國　立　交　通　大　學

電子工程學系 電子研究所碩士班

碩　士　論　文

在感知無線電網路中支分散式合作頻譜偵測

Distributed Cooperative Spectrum Sensing in Cognitive Radio Networks

研 究 生：游衛川

指導教授：簡鳳村　博士

中 華 民 國 九 十 七 年 七 月

Distributed Cooperative Spectrum Sensing in Cognitive Radio Networks

研究生：游衛川 Student: Wei-Chuan Yu
指導教授：簡鳳村 Advisor: Dr. Feng-Tsun Chien

國 立 交 通 大 學
電 子 工 程 學 系　電 子 研 究 所 碩 士 班
碩 士 論 文

A Thesis
Submitted to Department of Electronics Engineering & Institute of Electronics
College of Electrical and Computer Engineering
National Chiao Tung University
in Partial Fulfillment of the Requirements
for the Degree of Master
in
Electronics Engineering

June 2008

HsinChu, Taiwan, Republic of China

中華民國九十七年七月

在感知無線電網路中支分散式合作頻譜偵測

研究生: 游衛川　　　　　　　　　　指導教授: 簡鳳村 博士

國立交通大學

電子工程學系 電子研究所碩士班

摘要

感知無線電網路藉由動態頻譜存取而擁有更高的頻譜效率。因此，它將會成為未來無線通訊系統最常使用來減輕頻譜缺乏問題的技術。在感知無線電網路中頻譜偵測是主要且棘手的任務。然而，由於遮蔽、干擾、和無線通道的時變性質的影響，造成各別的感知無線電無法可靠和迅速地檢測出主要訊號是否存在、在本論文中，我們提出一簡單但有效率的合作式頻譜偵測且基於能量檢測。我們在次要的使用者與聯合中心之間考慮了兩個案例。一是只考慮通道雜訊，另外是考慮通道雜訊和干擾。最後，我們對修改的反射係數作最佳化，來找出最佳線性組合係數。藉由電腦模擬，我們觀察到所提出的合作式方法有較好的成果，而且藉由增加次要使用者的數目來改善偵測可靠性。

Distributed Cooperative Spectrum Sensing in Cognitive Radio Networks

Student: Wei-Chuan Yu   Advisor: Dr. Feng-Tsun Chien

Department of Electronic Engineering &
Institute of Electronics
National Chiao Tung University

Abstract

Cognitive radio network enables much higher spectrum efficiency by dynamic spectrum access. Therefore, it will be a popular technique for future wireless communications to mitigate the spectrum scarcity issue. Spectrum sensing is a main and tough task in cognitive radio networks. However, due to the effect of shadowing, fading, and time-varying nature of wireless channels, the individual cognitive radio may not be able to reliably and quickly detect the existence of a primary signal. In this thesis, we propose a simple yet efficient cooperation spectrum sensing based on energy detection, and consider the channel between the secondary user and fusion center in two cases. First, we consider only the channel noise between the secondary user and the fusion center (i.e., constant AWGN channel), and then we extend to consider both the perturbation noise and channel fading between the secondary user and the fusion center (i.e., fading channel). Our objective is to improve the detection performance while considering a realistic system environment. Finally, we optimize a modified deflection coefficient to find the optimal linear combining weights. From the simulations, we can observe that the proposed cooperation method has the better detection performance than the other methods, and the sensing reliability improves as the number of secondary users increase.

誌謝

這篇論文能夠順利完成，首先要感謝我的指導教授 簡鳳村 老師的細心指導。在這二年的研究生涯中，老師不僅在研究上給予適當的指導，提供了我許多寶貴的建議，讓我在研究時能有更多方面的考量，也使我在為人處世上獲益匪淺，同時也關心我們的日常生活。

另外，要感謝的是與我在通訊領域之中一起討論、研究的崑健與錫祺學長，不厭其煩的為我解惑，謝謝你們熱心地幫我解決了許多通訊方面相關的疑問。也感謝在研究上都一直共同打拼的自民、佳楓、永霖、宇哲、信榮，在修課中遇到問題時，大家互相討論，解決難題，讓我在這之中成長許多。也感謝實驗室的其他成員，在碩士這兩年中帶給我歡樂、成長，在生活上給我支持、勉勵，使我充滿信心。也感謝通訊電子與訊號處理實驗室(commlab)，提供了充足的軟硬體資源，讓我在研究中不虞匱乏。

最後，我要感謝我的父親塗金、母親寶蘭在我身後支持著我，對於我求學期間的幫助包容與體諒，讓我能夠積極的面對挑戰的決心與勇氣。

在此僅以這篇論文獻給所有幫助過我，陪伴我走過這一段歲月的師長，同學，朋友與家人，謝謝!

誌於 2008.7 風城交大

衛川

Contents

List of Figures

List of Tables

Chapter 1
Introduction

1.1 Significance

Wireless communication systems have been used widely for a long time. We know that spectrum plays an important role of the wireless communication. Without spectrum, no wireless communications would be possible. However, radio spectrum is a precious and limited resource, so we must use the limited spectrum efficiently. In order to improve the spectrum efficiency, cognitive radio networks is proposed to dynamically use the idle spectrum in a smart way [1]. According to the Federal Communications Commission (FCC) [2], cognitive radio (CR) is defined as the radio system that continuously performs spectrum sensing, dynamically identify unused spectrum, and then operate in those spectrum holes where the licensed (primary) radio systems are idle and use the spectrum only if communication does not interfere with the primary user. Therefore, the cognitive radio enables much higher spectrum efficiency by dynamic spectrum access, [3], [4]. And it is a potential technique for future wireless communications to mitigate the spectrum lack problem. This new communication system is referred to as neXt Generation (XG) or Dynamic Spectrum Access (DSA) network.

Spectrum sensing is an important task in the cognitive radio networks, and it needs to reliably and quickly detect the primary signal. Spectrum sensing should also monitor the activation of primary user in order for the secondary users to vacate the occupied spectrum. However, spectrum sensing is a difficult task because of the hostile nature of the wireless channel, such as shadowing, fading, and time-varying of wireless channels. To tackle that problem, recent research studies of spectrum sensing have focused on the detection of primary transmissions by cognitive radio devices.

Generally, the energy detector has been applied widely among these existing spectrum sensing techniques. This is because that it does not require any a priori knowledge of the primary signals and has much lower complexity than other detectors. In other words, if the secondary user has limited information about the primary signals, then the energy detector is optimal [5]. In chapter 3, we assume that the primary signal is unknown and we adopt energy detector.

1.2 Motivation

Cognitive radio is a potential technique for future wireless communications. However, the detection performance of spectrum sensing is usually dependent on destructive channel fading, since it is difficult to detect the primary signals in environments with deep fades. In order to improve the reliability of spectrum sensing, cooperative spectrum sensing exploiting the spatial diversity among secondary users has been proposed recently [3], [6]. A cooperation cognitive radio network would have a better detection performance by combining multiple sensing information from possibly correlated secondary users. In other words, cooperative spectrum sensing exploits the spatial diversity to have a better detection performance. Thus, it could reduce the probability of interfering with primary users.

Although the distributed detection has been studied since early 90's (e.g. [8], [9]), but the result might not be directly applied to cognitive radios, and the research of cooperative spectrum sensing is very limited. In [10], the voting rule is one of the simplest suboptimal solutions. It counts the number of cognitive radio nodes that decide for the presence of the signal and compares it with a given threshold. In [11], the fusion rule with OR logic operation was used to combine decisions from several secondary users. In [12], the hard decision with the AND operation and soft decision using the Neyman-Pearson criteria was proposed. It was shown that the soft decision combination of spectrum sensing outperforms hard decision combination. In [13], they exploited the fact that summing signals from two secondary users can increase the signal-to-noise ratio (SNR) as well as the detection reliability if the signals are correlated. In [14], they proposed the cooperation spectrum sensing method, and they assumed perfect control channel. In other words, they did not consider that channel fading and channel noise between the secondary user and fusion center. In practice, it is not realistic in the actual situation. Therefore, in this thesis, we propose the cooperation spectrum sensing and consider the channel in two cases. One is to consider only the channel noise between the secondary user and fusion center (i.e., constant AWGN channel), and the other is considered both of the channel noise and channel fading between the secondary user and fusion center (i.e., fading channel).

1.3 Contribution

In this thesis, we develop a simple yet efficient cooperation for spectrum sensing and consider the channel fading effects between the secondary user and fusion center in two cases. The global decision is based on simple energy detection over a linear combination of the local statistics from individual secondary user. The approach does not find optimal thresholds for individual nodes. Instead, we transmit the local test statistics through fading channel to the fusion center. Thus, the optimal threshold at the fusion center can be simply and jointly determined with the optimal linear combining weights. We derive the closed-form expressions of probabilities of detection and false alarm, and we can use the close-form expressions to make quick adaptations when some parameters change during the operation. Finally, we optimize a modified deflection coefficient to find the optimal linear combining weights and improve the detection performance. From the simulations, we can observe that the proposed cooperation method have the better detection performance then other methods, and the sensing reliability improves as the number of secondary users increase.

Chapter 2
Background Review

2.1 Cognitive Radio Networks

The material in this section is largely taken from [3].

2.1.1 Introduction to Cognitive Radio

The radio spectrum is a precious natural and limited resource, the use of which by transmitters and receivers is licensed by governments. Spectrum plays an important role of the wireless communication. Without spectrum, no wireless telecommunications or wireless internet services would be possible. Now, the telecommunication industry is a 1 Trillion (10^{12}) dollar per year industry. And the wireless part is growing very rapidly, while the wired telecommunication services are experiencing a relatively flat business. In 2006, the wired and wireless businesses were nearly equal in revenue. Spectrum is required to support these wireless communications. In the United States, the increase in cellular telephony demand is supported by increasing density of cellular infrastructure. But, in some region, the cellular infrastructure is at the peak capacity and increased infrastructure density is not feasible. In order to continue serving the market demand, we develop the cognitive radio networks that enable continued growth.

In November 2002, the Federal Communications Commission (FCC) published a report prepared by the Spectrum –Policy Task Force in the United States. Their objective is that manage this precious spectrum efficiently. The Task Force was a tem of FCC staff, and the team was high-level, multidisciplinary and professional. It was included economists, engineers, and attorneys from across the commission’s bureaus and offices. Among the Task Force major findings and recommendations, we can find that as follows in this report:

"In many bands, spectrum access is a more significant problem than physical scarcity of spectrum, in large part due to legacy command-and-control regulation that limits the ability of potential spectrum users to obtain such access."

Indeed, if we scan portions of the radio spectrum in urban areas, we would observe that:

1) some frequency bands in the spectrum are largely unoccupied most of the time;
2) some other frequency bands are only partially occupied;
3) the remaining frequency bands are heavily used.

The unused spectrum of primary user was called spectrum holes, and we define as follows:

A spectrum hole is a band of frequencies assigned to a primary user, but, at a particular time and specific geographic location, the band is not being utilized by that user.

Spectrum utilization can be improved significantly while a secondary user to access a spectrum hole unoccupied by the primary user at the right location and the time in question. Cognitive radio has been proposed to promote the efficient use of the spectrum by exploiting the unused spectrum holes.

What is the cognitive radio? Cognitive radio's objective is to improve utilization of the radio spectrum, we offer the following definition for cognitive radio.

Cognitive radio is an intelligent wireless communication system. It is aware of its surrounding environment (outside world), and uses the methodology of understanding-by-building to learn from the environment and adapt its internal states to statistical variations in the incoming RF stimuli by making corresponding changes in certain operating parameters (e.g., transmit-power, carrier frequency, and modulation strategy) in real-time, with two primary objectives in mind:

- highly reliable communication whenever and wherever needed;
- efficient utilization of the radio spectrum.

Now, we can say that cognitive radio can be represented by the six key steps as follows:

- awareness
- intelligence
- learning
- adaptivity
- reliability
- efficiency

Implementation of the six steps of combination is indeed feasible today, thank to the rapid advances in digital signal processing, networking , machine learning, computer software, and computer hardware.

In additional to the cognitive capabilities just mentioned, a cognitive radio is also endowed with re-configurability. Now, we see the re-configurability which provides the basis as follows:

- Adaptation of the radio interface so as to accommodate variations in the development of new interface standards.
- Incorporation of new applications and services as they emerge.
- Incorporation of updates in software technology.
- Exploitation of flexible heterogeneous services provided by radio networks.

This latter capability is provided by a platform known as Software-defined radio, upon which a cognitive radio is built. Software-defined radio (SDR) is a practical reality today, thank to the convergence of two key technologies: digital radio, and computer software.

2.1.2 Cognitive Task

For the re-configurability, a cognitive radio looks naturally to software-defined radio to perform this task. For other tasks of a cognitive kind, the cognitive radio looks to signal-processing and machine-learning procedures for their implementation. The cognitive process starts with the input stimuli and culminates with action.

In this section, we discuss the three cognitive radio tasks:

(1) Radio-scene analysis, which includes the following:

- estimation of interference temperature of the radio environment;
- detection of the spectrum holes.

(2) Channel identification, which includes the following:

- estimation of channel-state information (CSI);
- prediction of channel capacity for use by the transmitter.

(3) Transmit-power control and dynamic spectrum management.

Tasks (1) and (2) are performed in the receiver, and (3) is performed in the transmitter. Through interaction with the RF environment, these three tasks form a cognitive cycle, which is illustrated in Fig. 2-1.

From this brief discuss, it is showed that the cognitive radio's module in the transmitter must work in a harmonious manner with the cognitive radio's modules in the receiver. In order to maintain this harmony between the cognitive radio's transmitter and receiver at all time, we need a feedback channel connecting the receiver to the transmitter. Through the feedback channel, the receiver can be enabled to convey information on the performance of the forward link to the transmitter. Therefore, the cognitive radio system is necessarily an example of a feedback communication system.

One other comment is in order. A broadly defined cognitive radio technology accommodates a scale of differing degree of cognitive. At one end of the scale, the user may simply pick a spectrum hole and build its cognitive cycle around that spectrum hole. At the other end of scale, the user may employ multiple implementation technologies to build its cognitive cycle around a wideband spectrum hole or set of narrowband spectrum holes to provide the best expected performance by spectrum management and transmit -power control, and do so in the most highly secure manner possible.

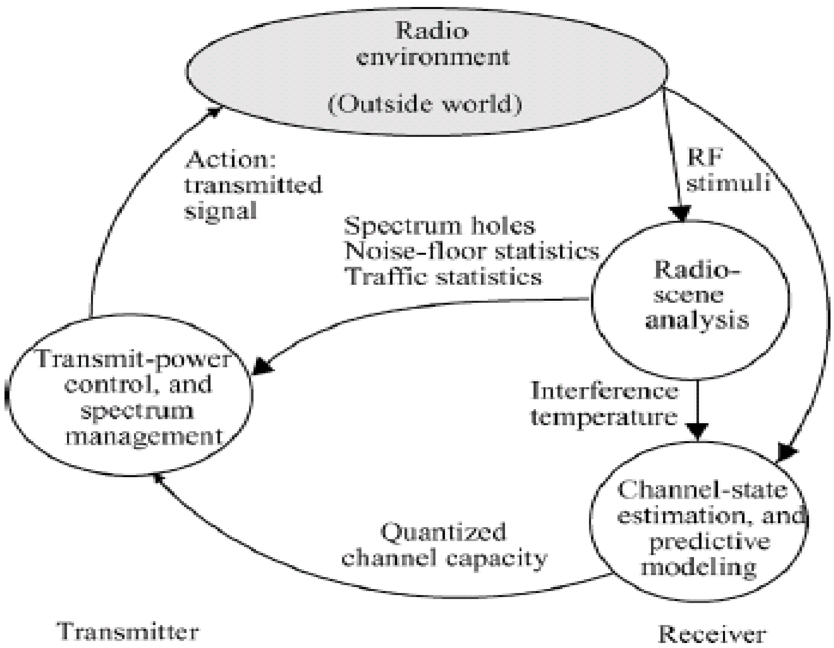

Fig. 2-1 Basic cognitive cycle.(The figure focuses on three fundamental cognitive tasks.) From [3]

2.1.3 Historical Notes

The history of cognitive radio was started in December 1901 by Guglielmo Marconi. And at that time the development of cognitive radio is still at a conceptual stage. But, as we looks to the future, we see that cognitive radio has the potential for making a significant difference to the way in which the radio spectrum can be accessed with improved utilization of the spectrum as a primary objective. Indeed, given its potential, cognitive radio can be described as a “disruptive, but unobtrusive technology.”

The two terms “cognitive radio” and “software-defined radio” were coined by Joseph Mitola. In an article published in 1999, Mitola described how a cognitive radio could enhance the flexibility of personal wireless services through a new language called the radio knowledge representation language (RKRL) [1]. The idea of RKRL was further expanded in Mitola’s own doctoral dissertation, which was presented at the Royal Institute of Technology, Sweden, in May 2000 [18]. This dissertation presents a conceptual overview of cognitive radio as an exciting multidisciplinary subject.

As mentioned earlier, the FCC published a report in 2002, which was aimed at the changes in technology and the significant impact that those changes would have on spectrum policy [19]. That report set the stage for a workshop on cognitive radio, which was held in Washington, DC, in May 2003. Those papers and reports that were presented at that workshop are at the web site listed under [20]. This workshop was followed by a conference on cognitive radio, which was held in Las Vegas，NV, in March 2004 [21].

2.2 Statistical Decision Theory

The material in this section is largely taken from [14].

The simplest detection problem is to decide whether a signal is present, which, as always, is embedded in noise, or only noise is present. An example of this problem is the detection of the primary signal based on cognitive radio network. Since we wish to decide between two possible hypotheses, signal and noise present versus only noise present, we call this the binary hypothesis testing problem. Our objective is to use the received data as efficiently as possible in making our decision and to be correct most of the time.

Now, assume that we observe a realization of a random variable whose PDF is either $N(0,1)$ or $N(1,1)$, where $N(\mu,\sigma^2)$ denotes a Gaussian PDF with mean μ and variance σ^2. We must decide if $\mu=0$ or $\mu=1$ based on a single observation $x[0]$. Each possible of μ can be though of a hypothesis, and our problem is to choose among two hypotheses. We can summarize as follows:

Binary Hypotheses Test

$$\begin{aligned} H_0 &: \mu=0 \qquad \text{nullhypothesis} \\ H_1 &: \mu=1 \qquad \text{alternative hypothesis} \end{aligned} \tag{2.1}$$

where H_0 is null hypothesis and H_1 is alternative hypothesis. The PDF under each hypothesis is shown in figure 2-2. However, a reasonable approach is to decide H_1 if $x[0]>1/2$. This is because if $x[0]>1/2$, it is more likely if H_1 is true. Then, our detector compares the observed sample with the threshold value (1/2). Now, we define two type errors. If we decide H_1 but H_0 is true, we call the Type I error. On the other hand, if we decide H_0 but H_1 is true, we call the Type II error. These two errors are shown in figure 2-2. The $P(H_i;H_j)$ is represented as that the probability of deciding H_i when H_j is true. (e.g., $P(H_1;H_0)=\Pr(x[0]>1/2;H_0)$).

From figure 2-3, we find that these two errors are unavoidable to some extent but can tradeoff by each other. Obviously, when the Type I error probability ($P(H_1;H_0)$) is decreased by changing the threshold, the Type II error probability ($P(H_0;H_1)$) is then increased. As the threshold changes, one error probability increases, while the other decreases. It is not possible to reduce both error probabilities simultaneously.

Now, we have the signal detection problem as follows:

$$\begin{aligned} H_0 &: x[0]=w[0] && \text{null hypothesis} \\ H_1 &: x[0]=s[0]+w[0] && \text{alternative hypothesis} \end{aligned} \qquad (2.2)$$

where $s[0]=1$ and $w[0]\sim N(0,1)$. We can define three probabilities. Deciding H_1 when H_0 is true can be thought as the false alarm. The $P(H_1;H_0)$ is the probability of false alarm which is denoted by P_f, and deciding H_1 when H_1 is true can be thought as the detection. The $P(H_1;H_1)$ is the probability of detection which is denoted by P_d. However, the other error $P(H_0;H_1)=1-P(H_1;H_1)$ can be thought of the probability of miss detection which is denoted by P_m. The P_f is usually a small value, and we often design the optimal detector to minimize the probability of miss detection (P_m) or maximize the probability of detection (P_d). Finally, we will summarize these probabilities in the table 2-1.

Table 2-1 Summary of probabilities.

False Alarm	Miss Detection	Detection
P_f	P_m	P_d
$P(H_1;H_0)$	$P(H_0;H_1)$	$P(H_1;H_1)$
Type I error	Type II error	$P_d = 1 - P_m$

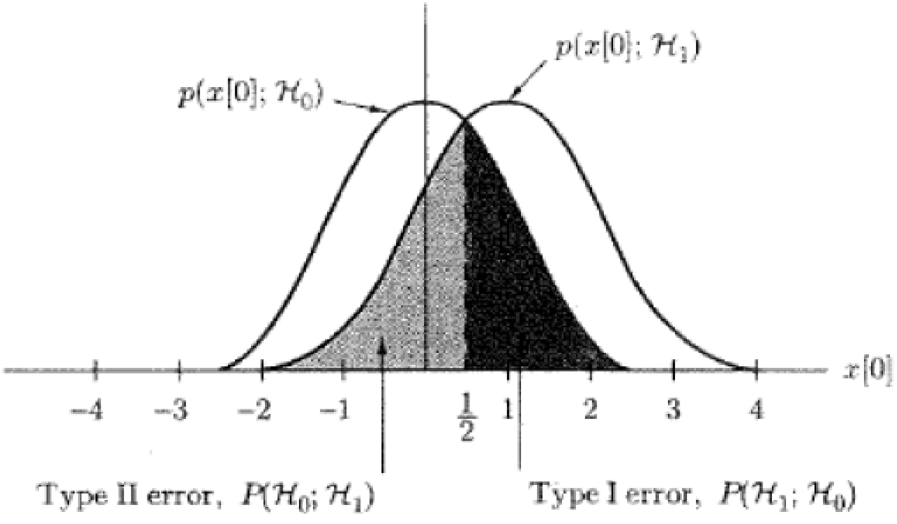

Fig. 2-2 Possible hypothesis testing error and their probability. From [14]

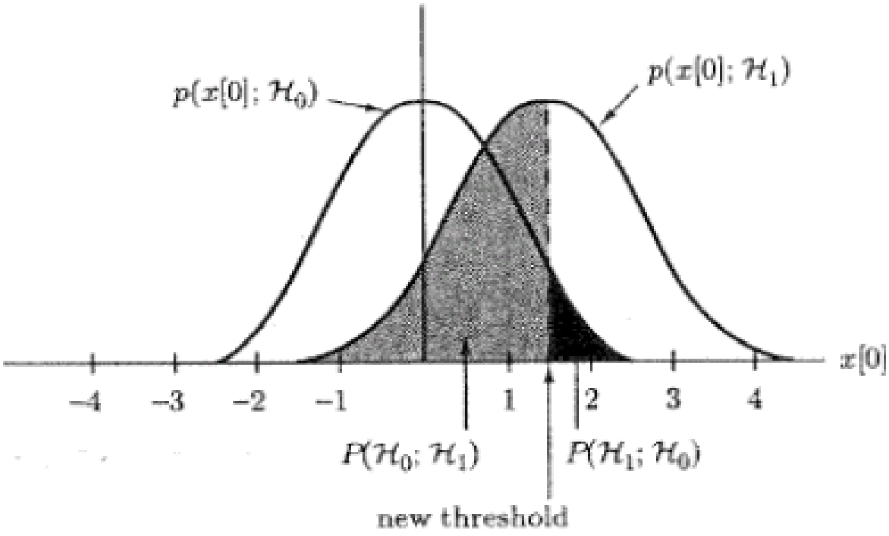

Fig. 2-3 Tradeoff errors by adjusting threshold. From [14]

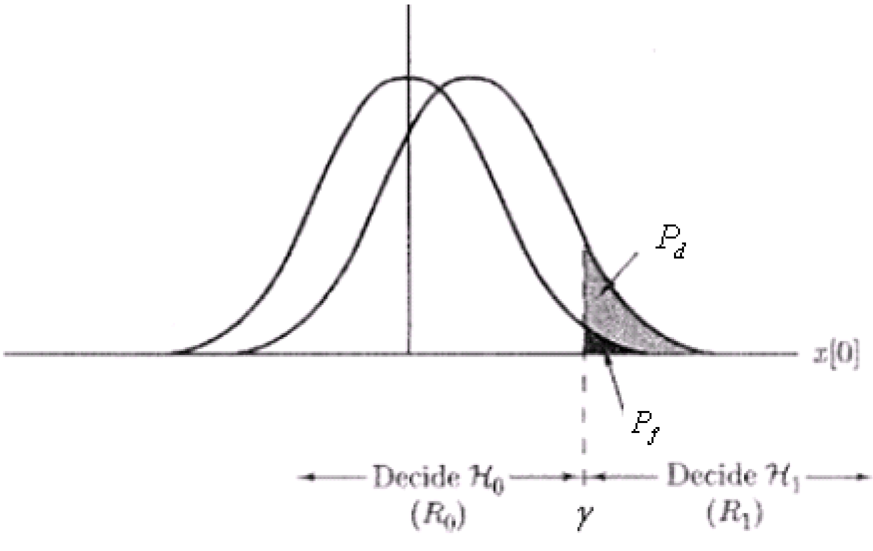

Fig. 2-4 Decision region and probabilities. From [14]

As the figure 2-4 illustrated, we can express the probability of false alarm and the probability of detection as follows:

$$\begin{aligned} P_f &= P(H_1; H_0) \\ &= \Pr(x[0] > \gamma; H_0) \\ &= \int_{\gamma}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}t^2} dt \\ &= Q(\gamma) \end{aligned} \tag{2.3}$$

where γ is the threshold value, and $Q(.)$ is the calculation of the tail probability of the zero mean and unit variance Gaussian random variable.

$$\begin{aligned} P_d &= P(H_1; H_1) \\ &= \Pr(x[0] > \gamma; H_1) \\ &= \int_{\gamma}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(t-1)^2} dt \\ &= Q(\gamma - 1) \end{aligned} \tag{2.4}$$

From (2.3) and (2.4), by changing the threshold we can trade off P_d and P_f. Now, we further consider the particularly useful hypothesis testing problem, and we call the mean-shifted Gauss-Gauss problem. We observe the value of a test statistic T and decide H_1 if $T > \gamma$ or H_0 if $T < \gamma$. The PDF of T is assumed as follows:

$$T \sim \begin{cases} N(\mu_0, \sigma^2) & H_0 \\ N(\mu_1, \sigma^2) & H_1 \end{cases} \tag{2.5}$$

where $\mu_1 > \mu_0$. Hence, we wish to decide between the two hypotheses that differ by a shift in the mean of T. For this type of detector, the detection performance is totally characterized by the deflection coefficient (d^2), and it is defined as follows:

$$d^2 = \frac{(E(T;H_1) - E(T;H_0))^2}{Var(T;H_0)} = \frac{(\mu_1 - \mu_0)^2}{\sigma^2} \tag{2.6}$$

In the definition, we know that a larger value of d^2 leads to a larger probability of detection (P_d). This is because that when the distance between μ_0 and μ_1 is larger, it would result in more accurate inference. In the case when $\mu_0 = 0$, $d^2 = \frac{\mu_1^2}{\sigma^2}$ may be interpreted as a signal-to-noise ratio (SNR). To find the dependence of detection performance on d^2 we have that

$$P_f = \Pr(T > \gamma; H_0) = Q(\frac{\gamma - \mu_0}{\sigma}) \tag{2.7}$$

$$\begin{aligned} P_d &= \Pr(T > \gamma; H_1) \\ &= Q(\frac{\gamma - \mu_1}{\sigma}) \\ &= Q(\frac{\mu_0 + \sigma Q^{-1}(P_{FA}) - \mu_1}{\sigma}) \\ &= Q\{Q^{-1}(P_{FA}) - (\frac{\mu_1 - \mu_0}{\sigma})\} \end{aligned} \tag{2.8}$$

Finally, we can obtain as follows:

$$P_d = Q\{Q^{-1}(P_f) - \sqrt{d^2}\} \tag{2.9}$$

The detection performance is therefore monotonic with the deflection coefficient. And we can summarize the detection performance by plotting P_d versus P_f. This type of performance summary is called the receiver operating characteristic (ROC). From figure 2-5, we can observe that as γ increases, P_f decreases and so does P_d. On the other hand, as γ decreases, P_f increases and so does P_d. The ROC always be above the 45° line. And when we increase the value of d^2 for a fixed value of P_f, the value of P_d also increases. In other words, a larger value of d^2 leads to a larger probability of detection(P_d). For $d \to \infty$, the idea ROC is attained ($P_d = 1$ for any P_f).

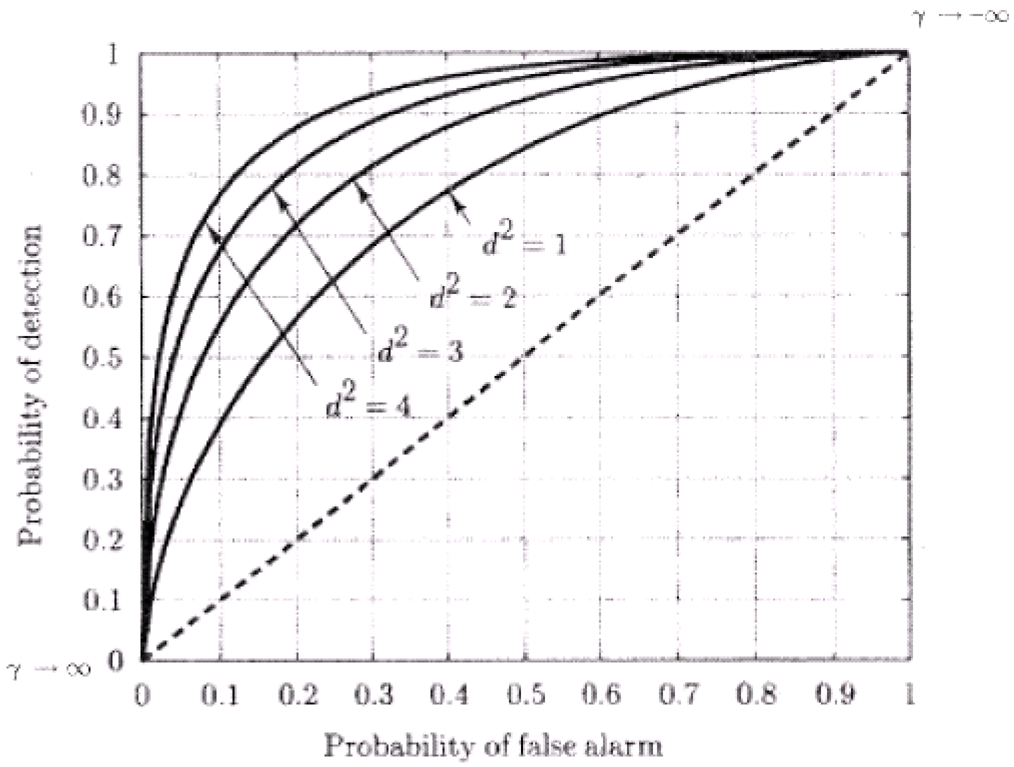

Fig. 2-5 Family of receiver operating characteristics. From [14]

2.3 Energy Detection

The material in this Chapter is largely taken from [15] and [16].

2.3.1 Introduction to Energy Detection

In many wireless communications, it is of great interest to check the presence and availability of an active communication link. What kind of detector do we adopt in the detection of a signal in the presence of the noise? The answer to the question is depended upon the knowledge of the transmitted signal characteristics and of the noise. When we has known that the transmitted signal has a known form and the noise is Gaussian, even with unknown parameters, the appropriate detector is chosen as the matched filter or its correlator equivalent. When the transmitted signal has an unknown form, it is sometimes appropriate to consider the signal as a sample function of a random process. When the transmitted signal statistics are known, we can often use this knowledge to design suitable detectors.

In the situation which is considered here, we have so little knowledge of the transmitted signal form, and we may make unreasonable assumptions about it. However, we consider that the transmitted signal is deterministic, although unknown in detail. And the spectral region is considered to be known. The noise is assumed to be additive white Gaussian noise with zero mean; the assumption of a deterministic signal represents that the input with the signal present is Gaussian but not zero mean.

If we have limited knowledge of the transmitted signal, it may seem appropriate to use an energy detector to detect the presence of the signal. The energy detector measures the energy in the input wave over a time interval. Due to only the signal energy matters (not its form), we can apply this result to any deterministic signal.

It is assumed here that the noise has a flat band-limited power density spectrum.

When the transmitted signal is absent, by means of a sampling theory, the energy in a finite time sample of the noise can be approximated by the sum of squares of statistically independent random variables which has zero means and equal variances. We can derive that this sum is a central chi-square distribution with the number of degrees freedom equal to twice the time-bandwidth product of the input. When the transmitted signal is present, by means of the sampling theory, the energy in a finite time sample of the transmitted signal and noise can be approximately by the sum of squares of random variables, where the sum has a non-central chi-square distribution with the same number of degrees freedom and a non-centrality parameter λ equal to the ratio of signal energy to two-sided noise spectral density.

2.3.2 Energy Detection in White Noise

The energy detector consists of a noise pre-filter, a square law device followed by a finite time integrator that is shown in figure 2-2. The output of the integrator at any time is the energy of the input to the squaring device over the interval T in the past. The noise pre-filter limits the noise bandwidth; the noise at the input to the squaring device has a band-limited, flat spectral density.

The detection is a binary hypothesis as follows:

$$\begin{aligned} H_0 &: \quad r(t) = n(t) \\ H_1 &: \quad r(t) = s(t) + n(t) \end{aligned} \tag{2.10}$$

where $r(t)$: the received signal.

$s(t)$: the transmitted signal.

$n(t)$: the noise which is zero-mean white Gaussian random process.

As figure 2-2, the received signal is first pre-filtered by an idea bandpass filter with transfer function

$$H(f)=\begin{cases}\dfrac{2}{\sqrt{N_0}}, & |f-f_c|\le W\\ 0, & |f-f_c|>W\end{cases} \tag{2.11}$$

where N_0 : one-sided noise power spectral density.

f_c : carrier frequency.

W : one-sided bandwidth (Hz).

to limit the average noise power and normalize the noise variance. Than, the output of the per-filter is squared and integrated over a time interval T . Finally, we produce a measure of the energy of the received waveform. The output of the integrator denoted by Y will be the test statistic to test the two hypotheses H_0 and H_1 .

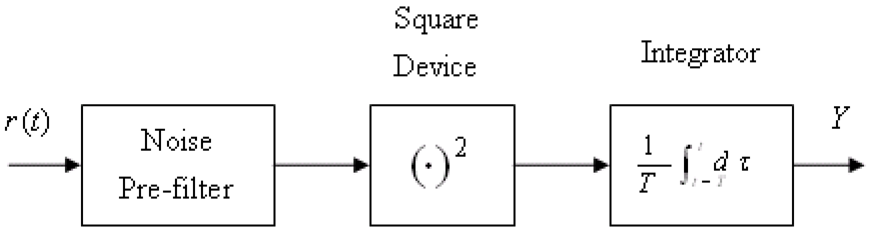

Fig. 2-6 Energy detection

According to the sample theorem, the noise process can be expressed as follows [22]:

$$n(t) = \sum_{i=-\infty}^{\infty} n_i \sin c(2Wt - i) \tag{2.12}$$

where $\sin c(x) = \frac{\sin(\pi x)}{\pi x}$ and $n_i = n(\frac{i}{2W})$

We can easily find that

$$n_i \sim N(0, N_0 W), \quad \text{for all } i \tag{2.13}$$

Over the time interval $(0, T)$, the noise energy can be approximated as follows [16]:

$$\int_0^T n^2(t)dt = \frac{1}{2W}\sum_{i=1}^{2u} n_i^2 \tag{2.14}$$

where $u = TW$: time-bandwidth product.

We assume that T and W are chosen to let u to be integer value. If we defined as follows:

$$n_i^{'} = \frac{n_i}{\sqrt{N_0 W}} \tag{2.15}$$

Then, the test statistic Y can be expressed as follows:

$$Y = \sum_{i=1}^{2u} n_i'^2 \qquad (2.16)$$

Y can be seen as the sum of squares of $2u$ standard Gaussian variables with zero mean and unit variance. So Y is a central chi-square distribution with $2u$ degrees of freedom.

The same approach is applied when the signal $s(t)$ is present. We replace each n_i by $n_i + s_i$ (where $s_i = s(\frac{i}{2W})$). And then, the test statistic Y is a non-central chi-square distribution with $2u$ degrees of freedom and a non-centrality parameter 2γ. ($\gamma = \frac{Es}{N_0}$: signal to noise ratio; $Es = \int_0^T s(t)dt$: signal energy.). Finally, we can express the test statistic as follows:

$$Y \sim \begin{cases} \chi_{2u}^2 & ,H_0 \\ \chi_{2u}^2(2\gamma) & ,H_1 \end{cases} \qquad (2.17)$$

The probability density function (PDF) of the test statistic Y can be expressed as

$$f_Y(y) \sim \begin{cases} \frac{1}{2^u \Gamma(u)} y^{u-1} e^{-\frac{y}{2}} & ,H_0 \\ \frac{1}{2} (\frac{y}{2\gamma})^{\frac{u-1}{2}} e^{-\frac{2\gamma+y}{2}} I_{u-1}(\sqrt{2\gamma y}) & ,H_1 \end{cases} \qquad (2.18)$$

where $\Gamma(\cdot)$ is the gamma function which defined as $\Gamma(u)=\int_0^\infty t^{u-1}e^{-t}dt$, and $I_v(\cdot)$ is the $v-th$ order modified Bessel function of the first kind, and it defined as

$$I_v(u)=\frac{(\frac{1}{2}u)^v}{\sqrt{\pi}\Gamma(v+\frac{1}{2})}\int_0^\pi e^{u\cos\theta}\sin^{2v}\theta\,d\theta \tag{2.19}$$

Now, we can obtain their mean and variance as follows:

$$E[Y]=\begin{cases}2u & ,H_0\\ 2\mathrm{u}+2\gamma & ,H_1\end{cases} \tag{2.20}$$

$$Var[Y]=\begin{cases}4u & ,H_0\\ 4\mathrm{u}+8\gamma & ,H_1\end{cases} \tag{2.21}$$

Therefore, we can compute that the probability of detection and false alarm as follows:

$$P_d=\Pr(Y>\lambda|H_1) \tag{2.22}$$

$$P_f=\Pr(Y>\lambda|H_0) \tag{2.23}$$

where λ is the decision threshold.

In chapter 3, for simplicity, we apply the central limit theory to the test statistic Y, and the probability distribution function of test statistic Y may be approximated as Gaussian distribution. This makes us easy to deal with P_d and P_f.

2.4 SNR Wall Reduction

The material in this Chapter is largely taken from [17].

When we use the energy detector, a significant problem is that it is suffers from an SNR wall when the noise power uncertainty is present [5],[23]. Caused by the noise uncertainty, the SNR wall is defined as an SNR threshold below which energy detection is absolutely impossible no matter how many samples are used. Now, we consider that there exists x dB uncertainty in noise power estimation, and then the actual noise power may take any value within $(\frac{\sigma_n^2}{\alpha}, \alpha\sigma_n^2)$, where $\alpha = 10^{\frac{x}{10}}$ and σ_n^2 is the estimated noise power. If the primary signal power is smaller then $(\alpha\sigma_n^2 - \frac{\sigma_n^2}{\alpha})$, then the energy detection will always fail. In other words, the SNR wall of energy detection is defined as follows:

$$SNR_{wall} = 10\log_{10}(\gamma_w) = 10\log_{10}(\alpha - \frac{1}{\alpha}) \quad (2.24)$$

where γ_w is SNR wall. Here we assume that the channels of the cognitive radio users experience block fading, and the block length is long enough so that errorless detection can be guaranteed if only the instantaneous SNR is greater then the SNR wall (γ_w). While the number of the secondary users increases, the probability that the instantaneous SNR on one of these users is greater then SNR wall increases. Once this probability exceeds the target overall probability of detection of the cognitive radio network, energy detection will work well. Therefore, cooperation equivalently decreases the SNR wall with a certain target probability of detection. Now, we will derive the equivalently SNR wall reduction achieved by cooperation among independently cognitive radio users.

Let $\bar{\gamma}_M$ be the minimum average SNR that meets the target overall probability of detection (P_{d_TAR}), when M independent secondary users are cooperating. In other words, $\bar{\gamma}_M$ is equivalent the SNR wall of a M-secondary users network with the target overall probability of detection (P_{d_TAR}). Miss detection will happen if and only if the instantaneous SNR of all cognitive radio users are below γ_w, so we can obtained as follows:

$$P_{m\_TAR} = 1 - P_{d\_TAR} = (\Pr(\gamma < \gamma_w))^M \qquad (2.25)$$

Here we use the Nakagami channel [24]. In this case, the CDF of the instantaneous SNR (γ) is obtained as follows:

$$\Pr(\gamma < \gamma_w) = P(m, \frac{m\gamma_w}{\bar{\gamma}_M}) \qquad (2.26)$$

where m is the Nakagami parameter, and $P(m,x) = \frac{1}{\Gamma(m)}\int_0^x e^{-t} t^{m-1} dt$ is the normalized lower incomplete gamma function, and $\Gamma(m)$ is the gamma function.

Now, we can obtain the equivalent SNR wall of a M-secondary users cooperative network as

$$\bar{\gamma}_M = \frac{m\gamma_w}{P^{-1}(m, P_{m\_TAR}^{1/M})} \tag{2.27}$$

where $P^{-1}(m, y)$ is the inverse normalized lower incomplete function. Finally, we can obtain the equivalent SNR wall reduction relative to the single user as follows:

$$\begin{aligned} SNR_{wall\_red} &= 10\log_{10}\left(\frac{\bar{\gamma}_1}{\bar{\gamma}_M}\right) \\ &= 10\log_{10}\left(\frac{P^{-1}(m, P_{m\_TAR}^{1/M})}{P^{-1}(m, P_{m\_TAR})}\right) \end{aligned} \tag{2.28}$$

According to (2.28), we observe that the SNR wall reduction increases with the number of secondary user (M), independent of γ_w. For fixed m and $P_{m,TAR}$,

$$\lim_{M\to+\infty} SNR_{wall\_red} = 10\log_{10}\left(\frac{P^{-1}(m,1)}{P^{-1}(m, P_{m\_TAR})}\right) = +\infty \tag{2.29}$$

which means that the equivalent SNR wall of the cognitive radio network can be reduced to any arbitrarily low level as long as a sufficient number of the cooperating secondary users. Therefore, we use this result to improve the detection performance in chapter 3, the simulation can be shown that as the number of the cooperating secondary users increase, the detection performance becomes better.

Chapter 3

Distributed Cooperative Spectrum Sensing for Two Cases

In this chapter, we propose an optimal linear cooperative structure for spectrum sensing in order to accurately detect the primary signal. In this structure, spectrum sensing is based on the linear combination of local statistics from individual cognitive radio, and we control the combining weights to combat the effect of channel fading. Our objective is to minimize the interference to the primary user while the secondary users access the licensed band. So we optimize the modified deflection coefficient at the fusion center in order to improve the detection performance.

3.1 System Model

We consider a cognitive radio networks with M secondary users. The binary hypothesis test for spectrum sensing at the k-th time instant is expressed as follows:

$$\begin{aligned} H_0 &: y_i(k) = v_i(k) \qquad & i = 1,2,.....,M \\ H_1 &: y_i(k) = h_i s(k) + v_i(k) \qquad & i = 1,2,.....,M \end{aligned} \tag{3.1}$$

where $s(k)$ is the signal transmitted by the primary user and $y_i(k)$ is the received signal by the i-th secondary user. The channel gain, h_i between each secondary user and the target primary user, is assumed to be fixed during a detection interval, and $v_i(k)$ denotes the zero-mean additive white Gaussian noise (AWGN), i.e.

$v_i(k) \sim CN(0, \sigma_v^2)$. Without loss of generality, $v_i(k)$, $s(k)$ and h_i are assumed to be independent of each other.

As illustrated in Fig. 3-1, we use the energy detection, since it doesn't require any a priori knowledge of primary signals and has much lower complexity then other detectors. Each secondary user computes its summary statistic u_i over a detection interval of 2n samples. i.e.

$$u_i = \sum_{k=0}^{2n-1} |y_i(k)|^2 \qquad i = 1,2,\ldots,M \tag{3.2}$$

The summary statistic $\{u_i\}$ are then transmitted to the fusion center through a fading channel and are corrupted by the zero-mean additive white Gaussian noise (AWGN), we can express as follows:

$$r_i = g_i u_i + n_i \qquad i = 1,2,\ldots,M \tag{3.3}$$

or

$$\begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_M \end{bmatrix} = \begin{bmatrix} g_i u_1 \\ g_2 u_2 \\ \vdots \\ g_M u_M \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \\ \vdots \\ n_M \end{bmatrix} \tag{3.4}$$

where the channel gain $\{g_i\}$ between secondary users and fusion center are additive white Gaussian noise with zero-mean and variance σ_g^2, and they are assumed to be fixed during a detection interval, and the channel noise $\{n_i\}$ are also additive white Gaussian noise with zero-mean and variance σ_n^2, i.e. $g_i \sim N(0,\sigma_g^2), n_i \sim N(0,\sigma_n^2)$. Finally, the fusion center computes the global test statistics, r_c as in (3.18), from the outputs $\{r_i\}$ of the individual secondary users in a linear combination manner, and then r_c is used to make a global decision.

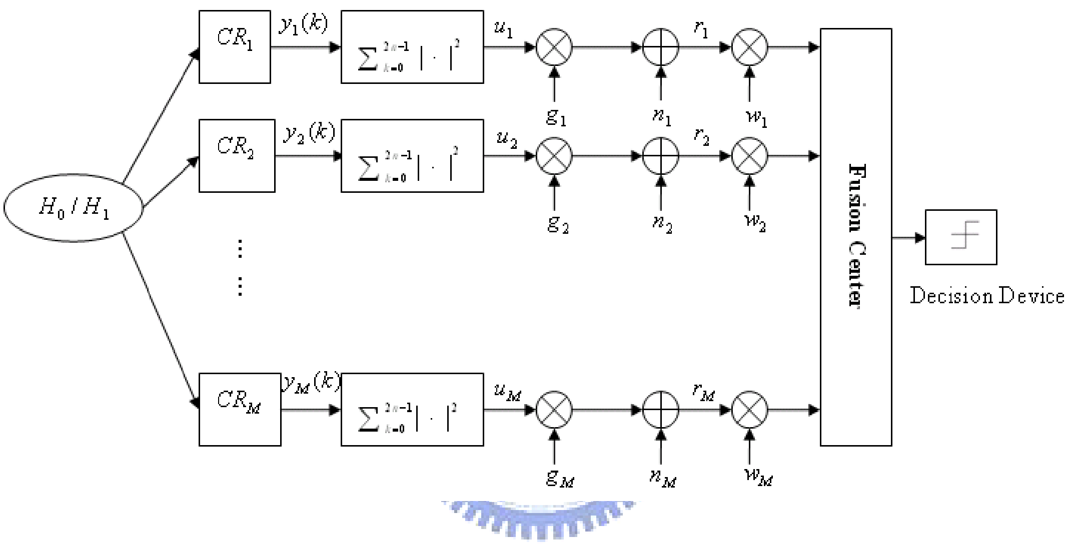

Fig. 3-1 A schematic representation of weighting cooperation for spectrum sensing in cognitive radio networks.

3.2 Cooperative Spectrum Sensing

In this section, we propose a optimal strategy for cooperative spectrum sensing. Because we do not know the prior knowledge of the primary signals (i.e. the secondary user has limited information of the primary signals), the energy detection is optimal and the simplest, so we adopt energy detection as the local sensing rule, which will be discussed as follows.

3.2.1 Local Sensing

We first consider local spectrum sensing at individual secondary users, and then we find out the local test statistics at each node. For the sequence of $2n$ samples over each detection interval, we define

$$E_s = \sum_{k=0}^{2n-1} |s(k)|^2 \tag{3.5}$$

which denotes the transmitted signal energy. The local test statistics of the i-th secondary user using energy detector are expressed as follows:

$$u_i = \sum_{k=0}^{2n-1} |y_i(k)|^2 \qquad i = 1,2,\ldots,M \tag{3.6}$$

Since u_i is the sum of the squares of $2n$ Gaussian random variables, so we can show that u_i / σ_v^2 is a central chi-square χ^2 distribution with $2n$ degrees of freedom if H_0 is true; otherwise, the u_i / σ_v^2 would be a non-central chi-square $\chi^2(\eta_i)$ distribution with $2n$ degrees of freedom and η_i is a non- centrality parameter. We can express as follows:

$$\frac{u_i}{\sigma_v^2} \sim \begin{cases} \chi_{2n}^2 & H_0 \\ \chi_{2n}^2(\eta_i) & H_1 \end{cases} \tag{3.7}$$

where

$$\eta_i = \frac{|h_i|^2 E_s}{\sigma_v^2} \tag{3.8}$$

is the local SNR (signal to noise ratio) at i-th secondary user. According to CLT (central limit theorem), if the number of samples is large enough, the test statistics u_i can be asymptotically normally distributed with mean

$$E[u_i] = \begin{cases} 2n\sigma_v^2 & H_0 \\ (2n+\eta_i)\sigma_v^2 & H_1 \end{cases} \tag{3.9}$$

and variance

$$Var(u_i) = \begin{cases} 4n\sigma_v^4 & H_0 \\ 4(n+\eta_i)\sigma_v^4 & H_1 \end{cases} \tag{3.10}$$

We can express simply as follows:

$$u_i \sim \begin{cases} N(2n\sigma_v^2, 4n\sigma_v^4) & H_0 \\ N((2n+\eta_i)\sigma_v^2, 4(n+\eta_i)\sigma_v^4) & H_1 \end{cases} \tag{3.11}$$

for $2n$ is large enough. Now, for a single-CR spectrum sensing scheme, the decision rule at each secondary user is given by

$$u_i \underset{H_0}{\overset{H_1}{\gtrless}} \gamma_i \qquad i = 1,2,\ldots,M \tag{3.12}$$

where γ_i is the corresponding decision threshold. Therefore, secondary user i will have the probabilities of detection and false alarm, and we can express as the following Q-function:

$$P_d^{(i)} = \Pr(u_i > \gamma_i | H_1) = Q(\frac{\gamma_i - E[u_i],_{H_1}}{Var(u_i),_{H_1}}) \tag{3.13}$$

and

$$P_f^{(i)} = \Pr(u_i > \gamma_i | H_0) = Q(\frac{\gamma_i - E[u_i],_{H_0}}{Var(u_i),_{H_0}}) \tag{3.14}$$

$Q(.)$: Calculates the tail probability of the zero mean unit variance Gaussian random variable, i.e. , $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{+\infty} \exp(-t^2/2)dt$.

In the cognitive radio networks, a larger probability of detection results in less interference to the primary users and a smaller probability of false alarm results in higher spectrum efficiency. This is because based on the assumption that if no primary signals are detected, the secondary users use the channel (such that interference is generated in case of miss-detection); if a primary signals is detected (possibly a false alarm), the secondary users are restrained to use the channel (such that spectrum is wasted in case of false alarm).

3.2.2 Global Detection

As illustrated in Fig. 3-1, we transmit the local test statistic $\{u_i\}$ to the fusion center via a channel and the zero-mean additive white Gaussian noise (AWGN),and then are multiplied by weights in a linear combination manner. Now, we consider the channel with two conditions. First, for simplification, we assume that the channel can be treated as constant AWGN channels which channel gains are constant one (i.e. $g_i = 1$). Second, we further consider that the channels are fading channels which channel gains are generated according to a normal distribution and assumed to be fixed during a detection interval.

I. Constant AWGN Cannel between Secondary User and Fusion Center

From (3.3) or (3.4), we assume that the channel gains are constant one (i.e. $g_i = 1$), then we can express as follows:

$$r_i = u_i + n_i \qquad i = 1,2,...,M \tag{3.15}$$

According to (3.11), since $u_i \sim N(E[u_i], Var(u_i))$, $n_i \sim N(0, \sigma_n^2)$, so the received statistics $\{r_i\}$ are normally distributed with mean

$$E[r_i] = \begin{cases} 2n\sigma_v^2 & H_0 \\ (2n+\eta_i)\sigma_v^2 & H_1 \end{cases}$$	(3.16)

and variance

$$Var(r_i) = \begin{cases} 4n\sigma_v^4 + \sigma_n^2 & H_0 \\ 4(n+\eta_i)\sigma_v^4 + \sigma_n^2 & H_1 \end{cases}$$	(3.17)

Once the fusion center receives $\{r_i\}$, a global test statistic r_c is calculated linearly as follows:

$$r_c = \sum_{i=1}^{M} w_i r_i = \underline{w}^T \underline{r}$$	(3.18)

where the weight vector $\underline{w} = (w_1, w_2, ..., w_M)^T$ satisfies $\|\underline{w}\|_2^2 = 1$ and $\|\cdot\|_2$ is the Euclidean norm. The weight vector is used to control the global spectrum detector. The combining weight for the signal from a particular user represents its contribution to the global decision. For example, if a CR generates a high-SNR signal that may lead to correct detection on its own, it should be assigned a larger weight coefficient. For the secondary users passing deep fading or shadowing, their weights are decreased in order to reduce their effect to the decision fusion. $\underline{r} = (r_1, r_2, ..., r_M)^T$ is the received vector. Since the received statistics $\{r_i\}$ are Gaussian random variables, so their linear combination is also Gaussian. Then, r_c is normally distributed with mean

$$E[r_c] = \begin{cases} 2n\underline{1}^T \underline{w}\sigma_v^2 & H_0 \\ (2n\underline{1} + \underline{\eta})^T \underline{w}\sigma_v^2 & H_1 \end{cases}$$	(3.19)

where $\underline{1}$ is a column vector that are all ones, and $\underline{\eta} = (\eta_1, \eta_2, ..., \eta_M)^T$ is the SNR vector ,and variance

$$\begin{aligned} Var(r_c) &= E(r_c - E[r_c])^2 \\ &= \underline{w}^T E[(\underline{r} - E[\underline{r}])(\underline{r} - E[\underline{r}])^T]\underline{w} \end{aligned}$$	(3.20)

Therefore, the variances for different hypothesis are given by

$$\begin{aligned} Var(r_c)_{,H_0} &= \underline{w}^T E[(\underline{r} - E[\underline{r}]_{H_0})(\underline{r} - E[\underline{r}]_{H_0})^T \vert H_0]\underline{w} \\ &= \underline{w}^T (4n\sigma_v^4 + \sigma_n^2) I \underline{w} \\ &= 4n\sigma_v^4 + \sigma_n^2 \end{aligned}$$	(3.21)

and

$$\begin{aligned} Var(r_c)_{,H_1} &= \underline{w}^T E[(\underline{r} - E[\underline{r}]_{H_1})(\underline{r} - E[\underline{r}]_{H_1})^T \vert H_1]\underline{w} \\ &= \underline{w}^T (4\sigma_v^4[nI + diag(\underline{\eta})] + \sigma_n^2 I)\underline{w} \end{aligned}$$	(3.22)

where I denotes the identity matrix, and $diag(.)$ is square diagonal matrix with the elements of a given vector on the diagonal. Therefore, we can express simply as follows:

$$r_c \sim \begin{cases} N(2n\underline{1}^T \underline{w}\sigma_v^2, 4n\sigma_v^4 + \sigma_n^2) & H_0 \\ N((2n\underline{1}+\underline{\eta})^T \underline{w}\sigma_v^2, \underline{w}^T(4\sigma_v^4[nI + diag(\underline{\eta})] + \sigma_n^2 I)\underline{w}) & H_1 \end{cases} \tag{3.23}$$

Finally, to make decision on the presence of the primary signal, the global test statistics r_c is compared with a threshold T_c.

$$r_c \underset{H_0}{\overset{H_1}{\gtrless}} T_c \tag{3.24}$$

And then, the probabilities of detection and false alarm at the fusion center can be expressed as

$$P_d^{(c)} = \Pr(r_c > T_c | H_1) = Q(\frac{T_c - E[r_c],_{H_1}}{Var(r_c),_{H_1}}) \tag{3.25}$$

and

$$P_f^{(c)} = \Pr(r_c > T_c | H_0) = Q(\frac{T_c - E[r_c],_{H_0}}{Var(r_c),_{H_0}}) \tag{3.26}$$

Next we will show that the channels are fading channels which channel gains are generated according to a normal distribution and assumed to be fixed during a detection interval.

II. Fading Cannel between Secondary User and Fusion Center

From (3.3) or (3.4), the channel gain $\{g_i\}$ between secondary users and fusion center are additive white Gaussian noise with zero-mean and variance σ_g^2, and they are assumed to be fixed during a detection interval, and the channel noise $\{n_i\}$ are also additive white Gaussian noise with zero-mean and variance σ_n^2, i.e. $g_i \sim N(0,\sigma_g^2), n_i \sim N(0,\sigma_n^2)$. And $u_i \sim N(E[u_i], Var(u_i))$. Without loss of generality, we assume that $\{g_i\}$, $\{n_i\}$, and $\{u_i\}$ are independent of each other. Therefore, at the fusion center receives $\{r_i\}$, a global test statistic r_c is calculated linearly as follows:

$r_c = \sum_{i=1}^{M} w_i r_i = \underline{w}^T \underline{r} = \sum_{i=1}^{M} w_i (g_i u_i + n_i)$	(3.27)

Once again, according to CLT (central limit theorem), if the number of secondary users (M) is large enough, the global test statistics r_c, can be asymptotically normally distributed with mean

$$E[r_c] = E[\underline{w}^T \underline{r}] = \underline{w}^T E[\underline{r}] = \underline{w}^T E\begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_M \end{bmatrix} = \underline{w}^T E\begin{bmatrix} g_1 u_1 + n_1 \\ g_2 u_2 + n_2 \\ \vdots \\ g_M u_M + n_M \end{bmatrix}$$	(3.28)

where { g_i } are assumed to be fixed during detection interval, and $n_i \sim N(0, \sigma_n^2)$, u_i can be asymptotically normally distributed

$$u_i \sim \begin{cases} N(2n\sigma_v^2, 4n\sigma_v^4) & H_0 \\ N((2n+\eta_i)\sigma_v^2, 4(n+\eta_i)\sigma_v^4) & H_1 \end{cases} \qquad (3.29)$$

So we can derive the mean as follows:

$$E[r_c] = \underline{w}^T E\begin{bmatrix} g_1 E[u_1] + E[n_1] \\ g_2 E[u_2] + E[n_2] \\ \vdots \\ g_M E[u_M] + E[n_M] \end{bmatrix}$$	(3.30)
$$E[r_c] = \begin{cases} 2n\underline{g}^T \underline{w}\sigma_v^2 & H_0 \\ (2n\underline{g} + \underline{\eta}_g)^T \underline{w}\sigma_v^2 & H_1 \end{cases}$$	(3.31)

where $\underline{g} = (g_1, g_2, ..., g_M)^T$, $\underline{\eta}_g = (\eta_1 g_1, \eta_2 g_2, ..., \eta_M g_M)^T$.

And variance

$$\begin{aligned} Var(r_c) &= E(r_c - E[r_c])^2 \\ &= \underline{w}^T E[(\underline{r} - E[\underline{r}])(\underline{r} - E[\underline{r}])^T]\underline{w} \\ &= \underline{w}^T K \underline{w} \end{aligned}$$	(3.32)

where $K = E[(\underline{r} - E[\underline{r}])(\underline{r} - E[\underline{r}])^T]$ is covariance matrix.

$$\begin{aligned} K &= E[(\underline{r} - E[\underline{r}])(\underline{r} - E[\underline{r}])^T] \\ &= E\left[\left(\begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_M \end{bmatrix} - \begin{bmatrix} E[r_1] \\ E[r_2] \\ \vdots \\ E[r_M] \end{bmatrix} \right) \left(\begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_M \end{bmatrix} - \begin{bmatrix} E[r_1] \\ E[r_2] \\ \vdots \\ E[r_M] \end{bmatrix} \right)^T \right] \\ &= E\left[\left(\begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_M \end{bmatrix} - \begin{bmatrix} E[g_1 u_1 - n_1] \\ E[g_2 u_2 - n_2] \\ \vdots \\ E[g_M u_M - n_M] \end{bmatrix} \right) \left(\begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_M \end{bmatrix} - \begin{bmatrix} E[g_1 u_1 - n_1] \\ E[g_2 u_2 - n_2] \\ \vdots \\ E[g_M u_M - n_M] \end{bmatrix} \right)^T \right] \\ &= E\left[\left(\begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_M \end{bmatrix} - 2n\sigma_v^2 \begin{bmatrix} g_1 \\ g_2 \\ \vdots \\ g_M \end{bmatrix} \right) \left(\begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_M \end{bmatrix} - 2n\sigma_v^2 \begin{bmatrix} g_1 \\ g_2 \\ \vdots \\ g_M \end{bmatrix} \right)^T \right] \\ &= \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1M} \\ a_{21} & a_{22} & & \\ \vdots & & \ddots & \\ a_{M1} & & & a_{MM} \end{bmatrix} \end{aligned}$$	(3.33)

Therefore, we can find the element of the covariance matrix for different hypothesis as follows:

$$a_{ij} = \begin{cases} 4ng_i^2\sigma_v^4 + \sigma_n^2 & ,i = j \\ 0 & ,i \neq j \end{cases} \quad \text{for } H_0$$	(3.34)

and

$$a_{ij} = \begin{cases} 4g_i^2(n+\eta_i)\sigma_v^4 + \sigma_n^2 & ,i = j \\ 0 & ,i \neq j \end{cases} \quad \text{for } H_1 \tag{3.35}$$

However, we can observe that the covariance matrix is diagonal matrix which non-diagonal element are all zeros. Then, we can derive the variance

$$\begin{aligned} Var(r_c) &= \underline{w}^T K \underline{w} \\ &= \begin{cases} \underline{w}^T[4n\sigma_v^4 diag(\underline{g}^2) + \sigma_n^2 I]\underline{w} & H_0 \\ \underline{w}^T[4n\sigma_v^4 diag[\underline{g}^2(n+\underline{\eta})] + \sigma_n^2 I]\underline{w} & H_1 \end{cases} \end{aligned} \tag{3.36}$$

where $\underline{g}^2 = (g_1^2, g_2^2, ..., g_M^2)^T$, $\underline{g}^2(n+\underline{\eta}) = [g_1^2(n+\eta_1), g_2^2(n+\eta_2), ..., g_M^2(n+\eta_M)]^T$. Finally, we can express that the global test statistics r_c are asymptotically normally distributed when M is large enough.

$$r_c \sim \begin{cases} N(2n\underline{g}^T \underline{w}\sigma_v^2, \underline{w}^T[4n\sigma_v^4 diag(\underline{g}^2) + \sigma_n^2 I]\underline{w}) & H_0 \\ N((2n\underline{g} + \underline{\eta}_g)^T \underline{w}\sigma_v^2, \underline{w}^T[4n\sigma_v^4 diag[\underline{g}^2(n+\underline{\eta})] + \sigma_n^2 I]\underline{w}) & H_1 \end{cases} \tag{3.37}$$

Then, to make decision on the presence of the primary signal, the global test statistics r_c is compared with a threshold T_c.

$$r_c \underset{H_0}{\overset{H_1}{\gtrless}} T_c \tag{3.38}$$

The probabilities of detection and false alarm at the fusion center can be expressed as

$$P_d^{(c)} = \Pr(r_c > T_c \mid H_1) = Q\left(\frac{T_c - E[r_c]_{,H_1}}{Var(r_c)_{,H_1}}\right) \tag{3.39}$$

and

$$P_f^{(c)} = \Pr(r_c > T_c \mid H_0) = Q\left(\frac{T_c - E[r_c]_{,H_0}}{Var(r_c)_{,H_0}}\right) \tag{3.40}$$

We see that the sensing performance of the linear detector depends largely on the weighting coefficient and the decision threshold. We next show how to design the optimal weight vector $\underline{w}$ in order to maximize the modified deflection coefficient.

3.3 Performance Optimization

For cognitive radio networks, the probabilities of detection and false alarm have unique relationship. Specifically, 1-$P_d^{(c)}$ represents the probability of interference from secondary users on the primary users. On the other hand, $P_f^{(c)}$ determines the upper bound on the spectrum efficiency, where a large $P_f^{(c)}$ usually results in low spectrum utilization. This is based on a typical assumption that if primary signals are detected, the secondary users do not use the licensed band, and if no primary signals are detected, the secondary users use the licensed band. In this section, we maximize the modified deflection coefficient in order to improve the detection performance.

From the mean and variance of r_c, we observe that the weight vector $\underline{w}$ plays an important role in controlling the PDF of the global test statistics r_c. To measure the effect of the PDF on the detection performance, we define a modified deflection coefficient as follows:

$$d_m^2 = \frac{(E[r_c],_{H_1} - E[r_c],_{H_0})^2}{Var(r_c),_{H_1}} \tag{3.41}$$

Now, we would like to maximize d_m^2 under the unit norm constraint to find the optimization of weight vector, i.e.

$$\begin{aligned} &\text{maximize} \quad d_m^2(\underline{w}) \\ &\text{subject to} \quad \|\underline{w}\|_2^2 = 1 \end{aligned} \tag{3.42}$$

Where $\|\cdot\|_2$ denotes the Euclidean norm. And then we consider two cases to find the weight coefficient. First, we consider that the channels are the constant AWGN channel, and we can obtain the follows from (3.41):

$$d_m^2(\underline{w}) = \frac{(\underline{\eta}^T \underline{w}\sigma_v^2)^2}{\underline{w}^T(4\sigma_v^4[nI + diag(\underline{\eta})] + \sigma_n^2 I)\underline{w}} \tag{3.43}$$

We solve the problem as follows. Since we have $4\sigma_v^4[nI + diag(\underline{\eta})] + \sigma_n^2 I \succ 0$, so we can know its square root can be expressed as

$$\begin{aligned} D &= (4\sigma_v^4[nI + diag(\underline{\eta})] + \sigma_n^2 I)^{1/2} \\ &= \begin{bmatrix} \sqrt{4\sigma_v^4(n+\eta_1)+\sigma_n^2} & & & \\ & \sqrt{4\sigma_v^4(n+\eta_2)+\sigma_n^2} & & \\ & & \ddots & \\ & & & \sqrt{4\sigma_v^4(n+\eta_M)+\sigma_n^2} \end{bmatrix} \end{aligned} \tag{3.44}$$

Where is a diagonal matrix, Applying the linear transformation $\underline{q} = D\underline{w}$ gives

$$\begin{aligned} d_m^2(\underline{w}) &= \frac{\sigma_v^4 \underline{q}^T D^{-1} \underline{\eta}\underline{\eta}^T D^{-1} \underline{q}}{\underline{q}^T \underline{q}} \\ &\overset{(a)}{\le} \sigma_v^4 \lambda_{\max}(D^{-1}\underline{\eta}\underline{\eta}^T D^{-1}) \end{aligned} \tag{3.45}$$

Where $\lambda_{\max}(\cdot)$ denotes the maximum eigenvalue of the matrix. Note that (a) follows the Rayleigh Ritz inequality and the equality is achieved if $\underline{q} = \underline{q}^o$, which is the eigenvector of the positive definite matrix $D^{-1}\underline{\eta}\underline{\eta}^T D^{-1}$ corresponding to the maximum eigenvalue. Therefore, we find the optimal solution of (3.42) is

$$\underline{w} = \frac{D^{-1}\underline{q}^o}{\left\| D^{-1}\underline{q}^o \right\|_2} \tag{3.46}$$

which maximizes the modified deflection coefficient.

Now, we further consider that the channel are fading channel which channel gains are generated according to a normal distribution and assumed to be fixed during a detection interval. With the same step, we would like to maximize d_m^2 under the unit norm constraint to find the optimization of weight vector, and then we can obtain the follows from (3.41)

$$d_m^2(\underline{w}) = \frac{(\underline{\eta}_g^T \underline{w}\sigma_v^2)^2}{\underline{w}^T(4\sigma_v^4[diag(\underline{g}^2(n+\underline{\eta}))] + \sigma_n^2 I)\underline{w}} \tag{3.47}$$

We solve the problem as the same, since we have $4\sigma_v^4[diag(\underline{g}^2(n+\underline{\eta}))] + \sigma_n^2 I \succ 0$, so we can also know its square root can be expressed as

$$\begin{aligned} D &= (4\sigma_v^4[diag(\underline{g}^2(n+\underline{\eta}))] + \sigma_n^2 I)^{1/2} \\ &= \begin{bmatrix} \sqrt{4\sigma_v^4 g_1^2(n+\eta_1)+\sigma_n^2} & & & \\ & \sqrt{4\sigma_v^4 g_2^2(n+\eta_2)+\sigma_n^2} & & \\ & & \ddots & \\ & & & \sqrt{4\sigma_v^4 g_M^2(n+\eta_M)+\sigma_n^2} \end{bmatrix} \end{aligned} \tag{3.48}$$

Applying the linear transformation $\underline{q} = D\underline{w}$ gives

$$d_m^2(\underline{w}) = \frac{\sigma_v^4 \underline{q}^T D^{-1} \underline{\eta}_g \underline{\eta}_g^T D^{-1} \underline{q}}{\underline{q}^T \underline{q}} \overset{(a)}{\leq} \sigma_v^4 \lambda_{\max}(D^{-1} \underline{\eta}_g \underline{\eta}_g^T D^{-1}) \tag{3.49}$$

Where $\lambda_{\max}(\cdot)$ denotes the maximum eigenvalue of the matrix. Note that (a) follows the Rayleigh Ritz inequality and the equality is achieved if $\underline{q} = \underline{q}^o$, which is the eigenvector of the positive definite matrix $D^{-1}\underline{\eta}_g \underline{\eta}_g^T D^{-1}$ corresponding to the maximum eigenvalue. Therefore, we find the optimal solution of (3.42) as the same step is

$$\underline{w} = \frac{D^{-1}\underline{q}^o}{\left\| D^{-1}\underline{q}^o \right\|_2} \tag{3.50}$$

which maximizes the modified deflection coefficient. we can prove by the simulation results below, a larger value of d_m^2 leads to a larger probability of detection.

3.4 Simulation Result

In this section, the proposed approach is simulated numerically and compare with some other existing approaches. Firstly, we consider three or ten secondary users (M=3 or M=10) in the cognitive radio networks, and the secondary users sense the frequency spectrum independently. The channel gain between each secondary user and the target primary user is generated by a complex normal distribution (i.e., $h_i \sim CN(0,1)$) and the channel noise between each secondary user and the target primary user are AWGN with zero mean and variance $\sigma_v^2 = 1$. For simplicity, we assume the channel gain { g_i } between secondary users and fusion center are constant AWGN ($g_i = 1$), and the channel noise { n_i } between secondary users and fusion center are AWGN with zero mean and variance $\sigma_n^2 = 1$. The transmitted primary signal has unit power $|s(k)|^2 = 1$ and the detection interval is $2n$ samples. The proposed cooperation schemes are compared with selection combining method (SC i.e., selecting the user with maximum SNR), equal gain combination method (EGC i.e., $w_i = \frac{1}{\sqrt{M}} \quad , i = 1,2,...,M$) and single cognitive radio.

Secondly, we further consider the channel gain { g_i } between secondary users and fusion center are generated by a normal distribution with zero mean and unit variance, and they are assumed to be fixed during the detection interval. We assume that the channel noise between each secondary user to the target primary user and secondary users to fusion center are, respectively, AWGN with zero mean and variance $\sigma_v^2 = 2$ and $\sigma_n^2 = 2$. Under the condition, we observe the effect of different number of secondary user (M) and different variance of channel noise between each secondary user to the target primary user or secondary users to fusion center.

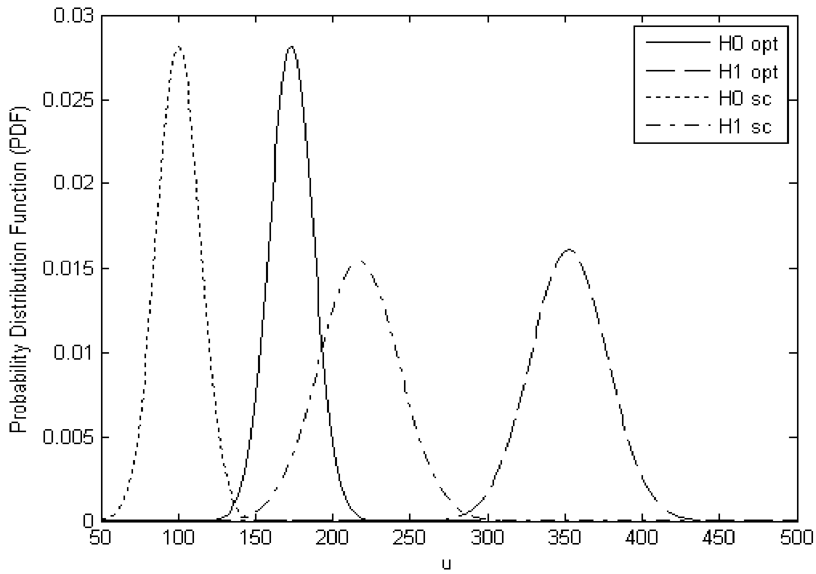

Fig. 3-2 The probability distribution function of the test statistics (u) under different hypotheses, with constant AWGN channel ($g_i = 1$) ,M=3, n=50, $\sigma_v^2 = 1$, and $\sigma_n^2 = 1$. The result is the average of 100 simulations.

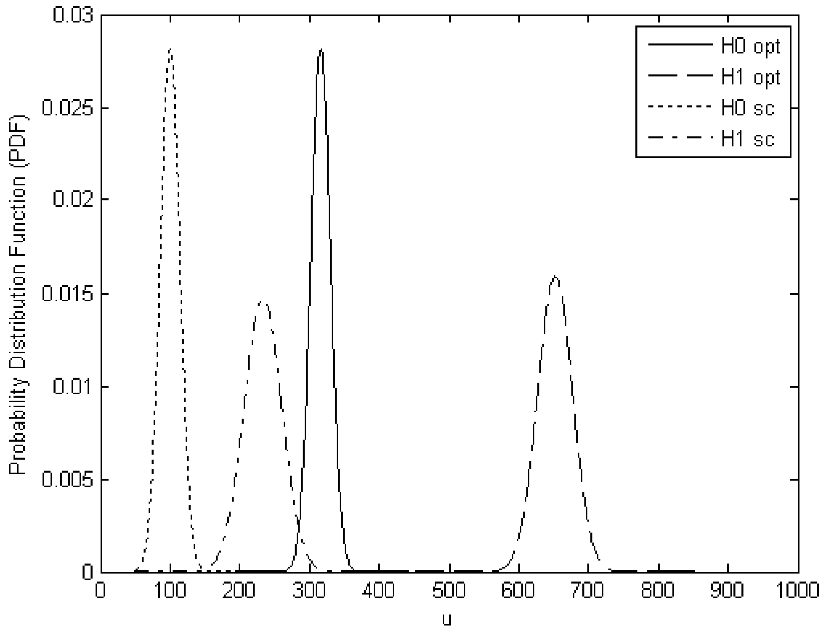

Fig. 3-3 The probability distribution function of the test statistics (u) under different hypotheses, with constant AWGN channel ($g_i = 1$) ,M=10, n=50, $\sigma_v^2 = 1$, and $\sigma_n^2 = 1$. The result is the average of 100 simulations.

From figure 3-2 and figure 3-3, we show that the probability distribution functions of the test statistics under different hypotheses. We compare the distribution of optimization of modified reflection coefficient with the distribution of single cognitive radio SC. We can observe that the distance between $\bar{u}_{opt.PDF,H_0}$ and $\bar{u}_{opt.PDF,H_1}$ is larger than the distance between $\bar{u}_{sc,H_0}$ and $\bar{u}_{sc,H_1}$. Also, we can find that the spread of $u_{opt.PDF,H_1}$ is narrower than that of u_{sc,H_1}. On the other word, the variance of $u_{opt.PDF,H_1}$ is smaller than the variance of u_{sc,H_1}. Further, when the numbers of secondary user are increased, we can observe that the distance between $\bar{u}_{opt.PDF,H_0}$ and $\bar{u}_{opt.PDF,H_1}$ become large.

According to above-mentioned, we obviously understand that the distribution of optimization of modified reflection coefficient and increased the numbers of secondary user would result in more accurate inference. These observations imply that the PDF optimization cooperation scheme outperforms any local spectrum sensing by individual secondary users.

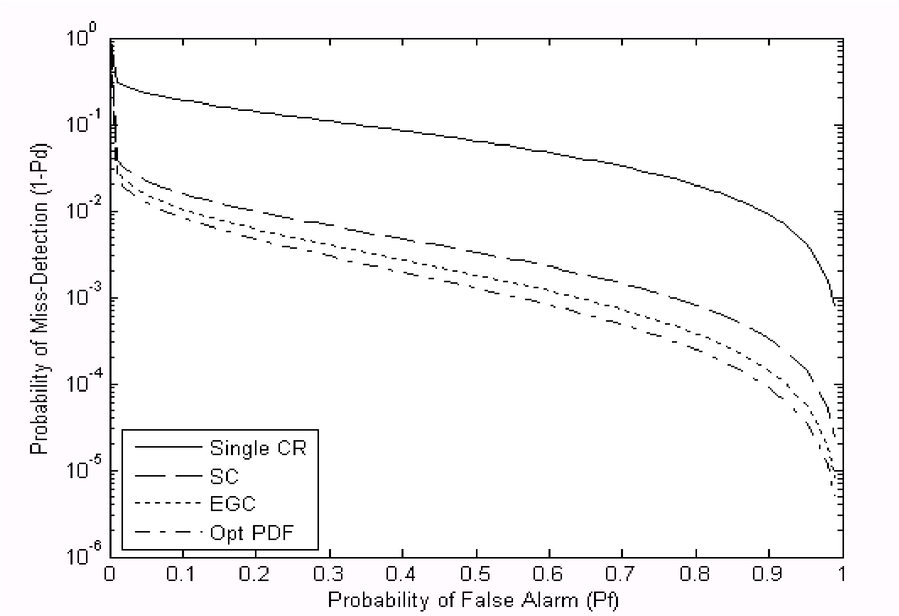

Fig. 3-4 The probability of miss-detection ($1-P_d$) vs. the probability of false alarm (P_f), with constant AWGN channel ($g_i = 1$) ,M=3, n=50, $\sigma_v^2 = 1$, and $\sigma_n^2 = 1$. The result is the average of 1000 simulations.

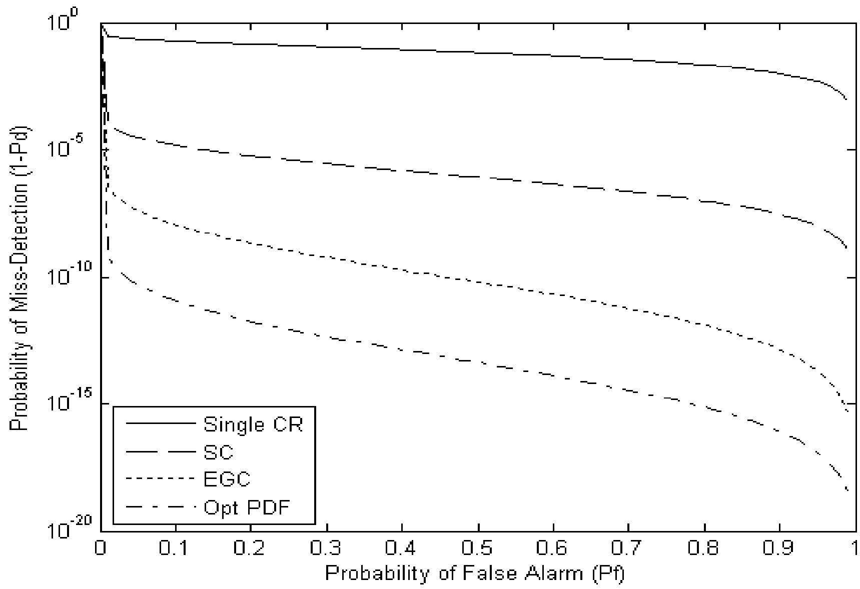

Fig. 3-5 The probability of miss-detection ($1-P_d$) vs. the probability of false alarm (P_f), with constant AWGN channel ($g_i = 1$) ,M=10, n=50, $\sigma_v^2 = 1$, and $\sigma_n^2 = 1$. The result is the average of 1000 simulations.

From figure 3-4 and figure 3-5, we plot the probability of miss-detection ($1-P_d$) versus the probability of false alarm (P_f) under various approaches, such as optimized modified reflection coefficient method, equal gain combination method (EGC, the corresponding weight coefficient is expressed as $w_i = \frac{1}{\sqrt{M}}$, $i = 1,2,..., M$), selection combining method (SC, selecting the user with maximum SNR), and single cognitive radio. The probability of miss-detection ($1-P_d$) versus the probability of false alarm (P_f) directly measures the interference level to the primary users for a given P_f. The simulation shows that the proposed optimized modified reflection coefficient method (denoted as opt PDF) lead to much less interference (much higher probability of detection) to the primary user than single cognitive radio, selection combining method, and equal gain combination method. Also, we can find that the cooperation schemes (opt PDF, EGC) outperform single cognitive radio schemes (SC, single CR). The cooperation gain is due to control the combining weight coefficient which sharp the probability distribution function. Further, we can observe that when the numbers of secondary user are increased, the cooperation gain become large.

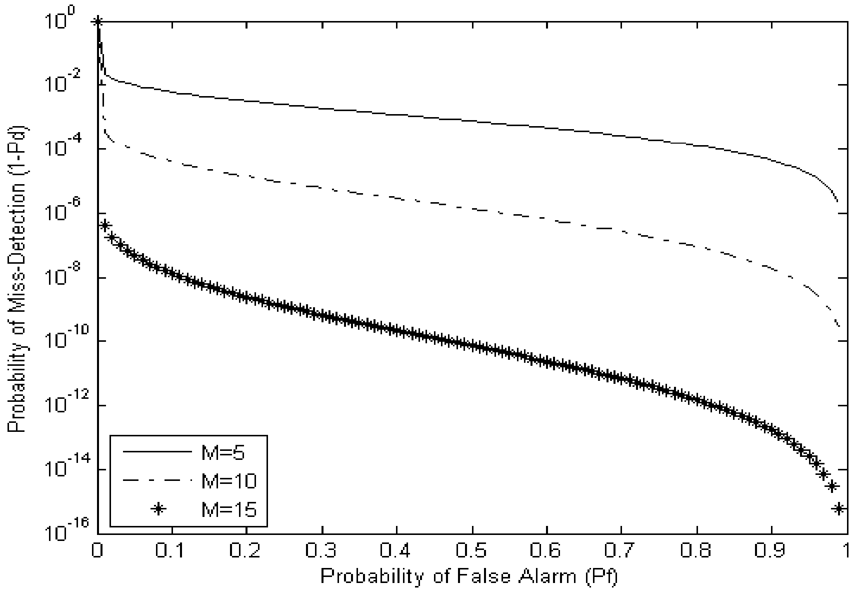

Fig. 3-6 The probability of miss-detection ($1-P_d$) vs. the probability of false alarm (P_f) under various M, with fading channel (g_i are generated according to a normal distribution and assumed to be fixed during a detection interval) , n=50, $\sigma_v^2 = 2$, and $\sigma_n^2 = 2$. The result is the average of 1000 simulations.

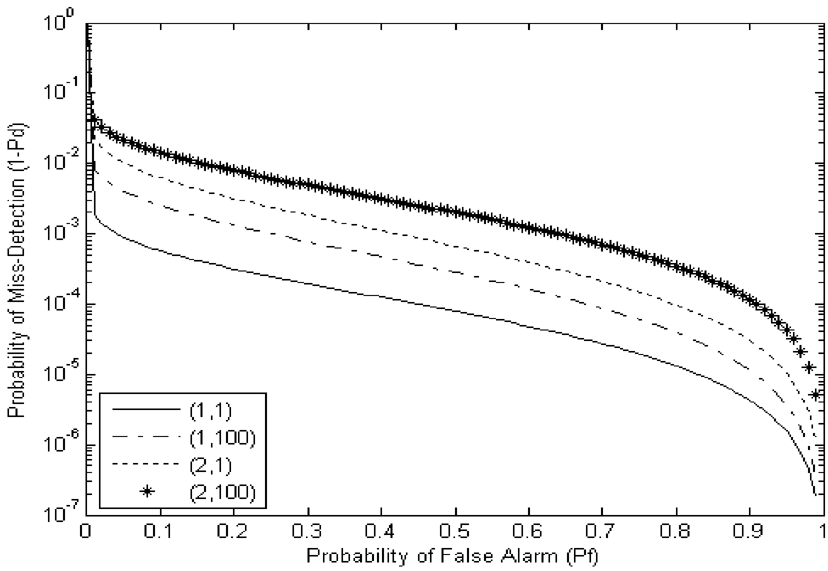

Fig. 3-7 The probability of miss-detection ($1-P_d$) vs. the probability of false alarm (P_f) under various (σ_v^2, σ_n^2), with fading channel (g_i are generated according to a normal distribution) ,M=5, n=50. The result is the average of 1000 simulations.

From figure 3-6, we plot the probability of miss-detection ($1-P_d$) versus the probability of false alarm (P_f) under different numbers of secondary user. We further consider the channel gain {g_i} between secondary users and fusion center are generated by a normal distribution with zero mean and unit variance, and they are assumed to be fixed during the detection interval. And, we observe that when the numbers of secondary user are increased, the performance become batter. In other words, under the same condition, the sensing reliability improves as the number of secondary users increase.

From figure 3-7, we plot the probability of miss-detection ($1-P_d$) versus the probability of false alarm (P_f) under different noise condition. And we consider the channel gain {g_i} between secondary users and fusion center are generated by a normal distribution with zero mean and unit variance, and they are assumed to be fixed during the detection interval. As we can observe, the detection performance degrades as the noise conditions become bad. We can also find that the channel noise between each secondary user to the target primary user is more sensitive to the detection performance than that of secondary users to fusion center.

From figure 3-8, we plot the probability of miss-detection ($1-P_d$) versus the probability of false alarm (P_f) under different cooperation schemes (opt PDF and EGC) and various M with fading channel (g_i are generated according to a normal distribution). We can obviously see that the EGC method has a severe detection performance in the fading channel between secondary user and fusion center, even if we increase the number of secondary user. In other words, the EGC cooperation scheme doesn't work due to the fading channel in that environment. However, we proposed cooperation scheme works well, and the sensing reliability improves as the number of secondary users increase.

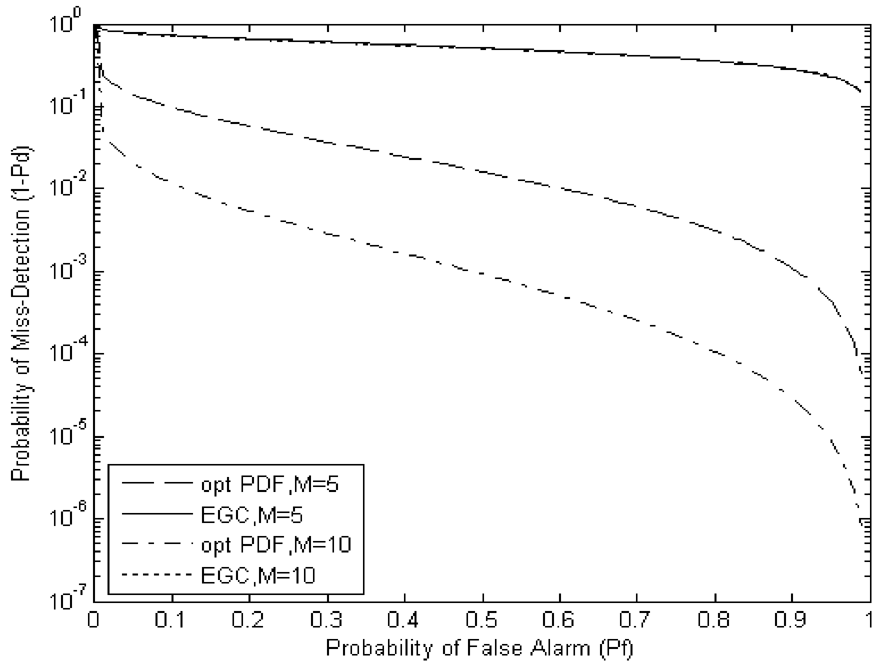

Fig. 3-8 The probability of miss-detection ($1-P_d$) vs. the probability of false alarm (P_f) under different cooperation schemes (opt PDF and EGC) and various M, with fading channel (g_i are generated according to a normal distribution), n=50, $\sigma_v^2 = 2$, and $\sigma_n^2 = 2$. The result is the average of 1000 simulations.

Chapter 4

Conclusion

Cognitive radio network enables much higher spectrum efficiency by dynamic spectrum access. Therefore, it will be a popular technique for future wireless communications to mitigate the spectrum scarcity issue. Spectrum sensing is a main and tough task in cognitive radio networks. However, due to the effect of shadowing, fading, and time-varying nature of wireless channels, the individual cognitive radios may not be able to reliably and quickly detect the existence of a primary signal. In this thesis, we propose a simple but efficient cooperation spectrum sensing based on energy detection and consider the channel between the secondary user and fusion center with two cases. One is considered only the channel noise between the secondary user and fusion center (i.e., constant AWGN channel), and the other is considered both of the channel noise and channel fading between the secondary user and fusion center (i.e., fading channel). Our objective is to improve the detection performance and to combat the channel fading effects. Finally, we optimize a modified deflection coefficient to find the optimal linear combining weights.

From the simulations, we obviously understand that the distribution of optimization of modified reflection coefficient and increased the numbers of secondary user would result in more accurate inference. Also, we can observe that the proposed cooperation method have the better detection performance then other methods (i.e., single CR, SC, and EGC). We can also find that the channel noise between each secondary user to the target primary user is more sensitive to the detection performance than that of secondary users to fusion center. Finally, the sensing reliability improves as the number of secondary users increase.

References

[1] J. Mitola, III and G. Q. Maguire,“Cognitive radio: Making software radios more personal,” *IEEE Pers. Commun,* vol. 6, pp.13-18, 1999.

[2] Fed. commun, Comm. 2003, Et docket-322.

[3] S. Haykin, “Cognitive radio: Brain-empowered wireless communications,” *IEEE Journal Selected Areas in Commun.*, vol. 23, no. 2, pp. 201-220, Feb. 2005.

[4] D. Cabric, S. M. Mishra, D. Willkomm, R. W. Brodersen, and A. Wolisz,“A cognitive radio approach for usage of virtual unlicensed spectrum,” *in Proc. 14th IST Mobile and Wireless Commun., Summit*, June 2005.

[5] A. Sahai, N. Hoven, and R. Tandra, “Some fundamental limits on cognitive radio,” in *Proc. Allerton Conf. Communication, Control, and Computing*, Oct.2004, pp. 131-136.

[6] D. Cabric, S. M. Mishra, and R. Brodersen, “Implementation issues in spectrum sensing for cognitive radios,” in *Proc. 38th Asilomar Conf. Signal, Systems and Computers,* Pacific Grove, CA, Nov. 2004, pp. 772-776.

[7] P. K. Varshney, *Distributed Detection and Data Fusion*. New York: Springer Verlag, 1997.

[8] R. S. Blum, S. A. Kassam, and H. V. Poor,” Distributed detection with multiple sensors: Part II- Advanced topics,” *Proc. IEEE*, vol. 85, pp. 64-79, Jan. 1997.

[9] V. Aalo and R. Viswanathan,“ Asymptotic performance of a distributed detection system in correlated Gaussian noise,” IEEE Trans. Signal Processing, vol. 40, pp. 211-213, Feb. 1992.

[10] A. Ghasemi and E. Sousa,“ Collaborative spectrum sensing for opportunistic access in fading environments,” in *Proc. IEEE Symp. New Frontiers in Dynamic Spectrum Access Networks*, Baltimore, MD, Nov. 2005, pp. 131-136.

[11] E. Vistotsky, S. Kuffner, and R. Peterson, “ On collaborative detection of TV transmissions in support of dynamic spectrum sharing,” in *Proc. IEEE Symp. New Frontiers in Dynamic Spectrum Access Networks*, Baltimore, MD, Nov. 2005, pp. 338-345.

[12] G. Ghurumuruhan and Y. Li, “ Agility improvement through cooperative diversity in cognitive radio,” in *Proc. IEEE GLOBECOM*, St. Louis, MO, Nov. 2005, pp. 2505-2509.

[13] Z Quan, S. Cui, and A. H. Sayed, “ An optimal strategy for cooperative spectrum sensing in cognitive radio networks,” in *Proc. IEEE GLOBECOM*, 2007.

[14] S. M. Kay, “Fundamentals of statistical signal processing,” Prentice Hall.

[15] F. F. Digham, M. -S. Alouini, and M. K. Simon, “On the energy detection of unknown signals over fading channels”, in *Proc. IEEE Int. Conf. on Commun*., May 2003, vol. 5, pp.3575-3579.

[16] H. Urkowitz, ”Energy detection of unknown deterministic signals ”, in *Proc. IEEE*, vol. 55, pp. 523-531, April 1967.

[17] J. Ma, and Y. Li, ”Soft combination and detection for cooperative spectrum sensing in cognitive radio networks”, in *Proc. IEEE GLOBECOM*, 2007.

[18] J. Mitola, “Cognitive radio: An integrated agent architecture for software defined radio,” Doctor of Technology, Royal Inst. Technol. (KTH), Stockholm, Sweden, 2000.

[19] Federal Communications Commission, “Spectrum Policy Task Force,” Rep. ET Docket no. 02-135, Nov. 2002.

[20] FCC, Cognitive Radio Workshop, May 19, 2003, [online]. Available: http://www.fcc.gov/searchtools.html.

[21]*Proc. Conf. Cogn. Radios*, Las Vegas, NV, Mar. 15-16, 2004.

[22] C.E. Shannon, “communication in the presence of noise,” Proc. IRE, vol. 37, pp. 10-21, Jan. 1949.

[23] R. Tandra and A. Sahai, “Fundamental limits on detection in low SNR under noise uncertainty,” *in Proc. Int. Conf. on wireless Networks, Commun., and Mobile Computing*, June 2005, vol. 1, pp. 464-469.

[24]G. L. Stuber, *Principles of Mobile Communication*, 2nd ed., Kluwer Academic Publishers, 2001.

自　　傳

游衛川, 西元 1983 年生於宜蘭縣。 西元 2006 年畢業於台灣台北淡江大學電機工程學系，之後進入交通大學電子研究所攻讀碩士學位，於 2008 年取得碩士學位。研究方向為無線通訊、感知無線電。