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碩 士 論 文

最大雜訊比之時域等化器設計

A TEQ Design Method That Maximizes Signal to Interference Ratio

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最大訊雜比之時域等化器設計

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摘　　要

在數位用戶迴路(DSL)中，可將頻帶分割以上傳訊號與下載訊號，此上傳與下載時皆有部分頻帶沒有用來傳輸。在此篇論文中，我們利用使用頻帶上的訊號能量對干擾雜訊能量之比值(訊雜比)，來設計時域等化器。我們將訊雜最佳化，由通道的頻率響應中可看出，此種方法能加強傳輸頻帶上的響應，在模擬中也可發現我們所提出之時域等化器設計不僅可以有效的縮短通道的等效長度也能夠有效的改善系統的傳輸率。

A TEQ Design Method That Maximizes Signal to Interference Ratio

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Abstract

In DSL (Digital Subscriber Lines) application, usually frequency division multiplexing is used to separate the upstream and downstream signals. In either direction of transmission, some of the frequency bands are not used for transmission. Earlier results show that minimizing interference by exploiting unused bands is usual for design TEQ. In this thesis, we also consider signal power in the objective function. The time domain equalizer (TEQ) is designed by maximizing signal to interference ratio. We will see that the incorporation of signal power will enhance the frequency response of the resulting channel response in the transmission bands. A better transmission rate can be achieved as demonstrated by simulation examples.

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Chapter 1

Introduction

The Discrete Fourier Transform (DFT) based Discrete MultiTone system (DMT) scheme is applied to asymmetric digital subscriber line (ADSL) and very high bit rate digital subscriber line (VDSL).[1]-[2]. In the DMT system, the input symbol block passes through the M-point inverse Discrete Fourier Transform (IDFT) at transmitter, and the receiver perform M-point DFT computation where M is the number of subchannel. After the signal through IDFT, a cyclic prefix of L samples is added to transmitter signal. When the channel order is less than the cyclic prefix (CP) length L, there is no Inter-Block Interference (IBI). On the other hand, if the channel order is larger than the cyclic prefix length, the channel coefficients of out of the CP length will cause interference. The interference would lower the signal to interference and noise ratio(SINR) and bit rate decrease. Because the channel is usually longer than CP length in DSL application, a time domain equalizer (TEQ) is usually inserted at the receiver to shorten the channel impulse response. The TEQ in DMT system plays an important role in the application of transmission over DSL channel.[1]-[2]. The design methods of TEQ affect the bit rate greatly.

Many TEQ methods have been proposed in the literature[3]-[13]. In [3], it proposed an optimal TEQ design method to maximize signal to interference ratio when channel is determinate. This method maximize the ratio of the energy inside the window to the energy out-of the window energy. The optimal TEQ with synchronization delay is found to maximize the ratio. This optimal solution can be solved as an eigen problem. In [4], the authors proposed a TEQ design that consider not only the energy outside the window but also the time index of the equivalent channel. In [5], TEQ design that minimizing the intersymbol interference and interblock interference is considered.

Many TEQ design methods that optimize the transmission rate have been reported, In [6], the author minimizes ISI and channel noise on the tones used for transmission to design the TEQ. The method of bit rate maximizing (BM) is a nonlinear solution, but a fast, near optimal solution of minimum-ISI [6] is proposed. It can be used in a practical system. The TEQ design methods of BM by using an approximation to the geometric SNR are proposed in (MGSNR)[7]-[8]. By optimizing the transmission rate using the adaptive algorithm, the TEQ designs are studied in [9]-[10]. Per-tone equalization for bit rate maximization is proposed in [11]. A filterbank approach to the design of TEQ for maximizing the bit rate is given in [12]. Many TEQ design are proposed in time domain. The TEQ response will effect the transmission rate [10]. The zeros of TEQ response at transmission bands will cause the poor total transmission rate. The frequency response TEQ response has large influence on bit rate. A semi-blind TEQ design method which maximizes the signal-to-interference ratio (SIR) using the training symbols in DSL initialization is given in [13]. For a given channel, a TEQ that minimizing partial interference of the so-called source tones to target tones, the channel cab be shortened.

In this thesis, we propose a frequency domain based design method of TEQ for DSL applications. The TEQ is designed by maximizing signal to interference ratio as in [3]. As in [14], partial interference from source to target tones is computed in the frequency domain. In addition, we consider signal power from the tones in the transmission bands. The incorporation of signal power help to enhance the frequency response of the TEQ in the transmission bands. As a result a higher bit rate can be achieved. Furthermore, the problem of maximizing signal to interference ratio can be formulated as an eigen problem. The optimal TEQ can be obtained in a closed form.

1.1 Outline

In Chapter 2, the block diagram and filterbank representation of DMT system model and introduction of VDSL will be shown. A survey of TEQ designs will be introduced in Chapter 3. The TEQ design with minimizing interference to signal power ratio method is proposed in Chapter 4. Chapter 5 shows some computer simulations and comparisons. Finally, Conclusions and discussions will be presented.

1.2 Notations

1. **Bold face** are used to represent the matrices or the vectors.

2. $\mathbf{A}^H$ denotes transpose-conjugate of $\mathbf{A}$.

3. The notation $\mathbf{I}_M$ is used to represent the $M \times M$ identity Matrix.

4. The notation $diag(\lambda_1, \lambda_2, \cdots, \lambda_L)$denotes an $M \times M$ diagonal matrix with the diagonal element equal to λ_k.

5. The notation $\mathbf{W}_M$ is used to represent the normalized $M \times M$ DFT matrix given be
$$[\mathbf{W}_M]_{kn} = \frac{1}{\sqrt{M}} e^{-j\frac{2\pi}{M}kn}$$
where $0 \leq k, n \leq M - 1$.

Chapter 2

System Model

2.1 DMT System Model

The block diagram of traditional DMT system is shown in Fig. 2.1. The transmitter and receiver perform respectively M-point IDFT and DFT computations, where M is the number of subchannels. The $M \times M$ DFT matrix is denoted by $\mathbf{W}$, with $[W]_{m,n} = e^{-j2\pi mn/M}/\sqrt{M}$. A cyclic prefix of length L is added after the parallel to serial (P/S) operation. The receiver includes the blocks z^d, where d is a parameter of synchronization delay. After removing CP at the receiver, the symbol passes through the serial to parallel ("S/P") operation to convert the serial sample sequence to parallel samples. Finally, the signal block passes the DFT system and M parallel one-tap frequency domain equalizers (FEQ). The FEQ for each subchannel is of one-tap coefficient $\frac{1}{\lambda_k}$, where $\lambda_k = \sum_{n=0}^{M-1} c_1(n) W_M^{kn}$. Then, the output symbols are obtained.

When the CP length is larger than the channel order, there is no Inter Block Interference (IBI). If not, the channel coefficients will out of L+1 samples will lead to interference and the transmission rate will decrease. Because of this situation, the time domain equalizer (TEQ) is inserted into the system at the receiver. To add TEQ can shorten the channel impulse response so that the equalized channel has most of the energy concentrated in a window of $L+1$ samples. In the DMT system, because the transmitted signals are real, the input symbols before the IDFT matrix need to have the conjugate symmetric property, i.e., x_0 is real and

$x_k(n) = x^*_{M-k}(n), k = 1, 2, ..., M-1$. in a block. As a result, the system only transmits $(\frac{M}{2} - 1)$ symbols in a block.

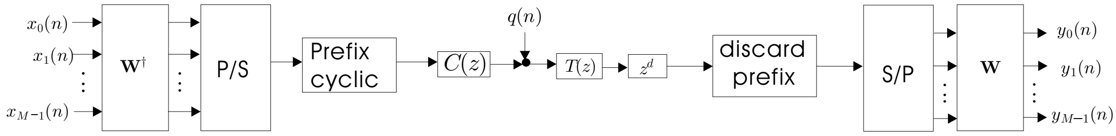


Figure 2.1: DMT System

2.2 VDSL System

The VDSL system use Frequency Division Duplexing (FDD) to separate upstream and downstream transmission. The 1U and 2U donate the upstream bands. The downstream bands are denoted 1D, 2D. The VDSL bands are shown in Fig. 2.2. Separating frequency of the VDSL band is given in Table 2.1.

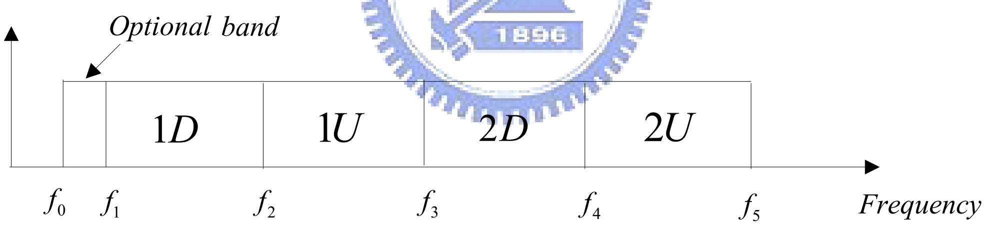


Figure 2.2: The band allocation of VDSL

Separating Frequencies	f_0	f_1	f_2	f_3	f_4	f_5
(MHz)	0.025	0.138	3.75	5.2	8.5	12

Table 2.1: VDSL band separating frequencies

The modulation shall use a maximum number of subchannels equal $\frac{M}{2} = 2^{n+8}$, where n is taken values of 0, 1, 2, 3 and 4. Disjoint subsets of the $\frac{M}{2}$ subchannels shall be defined for use in the downstream and upstream directions. The parameter "Δf" denotes the frequency spacing of subchannels. It shall be

4.3125kHz. The center frequencies of subchannels are $f = k \cdot \Delta f$ where k is taken the values of $0, 1, ..., M-1$. The downstream band 1D between 138kHz and 3.75MHz correspond to the tone set, $\{33 - 870\}$, and 2D between 5.2MHz and 8.5MHz corresponds to the tone set, $\{1206 - 1971\}$. The upstream band 1U between 3.75MHz and 5.2MHz is tone $\{871 - 1205\}$, and 2D between 8.5MHz and 12MHz is tone, $\{1272 - 2047\}$. The downstream and upstream band both can be transmission band but one is transmission band and the other must be unused band.

Chapter 3

Previous design methods

In this chapter, we will introduce a survey of four kinds of TEQ design methods. We will give design methods of maximum shortening SNR (MSSNR) in section 3.1 and minimum inter-symbol interference (Min-ISI) in section 3.2. The per-tone equalization design (PTEQ) is given in section 3.3 and frequency domain TEQ design method is given in section 3.4.

3.1 Maximum shortening SNR (MSSNR)

The MSSNR TEQ design method is to maximize the ratio of the energy in the largest consecutive $L + 1$ samples of effective channel to the energy in the remaining samples, where L is the CP length. The largest $L + 1$ samples will not necessarily start at the first sample. It would be stated at the delay point d. d is normally compensated for at the receiver by delaying the start of the received symbol. The equivalent channel is $h(n) = c(n) * t(n)$ where the coefficients of $h(n)$ can be rewritten as $\mathbf{h} = \mathbf{C}\mathbf{t}$. $\mathbf{C}$ is a convolution matrix which is composed of the original channel coefficients and $\mathbf{t}$ is a $T \times 1$ TEQ vector. The length of the equivalent channel $h(n)$ is $L+T-1$ where T is the length of TEQ.

$$
\begin{aligned}
\mathbf{h} &= \begin{pmatrix} h(0) \\ h(1) \\ \vdots \\ h(L-1) \\ h(L) \\ \vdots \\ h(L+T-2) \end{pmatrix}, \\
&= \begin{pmatrix} c(0) & 0 & \cdots & & 0 \\ c(1) & c(0) & \ddots & & \\ \vdots & \vdots & & & \\ c(L-1) & c(L-2) & \cdots & c(L-T+1) & c(L-T) \\ 0 & c(L-1) & \cdots & & c(L-T+1) \\ \vdots & \ddots & & & \vdots \\ 0 & \cdots & & 0 & c(L-1) \end{pmatrix} \begin{pmatrix} t(0) \\ t(1) \\ \vdots \\ t(T-1) \end{pmatrix}, \\
&= \mathbf{C}\mathbf{t}.
\end{aligned}
\tag{3.1}
$$

Let $\mathbf{h}_{win}$ represent a window of $L+1$ samples of $\mathbf{h}$ starting with sample d, and let $\mathbf{h}_{wall}$ represent the remaining $L+T-\nu-2$ samples of $\mathbf{h}$. These are shown as

$$
\begin{aligned}
\mathbf{h}_{win} &= \begin{pmatrix} h(d) \\ h(d+1) \\ \vdots \\ h(d+L) \end{pmatrix}, \\
&= \begin{pmatrix} c(d) & c(d-1) & \cdots & c(d-T+1 \\ c(d+1) & c(d) & \ldots & c(d-T+2) \\ \vdots & & \ddots & \vdots \\ c(d+L) & c(d+L-1) & \cdots & c(d+L-T+1) \end{pmatrix} \begin{pmatrix} t(0) \\ t(1) \\ \vdots \\ t(T-1) \end{pmatrix}, \\
&= \mathbf{C}_{win}\mathbf{t}.
\end{aligned}
\tag{3.2}
$$

$$
\begin{aligned}
\mathbf{h}_{wall} &= \begin{pmatrix} h(0) \\ h(d-1) \\ \vdots \\ h(d+L+1) \\ \vdots \\ h(L+T-2) \end{pmatrix} \\
&= \begin{pmatrix} c(0) & 0 & \cdots & 0 \\ \vdots & \ddots & & \\ c(d-1) & c(d-2) & \cdots & c(d-T) \\ c(d+L+1) & c(d+L) & \cdots & c(d+L-T+2) \\ \vdots & \ddots & & \\ 0 & \cdots & 0 & c(L-1) \end{pmatrix} \begin{pmatrix} t(0) \\ t(1) \\ \vdots \\ t(T-1) \end{pmatrix} \\
&= \mathbf{C}_{wall}\mathbf{t}.
\end{aligned}
\tag{3.3}
$$

We define the SIR is the ratio of energy inside the length of $L+1$ window to the outside the length of $L+1$ window. That is

$$
SIR = \max_{d} \frac{\sum_{n=d}^{d+L} h^2(n)}{\sum_{n=0, i\neq(d,\ldots,d+L)}^{\nu} h^2(n)} \tag{3.4}
$$

where ν is the length of channel.

By (3.2)-(3.4), the SIR can be expressed as

$$
SIR = \max \frac{\mathbf{h}_{win}^{\dagger}\mathbf{h}_{win}}{\mathbf{h}_{wall}^{\dagger}\mathbf{h}_{wall}} = \max \frac{\mathbf{t}^{\dagger}\mathbf{C}_{win}^{\dagger}\mathbf{C}_{win}\mathbf{t}}{\mathbf{t}^{\dagger}\mathbf{C}_{wall}^{\dagger}\mathbf{C}_{wall}\mathbf{t}} = \max \frac{\mathbf{t}^{\dagger}\mathbf{A}\mathbf{t}}{\mathbf{t}^{\dagger}\mathbf{B}\mathbf{t}}. \tag{3.5}
$$

where $\mathbf{A} = \mathbf{C}_{win}^{\dagger}\mathbf{C}_{win}$ and $\mathbf{B} = \mathbf{C}_{wall}^{\dagger}\mathbf{C}_{wall}$. The optimal TEQ shall be chosen to maximize $\mathbf{h}_{win}^{\dagger}\mathbf{h}_{win}$ while satisfying the constraint $\mathbf{h}_{wall}^{\dagger}\mathbf{h}_{wall} = 1$ The matrix $\mathbf{B}$ is Hermitian and positive semi-definite. We consider $\mathbf{B}$ is a positive definite matrix and invertible. It is a rare case when the determinant of matrix $\mathbf{B}$ is zero. The optimal TEQ $\mathbf{t}$ is the eigenvector of $\mathbf{B^{-1}A}$ corresponding to the maximum eigenvalue.

3.2 Minimum inter-symbol interference (Min-ISI)

We review the min-ISI method in this section. In order to use an equalizer in a practical system, the nonlinear optimization must be avoided. The min-ISI method don't need a globally optimal constrained nonlinear optimization solver to calculate the equalizer. In the DMT system, the output of TEQ at receiver is

$$y_k = c_k * t_k * x_k + t_k * q_k, \tag{3.6}$$

where x_k is transmitted signal, c_k is the impulse response of discrete channel, q_k is the discrete additive noise and t_k is the impulse response of TEQ. The transmitted signal is M points separated by cyclic prefix lengths of L. If we can use a equalizer to let the channel energy concentrated in a window of $L+1$ samples, the ISI free. Because we want the system to be free from ISI, we formulate a windowing function g_k to isolate the desire pare of c_k.

$$g_k = \begin{cases} 1, & \Delta \leq k \leq \Delta + L \\ 0, & otherwise \end{cases} \tag{3.7}$$

where Δ is a synchronization delay. Also, we can separate the signal, interference and noise terms by using the windowing function. That is

$$y_k = h_k^{signal} * x_k + h_k^{ISI} * x_k + t_k * q_k \tag{3.8}$$

where $h_k^{signal} = g_k(c_k * t_k)$ and $h_k^{ISI} = (1 - g_k)(c_k * t_k)$. By(3.8), the SNR in each subchannel is given by

$$SNR_i = \frac{|H_i^{signal}|^2 S_{x,i}}{|H_i^{ISI}|^2 S_{x,i} + |W_i|^2 S_{q,i}} \tag{3.9}$$

where $S_{x,i}$ is the M-point power spectrum of x_i and $S_{q,i}$ is the M-point power spectrum of q_i. Define the matrix-vector notation as

$$\begin{array}{rcl} H_i^{signal} & = & \mathbf{r}_i^{\dagger}\mathbf{GHt} \\ H_i^{ISI} & = & \mathbf{r}_i^{\dagger}\mathbf{DHt} \\ W_i & = & \mathbf{r}_i^{\dagger}\mathbf{Ft}, \end{array} \tag{3.10}$$

where

$$\begin{aligned}
\mathbf{t} &= \begin{pmatrix} t_0 & t_1 & \cdots & t_{T-1} \end{pmatrix}^\top, \\
\mathbf{H} &= \begin{pmatrix} h_0 & h_{-1} & \cdots & h_{-(T-1)} \\ h_1 & h_0 & \cdots & h_{-(T-2)} \\ \vdots & \vdots & \ddots & \vdots \\ h_{M-1} & h_{(M-2)} & \cdots & h_{-(M-T)} \end{pmatrix}, \\
\mathbf{G} &= diag\begin{pmatrix} g_0 & g_1 & \cdots & g_{M-1} \end{pmatrix}^\top, \\
\mathbf{D} &= \mathbf{I} - \mathbf{G}, \\
\mathbf{r}_i &= \begin{pmatrix} 1 & e^{j\frac{2\pi i}{M}} & \cdots & e^{j\frac{2\pi i(M-1)}{M}} \end{pmatrix}^\top,
\end{aligned} \tag{3.11}$$

$\mathbf{T}$ is length of TEQ.

Finally, the SNR can be rewritten as

$$SNR_i = \frac{|\mathbf{r}_i^\dagger \mathbf{GHt}|^2 S_{x,i}}{|\mathbf{r}_i^\dagger \mathbf{DHt}|^2 S_{x,i} + |\mathbf{r}_i^\dagger \mathbf{Ft}|^2 S_{q,i}}. \tag{3.12}$$

Our goal is to minimize the distortion power in each subchannel. Because the power is nonnegative, minimizing the sum of the distortion power of all subchannel can be viewed as minimizing the distortion power in each subchannel, which can be written as

$$\mathbf{t}^\top \mathbf{H}^\top \mathbf{D}^\top \sum_{i\in\Phi} (\mathbf{r}_i \frac{S_{x,i}}{S_{q,i}} \mathbf{r}_i^\dagger) \mathbf{DHt} = \mathbf{t}^\top \mathbf{At}. \tag{3.13}$$

To prevent minimization of the signal power, we constrain the signal path impulse response energy to one:

$$\|H_i^{signal}\|^2 = \mathbf{t}^\top \mathbf{H}^\top \mathbf{G}^\top \mathbf{GHt} = \mathbf{t}^\top \mathbf{Bt} = 1. \tag{3.14}$$

Then, minimum ISI becomes

$$\min_{\mathbf{t}} \mathbf{t}^\top \mathbf{At} \quad s.t. \ \mathbf{t}^\top \mathbf{Bt} = 1. \tag{3.15}$$

We decompose $\mathbf{B}$ using Cholesky Decomposition into $\mathbf{Q}^\dagger \mathbf{Q}$ if $\mathbf{B}$ is positive definite. Let $\mathbf{v} = \mathbf{Qt}$, the optimize problem becomes

$$\min_{\mathbf{v}} \frac{\mathbf{v}^\dagger \mathbf{Q}^{-\dagger} \mathbf{AQ}^{-1} \mathbf{v}}{\mathbf{v}^\dagger \mathbf{v}}. \tag{3.16}$$

the solution $\mathbf{v}$ is the eigenvector corresponding to the smallest eigenvalue of the matrix $\mathbf{Q}^{-\dagger}\mathbf{AQ}^{-1}$. And the optimum TEQ is obtained as

$$\mathbf{t} = \mathbf{Q}^{-1}\mathbf{v} \quad . \tag{3.17}$$

3.3 Per-tone equalization design (PTEQ)

PTEQ is a TEQ design method for maximizing the bit rate by using the filter bank approach. The optimum solution of PTEQ is closed form and it can be viewed as a theoretical upper bound for other TEQ design methods. The equivalent channel $h(n) = c(n) * t(n)$ where $c(n)$ is channel and it's length is ν and $t(n)$ is the TEQ of length T. Fig. 3.1 is the filterbank representation of DMT receiver. The DFT size of the DMT system is M, and the receiver filter $P_k(z)$ is written

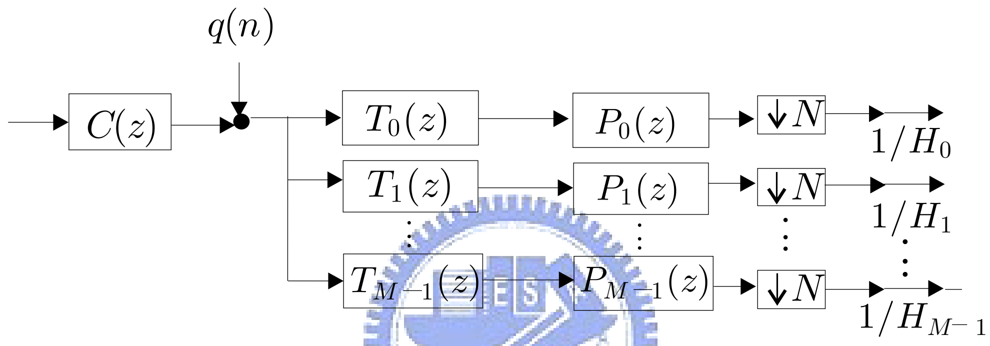


Figure 3.1: The filter bank representation of DMT receiver

as

$$P_k(z) = \sum_{i=L}^{M+L-1} e^{-\frac{j2\pi ki}{M}} z^i. \tag{3.18}$$

The scalars H_k are given by

$$H_k = C(e^{\frac{j2\pi}{M}k}) T_k(e^{\frac{j2\pi}{M}k}). \tag{3.19}$$

From Fig. 3.1, we know $t_k(n)$ is a TEQ of the k-th subchannel. Our goal is to design a TEQ method which have most energy within a specific window of length L. When the energy outside the window, it will generate inter-block ISI. Define the sequence of d_n

$$d_n = \begin{cases} 0, & n_w < n \le n_w + L, \\ 1, & 0 \le n \le n_w \ or \ n_w + L \le n \le \nu + T - 2, \end{cases} \tag{3.20}$$

where n_w is the beginning point of the desired window. The ISI of the k-th subchannel is given by

$$h_{isi,k}(n) = d(n)(c(n) * t_k(n)). \tag{3.21}$$

In Fig. 3.1, we see the output error caused by ISI and noise at the k-th subchannel is $e_k(n) = (e_{isi,k}(n) + e_{q,k}(n))_{\downarrow N}$, $N = M + L$, where

$$\begin{aligned} e_{isi,k}(n) &= p_k(n) * h_{isi,k}(n) * x(n)/H_k \\ e_{q,k}(n) &= p_k(n) * t_k(n) * q(n)/H_k. \end{aligned} \tag{3.22}$$

Because the decimator doesn't change the signal variance, we have

$$\sigma^2_{e_k} = \sigma^2_{isi,k} + \sigma^2_{q_k},$$

where $\sigma^2_{isi,k}$ and $\sigma^2_{q_k}$ are the variances of $e_{isi,k}(n)$ and $e_{q,k}(n)$ respectively. Assume the signal and noise are uncorrelated. Let $\mathbf{t_k}$ be the $T \times 1$ column vector. It consist of the k-th TEQ coefficients.

$$\mathbf{t}_k = (t_k(0)\ t_k(1)\ \ldots\ t_k(T-1))^T_{1\times T}.$$

Let $\mathbf{w}_k$ be the first L elements of the k-th row vector of the M-point DFT matrix,

$$\mathbf{w}_k = (1\ e^{-j\frac{2\pi}{M}k}\ \ldots\ e^{-j\frac{2\pi}{M}k(N_l)})_{1\times L}.$$

Let $\mathbf{C}$ be a $(\nu+T-1)\times T$ lower triangular Toeplitz matrix. The first column of $\mathbf{C}$ is given by

$$\begin{pmatrix} c_0 & c_1 & \cdots & c_{\nu-1} & 0 & \cdots & 0 \end{pmatrix}^\top.$$

Let $\mathbf{D}$ be a $(\nu+T-1)\times(\nu+T-1)$. The $\mathbf{D}$ is diagonal matrix with entries $d_{ii} = d(i)$. Therefore, the error variance can be represented as

$$\begin{aligned} \sigma^2_{isi,k} &= \frac{\sigma_x^2 \mathbf{t}_k^\dagger \mathbf{C}^\dagger \mathbf{D} \mathbf{P}_k^\dagger \mathbf{P}_k \mathbf{D} \mathbf{C} \mathbf{t}_k}{|C(e^{j2\pi k/M})|^2 \mathbf{t}_k^\dagger \mathbf{w}_k \mathbf{w}_k^\dagger \mathbf{t}_k} \\ \sigma^2_{\nu,k} &= \frac{\mathbf{t}_k^\dagger \tilde{\mathbf{P}}_k^\dagger \mathbf{R}_q \tilde{\mathbf{P}}_k \mathbf{t}_k}{|C(e^{j2\pi k/M})|^2 \mathbf{t}_k^\dagger \mathbf{w}_k \mathbf{w}_k^\dagger \mathbf{t}_k} \end{aligned} \tag{3.23}$$

where $\mathbf{R_q}$ is the autocorrelation of $q(n)$. $\mathbf{P_k}$ is lower triangular Toeplitz matrix and its first column is given by

$$\begin{pmatrix} e^{\frac{j2\pi}{M}k(M-1)} & \cdots & e^{\frac{j2\pi}{M}k} & 1 & 0 & \cdots & 0 \end{pmatrix}^\top.$$

The form of $\tilde{\mathbf{P}}_k$ and $\mathbf{P}_k$ are the same but the dimension of $\tilde{\mathbf{P}}_k$ is $(M+T-1)\times T$. We define

$$\mathbf{Q}_{isi,k} = \frac{\varepsilon_x \mathbf{C}^\dagger \mathbf{D} \mathbf{P}_k^\dagger \mathbf{P}_k \mathbf{D} \mathbf{C}}{|C(e^{j2\pi k/M})|^2}, \mathbf{Q}_{\nu,k} = \frac{\tilde{\mathbf{P}}_k^\dagger \mathbf{R}_q \tilde{\mathbf{P}}_k}{|C(e^{j2\pi k/M})|^2}.$$

The matrix $\mathbf{Q}_{isi,k}$ is semi-positive definite and the matrix $\mathbf{Q}_{q,k}$ is positive define for all k. The two matrices satisfy

$$\mathbf{Q}_{isi,M-k} = \mathbf{Q}^*_{isi,k}, \mathbf{Q}_{q,M-k} = \mathbf{Q}^*_{q,k} \tag{3.24}$$

for $k = 1, \ldots, \frac{M}{2} - 1$.

Finally, the optimization problem for PTEQ becomes

$$\min_{\mathbf{t}_k} \frac{\mathbf{t}_k^\dagger(\mathbf{Q}_{isi,M-k} + \mathbf{Q}_{q,k})\mathbf{t}_k}{\mathbf{t}_k^\dagger \mathbf{w}_k \mathbf{w}_k^\dagger \mathbf{t}_k}. \tag{3.25}$$

Use Cholesky decomposition $(\mathbf{Q}_{isi,M-k} + \mathbf{Q}_{q,k})$=$\mathbf{Q_k}^\dagger\mathbf{Q_k}$. Let $\mathbf{u}_k = \mathbf{Q}_k\mathbf{t}_k$. Then the problem can be rewritten as

$$\max_{\mathbf{u}_k} \frac{\mathbf{u}_k^\dagger \mathbf{Q}_k^{-1} \mathbf{w}_k \mathbf{w}_k^\dagger \mathbf{Q}_k^{-\dagger} \mathbf{u}_k}{\mathbf{u}_k^\dagger \mathbf{u}_k}. \tag{3.26}$$

We can obtain the optimum TEQ of each subchannel by solving

$$\mathbf{t}_k = \mathbf{Q}_k^{-1}\mathbf{u}_k, \tag{3.27}$$

where the solution $\mathbf{u_k}$ is the eigenvector corresponding to the largest eigenvalue of $\mathbf{Q}_k^{-1}\mathbf{w}_k\mathbf{w}_k^\dagger\mathbf{Q}_k^{-\dagger}$.

3.4 Frequency Domain Interference minimization TEQ Design [14]

In DSL (digital subscriber loops) applications, the upstream and downstream bands usually do not overlap. The frequency domain TEQ design method use the unused tones to design TEQ. In [14], interference from the set of tones (source tones) to another set of tones (target tones) is considered. When the source and target tones are null tones, the frequency response of TEQ will be free from zeros in the transmission band.

We consider the DMT system is in Fig. 2.1. The transmitter and receiver use M-point IDFT and DFT matrix, where M is the number of subchannel. Fig. 2.1 can be redrawn as Fig. 3.2, where the channel and TEQ are lumped together the

$$h(n) = c(n) * t(n) \tag{3.28}$$

where the length of $t(n)$ is Q and the length of $h(n)$ is N which is shorter than $N = M+L$ where L is CP length. The equivalent noise after is $q'(n) = q(n) * t(n)$. The matrix $\mathbf{F}_0$ and $\mathbf{F_1}$ are the prefix insertion and removal matrices.

$$\mathbf{F}_0 = \begin{bmatrix} \mathbf{0} & \mathbf{I_L} \\ \multicolumn{2}{c}{\mathbf{I_M}} \end{bmatrix}, \mathbf{F}_1 = \begin{bmatrix} \mathbf{0} & \mathbf{I_M} \end{bmatrix} \tag{3.29}$$

From Fig. 3.3, the system from $\mathbf{u}(n)$ to $\mathbf{s}(n)$ is LTI with the transfer matrix $\mathbf{H}(z)$ is

$$\mathbf{H}(z) = \begin{pmatrix} h(d) & \cdots & h(0) & z^{-1}h(N-1) & \cdots & z^{-1}h(d+1) \\ \vdots & & & & \ddots & \vdots \\ h(N-1) & & \ddots & & & z^{-1}h(N-1) \\ zh(0) & & & \ddots & & h(0) \\ \vdots & \ddots & & & & \vdots \\ zh(d-1) & \ddots & \ddots & & & h(d) \end{pmatrix}, \tag{3.30}$$

We can write $\mathbf{H}(z)$ as

$$\mathbf{H}(z) = \mathbf{H}_0 + z^{-1}\mathbf{H}_1 + z\mathbf{H}_{-1} \tag{3.31}$$

$$\mathbf{s}(n) = \mathbf{H}_0\mathbf{u}(n) + \mathbf{H_1}\mathbf{u}(n-1) + \mathbf{H_{-1}}\mathbf{u}(n+1) \tag{3.32}$$

We use such a representation of $\mathbf{H}(z)$. The Fig. 3.2 can redrawn as Fig. 3.3. There is interference from the previous block $\mathbf{H}_1\mathbf{u}(n-1)$ and interference from the next block $\mathbf{H}_{-1}\mathbf{u}(n+1)$ due to the equivalent channel taps $h(d+1), \ldots, h(N-1)$. Splitting the constant matrix $\mathbf{H}_0$ into two parts,

$$\mathbf{H}_0 = \mathbf{H}_{00} + \mathbf{H}_{01}.$$

Then we have

$$\begin{aligned} \mathbf{y}(n) &= \underbrace{\mathbf{W}\mathbf{F}_1\mathbf{H}_{00}\mathbf{F}_0\mathbf{W}^\dagger}_{\boldsymbol{\Lambda}}\mathbf{x}(n) + \underbrace{\mathbf{W}\mathbf{F}_1\mathbf{H}_{01}\mathbf{F}_0\mathbf{W}^\dagger}_{\mathbf{A}}\mathbf{x}(n) \\ &+ \underbrace{\mathbf{W}\mathbf{F}_1\mathbf{H}_1\mathbf{F}_0\mathbf{W}^\dagger}_{\mathbf{B}}\mathbf{x}(n-1) \\ &+ \underbrace{\mathbf{W}\mathbf{F}_1\mathbf{H}_{-1}\mathbf{F}_0\mathbf{W}^\dagger}_{\mathbf{C}}\mathbf{x}(n+1) + \underbrace{\mathbf{W}\mathbf{F}_1\mathbf{q}(n)}_{\mathbf{e}(n)}. \end{aligned} \tag{3.33}$$

Then $\mathbf{y}(n) = \mathbf{\Lambda x}(n) + \mathbf{A x}(n) + \mathbf{B x}(n-1) + \mathbf{C x}(n+1) + \mathbf{e}(n)$

We can see $\mathbf{\Lambda}$ is diagonal matrix, the k-th diagonal element represent the k-th subchannel gain. The matrix $\mathbf{A}$ is the interference from the other tones of the same block $\mathbf{x}(n)$. The elements of $\mathbf{B}$ and $\mathbf{C}$ represent the interference from the previous and the next block respectively. Consider the interference from a selected set of tones S (source tones) to a chosen set of tones T (target tones). An object function for this is

$$\phi = \Sigma_{\ell \epsilon S} \Sigma_{k \epsilon T} (|A_{k,\ell}|^2 + |B_{k,\ell}|^2 + |C_{k,\ell}|^2) \tag{3.34}$$

To minimize equation (3.34), note that the equivalent channel can be written as $\mathbf{h}$=$\mathbf{Qt}$. The elements of $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ can be expressed as

$$\begin{aligned} A_{k,\ell} &= \mathbf{a}_{k,\ell}\mathbf{t} = \mathbf{a}'_{k,\ell}\mathbf{h} = \mathbf{a}'_{k,\ell}\mathbf{Qt} \\ B_{k,\ell} &= \mathbf{b}_{k,\ell}\mathbf{t} = \mathbf{b}'_{k,\ell}\mathbf{h} = \mathbf{b}'_{k,\ell}\mathbf{Qt} \\ C_{k,\ell} &= \mathbf{c}_{k,\ell}\mathbf{t} = \mathbf{c}'_{k,\ell}\mathbf{h} = \mathbf{c}'_{k,\ell}\mathbf{Qt} \end{aligned} \tag{3.35}$$

where $\mathbf{t}$ is a $Q \times 1$ TEQ coefficients vector. $\mathbf{h}$ is the $N \times 1$ vector consisting of the coefficients of $h(n)$. So,the cost function can rewritten as

$$\phi = \mathbf{t}^\dagger \mathbf{U} \mathbf{t} \tag{3.36}$$

subject to $\mathbf{t}^\dagger \mathbf{t} = 1$ where

$$\mathbf{U} = \Sigma_{\ell \epsilon S} \Sigma_{k \epsilon T} (\mathbf{a}^\dagger_{k,\ell}\mathbf{a}_{k,\ell} + \mathbf{b}^\dagger_{k,\ell}\mathbf{b}_{k,\ell} + \mathbf{c}^\dagger_{k,\ell}\mathbf{c}_{k,\ell}).$$

We minimize the objective function (3.36) to find the optimal TEQ which is the eigenvector corresponding to the smallest eigenvalue of $\mathbf{U}$.

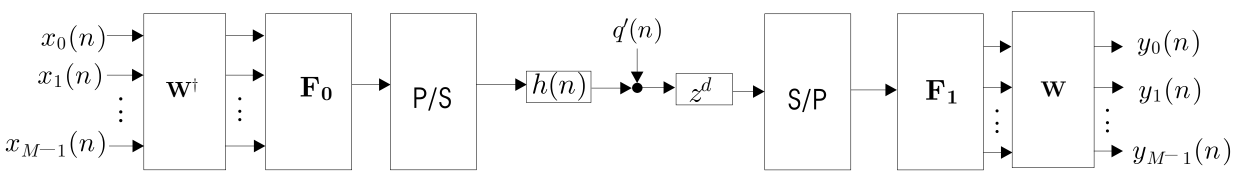


Figure 3.2: Equivalent DMT System

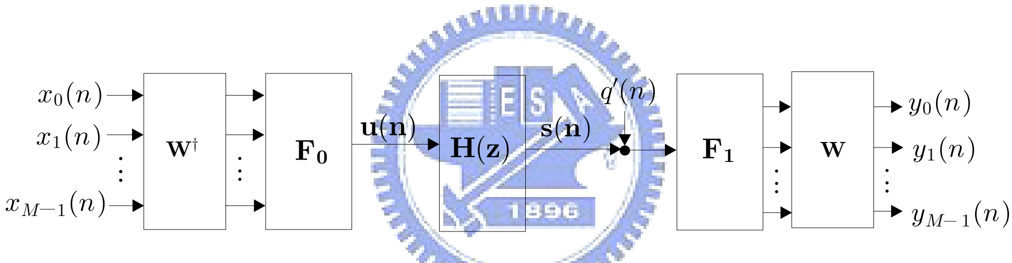


Figure 3.3: Equivalent DMT System

Chapter 4

Proposed TEQ Design

In this chapter, we proposed a TEQ design by maximizing signal to interference ratio for DMT system. We know that the TEQ design in [14] has good performance and properties. It compute partial interference from source to target tones in the frequency domain. In addition, we consider signal power from the tones in the transmission bands. The incorporation of signal power enhance the frequency response of the TEQ in the transmission bands. Therefore, we can obtain a higher bit rate.

From the derivation of the section 3.4 and the equivalent DMT system of Fig. 3.3, the receiver output vector $\mathbf{y}(n)$ is related to the transmitter input vector $\mathbf{x}(n)$ by

$$\begin{aligned}
\mathbf{y}(n) &= \underbrace{\mathbf{W}\mathbf{F}_1\mathbf{H}_{00}\mathbf{F}_0\mathbf{W}^\dagger}_{\mathbf{\Lambda}}\mathbf{x}(n) + \underbrace{\mathbf{W}\mathbf{F}_1\mathbf{H}_{01}\mathbf{F}_0\mathbf{W}^\dagger}_{\mathbf{A}}\mathbf{x}(n) \\
&+ \underbrace{\mathbf{W}\mathbf{F}_1\mathbf{H}_1\mathbf{F}_0\mathbf{W}^\dagger}_{\mathbf{B}}\mathbf{x}(n-1) \\
&+ \underbrace{\mathbf{W}\mathbf{F}_1\mathbf{H}_{-1}\mathbf{F}_0\mathbf{W}^\dagger}_{\mathbf{C}}\mathbf{x}(n+1) + \underbrace{\mathbf{W}\mathbf{F}_1\mathbf{q}(n)}_{\mathbf{e}(n)}.
\end{aligned} \tag{4.1}$$

We have assumed the VDSL symbols have been perfectly synchronized. Then $\mathbf{y}(n) = \mathbf{\Lambda}\mathbf{x}(n) + \mathbf{A}\mathbf{x}(n) + \mathbf{B}\mathbf{x}(n-1) + \mathbf{e}(n)$

The constant matrix $\mathbf{H}_1$ are relate with the interference of previous block. We spilt $\mathbf{H}_0$ into two parts,

$$\mathbf{H}_0 = \mathbf{H}_{00} + \mathbf{H}_{01}. \tag{4.2}$$

where the coefficients of $\mathbf{H}_{00}$ are $h_0, h_1, \cdots, h_L$ while $\mathbf{H}_{01}$ consists of $h_{L+1}, \cdots, h_{N-1}$.

In the expression of $\mathbf{\Lambda}$ in (4.1), the M by M matrix $\mathbf{F}_1\mathbf{H}_{00}\mathbf{F}_0$ is circulant with the first column given by

$$(h_0 \quad h_1 \quad \dots \quad h_L \quad 0 \quad \cdots \quad 0)^T$$

As a result, the product $\mathbf{W}\mathbf{F}_1\mathbf{H}_{00}\mathbf{F}_0\mathbf{W}^\dagger$ is a diagonal matrix. The diagonal terms of $\mathbf{\Lambda}$ are the M-point DFT of $(h_0, h_1, \cdots, h_L)$ and it is regarded as the signal gain. On the other hand, the off-diagonal elements of $\mathbf{A}$ represent the interference gain in the same block. The (k, ℓ)-th element $A_{k,\ell}$, for $k \neq \ell$, represents the interference of the ℓ-th tone to the k-th tone of the same block. Also the elements of $\mathbf{B}$ is the interference gain from the previous block. The (k, ℓ)-th element $B_{k,\ell}$, represents the interference of the ℓ-th tone to the k-th tone of the previous block. The sum $\Lambda_{kk} + A_{k,k}$ is the signal gain of the k-th tone. From (4.1), we see the $\mathbf{As}(n)$ and $\mathbf{Bs}(n-1)$ contain mostly interference and $\mathbf{\Lambda s}(n)$ contain mostly signal. In the interference minimizing method, it's only consider the null tones of the TEQ response and doesn't consider the TEQ response of transmission bands. For our proposed method, we add signal tones to maximize signal to interference ratio to emphasize the TEQ response of the transmission bands. The signal to interference ratio can be written as:

$$\frac{signalpower}{interferencepower}$$

Signal power is

$$\sum_{j=0}^{M-1} \left(|\Lambda_{j,j}|^2\right)$$

Interference Power is

$$\sum_{\ell=0}^{M-1} \sum_{k=0}^{M-1} \left(|A_{k,\ell}|^2 + |B_{k,\ell}|^2\right)$$

We consider our objective function to design TEQ.

$$\phi = \frac{\sum_{j \epsilon S_i} (|\Lambda_{j,j}|^2)}{\sum_{\ell \epsilon S} \sum_{\ell \epsilon T} (|A_{k,\ell}|^2 + |B_{k,\ell}|^2)} \tag{4.3}$$

.

For above, we consider partial interference of a selected set of source tones (S) to a chosen set of target tones (T) and signal power of a select set of signal tones

(Si). We observe that the elements of $\mathbf{\Lambda}$,$\mathbf{A}$ and $\mathbf{B}$ can be expressed in terms of the TEQ coefficients. In particular,

$$\Lambda_{k,k} = \mathbf{v}_{k,k}\mathbf{t}, A_{k,\ell} = \mathbf{a}_{k,\ell}\mathbf{t}, B_{k,\ell} = \mathbf{b}_{k,\ell}\mathbf{t} \tag{4.4}$$

where $\mathbf{t}$ is the $T \times 1$ vector consisting of the TEQ coefficients. Because $A_{k,\ell}$ is a linear combination of the coefficients of $h(n)$, it can represented as $A_{k,\ell} = \mathbf{a}'_{k,\ell}\mathbf{h}$ where $\mathbf{h}$ is the $N \times 1$ vector consisting of the coefficients of $h(n)$. The equivalent channel is $h(n) = c(n) * t(n)$, and we can write it as $\mathbf{h} = \mathbf{Qt}$, where $\mathbf{Q}$ is an $N \times T$ convolution matrix. Therefore, we have

$$A_{k,\ell} = \mathbf{a}'_{k,\ell}\mathbf{h} = \mathbf{a}'_{k,\ell}\mathbf{Qt}$$

Defining $\mathbf{a}_{k,\ell} = \mathbf{a}'_{k,\ell}\mathbf{Q}$, we have $A_{k,\ell}$ in the form (4.4). Similarly, we can express $\Lambda_{k,\ell}$ and $B_{k,\ell}$ as in (4.4). Using (4.4), we have $|A_{k,\ell}|^2 = \mathbf{t}^\dagger\mathbf{a}^\dagger_{k,\ell}\mathbf{a}_{k,\ell}\mathbf{t}$.

$$\mathbf{R} = \sum_{\jmath \epsilon Si} \left(\mathbf{v}^\dagger_{\jmath,\jmath}\mathbf{v}_{\jmath,\jmath}\right)$$

$$\mathbf{U} = \sum_{\ell \epsilon S}\sum_{k \epsilon T} \left(\mathbf{a}^\dagger_{k,\ell}\mathbf{a}_{k,\ell} + \mathbf{b}^\dagger_{k,\ell}\mathbf{b}_{k,\ell}\right)$$

Then the objective function given in (4.4) becomes

$$\phi = \frac{\mathbf{t}^\dagger\mathbf{Rt}}{\mathbf{t}^\dagger\mathbf{Ut}}$$

The problem of maximizing ϕ is equivalent to maximizing $\mathbf{t}^\dagger\mathbf{Rt}$, subject to $\mathbf{t}^\dagger\mathbf{Ut} = 1$. We can obtain the optimal TEQ $\mathbf{t}_{opt}$.

$$\mathbf{t}_{opt} = \arg\max_{t} \frac{\mathbf{t}^\dagger\mathbf{Rt}}{\mathbf{t}^\dagger\mathbf{Ut}}$$

Let $\mathbf{U} = \mathbf{Q}^\dagger\mathbf{Q}$

$$\mathbf{t}_{opt} = \arg\max_{t} \frac{\mathbf{t}^\dagger\mathbf{Rt}}{\mathbf{t}^\dagger\mathbf{Q}^\dagger\mathbf{Qt}}$$

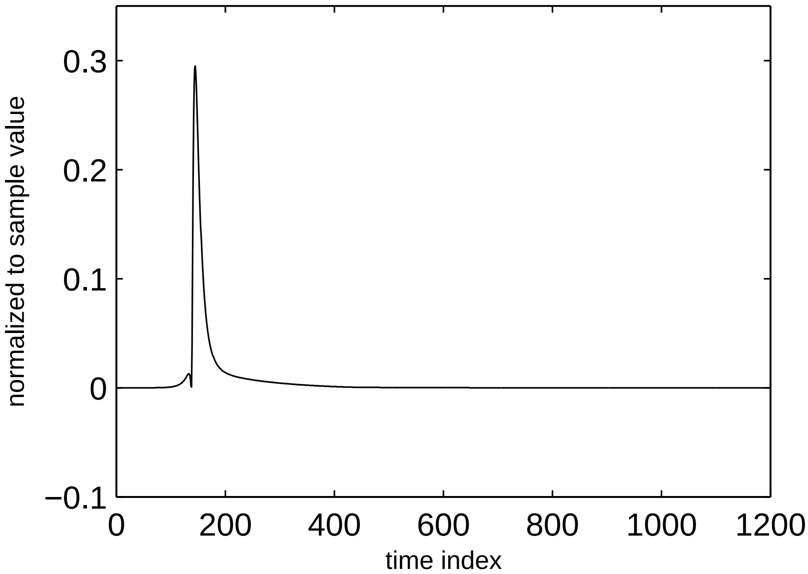
Let $\mathbf{r} = \mathbf{Qt}$

$$\mathbf{t}_{opt} = \mathbf{Q}^{-1}\arg\max_{\mathbf{r}} \frac{\mathbf{r}^\dagger\mathbf{Q}^{-\dagger}\mathbf{RQ}^{-1}\mathbf{r}}{\mathbf{r}^\dagger\mathbf{r}}$$

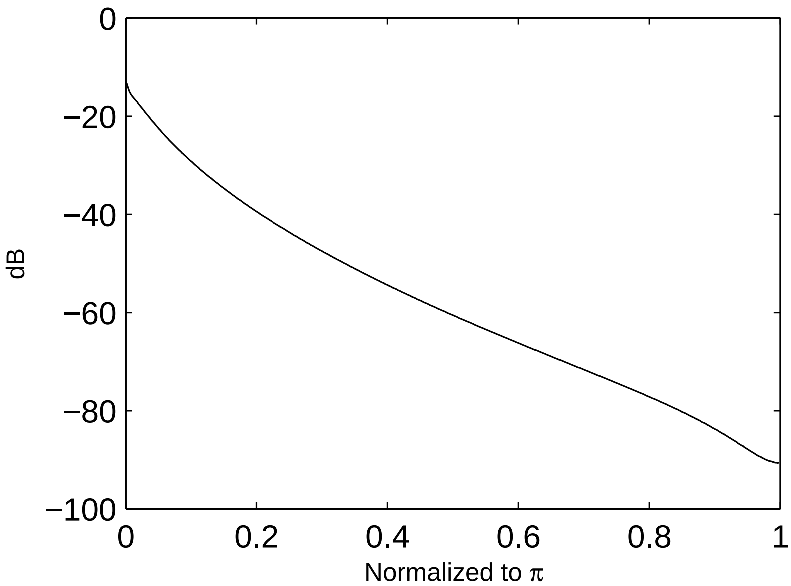
The solution of the equivalent problem can be obtained by solving the generalized eigenvector problem. The optimal TEQ $\mathbf{t}_{opt}$ can be maximized by finding the eigenvector corresponding to the largest eigen value of $\mathbf{U}^{-1}\mathbf{R}$.

A TEQ design example.

In this example, we use VDSL loop1 of 4500 feet for our simulation. The impulse response and magnitude response of loop 1 are shown respectively in Fig. 4.1.



(a)



(b)

Figure 4.1: VDSL loop1 (a) impulse response (b) magnitude response

Fig 4.2 is VDSL band allocation. There are two bands allocated for downstream transmission, denoted by " 1D " and " 2D " as the figure. These two bands correspond respectively to tones 33 to 870 and tones 1206 to 1971. For downstream transmission, the null tones are the tones in the upstream bands, denote by " 1U " and " 2U " in Fig 4.2. We consider downstream transmission in the simulation. We will compare the proposed TEQ method with the interference minimizing TEQ design method [14]. For the proposed method, we choose the target and source tones from null tones, and the signal tones from the two downstream transmission band. The source tones are chosen from the set $\{1008 - 1068\}$, a subset of tones in "1U". In order to reduce the complexity, the source set is $\{1008 - 1068\}$ decimated by 5. Similarly, the target set is the set of the last 60 null tones in " 2U " $\{1988 - 2048\}$ decimated by 5. The signal tones are from the downstream tones {33-870} and {1206-1971}, decimated by 5. The length of TEQ is 40 taps. The impulse response of the two equalized channels are shown in Fig. 4.3(a),(b). Both of the TEQ have effectively shorten the channel. The frequency responses of the two TEQ are shown in Fig. 4.4(a),(b). Our proposed method can enhance the TEQ frequency response in the transmission bands. For the two methods, the zeros are both located within the unused tones. The comparison of equivalent channel is in Fig. 4.5. Our proposed method enhance the frequency response of transmission bands. As a result, the bit allocations of proposed TEQ is higher than frequency domain TEQ design method. It is shown in Fig. 4.6. The transmission rate of proposed method is 72.28 Mbits/sec and the frequency domain TEQ design is 63.87 Mbits/sec.

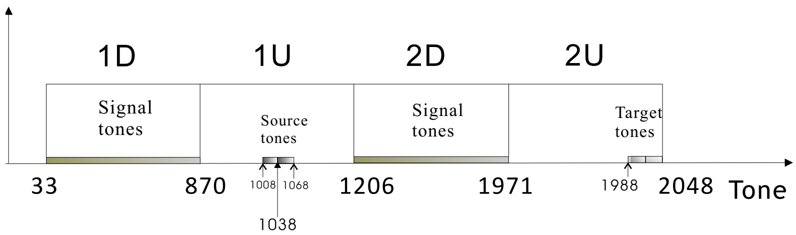
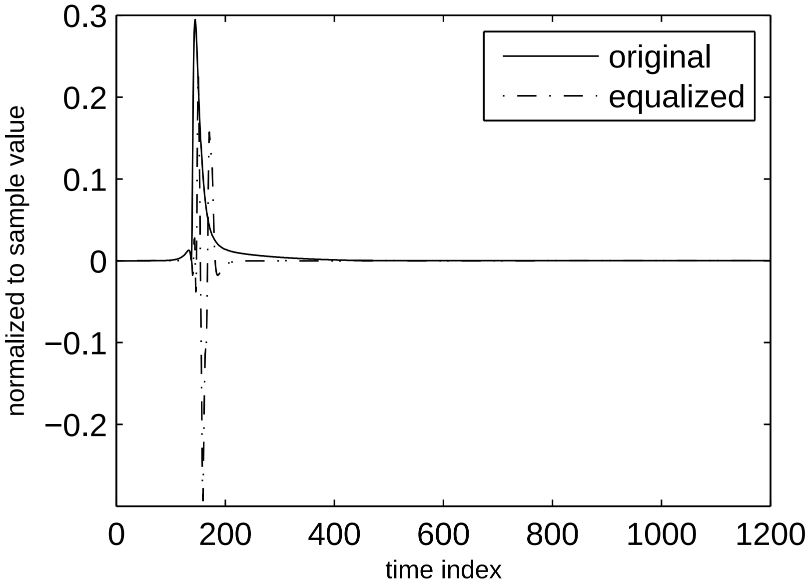
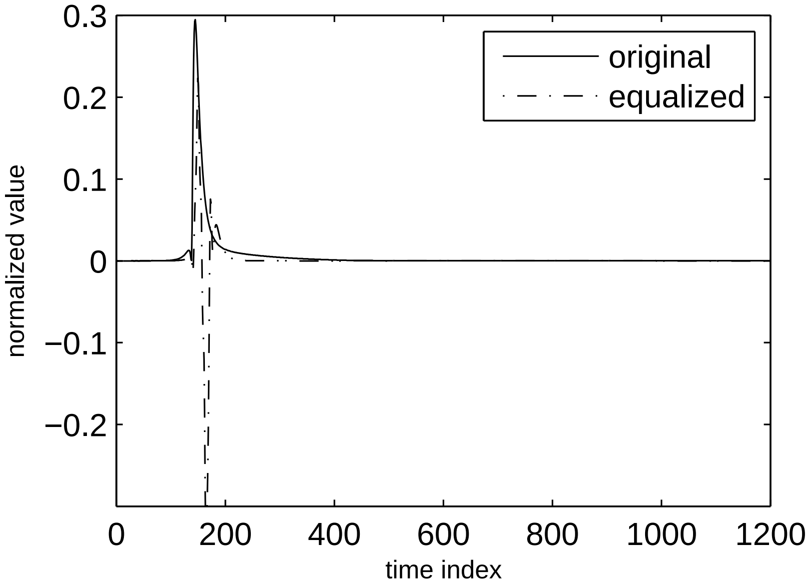


Figure 4.2: VDSL band allocation

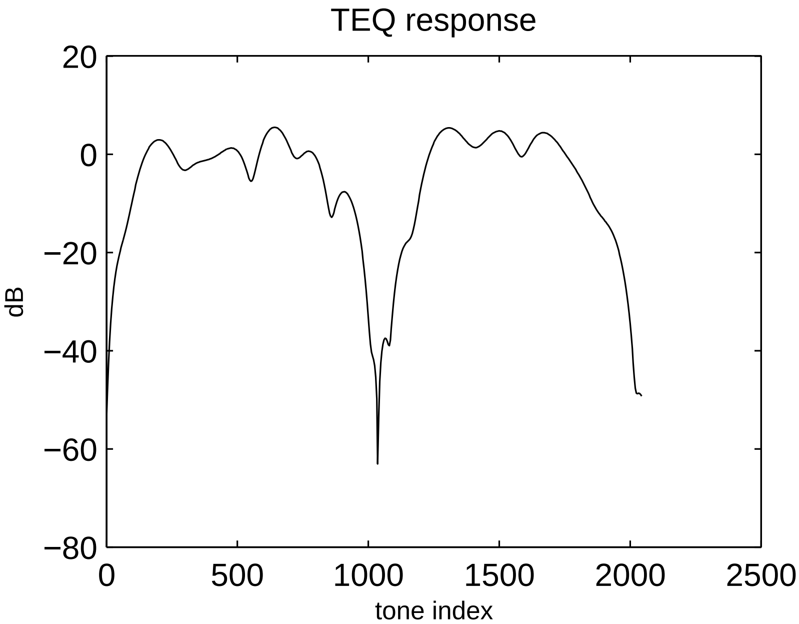


(a)

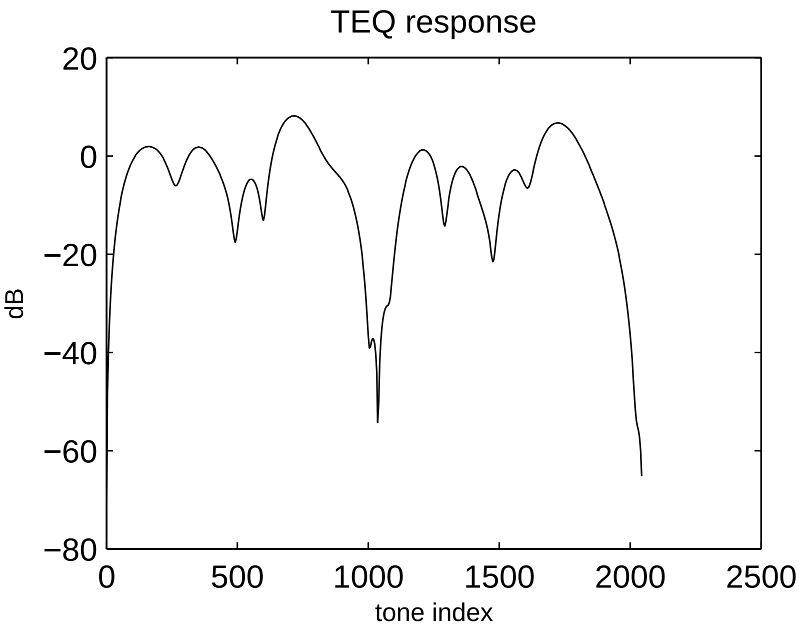


(b)

Figure 4.3: Impulse response of original channel and equalized channel (a) proposed method(b) frequency domain TEQ design [14]



(a)



(b)

Figure 4.4: Frequency responses of TEQ (a) proposed method(b) frequency domain TEQ design[14]

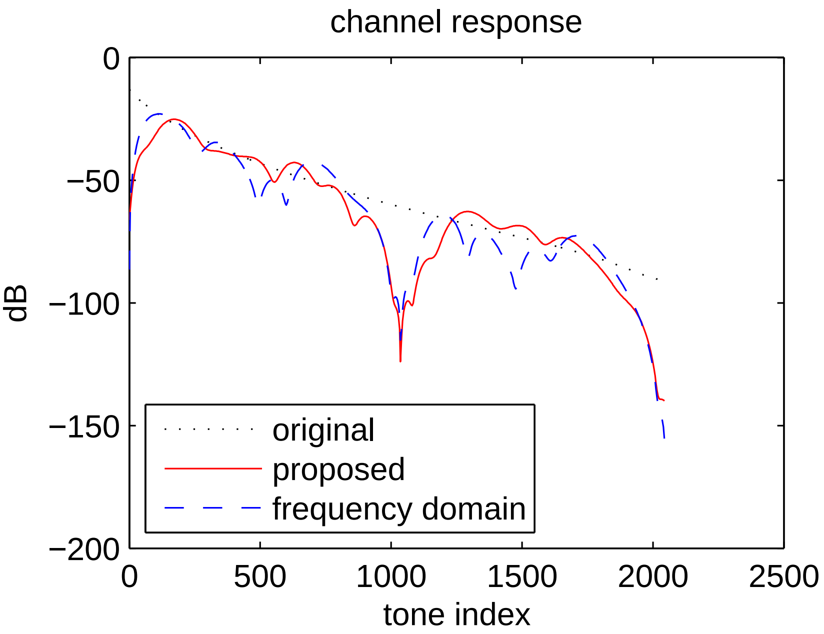


Figure 4.5: Frequency responses of the equivalent channel

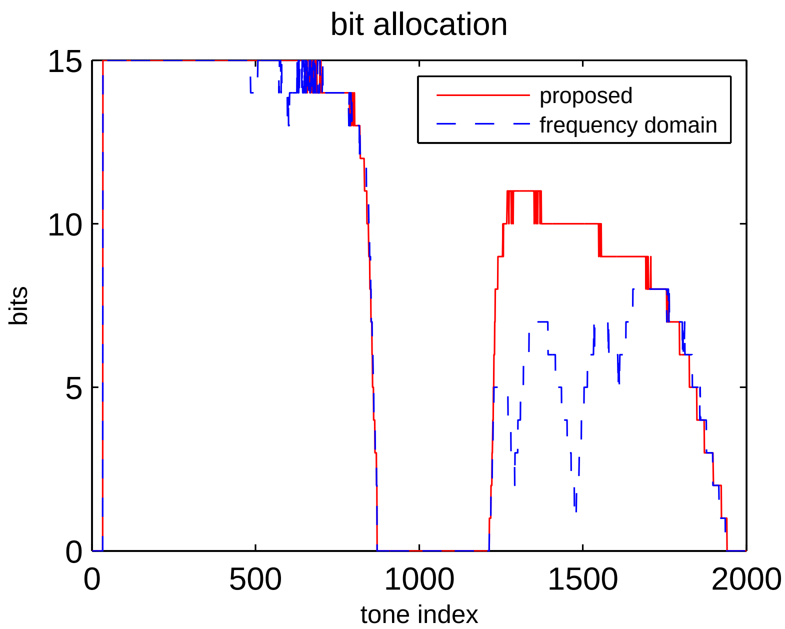


Figure 4.6: Bit allocations [14]

Chapter 5

Numerical Simulation

In this chapter, we use two performance measurements: SIR and bit rate, in our simulation. Two performance measurements in Section 5.1. The simulation environment is given in section 5.2. In section 5.3 and 5.4, we show the SIR and transmission rate comparison.

5.1 Measures of Performance

In our simulation, we use SIR and bit rate as our performance measurements.

SIR is usually used to evaluate the channel shortening effect. The measure of SIR is defined as

$$SIR = \max_{d} \frac{\sum_{i=d}^{d+L_{ch}} |h_i|^2}{\sum_{i=0, i \neq (d, \cdots, d+L_{ch})}^{L_{ch}} |h_i|^2} \tag{5.1}$$

where d is synchronization delay, L_{ch} is the length of equalized channel.

The number of bits allocated to the i-th subchannel is given by

$$b_i = \lfloor log_2(1 + \frac{SINR_i}{\Gamma}) \rfloor \tag{5.2}$$

The parameter, Γ represents the gap corresponding to the given symbol error rate. $SINR_i$ is the signal to interference and noise ratio of i-th tone. In our simulation the symbol error rate, $P_e = 10^{-5}$ and correspondingly $\Gamma = 4.7863$

where $\Gamma \approx \frac{1}{3}[Q^{-1}(P_e/4)]^2$. The transmission rate is equal to

$$\frac{1}{NT_s} \sum_{i=0}^{\frac{M}{2}-1} b_i \tag{5.3}$$

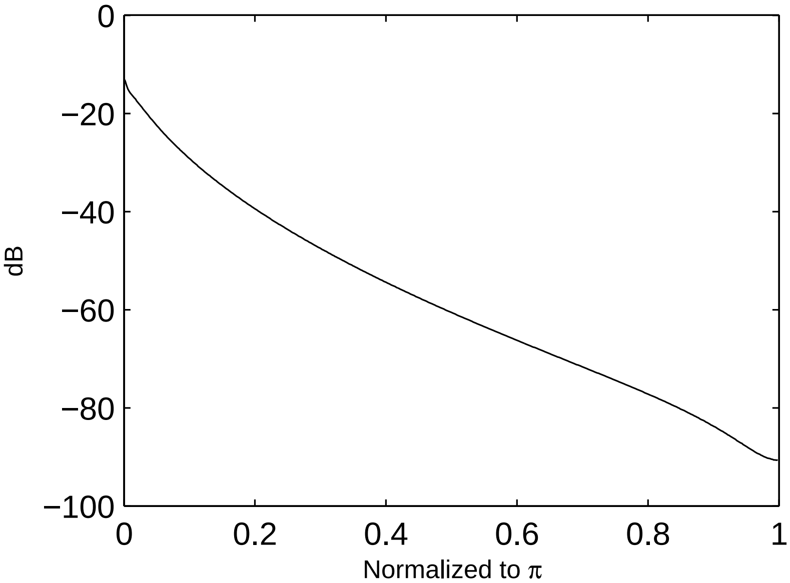
where $M = 4096$, $N = M + L = 4416$, and $f_s = \frac{1}{T_s} = 17.664$MHz. The max number of bits on each subchannel is 15.

5.2 Simulation Environment

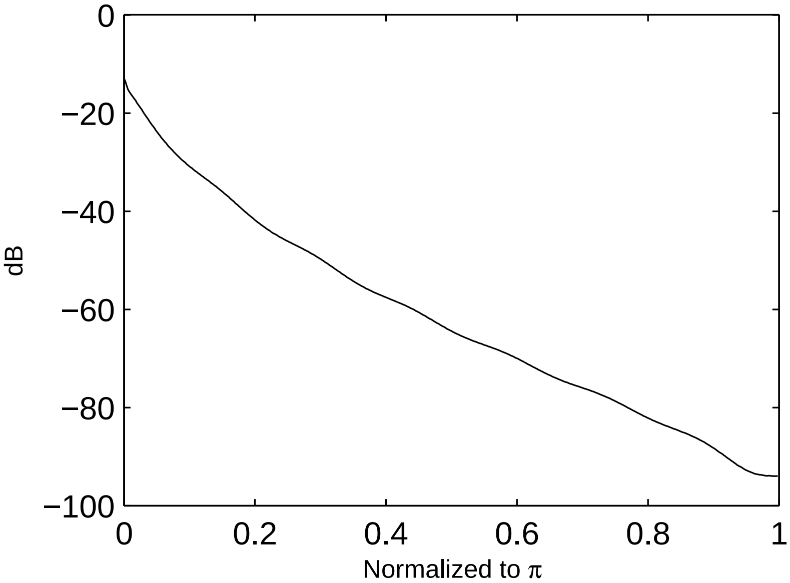
We use VDSL for our simulation. The DFT size is 4096, cyclic prefix length is 320, sampling rate is 17.664MHz. We consider downstream transmission, so the upstream tones are null tones. The tones are used for downstream transmission at $\{33 - 871\}$ and $\{1206 - 1971\}$. The rest tones are not used and these tones send zeros. The noise is composed of additive white Gaussian noise and crosstalk (FEXT and NEXT) generated from 20 VDSL disturbers. Seven VDSL test loops as given in [2] will be used in our simulation. The length of the seven loops are listed in Table. 5.1. The frequency response of loops are shown in Fig. 5.1(a)-(g)

Loop	Length(feet)
VDSL1	4500
VDSL2	4500
VDSL3	4500
VDSL4	4500
VDSL5	950
VDSL6	3250
VDSL7	4900

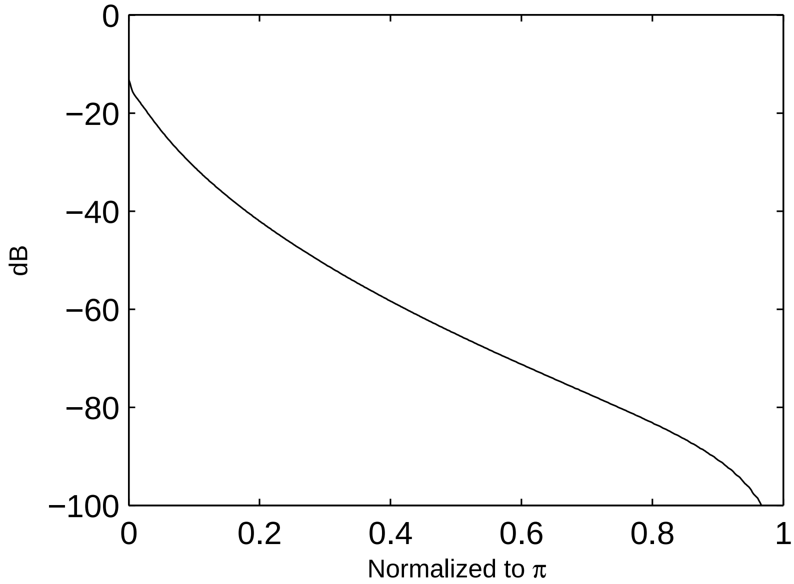
Table 5.1: VDSL test loop length


0
−20
−40
−60
−80
−100
dB
0
0.2
0.4
0.6
0.8
1
Normalized to π

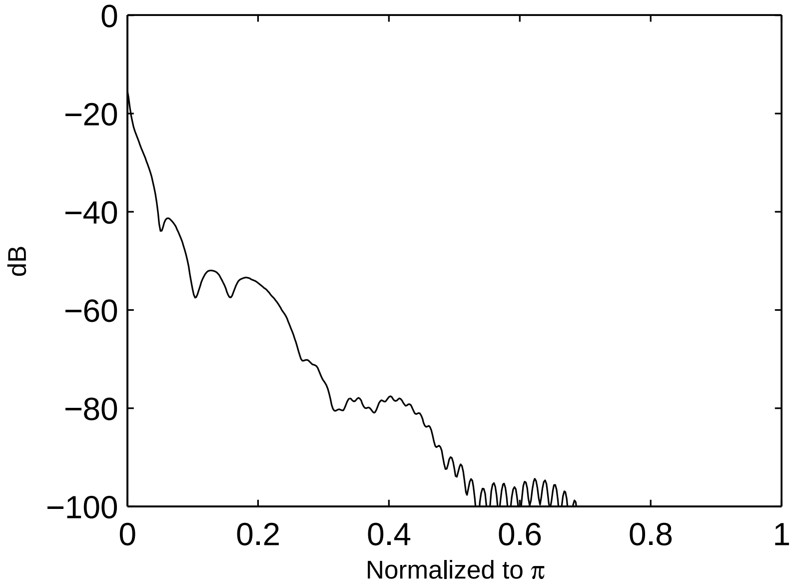
(a)


0
−20
−40
−60
−80
−100
dB
0
0.2
0.4
0.6
0.8
1
Normalized to π

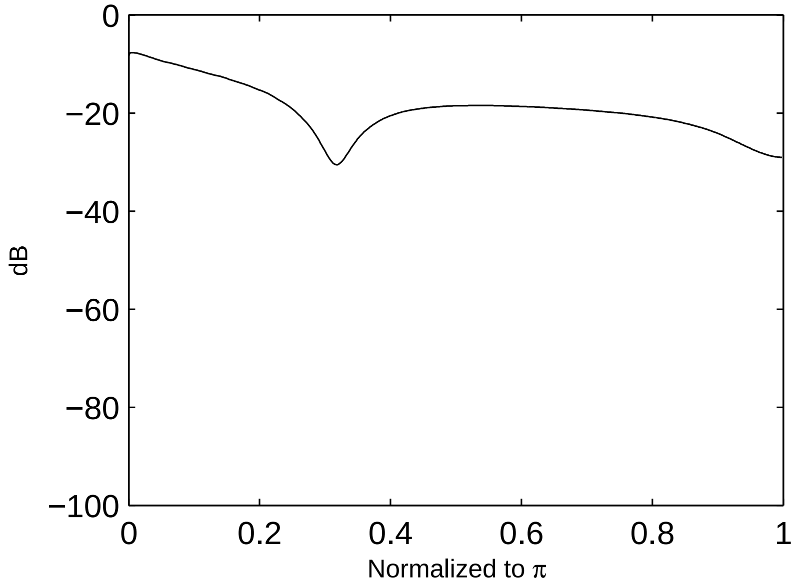
(b)


0
−20
−40
−60
−80
−100
dB
0
0.2
0.4
0.6
0.8
1
Normalized to π

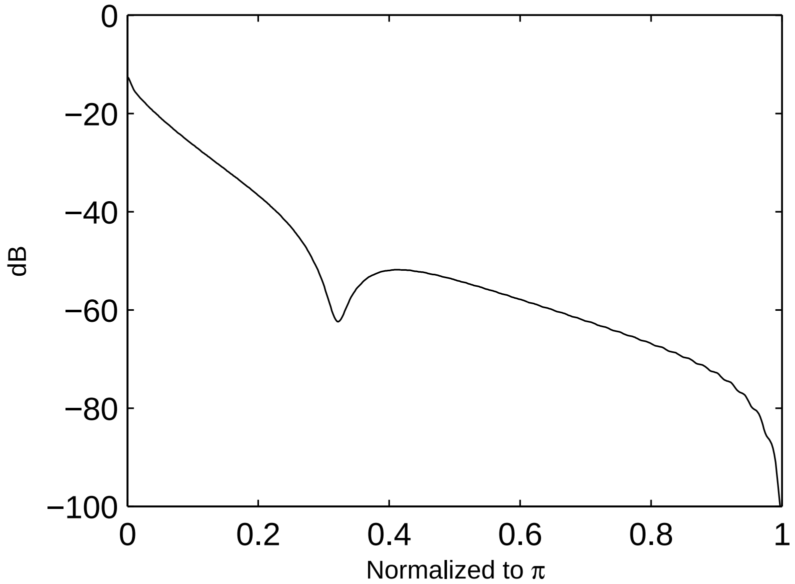
(c)


0
−20
−40
−60
−80
−100
dB
0
0.2
0.4
0.6
0.8
1
Normalized to π

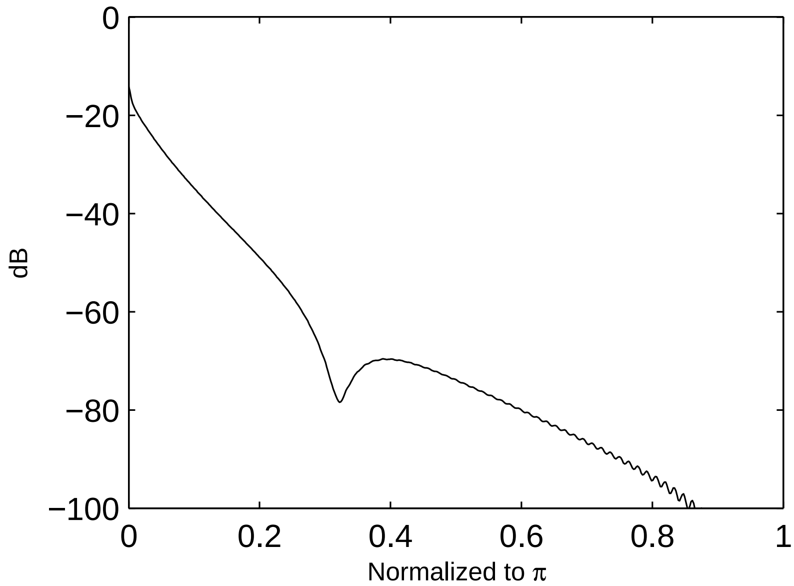
(d)


0
−20
−40
−60
−80
−100
dB
0
0.2
0.4
0.6
0.8
1
Normalized to π

(e)


0
−20
−40
−60
−80
−100
dB
0
0.2
0.4
0.6
0.8
1
Normalized to π

(f)



(g)

Figure 5.1: Magnitude responses of the VDSL test loops. (a) VDSL-1L, (b) VDSL-2L, (c) VDSL-3L, (d) VDSL-4L, (e) VDSL-5L, (f) VDSL-6L, (g) VDSL-7L.

5.3 SIR comparisons

In this section, we compare the SIR performance of MSSNR [3], Min.ISI [6], frequency domain TEQ design [14] and our proposed TEQ design method. The SIR performance computed used (5.1) are listed in Table. 5.2. The SIR is a good measure for channel shorting effect. We can observe our proposed TEQ method shorten the channel effectively.

Loop	proposed method	frequency [14] method	MSSNR [3]	Min.ISI [6]
VDSL1L	75.70	74.12	128.5	82.0
VDSL2L	69.88	63.69	121.5	91.1
VDSL3L	87.65	84.15	123.3	72.1
VDSL4L	69.83	53.55	101.4	52.9
VDSL5L	132.63	139.66	169.0	102.9
VDSL6L	91.17	96.48	122.9	85.4
VDSL7L	72.61	72.80	102.0	59.3

Table 5.2: SIR measure (dB) on VDSL loops

5.4 Transmission Rate comparisons

In this section, we evaluate the transmission of the proposed TEQ for the VDSL loops. The transmission rates of proposed TEQ design, frequency domain TEQ design, MSSNR, Minimum-ISI and Per-tone methods are listed in Table 5.3 and Table 5.4. From the tables, we observe our proposed TEQ design outperform MSSNR, Min-ISI and frequency domain TEQ design in transmission rates and close to the per-tone equalization method.

In Table 5.3, we choose the source tones, target tones and signal tones as in sec5.3. We use the same target and source tones for the frequency domain TEQ design method.

Loop	proposed method	frequency method	MSSNR	Min.ISI	PTEQ
VDSL1L	69.53	63.70	51.33	59.58	77.11
VDSL2L	62.68	57.97	40.07	51.44	73.51
VDSL3L	64.15	60.08	49.01	52.94	72.28
VDSL4L	40.95	36.82	35.55	12.91	48.47
VDSL5L	93.88	93.65	80.14	93.91	93.93
VDSL6L	75.25	72.13	66.63	66.56	83.42
VDSL7L	54.00	48.22	38.98	40.01	60.78

Table 5.3: Bit rate (Mbits/sec) on VDSL loops

In Table 5.4, we choose the different signal tones for seven types of the VDSL test loops to achieve higher bit rate when the target tones and source tones are fixed. The signal set is between $\{33-870\}$ and $\{1206-1971\}$. The source tones are $\{1008-1068\}$ with tone decimation by 5 and target tones at $\{1988-2048\}$ with tone decimation by 5. The signal tones are chosen as in Table 5.4:

Loop	proposed method	frequency method	MSSNR	Min.ISI	PTEQ
VDSL1L	72.28	63.70	51.33	59.58	77.11
VDSL2L	62.80	57.97	40.07	51.44	73.51
VDSL3L	65.16	60.08	49.01	52.94	72.28
VDSL4L	44.86	36.82	35.55	12.91	48.47
VDSL5L	93.89	93.65	80.14	93.91	93.93
VDSL6L	78.20	72.13	66.63	66.56	83.42
VDSL7L	54.00	48.22	38.98	40.01	60.78

Table 5.4: Bit rate (Mbits/sec) on VDSL loops

Loop	Set
VDSL1L	$\{33:5:870\}\ \{1206:4:1971\}$
VDSL2L	$\{33:4:870\}\ \{1206:5:1971\}$
VDSL3L	$\{33:4:870\}\ \{1206:4:1971\}$
VDSL4L	$\{33:5:870\}\ \{1206:4:1971\}$
VDSL5L	$\{33:5:870\}\ \{1206:4:1971\}$
VDSL6L	$\{33:5:870\}\ \{1206:4:1971\}$
VDSL7L	$\{33:5:870\}\ \{1206:5:1971\}$

Table 5.5: The signal Set on VDSL loops

Chapter 6

Conclusion

In this thesis, we proposed a new TEQ design method to increase transmission rate. We consider minimizing the interference to signal power ratio to design TEQ. The objective function can be simplified as a quadratic form of TEQ coefficients. And we can directly control the zeros of TEQ response by choosing the target tones and source tones. We also add signal tones to increase the transmission rate. The transmission bands would free from zeros using our proposed TEQ design and then the better transmission rate would be achieved. In our proposed TEQ design, the channel can be shortened effectively and we have much better transmission rate than many TEQ design methods, and very close to the per-tone equalization method.

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