Precision displacement measurement by active laser heterodyne interferometry

Yi-Jyh Lin and Ci-Ling Pan

We demonstrate an active laser heterodyne interferometer which can automatically compensate environmental disturbances and is capable of precision displacement measurement. Two Zeeman He–Ne lasers were employed. Laser II was used as the light source to stabilize the interferometer by using a piezomirror in a feedback loop. The hetereodyne signal of the two lasers was used to directly calibrate mirror displacement, with the wavelength of laser I tuned to compensate change in the interference signal due to displacement only. Separation of the contributions to the interference signal from displacement and external disturbances was accomplished by using polarization optics. This interferometer was used to measure the hysteresis of a piezoelectric transducer (PZT) over a dynamic range of 0.5 μm with a resolution of $\pm$0.8 nm. *Key words*: Metrology, laser.

I. Introduction

Traditionally, precision displacement measurements were performed using fringe counting-type laser interferometers.[1] If desired, dc drifts and low frequency phase noise in these interferometers could be eliminated to a large extent by employing various feedback schemes. For example, this can be achieved simply by altering the optical path length in the reference arm of a Michelson interferometer by physically moving a piezomirror.[2] Improved versions using flexure elements have also been recently demonstrated.[3] Another technique involves the use of optical fibers and compensating path length changes by straining the fiber.[4] Shajenko and Green[5] first proposed using a tunable laser for signal stabilization of optical interferometric hydrophones. This was experimentally demonstrated by Olsson *et al.*[6] using a dye laser with an electrooptical (EO) tuner. Employing widely tunable laser diodes as the light source, Yoshino and co-workers[7] improved fringe stability by 2 orders of magnitude and achieved displacement measurement over a dynamic range of 8–9 μm with a precision of 10–16 nm. In these so-called active laser interferometers, the change in the interference signal from displacement, etc. was compensated by tuning the frequency of the laser. The measurement resolution was limited by the linewidth of the laser, while the laser tuning range determined the measurement range.

It is not necessary to calibrate the wavelength of the tunable laser in these active interferometers if a heterodyne arrangement is employed. For example, Brillet and Hall[8] measured space anisotropy by locking a tunable laser to a cavity resonance and measuring the heterodyne beat frequency between the tunable laser and a second fixed frequency laser. The same technique was employed by Riis *et al.*[9] recently to calibrate the electrical response of piezoelectric elements used in scanning tunneling microscopes. Alternatively, Bjorklund *et al.*[10] used a FM sideband technique by locking a laser FM sideband to an interferometer resonance. In these works, extreme care was taken for environmental control. The Fabry-Perot interferometer setup used was also not suitable for general purpose displacement measurement.

Previously we showed preliminary results for a Zeeman He–Ne laser-based active laser interferometer.[11] The Zeeman beat frequency of the laser varies as a function of the laser wavelength and can be calibrated such that it provides a convenient frequency marker for the laser output.[12] Utilizing the extreme narrow linewidth and excellent frequency stability (<1 MHz) of the laser, we were able to demonstrate fringe stabilization and precision displacement measurements with subnanometer resolution. While the single-mode tuning range of this laser is only $\approx$300 MHz, displacement measurements over many wavelengths can be obtained by adjusting the optical path length difference (OPD)

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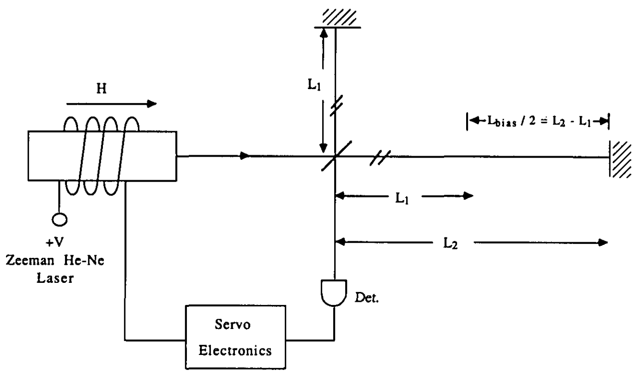

Fig. 1. Schematic representation of a generic active laser interferometer.

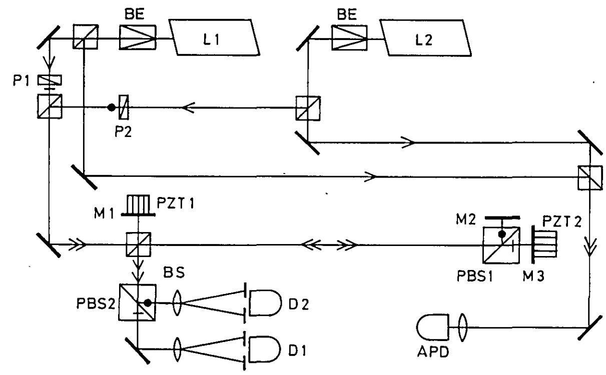

Fig. 2. Block diagram of the active laser heterodyne interferometer: $L1$, laser I; $L2$, laser II; BE, beam expanders; $P1$ and $P2$, polarizers; BS, beam splitter; $PBS1$ and $PBS2$, polarizing beam splitters; $D1$ and $D2$, photodiodes; and APD, avalanche photodiode.

of the two arms of the Michelson interferometer. One difficulty we encountered with this instrument was its extreme sensitivity to environmental disturbances, such as thermal drift, air flow, and mechanical vibrations. The laser would tune its wavelength to compensate change in the optical phase, regardless of its origin. The situation is particularly severe for an interferometer with long OPD. This is overcome to a large extent in this work by stabilizing the interferometer in a separate loop using a heterodyne technique. We present basic principles of this active laser heterodyne interferometer in Sec. II. Experimental techniques and results are presented in Secs. III and IV, respectively. Preliminary results were presented at the 1989 OSA Annual Meeting.[13]

II. Basic Principles and Experimental Methods

For convenience, let us consider a generic active laser interferometer using a Zeeman He–Ne laser as the light source and a Michelson interferometer as the displacement sensor (Fig. 1). The optical phase between the interfering beams of the two arms is given by

$$\phi = 2\pi \frac{\mathrm{OPD}}{\lambda}, \tag{1}$$

where λ is the wavelength of the laser. A variation of the OPD or λ will cause a phase change:

$$\Delta\phi = \frac{2\pi(\lambda\Delta\mathrm{OPD} - \mathrm{OPD}\Delta\lambda)}{\lambda^2}. \tag{2}$$

In an active laser interferometer, the phase change is nulled by wavelength tuning of the laser such that

$$\Delta\mathrm{OPD} = \left(\frac{\mathrm{OPD}}{\lambda}\right)\Delta\lambda = -\left(\frac{\mathrm{OPD}}{f}\right)\Delta f, \tag{3}$$

where $f = c/\lambda$ is the frequency of the laser. External disturbances such as air current, temperature fluctuations, etc., as well as the actual displacement x_m, contribute to ΔOPD. In this work we are interested in the measurement of displacement for which $\Delta\mathrm{OPD} \ll \mathrm{OPD}$ and the use of laser sources with a tuning range $\Delta f \ll f$. Equation (3) indicates that the range of displacement that can be measured for a given tuning range of the laser is linearly proportional to the OPD. Either subnanometer displacement or change in the OPD of many wavelengths can be measured by employing a laser with a sufficiently large tuning range or an appropriate choice of the OPD. The optical phase at the output of the interferometer can thus be viewed as biased by the OPD, which is the product of the index of refraction and the preset path length difference of the two arms, i.e., $\mathrm{OPD_{bias}} = nL_{\mathrm{bias}}$. If the environmental disturbances can be eliminated or neglected, we can write

$$x_m = -\frac{\mathrm{OPD_{bias}}}{2f}\Delta f = -\frac{L_{\mathrm{bias}}}{2f}\Delta f. \tag{4}$$

In Eq. (4), the displacement compensation ratio $x_m/\Delta f$ is essentially a constant for a given L_{bias}. For example, this ratio is 0.11 nm/MHz for $L_{\mathrm{bias}} = 10$ cm. That is, a displacement of 0.11 nm would be compensated by shifting the frequency of the laser by 1 MHz. Equation (4) also shows that the transfer function for the active laser interferometer is essentially linear.

The experimental arrangement of our active laser heterodyne interferometer is shown in Fig. 2. Two Zeeman He–Ne lasers[12,14,15] were employed. These were stabilized by phase locking the Zeeman beat signal to an external reference clock. The error signals were fed back as a current through a heater wound around the laser tube to control its cavity length. The Allan variance[16] of these lasers has been measured for averaging times τ between 10 ms and 100 s. It exhibits the familiar bathtub shape and the frequency stability was 3×10^{-10} for $\tau = 0.01$ s, 5×10^{-11} for τ between 0.1 and 1 s, and better than 10^{-10} for τ of the order of a few seconds.

Laser II serves as the light source for an auxiliary interferometer formed by mirrors M_1 and M_2 as well as beam splitter BS and polarizing beam splitter $PBS1$. Clearly, one arm of the auxiliary interferometer coincides with that of the main interferometer (formed by mirrors M_1, M_3, and the beam splitters). Mirrors M_2 and M_3 were carefully positioned relative to the polarizing beam splitter $PBS1$, such that the bias length of the other arm of the two interferometers as equal within a millimeter. The interference signals derived from these two interferometers were separated by using a pair of polarizing beam splitters and fed to detectors $D1$ and $D2$, respectively. Phase sensitive detection schemes were used to extract the error signal used for feedback control. The change in the optical phase detected by $D2$ was nulled by using piezomirror M_1 via a servo loop to compensate the change in the OPD because of environmental disturbances as well as frequency drift or fluctuation of laser II. As a result,

$$\frac{\Delta\mathrm{OPD}_n + \Delta\mathrm{OPD}_c}{\mathrm{OPD}} = -\frac{\Delta f_2}{f_2}, \tag{5}$$

where $\Delta\mathrm{OPD}_n$ and $\Delta\mathrm{OPD}_c$ were changes in the OPD because of environmental factors and compensational displacement from piezomirror M, f_2 is the output frequency of laser II, and Δf_2 is its fluctuation. In the main interferometer, nulling the optical phase by wavelength tuning of laser I results in

$$\frac{(\Delta\mathrm{OPD}_n + \Delta\mathrm{OPD}_c) + \Delta\mathrm{OPD}_m}{\mathrm{OPD}_{bias}} = -\frac{\Delta f_1}{f_1}, \tag{6}$$

where $\Delta\mathrm{OPD}_m$ is the change in the OPD because of displacement of piezomirror M_3, which we wish to measure. We have also assumed that $\Delta\mathrm{OPD}_n$ for the two interferometers is essentially the same. If follows from Eqs. (5) and (6) that

$$\Delta(f_1 - f_2) \equiv \Delta f_1 - \Delta f_2 = -\frac{f_1}{\mathrm{OPD}_{\mathrm{bias}}}\Delta\mathrm{OPD}_m. \tag{7a}$$

or

$$x_m = -\frac{L_{\mathrm{bias}}}{2f_1}\Delta(f_1 - f_2). \tag{7b}$$

Equation (7) differs from Eq. (4) in that heterodyne frequency $\Delta(f_1 - f_2)$ replaces Δf. It is no longer necessary to know the absolute frequency of either of the two lasers. The heterodyne frequency is also independent of the change in the OPD resulting from environmental disturbances. The range of $\Delta(f_1 - f_2)$ is limited by the tuning range of the lasers used ($\approx$300 MHz in our case). Experimentally, the heterodyne frequency of the two lasers was detected by an avalanche photodiode (APD) and extracted from the auxiliary output ports of a RF spectrum analyzer (RFSA) with sample-and-hold circuits. The access time for beat frequency measurement was 0.3 s. Frequency counters were also employed initially. In this case, broadband preamplifiers had to be used. The amplifier that was available to us generated spurious signals at the harmonic frequencies of $\Delta(f_1 - f_2)$. This rendered the use of counter for signal processing impossible. It should be feasible to replace the RFSA with appropriately designed counting circuits in future experiments.

III. Results and Discussions

We have determined the measurement time constant of the interferometer to be 1.2 s from the rise time of the change in the beat signal when a step voltage was applied to *PZT*2. This was limited by the thermal response of the laser, but sufficiently short for our purposes. The results reported here thus represent average displacement measured over this time inter-

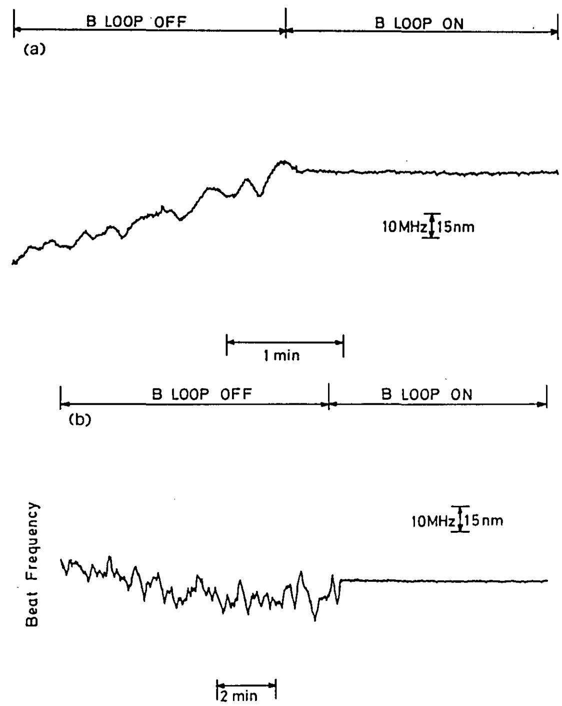

Fig. 3. (a) Residue fluctuation of the interferometer as represented by the beat frequency of the lasers with the feedback loop (*B* loop) for the auxiliary interferometer open or closed. (b) Same as (a) except that *PBS*1 was removed. (See Fig. 2 for reference.)

val, i.e., ~1.2 s. Figure 3(a) illustrates the measurement resolution of our interferometer for L_{bias} = 140 cm, corresponding to a displacement compensation ratio $x_m/\Delta f$ = 1.48 nm/MHz. With feedback loop *B* for the auxiliary interferometer closed, the residue fluctuation in the OPD of the interferometer was less than ±0.8 nm. Part of this residue error was attributed to the fact that, while the optical paths in the two interferometers were essentially the same, they were not identical. To check, we did the following experiment. The two arms of the main interferometer were BS–M_1 and *BS*–*PBS*1–*M*3, while those of the auxiliary interferometer were *BS*–*M*1 and *BS*–*PBS*1–*M*2, respectively (see Fig. 2). By removing *PBS*1, we could render the arms of the two interferometers exactly the same. In so doing, the residue noise reduced to ±0.4 nm [Fig. 3(b)].

It is not essential to use a frequency stabilized laser as the light source, laser II, in the auxiliary interferometer. Piezoelectric transducer PZT1 can, in principle, be used to compensate frequency drift of laser II also. Experimental verification of the above statement is shown in Fig. 4. The hysteresis of *PZT*2, which displaces mirror *M*3, measured by our apparatus is shown in Fig. 5. The specification of the PZT was 3 μm/1 kV. From our data it was determined that this ratio was 3.7 ± 0.8 nm/V. We note that the data points on the hysteresis loop were repeatable if we cycled the voltage applied to the PZT.

We now address the question of the error in the measurement of displacement employing our interferometer. Using Eq. (7), we can show that

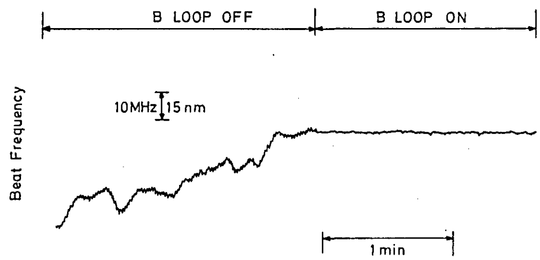

Fig. 4. Residue fluctuation of the interferometer as represented by the beat frequency of the lasers with the feedback loop (B loop) for the auxiliary interferometer open or closed. Laser III was free-runing.

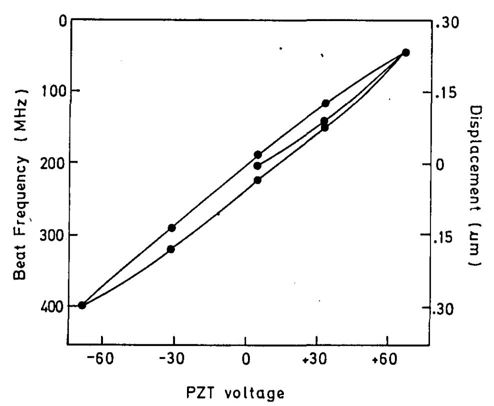

Fig. 5. Beat frequency as a function of the dc voltage applied to the piezoelectric transducer $PZT2$. The displacement was calculated using Eq. (7).

$$(\Delta x_m)^2 \cong \left(X_m \frac{\Delta K}{K}\right)^2 + (K\Delta f_{1\text{-}2})^2, \tag{8}$$

where Δx_m, ΔK, and $\Delta f_{1\text{-}2}$ are, respectively, the rms error in displacement x_m, displacement compensation ratio $K = -x_m/\Delta(f_1 - f_2) = -L_{\text{bias}}/2f_1$, and the beat frequency of the two lasers $\Delta(f_1 - f_2)$. The first term on the right-hand side of Eq. (8) represents the accuracy in the measured value of the displacement, while the second term determined the minimum displacement that can be measured. Since the frequency stability of our lasers was better than 10^{-9}, ΔK is dominated by ΔL_{bias} only, i.e., $\Delta K/K \approx \Delta L_{\text{bias}}/L_{\text{bias}}$. As an example, if $L_{\text{bias}} = 140$ cm, $\Delta L_{\text{bias}} = 2$ mm, $K = 1.48$ nm/MHz, $\Delta f_{1\text{-}2} = 0.5$ MHz, and $x_m = 500$ nm, we find that the accuracy of the measured value of the displacement is better than 0.15% and the resolution is ~0.5 nm. As a result, total error $\Delta x_m \cong 1$ nm. Thus, measurement with a dynamic range of ~0.5 μm with a combined error of 1 nm is possible with the present experimental arrangement. This is consistent with the data shown in Figs. 3 and 5.

In our interferometer, separation of the contributions to the interference signal because of displacement and external disturbances was accomplished by using polarization optics. We then expect that polarization leakage from a nonideal PBS would adversely affect the interferometer. Let the optical field amplitude incident on mirrors $M1$ and $M2$ be A_1 and A_2, respectively. We have shown that Eq. (7b) should be modified as[16]

$$x_m = -\eta \frac{L_{\text{bias}}}{2f_1} \Delta(f_1 - f_2), \tag{2}$$

where

$$\eta = 1 \pm \frac{A_2}{A_1} \cos\left(2\pi \frac{L_{\text{bias}}}{\lambda_1}\right)$$

and $A_2 \ll A_1$. Because of the presence of polarization leakage, displacement x_m is no longer linearly proportional to $\Delta(f_1 - f_2)$. Instead, it is modulated with a period of $\lambda_1/2$ and peak-to-peak variation of $2(A_2/A_1)$. The measurement resolution of the interferometer as expressed in Eq. (8) is affected in that the factor $\Delta K/K$ is modified: $(\Delta K/K)^2 = (\Delta\eta/\eta)^2 + (\Delta L_{\text{bias}}/L_{\text{bias}})^2$. We require that $\Delta\eta/\eta < 1/2(\Delta L_{\text{bias}}/L_{\text{bias}})$. The extinction ratio of the PBS used should then be better than 7×10^{-4} for $L_{\text{bias}} = 140$ cm and $\Delta L_{\text{bias}} = 2$ mm.

Finally, we note that the present interferometer can be used for tilt angle measurement of a reference plane. For this application, $PBS1$, M_2, and M_3 in Fig. 2 are replaced by the arrangement shown in Fig. 6. The orthogonally polarized beams from the two lasers are separated spatially by a distance D. By displacing the PZT, tilt angle θ is given by the ratio x_m/D. With $D = 4$ mm and $\Delta x_m = \pm 0.4$ mm, the angular resolution $\Delta\theta \cong 1 \times 10^{-7}$ rad.

IV. Conclusion

The basis concepts and experimental realization of an active laser heterodyne interferometer have been presented. The advantage of the conventional active interferometer is retained: displacement is obtained directly from frequency shift of the laser without post-detection processing. In our interferometer it is, however, not necessary to know the absolute frequency of the lasers because of the heterodyne arrangement used. Our interferometer is also relatively insensitive

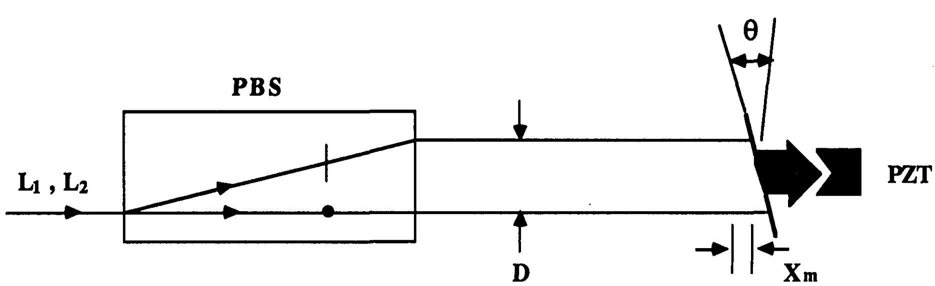

Fig. 6. Setup for the proposed method for tilt angle measurement with the present interferometer. Here, L_1 and L_2 are collinear orthogonally polarized beams from lasers I and II.

to environmental disturbances by using a novel design with main and auxiliary interferometers. To demonstrate its capabilities, we measured the displacement of a PZT over a range of $\sim 0.5\ \mu m$ with a resolution of $\pm 0.8\ \mu m$. A hysteresis loop of the PZT was observed which remained closed after repeated cycling. Displacement measurement over a range of many wavelengths or subnanometers in length should be possible with judicious choice of L_{bias} and narrow linewidth tunable lasers.

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