國立交通大學

電機學院通訊與網路科技產業研發碩士班

碩士論文

以多項式消除編碼為基礎之OFDM系統頻率估計

CFO Estimation for Polynomial Cancellation Coded OFDM Systems

研 究 生：洪千雅

指導教授：蘇育德　教授

中華民國 九十七 年八月

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以多項式消除編碼為基礎之 OFDM 系統頻率估計

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摘　　　　要

正交多頻分工(Orthogonal Frequency Division Multiplexing ,OFDM)技術近年來頗受到重視。OFDM 系統設計必需要考慮到載波頻率偏移(CFO)補償這個課題，因為載波頻率偏移會造成載波彼此之間的與干擾(ICI)，破壞載波的正交性並將大大的降低系統效能。載波彼此之間的干擾(ICI)會使載波上的信號強度衰減(attenuation)並產生相角旋轉 (phase rotation)。在許多消除 1
ICI 的方法中，ICI 自我消除法(ICI self-cancellation) 或稱為多項式消除編碼(polynomial cancellation coding, PCC)因其容易實現以及效能穩定而受到重視。然而，當載波頻率偏移大時，多項式消除編碼只能消除一小部份的 ICI。因此，我們提出以多項式消除編碼為基礎的載波頻率偏移估計。首先，以 PCC 做為事前編碼 (precoding)並解出初步的資料，接下來再以此資料估計載波頻率偏移(CFO)。藉由使用多項式消除編碼，直接估出載波頻率偏移(CFO)並加以補償，可以改善原本多項式消除編碼在載波頻率偏移(CFO)大時只能消除部份 ICI 的缺失。

CFO Estimation for Polynomial Cancellation Coded OFDM Systems

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Industrial Technology R & D Master Program of
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ABSTRACT

Orthogonal Frequency Division Multiplexing (OFDM) is being considered as a promising transmission technique. However, a carrier frequency offset (CFO) between the transmitter and receiver will results in ICI and thus destroy the orthogonality of the subcarrier and degrades the performance. ICI leads to attenuation and phase rotation of desired signal on each subcarrier. These impairments have already motivated several studies to find solutions. Among the several ICI reduction schemes, ICI self-cancellation or polynomial cancellation coding (PCC) scheme has received much attention due to its simplicity and its high robustness to frequency offset errors. However, for large CFO, PCC can only eliminate ICI to a certain extent. Thus, we propose new methods of PCC based CFO estimation to eliminate ICI for either low or high CFOs. First, we use PCC as precoding scheme and get the initial decoded data. Then CFO estimation is done by making use of the decode data. CFO estimation and compensation from PCC can overcome the drawback of PCC that only a certain part of ICI can be eliminated when CFO is large.

誌　　　　謝

本論文得以順利完成，首先要感謝我的指導老師蘇育德教授，在我研究所的生涯中，不厭其煩地賜予方向與教導。也謝謝蒞臨的口試委員們所提出的建議，使本論文得以更加完整。更感謝實驗室每一位優秀、熱心的學長姐與可愛的學弟妹們，你們總是在我最需要的時候，不吝伸出援手。

更感謝一直陪伴在我身邊的摯友，以及最關心我的家人們。

Contents

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1 Introduction

Orthogonal Frequency Division Multiplexing (OFDM) is being considered as a promising transmission technique for high data rate wireless communication systems because of its sufficient robustness to handle radio channel impairments and its bandwidth efficiency [1]. Despite its benefits, one of its impairments is intercarrier interference (ICI) and is considered as one limiting factor for efficient implementationof OFDM systems. An OFDM system uses a set of orthogonal subcarriers. This orthogonality allows the receiver to separate the data symbol on each subcarriers, which are overlapped at the transmitter to achieve spectral efficiency. The loss of orthogonality results in ICI. This ICI deteriorates the bit error rate of the system and introduces an error floor. This error floor cannot be removed by just increasing the transmit power. Generally, the cause for ICI can be grouped into following categories: (i) carrier frequency offset errors, (ii) phase noise, (iii) IQ mismatch at the receiver front end and (iv) time selectivity of the wireless channel.

The characteristics of the above causes and the degree of their effects are different. The first three causes are due to hardware imperfections and the later is due to unpredictable nature of wireless channel. Carrier frequency offset errors are caused by the mismatch of oscillators' frequencies between the transmitter and the receiver [2] where as phase noise is due to instability of an oscillator [2], [3]. Besides, the doppler frequency will also result in CFO. IQ mismatch refers to phase and gain imbalance between in-phase (I) and quadrature (Q) paths at the receiver front end [4], [5]. On the other hand, the fading channels generally exhibit both frequency selectivity and time selectivity. OFDM has been proposed to combat the frequency selectivity, but its performance might be affected by the time selectivity. The resulting time selective fading causes a loss of subcarrier orthogonality, thus resulting in ICI [6]. These ICI causes also lead to attenuation and phase rotation of desired signal on each subcarrier. These impairments have already motivated several studies to find solutions. Among the several ICI reduction schemes, ICI self-cancellation [7] or polynomial cancellation coding (PCC) scheme has received much attention due to its simplicity and its high robustness to frequency offset errors. In this technique, each data symbol is transmitted on two adjacent subcarriers with opposite polarity in order to cancel ICI. The data throughput of this scheme will therefore be half of that of conventional OFDM. Thus, this cancellation scheme is also referred to as rate-half repetition coding. This cancellation scheme is further extended to reduce more ICI by mapping data symbols onto a larger group of adjacent subcarriers [7], [8]. However, this further reduces the data throughput, despite more ICI reductions. In [9], rate 2/3 and 3/4 coding schemes has been proposed t o improve the data throughput with moderate ICI reduction. Authors in [l0] improved this cancellation scheme by transmitting the data symbols on k-th and (N-1-k)-th subcarriers, instead of adjacent subcarriers. This approach offers a frequency diversity because of the frequency separation between the same data symbols. In this paper, we will refer adjacent ICI self-cancellation scheme [7], [8] as "Adjacent Symbol Repetition (ASR)" and the non-adjacent ICI self-cancellation scheme [10] as "Symmetric Symbol Repetition (SSR)".

However, ICI self-cancellation schemes have been verified being effective at low frequency offsets. For high frequency offsets, the ICI self-cancellation can only eliminate ICI components to a certain extent. The remaining frequency errors still degrade the system significantly. The existing compensation methods based on the ICI self-cancellation for frequency offsets require either pilot information or training sequences [10]–[12]. Blind estimation methods are either only effective for small frequency offsets or with considerable computational complexity [13]–[17]. Therefore, a simple acquisition and tracking approaches for compensating carrier frequency offset prior to the ICI self-canceling demodulation is developed [18], and CFO is estimated and compensated in the range from 0 to 0.5.

So far as to CFO estimation, there have been a multitude of proposals for CFO compensation. A maximum likely hood estimate was proposed by Moose [19], based on the observation of two consecutive and identical symbols. Its maximum frequency acquisition range is $\pm 1/2$ subcarrier spacing because of mod 2π ambiguity.

Here we propose new methods to estimate CFO by making use of the decoded data symbol from polynomial cancellation coding.

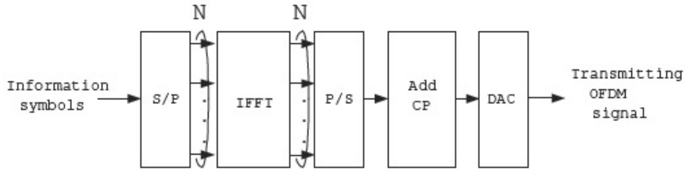

Figure 1: A typical OFDM modulator.

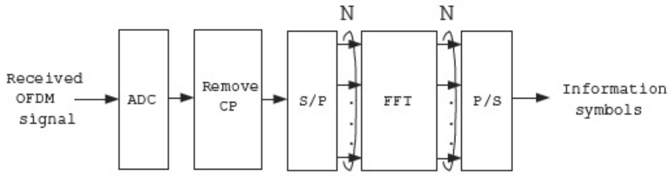

Figure 2: A typical OFDM demodulator.

2 Polynomial Cancellation Coded OFDM Systems

2.1 Introduction

Figure 2.1.1 plots a block diagram of a OFDM modulator where S/P and DAC are used to denote serial-to-parallel converter and digital-to-analog converter, respectively. The information symbols are used to modulate subcarriers vi N-points inverse discrete Fourier Transform (IDFT). The output of the IDFT (IFFT) block is converted to a serial complex block and a cyclic prefix(CP) is added to each block. The total duration of an OFDM symbol (frame) is equal to the length of the CP plus that of the IDFT symbol block. The CP is a copy of the tail part of the time-domain OFDM block and is attached to the front of the block. As long as the duration of the CP is longer than the channel impulse response, inter symbol interference (ISI) can be eliminated by the receiver through frequency domain excision.

An OFDM demodulator is shown in Figure 2.1.2. Based on the timing (frame) recovery subsystem output, the baseband receiver removes the CP part, takes discrete Fourier transform (DFT) on the remaining part and then compensate for the CFO and channel effect using information given by the frequency synchronization and channel estimation units before making decision on symbols modulated on each subcarriers, if no soft-decision channel decoding is needed. Parallel-to-serial conversion can be performed either before or after making symbol decision (detection).

2.2 System Model

Consider a frequency selective fading channel associated with an OFDM system with N subcarriers. The frequency domain signal at the receiver at the kth subcarrier,Y_k, is given by

$$Y_k = S_0 X_k + \sum_{l=0, l \neq k}^{N-1} S_{l-k} X_l + n_k, k = 0, 1, ..., N-1$$

Where n_k is a complex additive white Gaussian noise(AWGN) sequence and $Y_k$represents the symbol carried by the kth subcarrier.

Besides,

$$S_{l-k} = \frac{\sin(\pi(l + \epsilon - k))}{N \sin\left(\frac{\pi}{N}(l + \epsilon - k)\right)} \exp\left(j\pi(1 - \frac{1}{N})(l + \epsilon - k)\right)$$

is the ICI coefficient.

Moreover, consider a frequency selective fading channel associated with an OFDM system,

$$Y_k = S_0 H_k X_k + \sum_{l=0, l \neq k}^{N-1} S_{l-k} H_l X_l + n_k, \quad k = 0, 1, ..., N-1$$

H_l is the channel frequency response at the kth subcarrier.

2.3 Polynomial Cancellation Coding

The main idea of polynomial cancellation coding is to modulate one data symbol onto a group of subcarriers with predefined weighting coefficients. By doing so, the ICI signals generated within a group can be “self-cancelled” each other.

2.3.1 Overview

It has been shown in Figure1and Figure2 that both real and imaginary parts of the ICI coefficient are gradually changed with respect to the subcarrier index.It’s worthwhile to notice that although the peak the desired signal located is not smooth with respect to the neighboring index, but in our ASR and SSR scheme, we will extract the peak signal for CFO estimation and make use of gradually changed property of the subcarriers with only even k.

We will introduce these polynomial cancellation coding schemes :

- **ASR**(adjacent data-conversion method,1996)
- **SSR**(symmetric data-conversion method,2000)

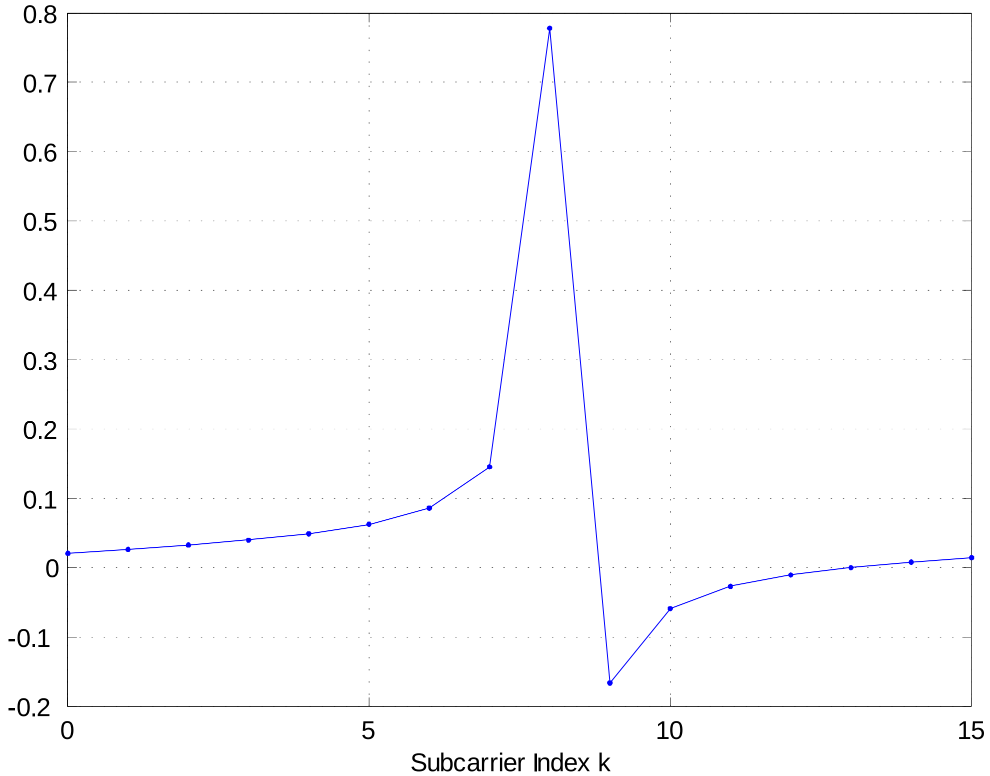

Figure 3: An example of $S(l-k)$ for $N=16, l=0$. Real part of $S(l-k)$.

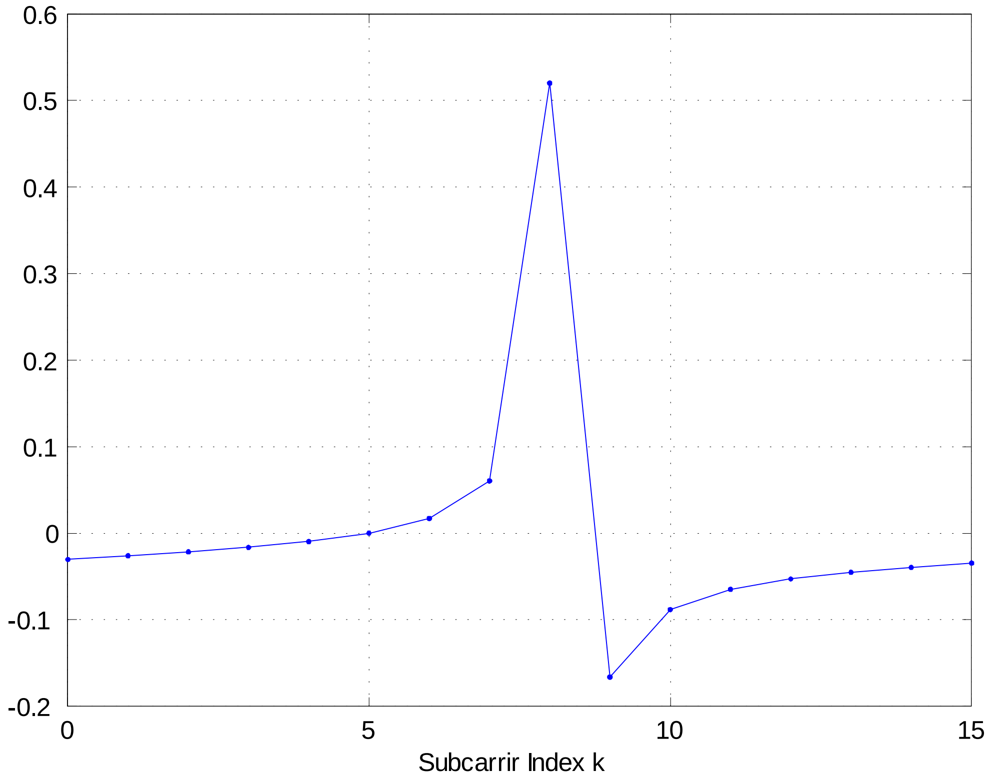

Figure 4: An example of $S(l-k)$ for $N=16, l=0$. Imaginary part of $S(l-k)$.

2.3.2 ASR (adjacent data-conversion method)

over AWGN Channel From Figure 1 and Figure 2, for the majority of $(l-k)$values, the difference between S_{l-k} and S_{l-k+1} is very small.

If a data pair $(X_0, -X_0)$ is modulated onto two adjacent subcarriers $(k, k+1)$, then the ICI signals generated by subcarrier k will be cancelled out significantly by the ICI generated by subcarrier $(k+1)$.

Therefore, the data block will be $\mathbf{X} = \left(X_0, -X_0, X_1, -X_1, ..., X_{\frac{N}{2}-1}, -X_{\frac{N}{2}-1}\right)$. We name this as modulation scheme of PCC. The received signal on subcarrier becomes

$$Y_k = \sum_{l=0, l\ even}^{N-2} \left(S_{l-k} - S_{l-k+1}\right) X_l + n_k, \quad k = 0, 1, ..., N-1$$

And on subcarrier $k+1$ becomes

$$Y_{k+1} = \sum_{l=0, l\ even}^{N-2} \left(S_{l-k-1} - S_{l-k}\right) X_l + n_{k+1}$$

In such a case, the ICI coefficient is denoted as

$$\acute{S}_{l-k} = S_{l-k} - S_{l-k+1}$$

By combining the received samples at the receiver, the effective coefficients are made to be small. It can be represented as

$$\begin{aligned}
\acute{Y}_k &= Y_k - Y_{k+1} \\
&= \sum_{l=0, l\ even}^{N-2} \left(2S_{l-k} - S_{l-k+1} - S_{l-k-1}\right) X_l + n_k - n_{k+1} \\
&= \left(2S_0 - S_{l-k+1} - S_{l-k-1}\right) X_k \\
&\quad + \sum_{l=0, l\ even, l\neq k}^{N-2} \left(2S_{l-k} - S_{l-k+1} - S_{l-k-1}\right) X_l + n_k - n_{k+1}
\end{aligned}$$

The corresponding ICI coefficient then becomes

$$S^{"} = 2S_{l-k} - S_{l-k+1} - S_{l-k-1}$$

Besides, we name this as demodulation scheme of PCC.

From Figure 4, we can see that for small CFO of 0.15, ASR performs well. However, if

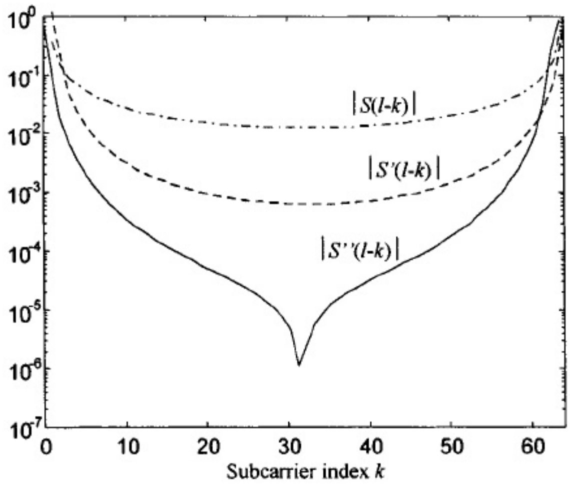

Figure 5: A comparison between $S(l-k), S^{'}(l-k)$ and $S^{"}(l-k), N=64$.

CFO increases to 0.4, the BER curve almost flows. It shows that when CFO is large, the PCC can only eliminate ICI to a certain extent.

Over Fading Channel For ASR over fading channel, similarly, consider the data block $\mathbf{X} = \left(X_0, -X_0, X_1, -X_1, ..., X_{\frac{N}{2}-1}, -X_{\frac{N}{2}-1}\right)$. The received signal on subcarrier becomes

$$Y_k = \sum_{l=0,\ l\ even}^{N-1} \left(S_{l-k}H_l - S_{l-k+1}H_{l+1}\right) X_l + n_k, \quad k = 0, 1, ..., N-1$$

where H_l is the channel frequency response at the k th subcarrier. And on subcarrier $k+1$ becomes

$$Y_{k+1} = \sum_{l=0,\ l\ even}^{N-1} \left(S_{l-k-1}H_l - S_{l-k}H_{l+1}\right) X_l + n_{k+1}$$

In such a case, the ICI coefficient is denoted as

$$\acute{S}_{l-k} = S_{l-k}H_l - S_{l-k+1}H_{l+1}$$

By combining the received samples at the receiver, the effective coefficients are made to be small. It can be represented as

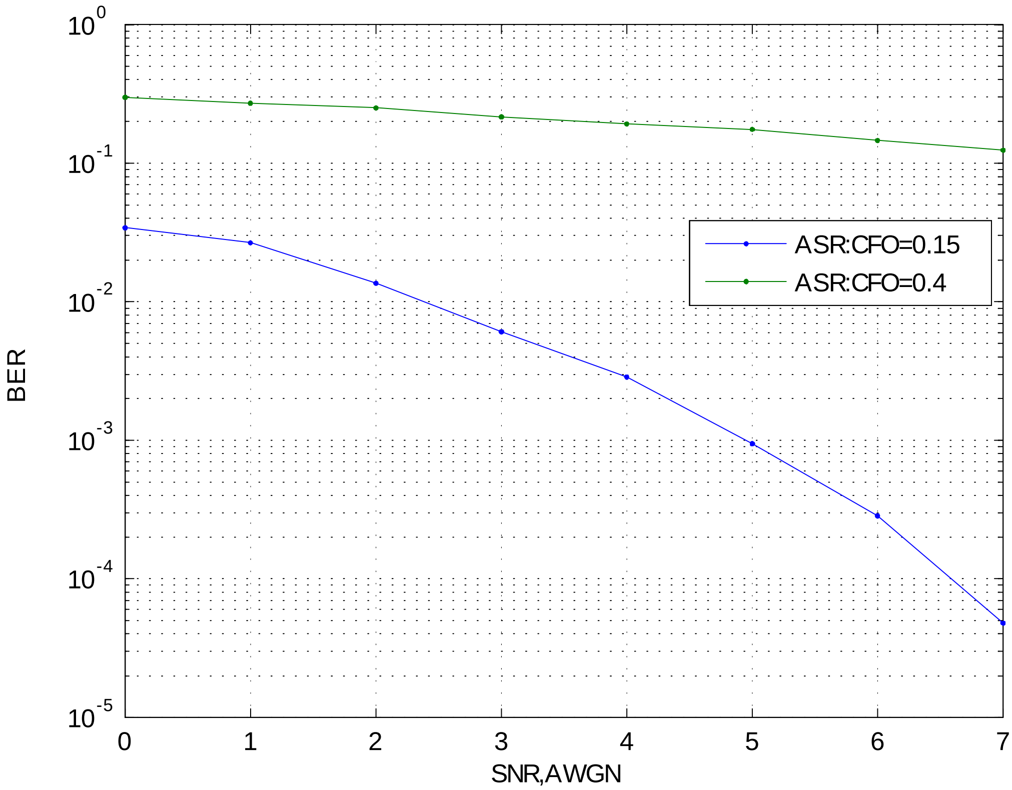

Figure 6: BER of ASR demodulation scheme over AWGN channel.

$$
\begin{aligned}
\acute{Y}_k &= Y_k - Y_{k+1} \\
&= \sum_{l=0,\ l\ even}^{N-1} \left(S_{l-k}(H_l + H_{l+1}) - S_{l-k+1}H_{l+1} - S_{l-k-1}H_l\right) X_l + n_k - n_{k+1} \\
&= S_0(H_k + H_{k+1}) - S_1 H_{k+1} - S_{-1} H_k X_k \\
&\quad + \sum_{l=0, l\neq k, l\ even}^{N-1} \left(S_{l-k}(H_l + H_{l+1}) - S_{l-k+1}H_{l+1} - S_{l-k-1}H_l\right) X_l + n_k - n_{k+1}
\end{aligned}
$$

The corresponding ICI coefficient then becomes

$$
S" = S_{l-k}(H_l + H_{l+1}) - S_{l-k+1}H_{l+1} - S_{l-k-1}H_l
$$

Figure 3 shows the amplitude comparison of $|S_{l-k}|$, $\left|\acute{S}_{l-k}\right|$,and $\left|S"_{l-k}\right|$ for $N = 64$ and $\epsilon = 0.3$ over AWGN channel. Notice the logarithmic scale on the vertical axis.For the majority of $(l-k)$values, $\left|\acute{S}_{l-k}\right|$ is much smaller than $|S_{l-k}|$, and the $\left|S"_{l-k}\right|$ is even smaller then $\left|\acute{S}_{l-k}\right|$.Thus, the ICI signals become smaller when applying the ASR scheme.

From Figure 4, we can see that the PCC can only eliminate ICI to a certain extent over

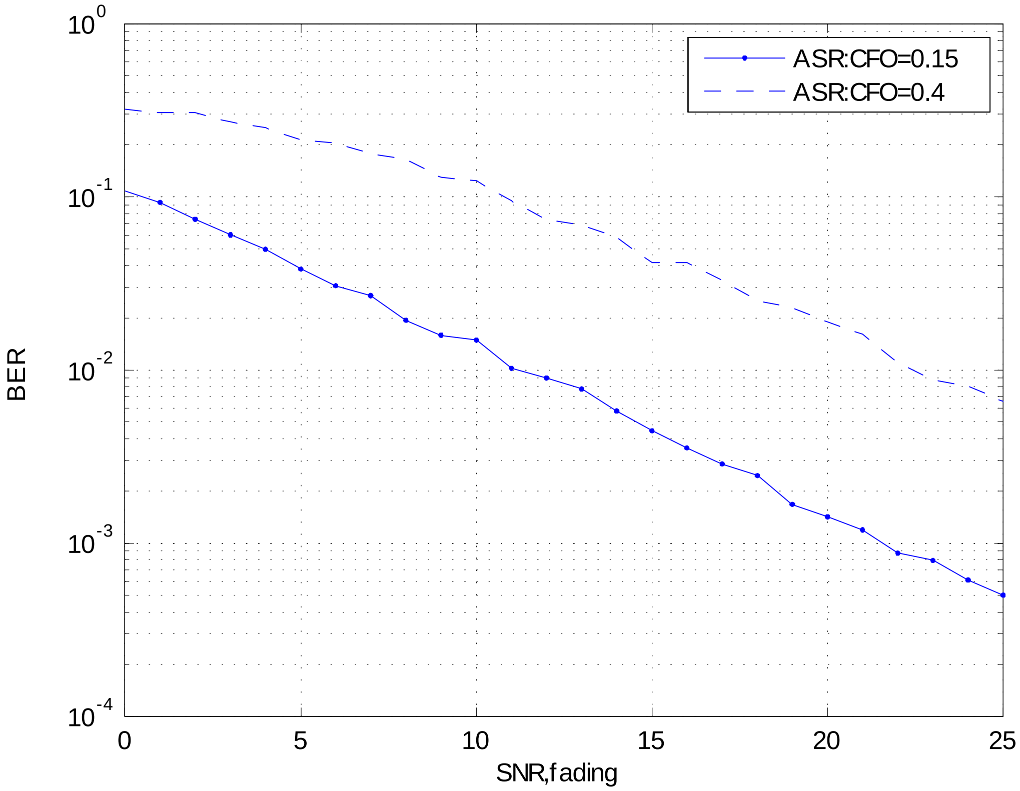

Figure 7: BER of ASR demodulation scheme over fading channel.

fading channel.

2.3.3 SSR (symmetric data-conversion method)

Over AWGN Channel The other feature of the ICI coefficients resulting from CFO error is that they are approximately symmetric. For the same modulated symbols with opposite polarity are transmitted on subcarrier k and $(N-k-1)$, and this is the key idea used in SSR scheme to reduce the ICI.

Then, the data block becomes $\mathbf{X}=\left(X_0, X_1, ..., X_{\frac{N}{2}-1}, -X_{\frac{N}{2}-1}, -X_1, -X_0\right)$.

We name this as modulation scheme of PCC. The received signal on subcarrier The received signal on subcarrier becomes

$$Y_k=\sum_{l=0}^{\frac{N}{2}-1}\left(S_{l-k}-S_{N-1-l-k}\right)X_l+n_k, \quad k=0,1,...,N-1$$

And on subcarrier $N-1-k$ becomes

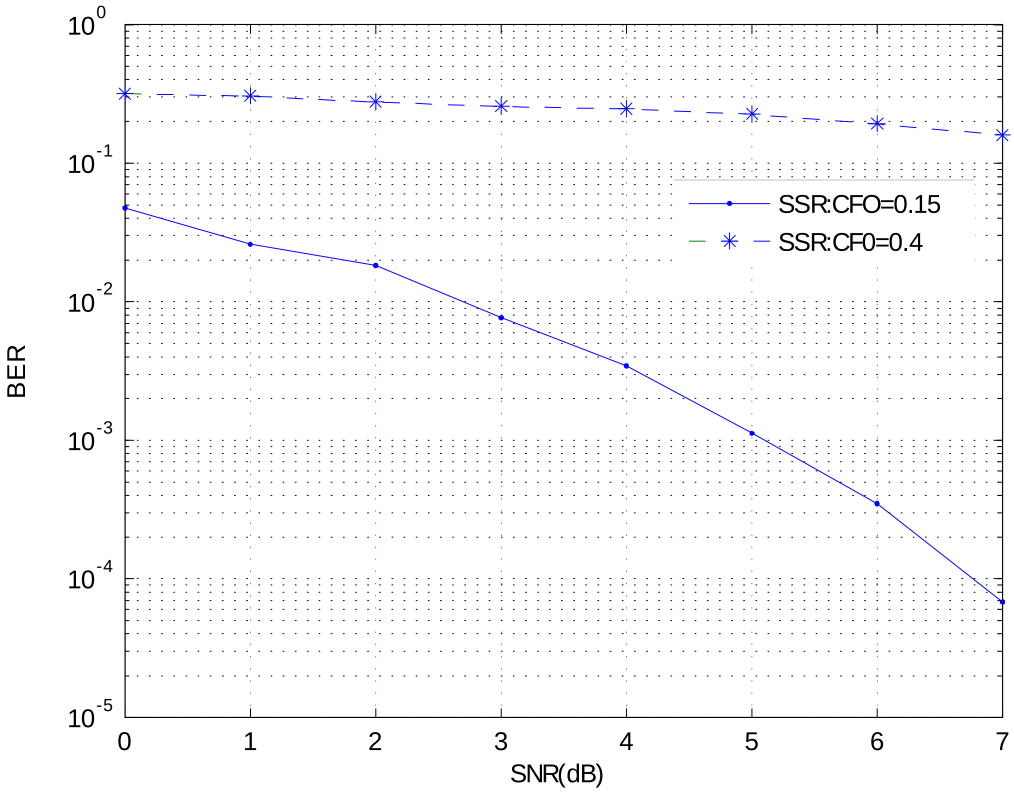

Figure 8: BER of SSR over AWGN channel.

$$Y_{N-1-k} = \sum_{l=0,l\neq k}^{\frac{N}{2}-1} \left(S_{l-(N-1-k)} - S_{-(l-k)}\right) X_l + n_{N-1-k}$$

The decision variable at the receiver becomes

$$\begin{aligned} \acute{Y}_k &= Y_k - Y_{N-k-1} \\ &= \left(S_{N-1} - S_{-(N-1)}\right) X_k \\ &\quad + \sum_{l=0,l\neq k}^{\frac{N}{2}-1} \left(S_{l-k} - S_{N-1-k-l} - S_{l-(N-1-k)} - S_{-(l-k)}\right) X_l + n_k - n_{N-1-k} \end{aligned}$$

We name this as demodulation scheme of PCC.

From Figure 4, we can see that for small CFO of 0.15, SSR performs well. However, if CFO increases to 0.4, the BER curve almost flows. It shows that when CFO is large, the PCC can only eliminate ICI to a certain extent.

over Fading Channel Consider the data block becomes $\mathbf{X} = \left(X_0, ..., X_{\frac{N}{2}-1}, -X_{\frac{N}{2}-1}, ..., -X_0\right)$.

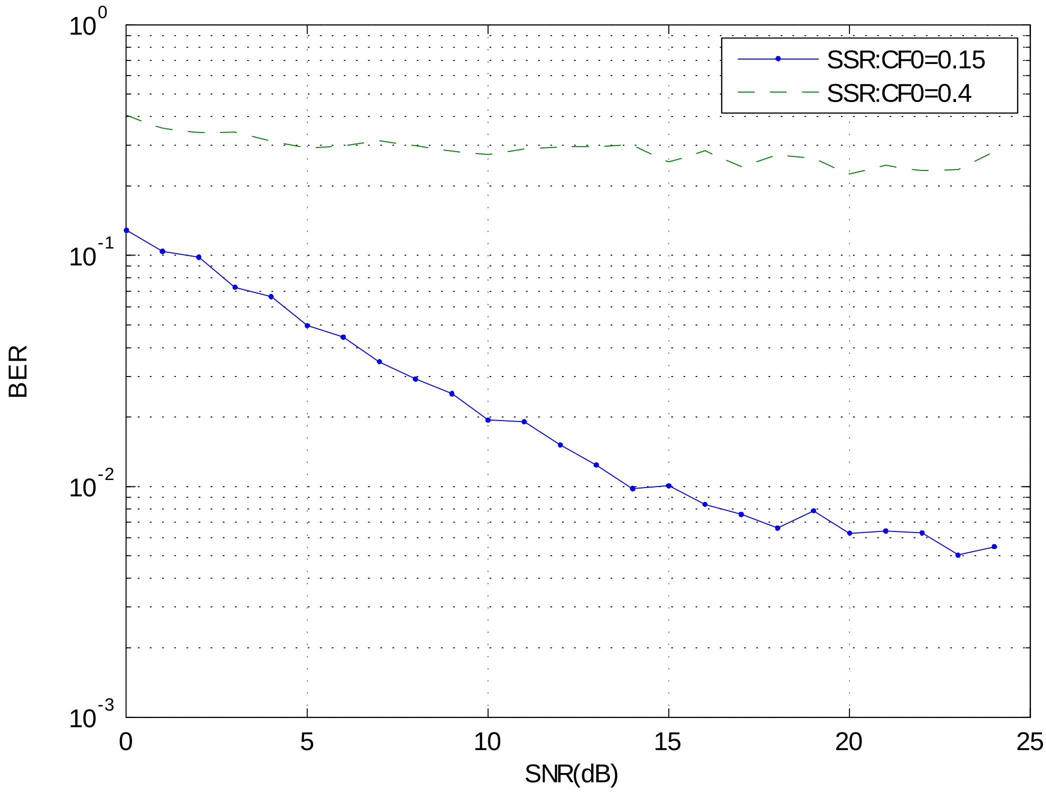

Figure 9: BER of SSR over fading channel.

The received signal on subcarrier The received signal on subcarrier becomes

$$Y_k = \sum_{l=0}^{\frac{N}{2}-1} \left(S_{l-k}H_l - S_{N-1-l-k}H_{N-1-l}\right) X_l + n_k, \quad k = 0, 1, ..., N-1$$

where H_l is the channel frequency response at the kth subcarrier. And on subcarrier $N-1-k$ becomes

$$Y_{N-1-k} = \sum_{l=0, l\neq k}^{\frac{N}{2}-1} \left(S_{l-(N-1-k)}H_l - S_{-(l-k)}H_{N-1-l}\right) X_l + n_{N-1-k}$$

The decision variable at the receiver becomes

$$\begin{aligned} \acute{Y}_k &= Y_k - Y_{N-k-1} \\ &= \left(S_0(H_k + H_{N-1-K}) - S_{-2k+N-1}H_{N-1-k} - S_{2k-(N-1)}H_k\right) X_k \\ &\quad + \sum_{l=0, l\neq k}^{\frac{N}{2}-1} \left(S_{l-k}H_l - S_{N-1-k-l}H_{N-1-l} - S_{l-(N-1-k)}H_l + S_{-(l-k)}H_{N-1-l}\right) X_l + n_k - n_{N-1-k} \end{aligned}$$

From Figure 4, we can see that the PCC can only eliminate ICI to a certain extent over fading channel.

3 CFO Estimation for ASR of Polynomial Cancellation Coding

3.1 Modulation Scheme over AWGN Channel

The data block will be $\mathbf{X} = \left(X_0, -X_0, .., X_{\frac{N}{2}-1}, -X_{\frac{N}{2}-1}\right)$.

Recall that

$$Y_k = \sum_{l=0,l\ even}^{N-2} \left(S_{l-k} - S_{l-k+1}\right) X_l + n_k, \quad k = 0, 2, ..., N-2$$

With X_k is derived from ICI self-cancellation scheme, and is deterministic.

According to the "Central Limit Theory", we approximate

$$\sum_{l=0,l\ even,l\neq k}^{N-2} \left(S_{l-k} - S_{l-k+1}\right) X_l \widetilde{=} 0$$

Sum up Y_k for $k = 0, 2, ..., N-2$, then $E[Y_k]$ becomes

$$E[Y_k] \widetilde{=} E[(S_0 - S_1)X_k] + E[n_k], \quad k = 0, 2, ..., N-2$$

When N is large,

$$E[n_k] \widetilde{=} 0$$

Then,

$$\frac{2}{N} \sum_{k=0,k\ even}^{N-2} \frac{Y_k}{X_k} \widetilde{=} S_0 - S_1$$

Denote

$$\Lambda(\epsilon) = S_0 - S_1$$

Then, the estimated $\hat{\epsilon}$ will be

$$\hat{\epsilon} = \arg\min_{\epsilon} \left(\Lambda(\epsilon) - \frac{2}{N} \sum_{k=0,k\ even}^{N-2} \frac{Y_k}{X_k}\right)$$

Next, we will discuss the real and imaginary parts of $\Lambda(\epsilon)$ respectively. We will deonote the real part of a variable X as $\mathrm{Re}\{X\}$, and the imaginary part of a variable X as $\mathrm{Im}\{X\}$.

Since $l-k=0$,

$$S_0 = \frac{\sin(\pi\epsilon)}{N\sin(\frac{\pi}{N}\epsilon)}\exp(j\pi\epsilon)$$

When N is large,$\sin(\frac{\pi}{N}\epsilon)\cong\frac{\pi}{N}\epsilon$

Then

$$\begin{aligned} S_0 &\cong \frac{\sin(\pi\epsilon)}{N\frac{\pi\epsilon}{N}}\exp(j\pi\epsilon) \\ &\cong \frac{\sin(\pi\epsilon)}{\pi\epsilon}\exp(j\pi\epsilon) \end{aligned}$$

Then,

$$\begin{aligned} \mathrm{Re}\{S_0\} &\cong \mathrm{Re}\left\{\frac{\sin(\pi\epsilon)}{\pi\epsilon}\exp(j\pi\epsilon)\right\} \\ &= \frac{\sin(\pi\epsilon)\cos(\pi\epsilon)}{\pi\epsilon} \\ &= \frac{\sin(2\pi\epsilon)}{2\pi\epsilon} \\ &= \mathrm{sinc}(\pi\epsilon) \end{aligned}$$

Next, with $l-k=1$, and N is large.

$$\begin{aligned} S_1 &= \frac{\sin(\pi(\epsilon+1))}{N\sin(\frac{\pi}{N}(\epsilon+1))}\exp(j\pi(\epsilon+1)) \\ &\cong \frac{\sin(\pi(\epsilon+1))}{\pi(\epsilon+1)}\exp(j\pi(\epsilon+1)) \end{aligned}$$

$$
\begin{aligned}
\mathrm{Re}\{S_1\} &\cong \mathrm{Re}\left\{\frac{\sin(\pi(\epsilon+1))}{\pi(\epsilon+1)}\exp(j\pi(\epsilon+1))\right\} \\
&= \frac{\sin(\pi(\epsilon+1))\cos(\pi(\epsilon+1))}{\pi(\epsilon+1)} \\
&= \frac{\sin(2\pi(\epsilon+1))}{2\pi(\epsilon+1)} \\
&= \frac{\sin(2\pi\epsilon)}{2\pi\epsilon}\frac{2\pi\epsilon}{2\pi(\epsilon+1)} \\
&= \mathrm{Re}\{S_0\}\frac{\epsilon}{(\epsilon+1)}
\end{aligned}
$$

As a result,

$$
\begin{aligned}
\mathrm{Re}\{\Lambda(\epsilon)\} &= \mathrm{Re}\{S_0 - S_1\} \\
&= \mathrm{Re}\{S_0\} - \mathrm{Re}\{S_1\} \\
&= \mathrm{Re}\{S_0\}\left(1 - \frac{\epsilon}{\epsilon+1}\right)
\end{aligned}
$$

Then,

$$
\hat{\epsilon} = \arg\min_{\epsilon} \mathrm{Re}\left\{\Lambda(\epsilon) - \frac{2}{N}\sum_{k=0,k\ even}^{N-2}\frac{\acute{Y}_k}{X_k}\right\}
$$

Next, consider the imaginary parts of $\Lambda(\epsilon)$:

$$
\begin{aligned}
\mathrm{Im}\{S_0\} &\cong \mathrm{Im}\left\{\frac{\sin(\pi\epsilon)}{\pi\epsilon}\exp(j\pi\epsilon)\right\} \\
&= \frac{\sin(\pi\epsilon)}{\pi\epsilon}\sin(\pi\epsilon)
\end{aligned}
$$

$$
\begin{aligned}
\mathrm{Im}\{S_1\} &= \mathrm{Im}\left\{\frac{\sin(\pi(\epsilon+1))}{\pi(\epsilon+1)}\exp(j\pi(\epsilon+1))\right\} \\
&= \frac{\sin(\pi(\epsilon+1))\sin(\pi(\epsilon+1))}{\pi(\epsilon+1)} \\
&= \frac{\sin(\pi\epsilon)\sin(\pi\epsilon)}{\pi\epsilon}\frac{\pi\epsilon}{\pi(\epsilon+1)} \\
&= \mathrm{Im}\{S_0\}\frac{\epsilon}{(\epsilon+1)}
\end{aligned}
$$

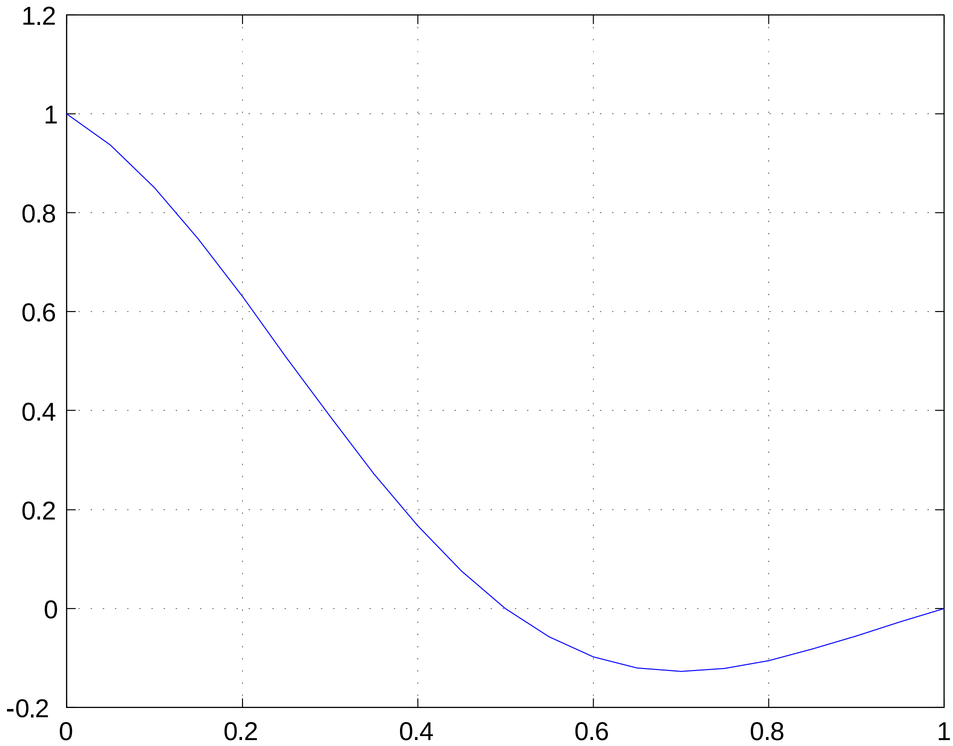

Figure 10: $\mathrm{Re}\{\Lambda(\epsilon)\}$ with $\Lambda(\epsilon) = S_0 - S_1$ of ASR modulation scheme when ϵ is from 0 to 1.

As a result,

$$\begin{aligned}
\mathrm{Im}\{\Lambda(\epsilon)\} &= \mathrm{Im}\{S_0 - S_1\} \\
&= \mathrm{Im}\{S_0\} - \mathrm{Im}\{S_1\} \\
&= \mathrm{Im}\{S_0\}\left(1 - \frac{\epsilon}{\epsilon + 1}\right)
\end{aligned}$$

Then, it becomes

$$\hat{\epsilon} = \arg\min_{\epsilon} \mathrm{Im}\left\{\Lambda(\epsilon) - \frac{2}{N}\sum_{k=0,k\ even}^{N-2} \frac{\acute{Y}_k}{X_k}\right\}$$

Refer to Figure 8 and Figure 9. Consider two regions of ϵ. The first region is from 0 to 0.5, and the second region is from 0.5 to 1. For the first region, $\mathrm{Re}\{\Lambda(\epsilon)\}$ is one-to-one mapping, but $\mathrm{Im}\{\Lambda(\epsilon)\}$ is not . For the second region, neither $\mathrm{Re}\{\Lambda(\epsilon)\}$ nor $\mathrm{Im}\{\Lambda(\epsilon)\}$ is one-to-one mapping. Therefore, we will use $\mathrm{Re}\{\Lambda(\epsilon)\}$ to estimate CFO for ϵ in the range of 0 to 0.5.

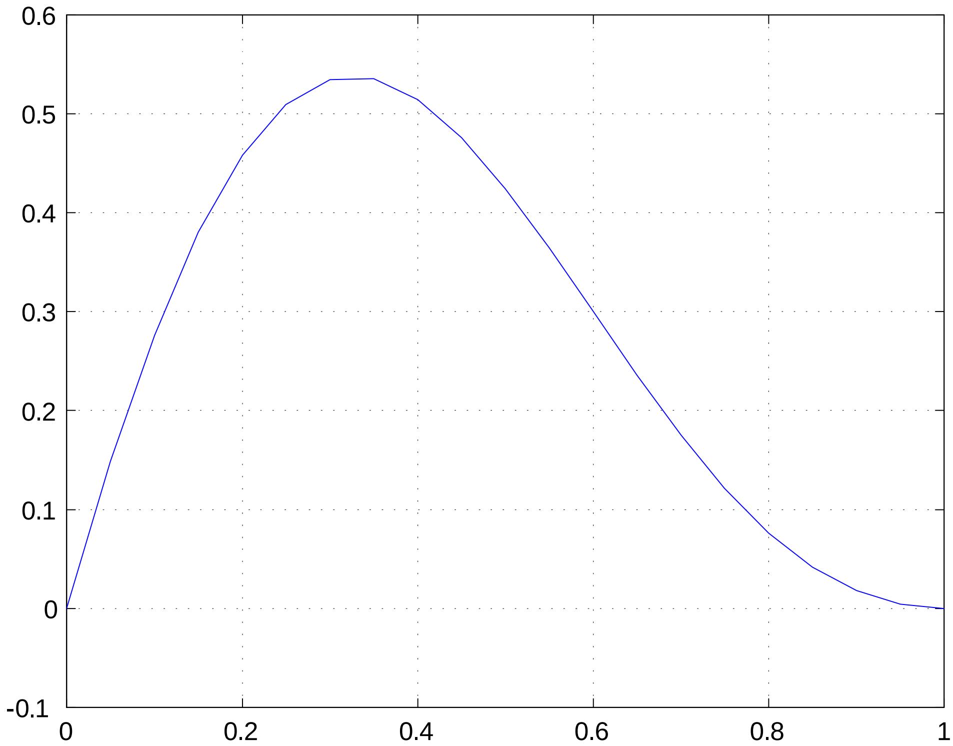

Figure 11: $\mathrm{Im}\{\Lambda(\epsilon)\}$ with $\Lambda(\epsilon) = S_0 - S_1$ of ASR modulation scheme when ϵ is from 0 to 1.

3.2 Modulation Scheme Over Fading Channel

Consider transmission over fading channel and apply ASR. The received signal on subcarrier becomes

$$
\begin{aligned}
\acute{Y}_k &= Y_k - Y_{k+1} \\
&= \left(S_0(H_k + H_{k+1}) - S_1 H_{k+1} - S_{-1} H_k\right) X_k \\
&\quad + \sum_{l=0, l\neq k, l\ even}^{N-1} \left(S_{l-k}(H_l + H_{l+1}) - S_{l-k+1} H_{l+1} - S_{l-k-1} H_l\right) X_l + n_k - n_{k+1}
\end{aligned}
$$

wherer $H_l, l = 0, 1, ..., N-1$ is the N point FFT of the delay profile. With X_k is derived from ICI self-cancellation scheme, and H_l is perfect known.

Applying the Central Limit Theorem, we try to approximate

$$
\sum_{l=0, l\neq k, l\ even}^{N-1} \left(S_{l-k}(H_l + H_{l+1}) - S_{l-k+1} H_{l+1} - S_{l-k-1} H_l\right) X_l + n_k - n_{k+1} \simeq 0
$$

Besides, $n_k - n_{k+1}$ is gaussian with zero mean.Then

$$\begin{aligned}
\acute{Y}_k &= Y_k - Y_{k+1} \\
&\cong \left(S_0(H_k + H_{k+1}) - S_1 H_{k+1} - S_{-1} H_k\right) X_k
\end{aligned}$$

Then

$$\begin{aligned}
E\left[\acute{Y}_k\right] &\cong E\left[\left(S_0(H_k + H_{k+1})\right) X_k\right] \\
&\cong S_0(H_k + H_{k+1}) E\left[X_k\right] \ , k = 0, 2, ..., N-2
\end{aligned}$$

$$\begin{aligned}
\frac{E\left[\acute{Y}_k\right]}{E\left[X_k\right]} &\cong S_0(H_k + H_{k+1}) - S_1 H_{k+1} - S_{-1} H_k \\
\frac{2}{N} \sum_{k=0}^{N-2} \frac{\acute{Y}_k}{X_k} &\cong S_0(H_k + H_{k+1}) - S_1 H_{k+1} - S_{-1} H_k \ , \\
k &= 0, 2, ..., N-2
\end{aligned}$$

Denote

$$\Lambda(\epsilon) = S_0(H_k + H_{k+1}) - S_1 H_{k+1} - S_{-1} H_k$$

Then ,the estimated $\hat{\epsilon}$ will be

$$\hat{\epsilon} = \arg\min_{\epsilon} \left(\Lambda(\epsilon) - \frac{2}{N} \sum_{k=0,k.even}^{N-2} \frac{\acute{Y}_k}{X_k} \right)$$

3.3 Demodulation Scheme over AWGN Channel

Here we use ASR, and make use of the PCC demodulation scheme. The data block will be $\mathbf{X} = \left(X_0, -X_0, X_1, -X_1, ..., X_{\frac{N}{2}-1}, -X_{\frac{N}{2}-1}\right)$.

Recall that

$$\begin{aligned}
\acute{Y}_k &= \left(2S_0 - S_1 - S_{-1}\right) X_k \\
&+ \sum_{l=0,l\ even,l\neq k}^{N-2} \left(2S_{l-k} - S_{l-k+1} - S_{l-k-1}\right) X_l + n_k - n_{k+1}
\end{aligned}$$

With X_k is derived from ICI self-cancellation scheme, and is deterministic.

According to the "Central Limit Theory", we approximate

$$\sum_{l=0,l\ even,l\neq k}^{N-2} (2S_{l-k} - S_{l-k+1} - S_{l-k-1}) \cong 0$$

Since both n_k is and n_{k+1} are AWGN noise, then $n_k - n_{k+1}$ is also AWGN noise.

Sum up Y_k for $k = 0, 2, ..., N-2$, then $E[\acute{Y}_k]$ becomes

$$E[\acute{Y}_k] \cong E[(2S_0 - S_1 - S_{-1})X_k] + E[n_k - n_{k+1}], \quad k = 0, 2, ..., N-2$$

When N is large,

$$E[n_k - n_{k+1}] \cong 0$$

Then,

$$\frac{2}{N} \sum_{k=0,k.even}^{N-2} \frac{\acute{Y}_k}{X_k} \cong 2S_0 - S_1 - S_{-1}$$

Denote

$$\Lambda(\epsilon) = 2S_0 - S_1 - S_{-1}$$

Then, the estimated $\hat{\epsilon}$ will be

$$\hat{\epsilon} = \arg\min_{\epsilon} \left(\Lambda(\epsilon) - \frac{2}{N} \sum_{k=0,k.even}^{N-2} \frac{\acute{Y}_k}{X_k} \right)$$

Recall that

$$\begin{aligned} \mathrm{Re}\{S_0\} &\cong \frac{\sin(2\pi\epsilon)}{2\pi\epsilon} \\ &= \mathrm{sinc}(\pi\epsilon) \end{aligned}$$

And

$$\mathrm{Re}\{S_1\} = \mathrm{Re}\{S_0\} \frac{\epsilon}{(\epsilon + 1)}$$

With $l-k=-1$, and N is large.

$$\begin{aligned} S_{-1} &= \frac{\sin(\pi(\epsilon-1))}{N\sin(\frac{\pi}{N}(\epsilon-1))}\exp(j\pi(\epsilon-1)) \\ &= \frac{\sin(\pi(\epsilon+1))}{\pi(\epsilon+1)}\exp(j\pi(\epsilon-1)) \end{aligned}$$

Then,

$$\begin{aligned} \mathrm{Re}\{S_{-1}\} &= \mathrm{Re}\left\{\frac{\sin(\pi(\epsilon-1))}{\pi(\epsilon-1)}\exp(j\pi(\epsilon-1))\right\} \\ &= \frac{\sin(\pi(\epsilon-1))\cos(\pi(\epsilon-1))}{\pi(\epsilon-1)} \\ &= \frac{\sin(2\pi(\epsilon-1))}{2\pi(\epsilon-1)} \\ &= \frac{\sin(2\pi\epsilon)}{2\pi\epsilon}\frac{2\pi\epsilon}{2\pi(\epsilon-1)} \\ &= \mathrm{Re}\{S_0\}\frac{\epsilon}{(\epsilon-1)} \end{aligned}$$

As a result,

$$\begin{aligned} \mathrm{Re}\left\{\Lambda(\epsilon)\right\} &= \mathrm{Re}\left\{2S_0 - S_1 - S_{-1}\right\} \\ &= 2\,\mathrm{Re}\{S_0\} - \mathrm{Re}\{S_1\} - \mathrm{Re}\{S_{-1}\} \\ &= \mathrm{Re}\{S_0\}\left(2-\frac{\epsilon}{\epsilon+1} - \frac{\epsilon}{\epsilon-1}\right) \end{aligned}$$

Then,

$$\hat{\epsilon} = \arg\min_{\epsilon} \mathrm{Re}\left\{\Lambda(\epsilon) - \sum_{k=0,k.even}^{N-2} \frac{\acute{Y}_k}{X_k}\right\}$$

Next, consider the imaginary parts of $\Lambda(\epsilon)$.

$$\mathrm{Im}\{S_0\} \simeq \frac{\sin(\pi\epsilon)}{\pi\epsilon}\sin(\pi\epsilon)$$

$$\mathrm{Im}\{S_1\} = \mathrm{Im}\{S_0\}\frac{\epsilon}{(\epsilon+1)}$$

Then,

$$
\begin{aligned}
\mathrm{Im}\{S_{-1}\} &= \mathrm{Im}\left\{\frac{\sin(\pi(\epsilon-1))}{\pi(\epsilon-1)}\exp(j\pi(\epsilon-1))\right\} \\
&= \frac{\sin(\pi(\epsilon-1))\sin(\pi(\epsilon-1))}{\pi(\epsilon-1)} \\
&= \frac{\sin(\pi\epsilon)\sin(\pi\epsilon)}{\pi(\epsilon-1)} \\
&= \frac{\sin(\pi\epsilon)\sin(\pi\epsilon)}{\pi\epsilon}\frac{\pi\epsilon}{\pi(\epsilon-1)} \\
&= \mathrm{Im}\{S_0\}\frac{\epsilon}{(\epsilon-1)}
\end{aligned}
$$

As a result,

$$
\begin{aligned}
\mathrm{Im}\{\Lambda(\epsilon)\} &= \mathrm{Im}\{2S_0 - S_1 - S_{-1}\} \\
&= 2\,\mathrm{Im}\{S_0\} - \mathrm{Im}\{S_1\} - \mathrm{Im}\{S_{-1}\} \\
&= \mathrm{Im}\{S_0\}\left(2 - \frac{\epsilon}{\epsilon+1} - \frac{\epsilon}{\epsilon-1}\right)
\end{aligned}
$$

Then,

$$
\hat{\epsilon} = \arg\min_{\epsilon} \mathrm{Im}\left\{\Lambda(\epsilon) - \frac{2}{N}\sum_{k=0,k.even}^{N-2}\frac{\acute{Y}_k}{X_k}\right\}
$$

Refer to Figure 10 and Figure 11. Consider two regions of ϵ. The first region is from 0 to 0.5, and the second region is from 0.5 to 1. For the first region, $\mathrm{Re}\{\Lambda(\epsilon)\}$ is one-to-one mapping, but $\mathrm{Im}\{\Lambda(\epsilon)\}$ is not . For the second region, neither $\mathrm{Re}\{\Lambda(\epsilon)\}$ nor $\mathrm{Im}\{\Lambda(\epsilon)\}$ is one-to-one mapping. Therefore, we will use $\mathrm{Re}\{\Lambda(\epsilon)\}$ to estimate CFO for ϵ in the range of 0 to 0.5.

3.4 Demodulation Scheme Over Fading Channel

Consider transmission over fading channel and apply ASR. The received signal on subcarrier becomes

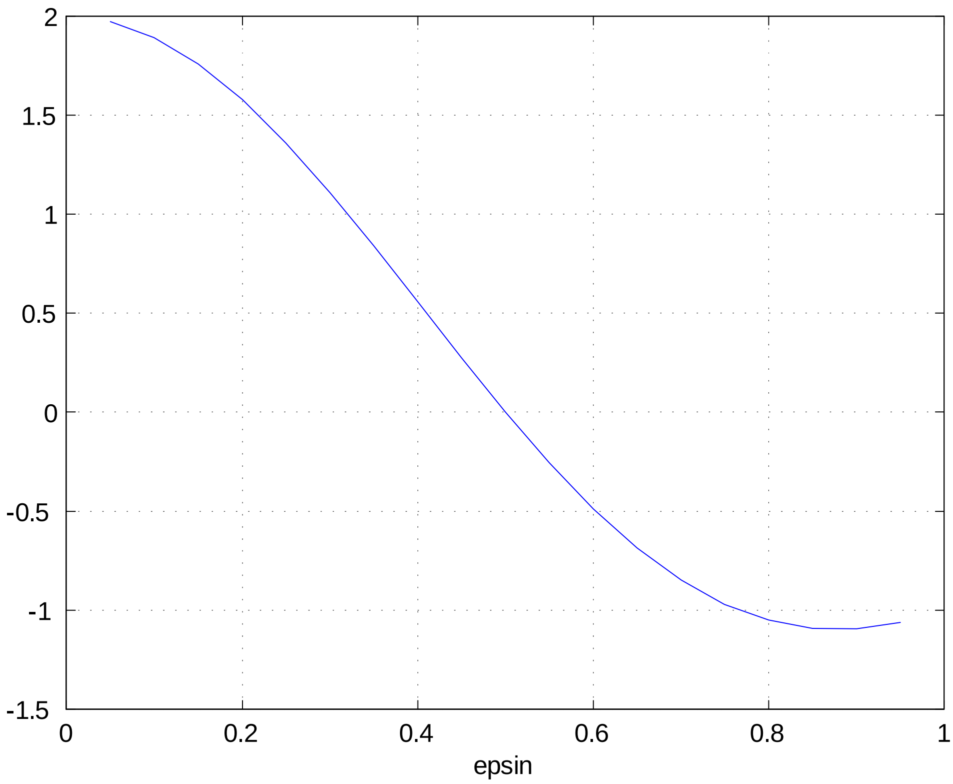

Figure 12: $\mathrm{Re}\{\Lambda(\epsilon)\}$ with $\Lambda(\epsilon) = 2S_0 - S_1 - S_{-1}$ of ASR demodulation scheme when ϵ is from 0 to 1.

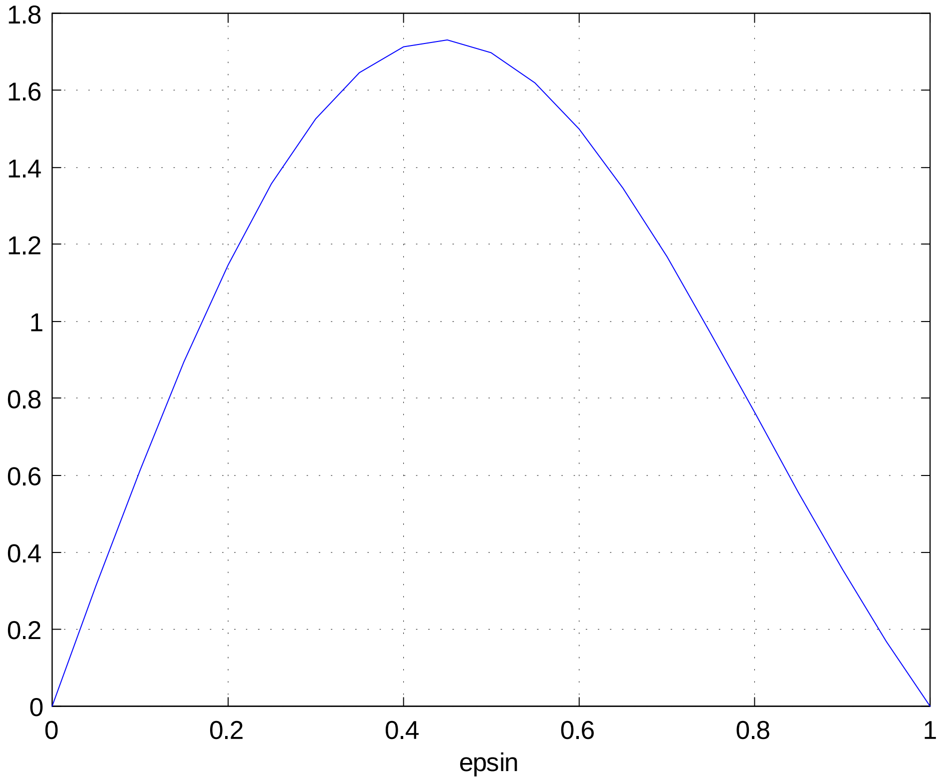

Figure 13: $\mathrm{Im}\{\Lambda(\epsilon)\}$ with $\Lambda(\epsilon) = 2S_0 - S_1 - S_{-1}$ of ASR demodulation scheme when ϵ is from 0 to 1.

$$
\begin{aligned}
\acute{Y}_k &= Y_k - Y_{k+1} \\
&= \left(S_0(H_k + H_{k+1}) - S_1 H_{k+1} - S_{-1} H_k\right) X_k \\
&+ \sum_{l=0, l\neq k, l\ even}^{N-1} \left(S_{l-k}(H_l + H_{l+1}) - S_{l-k+1} H_{l+1} - S_{l-k-1} H_l\right) X_l + n_k - n_{k+1}
\end{aligned}
$$

wherer $H_l, l = 0, 1, ..., N-1$ is the N point FFT of the delay profile. With X_k is derived from ICI self-cancellation scheme, and H_l is perfect known.

Applying the Central Limit Theorem, we try to approximate

$$
\sum_{l=0, l\neq k, l\ even}^{N-1} \left(S_{l-k}(H_l + H_{l+1}) - S_{l-k+1} H_{l+1} - S_{l-k-1} H_l\right) X_l + n_k - n_{k+1} \simeq 0
$$

Besides, $n_k - n_{k+1}$ is gaussian with zero mean.Then $\acute{Y}_k$ becomes

$$
\begin{aligned}
\acute{Y}_k &= Y_k - Y_{k+1} \\
&\simeq \left(S_0(H_k + H_{k+1}) - S_1 H_{k+1} - S_{-1} H_k\right) X_k
\end{aligned}
$$

Then

$$
\begin{aligned}
E\left[\acute{Y}_k\right] &\simeq E\left[\left(S_0(H_k + H_{k+1})\right) X_k\right] \\
&\simeq S_0(H_k + H_{k+1}) E\left[X_k\right] \ , k = 0, 2, ..., N-2
\end{aligned}
$$

$$
\begin{aligned}
\frac{E\left[\acute{Y}_k\right]}{E\left[X_k\right]} &\simeq S_0(H_k + H_{k+1}) - S_1 H_{k+1} - S_{-1} H_k \\
\frac{2}{N} \sum_{k=0}^{N-2} \frac{\acute{Y}_k}{X_k} &\simeq S_0(H_k + H_{k+1}) - S_1 H_{k+1} - S_{-1} H_k \ , \\
k &= 0, 2, ..., N-2
\end{aligned}
$$

Denote

$$
\Lambda(\epsilon) = S_0(H_k + H_{k+1}) - S_1 H_{k+1} - S_{-1} H_k
$$

Then ,the estimated $\hat{\epsilon}$ will be

$$\hat{\epsilon} = \arg\min_{\epsilon} \left(\Lambda(\epsilon) - \frac{2}{N} \sum_{k=0,k.even}^{N-2} \frac{\acute{Y}_k}{X_k} \right)$$

4 CFO Estimation for SSR of Polynomial Cancellation Coding

4.1 Modulation Scheme Over AWGN Channel

Here we use SSR. The data block will be $\mathbf{X} = \left(X_0, X_1.., X_{\frac{N}{2}-1}, -X_{\frac{N}{2}-1}, -X_1, -X_0\right)$.

Recall that

$$Y_k = \sum_{l=0}^{\frac{N}{2}-1} \left(S_{l-k} - S_{N-1-l-k}\right) X_l + n_k, \quad k = 0, 1, ..., \frac{N}{2} - 1$$

With X_k is derived from ICI self-cancellation scheme, and is deterministic.

According to the "Central Limit Theory", we approximate

$$\sum_{l=0, l\neq k,}^{\frac{N}{2}-1} \left(S_{l-k} - S_{N-1-l-k}\right) \cong 0$$

Sum up Y_k for $k = 0, 1, ..., \frac{N}{2} - 1$, then $E[\acute{Y}_k]$ becomes

$$E[\acute{Y}_k] \cong E[(S_0 - S_{N-1})X_k] + E[n_k], \quad k = 0, 1, ..., \frac{N}{2} - 1$$

When N is large,

$$E[n_k] \cong 0$$

Then,

$$\frac{2}{N} \sum_{k=0}^{\frac{N}{2}-1} \frac{\acute{Y}_k}{X_k} \cong S_0 - S_{N-1}$$

Denote

$$\Lambda(\epsilon) = S_0 - S_{N-1}$$

Then, the estimated $\hat{\epsilon}$ will be

$$\hat{\epsilon} = \arg\min_{\epsilon} \left(\Lambda(\epsilon) - \frac{2}{N} \sum_{k=0}^{\frac{N}{2}-1} \frac{\acute{Y}_k}{X_k}\right)$$

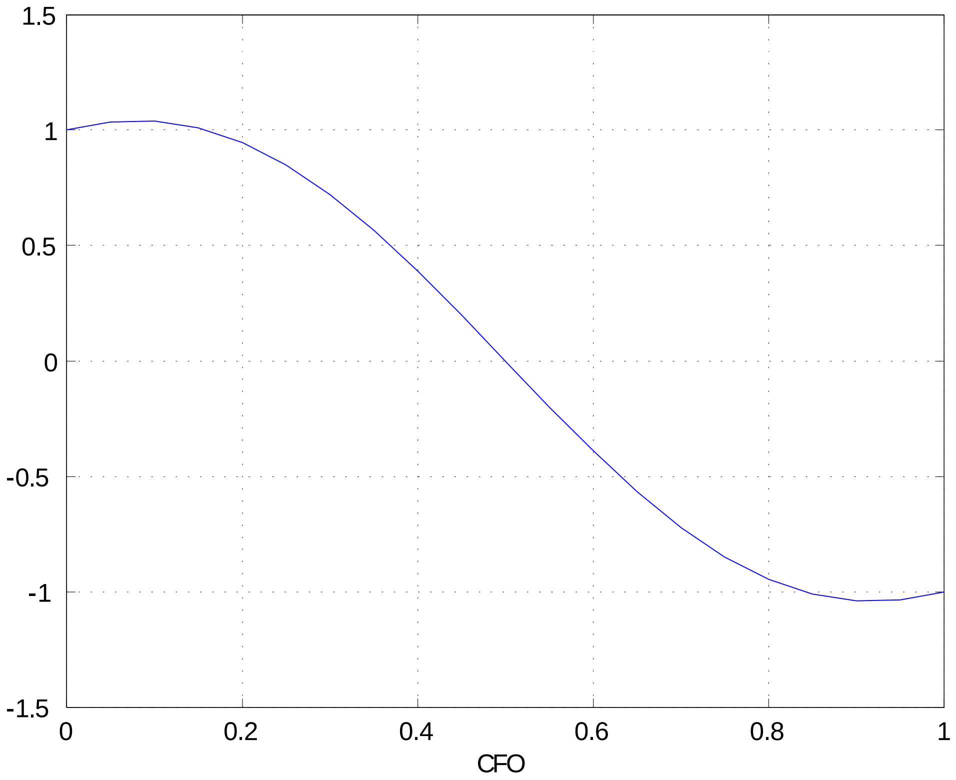

Figure 14: $\mathrm{Re}\{\Lambda(\epsilon)\}$ with $\Lambda(\epsilon) = S_0 - S_{N-1}$ of SSR modulation scheme when ϵ is from 0 to 1.

Refer to Figure 12 and Figure 13, both $\mathrm{Re}\{\Lambda(\epsilon)\}$ and $\mathrm{Im}\{\Lambda(\epsilon)\}$ are not strickly one-to-one mapping either when epsin is from 0 to 0.5 or when epsin is from 0 to 1. We will use $\mathrm{Im}\{\Lambda(\epsilon)\}$ to estimate CFO in the simulation.

4.2 Modulation Scheme Over Fading Channel

Consider transmission over fading channel and apply SSR. The received signal on subcarrier becomes

$$Y_k = \sum_{l=0}^{\frac{N}{2}-1} \left(S_{l-k}H_l - S_{N-1-l-k}H_{N-1-l}\right) X_l + n_k, \quad k = 0, 1, ..., \frac{N}{2} - 1$$

wherer $H_l, l = 0, 1, ..., N$ is the N point FFT of the delay profile. With X_k is derived from ICI self-cancellation scheme, and H_l is perfect known.

Applying the Central Limit Theorem, we try to approximate

$$\sum_{l=0, l\neq k}^{\frac{N}{2}-1} \left(S_{l-k}H_l - S_{N-1-l-k}H_{N-1-l}\right) \simeq 0$$

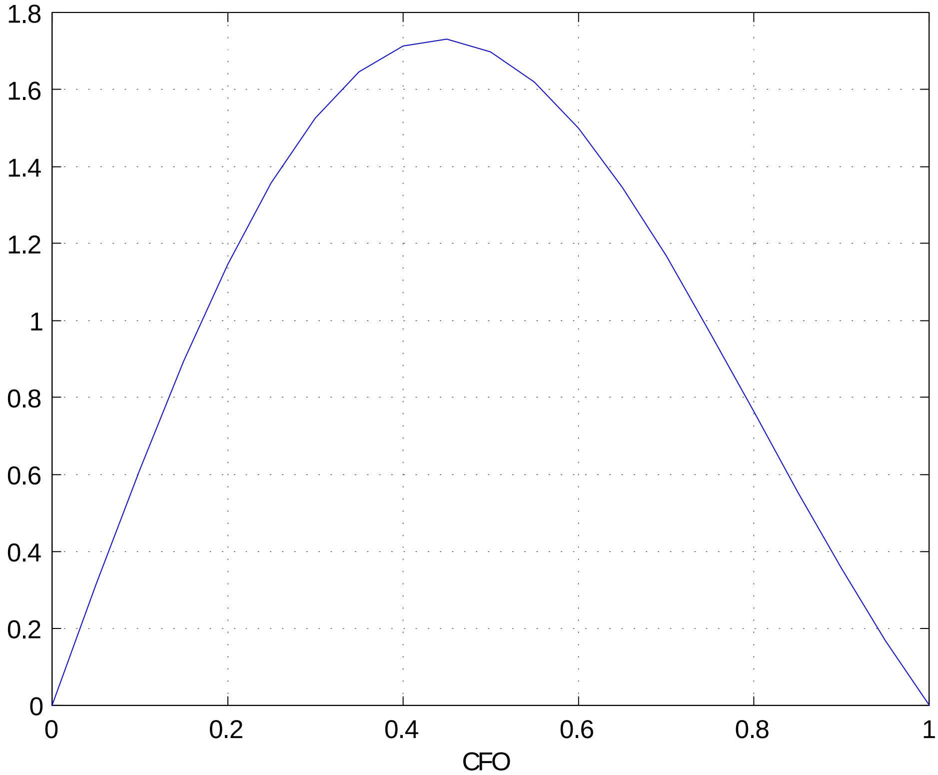

Figure 15: $\mathrm{Im}\{\Lambda(\epsilon)\}$ with $\Lambda(\epsilon) = S_0 - S_{N-1}$ of SSR modulation when ϵ is from 0 to 1.

Then

$$E[Y_k] \simeq E[(S_0 H_l - S_{N-1} H_{N-1-k}) X_k]$$

$$\begin{aligned} \frac{E[Y_k]}{E[X_k]} &\simeq S_0 H_l - S_{N-1} H_{N-1-k} \\ \frac{2}{N} \sum_{k=0}^{\frac{N}{2}-1} \frac{\acute{Y}_k}{X_k} &\simeq S_0 H_l - S_{N-1} H_{N-1-k} \text{ }, k = 0, 1, ..., \frac{N}{2} - 1 \end{aligned}$$

Denote

$$\Lambda(\epsilon) = S_0 H_l - S_{N-1} H_{N-1-k}$$

Then ,the estimated $\hat{\epsilon}$ will be

$$\hat{\epsilon} = \arg\min_{\epsilon} \left(\Lambda(\epsilon) - \frac{2}{N} \sum_{k=0}^{\frac{N}{2}-1} \frac{\acute{Y}_k}{X_k} \right)$$

4.3 Demodulation Scheme Over AWGN Channel

Here we use SSR, and make use of the PCC demodulation scheme. The data block will be $\mathbf{X} = \left(X_0, X_1.., X_{\frac{N}{2}-1}, -X_{\frac{N}{2}-1}, -X_1, -X_0\right)$.

Recall that

$$\begin{aligned} \acute{Y}_k &= \left(2S_0 - S_{N-1} - S_{-(N-1)}\right) X_k \\ &+ \sum_{l=0,l\neq k}^{\frac{N}{2}-1} \left(S_{l-k} - S_{N-1-k-l} - S_{l-(N-1-k)} - S_{-(l-k)}\right) X_l + n_k - n_{N-1-k} \end{aligned}$$

With X_k is derived from ICI self-cancellation scheme, and is deterministic.

According to the "Central Limit Theory", we approximate

$$\sum_{l=0,l\neq k}^{\frac{N}{2}-1} \left(S_{l-k} - S_{N-1-k-l} - S_{l-(N-1-k)} - S_{-(l-k)}\right) \simeq 0$$

Since both n_k is and n_{k+1} are AWGN noise, then$n_k - n_{N-1-k}$ is also AWGN noise.

Sum up Y_k for $k = 0, 1, ..., \frac{N}{2} - 1$, then $E[\acute{Y}_k]$ becomes

$$E[\acute{Y}_k] \simeq E[(2S_0 - S_{N-1} - S_{-(N-1)})X_k] + E[n_k - n_{k+1}], \quad k = 0, 1, ..., \frac{N}{2} - 1$$

When N is large,

$$E[n_k - n_{N-1-k}] \simeq 0$$

Then,

$$\frac{2}{N} \sum_{l=0,l\neq k}^{\frac{N}{2}-1} \frac{\acute{Y}_k}{X_k} \simeq 2S_0 - S_{N-1} - S_{-(N-1)}$$

Denote

$$\Lambda(\epsilon) = 2S_0 - S_{N-1} - S_{-(N-1)}$$

Then, the estimated $\hat{\epsilon}$ will be

$$\hat{\epsilon} = \arg\min_{\epsilon} \left(\Lambda(\epsilon) - \frac{2}{N} \sum_{l=0,l\neq k}^{\frac{N}{2}-1} \frac{\acute{Y}_k}{X_k}\right)$$

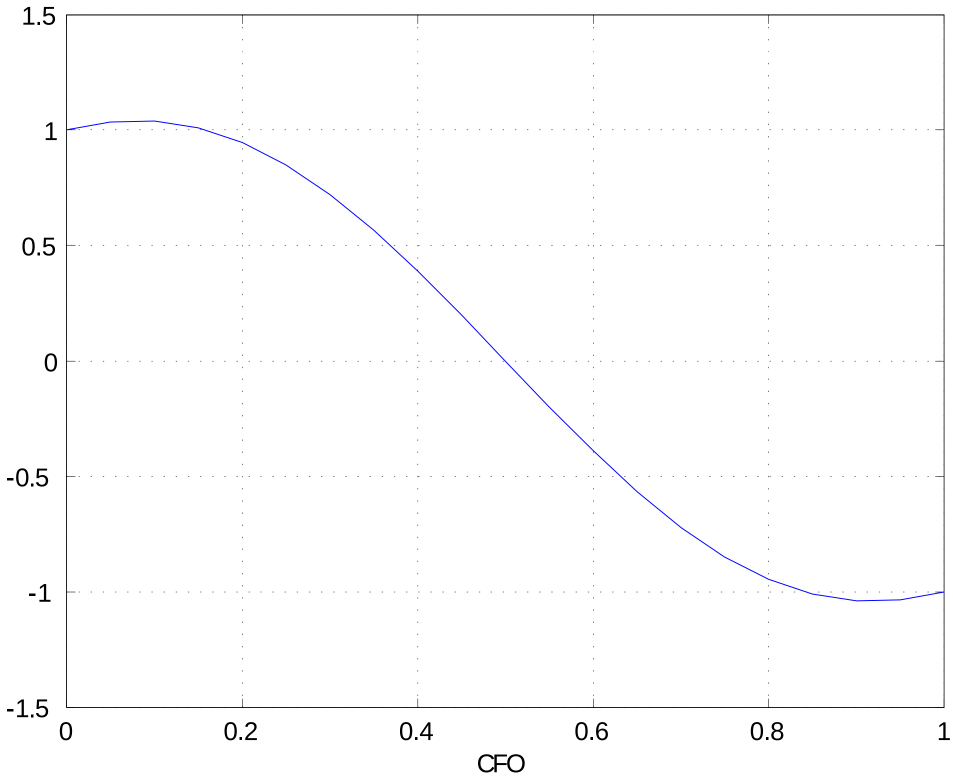

Figure 16: $\mathrm{Re}\{\Lambda(\epsilon)\}$ with $\Lambda(\epsilon) = 2S_0 - S_{N-1} - S_{-(N-1)}$ of SSR demodulation when ϵ is from 0 to 1.

Refer to Figure 14 and Figure 15, both $\mathrm{Re}\{\Lambda(\epsilon)\}$ and $\mathrm{Im}\{\Lambda(\epsilon)\}$ are not strickly one-to-one mapping either when epsin is from 0 to 0.5 or when epsin is from 0 to 1. We will use $\mathrm{Im}\{\Lambda(\epsilon)\}$ to estimate CFO in the simulation.

4.4 Demodulation Scheme Over Fading Channel

Consider transmission over fading channel and apply SSR. The received signal on subcarrier becomes

$$
\begin{aligned}
\acute{Y}_k &= Y_k - Y_{N-k-1} \\
&= \left(S_0(H_k + H_{N-1-K}) - S_{-2k+N-1}H_{N-1-k} - S_{2k-(N-1)}H_k\right) X_k \\
&\quad + \sum_{l=0, l\neq k}^{\frac{N}{2}-1} \left(S_{l-k}H_l - S_{N-1-k-l}H_{N-1-l} - S_{l-(N-1-k)}H_l + S_{-(l-k)}H_{N-1-l}\right) X_l + n_k - n_{N-1-k}
\end{aligned}
$$

wherer $H_l, l = 0, 1, ..., N$ is the N point FFT of the delay profile. With X_k is derived from ICI self-cancellation scheme, and H_l is perfect known.

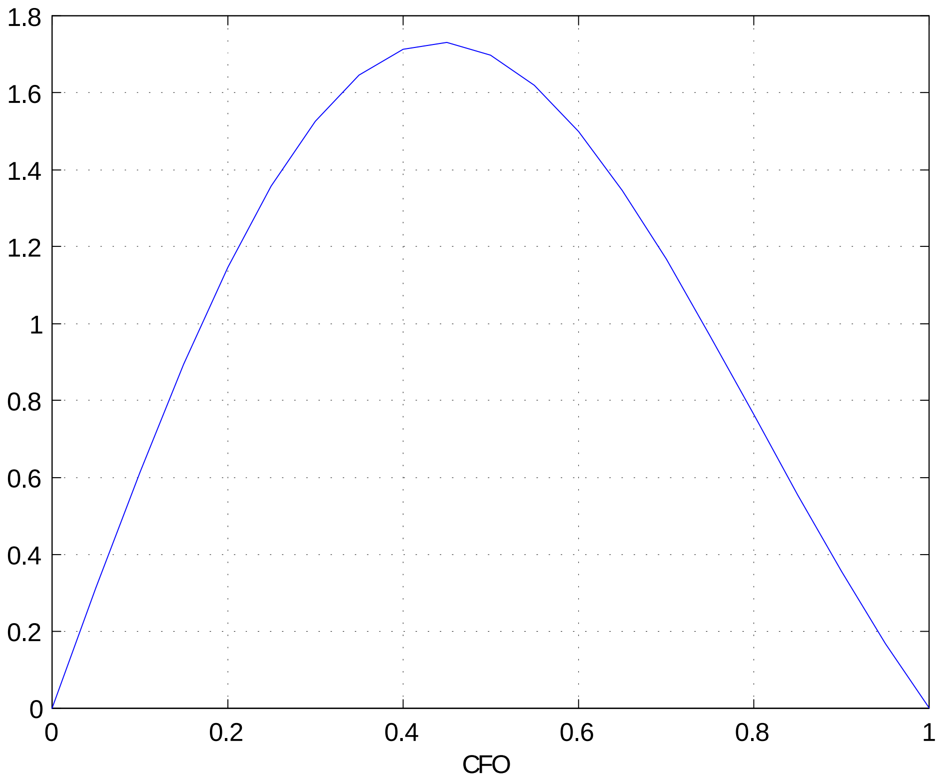

Figure 17: $\mathrm{Im}\left\{\Lambda(\epsilon)\right\}$ with $\Lambda(\epsilon) = 2S_0 - S_{N-1} - S_{-(N-1)}$ of SSR demodulation when ϵ is from 0 to 1.

Applying the Central Limit Theorem, we try to approximate

$$\left(S_{l-k}H_l - S_{N-1-k-l}H_{N-1-l} - S_{l-(N-1-k)}H_l + S_{-(l-k)}H_{N-1-l}\right) \tilde{=} 0$$

Besides, $n_k - n_{k+1}$ is gaussian with zero mean.Then the received signal becomes

$$\begin{aligned} \acute{Y}_k &= Y_k - Y_{N-k-1} \\ &\tilde{=} \left(S_0(H_k + H_{N-1-K}) - S_{-2k+N-1}H_{N-1-k} - S_{2k-(N-1)}H_k\right) X_k \end{aligned}$$

Then

$$E\left[\acute{Y}_k\right] \tilde{=} E\left[\left(S_0(H_k + H_{N-1-K}) - S_{-2k+N-1}H_{N-1-k} - S_{2k-(N-1)}H_k\right) X_k\right]$$

$$\begin{aligned} \frac{E\left[\acute{Y}_k\right]}{E\left[X_k\right]} &\tilde{=} S_0(H_k + H_{N-1-K}) - S_{-2k+N-1}H_{N-1-k} - S_{2k-(N-1)}H_k \\ \frac{2}{N}\sum_{k=0}^{\frac{N}{2}-1}\frac{\acute{Y}_k}{X_k} &\tilde{=} S_0(H_k + H_{N-1-K}) - S_{-2k+N-1}H_{N-1-k} - S_{2k-(N-1)}H_k\ , \\ k &= 0, 1, ..., \frac{N}{2} - 1 \end{aligned}$$

Denote

$$\Lambda(\epsilon) = S_0(H_k + H_{N-1-K}) - S_{-2k+N-1}H_{N-1-k} - S_{2k-(N-1)}H_k$$

Then ,the estimated $\hat{\epsilon}$ will be

$$\hat{\epsilon} = \arg\min_{\epsilon}\left(\Lambda(\epsilon) - \frac{2}{N}\sum_{k=0}^{\frac{N}{2}-1}\frac{\acute{Y}_k}{X_k}\right)$$

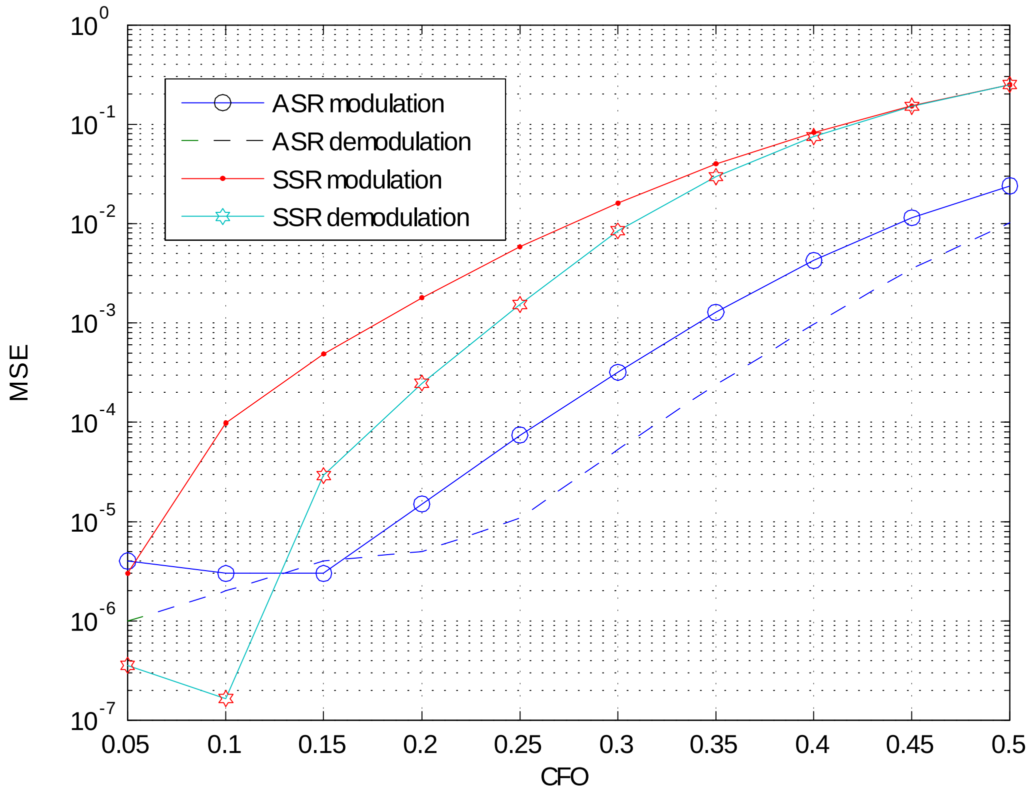

Figure 18: Comparisons of MSE by different schemes of PCC over AWGN channel at SNR=3dB.

5 Simulation Results and Discussion

The followings are the MSE (mean square error) of estimates CFOs. For fading channel, we use the vehicular profile of IEEE 802.16m.The demoulation scheme performs better than the modulation one, as shown in Figure 3. Meanwhile, ASR performs better than SSR in the estimation of CFO over AWGN channel.

However, when CFO is small, ie, smaller than 0.15, ASR doesn't do well compared with SSR. Recall Figure 13 and Figure 15, the imaginary part of desired signal is small when CFO is small. Thus the small desired signal will be affected easily by noise, and results in worse performance.In Figure 17 the modulation of ASR performs better than others. However, the demodulation scheme performs worse than modulation scheme. The reason is that when channel is selective, the subtraction of the desired signal will enhance the ICI term. Besides, Figure 17 also shows that the SSR deals with the selectivity of channel better than ASR.

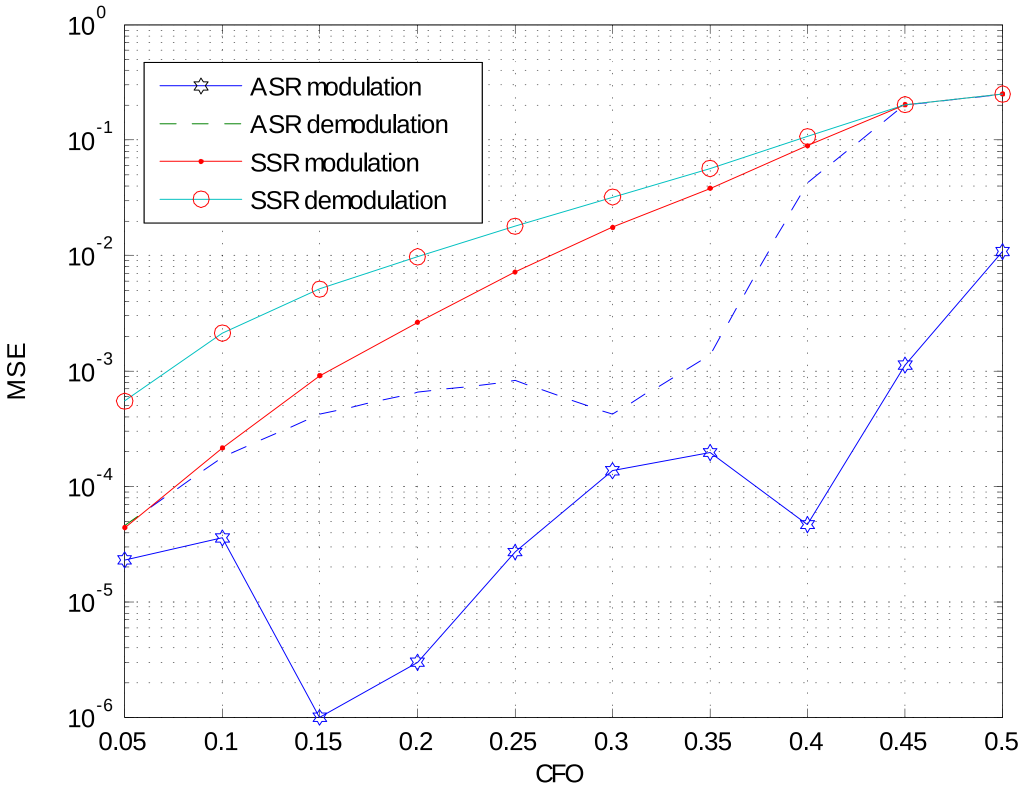

Figure 19: Comparisons of MSE by different schemes of PCC over fading channel at SNR=25dB.

6 Conclusion

In OFDM systems, a carrier frequency offset (CFO) between the transmitter and receiver will results in ICI and thus destroy the orthogonality of the subcarrier and degrades the performance. ICI leads to attenuation and phase rotation of desired signal on each subcarrier. These impairments have already motivated several studies to find solutions. Among the several ICI reduction schemes, ICI self-cancellation or polynomial cancellation coding (PCC) scheme has received much attention due to its simplicity and its high robustness to frequency offset errors. However, for large CFO, PCC can only eliminate ICI to a certain extent. Thus, we propose new methods of PCC based CFO estimation to eliminate ICI for either low or high CFOs. First, we use PCC as precoding scheme and get the initial decoded data. Then CFO estimation is done by making use of the decode data. CFO estimation and compensation from PCC can overcome the drawback of PCC that only a certain part of ICI can be eliminated when CFO is large.

Our methods show good performanceof CFO estimation and improve system performance. Besides, it's easy to be realized in real systems due to low computation.

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