Free-carrier absorption in n-type gallium arsenide in quantizing magnetic fields

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Free-carrier absorption in n-type GaAs in quantizing magnetic fields has been investigated quantum mechanically. It is assumed that the energy band of electrons in semiconductors is nonparabolic and the dominant scattering mechanism for electrons is acoustic-phonon scattering via deformation-potential coupling. When the radiation field is polarized parallel to the magnetic field, the free-carrier absorption coefficient assumes a complex value due to the ac radiation and phonon fields. Our results show that the real part of the absorption coefficient and the imaginary part of the absorption coefficient oscillate with the magnetic field. However, at very low temperatures, oscillations in the imaginary part are diminished and its value decreases rapidly with increasing magnetic field.

I. INTRODUCTION

Electromagnetic radiation is absorbed in semiconductors primarily through two distinct processes. In one process, electrons are excited from the valence band to the conduction band by radiation absorption. Such absorption may take place without the participation of phonons if the band gap is direct. However, if the band gap is indirect, the participation of phonons is required for momentum balance. This process of absorption is called interband absorption. In the second process, the electromagnetic radiation is absorbed by free electrons in the conduction band or quasifree holes in the valence band. For this latter process, electrons or holes are transferred to higher-energy levels in the same band by the absorption of the radiation. This process of absorption is called free-carrier absorption. It is essentially an intraband-transition phenomenon.

Free-carrier absorption in semiconductors is the dominant mechanism for the absorption of electromagnetic radiation with frequencies lower than those which give rise to interband transitions by free electrons in the conduction band or quasifree holes in the valence band.[1,2] The absorption of a photon by an electron in an intraband process is forbidden in a perfect crystal, since the probability of the absorption of photons by free electrons is zero. Thus free-carrier absorption does not take place if we consider only the case for the photon and the free electron.[2,3] However, such an intraband transition involving the absorption of a photon can occur if the free carrier gains the necessary momentum by scattering off phonons or other imperfections in the crystal. A quantum theory for free-carrier absorption of semiconductors in quantizing magnetic fields has been worked out for electrons in a parabolic energy band.[3,4] In quantizing magnetic fields, different mechanisms give rise to different magnetic-field dependences of the scattering rates.[5,6] It has been shown that the nonparabolicity of the energy band in semiconductors can be used to explain the longitudinal magnetoacoustic phenomena,[7,8] the transverse magnetoresistance in semiconductors,[9] the acoustic harmonic generation in piezoelectric semiconductors,[10] and Hall effect in nondegenerate semiconductors.[11] When the radiation field is polarized parallel to a dc magnetic field, the dependence of free-carrier absorption on the field only appears when the quantization of electronic energy levels in the magnetic field becomes important. This phenomenon occurs when the separation between adjacent Landau levels is becoming greater than either the collision broadening or the thermal broadening of these Landau levels.[12] In this paper, we shall study the effect of nonparabolicity in nondegenerate semiconductors on free-carrier absorption for the longitudinally polarized radiation with acoustic-phonon scattering via the deformation-potential coupling as the dominant scattering mechanism for electrons in the conduction band.

II. CALCULATIONS OF FREE-CARRIER ABSORPTION

The free-carrier absorption coefficient can be related to the transition probability for an electron to make an intraband transition absorbing a photon and simultaneously absorbing or emitting a phonon. This transition probability is given by the second-order perturbation theory,

$$W_i=(2\pi/\hbar)\sum_f [|\langle f|M_+|i\rangle|^2\delta(E_f-E_i-\hbar\Omega-\hbar\omega_q) + |\langle f|M_-|i\rangle|^2\delta(E_f-E_i-\hbar\Omega+\hbar\omega_q)] , \quad (1)$$

where $\langle f|M_{\pm}|i\rangle$ are the transition matrix elements for the interaction between electrons and the radiation field, and the electron-phonon interaction, E_i and E_f are the initial and final energies of electrons, and $\hbar\Omega$ and $\hbar\omega_q$ are energies of the photon and of the phonon, respectively. Then the free-carrier absorption coefficient is given by[3]

$$\alpha=\frac{\epsilon^{1/2}}{nc}\sum_i W_i f_i \ , \tag{2}$$

where ϵ is the dielectric constant, n is the index of refraction, and f_i is the free-carrier distribution function.

Using the Landau gauge for the vector potential $\mathbf{A}=(0,Bx,0)$, the eigenfunction and eigenvalue for electrons in a nonparabolic energy band with a magnetic field $\mathbf{B}$ directed along the z axis are given by

$$\Psi_{\mathbf{k}n}=[\exp(ik_y y+ik_z z)]\Phi_n\left[x-\frac{\hbar c}{eB}k_y\right] \tag{3}$$

and

$$E_{\mathbf{k}n}=-\tfrac{1}{2}E_g\{1-\{1+(4/E_g)[(n+\tfrac{1}{2})\hbar\omega_c+\hbar^2k_z^2/2m^*]\}^{1/2}\} \ , \tag{4}$$

respectively, where $\omega_c=|e|B/m^*c$ is the cyclotron frequency of the electron, Φ_n is the harmonic-oscillator wave function, k_y and k_z are the components of the electron wave vector in the y and z directions, respectively, and m^* is the effective mass of the electron.

From Eqs. (1)–(4) and Ref. 3, the transition probability can be obtained as

$$\begin{aligned}
W_i=\frac{16\pi^3 nk_B Te^3C^2B}{m^{*1/2}\epsilon c\Omega\rho V}\sum_{n_f} a_{n_f}^{3/2}&\{[E_g(a_{n_f}-a_{n_i})+2\hbar k_{zi}(\hbar\Omega/m^*)^{1/2}-\hbar^2k_{zi}^2/m^*a_{n_i}]-a_{n_f}\hbar\Omega\}^{-3/2}\\
&\times(\{2\hbar^{10}k_{zi}^{10}/m^{*2}a_{n_f}^2+\hbar^6k_{zi}^6(11\hbar^2k_{zi}^2/m^*a_{n_f}-4a_{n_f}\hbar\Omega)[E_g(a_{n_f}-a_{n_i})-\hbar^2k_{zi}^2/m^*a_{n_i}]\\
&+\hbar^4k_{zi}^4(19\hbar^2k_{zi}^2-8m^*a_{n_f}^2\hbar\Omega)[E_g(a_{n_f}-a_{n_i})-\hbar^2k_{zi}^2/m^*a_{n_i}]^2\\
&+\hbar^2k_{zi}^2m^*a_{n_f}(15\hbar^2k_{zi}^2-4m^*a_{n_f}^2\hbar\Omega)[E_g(a_{n_f}-a_{n_i})-\hbar^2k_{zi}^2/m^*a_{n_i}]^3\\
&+7\hbar^2k_{zi}^2m^{*2}a_{n_f}^2[E_g(a_{n_f}-a_{n_i})-\hbar^2k_{zi}^2/ma_{n_i}]^4\\
&+2m^{*3}a_{n_f}^3[E_g(a_{n_f}-a_{n_i})-\hbar^2k_{zi}^2/m^*a_{n_i}]^5\}^{-1}\\
&\times\{-2\hbar^8k_{zi}^{10}/m^*a_{n_f}+(\hbar^7k_{zi}^{10}/m^*a_{n_f}^2\Omega-12\hbar^6k_{zi}^8+4\hbar^5k_{zi}^6m^*a_{n_f}^2\Omega)\\
&\times[E_g(a_{n_f}-a_{n_i})-\hbar^2k_{zi}^2/m^*a_{n_i}]\\
&+(15\hbar^5k_{zi}^8/2a_{n_f}\Omega-22\hbar^4k_{zi}^6m^*a_{n_f}+8\hbar^3k_{zi}^4m^{*2}a_{n_f}^3\Omega)\\
&\times[E_g(a_{n_f}-a_{n_i})-\hbar^2k_{zi}^2/m^*a_{n_i}]^2\\
&+(16\hbar^3k_{zi}^6m^*/\Omega-24\hbar^2k_{zi}^4m^{*2}a_{n_f}^2+4\hbar k_{zi}^2m^{*3}a_{n_f}^4\Omega)\\
&\times[E_g(a_{n_f}-a_{n_i})-\hbar^2k_{zi}^2/m^*a_{n_i}]^3\\
&+(16\hbar k_{zi}^4m^{*2}a_{n_f}/\Omega-12k_{zi}^2m^{*3}a_{n_f}^3)[E_g(a_{n_f}-a_{n_i})-\hbar^2k_{zi}^2/m^*a_{n_i}]^4\\
&+(10k_{zi}^2m^{*3}a_{n_f}^2/\hbar\Omega-4m^{*4}a_{n_f}^4\hbar^{-2})[E_g(a_{n_f}-a_{n_i})-\hbar^2k_{zi}^2/m^*a_{n_i}]^5\\
&+(9m^{*4}a_{n_f}^3/2\hbar^3\Omega)[E_g(a_{n_f}-a_{n_i})-\hbar^2k_{zi}^2/m^*a_{n_i}]^6\\
&+(m^{*5}a_{n_f}^4/\hbar^5k_{zi}^2\Omega)[E_g(a_{n_f}-a_{n_i})-\hbar^2k_{zi}^2/m^*a_{n_i}]^7\}+k_{zi}^2/2a_{n_f}\hbar\Omega\\
&-m^*[E_g(a_{n_f}-a_{n_i})-\hbar^2k_{zi}^2/m^*a_{n_i}]/\hbar^3\Omega\\
&+m^*a_{n_f}/\hbar^2-m^{*2}a_{n_f}[E_g(a_{n_f}-a_{n_i})-\hbar^2k_{zi}^2/m^*a_{n_i}]^2/2\hbar^5\Omega k_{zi}^2) \ ,
\end{aligned} \tag{5}$$

where

$$a_n=[1+4\hbar\omega_c(n+\tfrac{1}{2})/E_g]^{1/2} \ . \tag{6}$$

Here the sum over n_f goes over only those values for which the term in the square root is positive for all k_{zi}, then we have the condition $a_{n_f}\le a_{n_i}+2\hbar\Omega/E_g$. For high magnetic fields, where the difference between a_{n_f} and a_{n_i} is small, one may consider only the low quantum numbers. For lower magnetic fields, one may expand the factor a_n and get the con-

dition $n_f \leq n_i + \Omega/\omega_c$. Using the Maxwell-Boltzmann distribution for the nondegenerate semiconductor in the presence of a magnetic field **B**

$$f_{kn} = \frac{n_0 \hbar}{(2\pi m^* k_B T)^{1/2}} \left[\sum_{l=0}^{\infty} a_l^{1/2} \exp\left[-\frac{E_g a_l}{2k_B T} \right] \right]^{-1} \exp\left[-\frac{E_g a_n + \hbar^2 k_z^2 / m^* a_n}{2k_B T} \right], \tag{7}$$

where n_0 is the density of free carriers, the absorption coefficient can be obtained as

$$\begin{aligned}
\alpha = {} & 8\pi^3 [k_B T/m^* \epsilon (E_g - \hbar\Omega)]^{1/2} (e^3 C^2 B n_0 / \rho v_s^2 c^2 \hbar^3 \Omega^2) \left[\sum_{n=0}^{\infty} a_n^{1/2} \exp(-E_g a_n / 2k_B T) \right]^{-1} \\
& \times \sum_{n_i} a_{n_i}^{1/2} \exp(-E_g a_n / 2k_B T) \\
& \times \sum_{n_f} a_{n_f}^{3/2} (a_{n_f} - a_{n_i})^{-1/2} \\
& \times \left\{ P(B,T;n_i,n_f) + Q(B,T;n_i,n_f) W\left[-\left[\frac{E_g (a_{n_f} - a_{n_i})}{2k_B T} \right]^{1/2} \right] \right. \\
& + R(B,T;n_i,n_f) W\left[\frac{-(\hbar\Omega a_{n_i})^{1/2} - [(E_g - \hbar\Omega)(a_{n_f} - a_{n_i})]^{1/2}}{(2k_B T)^{1/2}} \right] \\
& \left. + S(B,T;n_i,n_f) W\left[\frac{(\hbar\Omega a_{n_i})^{1/2} - [(E_g - \hbar\Omega)(a_{n_f} - a_{n_i})]^{1/2}}{(2k_B T)^{1/2}} \right] \right\},
\end{aligned} \tag{8}$$

where C is the deformation potential, ρ is the density of the semiconductor, v_s is the sound velocity, $W(z)$ is a function related to a complementary error function,[13] and functions P, Q, R, and S are functions of the magnetic field B, temperature T, a_{n_i}, and a_{n_f}.[14]

III. NUMERICAL RESULTS AND DISCUSSION

The relevant values of physical parameters for n-type GaAs are taken to be $m^* = 0.07 m_0$ (m_0 is the mass of free electron), $\epsilon = 12.9$, $n_0 = 1.73 \times 10^{15}$ cm^{-3}, $E_g = 1.51$ eV, $\rho = 5.32$ g/cm^3, $C = 7$ eV, $v_s = 3.6 \times 10^5$ cm/sec, and a photon frequency corresponding to that of the 10.6-μm radiation characteristic of a CO_2 laser. From Eq. (8), it can be seen that the absorption coefficient will be complex, that is, $\alpha = \mathrm{Re}(\alpha) + i\,\mathrm{Im}(\alpha)$. Since the radiation field and the phonon field could be an ac field, the interaction between the radiation field and electrons, or the electron-phonon interaction from the scattering process, will cause a complex expression for the absorption coefficient. In Fig. 1, we plot $\mathrm{Re}(\alpha)$ and $\mathrm{Im}(\alpha)$ as a function of the magnetic field. It can be seen that both $\mathrm{Re}(\alpha)$ and $\mathrm{Im}(\alpha)$ oscillate with the magnetic field in the intermediate-field region. When the temperature decreases, these oscillations of the absorption coefficient with the magnetic field will be diminished. As shown by the curve D for $T = 4.2$ K in Fig. 1, the oscillations of $\mathrm{Im}(\alpha)$ with the magnetic field will vanish and the value of $\mathrm{Im}(\alpha)$ decreases rapidly with increasing magnetic field. From Eq. (8) and Ref. 13, it can be approximated at a limit for high magnetic fields and low temperatures as

$$\begin{aligned}
\alpha \cong {} & 145\pi (m^* \epsilon)^{-1/2} (E_g - \hbar\Omega)^{1/2} (E_g^{5/4} k_B T)^{-1} \\
& \times (\hbar\omega_c)^{9/4} (|e|^3 C^2 B n_0 / \rho v_s^2 c^2 \hbar^3 \Omega^2).
\end{aligned} \tag{9}$$

Thus $\mathrm{Im}(\alpha)$ will vanish at this limit. However, $\mathrm{Re}(\alpha)$ still oscillates with the magnetic field in the lower-field region and it appears to be nondecreasing with the field at the high-field region.

We plot the absorption coefficient $|\alpha| = \{[\mathrm{Re}(\alpha)]^2 + [\mathrm{Im}(\alpha)]^2\}^{1/2}$ as a function of the magnetic field as shown in Fig. 2. It is shown that the absorption coefficient oscillates with the magnetic field and these oscillations are diminished with decreasing temperature. Moreover, these oscillations of the absorption coefficient with the magnetic field will vanish at high magnetic fields and the absorption coefficient decreases with decreasing temperature.

From our numerical results presented here, it can be seen that the oscillations of the absorption coefficient appear only for the magnetic fields in which the cyclotron frequency is smaller than the photon frequency, that is, $\Omega > \omega_c$. In this region of magnetic fields, the absorption of a photon can occur with the simultaneous emission or absorption of phonons in an electronic transition to the

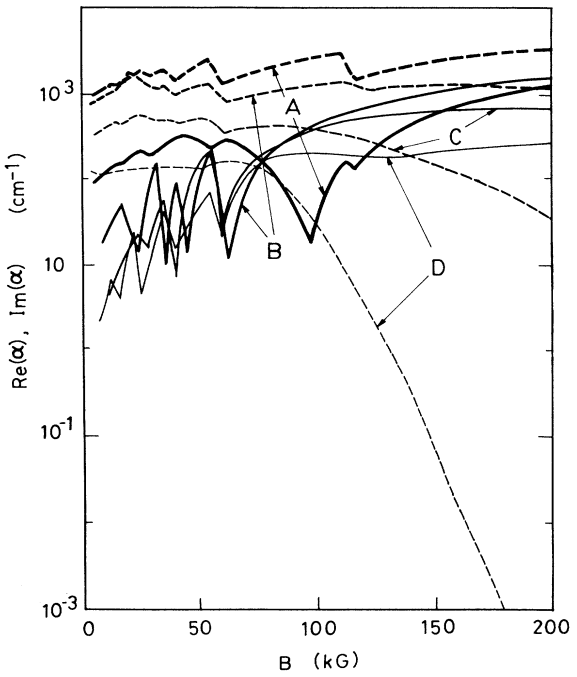

FIG. 1. Real part of the absorption coefficient $\mathrm{Re}(\alpha)$ (solid curves) and imaginary part of the absorption coefficient $\mathrm{Im}(\alpha)$ (dashed curves) in n-type GaAs as a function of magnetic field B in a radiation field of 10.6-μm wavelength for A: $T=300$ K; B: $T=77$ K; C: $T=19.7$ K; and D: $T=4.2$ K.

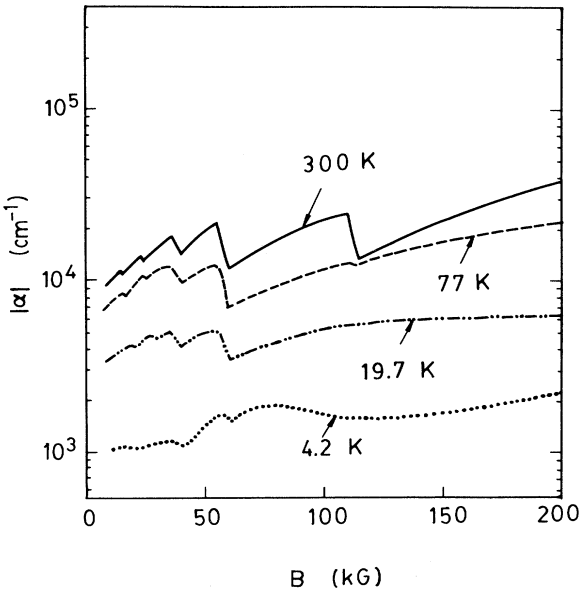

FIG. 2. Absorption coefficient $|\alpha|$ in n-type GaAs as a function of magnetic field B in a radiation field of 10.6-μm wavelength.

same or other Landau subbands. As the magnetic field increases, the cyclotron frequency ω_c becomes larger than the photon frequency Ω, then transition to a final state in the same subband can occur. Consequently, the absorption coefficient increases monotonically as a function of the magnetic field and there are no further oscillations in the absorption coefficient with the magnetic field. If the temperature decreases, the effect of lattice vibrations in solids will be reduced. Therefore the oscillations of the absorption coefficient with the magnetic field are diminished and the absorption coefficient decreases with decreasing temperature.

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