# Performance analysis of efficient concatenated coding scheme for ARQ protocols

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*Indexing terms: Codes and decoding, Communication, Data transmission*

**Abstract:** Benelli investigated the performance of two concatenated coding schemes for ARQ protocols. The two schemes differ in their strategies of retransmission. It was shown that the two schemes proposed by Benelli provide a significant improvement in throughput efficiency of ARQ protocols with respect to other similar schemes. The analysis presented by Benelli is complicated. An alternative analysis is provided. Some implementation problems of the Benelli's schemes are also discussed. An efficient retransmission strategy which avoids unnecessary retransmissions and maintains a fixed decoding complexity is proposed.

## 1 Introduction

Forward error correction (FEC) and automatic repeat request (ARQ) are common techniques used to handle transmission errors when data are transmitted over noisy channels. In practical applications where feedback is possible, ARQ techniques are often more preferable than FEC schemes because error detection requires much simpler decoding equipment and achieves a higher reliability than does error correction. When the channel is very noisy, the system throughput is smaller for ARQ techniques than FEC schemes because, in this situation, retransmissions will be requested too frequently. Concatenated coding schemes, which combine the concepts of FEC and ARQ [6, 7] can provide a high system throughput and maintain a high system reliability for communications over quite noisy channels.

Fig. 1 shows the structure of a communication system using a concatenated coding scheme. Two linear block

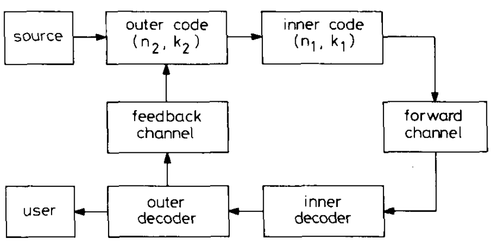


**Fig. 1** *Communication system using concatenated coding scheme*

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codes, denoted by $C_1$ and $C_2$, are used in the concatenated coding scheme. The inner code $C_1$ is of the type $(n_1, k_1)$ which can correct $t$ errors and detect $\lambda \geqslant t$ errors. The outer code $C_2$ is of the type $(n_2, k_2)$ which is used only for error detection. It is assumed that $n_2 = mk_1$, where $m$ is an integer. A message of $k_2$ bits is first encoded into a codeword of $n_2$ bits by code $C_2$. This codeword is divided into $m$ sub-blocks of $k_1$ bits. Each sub-block is then encoded by code $C_1$ into a codeword of $n_1$ bits, called a frame. Fig. 2 illustrates the format of a block obtained from the concatenated coding scheme.

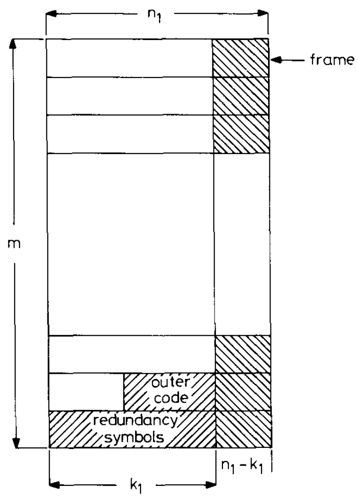


**Fig. 2** *Block format of concatenated codeword*

Benelli proposed and analysed two concatenated coding schemes for ARQ protocols [8]. These two schemes differ in their strategies of retransmission. It was shown that the throughput efficiencies of the stop-and-wait (SW), the go-back-$N$ (GBN) and the selective-repeat (SR) ARQ protocols can be significantly improved using the Benelli's schemes with respect to other similar schemes. The analysis provided in Reference 8 is rather complicated. The expression of the probability that a frame is decoded with an undetected error pattern given in Reference 8 (which was denoted as $P_{ud}$) is correct only when the inner code $C_1$ is used for error detection only. The results presented in Reference 8 should be modified if the code $C_1$ is used for both error detection and correction. Some implementation problems of Benelli's schemes will be discussed.

This paper provides an alternative analysis for the scheme 1 proposed by Benelli and proposes and analyses an efficient retransmission strategy which avoids unnecessary retransmissions and maintains a fixed decoding complexity. The proposed retransmission strategy is described, followed by the analysis. Numerical results and discussions are presented, and some conclusions are finally drawn.

## 2 Proposed scheme

The proposed concatenated coding scheme is a modification of scheme 1 proposed by Benelli [8]. Benelli's schemes are first reviewed. The receiver is assumed to contain a buffer for block storage and a register R capable of storing $m$ bits. The $m$ bits of R are initially set to zero. When a block is received for the first time, frames are analysed using the inner code $C_1$. If frame $i$ does not contain a detectable error pattern, then this frame is stored in buffer and the $i$th bit of R is set to one. After all the $m$ frames are analysed, a retransmission of the block is requested as long as there exist some bits of R whose contents are zeros. The retransmission strategies of scheme 1 and scheme 2 are different.

In scheme 1, the whole data block is retransmitted if the receiver requests a retransmission. The receiver analyses the $p$th frame only if the $p$th bit of R is zero. If the $p$th frame of the currently received block does not contain a detectable error pattern, then this frame is stored in buffer and the $p$th bit of R is set to one. After all the $m$ frames are examined, another retransmission is again asked if there are still some bits of R equal zero. If all the $m$ bits of R equal one, then the outer code $C_2$ is used to check whether or not the decoded block contains any error. The block is discarded if it is detected with error using code $C_2$. Otherwise, it is delivered to the user. A block is requested to be retransmitted all over again if it is discarded. In scheme 2, only the negatively acknowledged frames are retransmitted. These erroneous frames are repeated to maintain a constant block length. Fig. 3 shows an example of the retransmission strategy of scheme 2. The receiver analyses the copies of each retransmitted frame using code $C_1$. If a copy of the $p$th frame contains no error or a correctable error, then this copy is stored and the $p$th bit of R is set to one. Code $C_2$ is again used for error checking as soon as all the bits of R equal one. The block is delivered to the user if it is decoded without error by code $C_2$. Otherwise, it is discarded and requested to be retransmitted all over again.

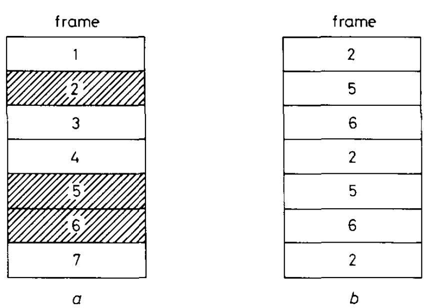


**Fig. 3** *Example of retransmission using Benelli's scheme 2*
▨ frame with detectable errors
□ frame without detectable errors
*a* First transmission format
*b* Second transmission format

Notice that, in scheme 2, the feedback from the receiver should include the number and the positions of the frames to be retransmitted.

There are problems in the Benelli's schemes. For example, in scheme 1, retransmission of frames which will not be analysed simply means a waste of bandwidth. It is also possible that some decoded frame, say frame 1, contains an undetected error in the first transmission when the block is asked to be retransmitted. In this case frame 1 is temporarily saved and successive retransmitted versions will not be analysed. Since frame 1 is erroneous, the block will eventually be discarded and asked to be retransmitted all over again. Obviously, if successive retransmitted versions are analysed, then the block may not be discarded and the throughput performance can thus be improved. The decoding process becomes much more complicated if every retransmitted frame is analysed because it is then possible that multiple copies of the same frame have to be stored and the outer code $C_2$ has to be used to check multiple combinations. Another situation which causes the same problem may occur in each retransmission using scheme 2. Suppose frame 1 is retransmitted with two copies which are decoded into different codewords by code $C_1$. If only one copy is selected, then the block may be finally discarded because the receiver has a nonzero probability to choose an erroneous copy. This problem cannot be neglected in practice unless the probability that a decoded frame contains an undetected error, i.e. $P_{ud}$, is extremely small for the inner code $C_1$.

To avoid wasting bandwidth and maintain a simple decoding process, it is proposed that only the frames decoded with detectable errors are retransmitted, each with one copy. Such a retransmission strategy clearly results in variable block length and when it is used in the GBN or the SR protocols, ACK or NAK of a block may arrive at the transmitter when it is transmitting another block. Assume that the transmission is stopped as soon as a NAK is received if the GBN protocol is adopted. The transmission is continued until the whole block is finished if the SR protocol is adopted. The investigated SR protocol is also assumed to be ideal, i.e. the receiver has a buffer and a register for each data block. The ideal SR protocol is studied because it sets an upper bound on the throughput performance of various ARQ protocols. When real implementation is considered where only finite buffers are available, the operation and performance of the SR protocol needs to be modified. To evaluate the performance of the proposed scheme, it is assumed that the communication channel is a memoryless binary symmetric channel (MBSC).

## 3 System performance on MBSC

An alternative analysis of the throughput efficiencies of the SW, the GBN and the SR protocols is provided using scheme 1 proposed by Benelli. As in Reference 8, it is assumed that the outer code $C_2$ is a perfect error detecting code for simplicity. Let $P_c$ and $P_{ud}$ denote the probabilities that a decoded frame contains no error and an undetected error using code $C_1$, respectively. Then the value of $P_c$ can be computed by [1]

$$P_c = \sum_{i=0}^{t} \binom{n_1}{i} p^i (1-p)^{n_1 - i} \tag{1}$$

where $p$ is the bit error probability. As mentioned before, the expression of $P_{ud}$ presented in Reference 8 is correct only when the inner code $C_1$ is used solely for error detection. The enumeration of $P_{ud}$ when code $C_1$ is used for both error detection and correction can be found in Reference 7. The result is

$$P_{ud} = \sum_{i=1}^{n_1} A_i P_f(i) \quad (2)$$

where $\{A_i, 0 \leqslant i \leqslant n_1\}$ is the weight distribution of code $C_1$ and $P_f(i)$ represents the probability that a decoded frame contains a specific undetectable error pattern of weight $i$

$$P_f(i) = \sum_{j=0}^{t} \sum_{k=0}^{\min(t-j, n_1-i)} \binom{i}{j}\binom{n_1-i}{k} \times p^{i-j+k}(1-p)^{n_1-i+j-k} \quad (3)$$

Consider the transmission of a particular data block. Let $X_i$, $i = 1, 2, \ldots, m$, denote the number of transmissions until frame $i$ is stored in buffer. It is not hard to see that the probability density function of $X_i$ is given by

$$P(X_i = j) = P_d^{j-1}(P_c + P_{ud}) \quad j = 1, 2, \ldots \quad (4)$$

where $P_d = 1 - P_c - P_{ud}$ represents the probability that a frame is decoded with detectable errors. To evaluate the throughput efficiencies of the three basic ARQ protocols using the concatenated coding scheme, it is necessary to compute the value $n_t$, which is the average number of transmissions until a block is correctly retrieved. Let $Y = \max(X_1, X_2, \ldots, X_m)$ represent the number of transmissions until the block is retrieved, i.e. the outer code $C_2$ is used for error checking. It can then be shown that for the memoryless binary symmetric channel

$$F_Y(y) = P(Y \leqslant y) = \prod_{i=1}^{m} P(X_i \leqslant y) = (1 - P_d^y)^m \quad (5)$$

As a result, the average number of transmissions until the block is retrieved is equal to

$$\bar{Y} = \sum_{y=1}^{\infty} P(Y \geqslant y) = \sum_{y=1}^{\infty} [1 - (1 - P_d^{y-1})^m] \quad (6)$$

which can be shown to be finite for $0 < P_d < 1$.

Let $P_{cr}$ denote the probability that the block contains no error when it is retrieved. Then, since the errors in frames are independent, it can easily be seen that

$$P_{cr} = [P_c/(P_c + P_{ud})]^m \quad (7)$$

Consequently, the average number of retrievals until the block is correctly received is equal to $1/P_{cr}$. The value $n_t$ can be computed by

$$n_t = \bar{Y}/P_{cr} \quad (8)$$

Assuming the round trip delay $N$ is a multiple of block transmission time, the throughput efficiencies of the SW, the GBN and the SR protocols using the concatenated coding scheme are

$$T_{SW} = \frac{k_2}{n_1 m n_t N} \quad (9)$$

$$T_{GBN} = \frac{k_2}{n_1 m(n_t N - N + 1)} \quad (10)$$

$$T_{SR} = \frac{k_2}{n_1 m n_t} \quad (11)$$

The performance of the proposed scheme is evaluated. Let $Y$ have the same meaning as defined previously and $Z = X_1 + X_2 + \cdots + X_m$ denote the total number of frames transmitted until the block is retrieved. It can be shown that $\bar{Z} = E(Z) = m/(P_c + P_{ud})$. Since the proposed scheme results in variable block length, the transmission time of a frame, called a slot, is selected as the time unit. When the proposed scheme is used in the GBN or the SR protocols, ACK or NAK of a block may arrive at the transmitter when it is transmitting another block. It is assumed that on the receipt of a NAK the transmission is immediately stopped if the GBN protocol is adopted. The transmission is continued until the whole block is transmitted if the SR protocol is adopted.

Let $N' = m(N-1)$ represent the number of slots between the end of a transmission and the receipt of its response. Let $W$ denote the total number of slots spent until the result of a retrieval, i.e. ACK or NAK, is received by the transmitter. The expectation of $W$ conditioning on $Y = y$ is then equal to

$$E(W \mid Y = y) = E(yN' + Z \mid Y = y) = N'y + E(Z \mid Y = y) \quad (12)$$

Unconditioning $Y$

$$\bar{W} = E(W) = N'\bar{Y} + \bar{Z} \quad (13)$$

The throughput efficiencies of the three basic ARQ protocols using the proposed scheme can therefore be computed by

$$T = \frac{k_2}{n_1 a_t} \quad (14)$$

where

$$a_t = \begin{cases} \bar{W}/P_{cr} & \text{SW} \\ (\bar{W}/P_{cr}) - N' & \text{GBN} \\ \bar{Z}/P_{cr} & \text{SR} \end{cases} \quad (15)$$

represents the average number of slots spent for a data block to be correctly received. Notice that the value of $P_{cr}$ appeared in the above equation is given in eqn. 7.

## 4 Numerical results

All the results presented assume an outer code of $n_2 = 1023$ and $k_2 = 993$. This outer code is assumed to be a perfect error detecting code. The Hamming codes (8, 4) and (16, 11) and the Golay code (24, 12) are used as the inner code. Whenever $n_2$ is not a multiple of $k_1$, the value of $m$ is chosen to be $\lceil n_2/k_1 \rceil$, the least integer which is greater than $n_2/k_1$. Schemes S1, S2, and S3 represent the scheme 1 proposed by Benelli, the proposed scheme, and the scheme presented in Reference 6.

Fig. 4 shows the throughput efficiency of the SW protocol using scheme S1 against the bit error probability $p$ for $N = 20$. Similar results are shown in Figs. 5 and 6 for the GBN and the SR protocols. It can be seen that the throughput efficiencies of the three basic ARQ protocols can be significantly improved using scheme S1 with respect to other similar schemes.

Fig. 7 shows the comparison of the throughput efficiency of the GBN protocol using schemes S1 and S2 for different inner codes with $t = 0$. Notice that $t = 0$ means the inner code is used for error detection only and thus requires a simpler implementation than the case when $t \neq 0$. It can be seen from this Figure that, compared with the classic GBN protocol, scheme S1 still provides a significant improvement for large values of bit error probability $p$. The proposed scheme is shown to be consistently better than scheme S1.

Figs. 8 and 9 show similar comparisons for the GBN and the SR protocols, respectively, for different inner codes with $t \neq 0$. The proposed scheme is consistently better than scheme S1 in system throughput, significantly so in Fig. 9. The tradeoff is clearly higher implementation complexity because the feedback in the proposed scheme must include the positions of negatively acknowledged frames.

Notice that, compared with scheme 1 studied in Reference 8, the proposed scheme does not reduce the number of retransmissions of any data block. It only saves the unnecessary retransmissions of frames which had been successfully decoded (either correctly or incorrectly) during previous transmissions. The improvement for the SW and the GBN protocols is therefore not as significant as the improvement for the SR protocol because of the

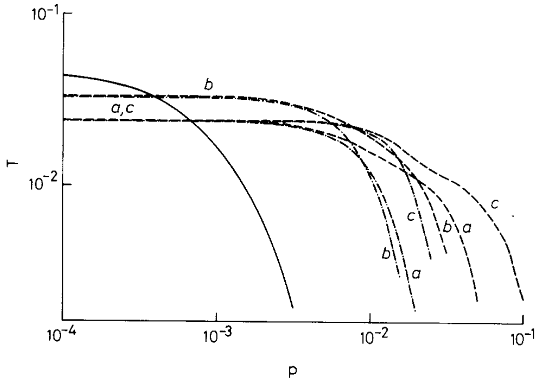


**Fig. 4** *Throughput of SW protocol against bit error probability*
Scheme S1
——— classical a: (8, 4); $t = 1$
—·—· S3 b: (16, 11); $t = 1$
- - - - S1 c: (24, 12); $t = 2$

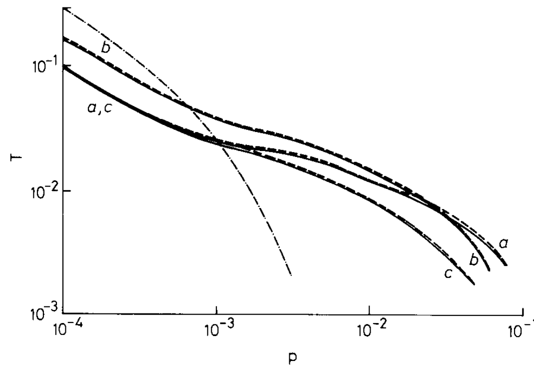


**Fig. 7** *Throughput of GBN protocol against bit error probability*
Scheme S2 a: (8, 4); $t = 0$
—·—· classical b: (16, 11); $t = 0$
——— S1 c: (24, 12); $t = 0$
- - - - S2

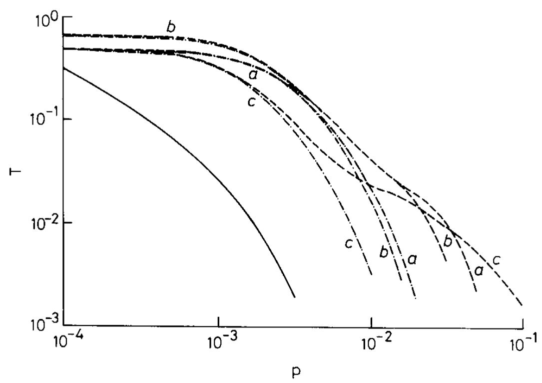


**Fig. 5** *Throughput of GBN protocol against bit error probability*
Scheme S1
——— classical a: (8, 4); $t = 1$
—·—· S3 b: (16, 11); $t = 1$
- - - - S1 c: (24, 12); $t = 1$

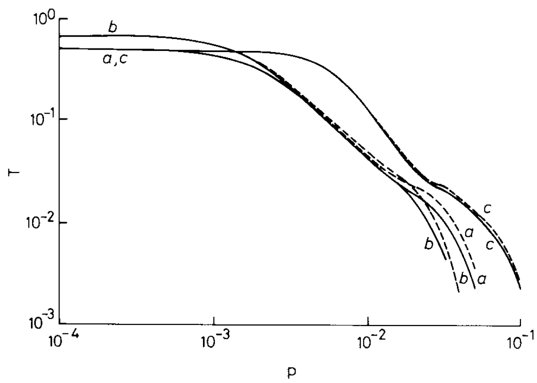


**Fig. 8** *Throughput of GBN protocol against bit error probability*
Scheme S2 a: (8, 4); $t = 1$
——— S1 b: (16, 11); $t = 1$
- - - - S2 c: (24, 12); $t = 1$

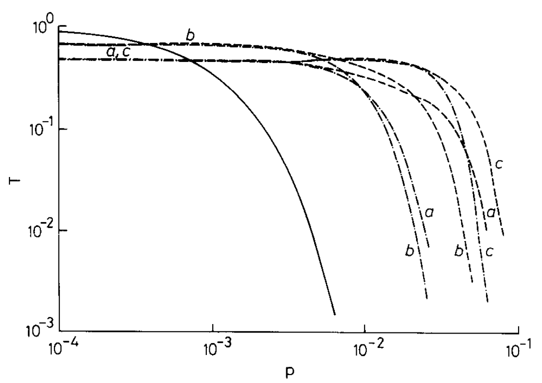


**Fig. 6** *Throughput of SR protocol against bit error probability*
Scheme S1
——— classical a: (8, 4); $t = 1$
—·—· S3 b: (16, 11); $t = 1$
- - - - S1 c: (24, 12); $t = 3$

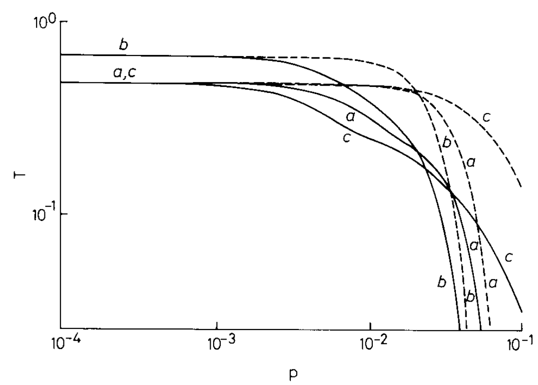


**Fig. 9** *Throughput of SR protocol against bit error probability*
Scheme S2 a: (8, 4); $t = 1$
——— S1 b: (16, 11); $t = 1$
- - - - S2 c: (24, 12); $t = 1$

cost of a large round-trip delay when a retransmission is requested (in our examples, $N = 20$). The number of retransmissions has to be reduced to significantly enhance the performance of the SW and the GBN protocols. An approach to achieve this is to transmit multiple copies of each frame or block contiguously to the receiver [9]. In scheme 2 proposed in Reference 8, multiple copies of frames which had not been successfully decoded are transmitted at each retransmission. This scheme is expected to reduce the number of retransmissions and achieve a better performance for the SW and the GBN protocols for channels having a high error rate and/or a large round-trip delay. Unfortunately, for high error rate channels, $P_{ud}$ is large. The outer code may therefore have to be used to check multiple combinations to achieve the high performance. As a consequence, the decoding procedure and buffer management become so complicated as to make this scheme impractical for real applications.

## 5 Conclusions

An alternative approach to evaluate the throughput efficiencies of the SW, the GBN and the SR ARQ protocols using the concatenated coding scheme 1 proposed by Benelli was provided. Some implementation problems of Benelli's schemes were also discussed. An efficient retransmission strategy which avoids unnecessary retransmissions and maintains a fixed decoding complexity was proposed. The throughput efficiencies of the SW, the GBN and the SR protocols using the proposed scheme were shown to be consistently better than those using scheme 1 suggested in Reference 8. The improvement is significant for the SR protocols. An interesting topic which can be further studied is to evaluate the performance of ARQ protocols using various concatenated coding schemes for communications over burst-error channels.

## 6 Acknowledgment

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