國立交通大學

電子工程學系　電子研究所

碩士論文

在多重輸入輸出-正交分頻多工的系統下有效率地內插方向性向量

Efficient Interpolation of Beamforming Vectors in MIMO-OFDM Systems

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中華民國九十七年五月

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摘要

在多重輸入輸出-正交分頻的系統下，為了使用所有通道容量，藉著奇異值分解出的方向性向量，可用在傳送端及接收端身上。 最佳化的方向性向量需要精準的通道狀態資訊給每一個正交分頻的頻率。在這篇論文裡，提出一個有限的回傳及有限的通道分解架構基於使用了適合的內插方式，使其降低來自方向性向量的計算量。在所提出的系統裡，接收端只需回傳在導向頻率上所估測出的通道資訊，以及傳送端使用這些通道資訊，藉著正交內插的方式來得到所有頻率上分解的結果。這是個個圓形內插的方法，且必須陪伴著重要的假設是奇異值在相鄰頻率間的改變差不多是線性的狀態。數值的模擬會展現出在適當的環境 ，如適當的快速傅立葉轉換的大小 、內插的間距等等，所提出的架構，在犧牲一點點的效能下可以大幅降低計算量。

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ABSTRACT

To fully exploit the channel capacity in multiple-input multiple-output orthogonal frequency division multiplexing (MIMO-OFDM) systems, beamforming vectors derived from SVD can be used at the transmitter and the receiver. The optimal beamforming requires accurate the channel state information for each OFDM subcarrier. This paper proposes a limited feedback and limited channel decomposition architecture by deploying a novel suitable interpolation method to reduce the computation cost of beamforming. In the proposed system, the receiver feedbacks only the estimated channel information on the pilot subcarrier and the transmitter use these channel information to get decomposition results on all subcarrier by an orthogonal interpolation method. This method is a circular interpolator with the important assumption that the singular modes on adjacent subcarriers approximately change in a linear way. Numerical simulations show that under certain circumstances, e.g., proper FFT size, interpolation gap etc., the proposed scheme can reduce computation efficiently while only sacrifice little performance.

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Chapter 1

■ Introduction

MIMO systems provide spatial diversity that can be used to get the multiplex gain, array gain, diversity gain, and interference reduction. When the channel state information (CSI) is not feedback from the receiver to the transmitter, using space-time coding [1] [2] can get the diversity gain. When the CSI is available at the transmitter, diversity can be obtained using beamforming design based on CSI at the transmitter and combination at the receiver. It can significantly improve system performance.

The beamforming technique is proposed for narrowband channels can be easily extended to frequency selective channels by employing orthogonal frequency division multiplexing (OFDM). The combination of MIMO and OFDM, known as MIMO-OFDM, converts a broadband MIMO channel into a series of parallel narrowband MIMO channels, one for each OFDM subcarrier.

In nonreciprocal channels, this requires that the receiver sends back to the transmitter CSI or the optimal beamforming vector for every active subcarrier. The subchannels are obtained from the discrete Fourier transform (DFT) of the sampled frequency-domain channel are significantly correlated. As a result, the beamforming vectors that determined by the subchannels are also substantially correlated [3] in frequency domain but also to time domain. In [3], it only says how to get the next tone beamforming vector and to describe phenomenon on SVD (Singular Value Decomposition). We propose a new beamforming scheme that sends back only a fraction of beamforming vectors or CSI information to the transmitter and generates the beamforming vectors for all subcarriers through interpolation at the transmitter [4]. We assume that the transmit power is equally assigned to all subcarriers and CP length is larger than channel delay length to avoid ISI (Inter-symbol Interference) and ICI (Inter-carrier Interference). Power allocation can be included in the system model with additional feedback but we defer this to future work.

The remaining of this thesis is organized as follows: in chapter 2, we show that the design criteria to beamforming vector. In chapter 3, we discuss what condition must be considered when CSI at transmitter and receiver. In chapter 4, we introduce the OSVDI (Orthogonal Singular Value Decomposition Interpolation) algorithm and what impairment has to consider. Finally, we generalize the OSVDI to the other decomposition and show the simulation result in chapter 5.

Chpter2

■ Design beamforming vector in MIMO-OFDM system

The MIMO-OFDM system show in **Fig(2.1)**. The multi-path channel are independent and identically distributed (i.i.d) complex Gaussian distribution with zero mean and unit variance. The received signal at the k-th subcarrier can be expressed by

$$R_k = Z_k^H \{H_k W_k I_k + N_k\} \qquad 1 \le k \le N \tag{1}$$

where

$$R_k = \begin{bmatrix} r_{1,k} & r_{2,k} & \cdots & r_{M_r,k} \end{bmatrix}^T \quad , \tag{2}$$

$$Z_k = \begin{bmatrix} z_{1,k} & z_{2,k} & \cdots & z_{M_r,k} \end{bmatrix} \tag{3}$$

where $z_{i,k} = [z_{1,i,k} \quad z_{2,i,k} \quad \cdots \quad z_{Mr,i,k}]^T$,

$$W_k = \begin{bmatrix} w_{1,k} & w_{2,k} & \cdots & w_{M_t,k} \end{bmatrix} \tag{4}$$

where $w_{i,k} = [w_{1,i,k} \quad w_{2,i,k} \quad \cdots \quad w_{Mr,i,k}]^T$,

$$I_k = \begin{bmatrix} I_{1,k} & I_{2,k} & \cdots & I_{M_t,k} \end{bmatrix}^T , \tag{5}$$

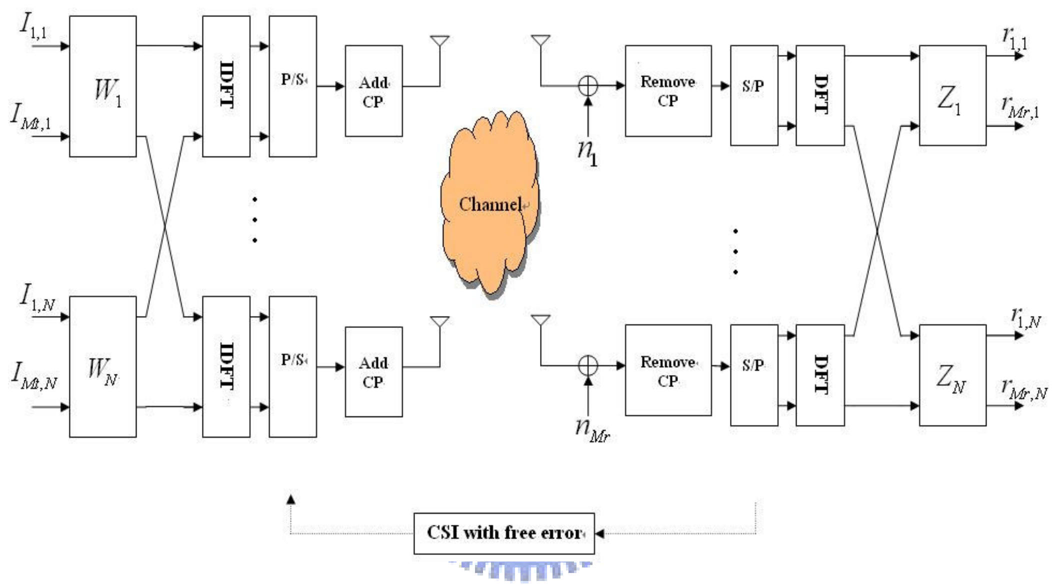
$$N_k = \begin{bmatrix} n_{1,k} & n_{2,k} & \cdots & n_{M_r,k} \end{bmatrix}^T , \tag{6}$$

and H_k is the frequency response of MIMO-OFDM channel at k-th subcarrier, and N is the IDFT/DFT sizes. The $N(k)$ is the $M_r \times 1$ noise vector that entries are independent and identically distribution complex Gaussian with zero mean and variance to be N_0, Let $E\left[\left|I_k\right|^2\right] = \varepsilon_s$ and $W_k^H W_k$ to be identity matrix where W_k is the pre-coder matrix working at k-th subcarrier. In order to avoid noise enhancement at receiver, we set $Z_k^H Z_k = I$ where Z_k to be de-coder matrix working at k-th subcarrier. Then the SNR for k-th subcarrier can be expressed as

$$SNR_k = \frac{\varepsilon_s \left| Z_k^H H_k W_k \right|^2}{N_0} \quad . \tag{7}$$

In order to maximum SNR_k, we know that beamforming vector at receiver can be written as

$$Z_k = \frac{H_k W_k}{\left\| H_k W_k \right\|} . \tag{8}$$



Fig(2.1) MIMO-OFDM system with Mt transmit antennas, Mr receive antennas and N subcarriers.

Chapter 3

■ **Communicaton system with CSI to Tx and Rx**

3.1 Communicaton system with full CSI to transmitter and receiver

We know MIMO-OFDM systems as many the MIMO system in different subcarrier working. We get a simplify subsystem working at k-th subcarrier and show in **Fig (3.1).** In next discussion, about the number of transmit and receiver antenna are equal to M.

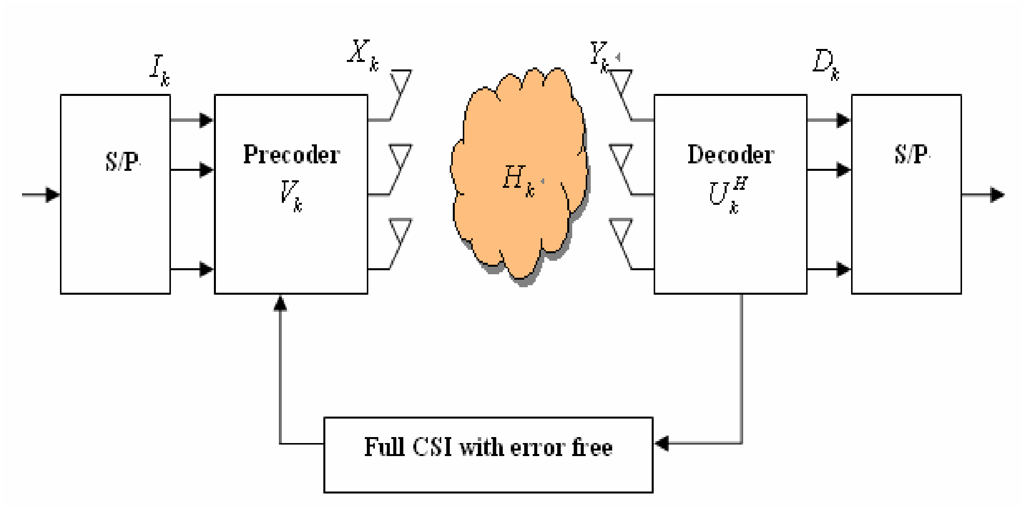


Fig (3.1) The subsystem working at k-th subcarrier and the number of transmitter is M and receiver antenna is M. Full CSI feedback to transmitter from receiver with error free

In MIMO systems, to get the full spatial multiplexing gain without symbol interference, a simple method is that the information product with beamforming vector at the transmitter and combination at the receiver. One way to get the beamforming vector is to decompose the channel by SVD (Singular Value Decomposition). The MIMO-OFDM channel with multipath at k-th subcarrier as

$$H_k = \begin{bmatrix} h_{11,k} & h_{12,k} & \cdots & h_{1M,k} \\ h_{21,k} & \cdots & & \\ & & & \\ h_{M1,k} & \cdots & & h_{MM,k} \end{bmatrix} \qquad (9)$$

and each path $h_{ji,k}$ are independent and identically distribution complex Gaussian with zero mean and unit variance and transmit from transmit antenna i to received antenna j. The channel matrix at k-th subcarrier can be decomposed as

$$H_k = U_k S_k V_k^H \tag{10}$$

where U_k, V_k and S_k define

$$U_k = \begin{bmatrix} u_{1,k} & u_{2,k} & \dots & u_{M,k} \end{bmatrix} \tag{11}$$

where $u_{i,k} = [u_{1,i,k} \quad u_{2,i,k} \quad \dots \quad u_{M,i,k}]^T$, (12)

$$V_k = \begin{bmatrix} v_{1,k} & v_{2,k} & \dots & v_{M,k} \end{bmatrix} \tag{13}$$

where $v_{i,k} = [v_{1,i,k} \quad v_{2,i,k} \quad \dots \quad v_{M,i,k}]^T$, (14)

and

$$S_k = \begin{bmatrix} \sigma_{1,k} & 0 & \dots & 0 \\ 0 & \sigma_{2,k} & 0 & \\ & & & \\ 0 & \dots & & \sigma_{M,k} \end{bmatrix} \tag{15}$$

where $\sigma_{i,j}$ is the i-th singular value at j-th subcarrier MIMO-OFDM channel with SVD decomposition.

The performance of the MIMO system as **Fig(3.1)** is determined by the correctiveness of CSI (Channel State Information) coming from receiver to transmitter, and it is very inefficiently if we perform SVD decomposition on H_k for all subcarrier. In this paper, we assume that CSI are available to transmitter via a low-rate, error-free, zero-delay feedback link from the receiver back to the transmitter. We will propose an algorithm for MIMO-ODFM system with lower computation and lower complex to interpolate beamforming vector, simulations show the effectiveness of the propose algorithm and compare with ideal case and other interpolation of beamforming vector.

According to MIMO system working at k-th subcarrier as **Fig(3.1)**, the transmit signal can be written as

$$X_k = [x_{1,k} \quad x_{2,k} \quad \dots \quad x_{M,k}]^T \tag{16}$$

that $x_{i,k}$ transmit from antenna i at k-th subcarrier and $X_k = V_k I_k$. The information I_k express as

$$I_k = [I_{1,k} \quad I_{2,k} \quad \dots \quad I_{M,k}]^T$$

and the receiver signal is

$$R_k = [R_{1,k} \quad R_{2,k} \quad \dots \quad R_{M,k}]^T \tag{17}$$

that $R_{i,k}$ is received at antenna i at k-th subcarrier, and can be express as

$$\begin{aligned} R_k &= \begin{bmatrix} R_{1,k} \\ R_{2,k} \\ \\ R_{M,k} \end{bmatrix} \\ &= \begin{bmatrix} h_{11,k} & h_{12,k} & \dots & h_{1M,k} \\ h_{21,k} & \dots & & \\ & & & \\ h_{M1,k} & \dots & & h_{MM,k} \end{bmatrix} \begin{bmatrix} x_{1,k} \\ x_{2,k} \\ \\ x_{M,k} \end{bmatrix} + \begin{bmatrix} n_{1,k} \\ n_{2,k} \\ \\ n_{M,k} \end{bmatrix} \\ &= H_k X_k + N_k \end{aligned} \tag{18}$$

that with AWGN noise express in (19) that each element are independent and identically distribution complex Gaussian with zero mean and N_0 variance.

$$N_k = \left[n_{1,k} \quad n_{2,k} \quad \dots \quad n_{M,k} \right]^T \tag{19}$$

The pre-coder V_k and de-coder U_k apply in (18), we can get decoder signal R_k as

$$\begin{aligned} & R_k \\ & = U_k^H Y_k \\ & = U_k^H (H_k X_k + N_k) \\ & = S_k I_k + U_k^H N_k \end{aligned} \tag{20}$$

that working at k-th subcarrier. It means that information about beamforming is full feedback to transmitter from receiver in order to get full diversity [7]-[10]. Observation equation of (20), we know that information signal of I_k can be transmitted to receiver without inter-symbol interference and no noise enhancement problem because decoder matrix of U_k is the orthogonal operator.

This communication system with full CSI to the transmitter and receiver, and that (20) has perfect form to resolve cross talk problem, we call this situation to be **Ideal Case**. But we know this communication system need to perform N times SVD and feedback N times with BPSK and coding for error free if FFT size is N. This is not only computation complex but also power consumption seriously.

3.2 Communicaton system with limited CSI to transmitter and receiver

In MIMO-OFDM system, if communication system consider as **Fig(3.1)**, then computation and power consumption increase with FFT size. In order to reduce not only computation complex but also power consumption seriously, we must reduce to perform SVD times and feedback times. Base on this mind, we are developed OSVDI (Orthogonal SVD Interpolation) at receiver and transmitter to interpolate decoder matrix of U_k and pre-coder matrix of V_k. The communication system as show in **Fig (3.2)** that working at k-th subcarrier.

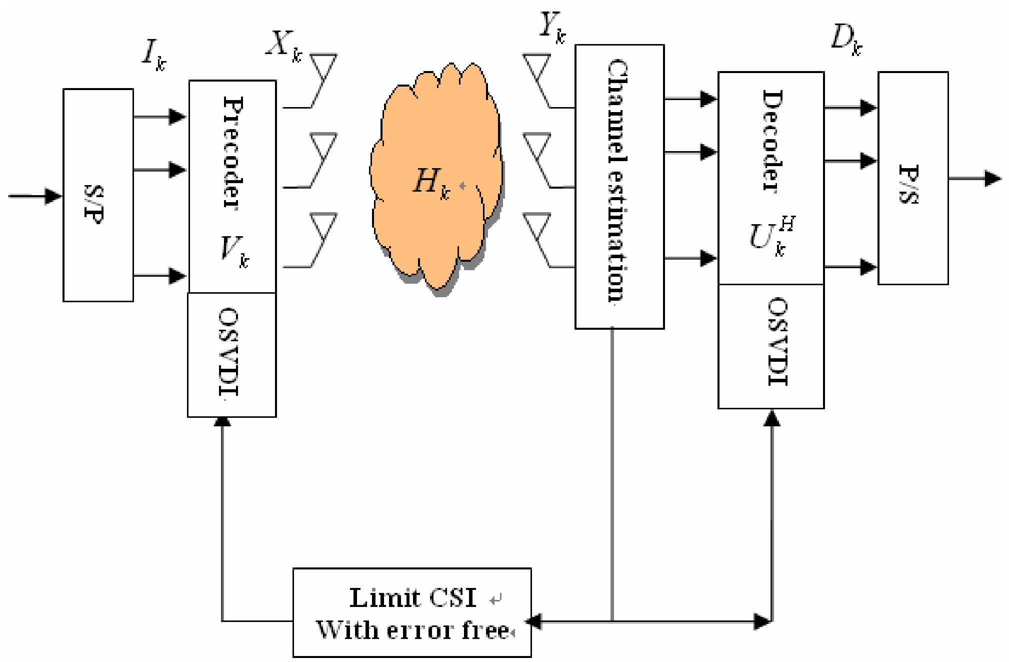


Fig (3.2). The simplify MIMO-OFDM system that work at k-th subcarrier and limited CSI feedback.

In this paper, in order to simplify problem, we assume that channel estimation is perfect. The receiver need to perform OSVDI to interpolate U_k only and feedback limited CSI to transmitter. In this paper, we assume transmit CSI from receiver is no impairment by BPSK modulation and addition channel coding to ensure this assumption. When the transmitter gets CSI from receiver, then perform OSVDI to interpolate V_k matrix only.

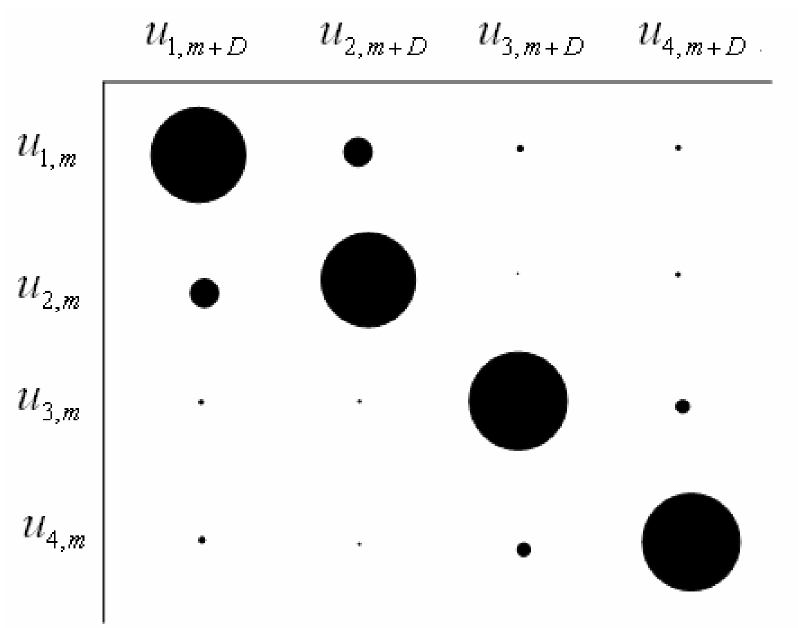
If OSVDI algorithm can efficient interpolate beamforming vector with a little performance loss than we can see that not only reduce feedback times but also reduce computation times for SVD operation by OSVDI (Orthogonal Singular Value Decomposition Interpolation). In next section, we will introduce OSVDI algorithm.

Chapter 4

■ Orthogonal singular value decomposition interplation

4.1 The correlation between beamforming vector and singular value

If we want to get the transform matrix from V_m to V_{m+D} or from U_m to U_{m+D} , the instinctive method can be derived from $U_{m+D} = U_m T_{m,m+D}$, then $T_{m,m+D} = U_m^H U_{m+D}$ and result can be show as **Fig (4.1)** where the value of D is the gap between adjacent pilot tone.



Fig(4.1) Using projection method to find out the correlation and permutation. The number of transmitter and receiver antenna is 4. The degree of spot size is corresponding to the degree of correlation.

We see a phenomenon as show in **Fig (4.2)** and corresponding to $T_{m,m+D}$ matrix state as show in **Fig (4.3).**

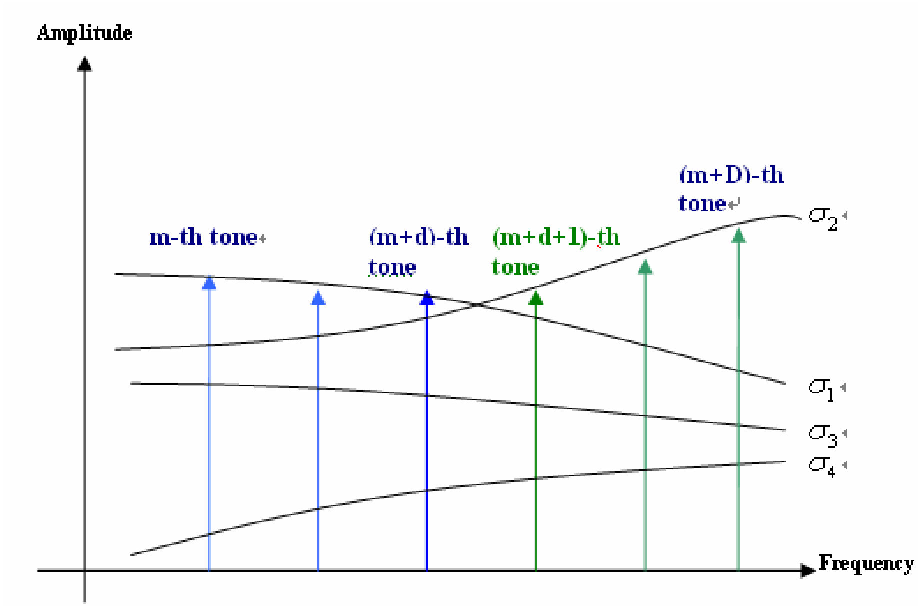
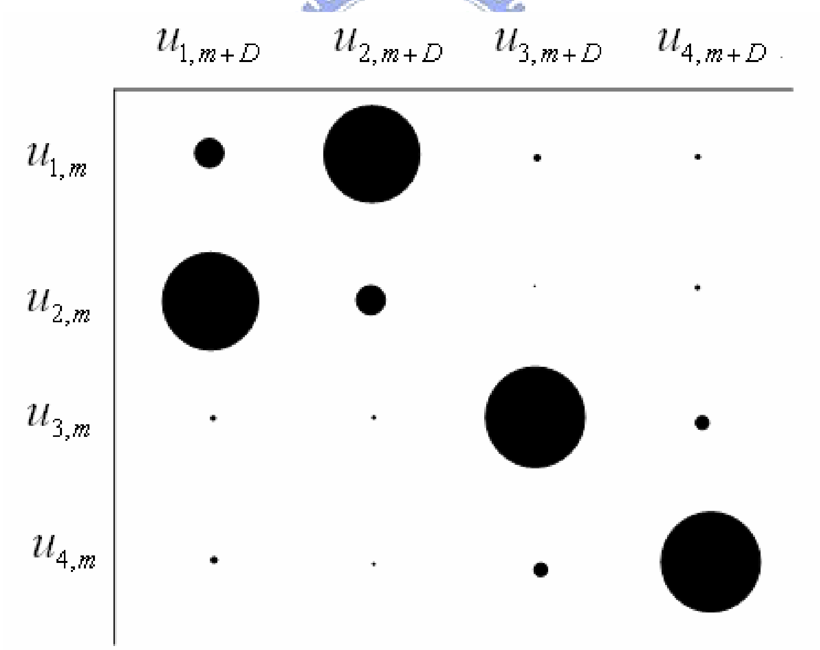


Fig (4.2) $Mr = M_t = 4$**, The order of singular value be switched.**



Fig(4.3) Using projection method to find out the correlation and permutation. The order of singular value has been changed.

In a small frequency range, the singular value linearly changes and has cross points as **Fig(4.2)** between (m+d)-th tone and (m+d+1)-th tone. We have an important assumption in this paper that each singular vector during a small frequency range linearly change from m-th subcarrier to (m+D)-th subcarrier, the phase linearly change uncorrelated of other singular vector.

If $D \neq 0$, observation of $T_{m,m+D}$ matrix, the element of $T_{m,m+D}$ matrix at diagonal line always has two properties. The first, the absolute value of these elements are bigger than the other element in the same column of $T_{m,m+D}$ matrix. And second, the absolute value of these elements is much closed to one with arbitrarily phase. Otherwise, if $D = 0$, these elements are unit norm with zero phase and the other absolute value are zero.

So we just pay attention to diagonal element of transform matrix and neglect the other element. When we just focus on diagonal line element, this mean that eigenvector are uncorrelated to each other. But sometime, we find an interest phenomenon that the biggest absolute values of $T_{m,m+D}$ matrix not happen to diagonal line as show in **Fig(4.2)** and **Fig(4.3)**. In **Fig(4.2)** and **Fig(4.3),** we know there has cross point from m-th subcarrier to (m+D)-th subcarrier. So we assume singular value is linear change when D is small, then we can estimate when the cross point will be occurred.

4.2 The orthogonal interpolation operation

For simplify this transform matrix, to interpolate U_{m+d} matrix between pilot tone of U_m and U_{m+D}, and to interpolate V_{m+d} by V_m and V_{m+D}. The OSVDI algorithm is to observe some of SVD characteristic and must hold as follow:

(a) The i-th and j-th column of U_m matrix are orthogonal.

(b) The i-th and j-th column of V_m matrix are orthogonal.

(c) Singular value in S_m matrix must in order.

(d) Change in Singular value is linear in small value of D.

Now, we define math form about pre-coder and de-coder matrix at (m+d)-th subcarrier, the de-coder matrix as

$$Z_{m+d} = \begin{cases} U_m \theta_{m,m+D,d} P_{m,m+D,d} & 0 < d < D/2 \\ U_{m+D} \theta_{m+D,m,d} P_{m+D,m,d} & D/2 < d < D \\ U_{m+d} & d = 0, D \end{cases} \tag{21}$$

and pre-coder matrix as

$$W_{m+d} = \begin{cases} V_m \phi_{m,m+D,d} P_{m,m+D,d} & 0 < d < D/2 \\ V_{m+D} \phi_{m+D,m,d} P_{m+D,m,d} & D/2 < d < D \\ V_{m+d} & d = 0, D \end{cases} \tag{22}$$

and U_m, V_m and S_m matrix define as (10)(12)(14).

The orthogonal operator matrix as $\theta_{i,j,d}$, $\phi_{i,j,d}$ and $P_{i,j,d}$ used to interpolate pre-coder matrix, de-coder matrix. We know that the pilot insert in i-th and j-th subcarrier and (i+d)-th subcarrier need to be interpolated.

In this paper, we assume the number of transmit antenna equal to receiver antenna M. Conclude above discuss, we know that we need insert pilot at i-th and j-th subcarrier. $\theta_{i,j,d}$ is phase rotation matrix and form as

$$\theta_{i,j,d} = \begin{bmatrix} e^{j\theta_{i,j,d,1}} & 0 & \ldots & 0 \\ 0 & e^{j\theta_{i,j,d,2}} & 0 & \ldots \\ & & & 0 \\ 0 & \ldots & 0 & e^{j\theta_{i,j,d,M}} \end{bmatrix} \tag{23}$$

and so is that to $\phi_{i,j,d}$, both them to hold that each singular vector and phase change during a small frequency region change from i-th subcarrier to j-th subcarrier are linearly increase or decrease and uncorrelated to the other singular vector, and that get entire element by

$$\theta_{i,j,d,l} = \begin{cases} \pm \dfrac{d * \tan^{-1}\left(\dfrac{\left\| imag(u_{l,i}^{H} u_{y,j}) \right\|}{\left\| real(u_{l,i}^{H} u_{y,j}) \right\|}\right)}{D} & j > i \\ \pm \dfrac{(D-d) * \tan^{-1}\left(\dfrac{\left\| imag(u_{l,i}^{H} u_{y,j}) \right\|}{\left\| real(u_{l,i}^{H} u_{y,j}) \right\|}\right)}{D} & i > j \end{cases} \tag{24}$$

and so is to $\phi_{i,j,d,l}$ and detail show in **APPENDIX** . If $y = l$, that mean permutation matrix $P_{i,j,d}$ is identity matrix. If $y \neq l$, y is decided by the form of $P_{i,j,d}$ because we have to select beamforming that high correlation between them.

The permutation matrix is to process the cross point when singular value order is changed during a small frequency region from (m+d)-th subcarrier to (m+d+1)-th subcarrier as **Fig(4.2)** and **Fig(4.3)**. Under the assumption that singular value is the linear change, we can estimate when the cross will be occurred and how to set up the permutation matrix to process this cross phenomenon. The permutation matrix of $P_{m,m+D,d}$ must be accumulative from m-th subcarrier to (m+d)-th subcarrier for holding the order of singular value and eigenvector if we interpolate (m+d)-th subcarrier. Above process method are also applied to $P_{m+D,m,d}$.

From equation (21) (22), an important feature to inserted operator is that these operators are orthogonal operation. So that these operation don't change the coordinate in order to hold SVD structure and avoid symbol interference in transmitted symbol.

Base on discussion before, we can think straight forward that if we want to interpolate more channel applied SVD, the phase of rotation matrix $\theta_{i,j,d}$ and $\phi_{i,j,d}$ can be assumed linear change between adjacent tones that gap in frequency domain is not fixed one. There is punish we must take from error of interpolation applied in Z_{m+d} or V_{m+d} matrix because of reducing transform matrix of $T_{m,m+D}$ and instead $\theta_{i,j,d}$, $\phi_{i,j,d}$ and $P_{i,j,d}$. It can be measured as

$$\Delta_u = \sum_{i=1}^{M}\sum_{j=1}^{M}\left\| u_{i,j,m+d} - z_{i,j,m+d} \right\|^2 \tag{25}$$

where $z_{i,j,m+d}$ is the element of Z_{m+d} at i-th row and j-th column and

$$\Delta_v = \sum_{i=1}^{M}\sum_{j=1}^{M}\left\| v_{i,j,m+d} - w_{i,j,m+d} \right\|^2 \tag{26}$$

where $w_{i,j,m+d}$ is the element of W_{m+d} at i-th row and j-th column.

But we must know that interpolation error not only come from (25) and (26) but also from residual impairment. The residual impairment that pre-coder matrix must match to de-coder matrix. We define a value as

$$\Gamma = \min_{j\in\{1,2...M\}} \frac{c_{j,j}}{\sum_{i=1}^{M} c_{i,j}} \tag{27}$$

where

$$c_{i,j} = \sum_{v=1}^{M} \left[z^{*}_{v,i,m+d} \left(\sum_{k=1}^{M} \left(h_{v,k,m+d} w_{k,j,m+d} \right) \right) \right] \tag{28}$$

to measure this error.

In general case, this value is very close to one and that is we hope if FFT size is large, and we will show that the value Γ is higher correlated with interpolation error of OSVDI algorithm by simulation result.

Chapter 5

■ Generalize OSVDI to other decomposition

5.1 Generalize OSVDI to other decomposition

Before introduction of OSVDI algorithm, we can understand this algorithm suitable for interpolation of orthogonal matrix because we insert operator are orthogonal type. So the OSVDI also can be applied to QR, GMD (Geometric Mean Decomposition) for reducing computation. And base on channel decomposition, to decouple the symbol interference we can use iterative process [5] [6].

The communication system is the same as **Fig(3.2)**, if we take QR decomposition to be major decomposition method to get beamforming vector and reduce computation complex at transmitter, then receiver decompose H_m^H as

$$H_m^H = \widetilde{Q}_m \widetilde{R}_m \tag{29}$$

to get $\widetilde{Q}_m$ and $\widetilde{R}_m$ matrix. Receiver has to feedback CSI about $\widetilde{Q}_m$ to transmitter and setting decoder matrix of Z_{m+d} to be identity matrix. Using OSVDI at transmitter to get pre-coder matrix of W_{m+d} as

$$W_{m+d} = \begin{cases} \widetilde{Q}_m \widetilde{\theta}_{m,m+D,d} & 0 < d < D/2 \\ \widetilde{Q}_{m+D} \widetilde{\theta}_{m+D,m,d} & D/2 < d < D \\ \widetilde{Q}_{m+d} & d = 0, D \end{cases} \tag{30}$$

where $\widetilde{\theta}_{m,m+D,d}$ and $\widetilde{\theta}_{m+D,m,d}$ are the phase rotation matrix using equation (23) and (24) that effect to $\widetilde{Q}_m$ and $\widetilde{Q}_{m+D}$, and the received signal can be refine as

$$R_{m+d} = \begin{cases} \left(\widetilde{R}_{m+d}^H \widetilde{Q}_{m+d}^H\right)\left(\widetilde{Q}_m \widetilde{\theta}_{m,m+D,d}\right) I_{m+d} + N_{m+d} & 0 < d < D/2 \\ \left(\widetilde{R}_{m+d}^H \widetilde{Q}_{m+d}^H\right)\left(\widetilde{Q}_{m+D} \widetilde{\theta}_{m+D,m,d}\right) I_{m+d} + N_{m+d} & D/2 < d < D \\ \widetilde{R}_{m+d}^H I_{m+d} + N_{m+d} & d = 0, D \end{cases} \tag{31}$$

and receiver only execute the iterative process on R_{m+d} to get information signal I_{m+d}. If decreasing computation at receiver is our major, receiver no feedback CSI to transmitter and using QR decomposition to H_m as

$$H_m = \hat{Q}_m \hat{R}_m . \tag{32}$$

At transmitter, the pre-coder matrix of W_{m+d} is to be identity matrix. At receiver, using OSVDI to get decoder matrix of Z_{m+d} as

$$Z_{m+d} = \begin{cases} \hat{Q}_m \hat{\theta}_{m,m+D,d} & 0 < d < D/2 \\ \hat{Q}_{m+D} \hat{\theta}_{m+D,m,d} & D/2 < d < D \\ \hat{Q}_{m+d} & d = 0, D \end{cases} , \tag{33}$$

where $\hat{\theta}_{m,m+D,d}$ and $\hat{\theta}_{m+D,m,d}$ are the phase rotation matrix using equation (23) and (24) that effect to $\hat{Q}_m$ and $\hat{Q}_{m+D}$, then received signal can be refine as

$$R_{m+d} = \begin{cases} \left(\hat{Q}_m \hat{\theta}_{m,m+D,d}\right)^H \left(\hat{Q}_{m+d} \hat{R}_{m+d}\right) I_{m+d} + N_{m+d} & 0 < d < D/2 \\ \left(\hat{Q}_{m+D} \hat{\theta}_{m+D,m,d}\right)^H \left(\hat{Q}_{m+d} \hat{R}_{m+d}\right) I_{m+d} + N_{m+d} & D/2 < d < D \\ \hat{R}_{m+d} I_{m+d} + N_{m+d} & d = 0, D \end{cases} . \tag{34}$$

Finally, we iterative process the signal R_{m+d} in order to decouple received signal and find out the I_{m+d} information. On the other hand, if we use GMD decomposition to design beamforming vector and decide reducing computation complex at transmitter. We decompose H_m^H as

$$H_m^H = \overline{Q}_m \overline{R}_m \overline{P}_m^H . \tag{35}$$

To set up the decoder matrix of Z_{m+d} be identity matrix and pre-coder matrix of W_{m+d} as

$$W_{m+d}=\begin{cases}\bar{Q}_m\bar{\theta}_{m,m+D,d} & 0<d<D/2\\ \bar{Q}_{m+D}\bar{\theta}_{m+D,m,d} & D/2<d<D\\ \bar{Q}_{m+d} & d=0,D\end{cases}, \tag{36}$$

where $\bar{\theta}_{m,m+D,d}$ and $\bar{\theta}_{m+D,m,d}$ are the phase rotation matrix using equation (23) and (24) that effect to $\bar{Q}_m$ and $\bar{Q}_{m+D}$, then received signal can be written as

$$R_{m+d}=\begin{cases}\left(\bar{Q}_{m+d}\bar{R}_{m+d}\bar{P}_{m+d}^H\right)^H\left(\bar{Q}_m\bar{\theta}_{m,m+D,d}\right)I_{m+d}+N_{m+d} & 0<d<D/2\\ \left(\bar{Q}_{m+d}\bar{R}_{m+d}\bar{P}_{m+d}^H\right)^H\left(\bar{Q}_{m+D}\bar{\theta}_{m+D,m,d}\right)I_{m+d}+N_{m+d} & D/2<d<D\\ \left(\bar{R}_{m+d}\bar{P}_{m+d}^H\right)^H I_{m+d}+N_{m+d} & d=0,D\end{cases}. \tag{37}$$

Received signal product with $\bar{P}_{m+d}$ and execute the iterative process on R_{m+d} to get I_{m+d}. If decreasing computation at receiver is our goal, receiver no feedback CSI to transmitter and using QR decomposition to H_m as

$$H_m=\bar{\bar{Q}}_m\bar{\bar{R}}_m\bar{\bar{P}}_m^H. \tag{38}$$

Let W_{m+d} is to be identity matrix and decoder matrix of Z_{m+d} as

$$Z_{m+d}=\begin{cases}\bar{\bar{Q}}_m\bar{\bar{\theta}}_{m,m+D,d} & 0<d<D/2\\ \bar{\bar{Q}}_{m+D}\bar{\bar{\theta}}_{m+D,m,d} & D/2<d<D\\ \bar{\bar{Q}}_{m+d} & d=0,D\end{cases}. \tag{39}$$

where $\bar{\bar{\theta}}_{m,m+D,d}$ and $\bar{\bar{\theta}}_{m+D,m,d}$ are the phase rotation matrix using equation (23) and (24) that effect to $\bar{\bar{Q}}_m$ and $\bar{\bar{Q}}_{m+D}$, then received signal can be written as

$$R_{m+d}=\begin{cases}\left(\bar{\bar{Q}}_m\bar{\bar{\theta}}_{m,m+D,d}\right)^H\left(\bar{\bar{Q}}_{m+d}\bar{\bar{R}}_{m+d}\bar{\bar{P}}_{m+d}^H\right)I_{m+d}+N_{m+d} & 0<d<D/2\\ \left(\bar{\bar{Q}}_{m+D}\bar{\bar{\theta}}_{m+D,m,d}\right)^H\left(\bar{\bar{Q}}_{m+d}\bar{\bar{R}}_{m+d}\bar{\bar{P}}_{m+d}^H\right)I_{m+d}+N_{m+d} & D/2<d<D\\ \left(\bar{\bar{R}}_{m+d}\bar{\bar{P}}_{m+d}^H\right)I_{m+d}+N_{m+d} & d=0,D\end{cases} \tag{40}$$

5.2 Simulation result

The simulation parameters as show in **TABLE I**, we use frequency selective fading channel and adjust FFT and D size to observe channel estimation error. We propose an algorithm not only uncomplicated to estimate channel state information but also estimate pre-coding and decoding matrix.

Transmit antenna	4
Receive antenna	4
FFT Size	128 / 256 / 512 / 1024 / 2048
D Size	2 / 3 / 5 / 7
Modulation	QAM
Multi-path Length	3

TABLE I. Simulation Parameter. Change FFT and D size.

The first we will show interpolation error by OSVDI algorithm as below **Fig (5.1)** and the measure value of MSE define as

$$MSE = \frac{1}{N} \sum_{m=1+0*D}^{1+r*D} \sum_{d=0}^{D} \sum_{i=1}^{M} \sum_{j=1}^{M} \left\| h_{i,j,m+d} - h'_{i,j,m+d} \right\|^2 \quad (41)$$

where $h'_{i,j,m+d} = \sum_{k=1}^{M} \left[z_{i,k,m+d} (\sigma_{k,m+d} w^*_{j,k,m+d}) \right]$ and the value of r is equal to $\left\lfloor \frac{N-1}{D} \right\rfloor$.

So we know that OSVDI performance is depend on size of D and FFT if multipath is fixed and the number of D will determine how degree of computation reduction. We discuss a measure value of Γ before. This value will determine the MSE value and closed to unit is better. Simulation result as **Fig (5.2).**

We can find out the relation between Γ and MSE are an inverse ratio. We have to know if FFT size increase or D decrease, the value of Γ will close to unit and MSE will decrease to zero. Finally, we show the communication system as **Fig (3.1)** and compare with communication system as **Fig (3.2)** to understand computation complex reduction by OSVDI cause degree of performance impairment.

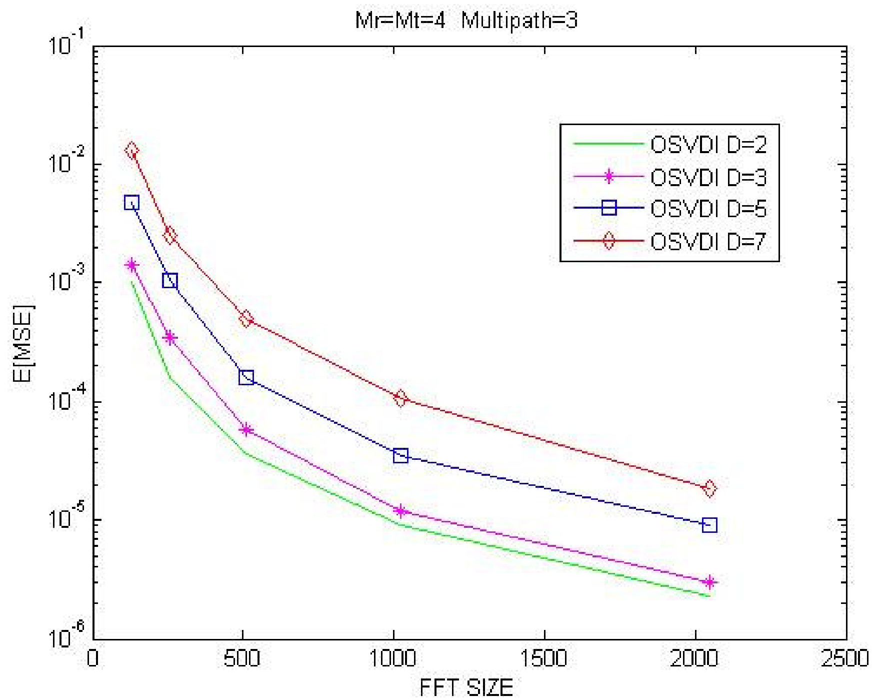


Fig (5.1) The number of transmitter and receiver antenna is equal to 4, frequency selective fading with multipath 3. The D is the gap size between adjacent pilots. To measure MSE come from OSVDI impairment.

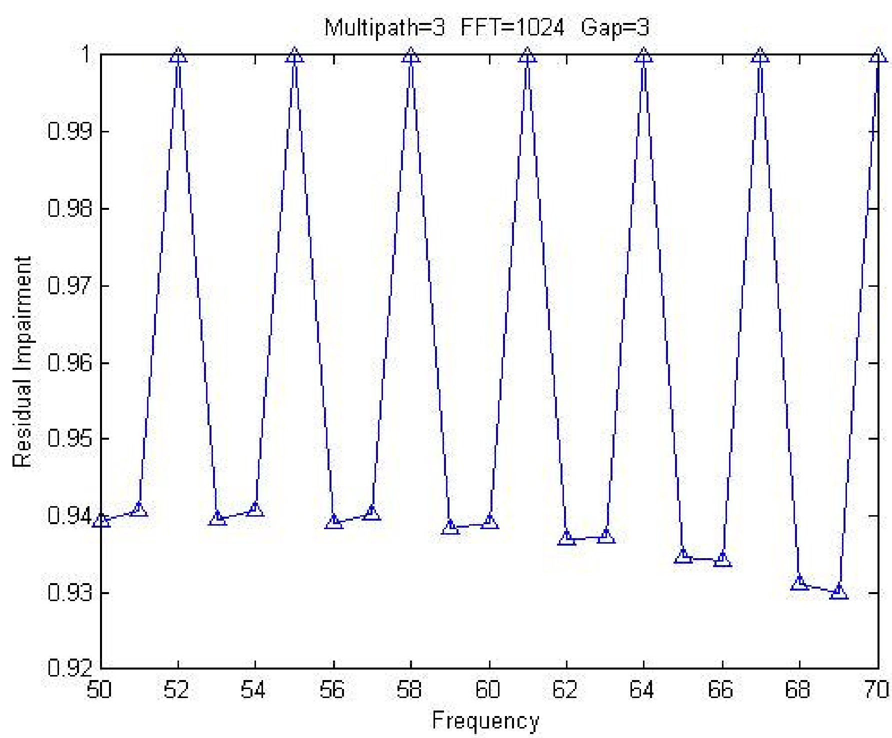


Fig (5.2) Residual impairment and measure as Γ, D=3

When we adjust FFT size, the performance result show in **Fig(5.3)** and parameter in **TABLE II**. When we adjust D size, the performance result show in **Fig(5.4)** and parameter in **TABLE III**. We will find there is an error floor happened due to small FFT size or large D caused larger OSVDI impairment.

Transmit antenna	4
Receive antenna	4
FFT Size	128 / 256 / 512 / 1024 / 2048
D Size	3
Modulation	QAM
Multi-path Length	3

TABLE II. Simulation Parameter. Fix D size and change

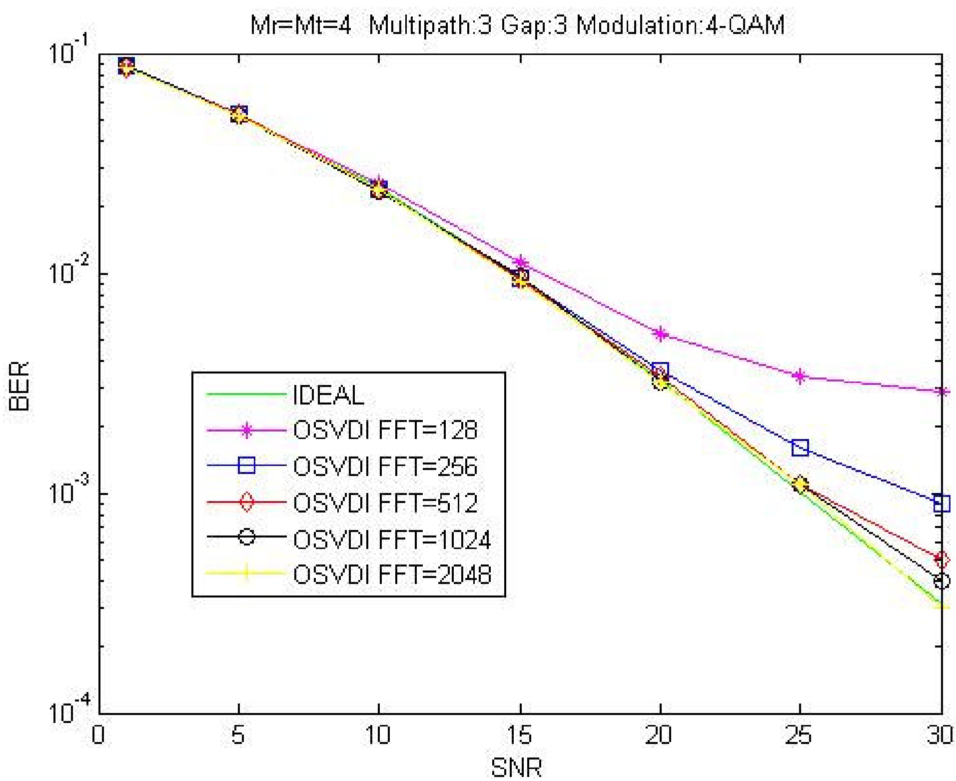


Fig (5.3). The MIMO-OFDM system with pre-coder and decoder.

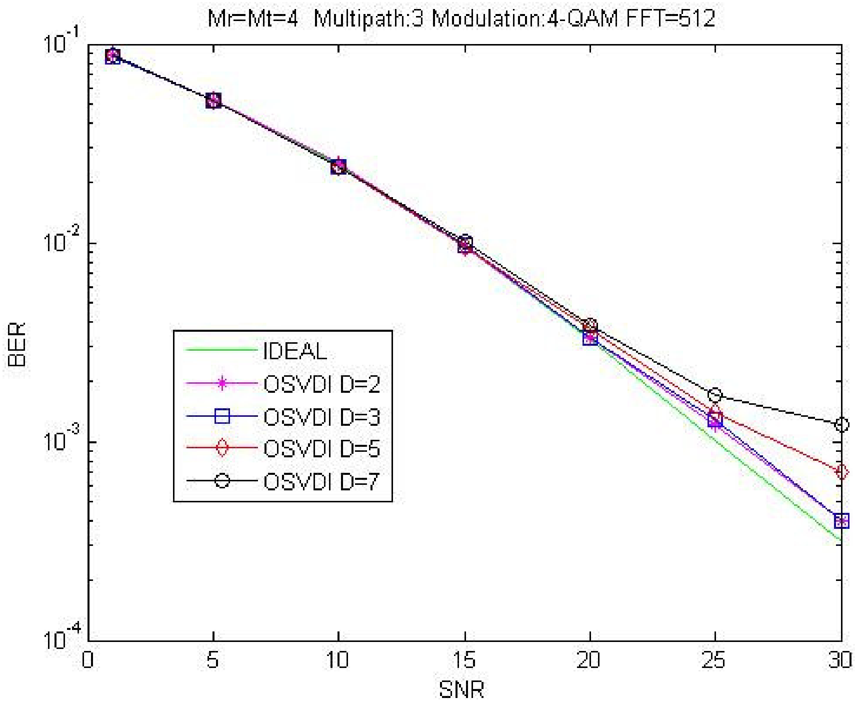


Fig (5.4) The MIMO-OFDM system with precoder and decoder.

Transmit antenna	4
Receive antenna	4
FFT Size	512
D Size	2 / 3 / 5 / 7
Modulation	QAM
Multi-path Length	3

TABLE III. Simulation Parameter Fix FFT size and change D size.

In detail discussion about **Fig(5.3)**, there have distinct and large error floor due to small FFT size. So we can set what FFT size is suitable base on degree of multipath. In detail discussion about **Fig(5.4)**, there have no distinct or large error floor. Because of correlation between pilots still strong even D is equal to 7 due to FFT size is large enough to sampling channel state information at frequency domain detailed.

Now, we will change our object of full multiplexing gain in order to compare with other algorithm of beamforming vector interpolation. Consider other MIMO-OFDM system and algorithm as [4]. We call this algorithm to be IBV (Interpolation of beamforming vector) and the major equations to interpolate beamforming vector as

$$w_{m+d}(\theta) = \frac{(1-\frac{d}{D})w_m + \frac{d}{D}e^{j\theta_{m+d}}w_{m+D}}{\left\|(1-\frac{d}{D})w_m + \frac{d}{D}e^{j\theta_{m+d}}w_{m+D}\right\|} \tag{42}$$

and

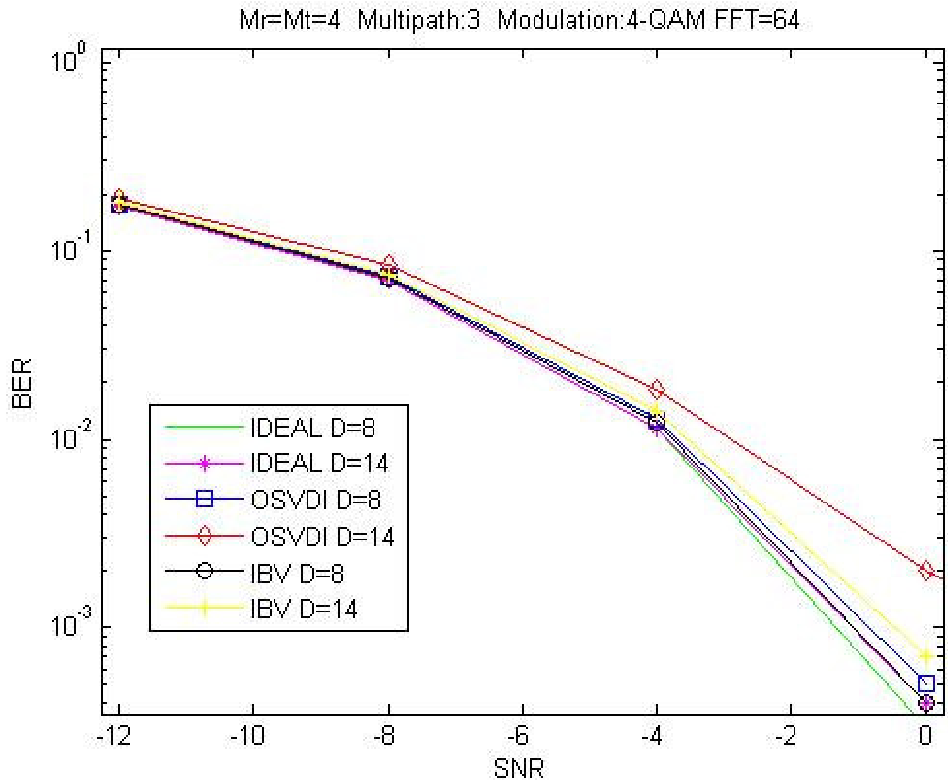
$$\theta_{m+d} = \arg\max_{\theta} \left\|H_{m+D/2}w_{m+D/2}(\theta)\right\|^2 \tag{43}$$

where $\theta = \{0, 2\pi/P, 4\pi/P, ..., 2(P-1)\pi/P\}$. In equation (42), there have division operation in order to normalize after interpolation by linear weight method. In equation (43), there use optimal criteria to maximum the worst channel gain in order to ensure maximum SNR.

If our MIMO-OFDM system as **Fig(2.1)** is changed to [4], we can see that data rate will decrease M_t time from NM_t but diversity gain will increase from one to M_t times. The simulation parameter as **Table IV** and simulation result show in **Fig(5.5)**

Transmit antenna	4
Receive antenna	4
FFT Size	64
D Size	8 14
Modulation	QAM
Multi-path Length	3

TABLE IV. Simulation parameter. Using small FFT size and two different D size.



Fig(5.5) The MIMO-OFDM system with precoder and decoder. Change D size and algorithm. Receiver has perfect CSI and feedback CSI to transmitter without error.

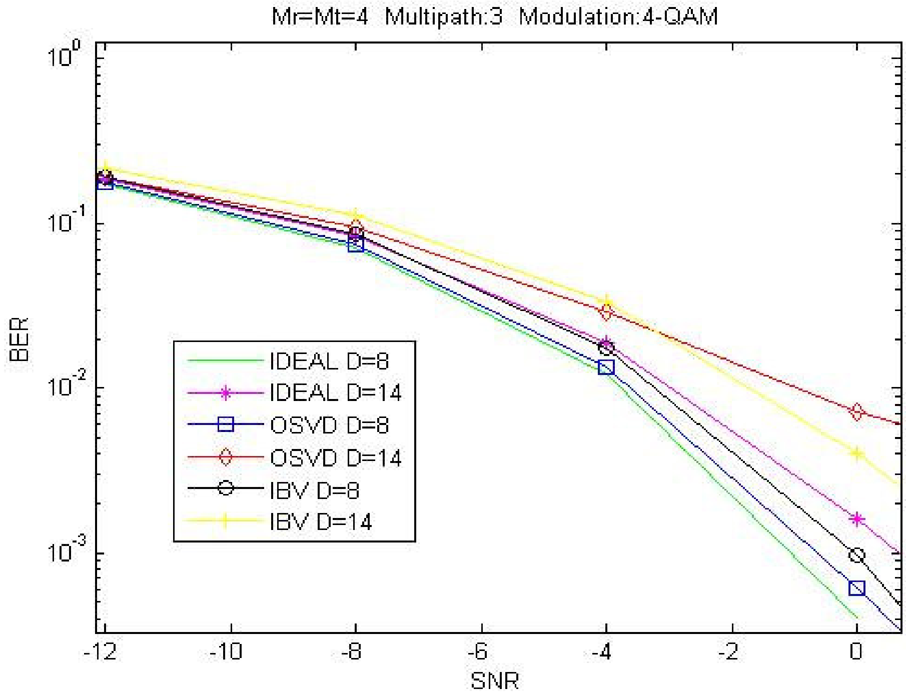
Observation of **Fig(5.5)**, we see performance of IBV is better than OSVDI. The reason will discuss and will show simulation result to prove our concern. The first, we develop OSVDI algorithm in order to maintain full multiplexing gain, so we insert orthogonal operation to interpolate beamforming vector under no damage of orthogonal property in pre-coder and de-coder but IBV is not. The second, our assumption of singular mode that linear change to be broken because of the size of D is too large compare with FFT size and the number of multi-path but IBV can dynamic adjust by optimal criteria. But we have to know if D is small, then performance of IBV and OSVDI are close to each other and usually practical communication system for example 802.16e WiMAX system set D to be small size.

The number of multiplication shows about in **TABLE V**. The IBV need $M(N-N/D)$ division for normalize interpolated beamforming vector and the OSVDI need $2N/D$ division to calculate phase. The computation complex of IBV comes from (42) (43), apply SVD in MIMO-ODFM channel, IDFT and DFT, combination at transmitter and receiver. The computation complex of OSVDI comes from (21) (22) (24), IDFT and DFT, apply SVD in MIMO-OFDM channel. We observe equation (42) (43), IBV need full CSI to calculate de-coder vector, need division operation in order to normalize interpolated pre-coder and force to find rotation phase by optimal criteria. But OSVDI algorithm is not, only need CSI on pilot tone because pre-coder and de-coder are interpolated, fewer division operation and instead by orthogonal operation and shift register, no force to find rotation phase by equation of (46).

	# of multiplication at Tx	# of multiplication at Rx
Ideal Case	$MN(1+\log_2 N)$	$MN(\log_2 N+1+M^2)$
IBV	$NM(\log_2 N+1+2\frac{D-1}{D})$	$MN\left[\log_2 N+1+(1+\frac{1}{D})\frac{M^2}{2}+(3+M)\frac{P}{D}\right]$
OSVDI	$NM(\log_2 N+1+2\frac{D-1}{D})$	$NM(\log_2 N+1+\frac{M^2}{D}+2\frac{D-1}{D})$

TABLE III. The number of multiplication.

Now, we will show another simulation result, simulation parameter as TABLE II but receiver only has perfect CSI on pilot tone and other tone using linear interpolation to get. The simulation result as **Fig(5.6)**.



Fig(5.6) The MIMO-OFDM system with precoder and decoder. Change D size and algorithm. Receiver only has perfect CSI on pilot tone and feedback CSI to transmitter without error.

In **Fig(5.6)**, we find OSVDI performance is better than IBV under $D=8$ but $D=14$ is not. In $D=8$ case, because IBV need more CSI than OSVDI, so performance of OSVDI is better than IBV. In $D=14$ case, IBV still need more CSI than OSVD but assumption of singular value to be linear change in OSVDI is broken. Also OSVDI has better performance in lower SNR but still have obvious error floor in high SNR.

Chap 6

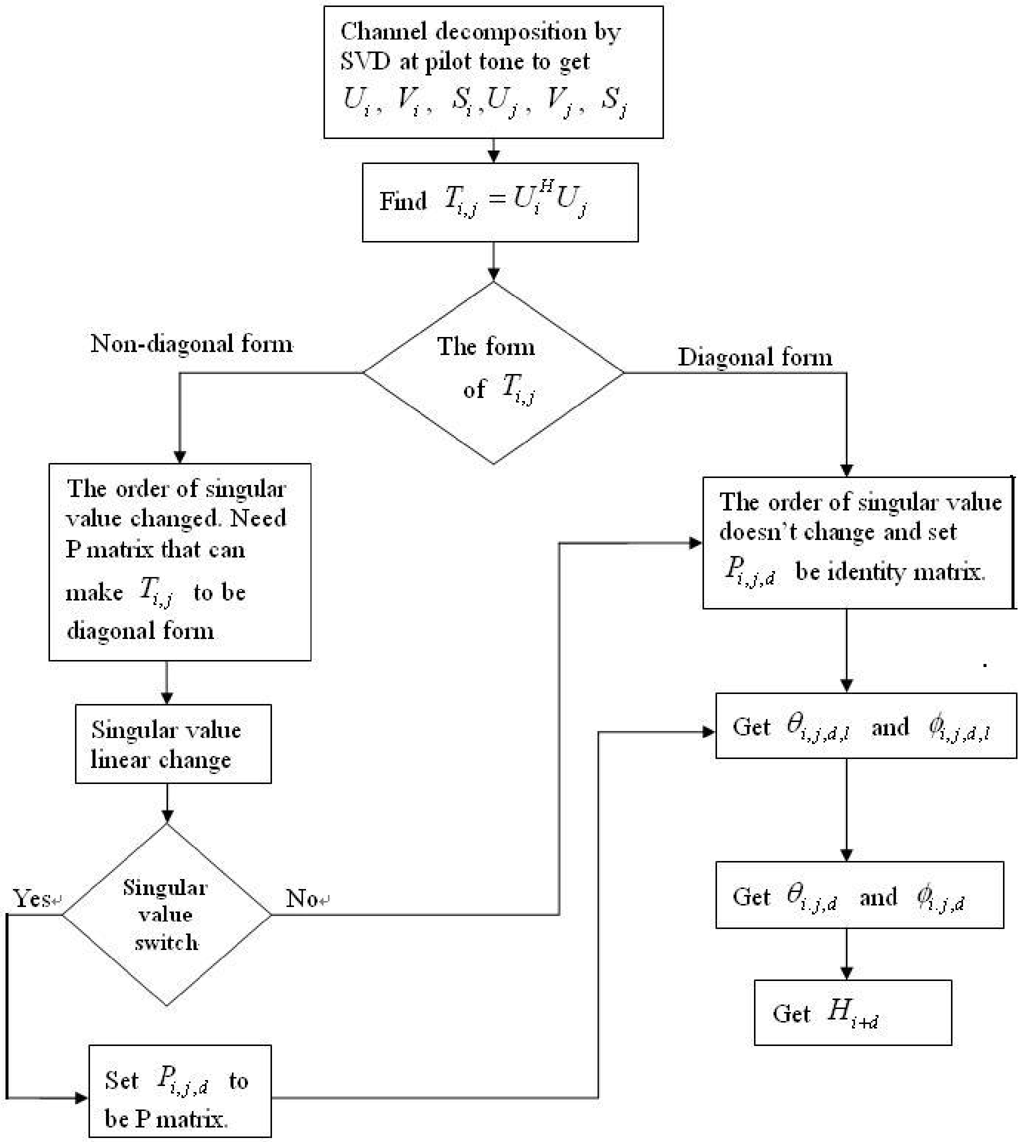
■ Conclution

We propose an algorithm OSVDI to interpolate pre-coder and de-coder matrix for reducing computation complex from SVD applying and feedback times. But how to chose FFT size, D size, the number of transmitter and receiver antenna become a trade off and how to improve residual impairment also become the main point. In this paper, pre-coder and de-coder has high correlation in adjacent frequency tone and we also base on these feature to interpolate pre-coder and de-coder by OSVDI. In the other word, we also know that pre-coder and de-coder has high correlation in adjacent time and also can be applied OSVDI in time domain.

The OSVDI algorithm can be applied to transmitter and receiver in order to reduce computation complex and feedback time, and the flowchart can be show as **Fig (6.1).**

We advance three measure value as (25) (26) and (27). Using (25) and (26) to measure error come from interpolation of OSVDI at receiver and transmitter. We have to know even error value that measure from (25) and (26) are small, but overall performance still have error floor because of residual impairment (27). Finally, we discuss about different algorithm of IBV and OSVDI. We know OSVDI algorithm is generalize, because it can be full spatial multiplex gain with diversity gain one or sacrifice some spatial multiplex gain in order to get diversity gain. Now, trade off between spatial multiplex gain and diversity gain also become our major.

Finally, we discuss about different algorithm of IBV and OSVDI. We know OSVDI algorithm is generalize, because it can be full spatial multiplex gain with diversity gain one or sacrifice some spatial multiplex gain in order to get diversity gain. Now, trade off between spatial multiplex gain and diversity gain also become our major.



Fig(6.1) OSVDI algorithm flowchart. Under $j > i$ and $\|j - i\| = D$. If $i > j$, then exchange the symbol i and j in this flowchart.

APPENDIX

In this paper, we know phase rotation from degree of zero to $\theta_{m,n,w}$ and change not seriously. In simulation result, we can observe that the measure of phase change more than $\frac{\Pi}{2}$ suddenly. The simulation parameter is in **TABLE IV** and measure value of $\theta_{m,n,v}$ show in **TABLE V**.

Transmit antenna	4
Receive antenna	4
FFT Size	1024
D Size	8
Modulation	QAM
Multi-path Length	4

TABLE IV. Simulation parameter. Fix FFT and D size.

$\theta_{1,9,d,l}$	$l=1$	$l=2$	$l=3$	$l=4$
$d=0$	0	0	0	0
$d=1$	-0.005219	-0.01334	-0.0448	-0.014986
$d=2$	-0.010504	-0.0267	-0.08775	-0.02976
$d=3$	-0.015859	-0.04008	-0.12887	-0.044367
$d=4$	-0.021287	-0.05348	-0.16817	3.0827
$d=5$	-0.026792	-0.06692	-0.20569	3.0684

TABLE V. The measure value of $\theta_{1,9,d,l}$, there have suddenly change in $l=4$ from $d=3$ to $d=4$.

The reason is that U, S, and V matrix from SVD is not unique. In order to simplify question, we consider 2X2 matrix and prove as

$$
\begin{aligned}
& H \\
&= \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \\
&= \begin{bmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \end{bmatrix} \begin{bmatrix} s_1 & 0 \\ 0 & s_2 \end{bmatrix} \begin{bmatrix} v_{11} & v_{12} \\ v_{21} & v_{22} \end{bmatrix}^H \\
&= \begin{bmatrix} u_{11} & -u_{12} \\ u_{21} & -u_{22} \end{bmatrix} \begin{bmatrix} s_1 & 0 \\ 0 & s_2 \end{bmatrix} \begin{bmatrix} v_{11} & -v_{12} \\ v_{21} & -v_{22} \end{bmatrix}^H
\end{aligned} \tag{44}
$$

So we know when $\theta_{m,m+D,d,l}$ is larger than $\pi/2$, is mean that the beamforming vector at transmitter rotate $\|\Pi\|$ and corresponding to beamforming vector also rotate $\|\Pi\|$ at receiver. We conclude above discussion, make a table shoe in **TABLE VI** and so is to $\theta_{m,m+D,d,l}$ under permutation matrix to be identity matrix.

Phase Choose	$real(u_{l,m}^H u_{l,m+D}) > 0$	$real(u_{l,m}^H u_{l,m+D}) < 0$
$imag(u_{l,m}^H u_{l,m+D}) > 0$	$\theta_{m,m+D,d,l} = \frac{d*\tan^{-1}\left(\frac{\left\|imag\left(u_{l,m}^H u_{l,m+D}\right)\right\|}{\left\|real\left(u_{l,m}^H u_{l,m+D}\right)\right\|}\right)}{D}$	$\theta_{m,m+D,d,l} = -\frac{d*\tan^{-1}\left(\frac{\left\|imag\left(u_{l,m}^H u_{l,m+D}\right)\right\|}{\left\|real\left(u_{l,m}^H u_{l,m+D}\right)\right\|}\right)}{D}$
$imag(u_{l,m}^H u_{l,m+D}) < 0$	$\theta_{m,m+D,d,l} = -\frac{d*\tan^{-1}\left(\frac{\left\|imag\left(u_{l,m}^H u_{l,m+D}\right)\right\|}{\left\|real\left(u_{l,m}^H u_{l,m+D}\right)\right\|}\right)}{D}$	$\theta_{m,m+D,d,l} = \frac{d*\tan^{-1}\left(\frac{\left\|imag\left(u_{l,m}^H u_{l,m+D}\right)\right\|}{\left\|real\left(u_{l,m}^H u_{l,m+D}\right)\right\|}\right)}{D}$

TABLE VI. Phase choose in OSVDI algorithm.

Additional, in **TABLE V,** we know that the phase is very small, so we have opportunity to reducing computation complex come from $\tan^{-1}(x)$. By Maclaurin series, we get

$$
\begin{aligned}
& \tan^{-1}(x) \\
&= \tan^{-1}(x_0) + \frac{1}{(1+x_0^2)}(x-x_0) + \frac{-x_0}{(1+x_0)^2}(x-x_0)^2 + \ldots
\end{aligned} \quad . \tag{45}
$$

In general case, the phase is small than 0.3, so we set $x_0 = 0.15$ and take to first order then

$$\tan^{-1}(x) \approx 0.0022 + 0.978 * x \quad . \tag{46}$$

Above discussion tell me why add negative to phase, how add negative and when we have add negative to phase in order to match phase change state. The $\phi_{m,m+D,d,l}$, $\phi_{m+D,m,d,l}$ and $\theta_{m+D,m,d,l}$ also hold these concept but $\theta_{m+D,m,d,l}$ and $\phi_{m+D,m,d,l}$ determine phase has different as (24).

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