Effect of Residual Stress on Cold-Formed Steel Column Strength

By C. C. Weng[1]

Abstract: The effect of residual stress on the flexural buckling strength of cold-formed steel columns is studied. Based on the results of 93 column tests and the measured residual stresses, significant relation is observed between the reduction of column strength, the magnitude and distribution of residual stress, and the flat-width ratio of the plate element of the cold-formed section. In this study, a new concept called the "second reduction" is developed to account for the effect of the residual stresses on the local buckling behavior of cold-formed steel columns. Based on this concept, a possible design procedure is outlined for the prediction of the flexural buckling strength of cold-formed steel columns. The theoretical predictions are found to be in good agreement with the test results.

Introduction

This is the third in a series of papers concerning the effect of residual stresses on the strength of cold-formed steel columns. The first paper (Weng and Pekoz 1990a) presented the results of 93 column tests and discussed the effect of some important parameters on the strength of cold-formed steel columns. In the second paper (Weng and Pekoz 1990b), the residual stresses in cold-formed steel sections were investigated. From the experimental results, an idealized residual stress distribution pattern for a cold-formed section was described. It was found that the magnitude and distribution of the residual stresses in cold-formed sections are quite different from those in hot-rolled shapes.

Based on the results presented in the aforementioned papers, the influence of residual stresses on the flexural buckling strength of cold-formed steel columns is investigated in this study.

Design Considerations

The design formulas used in the present AISI *Specification* (1986) for flexural buckling strength of cold-formed steel column are based on the Column Research Council's column curve (Johnston 1976). Since the CRC column curve was derived based on the residual stresses found in hot-rolled steel columns (Yang et al. 1952; Huber and Beedle 1954; Beedle and Tall 1960; Tall 1964), it is felt that the direct use of the Column Research Council (CRC) column curve for the design of cold-formed steel columns may not be appropriate due to the difference of residual stresses in these two groups of columns.

During the past few years, experimental results indicated that the American Iron and Steel Institute (AISI) column equations gave unconservative

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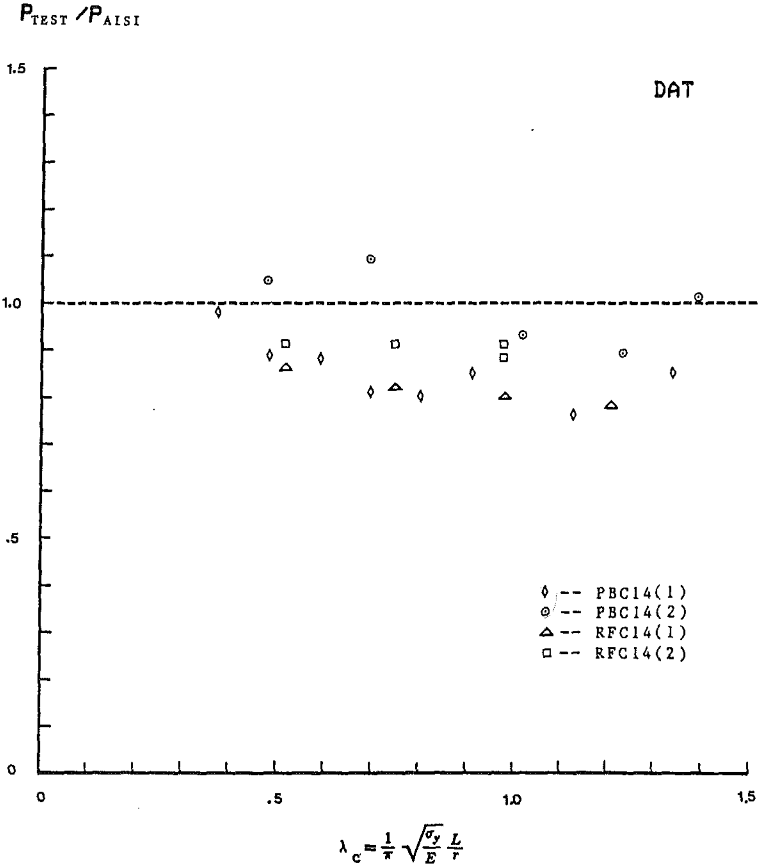
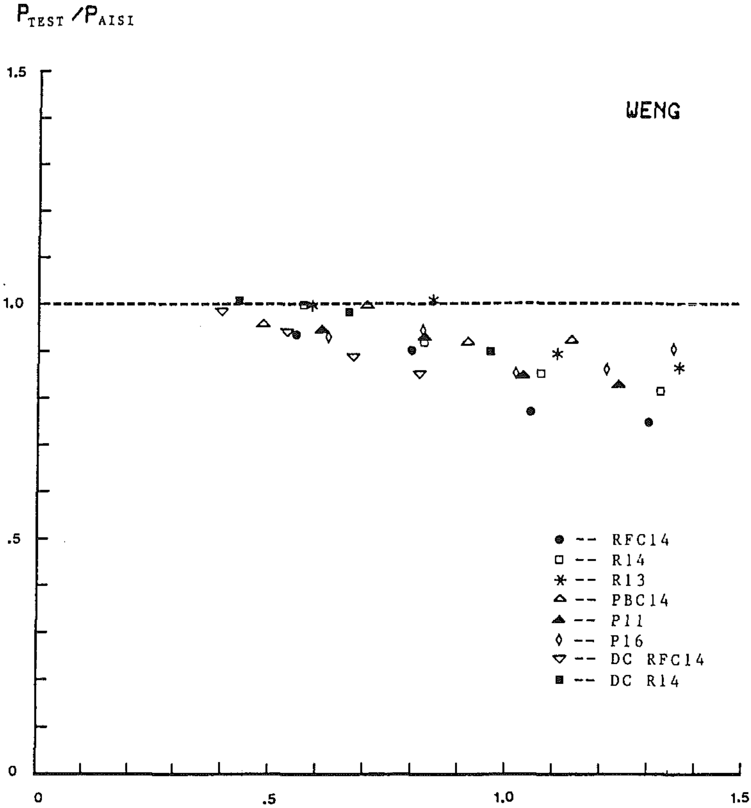


FIG. 1. Dat's (1980) Test Results versus AISI Predictions

predictions of the strength of some types of cold-formed steel columns. Figs. 1 and 2 show the column test results obtained by Dat (1980) and by Weng and Pekoz (1990a). It is seen that the AISI column equations overestimate the strength of these columns. In some cases, the differences between the test results and the AISI predictions can be larger than 15%. However, for some other types of columns tested by Dat (1980) including hat and channel sections, the values predicted by the AISI formulas were found to be satisfactory. Therefore, it is desirable to explain why the AISI column formulas gave unconservative predictions for some types of columns and good estimations for others, and to develop a better design approach for predicting the flexural buckling strength of cold-formed steel columns.

Idealized Residual Stress Distribution

A detailed description of the residual stresses measured from the tested specimens is presented in a paper by Weng and Pekoz (1990b). The follow-



$$\lambda_c = \frac{1}{\pi}\sqrt{\frac{\sigma_y}{E}}\frac{L}{r}$$

FIG. 2. Weng and Pekoz's (1990a) Test Results versus AISI Predictions

ing is a brief summary of the "idealized" residual stress distribution pattern in a cold-formed steel channel section (as shown in Fig. 3):

1. There are tensile residual stresses on the outside surface of the channel section, and compressive residual stresses on the inside surface.
2. Residual stresses are assumed to vary linearly through the plate thickness.
3. The increase in residual stress at the corner regions may be negated by the increase in yield stress. Thus, the residual stresses are assumed to be uniformly distributed along the perimeter of the section by neglecting the variations at the corners.
4. The magnitudes of the maximum tensile and compressive residual stresses are assumed to be the same and are conservatively taken as 50% of the yield stress of the material.

Important Observations

After an extensive study of the results of 93 column tests, an important correlation between the reduction of column strength, the flat-width ratios

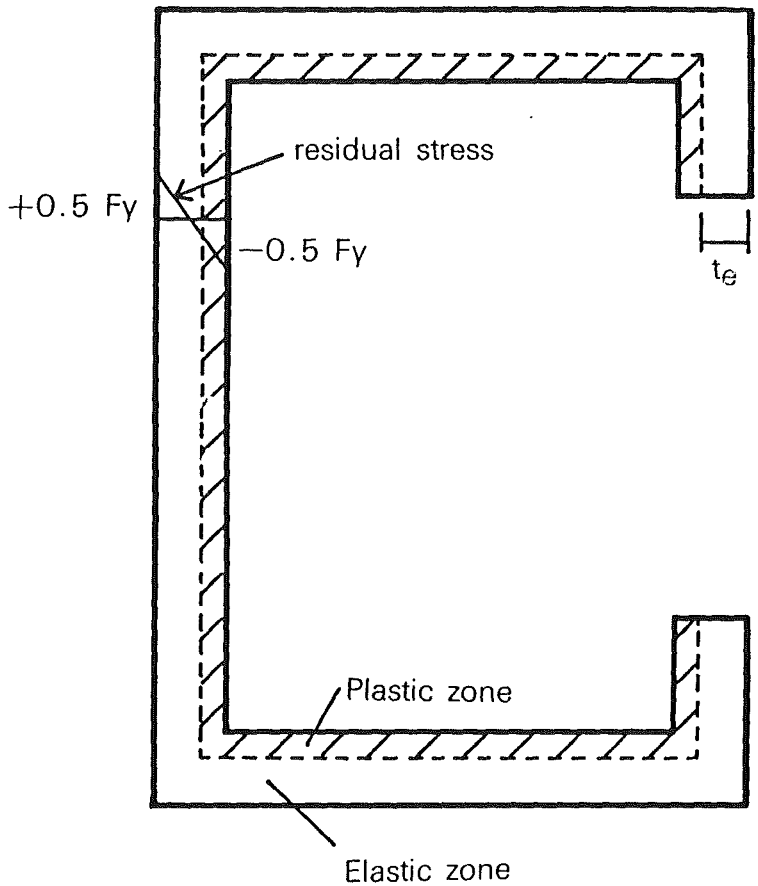


FIG. 3. Idealized Residual Stress Distribution Pattern for Cold-Formed Channel Section

of the component plate elements of the sections, and the residual stresses measured from the columns was observed.

Table 1 summarizes the flat-width ratios of the webs of the channel sections tested in this investigation as well as the maximum compressive residual strains measured from the webs. According to the AISI *Specification* (1986), the limiting value of the flat-width ratio for stiffened plate elements, $(W/t)_{lim}$, is $221/\sqrt{F_y}$. From this table, it is observed that for those columns showing lower strengths than the values predicted by the AISI formulas, the ratios of the limiting flat-width ratio to the flat-width ratio of the web of the section, $(W/t)_{lim}/(W/t)$, are all close to unity. On the other hand, for those columns showing good agreement with the AISI predictions, the values of the $(W/t)_{lim}/(W/t)$ are found to be much larger than unity.

In addition, it is observed that the residual stresses measured from the columns showing lower strengths are higher than the residual stresses measured from the columns showing good agreement with the AISI predictions.

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TABLE 1. Flat-Width Ratios and Maximum Compressive Residual Stresses of Test Specimens

Column (1)	W/t (2)	$(W/t)_{lim}/(W/t)$ (3)	ϵ_{rs} (10^{-6}) (4)	ϵ_{rs}/ϵ_y (5)	P_{TEST}/P_{AISI} (6)
RFC14	29.63	1.01	1,044	0.56	0.75–0.93
PBC14[a]	34.25	1.03	594	0.45	0.76–1.09
RFC14[a]	34.25	0.97	699	0.57	0.78–0.91
R14	30.70	1.02	758	0.45	0.82–0.99
P11	37.38	1.02	458	0.40	0.83–0.95
P16	36.48	1.07	458	0.40	0.86–0.95
R13	27.91	1.05	857	0.50	0.86–1.01
DC RFC14	29.63	1.01	1,044	0.56	0.85–0.99
DC R14	30.70	1.02	758	0.45	0.90–1.01
RFC13	26.70	1.15	428	0.24	0.90–1.04
PBC14	36.20	1.01	404	0.33	0.92–1.00
RFC11	21.87	1.59	381	0.28	0.92–1.13
P4100	19.67	1.56	314	0.18	0.93–1.11
P3300	13.48	2.19	402	0.21	0.95–1.13
DC14	13.48	2.45	496	0.27	0.98–1.23

[a]Dat's (1980) test specimens.

Note: W = flat width of web of section; t = thickness of the web of section; ϵ_{rs} = measured maximum compressive residual strain of web; ϵ_y = yield strain of web of section; P_{TEST} = column test result; P_{AISI} = column strength predicted by AISI equation; and $(W/t)_{lim} = 221/\sqrt{F_y}$ for stiffened plate element.

The magnitudes of the residual stresses are usually close to 50% of the yield stress of the material for the columns showing lower strengths than the AISI predictions.

These observations indicate that the proportioning of the cross-section dimensions and the magnitude of the residual stresses have a substantial influence on the reduction of the column strength. Based on the understanding of the magnitude and distribution of the residual stresses in cold-formed steel sections (Weng and Pekoz 1990b), it is possible that an axially compressed "fully effective section" may become "partially effective" due to the presence of residual stresses, especially when the W/t ratio of the component plate element of the section is close to the limiting value of the flat-width ratio.

Concept of Second Reduction

Based on the above observations, a new concept called the "second reduction" is developed to account for the effect of residual stresses on the local buckling behavior of cold-formed steel columns. This concept provides an explanation for the problem of the understrength of some types of cold-formed steel columns. Before explaining the concept of the second reduction, a term called the "first reduction" is introduced.

First Reduction

The first reduction represents the effect of residual stresses on the reduction of the "overall buckling strength" of steel columns. This effect has been

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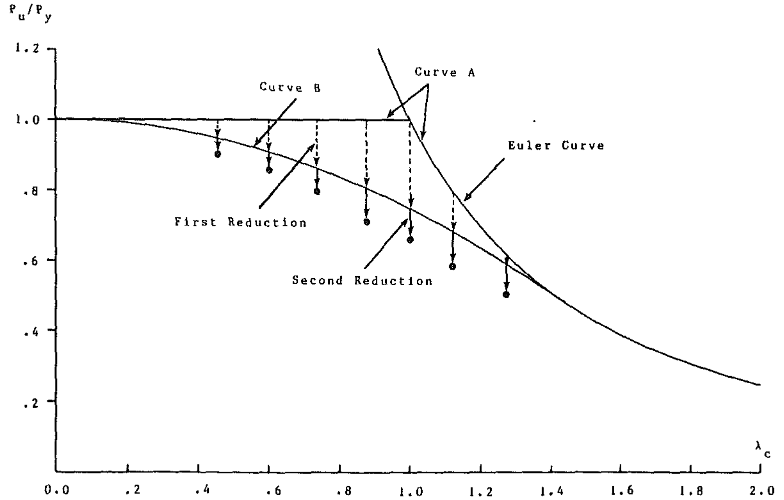


FIG. 4. First and Second Reduction of Column Strength Caused by Residual Stresses

taken into account during the development of the CRC column curve. As shown in Fig. 4, curve *A* represents the strength of straight columns without the influence of residual stresses. The first reduction refers to the reduction of the column strength from curve *A* to curve *B* due to the effect of residual stresses.

It is noted that the first reduction only accounts for the weakening of the overall column buckling strength caused by residual stresses. However, residual stresses may have an additional effect on the local buckling of cold-formed steel sections, which can result in a further reduction of the column strength.

Second Reduction

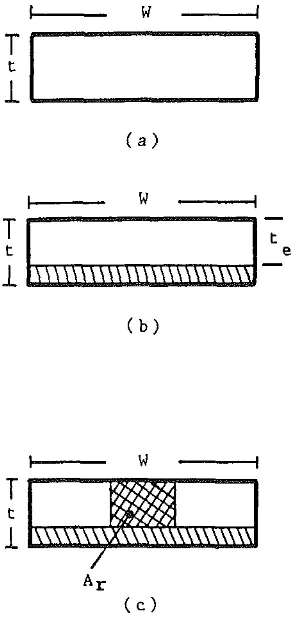
In order to obtain a better understanding of the behavior of residual stresses in cold-formed steel columns, the process of the yielding propagation of a component plate element of a fully effective section is illustrated in Figs. 5(*a*), (*b*), and (*c*), which is based on the idealized residual stress distribution pattern presented in the paper by Weng and Pekoz (1990).

As shown in Fig. 5(*a*), if there is no residual stress in the plate, the ultimate strength, P_u, of the fully effective section is

$$P_u = F_y \cdot A_g \qquad (1)$$

where A_g = the gross section area, and F_y = the yield stress of the material.

If residual stresses exist in a section and the section is axially compressed, the component plate element of the section may become partially yielded with a layer of elastic zone and layer of plastic zone zone as a result of the yielding propagation (Weng and Pekoz 1990b). Since the residual stresses are assumed to vary linearly through the plate thickness and distribute uniformly along the perimeter of the entire section (the variations of the residual stress and yield stress at the corner regions are neglected), the inelastic col-



$$P_u = F_y \cdot A_g$$

First Reduction

$$Fu = \frac{\pi^2 \cdot E}{(KL/r)^2} \cdot \tau$$

$$P_{u1} = F_u \cdot A_g$$

where $\tau = I_e/I = t_e/t$

Second Reduction

$$Fu = \frac{\pi^2 \cdot E}{(KL/r)^2} \cdot \tau$$

$$P_{u2} = F_u \cdot A_{eff}$$

where $A_{eff} = A_g - \Sigma(A_r)$

FIG. 5. Yielding Propagation and First and Second Reduction in Fully Effective Plate Element

umn buckling stress, F_u, can be found from

$$F_u = \left[\frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2}\right] \cdot \tau \quad (2)$$

where τ = a reduction factor for inelastic buckling.

As shown in Fig. 5(*b*), if only the first reduction is considered, the ultimate strength of the column, P_{u1}, can be obtained by multiplying the inelastic buckling stress by the gross area of the section. That is

$$P_{u1} = F_u \cdot A_g \quad (3)$$

However, by examining the behavior of the partially yielded plate element, which is originally fully effective, it is noted that the rigidity of the plate element is decreased due to the presence of a layer of yielded zone. The plate element may become partially effective if the flat-width ratio of the elastic zone, W/t_e, becomes larger than the limiting flat-width ratio, $(W/t)_{\lim}$. If W/t_e is larger than $(W/t)_{\lim}$, the cross-hatched part of the elastic zone, A_r, as shown in Fig. 5(*c*), should be subtracted from the gross area due to the effect of local buckling. Therefore, the effective area of the section, A_{eff}, becomes

$$A_{eff} = A_g - \Sigma(A_r) \quad (4)$$

Consequently, the ultimate strength of the locally buckled column, P_{u2}, is found to be

$$P_{u2} = F_u \cdot A_{eff} \quad (5)$$

It is obvious that the column strength P_{u2} is smaller than that obtained from the first reduction, P_{u1}. The reduction of the column strength from P_{u1} to P_{u2} is called the "second reduction."

As illustrated in Fig. 4, the reduction of column strengths from curve B to the test-data points is a result of the second reduction. The test-data points shown in the figure represent some typical test results of those columns showing lower strengths than the AISI predictions.

Magnitude of Second Reduction: Method of Reduced Thickness

To determine the magnitude of the second reduction, it is necessary to find the cross-hatched area, A_r, of the partially yielded and locally buckled plate element, as shown in Fig. 5(*c*). Since it is difficult to calculate A_r directly from the partially yielded plate element, a new approach called the "method of reduced thickness" is developed. This method uses an elastic plate of reduced thickness, t_r, to represent the behavior of a partially yielded plate such that Winter's (1970) effective width equation can be applied to determine the effective width, b, of the plate element. This method is illustrated in Figs. 6(*a*), (*b*), and (*c*).

Since the area A_r can not be obtained directly, an equivalent area, A_{eq}, is used which can be determined by using the effective width b obtained based on the reduced thickness t_r. Let

$$A_r = A_{eq} = t \cdot (W - b) \quad (6)$$

Then, the effective area of the section can be found from

$$A_{eff} = A_g = \Sigma[t \cdot (W - b)] \quad (7)$$

It is noted that the reduction of the thickness from t to t_r represents the subtraction of the equivalent area, A_{eq}, from the original area of the plate element.

The equation of the reduced thickness t_r has the form

$$t_r = \phi \cdot t \quad (8a)$$

$$\phi \leq 1.0 \quad (8b)$$

where ϕ = a reduction factor to be determined. The reduction factor can be related to the elastic thickness, t_e, as follows:

$$\phi = 1.0 - \alpha \cdot \left(\frac{t_e}{t}\right) \quad (9)$$

where α = a modification factor which depends on the stress level in the plate, and can be obtained empirically.

Since a maximum residual stress of $0.5F_y$ is assumed in the plate element,

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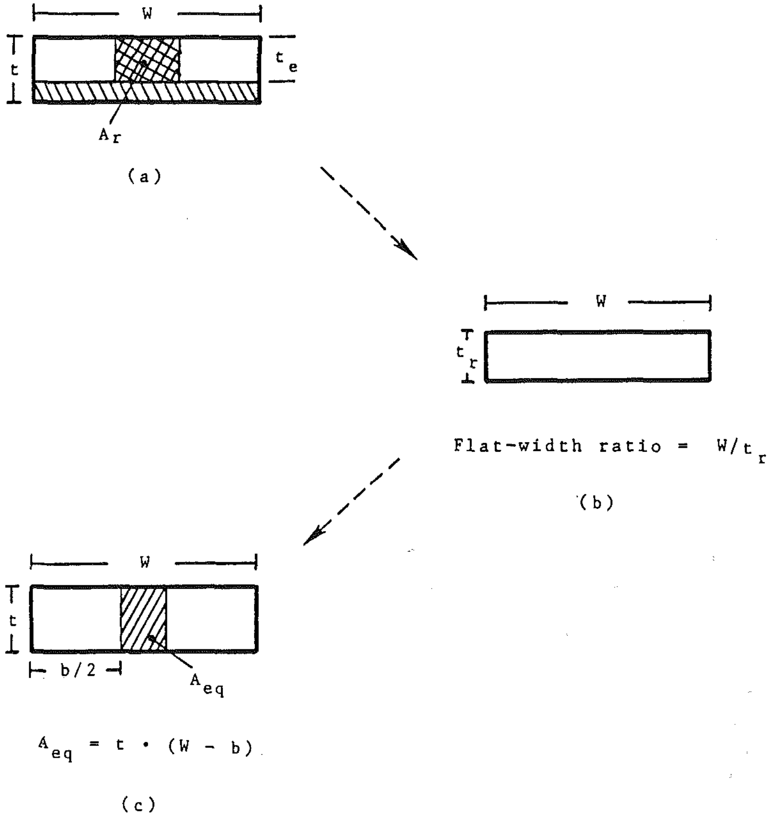

FIG. 6. Method of Reduced Thickness to Account for Effect of Second Reduction

the residual stress has no effect on the strength of the plate element when the applied stress is less than $0.5F_y$, and A_{eq} does not exist. Similarly, when the applied stress reaches F_y, the entire section has yielded and A_{eq} becomes zero. Hence, t_r equals t. In order to satisfy the conditions of t_r equals t when F_u/F_y is equal to 0.5 and 1.0, and to provide a good estimation of the column strength, several equations for the modification factor, α, were tried. The equation which provides the best fit between the predicted column strengths and the test results is found to be

$$\alpha = 0.5 - \frac{(0.005)}{\left[\left(\frac{F_u}{F_y}\right) - 0.49\right]} \qquad (10)$$

As derived in the paper by Weng and Pekoz (1990b), the equation of the elastic thickness, t_e, is

$$t_e = t \cdot \sqrt{2\left(1 - \frac{F_u}{F_y}\right)} \qquad (11)$$

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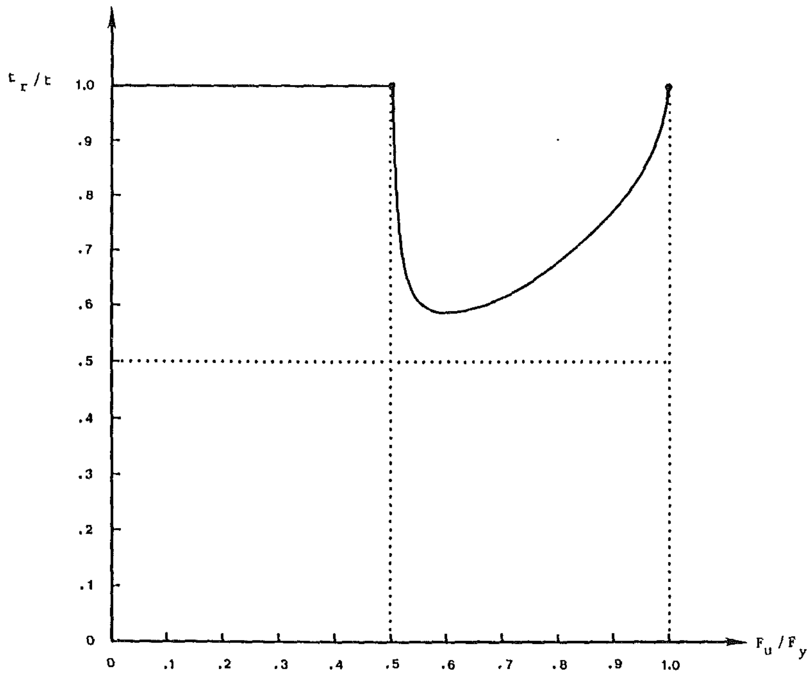

FIG. 7. Variation of Reduced Thickness, t_r

By substituting (10) and (11) into (9), the reduction factor, ϕ, becomes

$$\phi = 1 - \left(0.5 - \frac{0.005}{\left(\frac{F_u}{F_y}\right) - 0.49}\right) \cdot \sqrt{2\left(1 - \frac{F_u}{F_y}\right)} \quad \text{(12)}$$

Consequently, the reduced thickness, t_r, is found to be

$$t_r = t \cdot \left[1 - \left(0.5 - \frac{0.005}{\left(\frac{F_u}{F_y}\right) - 0.49}\right) \cdot \sqrt{2\left(1 - \frac{F_u}{F_y}\right)}\right] \quad \text{(13)}$$

The variation of t_r as a function of the stress ratio, F_u/F_y, is shown in Fig. 7.

Once the reduced thickness t_r is determined, the effective width b can be obtained by using Winter's effective equation. Then, the equivalent area, A_{eq}, and the effective area of the entire section, A_{eff}, can be calculated from (6) and (7), respectively. Finally, the strength of the column can be found from

$$P_{u2} = F_u \cdot A_{eff} \quad \text{(14)}$$

Possible Design Procedure

Based on the concept of second reduction, a possible design procedure

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for determining the flexural buckling strength of cold-formed steel columns is outlined as follows:

1. Find the Euler buckling stress, F_e

$$F_e = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2} \quad (15)$$

2. Check for elastic or inelastic buckling

If $F_e \leq 0.5F_y$,

$$F_u = F_e \quad (16a)$$

If $F_e > 0.5F_y$,

$$F_u = F_y\left[1 - \frac{F_y}{(4F_e)}\right] \quad (16b)$$

3. Determine the effective width, b, for each plate element of the section at the stress F_u:

a. Calculate

$$t_r = \phi \cdot t \quad (17)$$

$$\lambda = \left(\frac{1.052}{\sqrt{K}}\right) \cdot \left(\frac{W}{t}\right) \cdot \sqrt{\frac{F_u}{E}} \quad (18)$$

If ($F_u \leq 0.5F_y$ or $\lambda \geq 0.673$),

$$\phi = 1.0 \quad (19)$$

If ($F_u > 0.5F_y$ and $\lambda < 0.673$),

$$\phi = 1 - \left(0.5 - \frac{0.005}{\left(\frac{F_u}{F_y}\right) - 0.49}\right) \cdot \sqrt{2\left(1 - \frac{F_u}{F_y}\right)} \quad (20)$$

b. Calculate

$$\lambda_r = \left(\frac{1.052}{\sqrt{K}}\right) \cdot \left(\frac{W}{t_r}\right) \cdot \sqrt{\frac{F_u}{E}} \quad (21)$$

If $\lambda_r \leq 0.673$,

$$b = W \quad (22)$$

If $\lambda_r > 0.673$,

$$b = \rho \cdot W \quad (23)$$

where

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$$\rho = \frac{\left(1 - \frac{0.22}{\lambda_r}\right)}{\lambda_r} \quad (24)$$

4. Find the effective area of the section, A_{eff}

$$A_{eff} = A_g - \Sigma[t \cdot (W - b)] \quad (25)$$

5. Calculate the strength of the column under concentric loading, P_0,

$$P_0 = F_u \cdot A_{eff} \quad (26)$$

6. Determine the column strength when subjected to combined axial load and bending:

a. Find the shift of the centroid, X_s.

b. Calculate the bending capacity of the section, M_u.

c. Determine the Euler buckling load, P_e, based on the full, unreduced section.

d. Calculate the strength of the column, P_u, by using the interaction equation

$$\frac{P_u}{P_0} + \frac{P_u \cdot X_s}{M_u\left(\frac{1 - P_u}{P_e}\right)} = 1.0 \quad (27)$$

In Figs. 1 and 2, the ratios of the column test results to the AISI predictions, P_{TEST}/P_{AISI}, indicate that the AISI predictions are on the unconservative side. Many of the test results show a 10–25% lower strength than the AISI predictions.

On the other hand, the ratios of the column test results to the values predicted by the proposed design procedure, P_{TEST}/P_{PROP}, as shown in Figs. 8 and 9, indicate that a significant improvement has been achieved by using the proposed procedure to predict the column strength. It is observed that the proposed procedure reduces the column capacities computed on the basis of the AISI Specification to about 20%.

As shown in Tables 2 and 3, the mean values of P_{TEST}/P_{PROP} for Dat's (1980) and Weng and Pekoz's (1990a) column tests are between 0.95 and 1.11, and the coefficients of variation are less than 6.5%. Hence, the proposed design approach is satisfactory for predicting the flexural buckling strength of cold-formed steel columns.

It is noted that the second reduction occurs only when the following conditions are met:

1. The sum of the applied stress and the compression residual stress reaches the yield point of the material. In the procedure just outlined, the maximum residual stress in the column is taken as 0.5 F_y.

2. The elastic critical buckling stress, F_{cr}, of the component plate element of the section is higher than 0.5 F_y. When the critical buckling stress of the plate element is less than 0.5 F_y, the plate buckles before the sum of the applied stress and the residual stress reaches the yield point of the material.

It is also noted that when the calculated value of the W/t_r is less than the limiting value of the flat-width ratio, $(W/t)_{lim}$, the second reduction will not

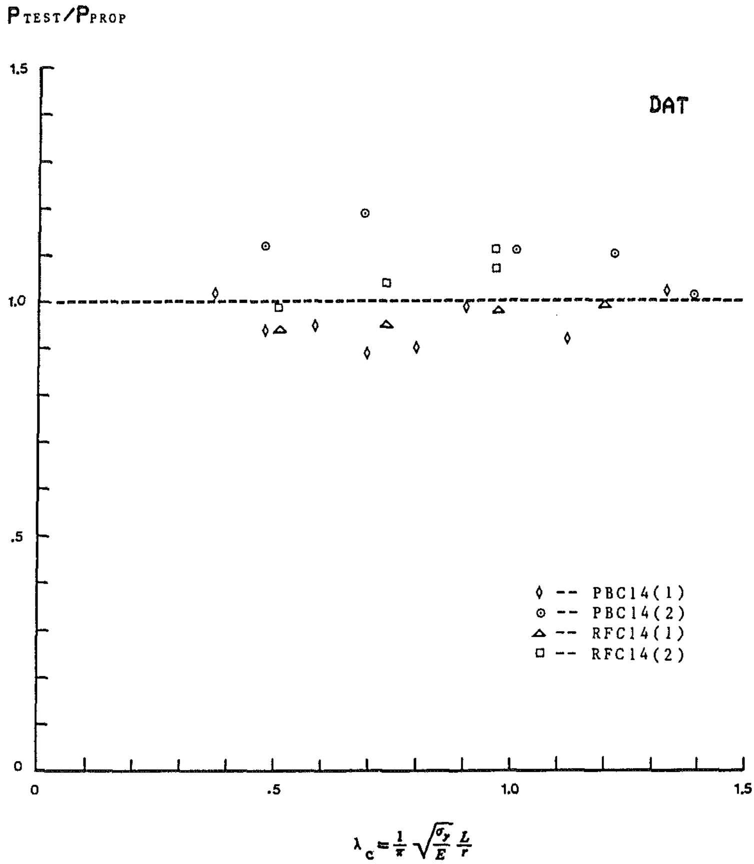

FIG. 8. Dat's (1980) Test Results versus Values Predicted by Proposed Method

occur. This may happen when the W/t ratio of a plate element is much smaller than $(W/t)_{lim}$.

Inelastic Column Equations without Second Reduction

It was mentioned that the design formula of the AISI Specification for column buckling in the inelastic range is based on the equation originally developed for hot-rolled sections. This equation is

$$\frac{F_u}{F_y} = 1.0 - \left(\frac{\lambda_c^2}{4}\right), \qquad (\lambda_c \leq \sqrt{2}) \qquad (28)$$

where

$$\lambda_c = \sqrt{\frac{F_y}{F_e}} \qquad (29)$$

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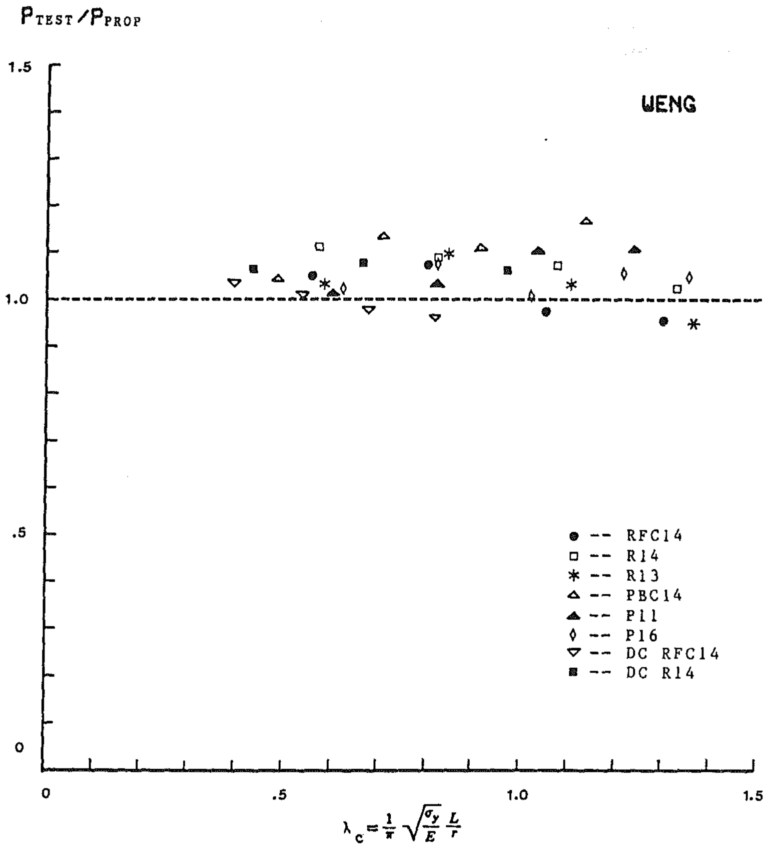

FIG. 9. Weng and Pekoz's (1990a) Test Results versus Values Predicted by Proposed Method

According to Yang et al. (1952), the inelastic buckling strength of a column can be found by multiplying the Euler load by the ratio of I_e/I, where I_e is the moment of inertia of the elastic part of the section and I is the moment of inertia of the entire section. The ratio of I_e/I can be regarded as a reduction factor, τ. Then, the equation of the inelastic buckling stress becomes

$$F_u = \left[\frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2}\right] \cdot \tau \qquad (30)$$

In order to determine the reduction factor, it is necessary to find the moment of inertia of the elastic part of the section. The elastic part of the section can be obtained from the pattern of the yielding propagation of the section caused by the residual stress and the applied compression stress.

For cold-formed steel columns, the inelastic buckling stress equation can

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TABLE 2. Comparison of Column Strengths Predicted by AISI Equations and by Proposed Method [Dat's (1980) Test Data]

Column number (1)	P_{TEST}/P_{AISI} (2)	P_{TEST}/P_{PROP} (3)
	(*a*) RFC14(1)	
1	0.91	0.99
2	0.91	1.04
3	0.91	1.11
4	0.88	1.07
Mean	0.90	1.05
Coefficient of variation	1.7%	4.9%
	(*b*) PBC14(2)	
1	0.98	1.02
2	0.89	0.94
3	0.88	0.95
4	0.81	0.89
5	0.80	0.90
6	0.85	0.99
7	0.76	0.92
8	0.85	1.02
Mean	0.85	0.95
Coefficient of variation	7.9%	5.3%
	(*c*) RFC14(2)	
1	0.82	0.95
2	0.80	0.97
3	0.78	0.99
4	0.86	0.94
Mean	0.82	0.96
Coefficient of variation	4.2%	2.3%
	(*d*) PBC14(1)	
1	1.06	1.12
2	1.09	1.19
3	0.93	1.11
4	0.89	1.10
5	1.01	1.01
Mean	1.00	1.11
Coefficient of variation	8.5%	6.2%

be derived by using the elastic thickness, t_e, obtained in the paper by Weng and Pekoz (1990b). Since the thickness of a cold-formed steel section is usually quite small, the moment of inertia of the section, I, can be found approximately by using the linear method. This leads to

$$I = I' \cdot t \qquad (31)$$

$$I_e = I' \cdot t_e \qquad (32)$$

where I' = the moment of inertia of the centerline of the section. Thus, the reduction factor, τ, becomes

TABLE 3. Comparison of Column Strengths Predicted by AISI Equations and by Proposed Method [Weng and Pekoz's (1990a) Test Data]

Column (1)	P_{TEST}/P_{AISI} (2)	P_{TEST}/P_{PROP} (3)
(*a*) RFC14		
1	0.934	1.053
2	0.903	1.073
3	0.772	0.974
4	0.748	0.957
Mean	0.839	1.014
Coefficient of variation	11.1%	5.6%
(*b*) R13		
1	0.997	1.033
2	1.009	1.099
3	0.893	1.032
4	0.863	0.938
Mean	0.941	1.026
Coefficient of variation	7.8%	6.4%
(*c*) P11		
1	0.948	1.006
2	0.922	1.031
3	0.850	1.106
4	0.832	1.108
Mean	0.888	1.063
Coefficient of variation	6.3%	4.9%
(*d*) DC RFC14		
1	0.989	1.038
2	0.945	1.012
3	0.894	0.979
4	0.853	0.960
Mean	0.920	0.997
Coefficient of variation	6.2%	3.5%
(*e*) R14		
1	0.993	1.113
2	0.918	1.089
3	0.852	1.073
4	0.816	1.025
Mean	0.895	1.075
Coefficient of variation	8.7%	3.5%
(*f*) PBC14		
1	0.956	1.042
2	0.994	1.133
3	0.921	1.107
4	0.924	1.169
Mean	0.949	1.113
Coefficient of variation	3.6%	4.8%
(*g*) P16		
1	0.938	1.022
2	0.947	1.078
3	0.855	1.009
4	0.866	1.057
5	0.905	1.048
Mean	0.902	1.043
Coefficient of variation	4.6%	2.7%
(*h*) DC R14		
1	1.010	1.063
2	0.984	1.078
3	0.901	1.066
Mean	0.965	1.069
Coefficient of variation	5.9%	0.8%

J. Struct. Eng. 1991.117:1622-1640.

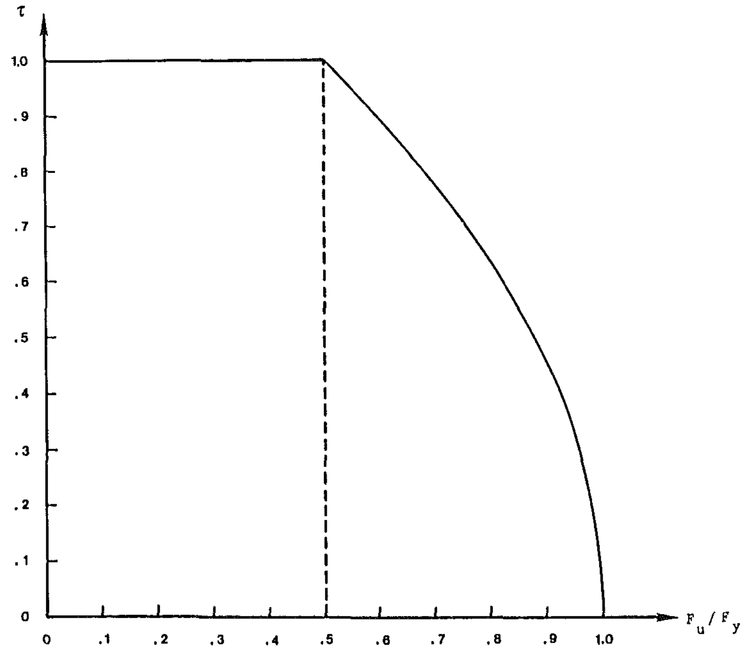

FIG. 10. Variation of Reduction Factor, τ, for Inelastic Buckling Strength

$$\tau = \frac{I_e}{I} = \frac{t_e}{t} \quad (33)$$

By using the equation derived for the elastic thickness, t_e, which is given in the paper by Weng and Pekoz (1990b), the reduction factor is found to be

$$\tau = \sqrt{2\left(1 - \frac{F_a}{F_y}\right)} \quad (34)$$

The reduction factor, τ, as a function of the applied stress is plotted in Fig. 10. Finally, the equation of the inelastic buckling stress for cold-formed steel columns becomes

$$F_u = \left[\frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2}\right] \cdot \sqrt{2\left(1 - \frac{F_a}{F_y}\right)} \quad (35)$$

Also, (35) can be written in terms of λ_c, which results in

$$\frac{F_u}{F_y} = \left(\frac{1}{\lambda_c^4}\right) \cdot (\sqrt{2\lambda_c^4 + 1} - 1), \qquad (\lambda_c \leq \sqrt{2}) \quad (36)$$

A comparison between the inelastic buckling stresses given by (36) and the AISI equation, (28), is presented in Table 4. It is observed that the

TABLE 4. Comparison of Inelastic Column Buckling Stresses Obtained by AISI and Derived Equations

λ_c (1)	F_u/F_y (AISI) (2)	F_u/F_y (derived) (3)	Difference (4)
0.2	0.990	0.999	1%
0.4	0.960	0.988	3%
0.6	0.910	0.942	3%
0.8	0.840	0.852	1%
1.0	0.750	0.732	3%
1.1	0.698	0.671	4%
1.2	0.640	0.612	5%
1.4	0.510	0.507	0.5%

Note: λ_c = Column slenderness parameter.

difference between these two equations is quite small, usually less than 5%.

However, it is noted that both the derived and the AISI equations do not take into account the effect of residual stresses on the local buckling behavior of cold-formed steel sections. This effect has been shown to be responsible for those columns showing lower strengths than the AISI predictions.

Therefore, if the presence of residual stresses does not cause a reduction of the local buckling strength of the section, the inelastic buckling equation originally developed for hot-rolled sections, (28), can be used satisfactorily for predicting the strength of cold-formed steel columns. This observation provides an explanation that the AISI formulas gave good estimations of the strength of some types of columns when compared with the column test results.

Summary and Conclusions

The major results obtained from this research are summarized as follows:

1. An explanation for the problem of the understrength of some types of cold-formed steel columns is presented.
2. The concept of the second reduction is introduced to account for the effect of residual stresses on the local buckling behavior of the component plate elements of a cold-formed section. This concept provides a further understanding of the influence of residual stresses on the strength of cold-formed steel columns.
3. A possible design procedure is outlined that gives satisfactory predictions of the flexural buckling strength of the columns tested in this investigation.
4. It is shown that if a second reduction in the calculation is not required, the inelastic buckling stress equation originally developed for hot-rolled sections can be used satisfactorily for cold-formed steel columns.

Appendix I. References

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APPENDIX II. NOTATION

The following symbols are used in this paper:

A_{eff} = effective section area;
A_{eq} = equivalent area;
A_g = gross section area;
A_r = area reduced due to second reduction;
b = effective width;
E = Young's modulus;
F_a = applied stress;
F_e = Euler buckling stress;
F_u = inelastic column buckling stress;
F_y = yield stress;
I = moment of inertia of gross area;
I_e = moment of inertia of elastic area;
k = column effective length factor;
L = column length;
P_{AISI} = column strength predicted by AISI equations;
P_{PROP} = column strength predicted by proposed equations;
P_{TEST} = column strength obtained from test;
P_u = ultimate column strength;
t = plate thickness;
t_e = elastic plate thickness;
t_r = reduced plate thickness;
W = flat width;
α = modification factor;
ϵ_{rs} = compressive residual strain;
ϵ_y = yield strain of material;
λ = plate slenderness parameter;
λ_c = column slenderness parameter;
τ = reduction factor for inelastic column buckling; and
ϕ = reduction factor for reduced thickness.