Error Analysis of Quaternion Transformations

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Three transformation formulas are derived that related the quaternions to the direction cosine matrix used in strapdown inertial systems. Tranformation errors associated with these formulas are fully analyzed. The drift errors evaluated under constant angular velocity have been shown to vary slightly among three different transformations. It is shown that the skew errors in three transformation schemes are not all intrinsically zero. Yet the scale errors may differ largely by two orders of magnitude among transformation schemes. This may become a selection criteria for selection of attitude transformation schemes.

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I. INTRODUCTION

In inertial navigation systems, the gyroscopes and accelerometers are often used to measure the angular velocity and specific force of the vehicle, respectively. While the inertial sensors are mounted directly to the vehicle in a strapdown system, the measured quantities are in body coordinates. In order to attain the navigation purposes, coordinate transformation matrix is needed to resolve the sensed specific force into the navigational reference frame for determining the velocity and position of the vehicle. Also, the attitude information may be extracted from the transformed direction cosine matrix. The establishment of an accurate mathematical transformation which is related to the gyro outputs is a vital computational problem in designing a strapdown inertial system.

Among the computational techniques for solving the transformation problem, two popular schemes are the direction cosine matrix and the quaternion [1, 2, 3]. In most practical strapdown systems, the quaternions (Euler parameters) are updated periodically at a fast rate and then the direction cosine matrix which are composed of the quaternions are calculated at a slower rate. The main advantages of this approach are that it requires less computing time, gives better accuracy, and avoids the singularity problem. These are inherent properties associated with the quaternion dynamics, since it contains only four parameters and uses the half angular increments [5].

Generally, for all kinds of transformation, there are three transformation errors: the skew error, the scale error and the drift error [1–3]. It is very important to investigate these errors before designing an adequate attitude algorithm to be built in the navigation computer. Early research workers [1–3] showed that the quaternion scheme is superior since it has less transformation errors than dirction cosine matrix. They claimed that the skew errors are inherently zero when the quaternion method is employed. We have found that this interesting property results from a particular transformation formula only. Since the transformation from quaternion to direction cosine matrix is not unique, hence the errors are varied in using different transformation formulas. The problem of selecting the best suitable form for strapdown inertial system applications gives us an impetus to reexamine the errors contained in different transformation formulas.

Three transformation formulas are derived here which can transform the quaternions into direction cosines. An error model associated with the computed direction cosine matrix is briefly discussed. Error analysis is fully evaluated analytically and tabulated for comparison. Some useful conclusions can be drawn from the analysis and discussions.

II. THREE TRANSFORMATION FORMULAS

Let $q = q_0 + q_1\hat{\imath} + q_2\hat{\jmath} + q_3\hat{k}$ be the rotation quaternion of the body axes with respect to the navigational reference axes, then any vector $\mathbf{v}$ that is transformed from body coordinates $\mathbf{v}^b$ into navigational coordinates $\mathbf{v}^n$ can be written as

$$\mathbf{v}^n = q\mathbf{v}^b q^* \tag{1}$$

where q^* is the conjugate of q. The equivalent transformation of using the direction cosine matrix C is given by

$$\mathbf{v}^n = C\mathbf{v}^b. \tag{2}$$

Using the quaternion algebra [1, 2, 5] in (1) and comparing with (2), yields the first transformation formula:

$$C_1 = \begin{bmatrix} q_0^2+q_1^2-q_2^2-q_3^2 & 2(q_1q_2-q_0q_3) & 2(q_1q_3+q_0q_2) \\ 2(q_1q_2+q_0q_3) & q_0^2-q_1^2+q_2^2-q_3^2 & 2(q_2q_3-q_0q_1) \\ 2(q_1q_3-q_0q_2) & 2(q_2q_3+q_0q_1) & q_0^2-q_1^2-q_2^2+q_3^2 \end{bmatrix}. \tag{3}$$

This equation has been widely employed for the analysis and design of strapdown inertial systems [1–4].

A second set of transformation equations can be readily obtained by letting $\boldsymbol{\phi} = [\phi_1,\phi_2,\phi_3]^{\mathrm{T}}$ be the rotation vector with magnitude (rotation angle) $\phi = (\phi_1^2+\phi_2^2+\phi_3^2)^{1/2}$, then the associated direction cosine matrix can be written as [6, 7]

$$C = I + \frac{\sin\phi}{\phi}[\boldsymbol{\phi}\times] + \frac{1-\cos\phi}{\phi^2}[\boldsymbol{\phi}\times]^2 \tag{4}$$

where I is the identity matrix and

$$[\boldsymbol{\phi}\times] = \begin{bmatrix} 0 & -\phi_3 & \phi_2 \\ \phi_3 & 0 & -\phi_1 \\ -\phi_2 & \phi_1 & 0 \end{bmatrix}$$

represents the equivalent vector cross-product operation associated with the rotation vector $\boldsymbol{\phi}$. In terms of the components of $\boldsymbol{\phi}$, the direction cosine matrix can be expressed as

$$C = \begin{bmatrix} 1-\frac{1-\cos\phi}{\phi^2}(\phi_2^2+\phi_3^2) & -\frac{\sin\phi}{\phi}\phi_3+\frac{1-\cos\phi}{\phi^2}\phi_1\phi_2 & \frac{\sin\phi}{\phi}\phi_2+\frac{1-\cos\phi}{\phi^2}\phi_1\phi_3 \\ \frac{\sin\phi}{\phi}\phi_3+\frac{1-\cos\phi}{\phi^2}\phi_1\phi_2 & 1-\frac{1-\cos\phi}{\phi^2}(\phi_1^2+\phi_3^2) & -\frac{\sin\phi}{\phi}\phi_1+\frac{1-\cos\phi}{\phi^2}\phi_2\phi_3 \\ -\frac{\sin\phi}{\phi}\phi_2+\frac{1-\cos\phi}{\phi^2}\phi_1\phi_3 & \frac{\sin\phi}{\phi}\phi_1+\frac{1-\cos\phi}{\phi^2}\phi_2\phi_3 & 1-\frac{1-\cos\phi}{\phi^2}(\phi_1^2+\phi_2^2) \end{bmatrix}. \tag{5}$$

Using the quaternions definition:

$$q_0 = \cos\frac{\phi}{2}$$
$$q_i = \frac{\phi_i}{\phi}\sin\frac{\phi}{2}, \qquad i = 1,2,3 \tag{6}$$

and $(1-\cos\phi = 2\sin^2\phi/2)$ in (5), after simplifying yields the second transformation formula:

$$C_2 = \begin{bmatrix} 1-2(q_2^2+q_3^2) & 2(q_1q_2-q_0q_3) & 2(q_1q_3+q_0q_2) \\ 2(q_1q_2+q_0q_3) & 1-2(q_1^2+q_3^2) & 2(q_2q_3-q_0q_1) \\ 2(q_1q_3-q_0q_2) & 2(q_2q_3+q_0q_1) & 1-2(q_1^2+q_2^2) \end{bmatrix}. \tag{7}$$

If we use the relation $[\boldsymbol{\phi}\times]^2 = \boldsymbol{\phi}\boldsymbol{\phi}^{\mathrm{T}} - \phi^2 I$ in (4), the transformation matrix can be expressed as

$$C = I\cos\phi + \frac{\sin\phi}{\phi}[\boldsymbol{\phi}\times] + \frac{1-\cos\phi}{\phi^2}\boldsymbol{\phi}\boldsymbol{\phi}^{\mathrm{T}}. \tag{8}$$

In terms of the components of $\boldsymbol{\phi}$, the direction cosine matrix can be rearranged as

$$C = \begin{bmatrix} \cos\phi+\frac{1-\cos\phi}{\phi^2}\phi_1^2 & -\frac{\sin\phi}{\phi}\phi_3+\frac{1-\cos\phi}{\phi^2}\phi_1\phi_2 & \frac{\sin\phi}{\phi}\phi_2+\frac{1-\cos\phi}{\phi^2}\phi_1\phi_3 \\ \frac{\sin\phi}{\phi}\phi_3+\frac{1-\cos\phi}{\phi^2}\phi_1\phi_2 & \cos\phi+\frac{1-\cos\phi}{\phi^2}\phi_2^2 & -\frac{\sin\phi}{\phi}\phi_1+\frac{1-\cos\phi}{\phi^2}\phi_2\phi_3 \\ -\frac{\sin\phi}{\phi}\phi_2+\frac{1-\cos\phi}{\phi^2}\phi_1\phi_3 & \frac{\sin\phi}{\phi}\phi_1+\frac{1-\cos\phi}{\phi^2}\phi_2\phi_3 & \cos\phi+\frac{1-\cos\phi}{\phi^2}\phi_3^2 \end{bmatrix}. \tag{9}$$

Using (6) and half-angle formulas ($\cos\phi = 2\cos^2(\phi/2) - 1$) and ($1 - \cos\phi = 2\sin^2\phi/2$) in (9), further simplification yields the third transformation formula:

$$C_3 = \begin{bmatrix} 2(q_0^2 + q_1^2) - 1 & 2(q_1q_2 - q_0q_3) & 2(q_1q_3 + q_0q_2) \\ 2(q_1q_2 + q_0q_3) & 2(q_0^2 + q_2^2) - 1 & 2(q_2q_3 - q_0q_1) \\ 2(q_1q_3 - q_0q_2) & 2(q_2q_3 + q_0q_1) & 2(q_0^2 + q_3^2) - 1 \end{bmatrix}. \tag{10}$$

Obviously, these three transformation equations (3), (7), and (10) are equivalent if the quaternions satisfy the condition of normality, i.e.,

$$q_0^2 + q_1^2 + q_2^2 + q_3^2 = 1. \tag{11}$$

It is seen that transformation matrices C_2 and C_3 need equal computation time (ten multiplications, twelve additions and nine scaling shifts). Three more additions and three less scaling shifts are required for C_1. Hence, C_2 and C_3 compute slightly faster than C_1.

III. ERROR MODEL OF DIRECTION COSINE MATRIX

Since the quaternions are transformed into a direction cosine matrix through various forms, the error analysis may be carried out for the corrsponding computed matrix. The skew, scale, and drift errors in a direction cosine matrix are now formulated for the convenience of error analysis.

Let B denote the true transformation matrix. By definition, B is orthogonal and also satisfy the orthonormal condition:

$$B^{\mathrm{T}}B = I \tag{12}$$

where B^{T} is the transpose of B. In a practical strapdown navigation system, assume the computed transformation matrix, C, is available, then C and B can be related by [3]

$$C = B(I + U + S) \tag{13}$$

where U and S represent the antisymmetric part and symmetric part of a small perturbed error matrix, respectively.

Using (12) and (13), it is easily found that, to the first-order approximation in S and U,

$$C^{\mathrm{T}}C = I + 2S \tag{14}$$

which physically shows that the diagonal elements of S are the scale errors and that the off-diagonal elements of $2S$ are the skew errors.

Equation (13) can also be written approximately as

$$C = B(I + U)(I + S). \tag{15}$$

Using (12) and taking matrix inverse in (15), yields

$$B^{\mathrm{T}}C(I + S)^{-1} = I + U. \tag{16}$$

Therefore, if the skew and scale errors in C are corrected, then the off-diagonal elements of U represent the drift errors of C relative to B by physical interpretation. In order to compute the matrix U more accurately, transposing (16) and using $S^{\mathrm{T}} = S$ and $U^{\mathrm{T}} = -U$, yields

$$(I + S)^{-1}C^{\mathrm{T}}B = I - U. \tag{17}$$

Subtracting (17) from (16) and using (12) and (13) and neglecting high-order approximations, it can be found that

$$U = \tfrac{1}{2}(B^{\mathrm{T}}C - C^{\mathrm{T}}B). \tag{18}$$

It is obvious, from (14), that skew and scale errors may be computed from matrix C alone; while from (18), evaluation of drift errors must rest upon B and C.

IV. ERROR ANALYSIS: SKEW AND SCALE ERRORS

The skew and scale errors can be found by applying (14) to (3), (7), and (10), respectively. The results are shown as follows: for matrix C_1, we have

$$\begin{aligned} C_1^{\mathrm{T}}C_1 &= I + 2S_1 \\ &= (q_0^2 + q_1^2 + q_2^2 + q_3^2)^2 I && (19) \\ &= I + [(q_0^2 + q_1^2 + q_2^2 + q_3^2)^2 - 1]I && (20) \end{aligned}$$

which shows that the skew error for i-axis of C_1 is

$$\epsilon_{1i} = 0, \qquad i = 1,2,3 \tag{21}$$

and the scale error for i-axis of C_1 is

$$\delta_{1i} = \tfrac{1}{2}[(q_0^2 + q_1^2 + q_2^2 + q_3^2)^2 - 1], \qquad i = 1,2,3. \tag{22}$$

Similarly, for matrix C_2, we have

$$\begin{aligned} C_2^{\mathrm{T}}C_2 &= I + 2S_2 \\ &= I + 4(q_0^2 + q_1^2 + q_2^2 + q_3^2 - 1) \\ &\quad \times \begin{bmatrix} q_2^2 + q_3^2 & -q_1q_2 & -q_1q_3 \\ -q_1q_2 & q_1^2 + q_3^2 & -q_2q_3 \\ -q_1q_3 & -q_2q_3 & q_1^2 + q_2^2 \end{bmatrix} \end{aligned} \tag{23}$$

which shows that the skew error for i-axis of C_2 is

$$\epsilon_{2i} = -4(q_0^2 + q_1^2 + q_2^2 + q_3^2 - 1)q_jq_k \qquad \begin{cases} i = 1,2,3 \\ j = 2,3,1 \\ k = 3,1,2 \end{cases}. \tag{24}$$

Note that i, j, and k are in permutative order. And the scale error for i-axis of C_2 is

$$\delta_{2i} = 2(q_0^2 + q_1^2 + q_2^2 + q_3^2 - 1)(q_j^2 + q_k^2) \qquad \begin{cases} i = 1,2,3 \\ j = 2,3,1 \\ k = 3,1,2 \end{cases}. \tag{25}$$

TABLE I
Quaternion Approximations

order	quaternions	
	q_0	$q_i \quad i = 1,2,3$
1	1	$\frac{1}{2}\phi_i$
2	$1 - \frac{1}{8}\phi^2$	$\frac{1}{2}\phi_i$
3	$1 - \frac{1}{8}\phi^2$	$\frac{1}{2}\phi_i(1 - \frac{\phi^2}{24})$
4	$1 - \frac{1}{8}\phi^2 + \frac{1}{384}\phi^4$	$\frac{1}{2}\phi_i(1 - \frac{\phi^2}{24})$

TABLE II
Skew Error

order	transformation matrix form		
	C_1	C_2	C_3
1	0	$-\frac{1}{4}\phi^2\phi_j\phi_k$	$\frac{1}{4}\phi^2\phi_j\phi_k$
2	0	$-\frac{1}{64}\phi^4\phi_j\phi_k$	$\frac{1}{64}\phi^4\phi_j\phi_k$
3	0	$\frac{1}{192}\phi^4\phi_j\phi_k$	$-\frac{1}{192}\phi^4\phi_j\phi_k$
4	0	$\frac{1}{4608}\phi^6\phi_j\phi_k$	$-\frac{1}{4608}\phi^6\phi_j\phi_k$

Finally, for matrix C_3, we have

$$\begin{aligned} C_3^{\mathrm{T}}C_3 &= I + 2S_3 \\ &= I + 4(q_0^2 + q_1^2 + q_2^2 + q_3^2 - 1) \\ &\quad \times \begin{bmatrix} q_0^2 + q_1^2 & q_1q_2 & q_1q_3 \\ q_1q_2 & q_0^2 + q_2^2 & q_2q_3 \\ q_1q_3 & q_2q_3 & q_0^2 + q_3^2 \end{bmatrix} \end{aligned} \tag{26}$$

which shows that the skew error for i-axis of C_3 is

$$\epsilon_{3i} = 4(q_0^2 + q_1^2 + q_2^2 + q_3^2 - 1)q_jq_k \qquad \begin{cases} i = 1,2,3 \\ j = 2,3,1 \\ k = 3,1,2 \end{cases} \tag{27}$$

and the scale error for i-axis of C_3 is

$$\delta_{3i} = 2(q_0^2 + q_1^2 + q_2^2 + q_3^2 - 1)(q_0^2 + q_i^2), \qquad i = 1,2,3. \tag{28}$$

Thus, it is observed from (20), (23), and (26) that all the skew and scale errors are zero for C_1, C_2 and C_3 if the quaternions satisfy the condition of normality (11). The skew errors associated with matrix C_1 are inherently zero, but it is not true all the time for C_2 and C_3. In addition, the skew errors in C_2 and C_3 are zero if the body rotation is along one of the principal axis. The scale errors are all equal in three axes of C_1, but they are not equal in three axes of C_2 and C_3. The skew error in C_2 and C_3 associated with the same direction has equal magnitude but in the opposite direction.

In a practical strapdown navigator, the computed quaternions are approximated by truncating the transcendental functions in (6). Table I gives the approximated quaternions for first order through fourth order [1]. Substituting the values of the approximated quaternions into (22), (24), (25), (27), and (28) the low-order approximation of skew and scale error for i-axis of C_1, C_2, and C_3 are listed in Table II and Table III, respectively.

Obviously, as comparing with C_1, the scale errors are twice larger and the skew errors are increasing a little in C_3. However, the large decrease by two orders of magnitude in scale errors in C_2 is achieved at the expense of a slight increase in skew errors. This effect may offer a longer renormalization period in correcting the orthgonality. In this respect, it is suitable for short range inertial guidance systems such as short range tactical missles.

TABLE III
Scale Error

order	transformation matrix form		
	C_1	C_2	C_3
1	$\frac{1}{4}\phi^2$	$\frac{1}{8}\phi^2(\phi_j^2 + \phi_k^2)$	$\frac{1}{2}\phi^2$
2	$\frac{1}{64}\phi^4$	$\frac{1}{128}\phi^4(\phi_j^2 + \phi_k^2)$	$\frac{1}{32}\phi^4$
3	$-\frac{1}{192}\phi^4$	$-\frac{1}{384}\phi^4(\phi_j^2 + \phi_k^2)$	$-\frac{1}{96}\phi^4$
4	$-\frac{1}{4608}\phi^6$	$-\frac{1}{9216}\phi^6(\phi_j^2 + \phi_k^2)$	$-\frac{1}{2304}\phi^6$

V. ERROR ANALYSIS: DRIFT ERRORS

In order to analyze the drift errors in a computed direction cosine matrix, the true transformation matrix B is required. Assuming that the direction of the angular velocity vector $\omega = [\omega_1,\omega_2,\omega_3]^{\mathrm{T}}$ is constant over the integration interval $(t, t + \Delta T)$, then the true matrix B is given as the right-hand side of (5) or (9) with

$$\phi_i = \int_t^{t+\Delta T} \omega_i \, dt, \qquad i = 1,2,3 \tag{29}$$

where ΔT represents the sampling period. The drift errors may be found by applying (18) to (3), (7) and (10), respectively. The results are as follows:

$$\begin{aligned} U_j &= \tfrac{1}{2}(B^{\mathrm{T}}C_j - C_j^{\mathrm{T}}B) \\ &= \begin{bmatrix} 0 & -\theta_{j3} & \theta_{j2} \\ \theta_{j3} & 0 & -\theta_{j1} \\ -\theta_{j2} & \theta_{j1} & 0 \end{bmatrix}, \qquad j = 1,2,3. \end{aligned} \tag{30}$$

TABLE IV
Drift Error

order	transformation matrix form		
	C_1	C_2	C_3
1	$-\frac{1}{12}\phi^2\phi_i$	$\frac{1}{6}\phi^2\phi_i$	$-\frac{1}{3}\phi^2\phi_i$
2	$\frac{1}{24}\phi^2\phi_i$	$\frac{1}{24}\phi^2\phi_i$	$\frac{1}{24}\phi^2\phi_i$
3	$\frac{1}{480}\phi^4\phi_i$	$-\frac{3}{960}\phi^4\phi_i$	$\frac{7}{960}\phi^4\phi_i$
4	$-\frac{1}{1920}\phi^4\phi_i$	$-\frac{1}{1920}\phi^4\phi_i$	$-\frac{1}{1920}\phi^4\phi_i$

For matrix C_1,

$$\theta_{1i} = q_0 q_i(1+\cos\phi) - q_0^2\left(\frac{\sin\phi}{\phi}\phi_i\right) + \left(\frac{\sin\phi}{\phi}q_i - \frac{1-\cos\phi}{\phi^2}q_0\phi_i\right) \times \sum_{n=1}^{3} q_n\phi_n, \qquad i = 1,2,3 \tag{31}$$

represents the drift error about i-axis of matrix C_1. By neglecting high order terms, it can be written as

$$\theta_{1i} = q_i(1+\cos\phi) - q_0\left(\frac{\sin\phi}{\phi}\phi_i\right), \qquad i = 1,2,3. \tag{32}$$

For matrix C_2, we have

$$\theta_{2i} = \theta_{1i} + (q_0^2 + q_1^2 + q_2^2 + q_3^2 - 1)\left(\frac{\sin\phi}{\phi}\phi_i\right), \qquad i = 1,2,3 \tag{33}$$

and similarly, for matrix C_3, we have

$$\theta_{3i} = \theta_{1i} - (q_0^2 + q_1^2 + q_2^2 + q_3^2 - 1)\left(\frac{\sin\phi}{\phi}\phi_i\right), \qquad i = 1,2,3. \tag{34}$$

It is evident that the drift errors in these three matrices are all equal if the condition of normality (11) is satisfied. Substituting the values of the approximated quaternions given in Table I into (32)–(34), the low order approximation of drift errors for i-axis of C_1, C_2, and C_3 are listed in Table IV. Note that the drift errors in C_2 and C_3 are only slightly different from that in C_1.

The transformation errors in matrix C_1 have been presented by Wilcox [1] and Mckern [2]; we list them in the tables for the convenience of comparison and completeness.

VI. CONCLUSIONS

We provide the derivation of three often encountered transformation formulas which transform quaternions into direction cosine matrix. Their analytic transformation errors are evaluated and tabulated for comparison.

It has been shown that the skew errors in these transformation matrices are not all zero. It is zero only for transformation matrix C_1. The scale errors are not equally distributed except for C_1. They are axis dependent. The transformation matrix C_2 has the smallest scale error which may enable the extension of the renormalization period. The drift errors are only slightly different in these three transformation matrices.

These results are helpful in the design of attitude algorithms for strapdown inertial navigation systems.

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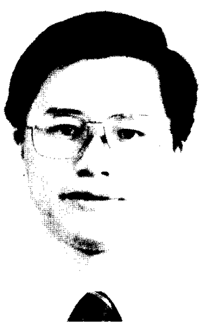

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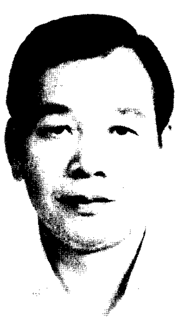

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