

# Maneuvering Target Tracking Using IMM Method at High Measurement Frequency

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**In tracking a maneuvering target by a radar system, the measurement noise is significantly correlated when the measurement frequency is high. In this paper, a simple decorrelation process is proposed to enhance the interacting multiple model (IMM) algorithm to track a maneuvering target with correlated measurement noise. It is found that the decorrelation process may improve system performance significantly, especially in velocity and acceleration estimations.**

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## I. INTRODUCTION

In tracking airborne or missile targets using noisy radar data, the measurement noise is usually assumed to be white and a conventional Kalman filter is frequently used for tracking the nonmaneuvering target. If the target is maneuvering, a situation when the target is suddenly accelerated by the pilot or missile guidance program, the conventional Kalman filter should be modified to keep the tracking performance. There have been several approaches for this modification so far [1-6]. Among them, the interacting multiple model (IMM) method [6] may provide rather good performance with efficient computation.

In practice, the measurement noises are not white. The noises are autocorrelated within a bandwidth of typically a few Hertz [7, 8]. When the measurement frequency is much lower than the error bandwidth, the successive errors are essentially uncorrelated, and can be treated as white noises. However, in many modern radar systems, the measurement frequency is usually high enough so that the correlation cannot be ignored. Rogers [8] treated the correlated noise as a first-order Markov process in the nonmaneuvering case. The noise can be decorrelated so that the conventional Kalman filter can work well after decorrelation. We extend this concept to the maneuvering case by deriving an efficient algorithm to decorrelate the measurement noise. It is found that significant improvement of the system performance can be obtained from the decorrelation process.

## II. PROBLEM FORMULATION

The target state is defined in the measurement vector (such as range, bearing and elevation in radar system) direction. Then, the tracking filter may work separately in each direction approximately. One single-direction operation is described in the following.

If the target is in a nonmaneuvering state, the target motion and the radar measurement can be modeled by a state with two-dimensional vector  $X_k (= [xx']_k^T)$  as follows.

$$X_{k+1} = \phi X_k + G w_k \quad (1)$$

$$Z_k = H X_k + v_k \quad (2)$$

where  $w_k$ ,  $v_k$ , and  $Z_k$  are the process noise, the measurement noise, and the measurement data, respectively.

When a maneuver occurs, an acceleration item  $Bu$  is applied in (1) such that

$$X_{k+1} = \phi X_k + Bu + G w_k. \quad (3)$$

Taking the acceleration variable  $u$  as part of the state vector, (3) and (2) can be described by

a three-dimensional vector state  $X_k^m$  given in the following.

$$X_{k+1}^m = \phi^m X_k^m + G^m w_k^m \quad (4)$$

$$Z_k = H^m X_k^m + \nu_k. \quad (5)$$

Here we consider the case that the target is tracked by the IMM algorithm [6]. The IMM algorithm has been implemented using models of different dimension: a second-order model which is dominating when the target is in nonmaneuvering state and one or several third-order models for the maneuvering state with different process noise levels. At least one third-order model having larger process noise than the true system must be used to respond to the rapid change of acceleration at the time of maneuver initiation.  $N$  Kalman filters should operate simultaneously in the IMM algorithm; each of the filter corresponds to a model. The probability of the model being correct is evaluated from measurement data and filter output. The weighted sum of all filter outputs with their probabilities being the weighting coefficients would be the overall system output. The detailed working procedure is described in the [6, Appendix].

### III. DECORRELATION PROCESS

In the case that the measurement frequency is high, the correlation in measurement noise cannot be ignored. Assume that the noise can be modeled as a first-order Markov process [8] given by

$$\nu'(t) = -\beta\nu(t) + \nu(t). \quad (6)$$

In discrete-time form, we have

$$\nu_{k+1} = \lambda\nu_k + \nu_k \quad (7)$$

where  $\lambda = e^{-\beta T}$ , and  $\nu_k$  is a white Gaussian noise.

To decorrelate the measurement noise, a new measurement  $Y_k$ , denoted as "artificial measurement" in [8], is generated. Let  $X_k^i$ ,  $\phi^i$ ,  $G^i$ ,  $w_k^i$ , and  $H^i$  denote the corresponding vectors or matrices in (1), (2), (4), or (5) for the  $i$ th model and  $\bar{\lambda}$  be the preset (estimated) value of noise-correlation, then the measurement equation can be rederived as follows.

$$Y_k = Z_k - \bar{\lambda}Z_{k-1} \quad (8)$$

$$\begin{aligned} &= (H^i X_k^i + \nu_k) - \bar{\lambda}(H^i X_{k-1}^i + \nu_{k-1}) \\ &= [H^i X_k^i - \bar{\lambda}H^i(\phi^i)^{-1}(X_k^i - G^i w_{k-1}^i)] \\ &\quad + (\nu_k - \bar{\lambda}\nu_{k-1}) \\ &= [H^i - \bar{\lambda}H^i(\phi^i)^{-1}]X_k^i + \bar{\lambda}H^i(\phi^i)^{-1}G^i w_{k-1}^i \\ &\quad + [(\lambda - \bar{\lambda})\nu_{k-1} + \nu_k]. \end{aligned} \quad (9)$$

Let

$$\hat{H}^i = H^i - \bar{\lambda}H^i(\phi^i)^{-1} \quad (10)$$

$$\hat{G}^i = \bar{\lambda}H^i(\phi^i)^{-1}G^i \quad (11)$$

$$\hat{\nu}_k^i = \hat{G}^i w_{k-1}^i + (\lambda - \bar{\lambda})\nu_{k-1} + \nu_k. \quad (12)$$

Then

$$Y_k = \hat{H}^i X_k^i + \hat{\nu}_k^i. \quad (13)$$

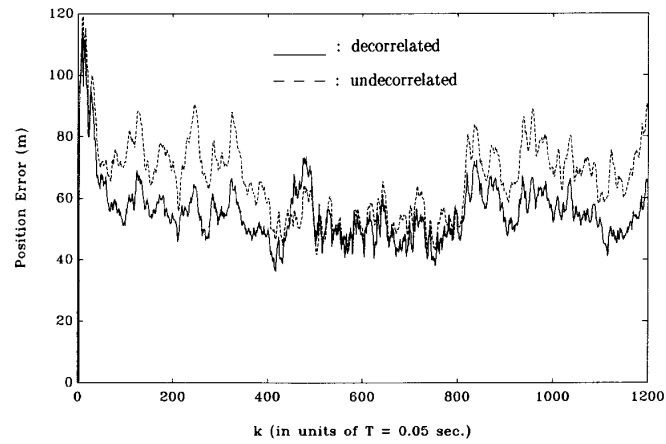
If  $\bar{\lambda} \approx \lambda$ , the new measurement noise  $\hat{\nu}_k^i$  would be white, but it is correlated with the process noise  $w_{k-1}^i$ . By reformulating the dynamic equation (1) or (4) properly, the process noise can be made to be uncorrelated with the new measurement noise. In most practical system, this procedure can be omitted with little degradation in performance since the item  $\hat{G}^i w_{k-1}^i$  is usually small. Thus, the IMM algorithm can be applied to the case with correlated measurement noise by the following substitutions:

$$\begin{aligned} H^i &\rightarrow \hat{H}^i; & \nu_k &\rightarrow \hat{\nu}_k^i; & Z_k &\rightarrow Y_k, \\ & & & & \text{for } i &= 1, 2, \dots, N. \end{aligned} \quad (14)$$

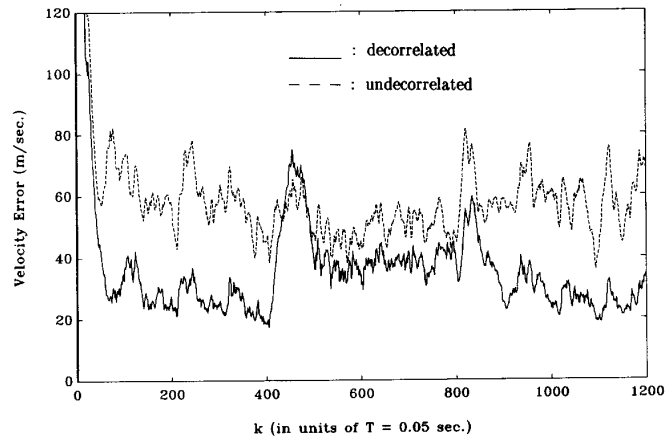
### IV. SIMULATION RESULTS

In the following, an example is used to demonstrate the effect of the decorrelation process. Some Monte Carlo simulations with 50 runs in each simulation are performed. The position of the target is measured every  $T = 0.05$  s. The target is generated to move with a constant velocity initially. At time interval  $k = [400, 800]$ , a constant acceleration  $u = 40$  (m/s<sup>2</sup>) is applied. After  $k = 800$ , the acceleration disappears and the target reverts to the constant velocity state. In the nonmaneuvering (constant velocity) periods, the correlation coefficient is assumed to be  $\beta = 4$  s<sup>-1</sup> such that the noise-correlation  $\lambda = 0.8187$  for  $T = 0.05$  s. When the target is in maneuvering (accelerating) state, the bandwidth of measurement noise would increase. Assume that  $\beta = 10$  s<sup>-1</sup> in maneuvering period such that  $\lambda = 0.6067$  for  $T = 0.05$  s. The process noise is assumed to be zero and the variance of the measurement noise is  $R = 100^2$  (m<sup>2</sup>). The coefficient matrices in (1), (2), (4), and (5) are given by

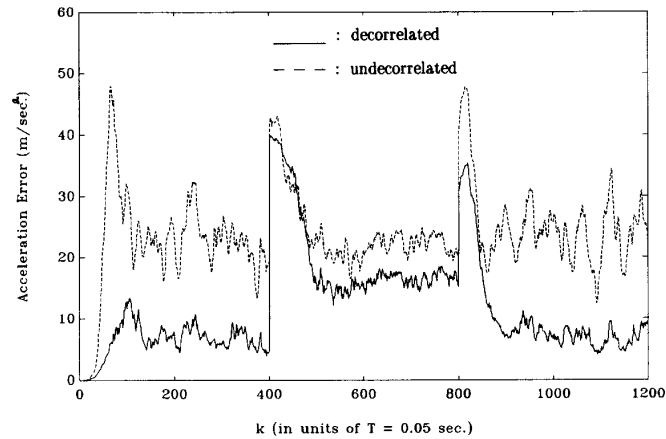
$$\begin{aligned} \phi &= \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix}; & H &= [1 \quad 0] \\ \phi^m &= \begin{bmatrix} 1 & T & T^2/2 \\ 0 & 1 & T \\ 0 & 0 & 1 \end{bmatrix} \\ G^m &= \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \\ H^m &= [1 \quad 0 \quad 0]. \end{aligned} \quad (15)$$



(a)



(b)



(c)

Fig. 1. Performances (rms error) of decorrelated and undecorrelated systems in IMM tracking method.

This target is tracked by the IMM algorithm with and without the decorrelation process, respectively. Let the IMM algorithm be composed of three filters corresponding to a second-order model with no process noise, a third-model with the variance of

the process noise  $Q$  and a third-order model with no process noise, respectively. The selection of  $Q$  is a tradeoff between the performance in steady state and the transient error as the maneuver initiates. In this simulation, the parameter  $Q$  is selected to be

$u^2 (= 40^2(\text{m/s}^2)^2)$ . The transition probability matrix between the three models is given by

$$P = \begin{bmatrix} 0.99 & 0.01 & 0.00 \\ 0.33 & 0.34 & 0.33 \\ 0.00 & 0.01 & 0.99 \end{bmatrix}. \quad (16)$$

Fig. 1 shows the performance of the decorrelated ( $\bar{\lambda} = 0.7$ ) and undecorrelated ( $\bar{\lambda} = 0$ ) systems for this simulation. It can be seen that the decorrelated system has better performance than the undecorrelated system when the target is in the nonmaneuvering state or in the steady state of the accelerating period. In the nonmaneuvering period, the improvement due to decorrelation is rather significant, especially in velocity and acceleration estimations. These large improvements in velocity and acceleration estimations are particularly useful in some tactical applications such as threat evaluation, the computation of the time of flight of a hostile missile, etc.

In Fig. 2, the steady state performances are shown as functions of the true value  $\lambda$  and the preset value  $\bar{\lambda}$  of noise-correlation in nonmaneuvering and maneuvering periods, respectively. Consider the nonmaneuvering case first. If the measurement noise is strongly correlated and is at least partially decorrelated, the system performance will usually be enhanced significantly from the decorrelation process. And, the performance will be only minorly degraded when the noise is over-decorrelated ( $\bar{\lambda} > \lambda$ ) besides a very large  $\bar{\lambda}$  (e.g.,  $\bar{\lambda} > 0.8$ ) is used. In the maneuvering case, some advantage can also be obtained by a proper decorrelation process but the improvement is generally not so significant as that in the nonmaneuvering case.

Fig. 3 shows the steady state performances of the perfectly decorrelated ( $\bar{\lambda} = \lambda$ ) and undecorrelated ( $\bar{\lambda} = 0$ ) systems as a function of the parameter  $Q$  and noise-correlation  $\lambda$ . For  $\lambda = 0.8$ , the improvements in position, velocity, and acceleration estimation in nonmaneuvering period due to decorrelation are about 20–30 percent, 56–67 percent, and 74–78 percent, respectively, and about 6–8 percent, 30–33 percent, and 34–44 percent, respectively, in the maneuvering period. The improvements are more significant for larger noise-correlation  $\lambda$  and are affected by some other parameters used in the simulations such as sampling time  $T$ , transition probability matrix  $P$ , etc. The process noise which is assumed to be zero above also dilutes the improvements. It should be noted that, if a large acceleration appears suddenly and a small parameter  $Q$  is used in the IMM algorithm, a large peak error would exist in the transient period and the decorrelated system may have larger peak error than the undecorrelated system. Thus, when the decorrelation process is employed, the parameter  $Q$  should be chosen properly (the same order of  $u^2$  or larger). In general, significant improvements can usually be obtained by applying the decorrelation

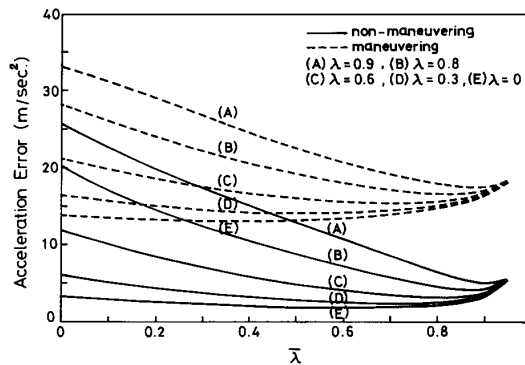
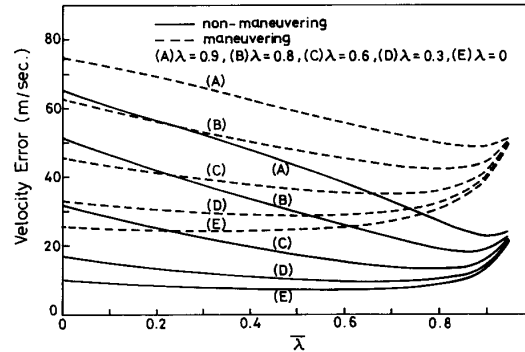
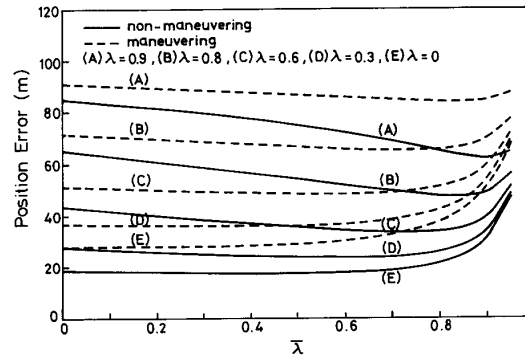


Fig. 2. Steady state performances (rms error) as functions of true value  $\lambda$  and the preset value  $\bar{\lambda}$  of noise-correlation in nonmaneuvering and maneuvering periods.

process to aid the IMM algorithm in tracking the maneuvering target at high measurement frequency.

## V. CONCLUSION

We consider the tracking problem of the maneuvering target at high measurement frequency. The measurement noise is significantly correlated when the measurement frequency is high in radar

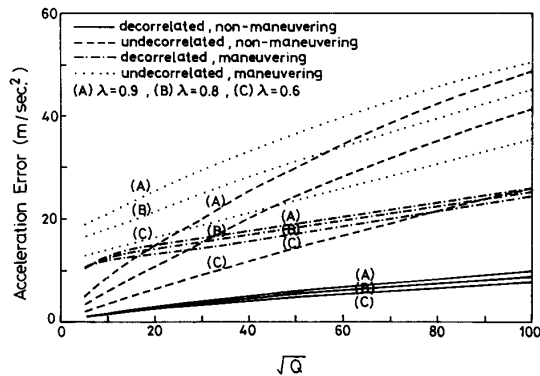
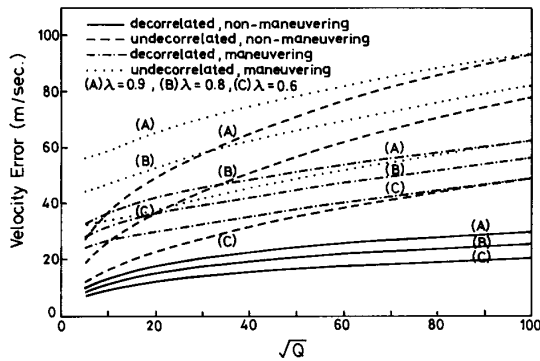
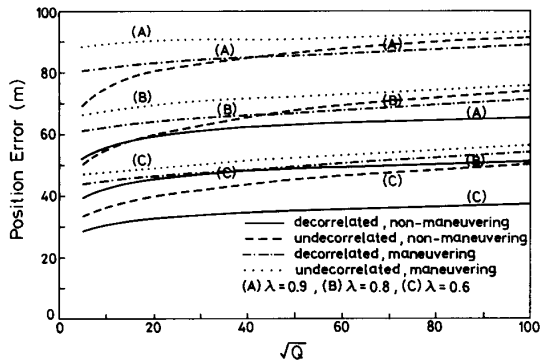


Fig. 3. Steady state performances (rms error) of the perfectly decorrelated and undecorrelated systems as functions of the parameter  $Q$  and noise-correlation  $\lambda$  in nonmaneuvering and maneuvering periods.

tracking system. A simple decorrelation process is proposed here to enhance the IMM algorithm to track the maneuvering target with correlated measurement noise. From the results of computer simulations, it can be found that the decorrelation process may improve system performance significantly, especially in velocity and acceleration estimations. These large improvements in velocity and acceleration are particularly useful in some tactical applications.

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