國立交通大學

電機學院通訊與網路科技產業研發碩士班

碩士論文

具中繼選擇合作式通信系統之極大訊雜比檢測機制

Maximum SNR Detection for Selection-Relaying Cooperative System

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具 中 繼 選 擇 合 作 式 通 信 系 統 之 極 大 訊 雜 比 檢 測 機 制

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摘　　要

合作式通訊系統就目前來說，是一個有用的方法來實現多輸入多輸出天線系統。但是，如何在目的端設計一個理想的接收機，使得此接收機的效能可以好到與以使用最大概似函數的方式設計的接收機的效能一樣是一個主要的議題。因為如此，在解碼與傳送的系統中我們在目的端設計一個接收機，且接收機可以組合接收到的信號。我們所提的架構是使用當中繼點會自行判斷是否要執行解碼與傳送的行為之臨界點選擇的中繼點，並且在目的端使用修正權重來減少因為中繼點決定錯誤的效應。我們也有試著去找尋適合中繼點使用的最佳臨界值在我們所建議的架構中。除此之外，我們有推導出此架構當使用二位元相位數位調變信號的理論位元錯誤率。我們將會發現理論錯誤率將會與模擬結果貼近。我們也有使用高信號雜訊比下的近似方式來簡化理論錯誤率。從這簡化版的理論錯誤率，我們可以得知建議系統的多樣性階數將會是在 1.5～2 之間，其結果與前人所做的結論相同。

Maximum SNR Detection for Selection-Relaying Cooperative System

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Industrial Technology R & D Master Program of
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ABSTRACT

The cooperative communication is to realize the MIMO system. The optimum receiver is a highly-complex ML receiver. Hence, the major issue is to design a receiver with low complexity at the destination node with comparable performance as that of the ML receiver. Therefore, we design a receiver to combine the received signals at the Destination node under Decode-and-Forward protocol. The proposed scheme uses the threshold-selection Relay and also uses the maximum SNR detection at the Destination node to minimize the effect that Relay made wrong decision. With this proposed scheme, we also try to find out the optimum threshold value of the Relay. Besides, we derive theoretical bit-error-rate (BER) with BPSK signals for the proposed scheme. We also show that the theoretical BER is tight to the simulated results. The high SNR approximation is made to simplify the theoretical BER, from which we could know that the diversity order of the proposed system lies in 1.5~2 which agrees with previous work.

Acknowledgment

I want to thank my advisor Dr. S. F. Hsieh first. Without his assistance, my thesis would not complete so smoothly. Second, I want to thank my parents and my family. With your support, I always have full energy to do my research after I spend time with you during weekend. Also, I want to thank my college and high school classmate. Thanks to you guys give me faith to finish my studies. Finally, thanks to my Lab friends. I can always be happy to do my research after we make fun to each other.

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Chapter 1

Introduction

As we know, the MIMO system is a very popular technology for wireless communication right now. But it's impractical to place 2 or more antennas in the portable device due to the limited device size area. Therefore, the cooperative communication has recently emerged as a promising alternative to form a virtual MIMO system and combat fading in a wireless channels in [1]-[5]. The basic idea is that users or nodes in a wireless network share their information and transmit cooperatively as a virtual antenna array, thus providing diversity without the requirement of additional antennas at each node. Figure 1.1 shows the general model of a cooperative communication system. In this figure, the Relays work differently when using different protocols.

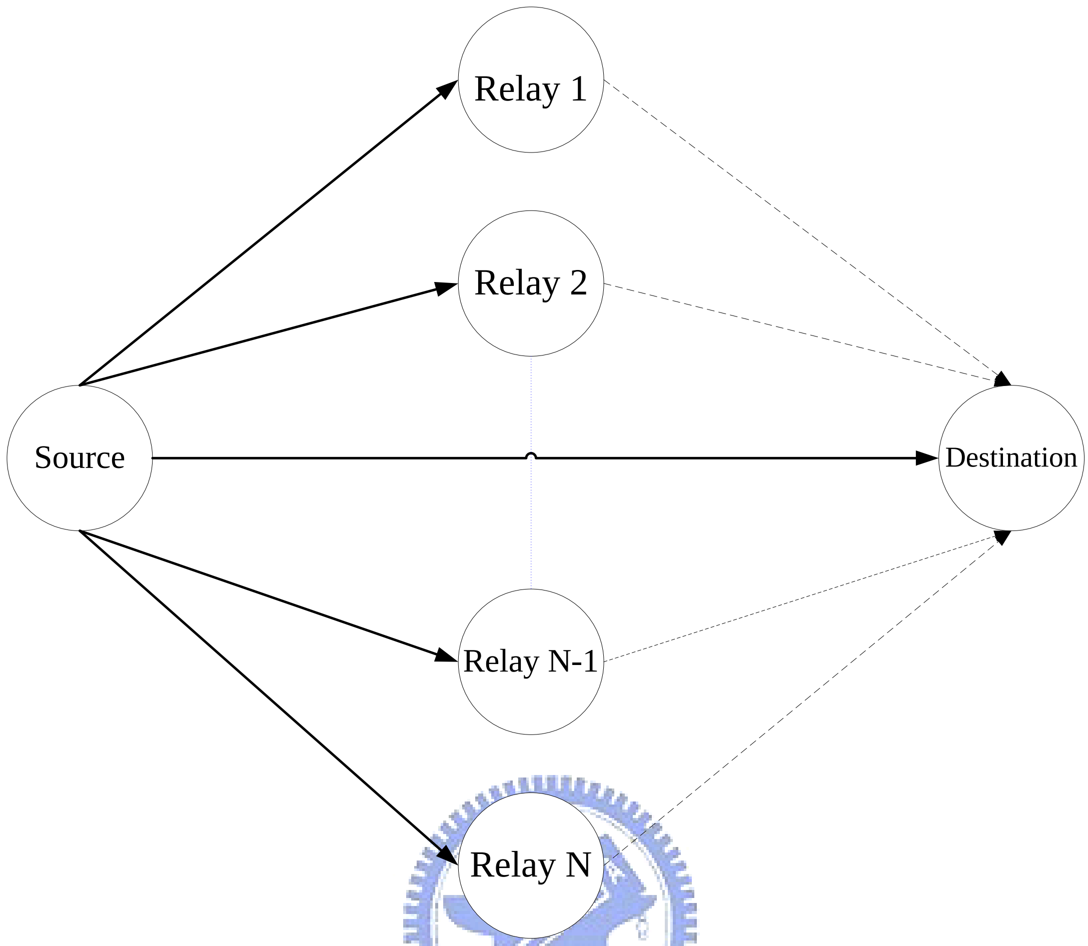

Figure 1.1 Cooperative system

For the relay working protocols, we could find out there are two protocols, Decode-and-Forward protocol and Amplify-and-Forward protocol in [3]-[5]. When the relays use the Decode-and-Forward protocol, the relays decode the received signal and forward the decoded signal to the destination node from [3] and [5]. When using Amplify-and-Forward protocol, the relays only amplify the received signal and forward to the destination from [4]. After the relays forward the signals to the destination, the destination detect the original signal from these N+1 signals. In [5], we could know the destination has two common detection methods, MRC receiver and ML receiver. The MRC receiver method has the low complexity property with bad BER performance. The ML receiver has the highly complexity property with great BER performance. We would describe these two detection methods in details at the chapter 2.

In this thesis, we only focus on the Decode-and-Forward protocol as the relay strategy and the relay uses the threshold value to evaluate the received signal quality in [6]. For reducing the detection complexity, we use the maximum SNR detection in [5] as our destination detection method. With this maximum SNR detection for selection relaying scheme, we would find out the performance is close to the ML receiver performance and reduce the detection complexity. From [6] and [8], we analyze the theoretical BER of the proposed scheme to verify the simulated results. From the theoretical BER, we would roughly know the theoretical BER depends on the destination received signal SNR and the threshold value. But we could not know the detail influence on these two parameters. Hence, we use the similar approximated method in [8] to simplify the theoretical BER. From the simplified theoretical BER, we would find out the proposed scheme diversity order which agrees with the previous work in [7].

The thesis is organized as follows. Chapter 2 is the system model for the cooperative system. Besides, the problems of the cooperative system are considered in detail. Chapter 3 is the main part of this thesis, and it focuses on the methods.

Chapter 4 is the BER analysis of the proposed cooperative system architecture and also we derive the approximated BER of the proposed architecture. Chapter 5 is simulation results and discussions about the effect of the proposed architecture. Finally, our conclusions are given in the chapter 6.

Chapter 2

The Cooperative System Model

In the Chapter 1, we talk about the cooperative system briefly. In this chapter, we will explain two Relay protocols, the Decode-and-Forward protocol and the Amplify-and-Forward protocol in detail under the cooperative system. Also we will introduce the methods that the destination node often uses to combine the received signals from the Source and the Relay node.

2.1 The Decode-and-Forward Protocol

We can use Figure 2.1 to describe how the Decode-and-Forward protocol works under the cooperative system.

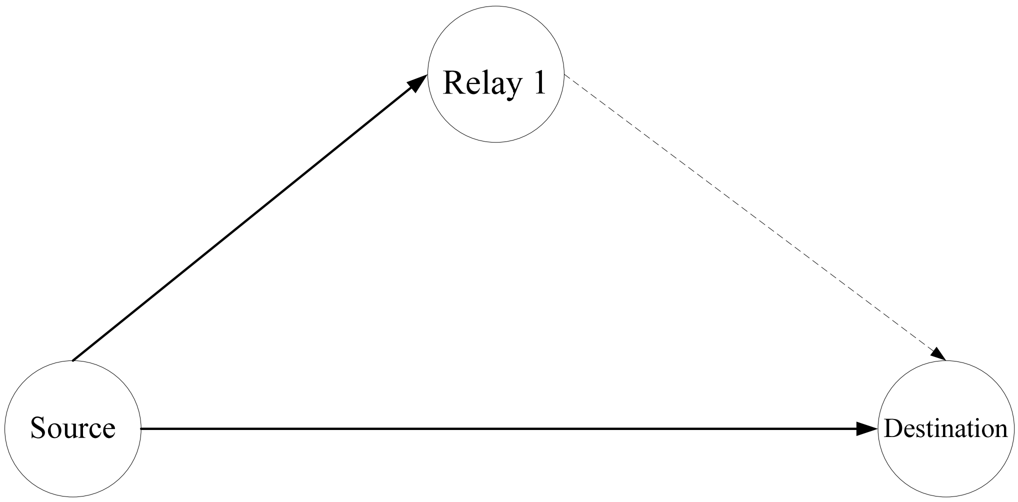

Figure 2.1 The basic cooperative system model

In the cooperative system, it has two kinds of the transmission mode. First, the solid line says the Source broadcasts the signal to the Relay and Destination node and it is called Phase 1. We can show the received signals into these form：

$$y_{s,r} = \sqrt{P_1} h_{s,r} x + n_{s,r} \,, \tag{2.1.1}$$

$$y_{s,d} = \sqrt{P_1} h_{s,d} x + n_{s,d} \tag{2.1.2}$$

As we can see, the signal from the Source is transmitted to the Relay and the Destination via the Source-Relay channel $h_{s,r}$ and the Source-Destination channel $h_{s,d}$. Second, the dashed line says the Relay decodes the received signal and forwards the decoded signal to the Destination node via the Relay-Destination channel h_{rd} and it calls Phase 2. We also can show the received signal into this form：

$$y_{rd} = \sqrt{P_2} h_{rd} \tilde{x} + n_{rd} \,, \tag{2.1.3}$$

Therefore the Destination node will combine these two signals from these two phases to improve the receiver performance.

2.2 The Amplify-and-Forward Protocol

We also can use the Figure 2.1 to describe the Amplify-and-Forward protocol and the only difference between these two protocols is in phase 2. In Phase 2, after receiving the signal the Relay only amplifies the received signal and transmits the amplified signal to the Destination node via the Relay-Destination channel. We can show the received signal from the Relay at the Destination node：

$$y_{r,d} = \beta\sqrt{P_2}h_{r,d}y_{s,r} + n_{r,d}, \qquad (2.2.1)$$

In Eq. (2.2.1), β is the amplification factor which is related to the signal power and noise power for the Amplify-and-Forward protocol. Compared to the Eq. (2.1.3), the relay doesn't estimate the original signal. But in the Decode-and-Forward protocol, the relay actually estimates the original signal and transmits the decoded signal in the next stage. From these two different relay working modes and [9], we could know under the Decode-and-Forward protocol the diversity order is small than the other protocol due to the estimation.

2.3 The Destination Combining Methods

After the Destination node receives the signals from the Phase 1 and Phase 2, the Destination will try to combine the received the signals and decode the combined signal. There are several popular combining methods. Here, we introduce the MRC combining and ML combining methods.

2.3.1 MRC Combining Method

The purpose of the MRC combining method is maximizing the received signal power to noise power ratio i.e. SNR. Therefore from [5], the weighting coefficients for each received signal can be shown into this form：

$$w_{s.d} = \frac{\sqrt{P_1} h_{s,d}^*}{\sigma_{s,d}^2}$$
$$w_{r,d} = \frac{\sqrt{P_2} h_{r,d}^*}{\sigma_{r,d}^2} \quad (2.3.1)$$

In Eq. (2.3.1), the $\sigma_{i,j}^2$ means the noise power that the Destination node receives the signal which contains the noise in Phase 1 and Phase 2 individually. Based on the Eq. (2.3.1), the receiver scheme could be shown as：

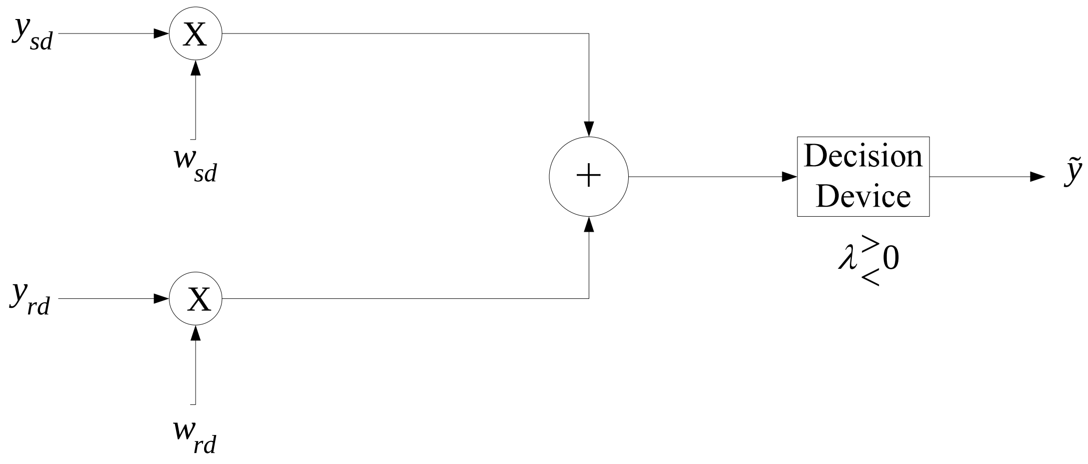

Figure 2.2 The destination MRC receiver scheme

2.3.2 ML Combining Method

The ML (Maximum Likelihood) combining method is the optimum solution for the Destination node from [3], [5] and [7]. First, we assume the symbol transmitted from the Source node has equal probability. Then the optimum receiver can be designed by the log-likelihood ratio of the received signal posterior probability. In [9], for BPSK the log-likelihood ratio can be defined as：

$$l^s(\mathrm{y}) = \ln \frac{p_{y_{s,d}|x_s}\left(y|x_1\right)}{p_{y_{s,d}|x_s}\left(y|x_0\right)}$$
$$l^r(\mathrm{y}) = \ln \frac{p_{y_{r,d}|x_s}\left(y|x_1\right)}{p_{y_{r,d}|x_s}\left(y|x_0\right)} \qquad (2.3.2)$$

to represent the received signals directly from the Source and the Relay, respectively. With these two definitions, the Destination ML decision rule can be written as

$$\hat{m} = \arg \max \; l^s\left(y_{s,d}\right) + l^r\left(y_{r,d}\right) \tag{2.3.3}$$

With the Eq, (2.3.3), we could use the similar concept in [5] and [9] to derive the ML receiver. Because of the Decode-and-Forward protocol, the relay may make decision errors. We should reconsider about $l^r(y)$ more seriously. First, based on the relay decision errors, the log-likelihood can be shown as：

$$\hat{l}^r_{m'}(y) = \ln \frac{p_{y_{r,d}|x_r}\left(y \mid x_{m'}\right)}{p_{y_{r,d}|x_r}\left(y \mid x_0\right)} \quad m' = 0,1 \tag{2.3.4}$$

And the transition probability of the Relay making decision errors is defined as：

$$P^r_{m',m} = \Pr\left[x_r = x_{m'} \mid x_s = x_m\right]. \tag{2.3.5}$$

According to these two parameters, the numerator of $l^r(y)$ can be rewritten as：

$$p_{y_{r,d}|x_s}\left(y \mid x_1\right) = \sum_{m'=0}^{1} P^r_{m',1} \exp(\hat{l}^r_{m'}(y)) \tag{2.3.6}$$

Similarly, the denominator has a similar form as Eq. (2.3.6). Therefore, we can extend $l^r(y)$, and design the receiver based on this result. Therefore, the ML receiver will contain one set of the weighting gain and follow a non-linear mapping function block which agrees with the result in [5]. The set of the weighting gains and the non-linear mapping can be shown as followed：

$$w_{s.d} = \frac{\sqrt{P_1} h_{s,d}^*}{\sigma_{s,d}^2}$$
$$w_{r,d} = \frac{\sqrt{P_2} h_{r,d}^*}{\sigma_{r,d}^2} \quad (2.3.7)$$

$$f(t) = \ln\left[\frac{\varepsilon_r + (1-\varepsilon_r)e^t}{(1-\varepsilon_r) + \varepsilon_r e^t}\right] \quad (2.3.8)$$

"ε_r", mean the transition probability of the Relay making decision errors.

Based on these two results, the receiver scheme would be shown as：

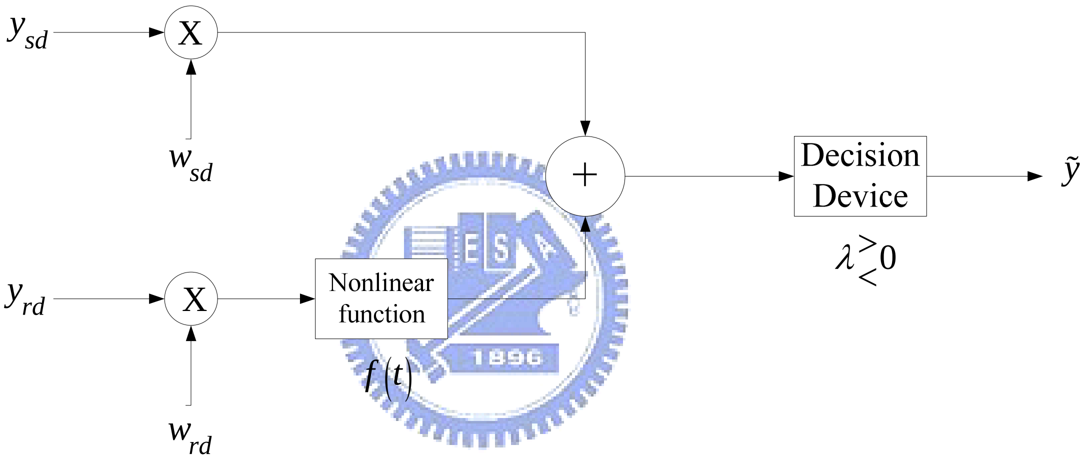

Figure 2.3 The destination ML receiver scheme

Compared to the Figure 2.2, the ML receiver has an extra nonlinear mapping function for the relay signal. The function $f(t)$ serves as a limiter which minimizes the contribution from the Relay when it is unreliable. Therefore, how to design a receiver at the Destination with comparable performance as the ML receiver without using any non-linear mapping function is the major challenge in the cooperative system.

Chapter 3

Maximum SNR Detection for Selection Relay

At the end of the Chapter 2, we say the non-linear function works like a limiter. Therefore, it will minimize the contribution from the Relay when it is unreliable. The Figure 3.1 shows the performance comparison between the MRC and ML receivers and the channel gains $h_{s,d}, h_{s,r}, h_{r,d}$ we use are mutually independent zero-mean, complex Gaussian random variables with variances set to 1. The noises $z_{s,d}, z_{s,r}, z_{r,d}$ are also mutually independent zero-mean, complex Gaussian random variables with variances set to 1. Also the PNR means the total system power ($P = P_1 + P_2$) to noise power ratio in dB. The result is shown below：

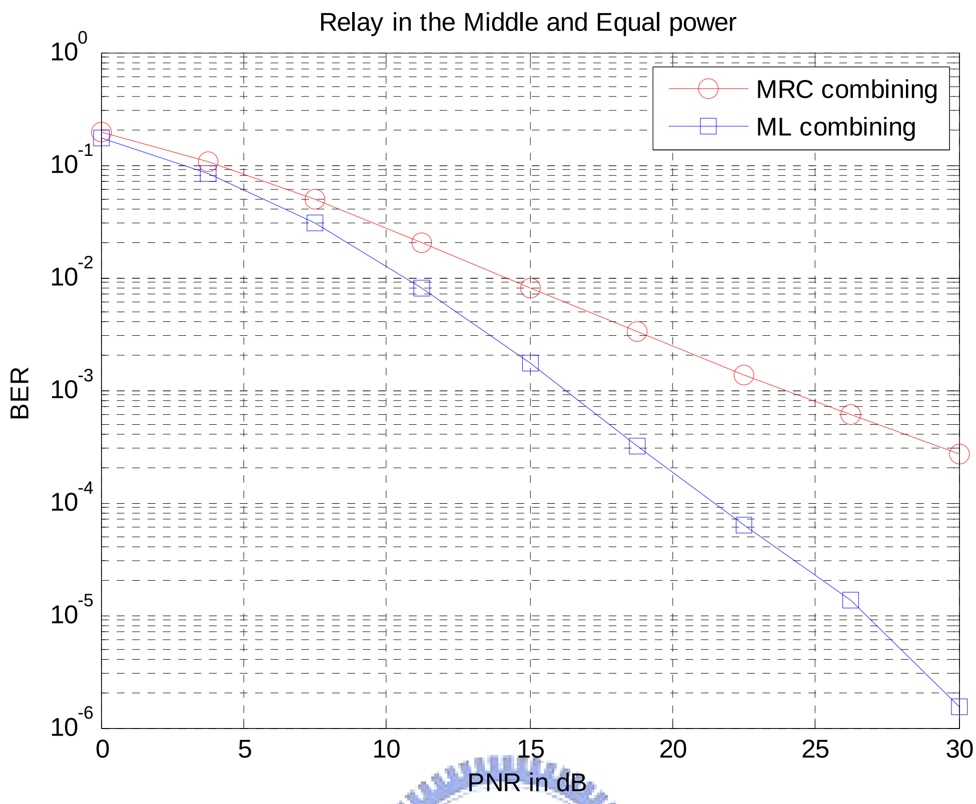

Figure 3.1 Performance comparisons of MRC and ML combining methods

Because of knowing the relay decision error probability, we know that the ML receiver uses this information to mitigate the error propagation. Therefore, from the Figure 3.1, the performance has greatly improved by using ML receiver than using MRC receiver. There are several works which improve the Destination receiver performance by different ways. We describe [5] and [6] they use in the next sections.

3.1 The Selection Relay Method

In [6], they propose a selection Relay under the Decode-and-Forward protocol at the cooperative system. The Relay uses the threshold value to evaluate the received signal reliability. If the received signal power is larger than the threshold value, then the Relay will decode the received signal and forward the decoded signal to the Destination. Otherwise, the Relay just suspends the Decode-and-Forward protocol. The Figure 3.2 shows the algorithm.

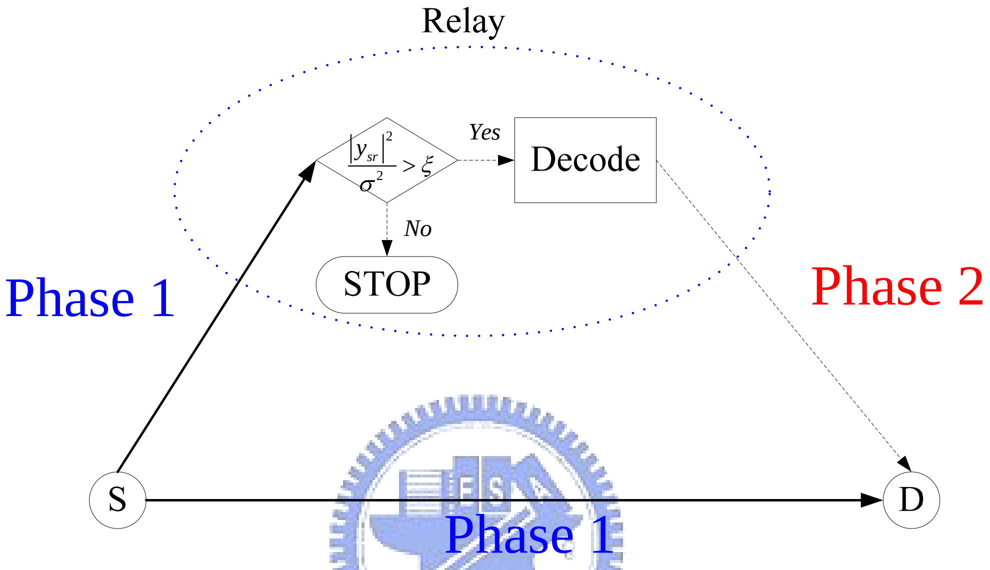

Figure 3.2 Threshold-selection relay system

Under this algorithm, the Destination node would have two types of the received signal and they could be shown as follows：

When the relay received signal power is smaller than the threshold value, the relay would stop the Decode-and-Forward protocol. With this condition, the destination received signal would be：

$$y_{sd} = \sqrt{P_1} h_{sd} x + n_{sd} \tag{3.1.1}$$

When the relay received signal power is larger than the threshold value, the relay stop to function the Decode-and-Forward protocol. Therefore, the destination received signal would be：

$$y_{sd} = \sqrt{P_1} h_{sd} x + n_{sd}$$
$$y_{rd} = \sqrt{P_2} h_{rd} \hat{x} + n_{rd} \quad (3.1.2)$$

The Destination uses the MRC combining method to estimate the symbol transmitted by the Source. Under this algorithm, the simulation results show improved performance than using the traditional cooperative system. Also, the performance is getting better and better when the threshold value increases. Figure 3.3 reflects the conclusions they made.

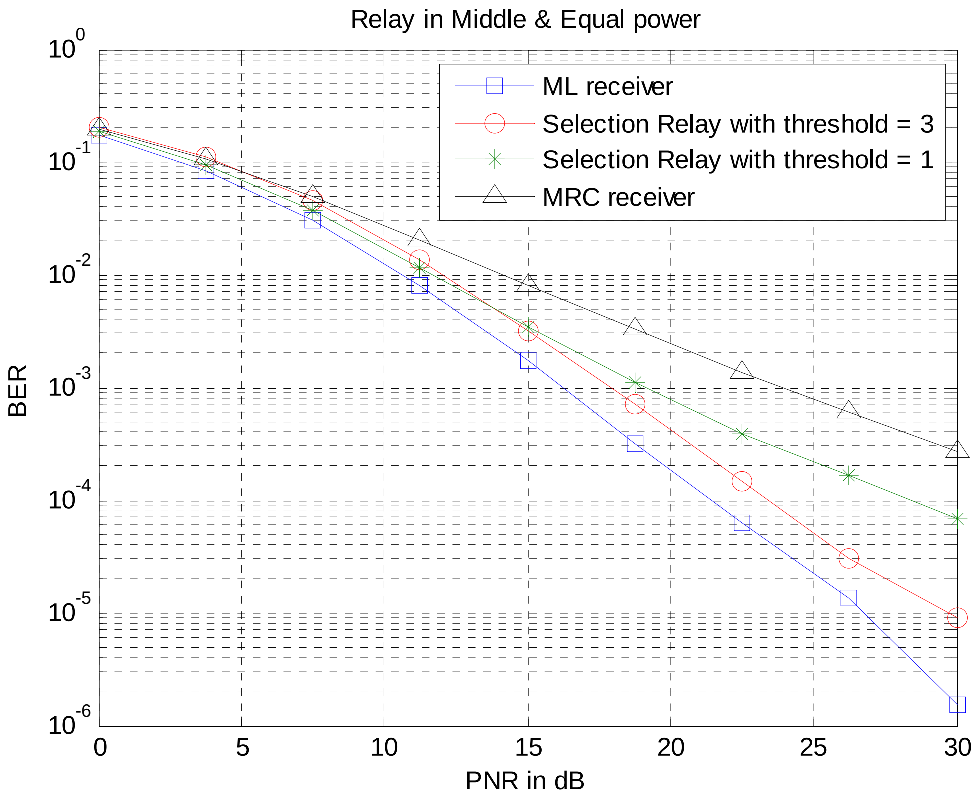

Figure 3.3 Improvement of selection relaying

The Performance analysis is provided for complex system with the BPSK signal. The reason that the small threshold value performance is worse than the large threshold value is the small threshold value will cause the Relay tends to forward the most of its received information which in turn increases the chance that incorrect symbol is sent to the Destination node. Therefore, the threshold-selection Relay improves the traditional MRC performance and is getting close to the ML performance when the threshold value increases.

But the performance will be bounded when the threshold value and power ratio is large enough. They discuss about this phenomenon and also choose a case that the location of the Relay is close to the Destination. And the simulation result is shown at the Figure 3.4 and Figure 3.5.

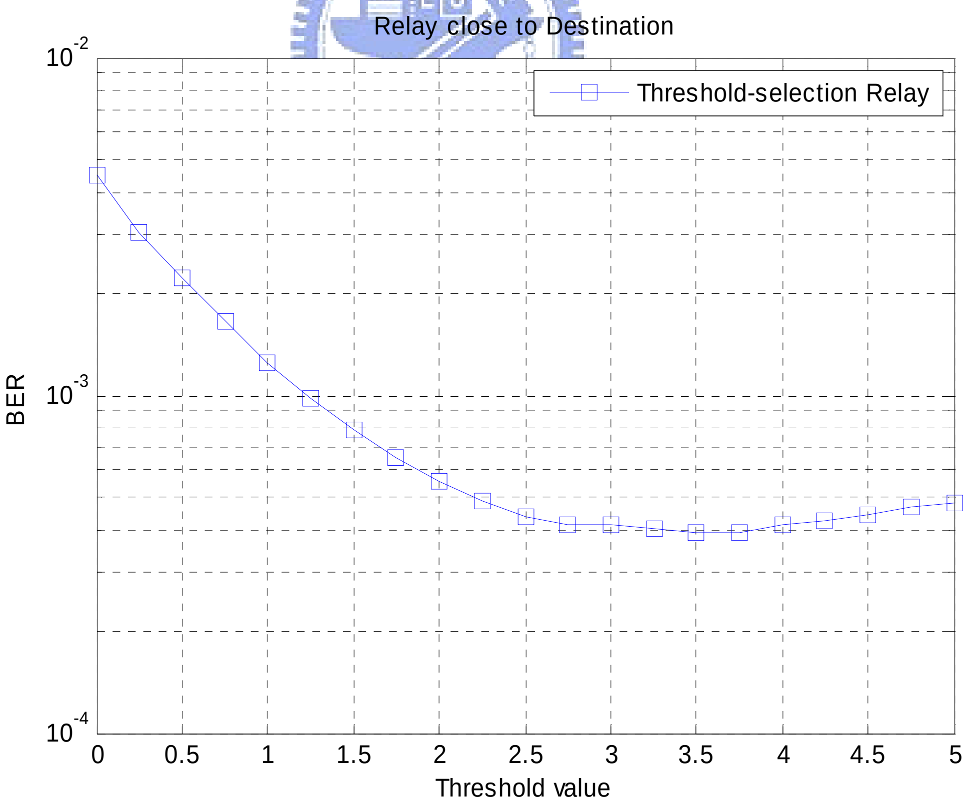

Figure 3.4 Boundary of the threshold value

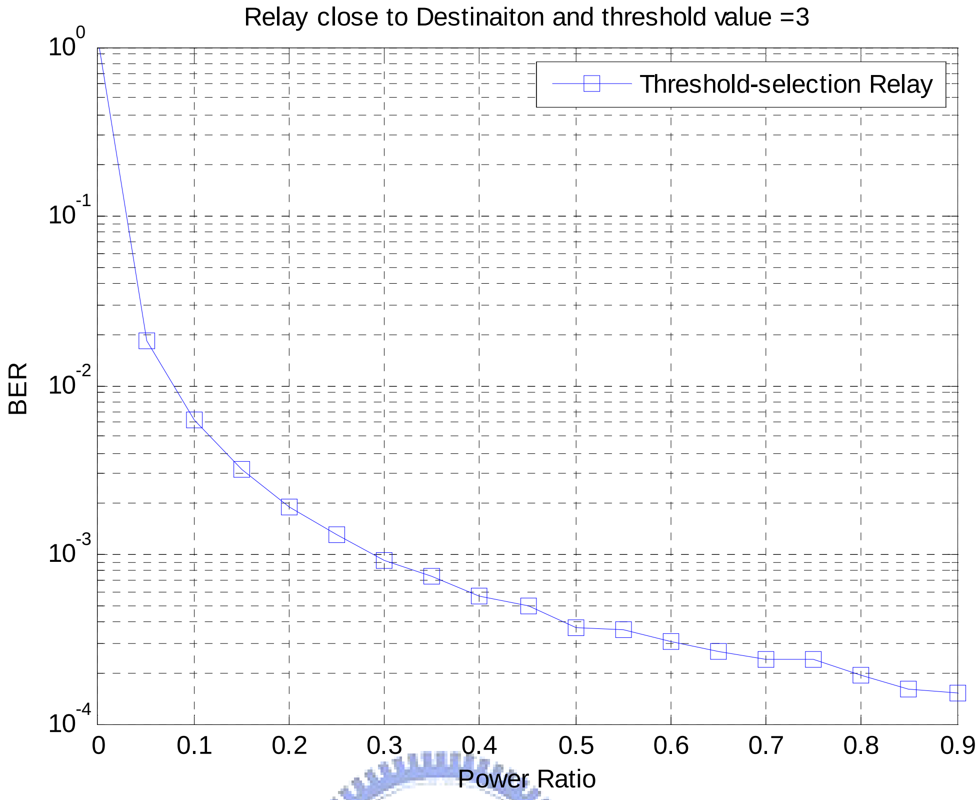

Figure 3.5 Boundary of the power ratio

As we can see, in the Figure 3.4 the performance is bounded when the threshold value is large than 3. Also in the Figure 3.5, the performance is bounded when the power ratio is large than 0.85. Therefore, when the Relay is close to Destination, they claim the optimum threshold value and power ratio are 3 and 0.85.

3.2 Maximum SNR Detection

There is another way to improve the traditional MRC performance. In [5], the Relay always executes the Decode-and-Forward protocol without any selection function. And they focus on the receiver at the Destination node based on the rule which maximizes the SNR of the slicer input. Therefore, the Destination receiver structure change into a set of MRC weighting gain and a gain function block for the received signal $y_{r,d}$. Figure 3.6 shows the structure.

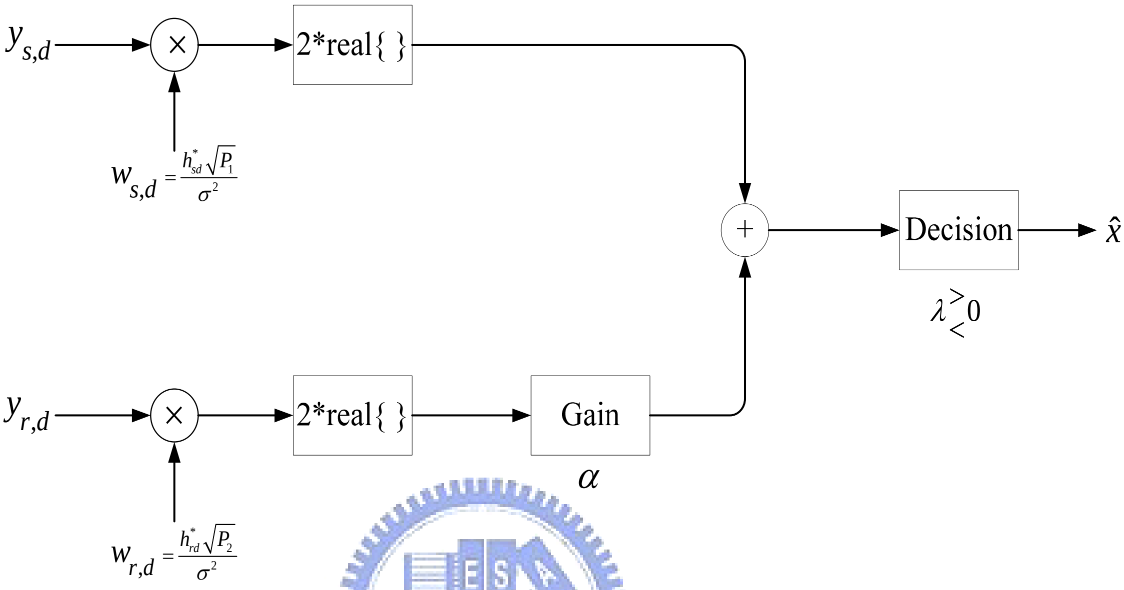

Figure 3.6 Maximum SNR detection structure

For obtaining the Gain α, they first examine the Relay decision $\hat{x}$. The Relay decision $\hat{x}$ can be written as：

$$\hat{x} = x_s + e \tag{3.2.1}$$

In Eq. (3.2.1), e is a random variable capturing the effects of decision errors. Also, if the symbol x_s is using the equal probability BPSK signal and $x_s \in \{x_1, x_{-1}\}$.

Therefore, we can yield the random variable properties by using some calculations.

$$E\left[e|x_s\right]=\begin{cases} p*\left(x_{-1}-x_1\right) & \text{if } x_s=x_1 \\ p*\left(x_1-x_{-1}\right) & \text{if } x_s=x_{-1} \end{cases} \tag{3.2.2}$$

And from the Eq. (3.2.2), the variance of the random variable is

$$\sigma_e^2=p*\left(x_1-x_{-1}\right)^2 \tag{3.2.3}$$

According to these properties, the new signal constellation of the Relay decoded symbol can be written as：

$$\begin{aligned} \hat{x}_1&=(1-p)x_1+px_{-1} \\ \hat{x}_{-1}&=(1-p)x_{-1}+px_1 \end{aligned} \tag{3.2.4}$$

According to this new constellation, the $\hat{x}$ transmitted by the Relay can be rewritten as $\hat{x}=\tilde{x}_s+\tilde{e}$, where

$$\begin{aligned} \tilde{x}_s&=x_s+E\left[e|x_s\right] \\ \tilde{e}&=e-E\left[e|x_s\right] \end{aligned}. \tag{3.2.5}$$

Based on these derivations, the $y_{r,d}$ can be written as

$$y_{r,d}=\sqrt{P_2}h_{r,d}\left(\tilde{x}_s+\tilde{e}\right)+n_{r,d} \tag{3.2.6}$$

From Eq. (3.2.6), we can get the weighting gain for this received signal, i.e.

$$w_{r,d}=\frac{h_{r,d}^*\sqrt{P_2}\left(1-2p\right)}{\left|h_{r,d}\right|^2 P_2\sigma_e^2+\sigma^2} \tag{3.2.7}$$

We can extract the traditional MRC weighting gain and the gain function from the Eq. (3.2.7). The result is shown below：

$$w_{r,d} = \frac{h_{r,d}^{*}\sqrt{P_2}}{\sigma^2}$$
$$\alpha = \frac{(1-2p)}{\gamma_{r,d}\sigma_e^2 + 1} \tag{3.2.8}$$

According to the Eq. (3.2.8) and the traditional MRC weighting gain $w_{s,d}$, the Gain structure improves the performance of the traditional MRC scheme. The simulation result is shown in Figure 3.7.

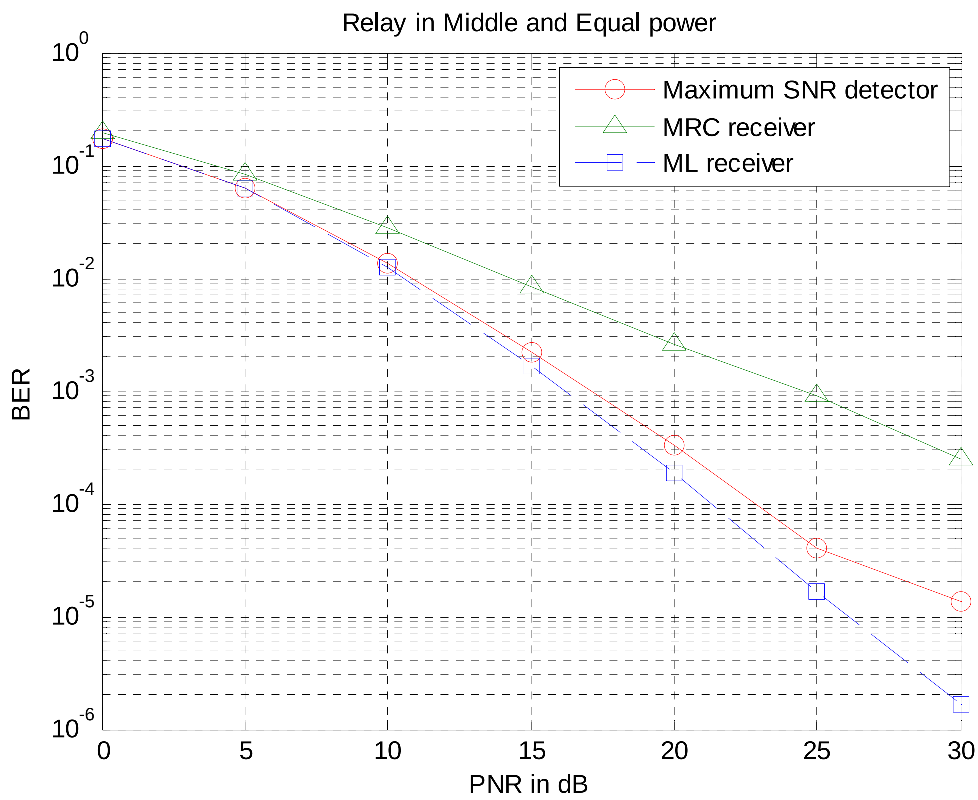

Figure 3.7 Improvement of maximum SNR detector

This method can improve the Destination receiver performance. But around the high PNR area, the new method seems to be worse than the ML method by about 3 or 4 dB.

3.3 The Proposed Architecture

According to these two methods in sections 3.1 and 3.2, they have their own advantage. The threshold-selection Relay only needs a threshold value to evaluate the received signal good enough and the maximum SNR detection method use a gain α to adjust the influence of the Relay decision errors. The disadvantage for first method is the smaller threshold value, the worse performance. Therefore, we could combine these two methods to combat the disadvantage in the first method. Figure 3.8 shows the combined scheme.

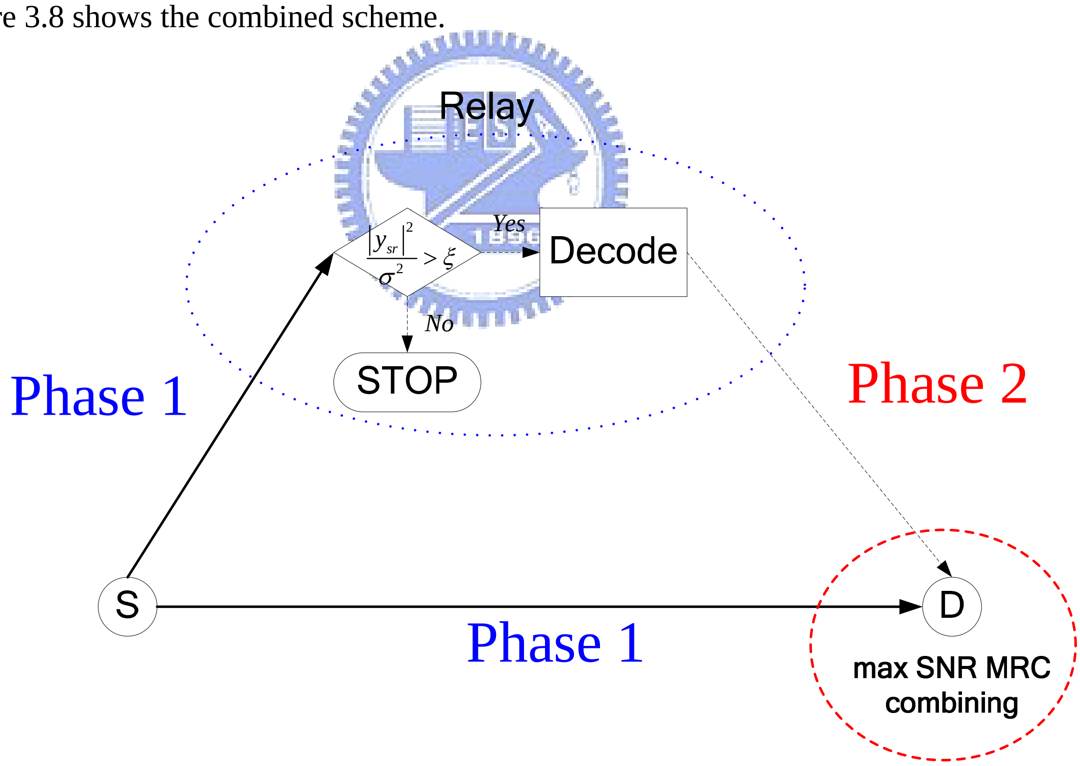

Figure 3.8 Proposed architecture

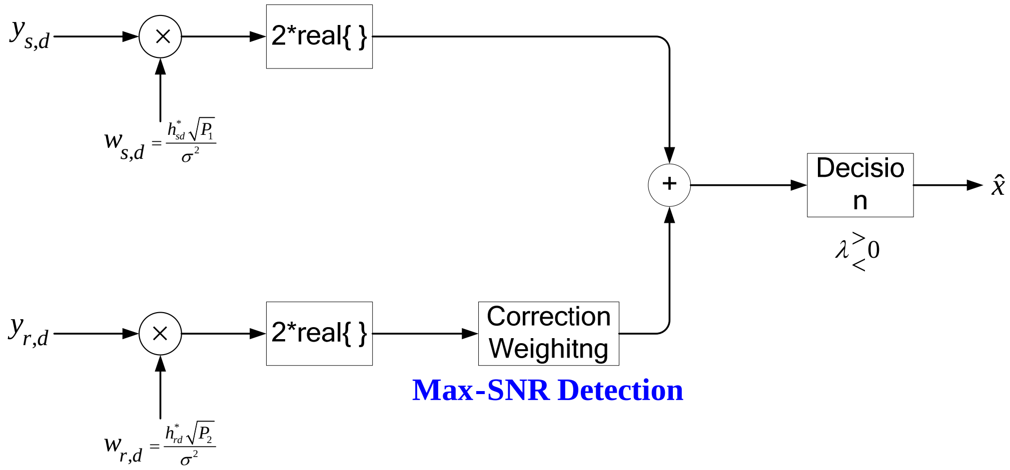

Figure 3.9 max-SNR MRC with correction weighting

From the Figure 3.8 and 3.9, we can use the advantage of second method to mitigate the small threshold phenomenon in first method. Table 3.1 shows the comparison between the proposed system and previous works.

Functionality	Ref. [5]	Ref. [6]	Combined scheme
Threshold value at the Relay	O	X	O
Gain factor at Destination receiver	X	O	O

Table 3.1 Differences between three methods

3.4 Two-Threshold Values in Selection Relay

We also offer another method to improve the selection-relay performance. The concept of this method is using the two different threshold values to measure the received signal's quality. There are three working status in the relay node. When the relay received signal power is lower than the small threshold value, $\mathrm{thr}_{\mathrm{low}}$, the relay don't work. When the relay received signal power is larger than large threshold value, $\mathrm{thr}_{\mathrm{high}}$, the relay would decode and forward the signal. Otherwise, the relay would amplify and forward the received signal. For the relay works in the amplify-and-forward protocol, we use the amplified factor in [5]. To satisfy the output power constraint, the relay amplifier can operate at a maximum gain satisfying

$$\beta = \sqrt{\frac{P_2}{\left|h_{sr}\right|^2 P_1 + \sigma^2}} \tag{3.4.1}$$

The corresponding destination weighting can be implemented as

$$\begin{aligned} w_{sd} &= \frac{h_{sd}^* \sqrt{P_1}}{\sigma^2} \\ w_{rd} &= \frac{h_{rd}^* \beta^* h_{sd}^* \sqrt{P_1}}{\left(\left|h_{rd}\right|^2 \left|\beta\right|^2 + 1\right)\sigma^2} \end{aligned} \tag{3.4.2}$$

With these equations, we could easily use the MRC combiner as the destination receiver. For the relay works in the decode-and-forward protocol, we use the same method in section 3.3.

Chapter 4

Theoretical BER of The Max-SNR Selection-Relay Architecture

In this chapter, we derive the theoretical BER of the max-SNR selection-relay scheme in detail. Also, we find the approximation for the theoretical BER to obtain the diversity order.

4.1 Theoretical BER Analysis

Before we derive the theoretical BER, we should make some assumptions first. The signal model of the cooperative system is just the same as the Chapter 2. We assume the channel gains $h_{s,d}, h_{r,d}, h_{s,r}$ are mutually independent complex Gaussian with zero-mean, variance equal to $\sigma_{s,d}^2, \sigma_{s,r}^2, \sigma_{r,d}^2$ random variables. And the noises are also mutually independent complex Gaussian with zero-mean and variances equal to σ^2. The signal we use is the BPSK symbol with $x_s \in \{x_1, x_{-1}\}$. With these assumptions, we can derive the theoretical BER.

4.1.1 Classification of The Proposed System Scenarios

From the Figure3.9, the working status of the Relay depends on the Relay received signal power which is transmitted from the Source. Therefore, there are two scenarios of the received signals at the Destination in the proposed system. The scenario 1 means the Relay received signal power lower than the threshold value. The scenario 2 is the opposite side, i.e. the Relay received signal power is larger than the threshold value. In the scenario 1 the Destination only has one signal which is transmitted from the Source. In the scenario 2, the Destination will have two signals and these two signals come from different ways. One is from the Source. The other one is from the Relay.

Hence, in the Scenario 1 the Destination receiver can be regarded as Figure 4.1.

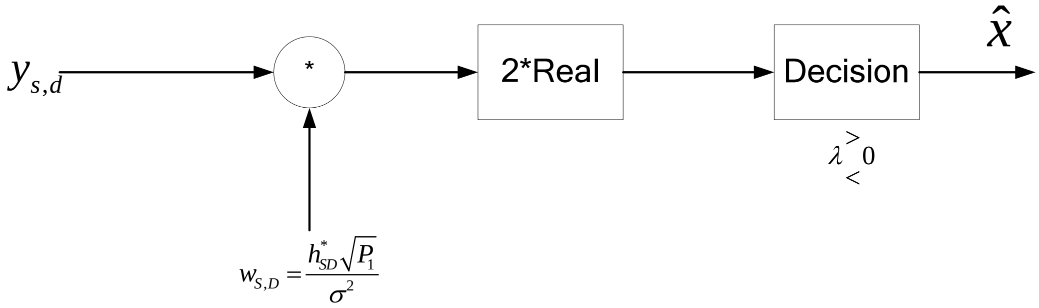

Figure 4.1 Scenario 1 destination receiver

In Scenario 2, the Destination receiver can be regarded as Figure 3.9. It is very important to know the Destination receiver type while we derive the theoretical BER. With these two receiver type, we will know the proposed system theory BER which depends on the received signal power at the Relay. And the theoretical BER would be shown as follows：

$$P_e = P(\phi_1) P_{e|\phi_1} + P(\bar{\phi}_1) P_{e|\bar{\phi}_1} \tag{4.1.1}$$

In Eq. (4.1.1), $P(\phi_1)$ denotes the occurrence when the relay received signal power is small than threshold value and $P_{e|\phi_1}$ denotes the system BER when the relay received signal power is small. $P(\bar{\phi}_1)$ and $P_{e|\bar{\phi}_1}$ mean the occurrence and the system BER of the opposite scenario.

4.1.2 Occurrence probability of two Scenarios

We use the similar derivation in [6] and [8] to obtain the theoretical BER. First, we could derive the occurrence of these two scenarios. From Eq. (2.1.1), we could know the Relay received signal model. Because $h_{s,r}$ and $n_{s,r}$ are the complex Gaussian random variables with zero-mean and different variances. The received signal $y_{s,r}$ is also the complex Gaussian random variable with zero-mean and variance equals to $P_1\sigma_{s,r}^2+\sigma^2$. Therefore, the received signal power $\left|y_{s,r}\right|^2$ is an exponential distribution random variable with $\lambda_1=\frac{1}{P_1\sigma_{s,r}^2+\sigma^2}$. With this property, we can easily obtain the occurrence probability of the low-SNR scenario 1

$$\begin{aligned} P(\phi_1) &= \text{Prob}\left(\left|y_{sr}\right|^2 \le \xi\sigma^2\right) \\ &= \int_0^{\xi\sigma^2} \lambda_1 e^{-\lambda_1 x} dx \\ &= 1-\exp\left(-\lambda_1\xi\sigma^2\right) \end{aligned} \tag{4.1.2}$$

The occurrence probability of the high-SNR scenario 2 is

$$\begin{aligned} P(\bar{\phi}_1) &= 1-P(\phi_1) \\ &= \exp\left(-\lambda_1\xi\sigma^2\right) \end{aligned} \tag{4.1.3}$$

4.1.3 Error probability of two Scenarios

First we can use the Figure 4.2 to analyze the error probability of the scenario 1.

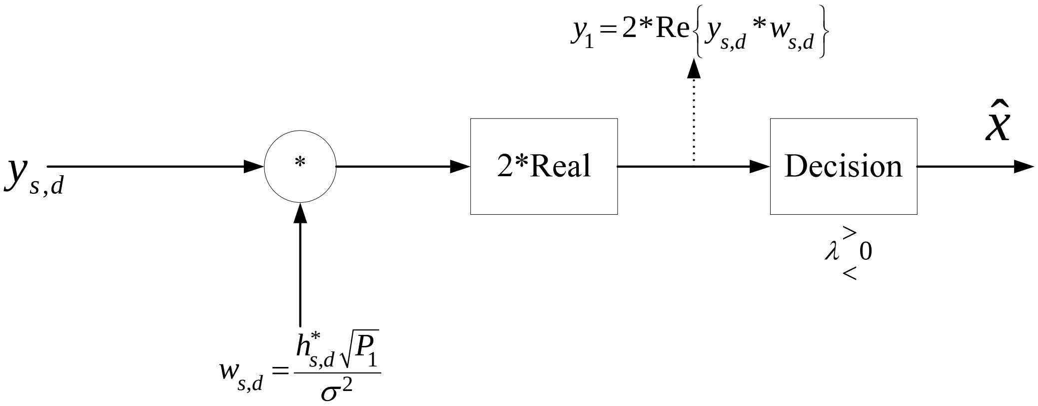

Figure 4.2 Scenario 1 destination receiver

From Eq. (2.1.2), we could know the destination received signal model. If the symbol $x=1$ and the channel gain $h_{s,d}$ is known, the properties of y_1 can be obtained easily. The detailed derivation is shown as follows：

$$\begin{aligned} y_1 &= 2*\mathrm{Re}\left\{ y_{s,d} * w_{s,d} \right\} \\ &= 2*\mathrm{Re}\left\{ \frac{P_1 \left| h_{s,d} \right|^2}{\sigma^2} + \frac{\sqrt{P_1} h_{s,d}^*}{\sigma^2} n_{s,d} \right\} \\ &= 2*\left[\frac{P_1 \left| h_{s,d} \right|^2}{\sigma^2} + \frac{\sqrt{P_1}}{\sigma^2} \mathrm{Re}\left\{ h_{s,d}^* n_{s,d} \right\} \right] \end{aligned} \quad (4.1.4)$$

Because of $\mathrm{Re}\left\{ h_{s,d}^* n_{s,d} \right\}$ is a real Gaussian random variable with zero mean and variance equal to $\frac{\left| h_{s,d} \right|^2 \sigma^2}{2}$, the y_1 is also a real Gaussian random variable with the mean and the variance equal to Eq. (4.1.5) and Eq. (4.1.6).

$$\mu_1 = E\left[y \right] = 2 \frac{P_1 \left| h_{s,d} \right|^2}{\sigma^2} \quad (4.1.5)$$

$$\sigma_1^2 = \text{var}\left[y\right] = 2\frac{P_1\left|h_{s,d}\right|^2}{\sigma^2} \tag{4.1.6}$$

According to these two statistical properties, we can know the probability density function of the y_1.

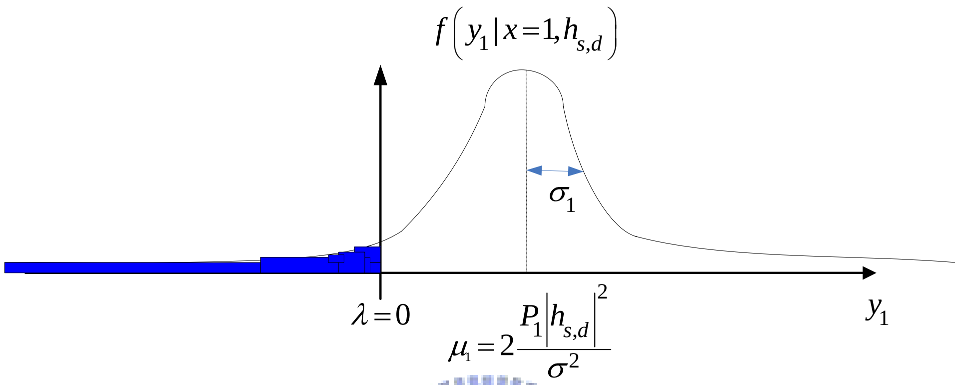

Figure 4.3 Illustration of y_1 's PDF

From Figure 4.3, we can realize the error probability of y is the painted area if the symbol x equals to 1. Hence, the conditional error probability is shown below.

$$\begin{aligned} P_{e|\phi_1}^{h_{sd}} &= \int_{-\infty}^{0} f\left(y_1 | x=1, h_{s,d}\right) dy \\ &= \frac{1}{\sqrt{2\pi}} \int_{\frac{\mu_1}{\sigma_1}}^{\infty} \exp\left(-\frac{t^2}{2}\right) dt = Q\left(\frac{\mu_1}{\sigma_1}\right) \end{aligned} \tag{4.1.7}$$

From Eq. (4.1.7) and [12] , we know $P_{e|\phi_1}$ has been approved after averaging the channel effect：

$$P_{e|\phi_1} = E_{h_{sd}}[Q(\sqrt{\frac{2P_1|h_{sd}|^2}{\sigma^2}})]$$
$$= \frac{1}{2}(1 - \sqrt{\frac{P_1\sigma_{sd}^2 / \sigma^2}{1 + P_1\sigma_{sd}^2 / \sigma^2}}) \qquad (4.1.8)$$

Before we start to derive the error probability of the scenario 2, we should analyze the component of $P_{e|\bar{\phi}_1}$ in Figure 4.4.

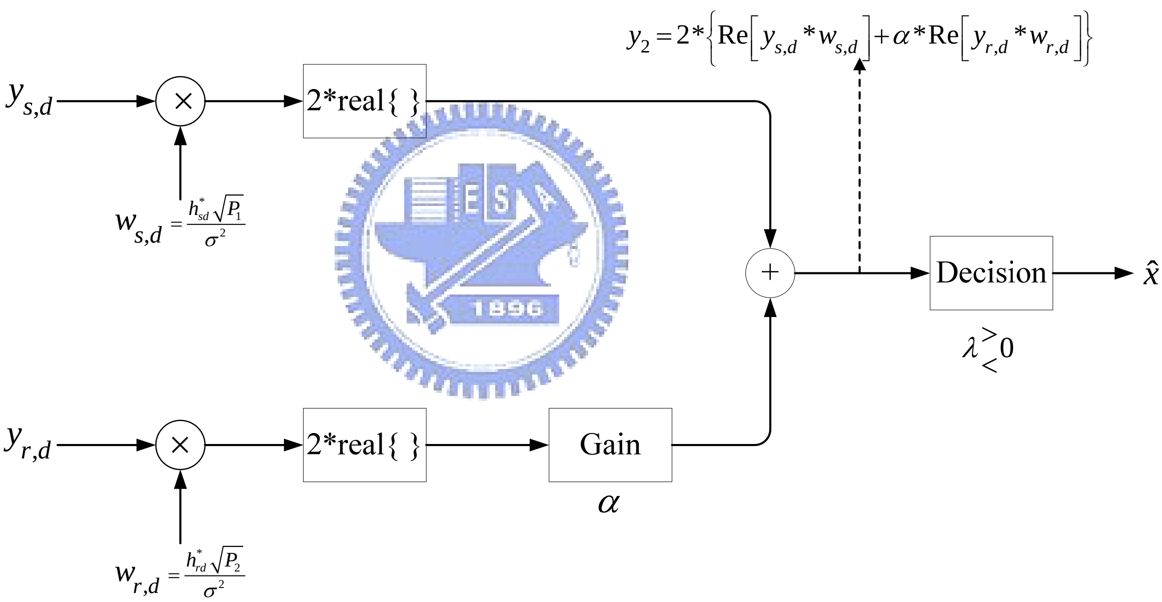

Figure 4.4 Scenario 2 destination receiver scheme

The received signals are rewritten from Eq. (2.1.2) and (2.1.3) here：

$$y_{s,d} = \sqrt{P_1}h_{s,d}x + n_{s,d} \qquad (4.1.9)$$

$$y_{r,d} = \sqrt{P_2}h_{r,d}\hat{x} + n_{r,d}. \qquad (4.1.10)$$

From Eq. (4.1.10), the $\hat{x}$ may not be the same as the original Source symbol due to the decision error at the Relay. Therefore, $\hat{x}$ will decode into this form：

$$\hat{x} = \begin{cases} x & \text{with probability } (1-p) \\ -x & \text{with probability } p \end{cases} \quad (4.1.11)$$

where p denotes the relay BER. Based on the Eq. (4.1.11), we realize that we need two forms to analyze the y_2. That is, y_2 is rewritten as：

$$y_{2,\text{correct}} = 2*\left[\frac{P_1|h_{s,d}|^2}{\sigma^2} + \alpha\frac{P_2|h_{r,d}|^2}{\sigma^2}\right]x + 2*\left[\frac{\sqrt{P_1}}{\sigma^2}\text{Re}\left[h_{s,d}^* n_{s,d}\right] + \alpha\frac{\sqrt{P_2}}{\sigma^2}\text{Re}\left[h_{r,d}^* n_{r,d}\right]\right] \quad (4.1.12)$$

$$y_{2,\text{wrong}} = 2*\left[\frac{P_1|h_{s,d}|^2}{\sigma^2} - \alpha\frac{P_2|h_{r,d}|^2}{\sigma^2}\right]x + 2*\left[\frac{\sqrt{P_1}}{\sigma^2}\text{Re}\left[h_{s,d}^* n_{s,d}\right] + \alpha\frac{\sqrt{P_2}}{\sigma^2}\text{Re}\left[h_{r,d}^* n_{r,d}\right]\right] \quad (4.1.13)$$

Eq. (4.1.12) will accompany with a probability $(1-p)$ which the Relay make a correct decision, and the Eq. (4.1.13) will accompany with a probability p which the Relay make a wrong decision. Besides, the gain factor α in [5] is defined as

$$\alpha = \frac{(1-2p)}{1+\gamma_{r,d}\sigma_e^2} \quad (4.1.14)$$

With these three equations in (4.1.11), (4.1.12) and (4.1.13) we know the error probability of the scenario 2 will depend on the error probability of the Relay. Hence, the error probability of the scenario 2 could be shown as follows：

$$P_{e|\bar{\phi}_1} = (1-p)P_{b1} + pP_{b2} \quad (4.1.15)$$

From Eq. (4.1.15), we should know the error probability of the Relay first before we derive the error probability of the scenario 2.

To derive the error probability p of the Relay, we can use Figure 4.5 to analyze.

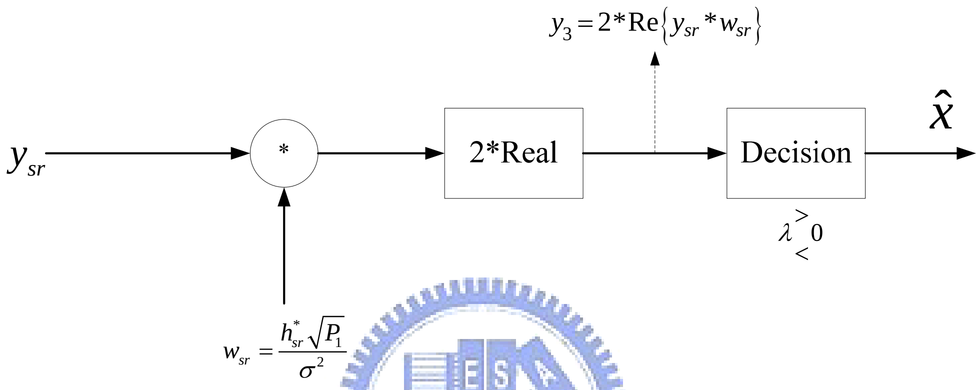

Figure 4.5 Relay receiver scheme

First, we could start to analyze $y_{s,r}$ from Eq. (2.1.1). Given the channel gain $h_{s,r}$ is known and the symbol x equals to 1, y_3 is shown as follows

$$y_3 = 2\mathrm{Re}\left[w_{s,r} y_{s,r}\right]$$
$$= 2\left(\frac{P_1 \left|h_{s,r}\right|^2}{\sigma^2} + \frac{\sqrt{P_1}}{\sigma^2} \mathrm{Re}[h_{s,r}^* n_{s,r}] \right) \qquad (4.1.16)$$

From Eq. (4.1.22), we know $\mathrm{Re}\left[h_{s,r}^* n_{s,r}\right]$ is a real Gaussian random variable with zero mean and variance equals to $\frac{\left|h_{s,r}\right|^2 \sigma^2}{2}$. Therefore, we can know the statistical properties of the y_3 as follows

$$\mu_3 = E\left[y_3\right] = \frac{2P_1\left|h_{s,r}\right|^2}{\sigma^2} \tag{4.1.17}$$

$$\sigma_3^2 = \text{var}\left[y_3\right] = \frac{2P_1\left|h_{s,r}\right|^2}{\sigma^2} \tag{4.1.18}$$

Based on these properties, the probability density function of y_3 is

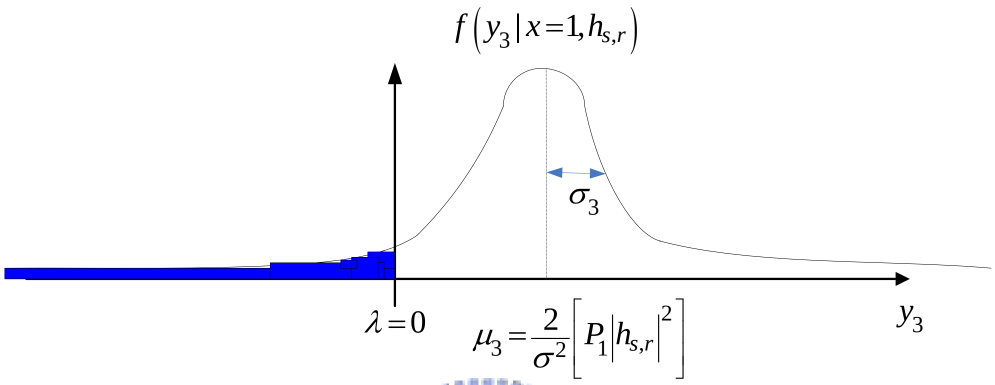

Figure 4.6 Illustration of y_3's PDF

The conditional error probability can be derived by calculating the painted area.

That is,

$$\begin{aligned} p^{h_{s,r}} &= \int_{-\infty}^{0} f\left(y_3 \mid x=1, h_{s,r}\right) dy \\ &= Q\left(\frac{\mu_2}{\sigma_2}\right) \end{aligned} \tag{4.1.19}$$

Then we average the channel gain effect to obtain the error probability of the Relay .

$$p = E_{h_{sr}}\left[Q\left(\sqrt{\frac{P_1\left|h_{sr}\right|^2}{\sigma^2}}\right)\right] \tag{4.1.20}$$

When we average the channel gain effect, we should also notice that the received signal power should be larger than the threshold value. Therefore, the lower limit of the integration in the Eq. (4.1.20) should meet this criterion. From the statistical viewpoint, the received power could be calculated as

$$\left|y_{s,r}\right|^2 = P_1\left|h_{s,r}\right|^2 + \sigma^2 \tag{4.1.21}$$

Because of the criterion is $\left|y_{s,r}\right|^2 \geq \xi\sigma^2$, the $\gamma_{s,r}$ need to meet this rule

$$\gamma_{s,r} \geq \left(\xi - 1\right) \tag{4.1.22}$$

With Eq. (4.1.22), the lower limit of the integral in Eq. (4.1.20) would be

$$lower_{\text{limit}} = \begin{cases} 0 & \text{if } \xi \leq 1 \\ \xi - 1 & \text{if } \xi > 1 \end{cases} \tag{4.1.23}$$

Therefore, the error probability of the Relay is shown as follows：

when $\xi \leq 1$,

$$p_{\xi\leq 1} = \frac{1}{2}\left(1 - \sqrt{\frac{P_1\sigma_{s,r}^2 / \sigma^2}{1 + P_1\sigma_{s,r}^2 / \sigma^2}}\right) \tag{4.1.24}$$

when $\xi > 1$,

$$p_{\xi>1} = \frac{1}{2}\left[\frac{erf\left(\sqrt{a(b+1)}\right)}{\sqrt{b+1}} + e^{-ba}erfc\left(\sqrt{a}\right) - \frac{1}{\sqrt{b+1}}\right]$$

$$\text{where } a = \xi - 1 \text{ , } b = \frac{1}{\bar{\gamma}_{sr}} = \frac{\sigma^2}{P_1\sigma_{sr}^2}$$

(4.1.25)

After we obtain the error probability of the Relay, we derive the error probability P_{b1} from Eq. (4.1.12). Given the channel gains $h_{s,d}$ and $h_{r,d}$ are known and the symbol x equals to 1, the Eq. (4.1.12) would be

$$\begin{aligned} y_2 &= 2*\left[\frac{P_1\left|h_{s,d}\right|^2}{\sigma^2}+\alpha\frac{P_2\left|h_{r,d}\right|^2}{\sigma^2}\right] \\ &+2*\left[\frac{\sqrt{P_1}}{\sigma^2}\mathrm{Re}\left[h_{s,d}^*n_{s,d}\right]+\alpha\frac{\sqrt{P_2}}{\sigma^2}\mathrm{Re}\left[h_{r,d}^*n_{r,d}\right]\right] \end{aligned} \tag{4.1.26}$$

From Eq. (4.1.26), we know $\mathrm{Re}\left[h_{s,d}^*n_{s,d}\right]$ and $\mathrm{Re}\left[h_{r,d}^*n_{r,d}\right]$ are the real Gaussian random variables with zero mean and variances equal to $\frac{\left|h_{s,d}\right|^2\sigma^2}{2}$ and $\frac{\left|h_{r,d}\right|^2\sigma^2}{2}$ individually.

Therefore, y_2 's statistical parameters are

$$\begin{aligned} \mu_{2,correct} &= E\left(y_{2,correct}\right) \\ &= \frac{2}{\sigma^2}\left[P_1\left|h_{s,d}\right|^2+\alpha P_2\left|h_{r,d}\right|^2\right] \end{aligned} \tag{4.1.27}$$

$$\sigma_{2,correct}^2 = \frac{2}{\sigma^2}\left[P_1\left|h_{s,d}\right|^2+\alpha^2 P_2\left|h_{s,d}\right|^2\right] \tag{4.1.28}$$

Based on these properties, we can know the probability density function and obtain the error probability P_{b1}.

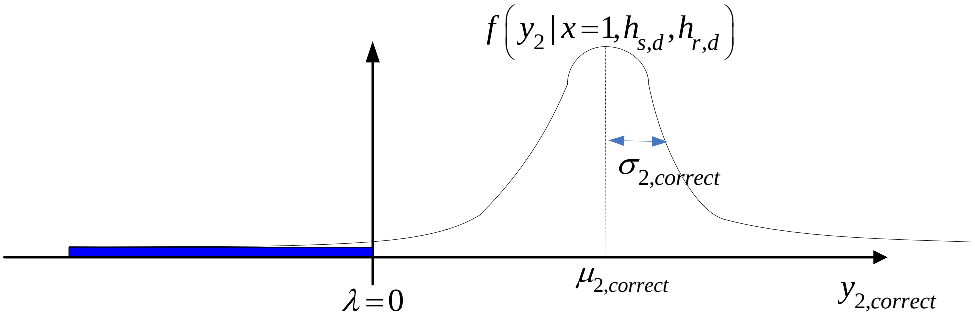

Figure 4.7 Illustration of $y_{2,correct}$'s PDF

We can obtain the conditional error probability by calculating the painted area.

$$P_{b1}^{h_{s,d},h_{r,d}} = \int_{-\infty}^{0} f\left(y_2 | x=1, h_{s,d}, h_{r,d}\right) dy$$
$$= Q\left(\frac{\mu_{2,correct}}{\sigma_{2,correct}}\right) \tag{4.1.29}$$
$$= Q\left(\frac{\sqrt{2}\left(\gamma_{s,d} + \alpha\gamma_{r,d}\right)}{\sqrt{\gamma_{s,d} + \alpha^2\gamma_{r,d}}}\right)$$

Therefore, we can obtain P_{b1} by averaging the channel gain effects. That is,

$$P_{b1} = E\left[P_{b1}^{h_{s,d},h_{r,d}}\right] \tag{4.1.30}$$

From Eq. (4.1.30), we use the MatLab to calculate the double integration of the expectation.

We can use the same method to derive the error probability P_{b2}. From Eq. (4.1.18), it is obviously known that the difference between Eq. (4.1.12) & (4.1.13) is the amplitude of the symbol. Hence, from Eq. (4.1.13) the mean of y_2 's statistical properties changes into

$$\mu_{2,wrong} = E\left[y_{2,wrong} \right]$$
$$= \frac{2}{\sigma^2}\left[P_1 \left| h_{s,d} \right|^2 - \alpha P_2 \left| h_{r,d} \right|^2 \right] \quad (4.1.31)$$

$$\sigma^2_{2,wrong} = \frac{2}{\sigma^2}\left[P_1 \left| h_{s,d} \right|^2 + \alpha^2 P_2 \left| h_{r,d} \right|^2 \right] \quad (4.1.32)$$

From Eq, (4.1.31) and (4.1.32), we would know the variance is the same as $\sigma^2_{2,correct}$ but the mean changes. With these two results, the probability density function can be shown as follows

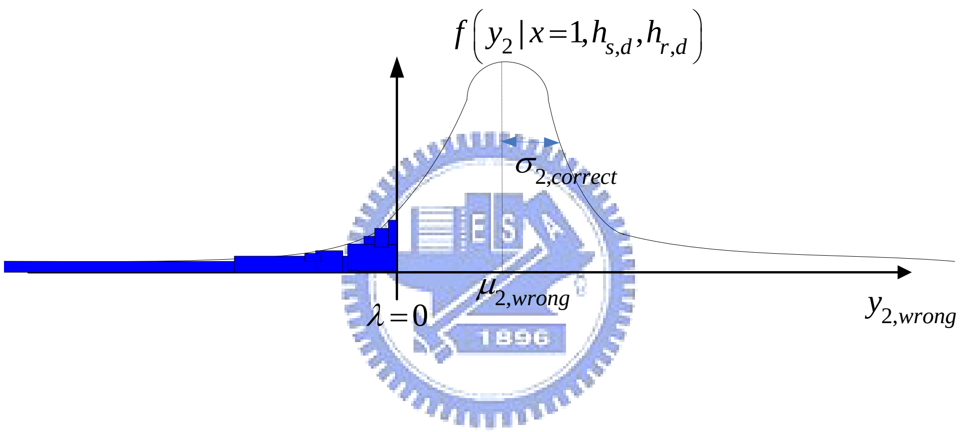

Figure 4.8 Illustration of $y_{2,wrong}$'s PDF

We can derive the error probability by these properties and Eq. (4.1.29). We can obtain

$$P_{b2} = E\left[P_{b2}^{h_{s,d},h_{r,d}} \right] \quad (4.1.33)$$

Same as Eq. (4.1.30), we use the MatLab to calculate the expectation in Eq, (4.1.33).

With equations (4.1.2), (4.1.3), (4.1.8), (4.1.24), (4.1.25), (4.1.30), (4.1.33), equation (4.1.1) would be shown as

$$P_e = \left(1 - e^{-\frac{\xi\sigma^2}{P_1\sigma_{sr}^2+\sigma^2}}\right)\left(\frac{1}{2}\left(1-\sqrt{\frac{\bar{\gamma}_{sd}}{1+\bar{\gamma}_{sd}}}\right)\right)$$

$$+e^{-\frac{\xi\sigma^2}{P_1\sigma_{sr}^2+\sigma^2}}\left((1-p)E\left[Q\left(\frac{\sqrt{2}\left(\gamma_{sd}+\alpha\gamma_{rd}\right)}{\sqrt{\gamma_{sd}+\alpha^2\gamma_{rd}}}\right)\right]+pE\left[Q\left(\frac{\sqrt{2}\left(\gamma_{sd}-\alpha\gamma_{rd}\right)}{\sqrt{\gamma_{sd}+\alpha^2\gamma_{rd}}}\right)\right]\right)$$

$$\text{where } p = \begin{cases} \frac{1}{2}\left(1-\sqrt{\frac{\bar{\gamma}_{sr}}{1+\bar{\gamma}_{sr}}}\right) & \xi \le 1 \\ \frac{1}{2}\left[\frac{erf\,(\sqrt{a(b+1)})}{\sqrt{b+1}} + e^{-ba} erfc\left(\sqrt{a}\right) - \frac{1}{\sqrt{b+1}}\right] & \xi > 1 \end{cases}$$

$$\text{where } \ a=\xi-1 \ b=\frac{1}{\bar{\gamma}_{sr}}$$

From the above theoretical error probability, we could know when α equals to 1 the theoretical error probability would meet the traditional MRC result in [6].

4.2 High SNR Approximation Analysis

In this section, we try to approximate the theoretical BER. Therefore, we could get the diversity order from the simplified theoretical BER.

4.2.1 The Approximated Theoretical BER

We use the high SNR condition to obtain the diversity order of proposed scheme. Before we start to derive, we should know the meaning of the diversity order. In [8], the definition of the diversity order is the negative exponent of the average BER plotted in a log-log scale when the average SNR tends to infinity, i.e.

$$P_e \propto (\bar{\gamma}_s)^{-G_d} \quad where\ \bar{\gamma}_s \to \infty \tag{4.2.1}$$

With this definition, we parameterize on the three SNRs first, i.e.

$$\bar{\gamma}_{sd} = c_1\bar{\gamma}_s,\ \ \bar{\gamma}_{rd} = c_2\bar{\gamma}_s,\ \ \bar{\gamma}_{sr} = c_3\bar{\gamma}_s$$

where $\bar{\gamma}_s$ denotes the average SNR. We also assume the Relay is close to the Source to achieve the high SNR status. With this assumption, we would know the relay BER p is almost zero and the correction weighting α in Eq. (4.1.14) is close to 1. With the above assumption, we could approximate equation (4.1.1) namely the theoretical BER of the proposed system. That is,

$$P_e \approx P(\phi_1)P_{e|\phi_1} + P_{e|\bar{\phi}_1} \tag{4.2.2}$$

where $P_{e|\bar{\phi}_1} \approx P_{b1} + pP_{b2}$.

Therefore, the occurrence probability of the scenario 1 could be simplified by using Taylor Expansion. That is,

$$\exp(x) = \sum_{n=0}^{\infty} \frac{x^n}{n!} \tag{4.2.3}$$

With this property, Eq. (4.1.3) could be shown as follows：

$$\begin{aligned} \because\ & \exp\left(-\frac{\xi}{c_3\bar{\gamma}_s+1}\right) \\ & = 1 - \frac{\xi}{c_3\bar{\gamma}_s+1} + \left(\frac{\xi}{c_3\bar{\gamma}_s+1}\right)^2 - \left(\frac{\xi}{c_3\bar{\gamma}_s+1}\right)^3 + \cdots \\ & \approx 1 - \frac{\xi}{c_3\bar{\gamma}_s+1} \quad \text{when } \bar{\gamma}_s \gg 1 \end{aligned}$$

$$P\left(\phi_1\right) \approx \frac{\xi}{\left(1+c_3\bar{\gamma}_s\right)}. \tag{4.2.4}$$

With Eq. (4.2.4), we could know the occurrence probability of the scenario 2. That is,

$$\begin{aligned} P\left(\bar{\phi}_1\right) &= 1 - P\left(\phi_1\right) \\ &\approx 1 - \frac{\xi}{c_3\bar{\gamma}_s + 1} \end{aligned}. \tag{4.2.5}$$

According to the Eq. (4.1.12), the error probability of the scenario 1 can be simplified by using the similar method. That is,

$$\because \quad \sqrt{\frac{c_1\bar{\gamma}_s}{1+c_1\bar{\gamma}_s}} = 1 - \frac{1}{2c_1\bar{\gamma}_s} + O\left(\frac{1}{\left(c_1\bar{\gamma}_s\right)^2}\right)$$

$$\begin{aligned} P_{e|\phi_1} &= \frac{1}{2}\left(1 - \sqrt{\frac{P_1\sigma_{s,d}^2 / \sigma^2}{P_1\sigma_{s,d}^2 / \sigma^2}}\right) \\ &\approx \frac{1}{4c_1\bar{\gamma}_s} \quad \text{when } \bar{\gamma}_s \gg 1 \end{aligned}. \tag{4.2.6}$$

Now we can approximate the error probability of the scenario 2 from the Eq. (4.2.1), i.e. $P_{e|\bar{\phi}_1} \approx P_{b1} + pP_{b2}$. With the assumption of the negligible relay BER $p \to 0$ and the correction weighting $\alpha \approx 1$, we could simplify the conditional

BER P_{b1} and P_{b2}. The reason why we make this assumption is we hope the conditional error probability $P_{b1}^{h_{s,d},h_{r,d}}$ and $P_{b2}^{h_{s,d},h_{r,d}}$ which have a simplified form like $Q(\sqrt{x})$, where x is an integer. Based on these assumptions, we can start to simplify P_{b1} and P_{b2}.

$P_{b1}^{h_{s,d},h_{r,d}}$ is shown as follows,

$$P_{b1}^{h_{s,d},h_{r,d}} = Q\left(\sqrt{2\left(\gamma_{s,d}+\gamma_{r,d}\right)}\right)$$
$$= \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \exp\left(-\frac{\gamma_{s,d}+\gamma_{r,d}}{\sin^2\theta}\right) d\theta$$

Therefore, P_{b1} can be obtained by averaging the channel gains.

$$P_{b1} = E\left[P_{b1}^{h_{s,d},h_{r,d}}\right]$$
$$= \int_0^\infty \int_0^\infty \lambda_1\lambda_2 P_{b1}^{x,y} e^{\lambda_1 x} e^{\lambda_2 y} dxdy \quad \text{with } t = \frac{-1}{\sin^2\theta}$$
$$= \frac{1}{\pi}\int_0^{\frac{\pi}{2}}\int_0^\infty\int_0^\infty \lambda_1\lambda_2 e^{t(x+y)} e^{\lambda_1 x} dx e^{\lambda_2 y} dy d\theta$$
$$\text{where } \lambda_1 = (c_1\bar{\gamma}_s)^{-1},\ \lambda_2 = (c_2\bar{\gamma}_s)^{-1}$$

Because of

$$\int_0^\infty \lambda_1 e^{tx} e^{-\lambda_1 x} dx = \frac{\lambda_1}{\lambda_1 - t} = \frac{1}{1-t/\lambda_1}$$

Therefore, P_{b1} would be

$$P_{b1} = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \left(1+\frac{\bar{\gamma}_{s,d}}{\sin^2\theta}\right)^{-1} \left(1+\frac{\bar{\gamma}_{r,d}}{\sin^2\theta}\right)^{-1} d\theta \quad \text{if } \bar{\gamma}_{s,d} \neq \bar{\gamma}_{r,d}$$

$$P_{b1} = \frac{1}{2(a_1 - a_2)}\left[-\frac{a_1^{3/2}}{\sqrt{a_1+1}} + (a_1 - a_2) + \frac{a_2^{3/2}}{\sqrt{a_2+1}} \right]$$

$$\text{where} \quad a_1 = c_1\bar{\gamma}_s, a_2 = c_2\bar{\gamma}_s \tag{4.2.7}$$

We can simplfy P_{b2} as follows

$$P_{b2}^{h_{s,d},h_{r,d}} = Q\left(\sqrt{\frac{2(\gamma_{s,d} - \gamma_{r,d})^2}{(\gamma_{s,d} + \gamma_{r,d})}}\right)$$

$$P_{b2} = E\left[Q\left(\sqrt{\frac{2(\gamma_{s,d} - \gamma_{r,d})^2}{(\gamma_{s,d} + \gamma_{r,d})}}\right)\right] \tag{4.2.8}$$

The error probability of the Relay could be simplified as follows. From the results of Eq. (4.1.24) and (4.1.25), we know the relay BER p has two types. Therefore, we could use the similar method to simplified the Eq. (4.1.24) and it can be shown：

when $\xi \le 1$ and $\bar{\gamma}_s \gg 1$,

$$p \approx \frac{1}{4c_3\bar{\gamma}_s} \tag{4.2.9}$$

From Eq. (4.1.25), we could simplify the equation by using Taylor expansion and the high SNR condition. That is,

$$\frac{1}{\sqrt{b+1}} \approx 1-\frac{b}{2},\ e^{-ba} \approx 1-ba$$

$$\text{and}\ erfc(x) \approx \frac{e^{-x^2}}{\sqrt{\pi}}\left(\frac{1}{x}-\frac{1}{2x^{3/2}}\right)$$

we could obtain the simplified form of the Eq. (4.1.25)：

when $\xi > 1$ and $\bar{\gamma}_s \gg 1$,

$$p \approx 0.5\left\{(1-ba)erfc(\sqrt{a})-\left(1-\frac{b}{2}\right)\left[\frac{e^{-a(b+1)}}{\sqrt{\pi}}\left(\frac{1}{\sqrt{a}}-\frac{1}{2a^{3/2}}\right)\right]\right\}$$

$$\text{where}\ a=\xi-1, b=\left(c_3\bar{\gamma}_s\right)^{-1}$$

(4.2.10)

With these approximated equations (4.2.4), (4.2.5), (4.2.6), (4.2.7), (4.2.8), (4.2.9), (4.2.10), we can substitute into equation (4.2.2) to obtain the final form of the approximated theoretical BER.

$$P_e \approx \frac{\xi}{\left(1+c_3\bar{\gamma}_s\right)}\frac{1}{4c_3\bar{\gamma}_s}+$$

$$\left(\left(\frac{1}{2(a_1-a_2)}\left[-\frac{a_1^{3/2}}{\sqrt{a_1+1}}+(a_1-a_2)+\frac{a_2^{3/2}}{\sqrt{a_2+1}}\right]\right)+pE\left[Q\left(\frac{(\gamma_{sd}-\gamma_{rd})}{\sqrt{\gamma_{sd}+\gamma_{rd}}}\right)\right]\right)$$

$$\text{where}\ a_1=c_1\bar{\gamma}_s, a_2=c_2\bar{\gamma}_s$$

(4.2.11)

$$\text{where } p \approx \begin{cases} \dfrac{1}{4c_3\bar{\gamma}_s} & ,\xi \le 1 \\ 0.5\left\{(1-ba)\,erfc\left(\sqrt{a}\right)-\left(1-\dfrac{b}{2}\right)\left[\dfrac{e^{-a(b+1)}}{\sqrt{\pi}}\left(\dfrac{1}{\sqrt{a}}-\dfrac{1}{2a^{3/2}}\right)\right]\right\} & ,\xi > 1, \end{cases}$$

$$\text{where } a=\xi-1, b=\left(c_3\bar{\gamma}_s\right)^{-1}$$

From Eq. (4.2.11) and Eq. (4.2.1), we could know the diversity order of first term is 2 but we can't know the exactly diversity order in the second term. Therefore we need make an approximation again to obtain the diversity order of the second term.

4.2.2 Derivation of Proposed System Diversity Order

For the purpose of knowing the diversity order of the proposed system, we need to approximate the theoretical BER in Eq. (4.2.2) much further. We use the similar method in [8] to approximate the second term of the simplified theoretical BER.

For P_{b1} , we could use the Chernoff bound to obtain another form of the conditional BER i.e.

$$P_{b1}^h = Q\left(\sqrt{2\left(\gamma_{sd}+\gamma_{rd}\right)}\right) = \int_a^\infty \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt$$
$$= \int_{-\infty}^{\infty} f_X(x) u(x-a) dx$$

where $f_X(x)$ denotes the Gaussian probability density function, $u(x)$ means the step function and a denotes the destination received SNR, $\sqrt{2(\gamma_{sd}+\gamma_{rd})}$. With this equation, we could obtain the upper bound on P_{b1}, i.e.

$$P_{b1}^{h} \le \int_{-\infty}^{\infty} f_X(x) e^{t(x-a)} dx = e^{-at} e^{\frac{t^2}{2}} \qquad (4.2.12)$$

There's a minimum value in Eq. (4.2.12) by differentiating with respect to t and occurs when t equals the destination received SNR. With this result, Eq. (4.2.12) would be simplified into

$$P_{b1}^{h} \le e^{-(\gamma_{sd}+\gamma_{rd})} \qquad (4.2.13)$$

After we average the channel effect, the Eq. (4.2.13) would be

$$P_{b1} \le E\left[e^{-(\gamma_{sd}+\gamma_{rd})}\right]$$

$$= \int_0^{\infty}\int_0^{\infty} e^{-(\gamma_{sd}+\gamma_{rd})} p(\gamma_{sd}) p(\gamma_{rd}) d\gamma_{sd} d\gamma_{rd}$$ wher

$$= \frac{\lambda_1}{1+\lambda_1}\frac{\lambda_2}{1+\lambda_2}$$

e λ_1 denotes $(c_1\bar{\gamma}_s)^{-1}$ and λ_2 denotes $(c_2\bar{\gamma}_s)^{-1}$.

$$\Rightarrow P_{b1} \le \frac{1}{1+c_1\bar{\gamma}_s}\frac{1}{1+c_2\bar{\gamma}_s} \propto (\bar{\gamma}_s)^{-2} \qquad (4.2.14)$$

According to Eq. (4.2.1) and (4.2.14), the diversity order of P_{b1} is equal to 2.

For the error probability of the Relay, we use the Q-function property to obtain the approximation. The property is

$$Q(x) \le \frac{1}{\sqrt{2}\pi x} e^{-\frac{x^2}{2}} \le \frac{1}{2} e^{-\frac{x^2}{2}} \tag{4.2.15}$$

With this inequality (4.2.15), the conditional approximated error probability is

$$p^h = Q\left(\sqrt{2\gamma_{sr}}\right) \le \frac{1}{2} e^{-\gamma_{sr}} \tag{4.2.16}$$

Hence, we average the channel effect of Eq. (4.2.16), we could obtain

$$p \le E\left[\frac{1}{2} e^{-\gamma_{sr}}\right] \le \frac{\lambda_3}{1+\lambda_3} e^{-(\lambda_3+1)d},$$

$$\therefore p \le \frac{1}{1+c_3\bar{\gamma}_s} e^{-d} \propto \left(\bar{\gamma}_s\right)^{-1} \tag{4.2.17}$$

where λ_3 denotes $(c_3\bar{\gamma}_s)^{-1}$ and d denotes 0 or $\xi - 1$. From Eq. (4.2.1) and (4.2.17), we know the diversity order of the Relay error probability is 1.

For P_{b2}, we also use the Q-function inequality in (4.2.15) to obtain the upper bound. Therefore, P_{b2}^h would be

$$\begin{aligned} P_{b2}^h &= Q\left(\frac{\sqrt{2}\left(\gamma_{sd} - \gamma_{rd}\right)}{\sqrt{\gamma_{sd} + \gamma_{rd}}}\right) \\ &\le \frac{1}{2} \exp\left(-\frac{\left(\gamma_{sd} - \gamma_{rd}\right)^2}{\left(\gamma_{sd} + \gamma_{rd}\right)}\right) \end{aligned} \tag{4.2.18}$$

From the inequality (4.2.18), we could obtain the P_{b2} by averaging the channel effect, i.e.

$$P_{b2} = E\left[P_{b2}^{h}\right]$$
$$\le \int_0^\infty \int_0^\infty \frac{1}{2} \exp\left(-\frac{\left(\gamma_{sd} - \gamma_{rd}\right)^2}{\left(\gamma_{sd} + \gamma_{rd}\right)} \right) p\left(\gamma_{sd}\right) p\left(\gamma_{rd}\right) d\gamma_{sd} d\gamma_{rd}$$

(4.2.19)

From the inequality in (4.2.19), we could use the long division method to obtain another form in the exponential part, i.e.

$$\exp\left(-\frac{\left(\gamma_{sd} - \gamma_{rd}\right)^2}{\left(\gamma_{sd} + \gamma_{rd}\right)} \right) = \exp\left(-\left(\gamma_{sd} + \gamma_{rd} - \frac{4\gamma_{sd}\gamma_{rd}}{\left(\gamma_{sd} + \gamma_{rd}\right)} \right) \right)$$

Therefore, the inequality would be

$$P_{b2} \le \int_0^\infty f_A\left(\gamma_{rd}\right) p\left(\gamma_{rd}\right) d\gamma_{rd}$$

where

$$f_A\left(\gamma_{rd}\right) = \int_0^\infty \frac{1}{2} e^{-\left(\gamma_{sd} + \gamma_{rd} - \frac{4\gamma_{sd}\gamma_{rd}}{\gamma_{sd} + \gamma_{rd}}\right)} p\left(\gamma_{sd}\right) d\gamma_{sd}$$

From the $f_A\left(\gamma_{rd}\right)$, we could average the channel effect γ_{sd} first. Due to the decreasing property of the exponential term, we make an approximation and change $\frac{4\gamma_{sd}\gamma_{rd}}{\gamma_{sd} + \gamma_{rd}}$ to $2\sqrt{\gamma_{sd} + \gamma_{rd}}$. Therefore, $f_A\left(\gamma_{rd}\right)$ is rewritten as

$$f_A\left(\gamma_{rd}\right) \le \int_0^\infty \frac{1}{2} e^{-\left(\gamma_{sd} + \gamma_{rd} - 2\sqrt{\gamma_{sd}\gamma_{rd}}\right)} p\left(\gamma_{sd}\right) d\gamma_{sd}$$
$$= \int_0^\infty b_1 \frac{1}{2} e^{-\left(\sqrt{x} - d\right)^2} e^{-bx} dx,$$

where x denotes γ_{sd}, b_1 denotes $(c_1\bar{\gamma}_s)^{-1}$ and d denotes $\sqrt{\gamma_{rd}}$. After integration, $f_A(\gamma_{rd})$ would be shown as：

$$f_A(\gamma_{rd}) \le b_1 e^{-d^2}\left[\frac{1}{1+b_1}+\frac{d\sqrt{\pi}e^{d^2/1+b_1}}{(1+b_1)^{3/2}}\left(1-erf\left(-\frac{d}{\sqrt{1+b_1}}\right)\right)\right]$$

We could use the Q function property,

$$Q(x)=1-erf(x) \qquad \text{(p.1)}$$

With this property and change $\sqrt{\pi}$ to 2, we could rewrite $f_A(\gamma_{rd})$ into

$$f_A(\gamma_{rd}) \le \frac{b_1}{1+b_1}e^{-d^2}+b_1 e^{-d^2}\left[\frac{2de^{d^2/1+b_1}}{(1+b_1)^{3/2}}Q\left(-\frac{\sqrt{2}d}{\sqrt{1+b_1}}\right)\right]$$

Due to the Q function property,

$$Q(-x)=1-Q(x) \qquad \text{(p.2)}$$

With this Eq. (p.2), we rewrite $f_A(\gamma_{rd})$ into

$$f_A(\gamma_{rd}) \le \frac{b_1}{1+b_1}e^{-d^2}+b_1 e^{-d^2}\left[\frac{de^{d^2/1+b_1}}{(1+b_1)^{3/2}}\left(1-Q\left(\frac{\sqrt{2}d}{\sqrt{1+b_1}}\right)\right)\right]$$

$$\le \frac{b_1}{1+b_1}e^{-d^2}+\frac{db_1}{(1+b_1)^{3/2}}e^{-\frac{d^2 b_1}{1+b_1}} \qquad (4.2.20)$$

From the inequality in (4.2.20), we could obtain the upper bound of the P_{b2} after averaging the γ_{rd} channel effect individually,

$$\frac{b_1}{1+b_1}\int_0^{\infty}\lambda_1 e^{-(\lambda_1+1)x}dx=\frac{b_1}{1+b_1}\frac{\lambda_1}{1+\lambda_1}$$

$$=\frac{1}{(1+c_1\bar{\gamma}_s)(1+c_2\bar{\gamma}_s)} \quad (4.2.21)$$

$$\frac{\lambda_1 b_1}{(1+b_1)^{3/2}}\int_0^{\infty}\sqrt{x}e^{-\left(\lambda_1+\frac{b_1}{1+b_1}\right)x}dx\le\frac{\lambda_1 b_1}{(\lambda_1+b_1+\lambda_1 b_1)^{3/2}}$$

$$=\frac{\left(c_1 c_2\bar{\gamma}_s^2\right)^{1/2}}{(c_1\bar{\gamma}_s+c_2\bar{\gamma}_s+1)^{3/2}} \quad (4.2.22)$$

From Eq. (4.2.21) and (4.2.22), both of them are proportional to $\bar{\gamma}_s$ with exponent order 2 and 1/2. Due to a sum is dominated by the term with the lowest diversity exponent, the diversity of P_{b2} is 1/2.

With the results of Eq. (4.2.14), (4.2.17), (4.2.21) and (4.2.22), the approximated BER would be

$$P_e\approx\frac{\xi}{(1+c_3\bar{\gamma}_s)}\frac{1}{4c_1\bar{\gamma}_s}+$$

$$\left(\frac{1}{(1+c_1\bar{\gamma}_s)(1+c_2\bar{\gamma}_s)}+\frac{e^{-d}}{1+c_3\bar{\gamma}_s}\left(\frac{1}{(1+c_1\bar{\gamma}_s)(1+c_2\bar{\gamma}_s)}+\frac{\left(c_1c_2\bar{\gamma}_s^2\right)^{1/2}}{(1+c_1\bar{\gamma}_s+c_2\bar{\gamma}_s)^{3/2}}\right)\right)$$

From the above approximated BER, we could find out the diversity of the proposed system under the high SNR condition would be

$$1.5 \leq G_d \leq 2 \tag{4.2.26}$$

This result is consistent with that of the diversity in [7]. Based on [7], the diversity of the uncoded Relay under Decode-and-Forward protocol would be 1.5~2 in the cooperative system with only one Relay to retransmit the Source signal.

Chapter 5

Computer Simulations

In this chapter, we show the performance of the proposed scheme under different Relay locations. Here, our channel gains are modeled as the mutually independent, complex Gaussian random variables, $CN\left(0,\sigma_{i,j}^2\right)$, where i, j indicate the Source (s), Relay (r) and Destination (d). There are three Relay locations which are "Relay in Middle", "Relay close to Source", "Relay close to Destination". We adjust the $\sigma_{i,j}^2$ value to represent the different Relay locations. Here, we set the $\sigma_{s,r}^2$ as 10 and $\sigma_{s,d}^2 = \sigma_{r,d}^2$ as 1 to realize the Relay close to Source condition. Also we set the $\sigma_{r,d}^2$ as 10 and $\sigma_{s,r}^2 = \sigma_{s,d}^2$ as 1 to realize the Relay close to Destination condition. For the Relay in Middle condition, we set all the channel gain variances are equal to 1. The noise power is also modeled as the mutually independent complex Gaussian random variable. But the variance we use is equal to 1 for all receiver nodes. The power for the Phase 1 and Phase 2 is allocated by the parameter r, i.e. $P_1 = r * P$ and $P_2 = (1-r)P$. Here, the powers we use in the Phase 1 and Phase 2 are equal. The background of cooperative system is using threshold-selection relay scheme. We use 4 threshold values which is $\xi = 0.5$, 1, 3 and 4 to see the performance variation and the simulated results would be shown into three groups. We use Eq. (4.1.39) to verify the proposed scheme simulated results. We also provide the simulations to find out the influence of the relay threshold value on the system BER.

5.1 Under Relay in Middle Status

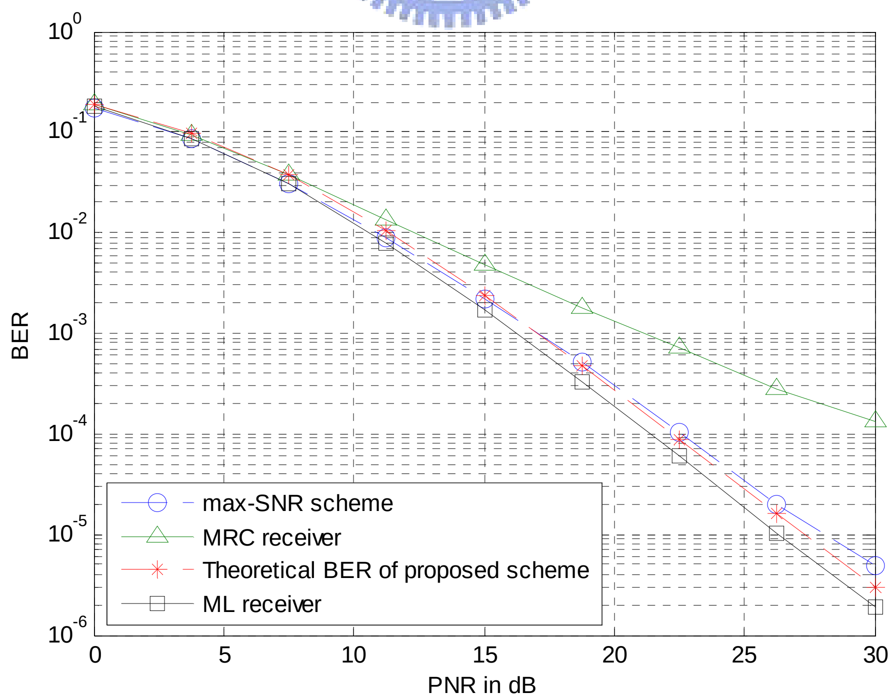

Figure 5.1 Threshold =0.5 with relay in middle

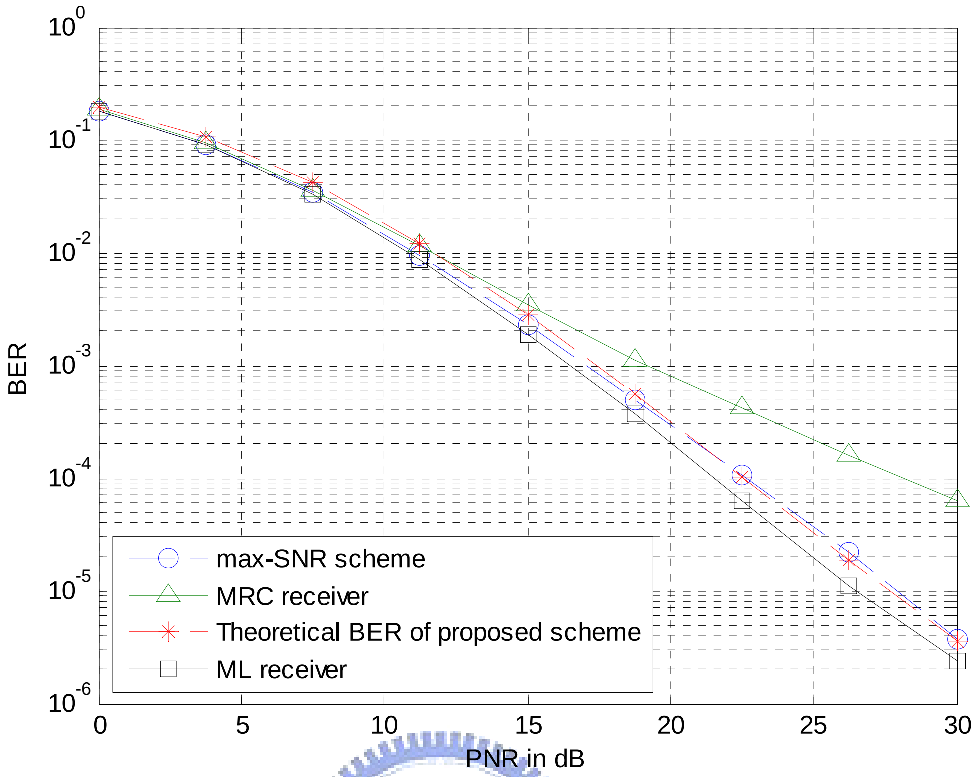

Figure 5.2 Threshold =1 with relay in middle

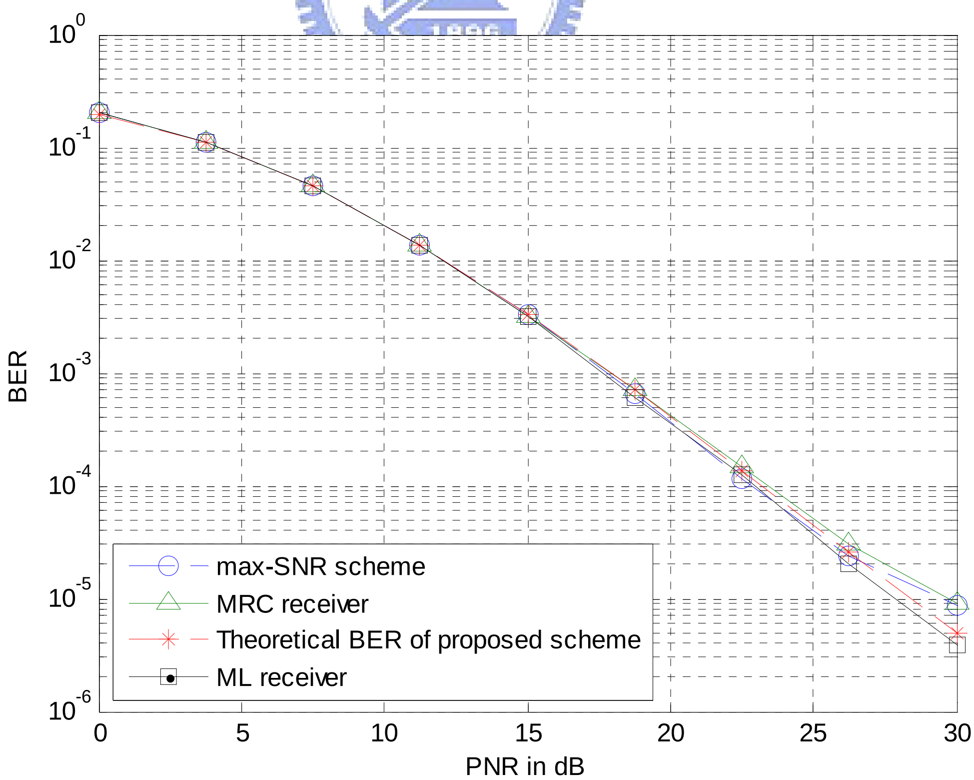

Figure 5.3 Threshold =3 with relay in middle

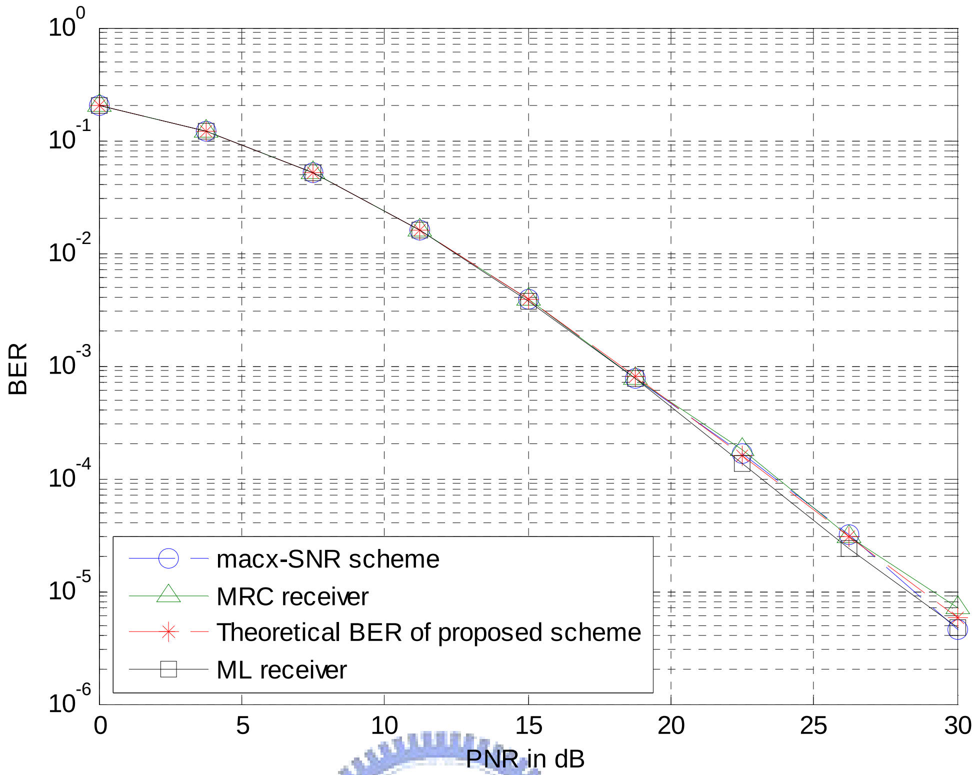

Figure 5.4 Threshold =4 with relay in middle

As we can see from the Figure (5.1) ~ Figure (5.4), the performance of the proposed system could improve the performances which is close to the ML receiver's result under small threshold value area and the simulated results are matched to the theoretical BER which verifies the proposed scheme. But the performance will be same as the Selection-Relay for the large threshold value. Also from the simulated results, we could find out the diversity order of max-SNR detection for selection relaying. Because of the diversity order of ML receiver and MRC receiver are 2 and 1.5 which proved in [7]. Also the simulated results lie between these simulated results. With these results, we could know the diversity order of proposed scheme matches to the conclusion we made. For the threshold value's influence, we set the PNR to 20 dB which means after receiving signals the destination would have 10 dB powers from the source and relay, respectively. The computer simulation is shown as follows

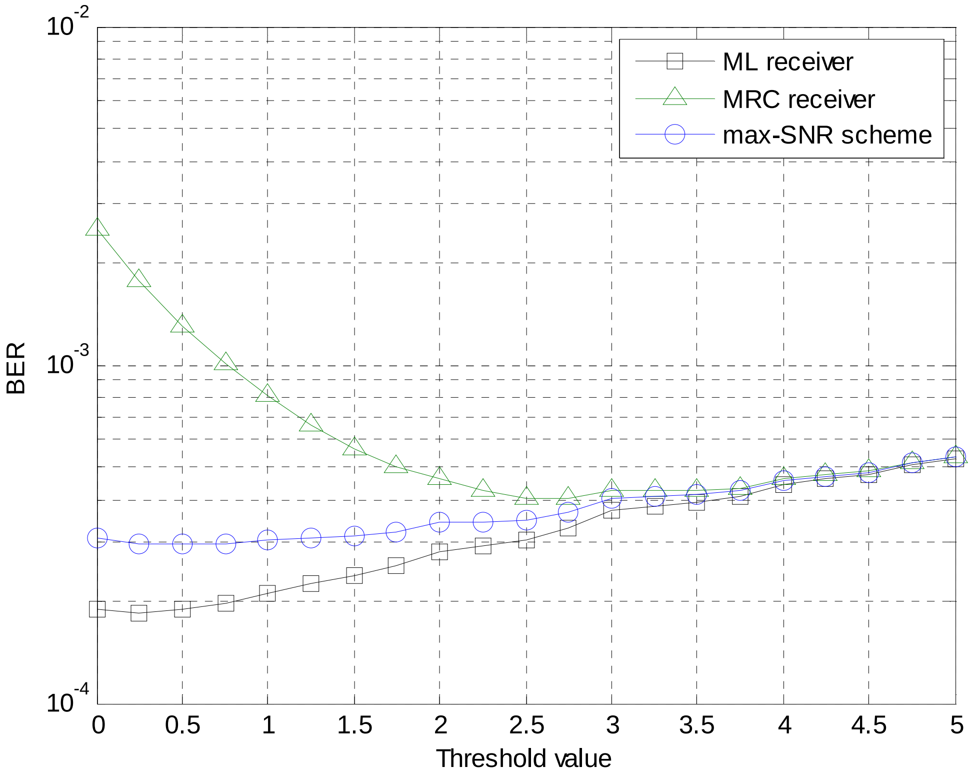

Figure 5.5 Influence of threshold value with relay in middle

Because the threshold value acts like a supervisor and it would increase the accuracy of transmitting the original symbol. With this reason, from Figure 5.5, we could know the performances of three receivers would merge during the large threshold utilization. But when the threshold value turns to the large value, the utilization of the relay-destination channel would drop. With this reason, the performances of three receivers would degrade when the threshold value increase.

Besides, when the threshold value turns to the small one, the MRC receiver's performance would degrade enormously due to error propagation. Hence, with the max-SNR scheme, we could reduce the threshold value for utilizing the relay-destination channel frequently and improve the MRC performance.

5.2 Under Relay Close to Source Status

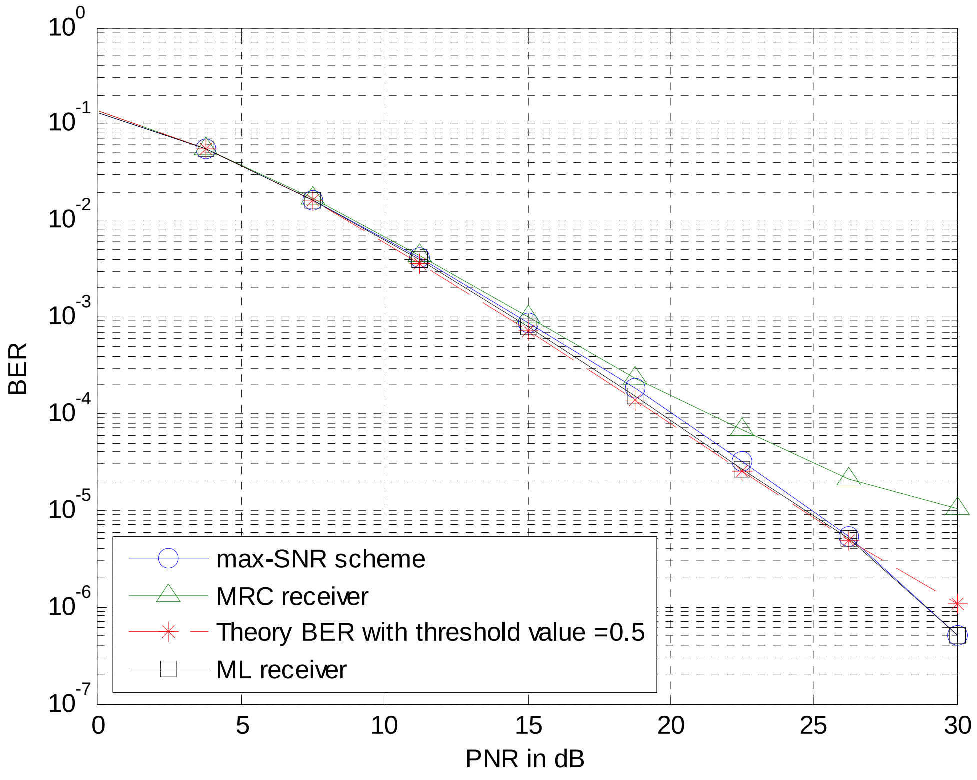

Figure 5.6 Threshold =0.5 with relay close to source

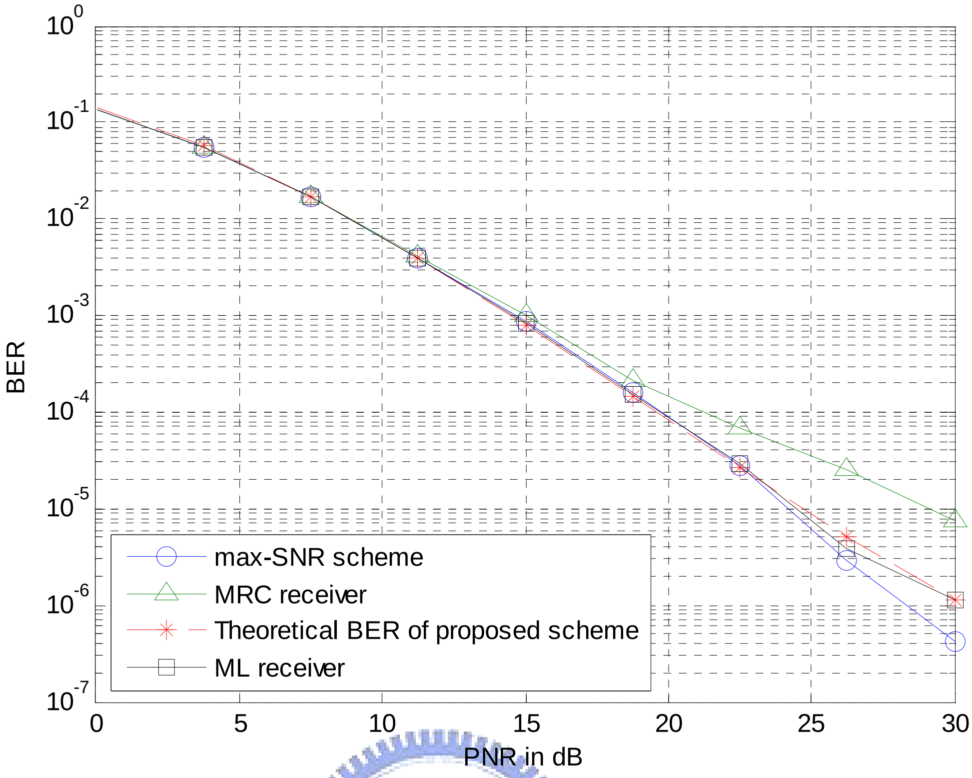

Figure 5.7 Threshold =1 with relay close to source

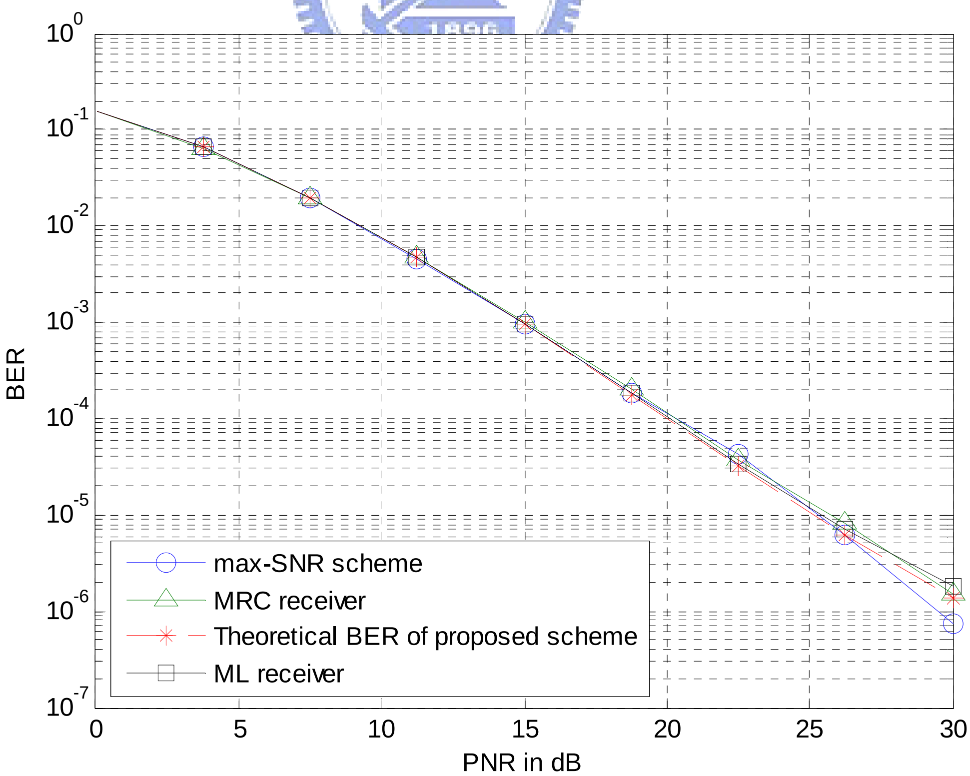

Figure 5.8 Threshold =3 with relay close to source

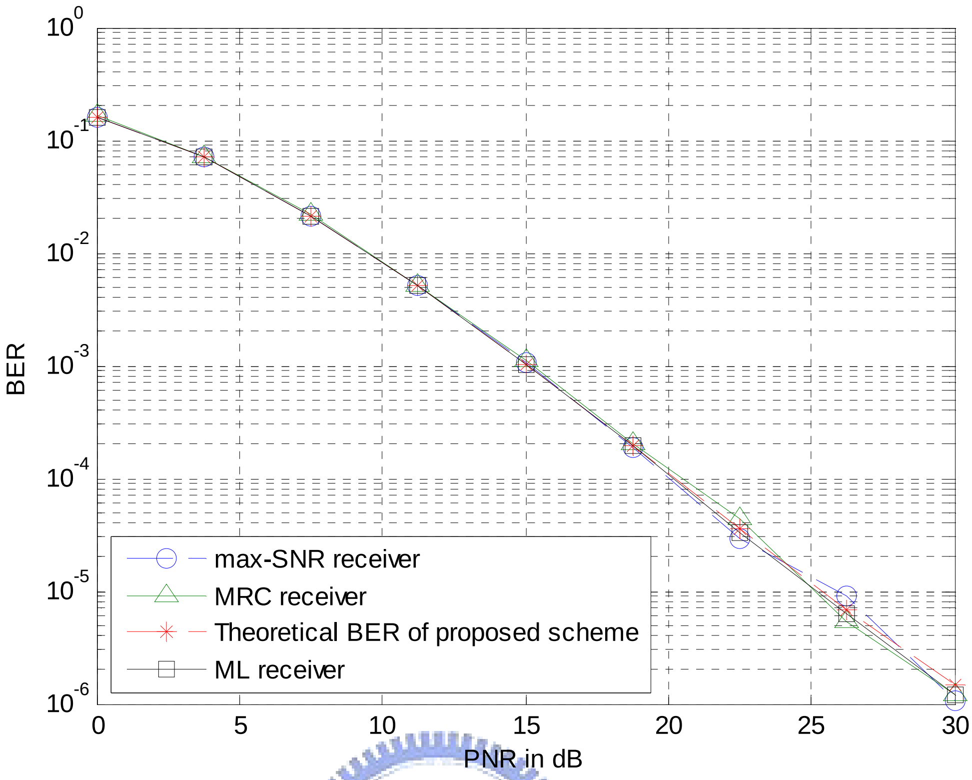

Figure 5.9 Threshold =4 with relay close to source

From Figure (5.6) ~ Figure (5.9), we could know the performances of three receivers have similar results under relay in middle status. But the performance of the proposed system which is so closed to the ML combining is that the decoded signals transmitted by the Relay are the original signals due to Relay close to Source. For the threshold value's influence, we use the same parameter in the relay in middle status. The simulation result of threshold value's influence is shown as follows.

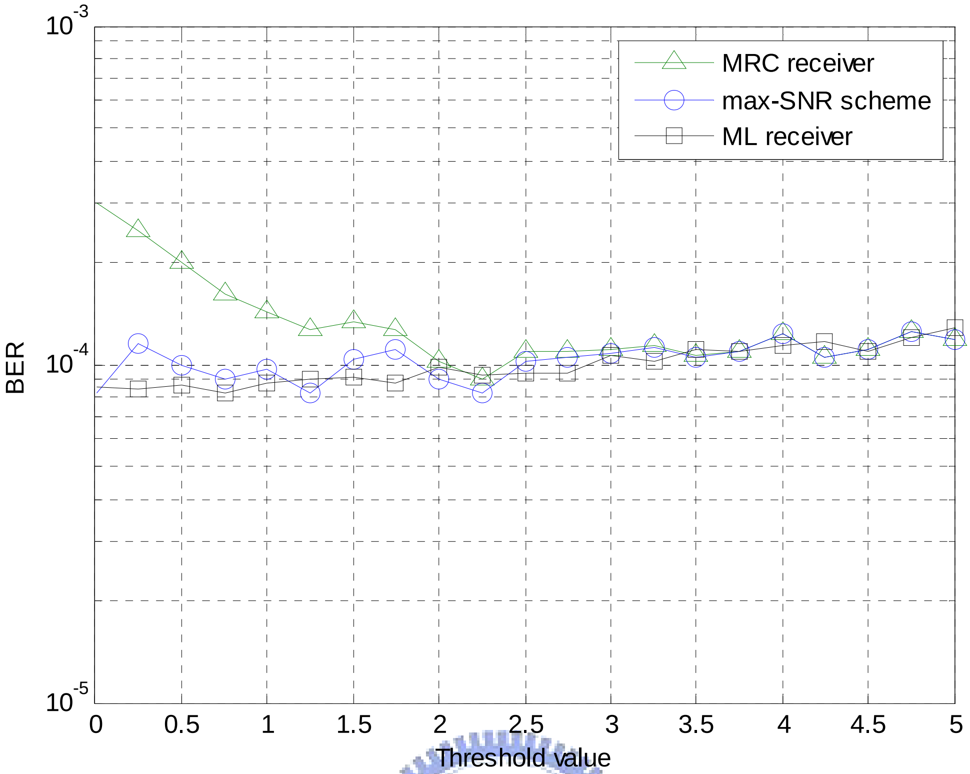

Figure 5.10 Influence of threshold value with relay close to source

Like the result in the relay in middle status, the performances of three receivers would merge during large threshold value utilization. But due to the condition, the utilization of the relay-destination channel would raise and the decoded signal would be a copy of the original symbol of the source. Hence, the destination would collect two useful data for recovering the original symbol.

5.3 Under Relay Close to Destination Status

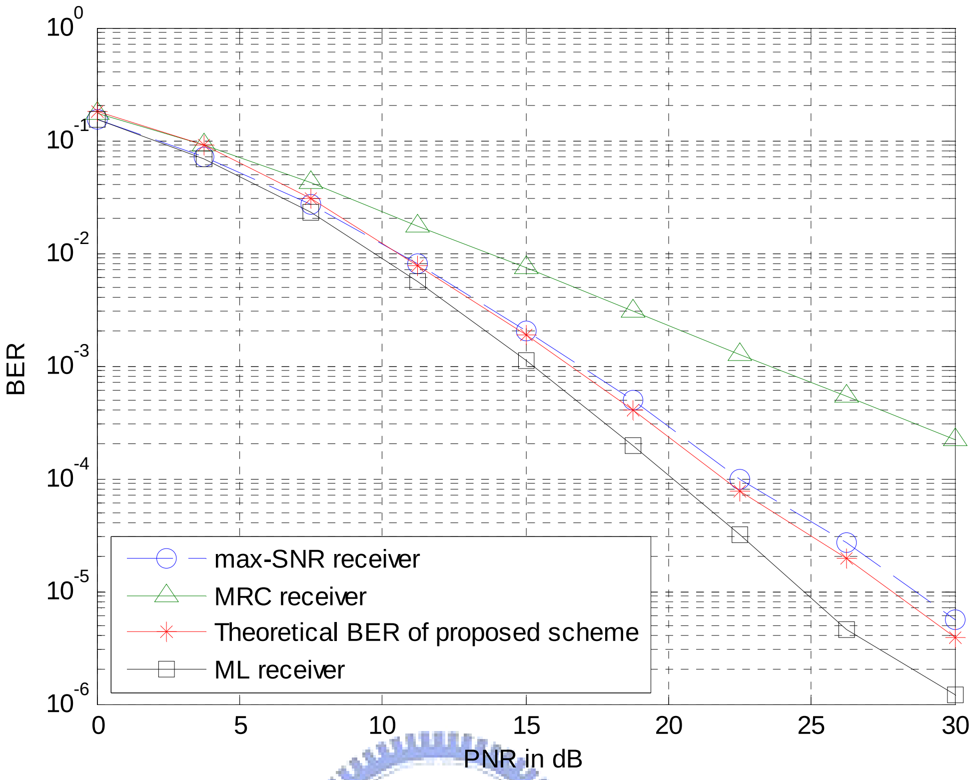

Figure 5.11 Threshold =0.5 with relay close to destination

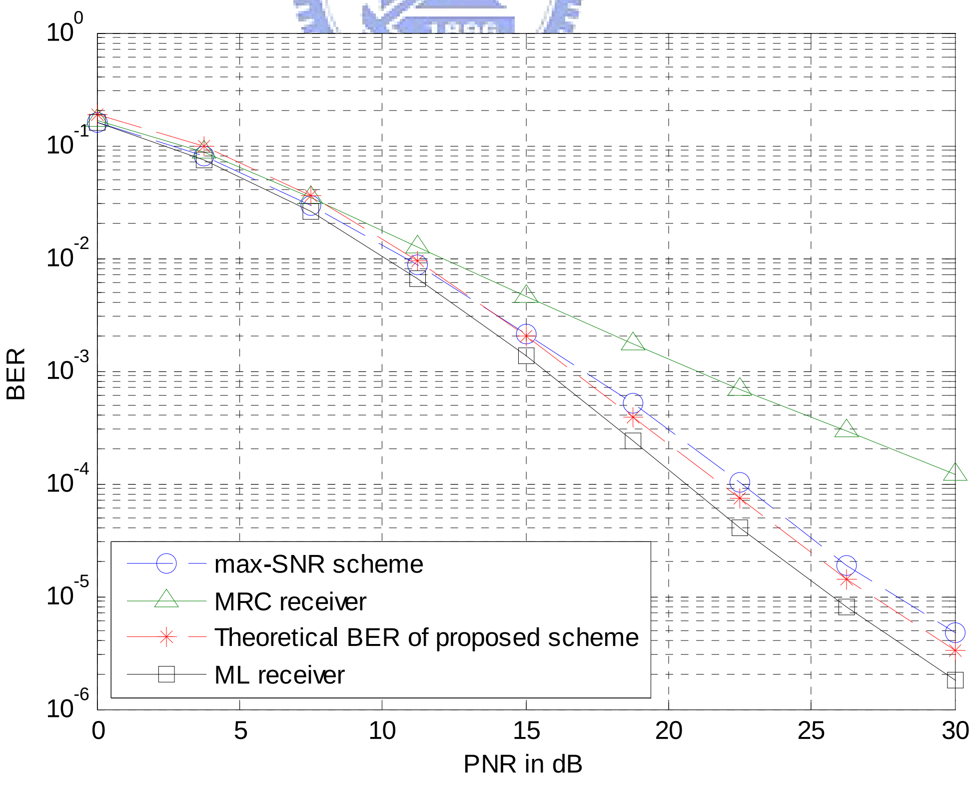

Figure 5.12 Threshold =1 with relay close to destination

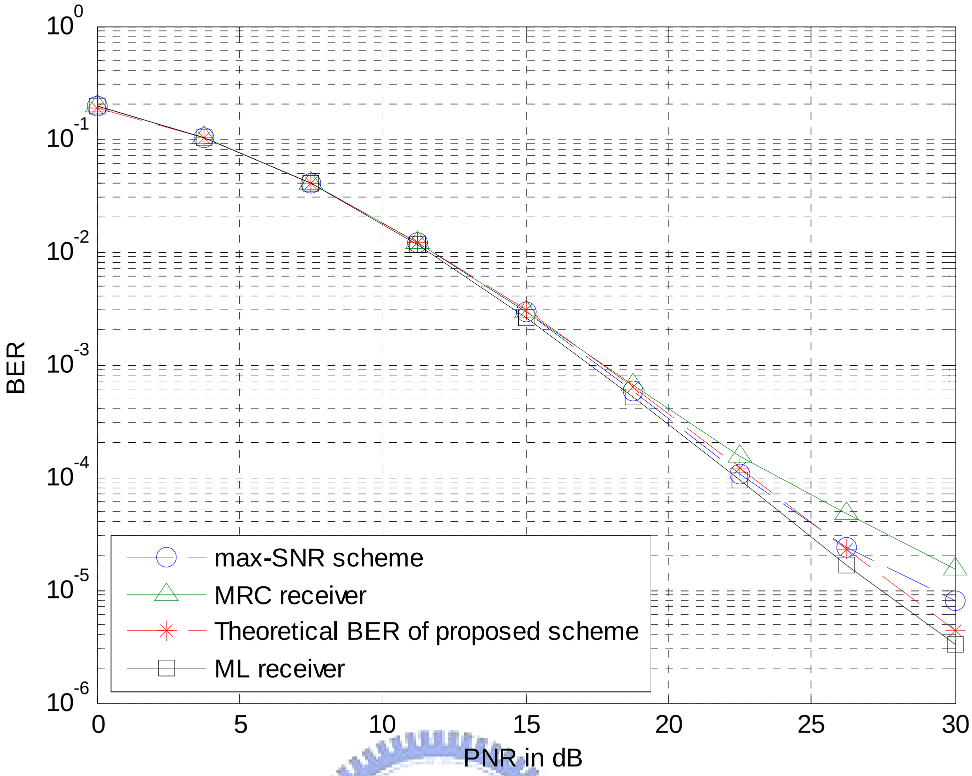

Figure 5.13 Threshold =3 with relay close to destination

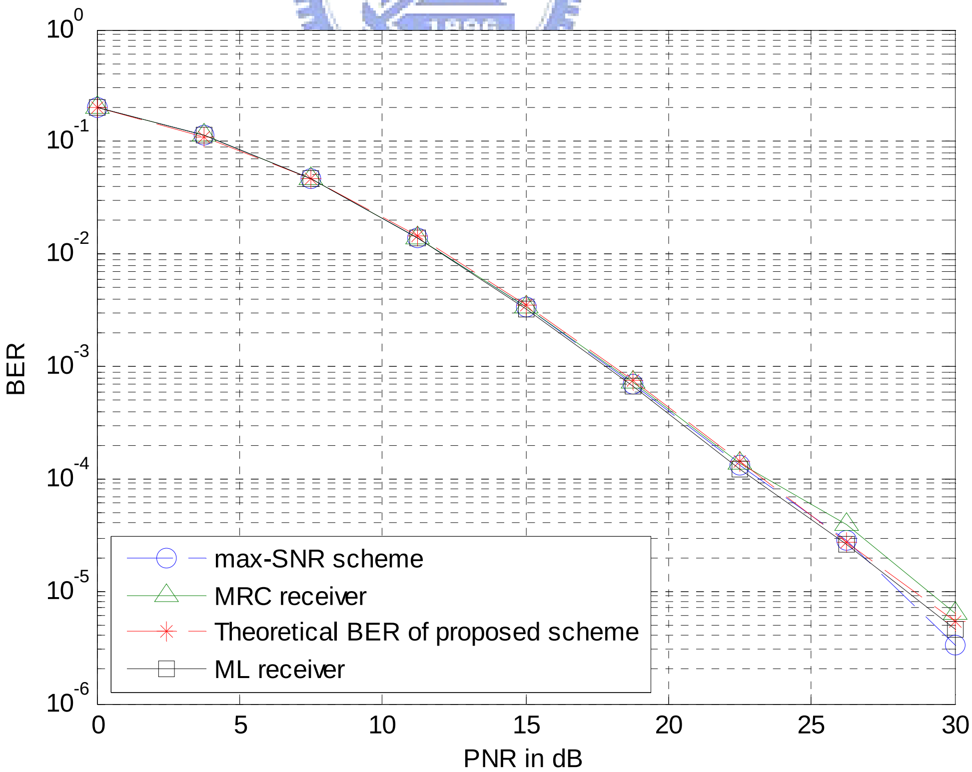

Figure 5.14 Threshold =4 with relay close to destination

From Figure (5.11) ~ Figure (5.14), the performance of the proposed system also has improved at the small threshold value area. But due to the condition that the Relay is close to Destination, the proposed system would need more 3 dB to meet the performance of ML combining during small threshold value utilization. Same parameter used in the sections 5.1 and 5.2, the simulation of the threshold value's influence is shown at below：

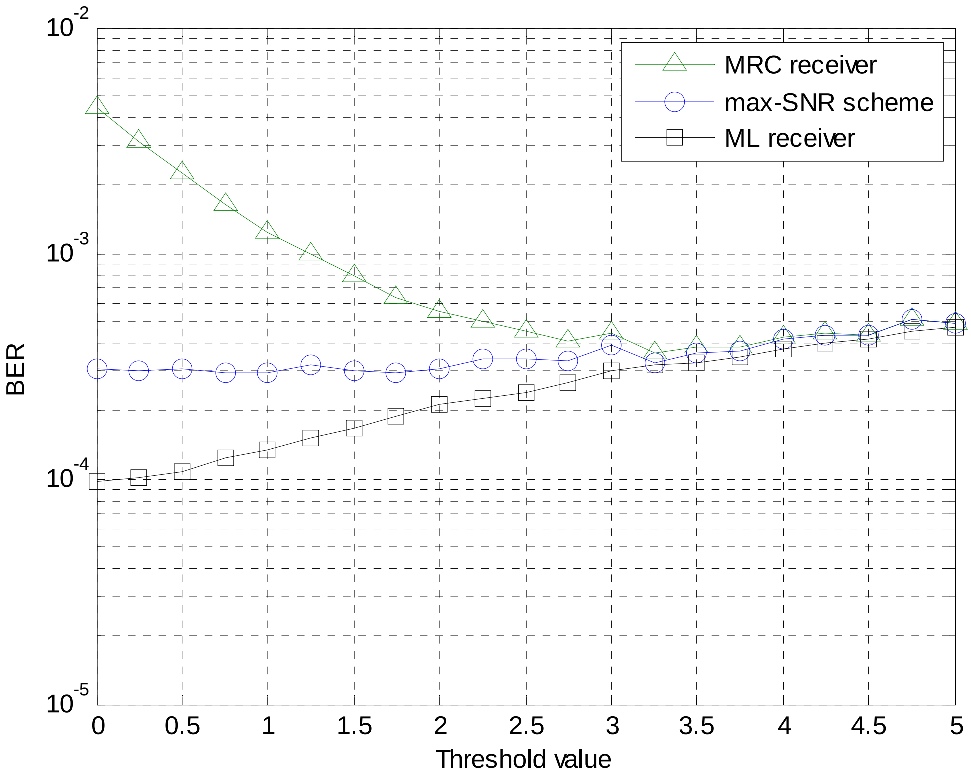

Figure 5.15 Influence of threshold value with relay close to destination

Like the result mentioned in the sections 5.1 and 5.2, the performances of three receivers would merge during large threshold value utilization. But due to the relay close to destination status, the BER of the relay would raise up. The destination would estimate wrong due to receive two copy different symbols in the detection device.

5.4 High SNR Approximation Simulations

We also could use equation (4.2.2) to verify our simulation results. Due to the assumption of high SNR status in section 4.2, we use the relay close to source status as our verifications. The results are shown as follows.

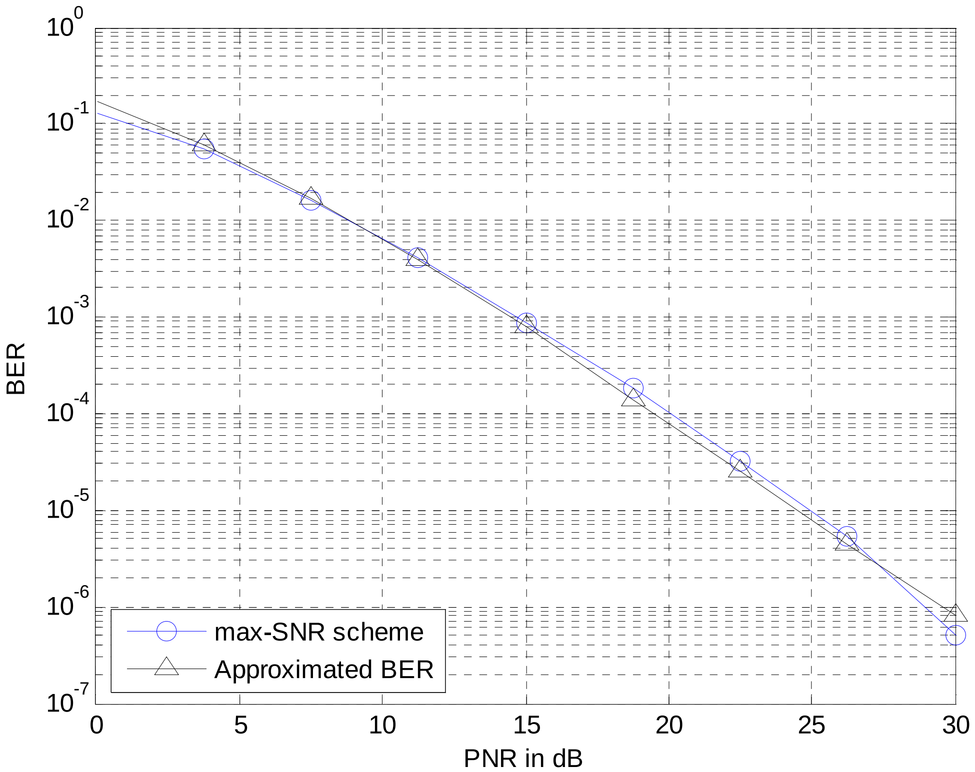

Figure 5.16 Threshold =0.5 for high SNR approximation

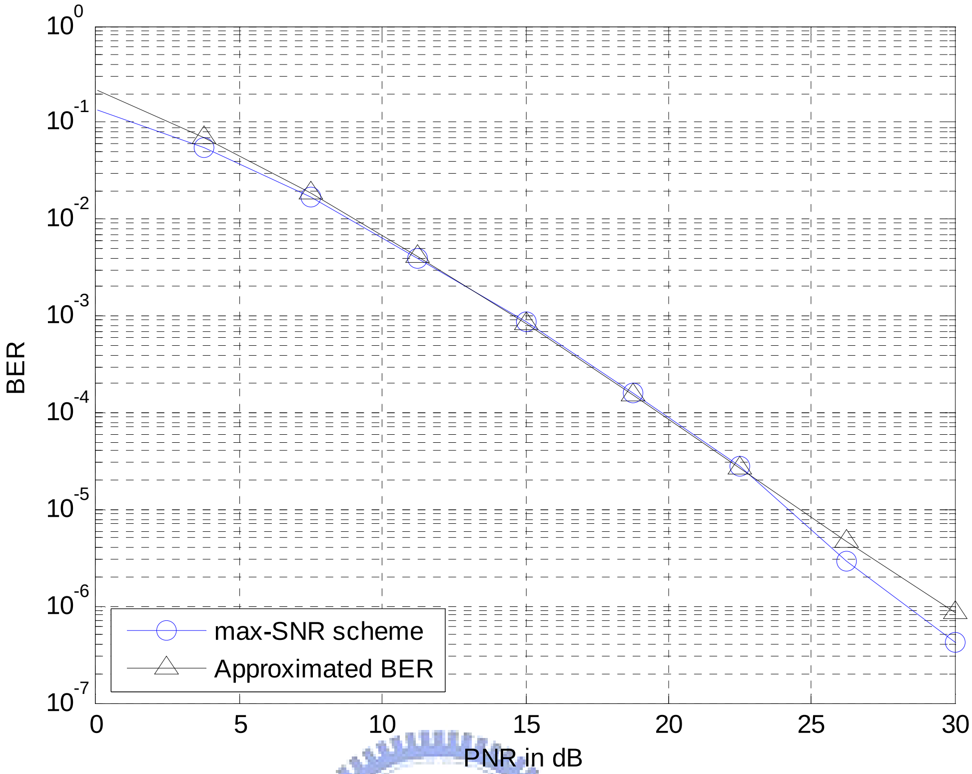

Figure 5.17 Threshold =1 for high SNR approximation

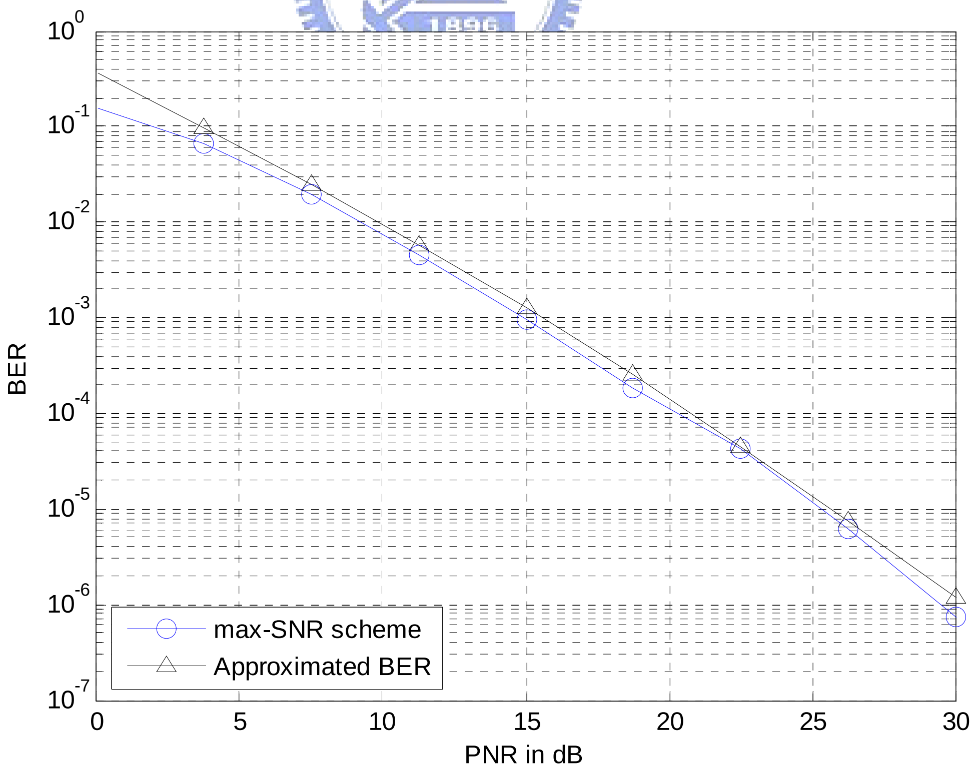

Figure 5.18 Threshold =3 for high SNR approximation

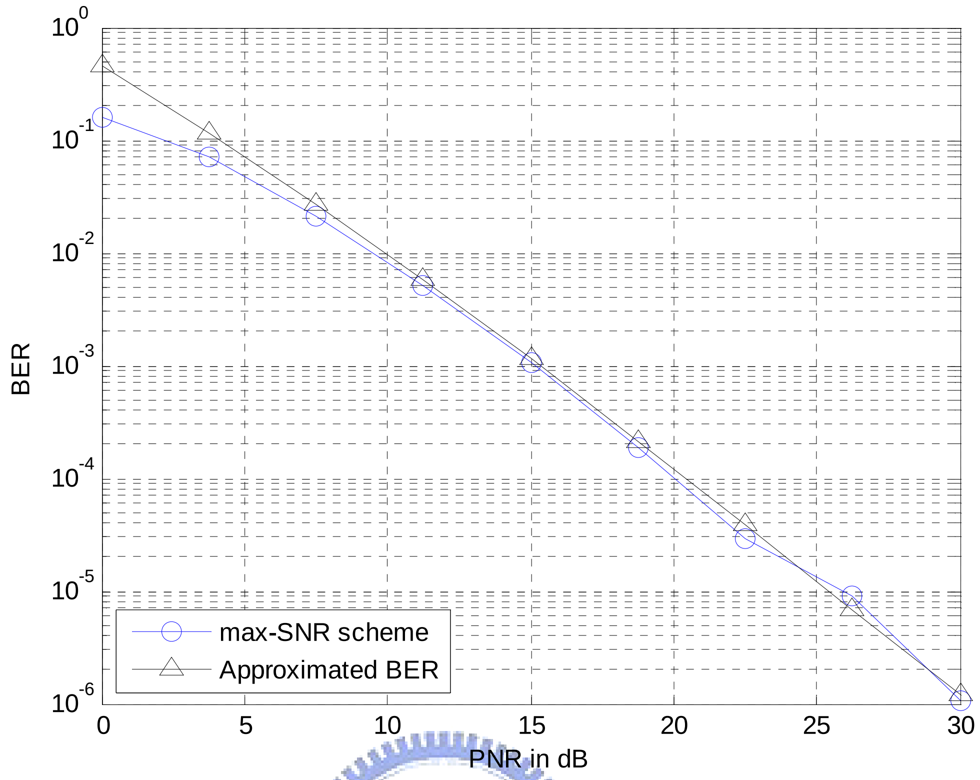

Figure 5.19 Threshold =4 for high SNR approximation

From Figure (5.16) ~ Figure (5.19), the approximated theory BER would approximate the simulation results during High SNR assumption.

5.5 Under 2 Threshold Values in Selection Relay

Under this concept, we make the large threshold value to 1 and small threshold value to 0.5. With these parameters, the simulated result would be shown as follow：

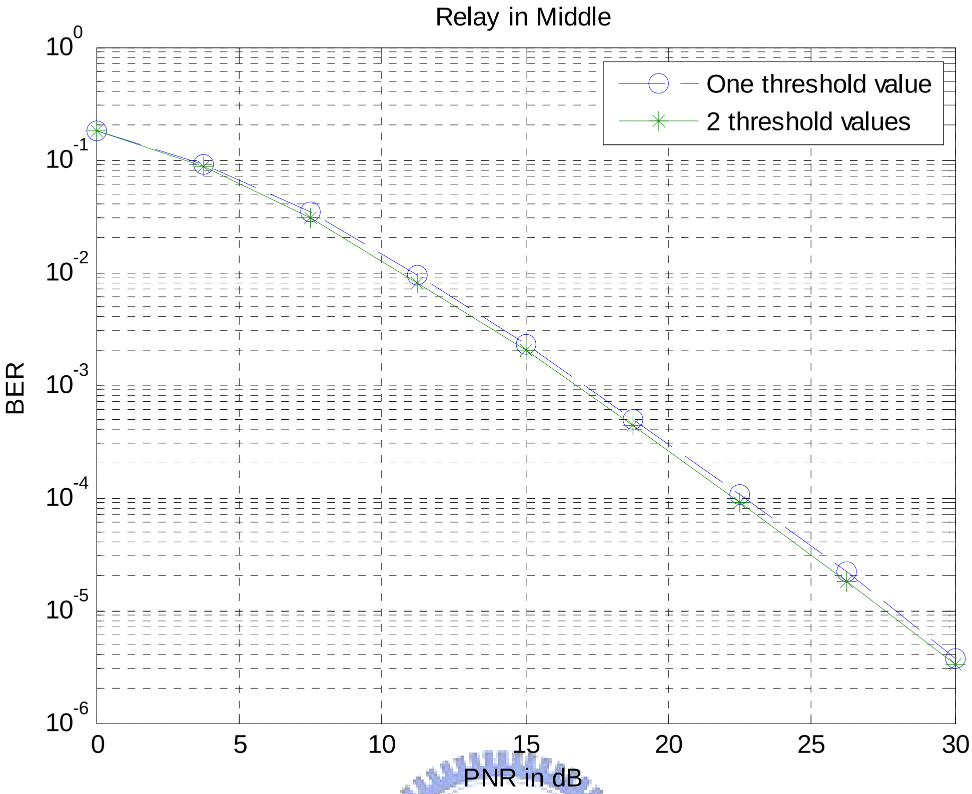

Figure 5.20 Threshold =1 with combo scheme

From Figure 5.20 and under threshold value equals to 1, we would know the two threshold value in selection relay improves the traditional MRC receiver in the selection relay cooperative system. With two stages comparison, we could make sure the quality of signal transmitted by the relay.

Chapter 6

Conclusions

In this thesis, we propose a threshold-selection relay with maximum SNR detection at the destination for the transmitted signal from the relay. It improves the performance of threshold-selection relay in case of small threshold value. It also successfully reduces the threshold value at the relay. The optimum threshold value at the relay is 0.5 when the relay places in the middle. The optimum threshold value would change in cases of different relay locations and/or different power ratios of the transmitted signal power. Also the diversity of proposed scheme attains to 1.5~2 which meets the result in [7] without using any non-linear mapping function at the destination.

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