The oscillation condition reduces to

$$2(\alpha^*)^2|r_1r_2|\cos[(k_1 + k_2)l + \xi] + R_1R_2[(\alpha^*)^2 - \kappa^2(\beta^*)^2]^2 + 1 - \kappa^2(\beta^*)^2(R_1 + R_2) = 0. \quad (17)$$

By separating Eq. (17) into real and imaginary parts, we can reduce it to

$$(1 - R_1R_2)\Delta k(\Delta k^2 + 4\kappa^2)^{1/2}\sin(\Delta k^2 + 4\kappa^2)^{1/2}l = 0, \quad (18a)$$

$$2\sqrt{R_1R_2}(\Delta k^2 + 4\kappa^2)\cos[(k_1 + k_2)l + \xi] + (R_1R_2 + 1) \times \left(\Delta k^2\cos(\Delta k^2 + 4\kappa^2)^{1/2}l + 4\kappa^2\cos^2\frac{(\Delta k^2 + 4\kappa^2)^{1/2}}{2}l\right) - 4\kappa^2(R_1 + R_2)\sin^2\frac{(\Delta k^2 + 4\kappa^2)^{1/2}}{2}l = 0. \quad (18b)$$

Note that when $\Delta k = 0$ Eq. (18a) is always satisfied and Eq. (18b) reduces to the result of Ref. 12.

For $R_1R_2 < 1$ and $\Delta k \neq 0$, $\sin(\Delta k^2 + 4\kappa^2)^{1/2}l = 0$, and Eq. (18b) reduces to

$$\cos[(k_1 + k_2)l + \xi] = -\frac{1 + R_1R_2}{2\sqrt{R_1R_2}}$$

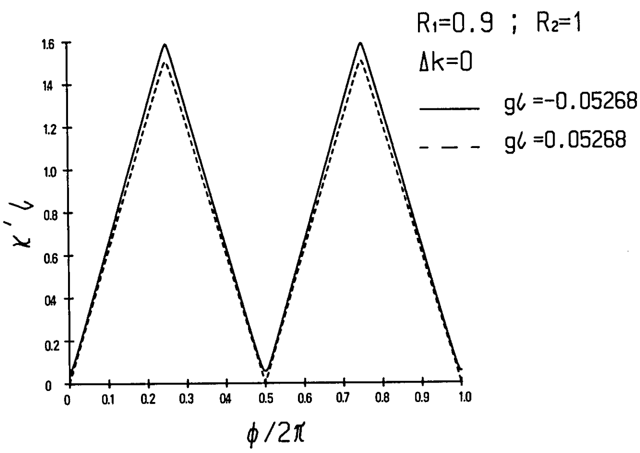

Fig. 7. Threshold parametric gain $\kappa'l$ required for oscillation as a function of normalized cavity length $\phi \equiv k(l + a + b)$ with $r_2 = -1$ and $r_1 = \sqrt{0.9}$ for linear gain ($gl = 0.052\,68$) and linear loss ($gl = 0.052\,68$).

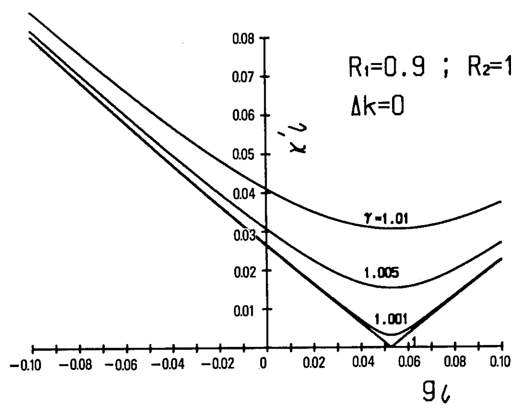

Fig. 8. Parametric gain $\kappa'l$ versus linear gain gl at threshold oscillation conditions for varius γ, where $kd = \gamma\pi$, $r_2 = -1$, and $r_1 = \sqrt{0.9}$.

or

$$\Delta k^2\{2\sqrt{R_1R_2}\cos[(k_1+k_2)l+\xi]+R_1R_2+1\} + 4\kappa^2\{2\sqrt{R_1R_2}\cos[(k_1+k_2)l+\xi]-R_1-R_2=0, \quad (19)$$

which is not possible for $R_1R_2 < 1$. Hence, when $a = b = 0$, oscillation can occur only for $\Delta k = 0$. Hence nondegenerate oscillation is not possible if the nonlinear medium fills the entire Fabry–Perot cavity.

For condition (2) of this section, let $r_1 = \sqrt{R_1}$, $r_2 = -1$, and $\kappa^2 = \kappa_1\kappa_2^*$, $ab \neq 0$, where R_1 is a real number and $\kappa_i^* = \omega_i/2\sqrt{\mu/\varepsilon}\chi^3\mathcal{A}_5\mathcal{A}_6$. Then the oscillation condition simplifies to

$$\kappa^2(\beta^*)^2\{R_1\exp[i\Delta k(b-a)]+\exp[-i\Delta k(b-a)]\} - \exp[i\Delta k(a+b)] - R_1[(\alpha^*)^2-\kappa^2(\beta^*)^2]^2\exp[-i\Delta k(a+b)] = -2\sqrt{R_1}(\alpha^*)^2\cos[(k_1+k_2)(l+a+b)]. \quad (20)$$

Comparing the imaginary parts of Eq. (20), we obtain

$$4\kappa^2\sin^2\frac{(\Delta k^2+4\kappa^2)^{1/2}}{2}l\sin\Delta k(b-a) + \left(\Delta k^2\cos(\Delta k^2+4\kappa^2)^{1/2}l + 4\kappa^2\cos^2\frac{(\Delta k^2+4\kappa^2)^{1/2}}{2}l\right) \times \sin\Delta k(a+b) + \Delta k(\Delta k^2+4\kappa^2)^{1/2}\sin(\Delta k^2+4\kappa^2)^{1/2}l \times \cos\Delta k(a+b) = 0, \quad (21)$$

and, comparing the real parts of Eq. (20), we obtain

$$\frac{2r_1}{1+R_1}(\Delta k^2+4\kappa^2)\cos[(k_1+k_2)(l+a+b)] = -4\kappa^2\sin^2\frac{(\Delta k^2+4\kappa^2)^{1/2}}{2}l\cos\Delta k(b-a) + \left[\Delta k^2\cos(\Delta k^2+4\kappa^2)^{1/2}l + 4\kappa^2\cos^2\frac{(\Delta k^2+4\kappa^2)^{1/2}}{2}l\right] \times \cos\Delta k(a+b) - \Delta k(\Delta k^2+4\kappa^2)^{1/2} \times \sin(\Delta k^2+4\kappa^2)^{1/2}l\sin\Delta k(a+b). \quad (22)$$

By canceling $\Delta k(b-a)$ and from Eqs. (21) and (22), we can obtain a single formula that is a function of κl, Δkl, $(k_1+k_2)l$, and $(a+b)/l$ only. Then we can use Mueller's iteration scheme of successive bisection and interpolation to find κl for a given Δkl, $(a+b)/l$, and $(k_1+k_2)l$. Actually, because $(k_1+k_2)l$ is much greater than Δkl, when we change $(a+b)/l$ the term $\cos(k_1+k_2)(l+a+b)$ on the left-hand side of Eq. (22) will oscillate much faster than $\cos\Delta k(b+a)$ and $\sin\Delta k(b+a)$ of Eq. (21) and the right-hand side of Eq. (22). So, when a pumping frequency is chosen [i.e., $(k_1+k_2)l$ = constant], we can choose X and Y as independent variables such that

$$X = \sin[\Delta k(a+b)], \quad Y = \cos[(k_1+k_2)(l+a+b)] \quad (23)$$

and introduce integers (p and q) such that

$$1+\frac{a+b}{l} = \frac{1}{(k_1+k_2)}(\cos^{-1}Y+2\pi q), \quad \Delta kl = \frac{\sin^{-1}X+2\pi p}{(a+b)/l}. \quad (24)$$

Then we can solve κl by using Eqs. (21) and (22). Figure 9 shows the X–Y plane versus nonlinear parametric gain κl for $(k_1+k_2)l = 10\,000$, $(p,q) = (1,10\,000)$, $r_1 = \sqrt{0.9}$, $r_2 = -1$, and reflection index $n = 1.62$ at the oscillation condition. Note that q must be greater than $(k_1+k_2)l$ such that $(a+b)/l$ is positive, that, using Eq. (21), we can obtain $(b-a)/l$, and that Δkl is dependent on (p,q), so that the solution of Eqs. (21) and (22) for κl is also dependent on (p,q).

Condition (6). $g \neq 0, \Delta k \neq 0, \kappa_1\kappa_2^* = \kappa^2, R_1R_2 \neq 0$
Condition (6) is the case of NDFWM in the transparent nonlinear medium that exhibits linear and parametric gain and is bounded by two conventional mirrors.

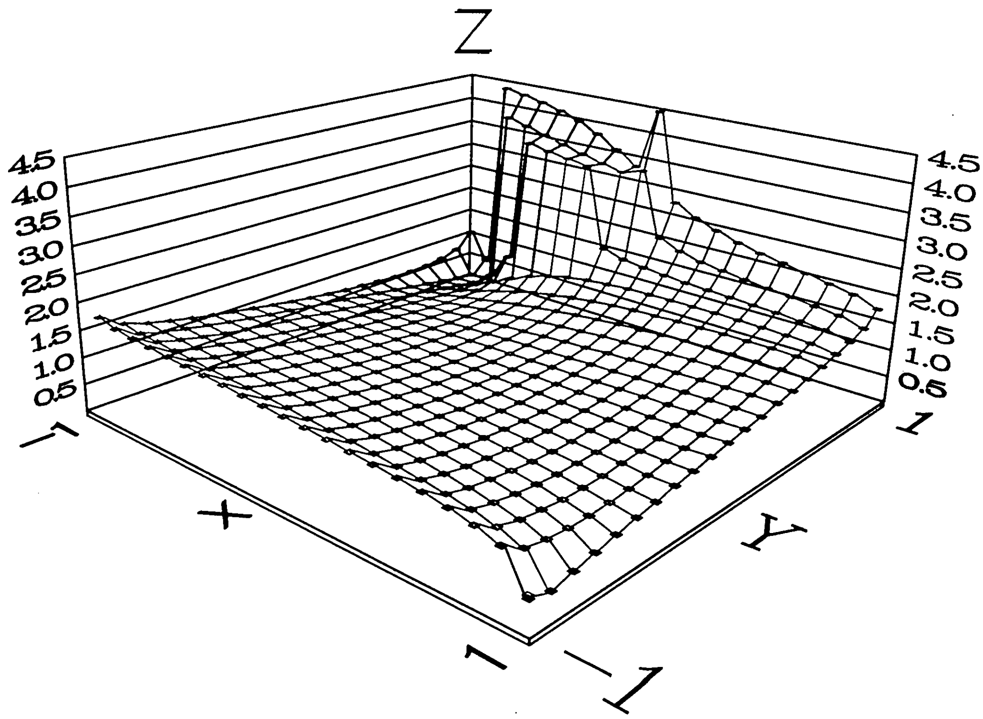

Fig. 9. X–Y plane versus nonlinear parametric gain κl for $(k_1+k_2)l = 10\,000$, $(p,q) = 1,10\,000)$, $r_1 = \sqrt{0.9}$, $r_2 = -1$, and $n = 1.62$ at the oscillation condition, where $X = \sin[\Delta k(a+b)]$, $Y = \cos[(k_1+k_2)(l+a+b)]$, and (p,q) is defined in Eq. (24).

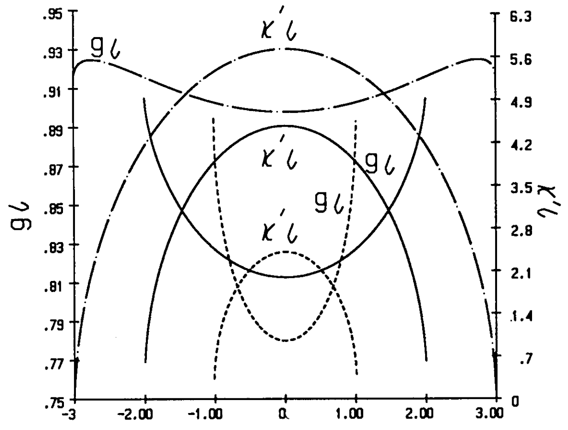

Fig. 10. Normalized wavelength detuning Ψ versus parametric gain $\kappa' l$ and linear gain gl for the oscillation condition at $\cos[(k_1 + k_2)l] = 1$, $R_1 = 0.4$, $R_2 = 0.4$, and $a = b = 0$.

For example, let $a = b = 0$ and $\kappa_1\kappa_2{}^* = \kappa^2$; r_1 and r_2 are real. The oscillation condition reduces to

$$R_1R_2[\alpha^2 - \kappa_1\kappa_2{}^*\beta^2]^2 + 1 - \kappa_1\kappa_2{}^*\beta^2(R_1 + R_2) + 2\alpha^2 r_1 r_2 \cos[(k_1 + k_2)l] = 0. \quad (25)$$

Equation (25) can also be separated into two equations for real and imaginary parts and functions of $\cos[(k_1 + k_2)l]$, $\kappa' l$, gl, and Δkl. Then we can use Brown's method for determination of $\kappa' l$ and gl from these two equations for constant $\cos[(k_1 + k_2)l]$. For example, Fig. 10 shows the normalized wavelength detuning Ψ versus parametric gain $\kappa' l$ and linear gain gl for the oscillation condition at $\cos[(k_1 + k_2)l] = 1$ and $R_1 = 0.4$, $R_2 = 0.4$. Because it is a multiple-valued solution, it is possible to have many pairs of gl and $\kappa' l$ for a particular value of Ψ (not shown in this figure).

CONCLUSION

We have developed a general theory of electromagnetic propagation in phase-conjugate oscillators. Specifically, we have treated the propagation of electromagnetic radiation in a resonator bounded by one conventional reflector and one phase-conjugate mirror and resonators containing an intracavity phase-conjugate element. Wavelength detuning and linear and parametric gains are considered. Phase-conjugate power reflectivity, coherent power reflectivity, and threshold oscillation conditions are derived. The results indicate that (1) nondegenerate oscillation is possible ($\Delta k \neq 0$) for the cases in which $gl = 0$ (as in Fig. 1 for $b \neq 0$, Fig. 6 for $ab \neq 0$, and all cases of $gl \neq 0$) and oscillation will occur only at pump frequency ($\Delta k = 0$) for the cases in which $gl = 0$ (as in Fig. 1 for $b = 0$ and Fig. 6 for $ab = 0$); (2) we can use small $\kappa' l$ to generate self-oscillation and simultaneously produce the coherent and phase-conjugate waves.

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