國 立 交 通 大 學

電信工程學系碩士班
碩士論文

應用於多用戶編碼協力式通訊之
高頻譜效率空時協定

Spectrally Efficient Space-Time Protocols for Multi-user Coded Cooperative Communications

研 究 生：王雋甫　　Student: Chun-Fu Wang

指導教授：李大嵩 博士　　Advisor: Dr. Ta-Sung Lee

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中 華 民 國 九 十 七 年 六 月

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A Thesis
Submitted to Department of Communication Engineering
College of Electrical and Computer Engineering
National Chiao Tung University
In Partial Fulfillment of the Requirements
For the Degree of
Master of Science
In
Communication Engineering
June 2008
Hsinchu, Taiwan, Republic of China

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摘要

在協力式通訊系統中，給中繼站使用的額外通道所造成的頻譜效率損失，是其主要的缺點。近年來，有一種高頻譜效率的解決方案被提出來，稱作「編碼協力式通訊」。在本論文中，吾人的目標為設計基於編碼協力式通訊的空時協定，藉此達到更高的系統可靠度。吾人將提出兩種不同的協定: 空時（ST）及碼分配式（CP）編碼協力式通訊，以上兩種協定皆能達到比傳統編碼協力式通訊更好的表現，同時還能維持相同的傳輸速率以及功耗。透過成偶比對錯誤率的推導，吾人證明這兩種通訊協定皆能達到最大多樣增益。此外，針對用戶之間通道不良的情況，吾人也提出了穩健的方法及其分析。值得一提的是，雖然本論文僅對於雙用戶及四用戶的情況做介紹，吾人很容易可將之延伸到其它用戶數量。此外，協力式編碼的架構以及空時碼的選擇，也可以彈性地針對不同的需求來修改。

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Abstract

A major drawback of cooperative communication is the spectral efficiency loss due to additional channel needs for the relay. Recently, coded cooperation had been proposed as a spectrally efficient solution for user cooperation. In this thesis, our aim is to design space-time protocols under coded cooperation that achieve higher system reliability. We introduce two modified protocols: space-time (ST) coded cooperation and code partitioning (CP) coded cooperation. These protocols achieve remarkable gains comparing to the performance of conventional coded cooperation while maintaining the same data rate and transmit power. We conduct performance analysis in terms of pairwise error probabilities, showing that both protocols achieve extra diversity gain guaranteed by the space-time code used. Robust algorithms in case of poor inter-user link are also developed and analyzed. Although we demonstrate the case of two and four users, extension to other number of users is straightforward. Different types of code structures for cooperation and space-time coding are also flexible to choose depending on various needs.

Acknowledgement

I would like to express my deepest gratitude to my advisor, Dr. Ta-Sung Lee, for his enthusiastic guidance and great patience, especially in the training of oral presentation. I learned a lot from his positive attitude in many areas. Thanks are also offered to all members in the Communication System Design and Signal Processing (CSDSP) Lab.

At last but not least, I would like to show my sincere thanks to my family for their invaluable love and support.

Contents

List of Figures

Acronym Glossary

AF	amplify-and-forward
AWGN	additive white Gaussian noise
BER	bit error rate
BPSK	binary phase shift keying
CP	code partitioning
CRC	cyclic redundancy check
DF	decode-and-forward
FER	frame error rate
MGF	moment generating function
MIMO	multiple-input multiple-output
MISO	multiple-input single-output
ML	maximum likelihood
OSTBC	orthogonal space-time block code
PDF	probability density function
RCPC	rate-compatible punctured convolutional
SNR	signal-to-noise ratio
ST	space-time
STBC	space-time block code
STC	space-time coding
STTD	space-time transmit diversity
TDMA	time division multiple access

Notations

$(\cdot)^T$	transpose operator
$(\cdot)^H$	Hermitian operator
$E\{\cdot\}$	expectation operator
$\|\cdot\|$	Euclidean norm operator
$\overline{(\cdot)}^h$	statistical averaging over a random variable
E_s	transmit energy per bit
K	data block length
N	frame length
N_0	noise variance
R	code rate
U_u	*u*th user
$\mathbf{c}$	transmit codeword
$\mathbf{h}$	channel coefficients
$\mathbf{n}$	noise vector
$\mathbf{r}$	received signal
$\mathbf{s}$	transmit signal
$\mathbf{z}$	detected signal

Chapter 1

Introduction

Various techniques are being investigated for meeting the goals of next generation wireless communications. Among these techniques, diversity is of great interests. In a multi-antenna communication system, one way to achieve diversity is to apply space-time coding (STC). For example, space–time trellis coding (STTC) is proposed in [1], which combines signal processing at the receiver with coding techniques for multiple transmit antennas. Space–time block coding (STBC), first discovered by Alamouti [2] and generalized in [3] and [4] to an arbitrary number of transmit antennas, is able to achieve the full diversity promised by the transmit and receive antennas.

However, mobile devices may not be able to support multiple antennas due to size or other hardware constraints. In this case, since most wireless communication systems are operated in a multi-user scheme, the idea of *user cooperation* was born [5] [6]. The basic concept is shown in Fig. 1-1, in which every user has its partners. The users are responsible for transmitting not only their own data, but also the data of their partners they received and detected [7]. Diversity can be achieved since each mobile user sees an independent channel to the base station.

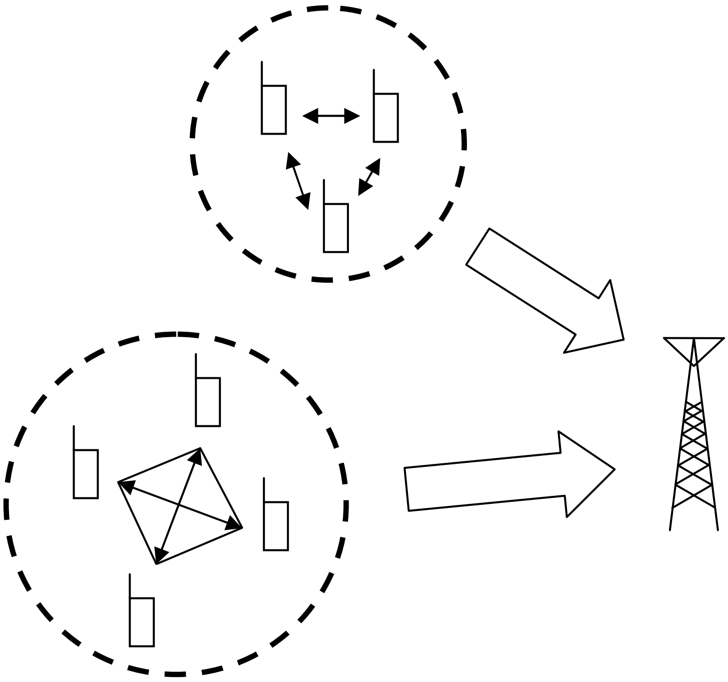

Fig. 1-1. Cooperation between mobile users

This form of space diversity is referred to as *cooperative diversity* in [8] (cf. *user cooperation diversity* of [6]). Space-time codes can be applied to the “virtual array” formed by the users. Anghel et al. extended the cooperative system by implementing a distributed Alamouti space-time code based on the multi-user scheme [9]. It is shown in [10] that cooperative diversity with appropriately designed codes realizes full spatial diversity gain.

Common information at distributed points is required for applying distributed space-time codes. In other words, users have to exchange their data before cooperating. However, half-duplex systems which are commonly used in real world devices need additional channel for the transmission from user to partners, thus suffering from the loss in spectral efficiency [11][12].

Many solutions are proposed to reduce the above spectral loss (Fig. 1-2). The authors of [13] propose a network coding scenario for communication between two users. In [14] it is extended to the case of two users transmitting to one destination, where multiple-access relay channel is used. In [15], the authors suggest spatially reusing the relay time slot. Although it causes interference between the users, this protocol has a spectral efficiency of $K/(K+1)$ and is close to 1 when K is large.

In the above cooperative models, the partner only repeats what it received. Recently, a different framework called *coded cooperation* was proposed ([16][17] and [18]), where signals are not repeated by the partner. They assume that the user data is protected by channel coding (which is commonly used in wireless communications) and partition the codeword into two sets: one set is transmitted by its own user and the other is transmitted by the partner. A sketch of system design of the mentioned cooperative models is given in Fig. 1-2.

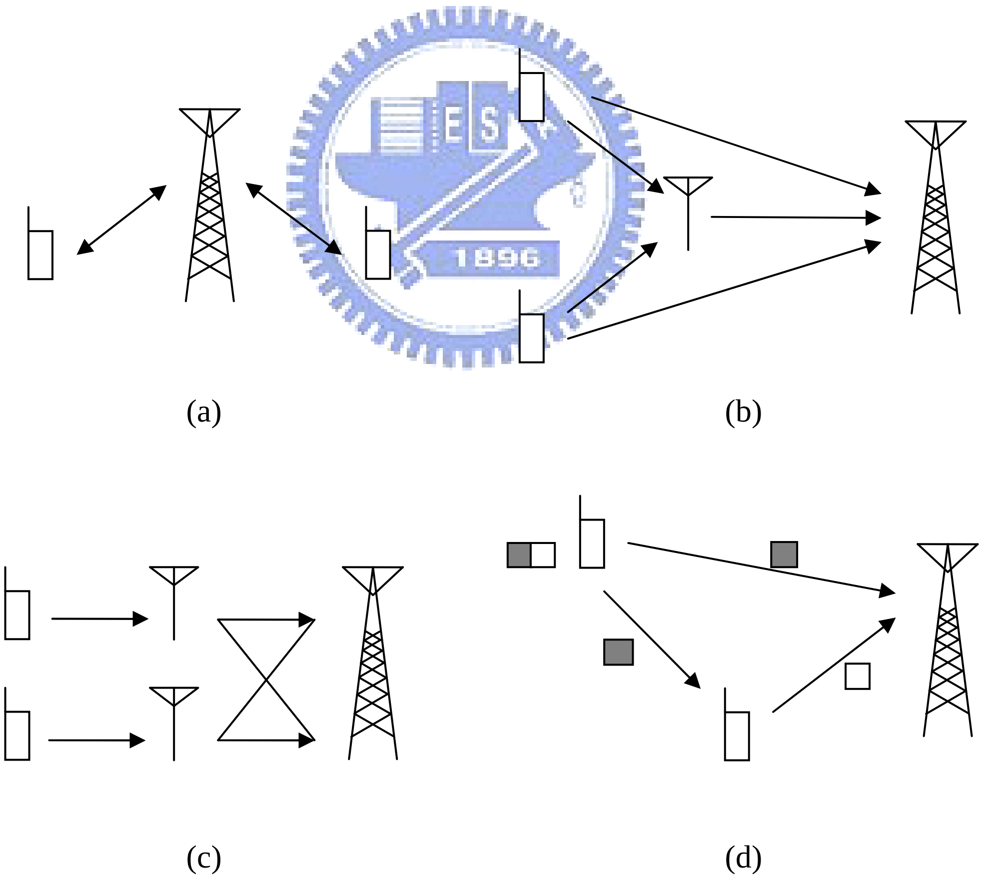

Fig. 1-2. System design of (a) Network coding [13], (b) Network coding [14], (c) Spatial reuse [15], (d) Coded cooperation[16]

Since there is no repetition of the codeword, the spectral efficiency of coded cooperation is 1. The works in [16]-[18] prove that it can achieve full diversity guaranteed by the number of users.

Space-time coding can also be applied to coded cooperation. In [16], the authors suggest the use of space-time code which allows the users to capture better space-time diversity under fast fading. In [19], space-time overlay coding is used to achieve higher diversity gain when the partner nodes and their destination have multiple antennas.

In this thesis, our main goal is to design and analyze the space-time coded cooperation protocols for single-antenna terminals under slow fading environments. It is intuitive to combine coded cooperation with distributed space-time code since it needs only half the time to complete the “data exchange” process comparing to conventional cooperation. In particular, we propose two modified protocols that achieve higher diversity order than conventional coded cooperation without the need for more antennas or channel resource.

The remainder of the thesis is organized as follows. We give a brief review of the basic model of conventional cooperative scheme and coded cooperative scheme in Chapter 2, where concept of the proposed protocol will also be given. The first protocol will be described in Chapter 3. Bit error rate (BER) and frame error rate (FER) will be analyzed and given along with simulation results. Also, algorithms addressing the cases of data exchange failure will be considered. The second protocol will be described in Chapter 4. This new scheme is developed based on the discovery in Chapter 3. Performance analysis and simulation results will be given, and pros and cons of these two protocols will be discussed. Finally, we summarize the contributions of our works and give an outline of possible extended research in Chapter 5.

Chapter 2

System model and Problem Formulation

In this Chapter we review the concept and model of conventional wireless cooperative communication and coded cooperative communication. We introduce the problem of loss in spectral efficiency due to the additional channel uses in half-duplex systems under conventional cooperative scheme. Then, a different scheme called coded cooperation which compensates the spectral efficiency loss will be given. Finally, we introduce the proposed cooperation scheme to further utilize the benefits in cooperation, thus achieves higher transmission reliability.

For the model of the cooperative system throughout this thesis, a narrow-band transmission in frequency-flat and slow fading channel with additive noise is considered to isolate the benefits of cooperation diversity. Extension to frequency- and time-selective fading channel can be naturally done, but the cooperation gain will not be substantial since other kind of diversity can be exploited in that system.

2.1 Review of Cooperation Network

Consider a system in Fig. 2-1 which two users U_1, U_2 (ex. mobile phones) are communicating with a destination D (base station). We assume all terminals are equipped with single antenna.

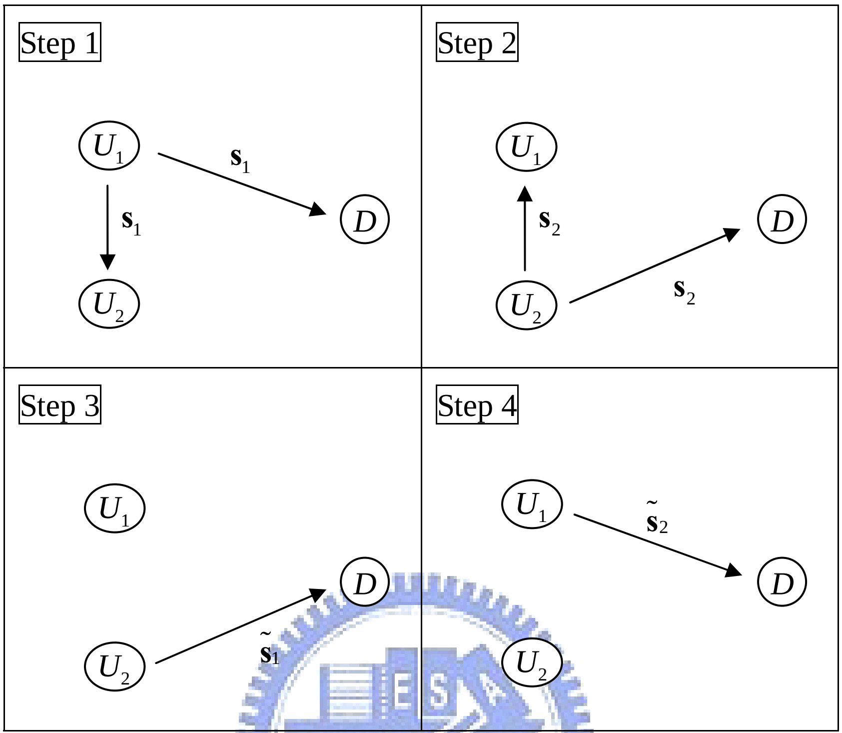

Fig. 2-1. Protocol of two-user cooperation

We define step 1, 2 as "*broadcast phase*", where two users separately transmit their information to the destination and the other user. We call it broadcast phase because it utilizes the broadcast nature of wireless channel. Step 3, 4 are defined as "*relay phase*", where the two user act as a "relay" of each other, passing the information received from other user to the destination. Throughout the 4-step transmission, two copies of one user data will be transmitted by different user in broadcast phase and relay phase, separately. There are commonly two relaying protocols: *amplify-and-forward* (AF) and *decode-and-forward* (DF). In amplify-and-forward mode, relays amplify their received signals subject to their own power constraint. In decode-and-forward mode, relays fully decode the information and retransmit it to the destination. Both of these protocols achieve full diversity (of order 2 under 2-user case) in appropriate conditions. [20]

In these cooperation protocols, relays have to do both receiving and transmitting. To preserve time for the relaying process, a straightforward way is to make relays receiving and transmitting at the same time. However, implementation of this kind of full-duplex system (transmitting and receiving at the same time in the same frequency band) is less practicable due to the limitation of current radio hardware. Instead, half-duplex operation is commonly used in such cooperation protocols. That means broadcast and relay phase need to be separated into different channel uses. An example is shown in Fig. 2-2. Assuming time division multiple access (TDMA) is used, a total of 4 time slots are needed for each user to transmit one data.

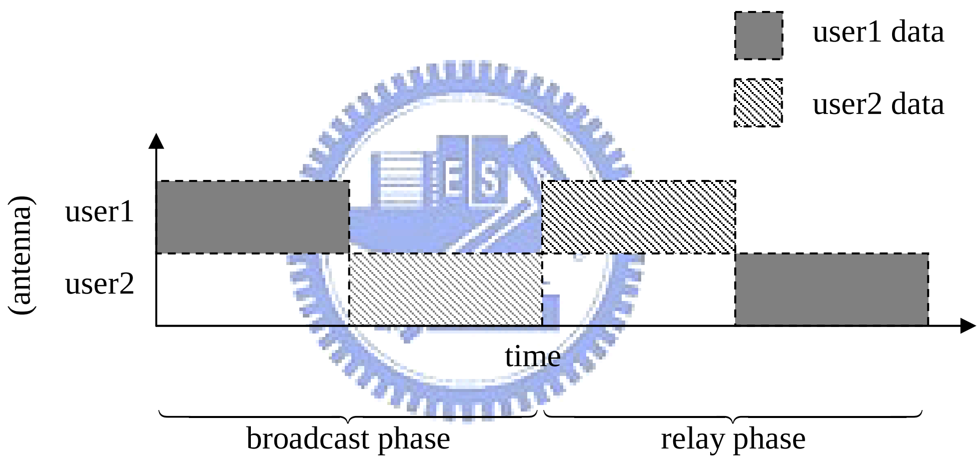

Fig. 2-2. Channel use in half-duplex two-user cooperation (TDMA)

Define the channel coefficients between user U_u and destination as h_u, where $u = 1,2$. Assume all channels are independent, flat fading and quasi-static over one phase. Assuming binary phase-shift keying (BPSK) modulation is used, we define the baseband-equivalent discrete-time symbols transmitted by user U_u as $\mathbf{s}_u = \begin{bmatrix} s_{u,1} & s_{u,2} & \cdots & s_{u,N} \end{bmatrix}^T$, where $s_{u,i} \in \{-1,+1\}$, $u = \{1,2\}$, $i = 1,\cdots,N$ and N is the frame length. The signal received by destination is

$$\mathbf{r}_u = h_u \sqrt{E_s}\mathbf{s}_u + \mathbf{n}_u \tag{2.1}$$

where E_s is the transmit energy per bit, h_u is the fading coefficient between user U_u and destination with $|h_u| \sim \text{Rayleigh}(\sigma)$, $h_u = X + jY$ and $X, Y \sim N(0, \sigma^2)$. $\mathbf{n}_u$ is a vector of additive white Gaussian noise (AGWN) with zero mean and variance $\frac{N_0}{2}$ per dimension.

Now we denote $\mathbf{r}_u^{(b)}$ as the received signal for user U_u data at destination in broadcast phase. In the same way, $\mathbf{r}_u^{(r)}$ denotes the received signal for user U_u data at destination in relay phase. Thus the received signal at the destination in the four time slots can be written as

$$\begin{cases} \mathbf{r}_1^{(b)} = h_1^{(b)} \sqrt{E_s} \mathbf{s}_1 + \mathbf{n}_1^{(b)} \\ \mathbf{r}_2^{(b)} = h_2^{(b)} \sqrt{E_s} \mathbf{s}_2 + \mathbf{n}_2^{(b)} \end{cases}, \quad \begin{cases} \mathbf{r}_2^{(r)} = h_1^{(r)} \sqrt{E_s} \tilde{\mathbf{s}}_2 + \mathbf{n}_2^{(r)} \\ \mathbf{r}_1^{(r)} = h_2^{(r)} \sqrt{E_s} \tilde{\mathbf{s}}_1 + \mathbf{n}_1^{(r)} \end{cases} \tag{2.2}$$

Note that the channel coefficients are added with superscripts (b) and (r) to distinguish the two phases. They are also added to the noise vector. The transmitted symbols at the relays $\tilde{\mathbf{s}}_1, \tilde{\mathbf{s}}_2$ are added with tilde because they are not always equal to the transmitted symbols at the source. In AF mode, $\tilde{\mathbf{s}}_1, \tilde{\mathbf{s}}_2$ are normalized signal received from the source, that is

$$\begin{cases} \tilde{\mathbf{s}}_1 = \dfrac{\mathbf{r}_{1,2}}{\sqrt{E\left\{|\mathbf{r}_{1,2}|^2\right\}}} = \dfrac{h_{1,2}\sqrt{E_s}\mathbf{s}_1 + \mathbf{n}_{1,2}}{\sqrt{E_s |h_{1,2}|^2 + N_{1,2}}} \\ \tilde{\mathbf{s}}_2 = \dfrac{\mathbf{r}_{2,1}}{\sqrt{E\left\{|\mathbf{r}_{2,1}|^2\right\}}} = \dfrac{h_{2,1}\sqrt{E_s}\mathbf{s}_2 + \mathbf{n}_{2,1}}{\sqrt{E_s |h_{2,1}|^2 + N_{2,1}}} \end{cases} \tag{2.3}$$

Note that $\mathbf{r}_{1,2}$ and $\mathbf{r}_{2,1}$ denote the received signals at user U_2 and U_1, respectively. These can be written in the form of

$$\begin{cases} \mathbf{r}_{1,2} = h_{1,2}\sqrt{E_s}\mathbf{s}_1 + \mathbf{n}_{1,2} \\ \mathbf{r}_{2,1} = h_{2,1}\sqrt{E_s}\mathbf{s}_2 + \mathbf{n}_{2,1} \end{cases} \tag{2.4}$$

where $h_{1,2}, h_{2,1}$ denote the channel coefficients from user U_1 to U_2 and from U_2 to U_1, respectively. $\mathbf{n}_{1,2}, \mathbf{n}_{2,1}$ denote the noise received by the relay, and $N_{1,2}, N_{2,1}$ is the noise power. In DF mode, the relays and the source will transmit the same information if the relays decode what they received correctly, that is, $\tilde{\mathbf{s}}_1 = \mathbf{s}_1$, $\tilde{\mathbf{s}}_2 = \mathbf{s}_2$.

Half-duplex cooperation protocols suffer from the loss in spectral efficiency due to the two channel uses required for the transmission from the source to the relay and relay to destination, which causes a pre-log factor 1/2 in the corresponding rate expressions [11].

The protocol can be easily extended to more user cases, for example, Fig. 2-3 demonstrates a 4-user scheme. A diversity order of 4 can be achieved for each user's data, but note that the length of the relay phase is three times longer than in 2-user case. Thus the loss of spectral efficiency is even severer.

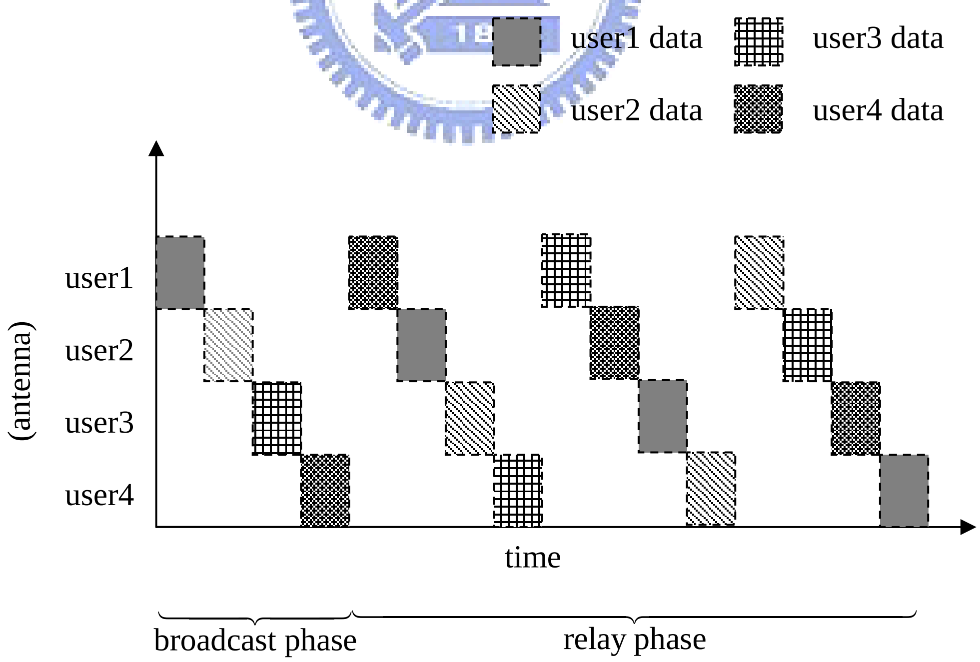

Fig. 2-3. Channel use in half-duplex four-user cooperation (TDMA)

2.2 Review of Coded Cooperation

One way to avoid the additional channel use is using *coded cooperation* ([16][17] and [18]). Coded cooperation combines the idea of user cooperation with channel coding, which is commonly used in wireless communication systems. Assume data of all users are protected by channel coding. We call the channel code used as *base code*. Apart from the conventional cooperation protocol mentioned in Section 2.1 where relays *repeat* the received data, relays in coded cooperation decode and re-encode the received data using different encoder, then transmit it to the destination. More specifically, the base code is partitioned into two sub-codes: one is transmitted by the source and the other is transmitted by the relay, we call them *broadcast sub-code* and *relay sub-code*, respectively. The basic idea is that the data exchanged between users is encoded by a shorter sub-code (broadcast sub-code) to preserve channel resource. Relay decodes the received data and encodes it into another sub-codeword (relay sub-code), then transmits it to the destination. The destination combines the codewords received from source and rebuilds the original base codeword for decoding.

It is important to note that relay needs to successfully decode the data received from the source to do cooperation. The data is now only protected by shorter codes, but it is tolerable since the channel between users is usually better than the channel to the destination (considering cellular phone system for instance, the distance to nearby mobile user are much shorter than the distance to the base station)

Consider 2-user case. For a rate-R base code, assuming there are K data bits per block, we have $N = \frac{K}{R}$ coded bits. The N-bits codeword is divided into two sub-codewords (for broadcast and relay phase, respectively). The length of each sub-code needs not to be the same. We define $N^{(b)}, N^{(r)}$ as the lengths of the sub-codes for broadcast and relay phase, respectively. Note that $N^{(b)} + N^{(r)} = N$. We focus on TDMA scheme which is similar to the scheme in Section 2.1. In the broadcast phase, each user transmits its broadcast sub-codeword. If the relay successfully decodes the source data, it re-encodes the data using relay sub-code and transmits. Since the total transmitted bits of a data block is still *N*, no more channel resource is needed compared to direct transmission scheme, that is, no spectral efficiency loss.

2.2.1 Code Selection

Various channel coding methods can be used in the coded cooperation protocols. For example, the base code may be a block or convolutional code or a combination of both [16]. Partitioning of the codeword can be achieved by puncturing, product codes or other forms of concatenation. Convolutional codes with puncturing are used in [16]~[18], but with different generators: [16] uses a rate-compatible punctured convolutional (RCPC) code proposed in [21], where [18] uses a self-designed code for specific purpose.

For the models in [18], the authors use convolutional codes with rate-1/4. The generator is [15 17 13 15], the codeword length is 260 bits and the puncturing patterns are [1 1 0 0] and [0 0 1 1]. Thus we have $N^{(b)} = N^{(r)} = 130$ and $R^{(b)} = R^{(r)} = \frac{1}{2}$. The structure of the codeword is given below:

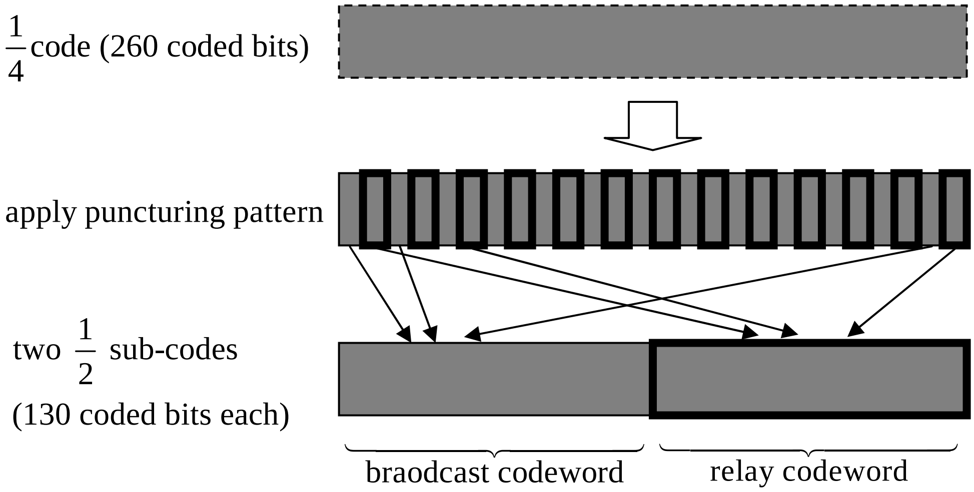

Fig. 2-4. Structure of codeword in coded cooperation

2.2.2 Transmission Model

Based on the code structure given above, we denote the broadcast and relay sub-codewords of user U_1 as $\mathbf{c}_1^{(b)},\mathbf{c}_1^{(r)}$, respectively. In the same way, $\mathbf{c}_2^{(b)},\mathbf{c}_2^{(r)}$ denote the sub-codewords for user U_2. The transmission model is similar to section 2.1, but different codewords are transmitted from the source and the relay. (Fig. 2-5)

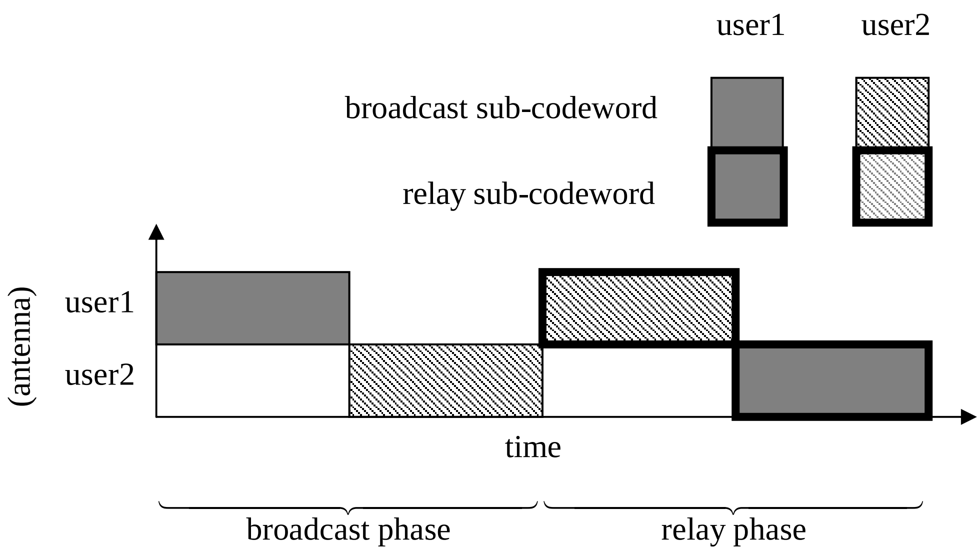

Fig. 2-5. Channel use of coded cooperation with two users (TDMA)

Thus the received signal at the destination can be modeled as

$$\begin{cases} \mathbf{r}_1^{(b)} = h_1^{(b)}\sqrt{E_s}\mathbf{c}_1^{(b)} + \mathbf{n}_1^{(b)} \\ \mathbf{r}_2^{(b)} = h_2^{(b)}\sqrt{E_s}\mathbf{c}_2^{(b)} + \mathbf{n}_2^{(b)} \end{cases}, \quad \begin{cases} \mathbf{r}_2^{(r)} = h_1^{(r)}\sqrt{E_s}\mathbf{c}_2^{(r)} + \mathbf{n}_2^{(r)} \\ \mathbf{r}_1^{(r)} = h_2^{(r)}\sqrt{E_s}\mathbf{c}_1^{(r)} + \mathbf{n}_1^{(r)} \end{cases} \tag{2.5}$$

Note that only DF mode can be used in coded cooperation, since the relay has to decode the received codeword in order to re-encode it into another codeword.

The model can be extended to more user cases. However, it has similar problem to conventional cooperation. Longer relay phase is needed for all users to transmit the relay sub-codeword. It decreases the spectral efficiency and makes coded cooperation less attractive.

2.3 Proposed Protocols Based on Coded Cooperation

Considering the transmission model of coded cooperation (Fig. 2-5), there are several empty slots in both broadcast phase and relay phase. In broadcast phase, the transmission of different user's data cannot be overlapped. One reason is that the destination is equipped with only one antenna and is unable to separate the signals of different users if they overlapped; the other reason is that a user has to listen to other user data in order to perform relaying.

However, in relay phase, all users already know each other's data (if decoded successfully). Thus some changes can be done here in order to improve the performance. We will introduce two modifications of the coded cooperation protocols, which we called *space-time (ST) coded cooperation* and *code partition (CP) coded cooperation*. Detailed descriptions, simulations and the performance analyses of the protocols will be given in Chapter 3 and Chapter 4 for ST-coded and CP-coded cooperation, respectively.

2.4 Summary

A comparison of all transmission protocols mentioned in this chapter is shown in Fig. 2-6 to make the difference of spectral efficiency more clear. The spectral efficiency loss in conventional 2-user cooperation protocol is clear shown in the figure. It uses twice more time slots compared to other protocols. Coded cooperation uses just half of the time but still achieves same diversity as conventional cooperation. For the proposed modification of coded cooperation protocols, we aim at the relay phase to achieve higher transmission reliability.

In this chapter we review the system model of conventional cooperation and coded cooperation under multi-user schemes. It is shown that by using coded cooperation with appropriate choice of codes, we can avoid the loss of spectral efficiency while still achieving full diversity. Then we point out the potential of extracting more benefits from coded cooperation by using more reliable transmission in relay phase. Two modified protocols which can acquire higher diversity order (>2) without losing spectral efficiency are introduced.

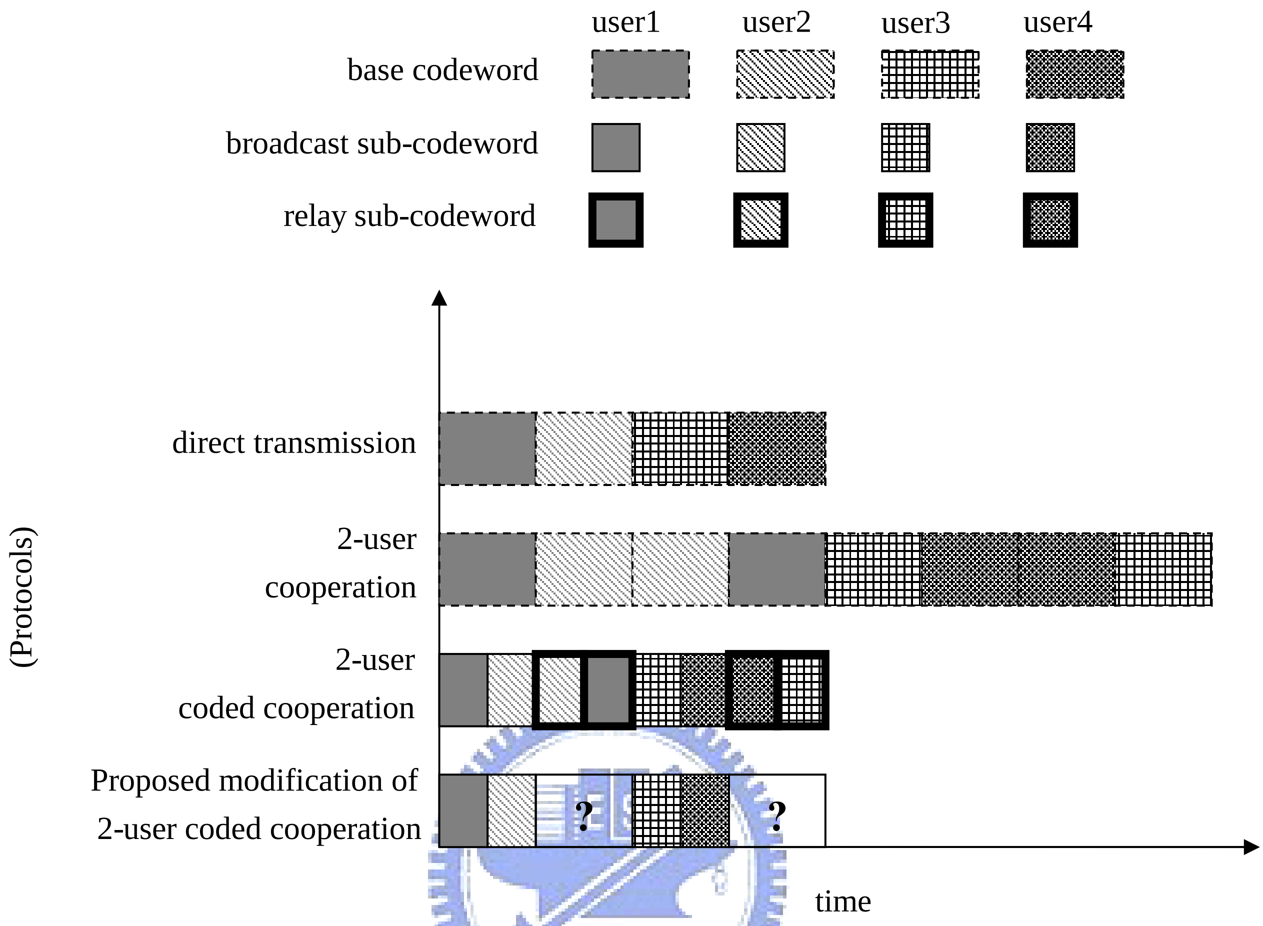

Fig. 2-6. Comparison of channel uses for cooperation protocols (TDMA)

Chapter 3

Spectrally Efficient Multi-user Coded Cooperation Using Space-Time Code

In this Chapter we introduce the first modification of coded cooperation: space-time (ST) coded cooperation. A detailed description of how this protocol works will be given. Performance bounds will be analyzed and showed along with simulation results. Cases when a user does not successfully decode the data from other users are also considered. We focus on 2-user coded cooperation with Alamouti space-time code for simplicity, but extension to more user cases with other types of space-time codes is straightforward.

3.1 Protocols of ST-Coded Cooperation

In coded cooperation, since all users know each other's data after the broadcast phase, we can treat the users as "virtual antennas". Since that, applying space-time code in relay phase to achieve transmit diversity is possible. In the following Sections we'll give an example that shows how 2-user ST-coded cooperation works.

3.1.1 Code Structure and System Model

Since there are two users in the cooperation scheme, we can apply Alamouti code [2] in the transmission of relay phase, the protocol is shown in Fig. 3-1.

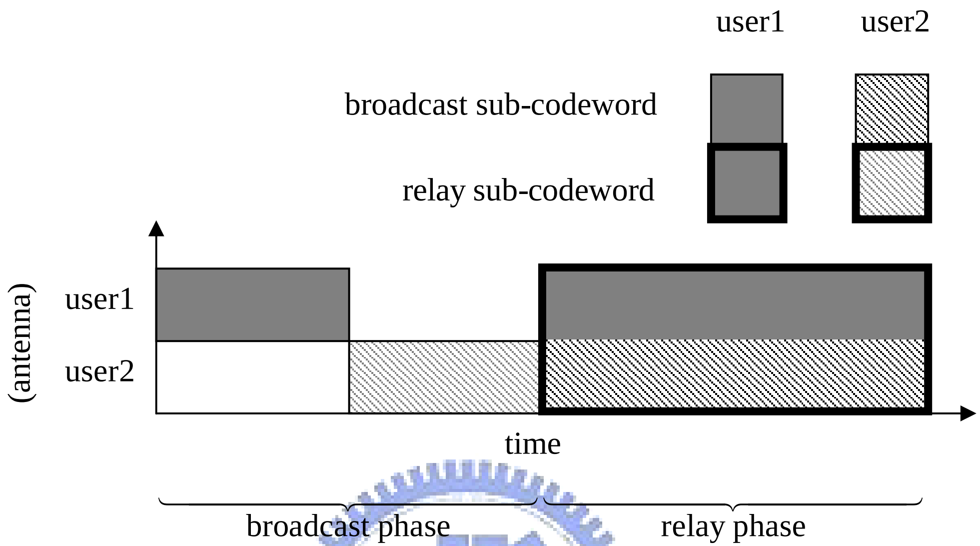

Fig. 3-1. Channel use of ST-coded cooperation with two users (TDMA)

Assume the same quasi-static fading channel as in Chapter 2, that is, the fading coefficient remains constant during the transmission of the space-time code (one phase). A diversity gain of 2 is extracted by the use of Alamouti code in the relay phase, thus we can expect a total diversity order of 3 for each user data since the broadcast sub-codeword sees independent channel with that seen by relay sub-codeword.

In the broadcast phase, there's no change to the transmission model comparing with (2.5), so it can be written as

$$\begin{cases} \mathbf{r}_1^{(b)} = h_1^{(b)} \sqrt{E_s}\, \mathbf{c}_1^{(b)} + \mathbf{n}_1^{(b)} \\ \mathbf{r}_2^{(b)} = h_2^{(b)} \sqrt{E_s}\, \mathbf{c}_2^{(b)} + \mathbf{n}_2^{(b)} \end{cases} \tag{3.1}$$

In the relay phase, Alamouti space-time code is applied, the code matrix of the ith element of the sub-codeword is

$$\mathbf{C}_i = \begin{bmatrix} c_{1,i}^{(r)} & -\left(c_{2,i}^{(r)}\right)^* \\ c_{2,i}^{(r)} & \left(c_{1,i}^{(r)}\right)^* \end{bmatrix} \tag{3.2}$$

where the subscript i denotes the ith element. ($c_{1,i}^{(r)}$ is the ith element of the vector $\mathbf{c}_1^{(r)}$)

Thus the received signal during relay phase is

$$\begin{cases} r_{1,i}^{(r)} = h_1^{(r)}\sqrt{\frac{E_s}{2}}c_{1,i}^{(r)} + h_2^{(r)}\sqrt{\frac{E_s}{2}}c_{2,i}^{(r)} + n_{1,i}^{(r)} \\ r_{2,i}^{(r)} = -h_1^{(r)}\sqrt{\frac{E_s}{2}}\left(c_{2,i}^{(r)}\right)^* + h_2^{(r)}\sqrt{\frac{E_s}{2}}\left(c_{1,i}^{(r)}\right)^* + n_{2,i}^{(r)} \end{cases} \tag{3.3}$$

Define channel matrix of the relay phase as

$$\mathbf{H}_r = \begin{bmatrix} h_1^{(r)} & h_2^{(r)} \\ \left(h_2^{(r)}\right)^* & -\left(h_1^{(r)}\right)^* \end{bmatrix} \tag{3.4}$$

then (3.3) can be written in matrix form

$$\begin{aligned} \mathbf{r}_i^{(r)} &= \sqrt{\frac{E_s}{2}}\begin{bmatrix} h_1^{(r)} & h_2^{(r)} \\ \left(h_2^{(r)}\right)^* & -\left(h_1^{(r)}\right)^* \end{bmatrix}\begin{bmatrix} c_{1,i}^{(r)} \\ c_{2,i}^{(r)} \end{bmatrix} + \begin{bmatrix} n_{1,i}^{(r)} \\ \left(n_{2,i}^{(r)}\right)^* \end{bmatrix} \\ &= \sqrt{\frac{E_s}{2}}\mathbf{H}_r\mathbf{c}_i^{(r)} + \mathbf{n}_i^{(r)} \end{aligned} \tag{3.5}$$

where $\mathbf{r}_i^{(r)} = \left[r_{1,i}^{(r)} \quad \left(r_{2,i}^{(r)}\right)^*\right]^T$, $\mathbf{c}_i^{(r)} = \left[c_{1,i}^{(r)} \quad c_{2,i}^{(r)}\right]^T$, $\mathbf{n}_i^{(r)} = \left[n_{1,i}^{(r)} \quad \left(n_{2,i}^{(r)}\right)^*\right]^T$. Multiplying (3.5) by $\mathbf{H}_r^H$, we have

$$\begin{aligned} \tilde{\mathbf{r}}_i^{(r)} &= \left[\tilde{r}_{1,i}^{(r)} \quad \tilde{r}_{2,i}^{(r)}\right]^T = \mathbf{H}_r^H\mathbf{r}_i^{(r)} \\ &= \sqrt{\frac{E_s}{2}}\begin{bmatrix} \left|h_1^{(r)}\right|^2 + \left|h_2^{(r)}\right|^2 & 0 \\ 0 & \left|h_1^{(r)}\right|^2 + \left|h_2^{(r)}\right|^2 \end{bmatrix}\begin{bmatrix} c_{1,i}^{(r)} \\ c_{2,i}^{(r)} \end{bmatrix} + \tilde{\mathbf{n}}_i^{(r)} \end{aligned} \tag{3.6}$$

where $\tilde{\mathbf{n}}_i^{(r)} = \mathbf{H}_r^H\mathbf{n}_i^{(r)}$. Note that $\tilde{\mathbf{r}}_i^{(r)}$ is a vector which contains the ith detected symbols of relay sub-codewords for both users.

Assuming ML detection, from (3.1) and (3.6) we can write the detected signals for user U_u $(u \in \{1,2\})$ in broadcast phase as

$$\mathbf{z}_u^{(b)} = \left(h_u^{(b)}\right)^* \mathbf{r}_u^{(b)} = \sqrt{E_s}\left|h_u^{(b)}\right|^2 \mathbf{c}_u^{(b)} + \tilde{\mathbf{n}}_u^{(b)} \tag{3.7}$$

where $\tilde{\mathbf{n}}_u^{(b)} = \left(h_u^{(b)}\right)^* \mathbf{n}_u^{(b)}$. Also, the detected signal in relay phase is

$$\begin{aligned} \mathbf{z}_u^{(r)} &= \sqrt{\frac{E_s}{2}}\left(\left|h_1^{(r)}\right|^2 + \left|h_2^{(r)}\right|^2\right)\mathbf{c}_u^{(r)} + \tilde{\mathbf{n}}_u^{(r)} \\ &= \sqrt{\frac{E_s}{2}}\left\|\mathbf{h}^{(r)}\right\|_F^2 \mathbf{c}_u^{(r)} + \tilde{\mathbf{n}}_u^{(r)} \end{aligned} \tag{3.8}$$

where $\mathbf{h}^{(r)} = \left[h_1^{(r)} \quad h_2^{(r)}\right]^T$. Base codeword can be rebuilt by combing the detected signals.

Consider the case of more than two users. For example, a 4×4 *orthogonal space-time block code* (OSTBC) can be used in 4-user coded cooperation, the protocol is shown below:

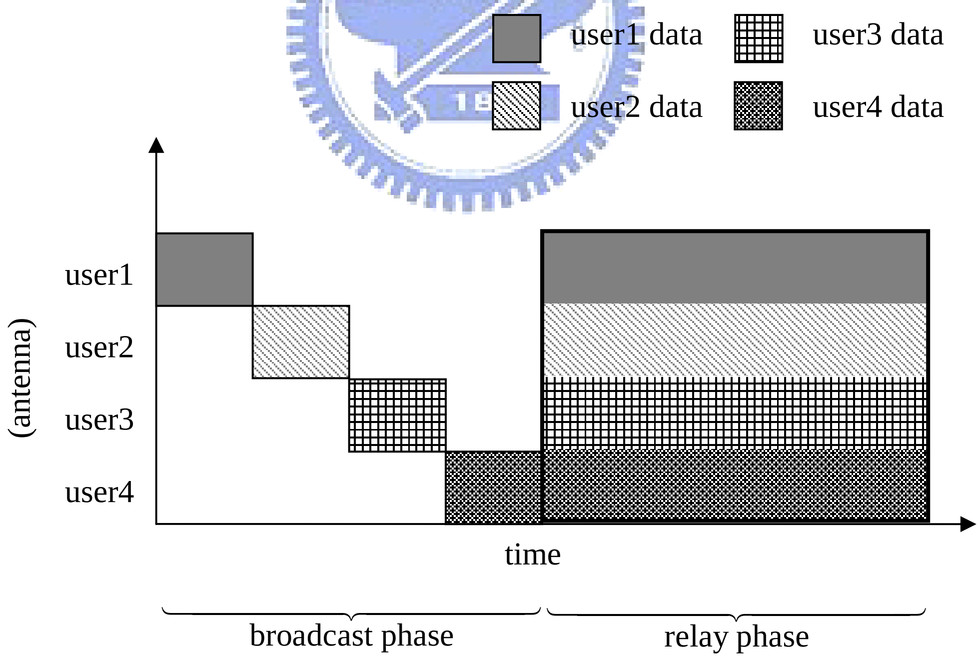

Fig. 3-2. Channel use of ST-coded cooperation with four users (TDMA)

The major advantage of ST-coded cooperation is that space-time code with higher diversity order can be applied when more users are involved. There is no loss of spectral efficiency as long as the space-time code used is of rate equal or higher than 1.

3.1.2 Case of Data Exchange Failure

For a wireless cooperative communication system, it is always possible that data exchanged between two users are corrupted due to deep fading and cannot be decoded successfully. We call this event *data exchange failure*.

The effect of data exchange failure to the coded cooperation can not be ignored. When it happens, cooperation can not be done since users don't know each other's information. Both users have to transmit their own relay sub-codewords by themselves (*no cooperation mode*). More specifically, if user U_2 can't decode the broadcast sub-codeword from U_1 correctly, it will notify U_1 by one bit of information to let U_1 transmit its relay sub-codeword itself. Meanwhile, U_2 will also transmit its relay sub-codeword itself. Cyclic redundancy check (CRC) can be used for error detection.

Consider a 4-user scheme using a 4×4 OSTBC. Note that every user has to know the data of all other users, that is, all users have to successfully decode the data from other users in order to perform cooperation. That gives rather high requirements for inter-user channels. The effect of data exchange failure to the overall performance will be analyzed in next Section.

3.2 Performance Bounds of ST-Coded Cooperation

We present an analytical methodology in this Section for evaluating performance of the proposed Alamouti ST-coded cooperation protocol. Only the performance of user U_1 data is considered since the error rates of the two user data are statistically equal. The pairwise error probability is calculated using the technique from Simon and Alouini [22], then we determine the union bounds for the overall bit error rate (BER) and frame error rate (FER) using weight enumerating function and the tools given by E. Malkamäki and H. Leib [25]. These bounds will be shown and compared with the simulation result.

3.2.1 Pairwise Error Probability

Since we focus on the performance of single user throughout this Section, some notations in the equations can be simplified. Thus we can rewrite the detected signal for U_1 in broadcast phase as (from (3.7))

$$z_i^{(b)} = h_1^{(b)} \sqrt{E_s} c_i^{(b)} + \tilde{n}_i^{(b)} \tag{3.9}$$

where $i = 1, \cdots, N_b$. The index i denotes the ith element of the detected signal. From (3.6), the detected signal for U_1 in relay phase can be rewritten as

$$z_i^{(r)} = \sqrt{\frac{E_s}{2}} \left\| \mathbf{h}^{(r)} \right\|_F^2 c_i^{(r)} + \tilde{n}_i^{(r)} \tag{3.10}$$

where $i = 1, \cdots, N_r$. Sub-codewords are rearranged according to the puncturing pattern and are combined at the destination to rebuild the base codeword. The base codeword can be represented as

$$\mathbf{z} = \begin{cases} \mathbf{z}^{(b)}, & \text{when } i \in \chi_b \\ \mathbf{z}^{(r)}, & \text{when } i \in \chi_r \end{cases} \tag{3.11}$$

Note that χ_b, χ_r are the sets of indexes which belong to the broadcast and the relay sub-codewords, respectively.

Let γ_b, γ_r be the received SNRs for broadcast and relay sub-codewords, respectively. It is straightforward to find that

$$\begin{cases} \gamma_b = \dfrac{\left|h_1^{(b)}\right|^2 E_s}{N_0} \\ \gamma_r = \dfrac{\left\|\mathbf{h}^{(r)}\right\|_F^2 E_s}{2N_0} \end{cases} \tag{3.12}$$

Since ML detection is used at the receiver, the corresponding symbol error probability is given by [23]

$$P_s(i) \approx \begin{cases} \widetilde{N}_e Q\left(\sqrt{\dfrac{\gamma_b d_{\min}^2}{2}}\right), & \text{when } i \in \chi_b \\ \widetilde{N}_e Q\left(\sqrt{\dfrac{\gamma_r d_{\min}^2}{2}}\right), & \text{when } i \in \chi_r \end{cases} \tag{3.13}$$

where $\widetilde{N}_e$ is the number of nearest neighbors and $d_{\min}$ is the minimum distance of separation of the underlying scalar constellation. Since BPSK modulation is assumed, we have $\widetilde{N}_e = 1$ and $d_{\min}^2 = 4$

Define the transmitted base codeword as $\mathbf{x} = [x_1, x_2, \cdots, x_N]$, the probability that z_i is decided as an erroneous symbol $\widehat{x_i} \neq x_i$ conditioned on known channel $\mathbf{H} = \begin{bmatrix} h_1^{(b)} & h_1^{(r)} \\ h_2^{(b)} & h_2^{(r)} \end{bmatrix} = \begin{bmatrix} \mathbf{h}^{(b)} & \mathbf{h}^{(r)} \end{bmatrix}$ is

$$P\left(z_i \to \widehat{x_i}, \widehat{x_i} \neq x_i \middle| \mathbf{H}, x_i\right) = P_e \approx \begin{cases} Q\left(\sqrt{\dfrac{2E_s}{N_0}\left|h_1^{(b)}\right|^2}\right), & \text{when } i \in \chi_b \\ Q\left(\sqrt{\dfrac{E_s}{N_0}\left\|\mathbf{h}^{(r)}\right\|_F^2}\right), & \text{when } i \in \chi_r \end{cases} \tag{3.14}$$

Base on above equation, we have the conditioned pairwise error probability:

$$P\left(\hat{\mathbf{x}} \neq \mathbf{x} \middle| \mathbf{H}, \mathbf{x}\right) = Q\left(\sqrt{\frac{E_s}{N_0}\left(\sum_{i \in \eta_b} 2\left|h_1^{(b)}\right|^2 + \sum_{i \in \eta_r}\left\|\mathbf{h}^{(r)}\right\|_F^2\right)}\right) \tag{3.15}$$

where η_b is the sub set of $i \in \chi_b$ that $\widehat{x_i} \neq x_i$ and η_r is the sub set of $i \in \chi_r$

that $\widehat{x_i} \neq x_i$. Thus the size of η_b are equal to the Hamming distance between the broadcast sub-codeword in $\hat{\mathbf{x}}$ and $\mathbf{x}$. In the same way, the size of η_r are equal to the Hamming distance between the relay sub-codeword in $\hat{\mathbf{x}}$ and $\mathbf{x}$.

Eq. (3.15) can be further simplified to

$$P\left(\hat{\mathbf{x}} \neq \mathbf{x} \middle| \mathbf{H}, \mathbf{x}\right) = Q\left(\sqrt{\frac{E_s}{N_0}\left(2d_b\left|h_1^{(b)}\right|^2 + d_r\sum_{u=1}^{2}\left|h_u^{(r)}\right|^2\right)}\right) \tag{3.16}$$

where d_b, d_r are the sizes of η_b and η_r, respectively.

Since Q function can be replaced by exponential form:

$$Q(x) = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \exp\left(-\frac{x^2}{2\sin^2\theta}\right) d\theta \tag{3.17}$$

we have

$$\begin{aligned} P\left(\hat{\mathbf{x}} \neq \mathbf{x} \middle| \mathbf{H}, \mathbf{x}\right) &= \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \exp\left(-\frac{\frac{E_s}{N_0}\left(2d_b\left|h_1^{(b)}\right|^2 + d_r\sum_{u=1}^{2}\left|h_u^{(r)}\right|^2\right)}{2\sin^2\theta}\right) d\theta \\ &= \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \exp\left(-\frac{E_s d_b\left|h_1^{(b)}\right|^2}{N_0\sin^2\theta}\right)\prod_{u=1}^{2}\exp\left(-\frac{E_s d_r\left|h_u^{(r)}\right|^2}{2N_0\sin^2\theta}\right) d\theta \end{aligned} \tag{3.18}$$

Hence, the pairwise error probability is

$$P\left(\hat{\mathbf{x}} \neq \mathbf{x} \middle| \mathbf{x}\right) = \overline{\frac{1}{\pi}\int_0^{\frac{\pi}{2}} \exp\left(-\frac{E_s d_b\left|h_1^{(b)}\right|^2}{N_0\sin^2\theta}\right)\prod_{u=1}^{2}\exp\left(-\frac{E_s d_r\left|h_u^{(r)}\right|^2}{2N_0\sin^2\theta}\right) d\theta}^{\mathbf{H}} \tag{3.19}$$

The overbar denotes statistical averaging over the random variable. Since channels between the destination and the users are assumed independent, the averaging can be performed separately, thus

$$P\left(\hat{\mathbf{x}} \neq \mathbf{x} \middle| \mathbf{x}\right) = \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \overline{\exp\left(-\frac{E_s d_b \left|h_1^{(b)}\right|^2}{N_0 \sin^2 \theta} \right)}^{\left|h_1^{(b)}\right|} \prod_{u=1}^{2} \overline{\exp\left(-\frac{E_s d_r \left|h_u^{(r)}\right|^2}{2N_0 \sin^2 \theta} \right)}^{\left|h_u^{(r)}\right|} d\theta \quad (3.20)$$

Define a random variable α that is statistically equal to the channel coefficient $\left|h_1^{(b)}\right|$, the first term in (3.20) which averages $\left|h_1^{(b)}\right|$ over a exponential function can be written as

$$\begin{aligned} \overline{\exp\left(-\frac{E_s d_b \left|h_1^{(b)}\right|^2}{N_0 \sin^2 \theta} \right)}^{\left|h_1^{(b)}\right|} &= \overline{\exp\left(-\frac{E_s d_b \alpha^2}{N_0 \sin^2 \theta} \right)}^{\alpha} \\ &= \int_0^{\infty} \exp\left(-\frac{E_s d_b \alpha^2}{N_0 \sin^2 \theta} \right) P_\alpha\left(\alpha\right) d\alpha \end{aligned} \quad (3.21)$$

Now let γ be the instantaneous received SNR (i.e. $\gamma = \alpha^2 E_s / N_0$), we have

$$\overline{\exp\left(-\frac{E_s d_b \alpha^2}{N_0 \sin^2 \theta} \right)}^{\alpha} = \int_0^{\infty} \exp\left(-\frac{d_b \gamma}{\sin^2 \theta} \right) P_\gamma\left(\gamma\right) d\gamma \quad (3.22)$$

The integral above is in the form of a Laplace transform of $P_\gamma\left(\gamma\right)$ and is the moment generating function (MGF) of γ, thus (3.22) can be written as

$$\overline{\exp\left(-\frac{E_s d_b \alpha^2}{N_0 \sin^2 \theta} \right)}^{\alpha} = M_\gamma\left(-\frac{d_b}{\sin^2 \theta} \right) \quad (3.23)$$

In Rayleigh fading channel, the PDF of instantaneous SNR per bit is

$$P_\gamma\left(\gamma\right) = \frac{1}{\bar{\gamma}} \exp\left(-\frac{\gamma}{\bar{\gamma}} \right), \qquad \gamma \geq 0 \quad (3.24)$$

where $\bar{\gamma}$ is the average SNR per bit. The Laplace transform of the PDF is

$$M_\gamma\left(-s\right) = \frac{1}{1 + s\bar{\gamma}}, \qquad s > 0 \quad (3.25)$$

Substituting (3.25) into (3.23), we have

$$\overline{\exp\left(-\frac{E_s d_b \alpha^2}{N_0 \sin^2\theta}\right)^{\alpha}} = \left(1+\frac{d_b \overline{\gamma}}{\sin^2\theta}\right)^{-1} \tag{3.26}$$

In the same way, the other term of (3.20) can be evaluated:

$$\overline{\exp\left(-\frac{E_s d_r \left|h_u^{(r)}\right|^2}{2N_0 \sin^2\theta}\right)^{\left|h_u^{(r)}\right|}} = \overline{\exp\left(-\frac{E_s d_r \alpha^2}{2N_0 \sin^2\theta}\right)^{\alpha}} = \left(1+\frac{d_r \overline{\gamma}}{2\sin^2\theta}\right)^{-1} \tag{3.27}$$

Substituting (3.26), (3.27) into (3.20) and note that the average SNR in different channels may not be the same, thus we separate it by $\overline{\gamma}_1^{(b)}, \overline{\gamma}_1^{(r)}, \overline{\gamma}_2^{(r)}$, which represent the average SNR between user U_1 and destination in broadcast phase, and the average SNR between U_1, U_2 and destination in relay phase, respectively. Thus we have

$$P\left(\hat{\mathbf{x}} \neq \mathbf{x} \middle| \mathbf{x}\right) = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \left(1+\frac{d_b \overline{\gamma}_1^{(b)}}{\sin^2\theta}\right)^{-1} \left(1+\frac{d_r \overline{\gamma}_1^{(r)}}{2\sin^2\theta}\right)^{-1} \left(1+\frac{d_r \overline{\gamma}_2^{(r)}}{2\sin^2\theta}\right)^{-1} d\theta \tag{3.28}$$

From (3.28) it is clear that a diversity order of 3 is achieved as long as $d_b, d_r \neq 0$, that is, the Hamming distances between broadcast, relay sub-codewords and the transmitted sub-codewords are not zero. The diversity is gained by the use of Alamouti space-time code in the relay sub-codeword, and by the independent channel seen in broadcast phase.

3.2.2 Bit and Block Error Rate

A union bound for the BER and FER can be calculated using weight enumerating functions. In traditional approach [24], the first step is finding the first-error-event probability. Assume all-zero path is the correct path, we want to find the probability that a path through the trellis with Hamming distance *d* from the all-zero path is the survivor. The second step is taking the summation of the first-error-event probabilities over all possible Hamming distances. Note the Hamming distance between a path and all-zero path is also the weight of that path. Recall that (3.28) is a function of Hamming distances d_b, d_r and average SNRs $\bar{\gamma}_1^{(b)}, \bar{\gamma}_1^{(r)}, \bar{\gamma}_2^{(r)}$. Now we assume the SNRs are given, (3.28) can be rewritten as a pairwise error probability function of only the Hamming distances

$$P_2\left(d_b, d_r\right) = P\left(\hat{\mathbf{x}} \neq \mathbf{x} \middle| \mathbf{x}, \boldsymbol{\gamma}\right) \tag{3.29}$$

where $\boldsymbol{\gamma}$ is a SNR vector with elements $\bar{\gamma}_1^{(b)}, \bar{\gamma}_1^{(r)}$ and $\bar{\gamma}_2^{(r)}$. Denoting the number of paths that the broadcast and relay sub-codewords have weights d_b, d_r by $a\left(d_b, d_r\right)$, we can bound the first-error-event probability by

$$P_e \leq \sum_{d_1=1}^{\infty} \sum_{d_2=1}^{\infty} a\left(d_b, d_r\right) P_2\left(d_b, d_r\right) \tag{3.30}$$

where on the right-hand side, we have included all paths through the trellis that merge with the all-zero path.

To calculate BER, first define $b\left(d_b, d_r\right)$ as the total number of bit errors in paths that the broadcast and relay sub-codewords have weights d_b, d_r, then the union bound of BER is

$$P_b \leq \sum_{d_b=1}^{\infty} \sum_{d_r=1}^{\infty} b\left(d_b, d_r\right) P_2\left(d_b, d_r\right) \tag{3.31}$$

However, it is shown in [25] that the union bound approach was found to provide quite loose bounds. This is because there is no dominant error event in (3.31), even at high SNR region. Therefore, a modification to this method is proposed by [25] to obtain a much tighter bound. It is done by limiting the conditioned union bound on the bit error probability before averaging over the fading matrix. Thus we need the pairwise error probability conditioned on a given channel, which is given in (3.18). Rewrite (3.18) as

$$P_2(\alpha_1,\alpha_2,\alpha_3,d_b,d_r)=\frac{1}{\pi}\int_0^{\frac{\pi}{2}}\exp\left(-\frac{E_s d_b \alpha_1^2}{N_0\sin^2\theta}\right)\exp\left(-\frac{E_s d_r \alpha_2^2}{2N_0\sin^2\theta}\right)\exp\left(-\frac{E_s d_r \alpha_3^2}{2N_0\sin^2\theta}\right)d\theta \tag{3.32}$$

where $\alpha_1=\left|h_1^{(b)}\right|,\alpha_2=\left|h_1^{(r)}\right|,\alpha_3=\left|h_2^{(r)}\right|$. Then we sum over all possible distances to obtain the bound of bit error probability which is a function of known channel coefficients:

$$P_b(\alpha_1,\alpha_2,\alpha_3)\le\min\left[\frac{1}{2},\sum_{d_b=1}^{\infty}\sum_{d_r=1}^{\infty}b(d_b,d_r)P_2(\alpha_1,\alpha_2,\alpha_3,d_b,d_r)\right] \tag{3.33}$$

The bit error probability is limited to 1/2 because in practice, the maximum error rate using Viterbi decoder is 1/2. By averaging over the channel coefficients, the new bound can be obtained:

$$P_b\le\int_{\alpha_1}\int_{\alpha_2}\int_{\alpha_3}P_b(\alpha_1,\alpha_2,\alpha_3)P_\alpha(\alpha_1)P_\alpha(\alpha_2)P_\alpha(\alpha_3)\,d\alpha_3 d\alpha_2 d\alpha_1 \tag{3.34}$$

A much tighter bound can be obtained this way, but the order of the integrations and the summations can not be exchanged in (3.34) due to the min operator, thus numerical integral is needed to calculate the results. The bounds will be compared with simulation results in Section 3.3.

Frame error rate can be evaluated in similar way. For a data block of K bits, given the event error probability P_e, the FER can be bounded as [26]

$$P_f\le 1-(1-P_e)^K\le KP_e \tag{3.35}$$

The traditional approach gives loose bound for the FER, thus the limit-before-averaging technique is applied. We first calculate the bound of event error probability conditioned on given channel coefficients:

$$P_e(\alpha_1,\alpha_2,\alpha_3)\le\min\left[1,\sum_{d_b=1}^{\infty}\sum_{d_r=1}^{\infty}a(d_b,d_r)P_2(\alpha_1,\alpha_2,\alpha_3,d_b,d_r)\right] \tag{3.36}$$

This time the event error rate is limited to 1 because the maximum error rate is 1 in practice. Apply (3.36) to (3.35) and average over the channel coefficients we get

$$P_f \le 1-\int_{\alpha_1}\int_{\alpha_2}\int_{\alpha_3}\left(1-P_e\left(\alpha_1,\alpha_2,\alpha_3\right)\right)^K P_\alpha\left(\alpha_1\right)P_\alpha\left(\alpha_2\right)P_\alpha\left(\alpha_3\right)d\alpha_3 d\alpha_2 d\alpha_1 \quad (3.37)$$

Again, the above equation needs to be carried out numerically in this case due to the min operator.

3.2.3 Impact of Data Exchange Failure

Now consider the impact to the performance due to data exchange failure. We focus on 2-user case first. Since the transmissions from user U_1 to U_2 and from U_2 to U_1 are on the same frequency band and the same coherence interval, we assume they see equal channels due to the reciprocity theorem [27]. Denoting $P_{f,u}$ as the rate of the data exchange failure between U_1 and U_2, the probability of successfully cooperation between two users can be written as

$$P_C = 1 - P_{f,u} \quad (3.38)$$

Denote the FER of 2-user cooperation with no data exchange failure as $P_{f,2user}$ and the FER which data exchange always fails as $P_{f,nocoop}$. We have shown how to calculate $P_{f,2user}$ in previous two Sections. The calculation of $P_{f,nocoop}$ can be done simply by setting $h_2^{(r)}$ to $h_1^{(r)}$ in (3.18). Thus the overall frame error rate for U_1 data is

$$\begin{aligned} P_f &= P_{f,2user}P_C + P_{f,nocoop}\left(1-P_C\right) \\ &= P_{f,2user}\left(1-P_{f,u}\right) + P_{f,nocoop}P_{f,u} \end{aligned} \quad (3.39)$$

Since $P_{f,2user}$ has diversity order of 3 but $P_{f,nocoop}$ has only 2, at high SNR region, the frame error rate will be dominated by $P_{f,nocoop}$ and (3.39) can be bounded as

$$P_f > P_{f,nocoop} P_{f,u} \tag{3.40}$$

It can be seen that the diversity gain decreases at high SNR region.

Now consider a 4-user scheme. Note that all users have to decode data from others correctly to apply 4×4 OSTBC. Assuming the rates of data exchange failure are equal between all users, the probability of successfully cooperation can be written as

$$P_{C,4} = \left(1 - P_{f,u}\right)^6 \tag{3.41}$$

Thus we have the overall FER in 4-user case as

$$\begin{aligned} P_f &= P_{f,4user} P_{C,4} + P_{f,nocoop}\left(1 - P_{C,4}\right) \\ &= P_{f,4user}\left(1 - P_{f,u}\right)^6 + P_{f,nocoop}\left(1 - \left(1 - P_{f,u}\right)^6\right) \end{aligned} \tag{3.42}$$

where $P_{f,4user}$ is the FER with 4-user coded cooperation and no data exchange failure. The overall FER can be bounded as follow in high SNR region

$$P_f > P_{f,nocoop}\left(1 - \left(1 - P_{f,u}\right)^6\right) \tag{3.43}$$

Again, we can expect the loss in diversity order at high SNR region, what's more, note that $\left(1 - \left(1 - P_{f,u}\right)^6\right)$ is much higher than $P_{f,u}$, the degradation of performance will be larger compared to 2-user case. It will be shown in Fig. 3-4 in next Section.

To this problem, an adaptive scheme can be used to compensate the huge performance loss in 4-user case. It is unnecessary to make all users back to no cooperation mode if some of them still received others' data correctly. Instead, other space-time codes can be applied to the users who is capable for cooperation. For example, if there are only two users successfully exchanged information with each other, an Alamouti space-time code is applied instead of 4×4 OSTBC; if there are three, a 3-user space-time code can be used. For simplicity, consider only using

Alamouti code and 4×4 OSTBC in the adaptive scheme, the probability of frame error becomes

$$P_f = P_{f,4user}P_{C,4} + P_{f,2user}P_{C,2} + P_{f,nocoop}P_{C,0} \tag{3.44}$$

where $P_{C,4}$ is the probability that all four users exchanged information successfully, which is equal to (3.41); $P_{C,2}$ is the probability of any user successfully exchanged data with U_1; $P_{C,0}$ is the probability of no user successfully exchanged data with U_1. They are

$$\begin{aligned} P_{C,0} &= P_{f,u}^3 \\ P_{C,4} &= \left(1-P_{f,u}\right)^6 \\ P_{C,2} &= 1-P_{C,4}-P_{C,0} \end{aligned} \tag{3.45}$$

respectively. Thus (3.44) can be written as

$$P_f = P_{f,4user}\left(1-P_{f,u}\right)^6 + P_{f,2user}\left(1-\left(1-P_{f,u}\right)^6 - P_{f,u}^3\right) + P_{f,nocoop}P_{f,u}^3 \tag{3.46}$$

The FER at high SNR region can be bounded as

$$P_f \approx P_{f,nocoop}P_{f,u}^3 \tag{3.47}$$

Note the difference between (3.43) and (3.47). The probability that users go back to no cooperation mode is much lower comparing to the probability in non-adaptive scheme, thus provides a better performance. Simulations of the adaptive scheme will be presented in Fig. 3-5 in next Section.

3.3 Computer Simulations

In this Section we simulate the proposed space-time coded cooperation protocols and compare with other protocols mentioned in Chapter 2. All systems are with equal code rate *R* and hence equal data rate. We use a rate-1/4 base code with generator [15

17 13 15] used by [18]. For the conventional coded cooperation and the proposed ST-coded cooperation, the puncture patterns are [1 1 0 0] for the broadcast sub-codeword, [0 0 1 1] for the relay sub-codeword. Binary phase-shift keying (BPSK) modulation is used. The frame size is 260 bits. We consider the case that both nodes communicate with the same destination. Each user and the destination are equipped with a single antenna. The channel is slow Rayleigh fading channel with AWGN.

Fig. 3-3 plots the analytical bound and simulation results of FER as a function of the transmit SNR. Similar results could be obtained in terms of BER. Perfect inter-user channel is assumed so there is no data exchange failure. Investigating the FER at high SNR region and comparing with the analytical bounds (dash line with diamonds), it is clear that the proposed ST-coded cooperation (line with diamonds) achieves full diversity as we expected in Section 3.2. Comparing the performance of the proposed protocol with conventional coded cooperation (line with squares) at FER of 10^{-3}, the proposed ST-coded cooperation using Alamouti code achieves nearly 4dB gain over the conventional coded cooperation. If more users join the cooperation, higher order space-time code can be used to gain more diversity. For the case of four users (line with down triangles), additional gain of 2.5dB is achieved comparing to 2-user case.

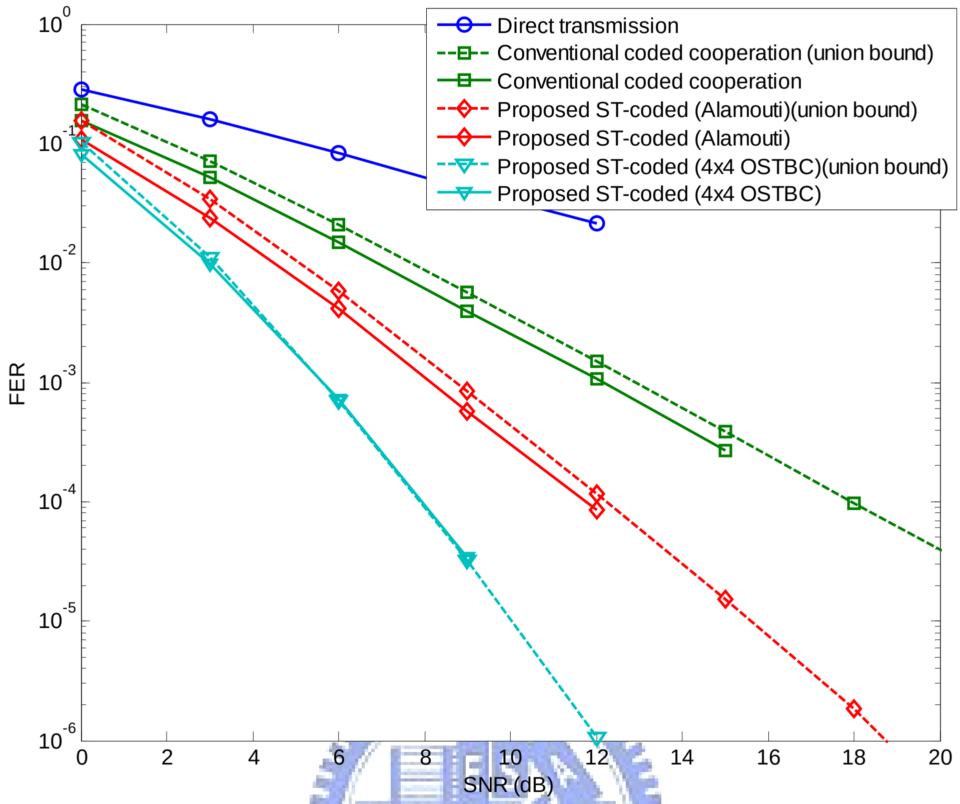

Fig. 3-3. Simulations and bounds of frame error rate (FER) in ST-coded cooperation. Equal uplink SNR, base code [15 17 13 15]

Fig. 3-4 shows the impact of data exchange failure to the overall FER. We assume that the rate of data exchange failure between users is 0.1. Analytical bounds based on Section 3.2.3 are shown in the figure. It can be seen that the simulation result is consistent with the analytical bound.

Lines with diamonds and down triangles are the same as the simulation results in Fig. 3-3, that is, ST-coded cooperation with no data exchange failure. Diamonds and down triangles with no lines are the simulation result when data exchange failure is considered. From the figure we can see that the proposed ST-coded cooperation protocols lose their diversity at high SNR region. At that region, they seem to have diversity of order 2 because the two sub-codewords still experience independent channels in broadcast and relay phase.

Despite the loss in diversity, the proposed 2-user ST-coded cooperation still has advantages over the conventional one. But the performance of 4-user scheme is even worse than 2-user case. As mentioned in Section 3.2.3, cooperation among four users with imperfect inter-user channel experiences severe performance degradation since the data exchange between 4 users is hardly all successful. To this problem, we'll show in Fig. 3-5 the performance improvements of 4-user cooperation with adaptive protocol.

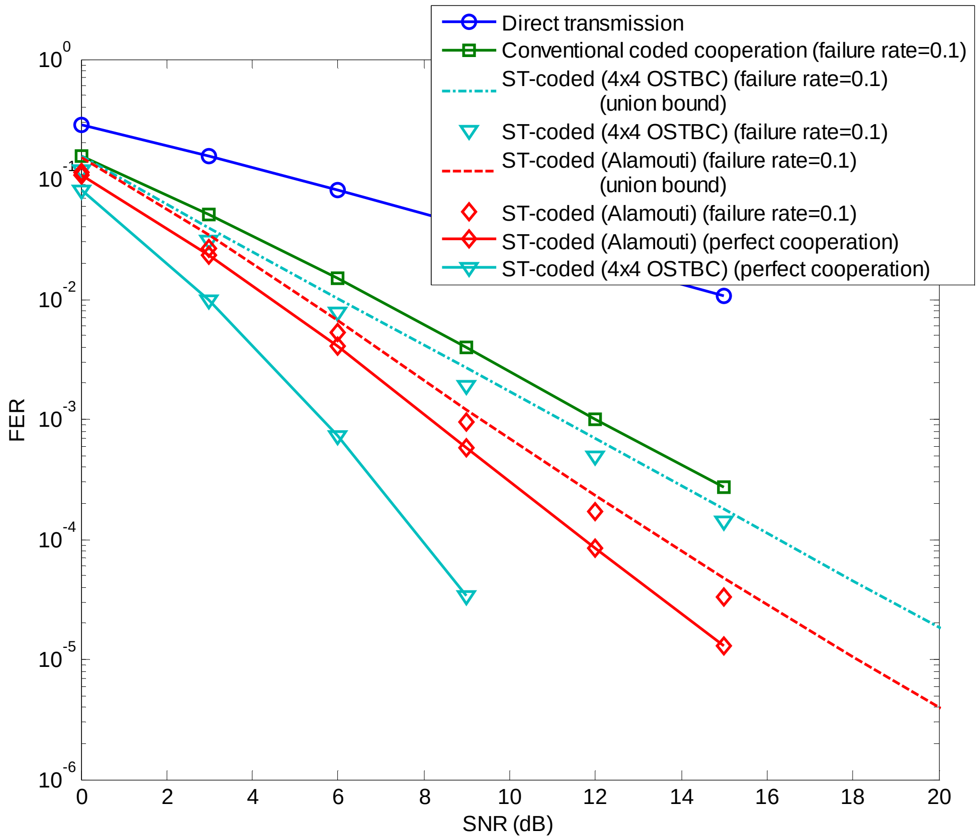

Fig. 3-4. Frame error rate (FER) with imperfect inter-user channels. Equal uplink SNR, generator [15 17 13 15], inter-user FER=0.1

Comparing the performance of 4-user case (down triangles) in Fig. 3-4 and Fig. 3-5, we can see clearly the improvement by applying adaptive protocol mentioned in

Section 3.2.3. The algorithm assures the performance under 4-user scheme not worse than the case when only 2-user Alamouti code is applied. Meanwhile, it still benefits from the use of 4×4 OSTBC when the data is exchanged successfully. For the case of data exchange failure rate=0.1, the average probability of 4-user cooperation is about 0.531, 2-user cooperation is about 0.468 and the probability of no cooperation is only 0.001.

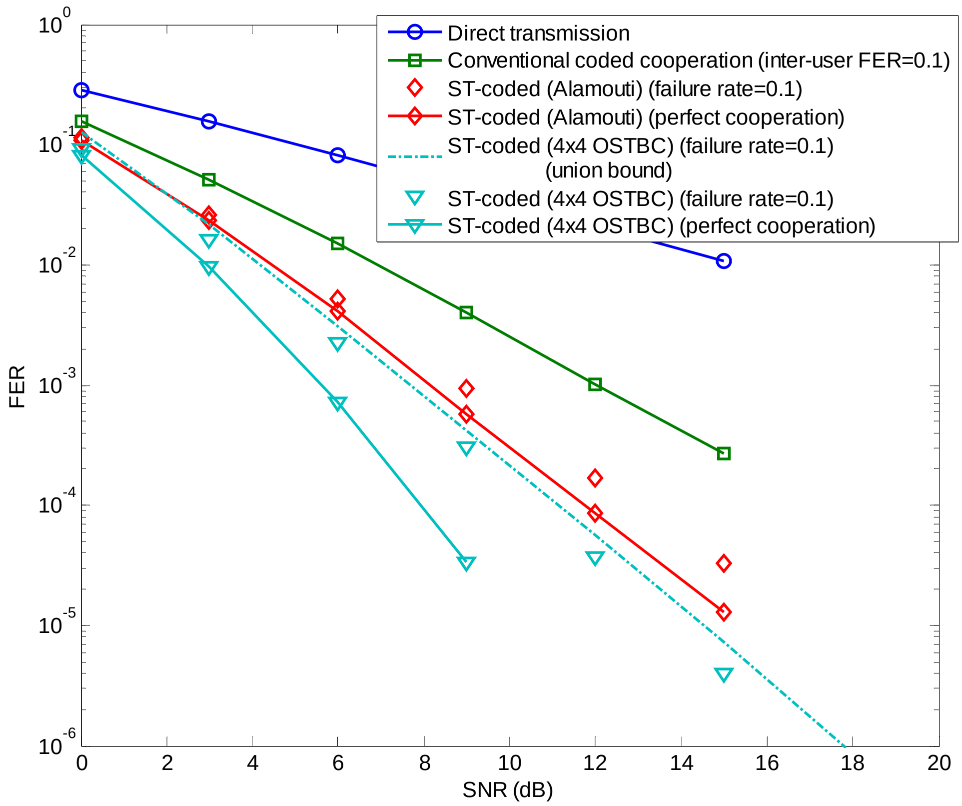

Fig. 3-5. Frame error rate (FER) with imperfect inter-user channels. Equal uplink SNR, generator [15 17 13 15], inter-user FER=0.1, adaptive algorithm

3.4 Summary

In this Chapter we give a detailed description of the proposed space-time coded cooperation protocol. We show the diversity gain in the case of 2-user coded cooperation with Alamouti space-time code by evaluating the pairwise error probability. Extension to other space-time code is straightforward. The proposed protocol can utilizes full diversity gain from the used space-time code. Tight union bounds for the BER as well as the FER are given by using the weight enumerating function and the limit-before-averaging technique. Both analytical and simulation results have been shown to prove the performance gain. We also consider the impact of imperfect inter-user channel to the proposed protocol and give an adaptive way to reduce the performance loss.

Chapter 4

Spectrally Efficient Multi-user Coded Cooperation using Code Partitioning

In this Chapter we introduce the second modification of coded cooperation: code-partition (CP) coded cooperation. The proposed protocols still achieve great system reliability while maintaining equal spectral efficiency as non-cooperation protocols. The CP-coded cooperation has similar performance to the protocols in Chapter 3. Besides, it has the advantages of less complexity and lower requirements for inter-user channel.

4.1 Protocols of CP-Coded Cooperation

Look at the performance bound in eq. (3.28), we can see that the diversity gain comes from the independent channels of user U_1 in the broadcast phase and relay phase. Further more, additional diversity is gained from U_2 channel by using Alamouti space-time code in relay phase. Thus the total diversity gain compared to direct transmission protocol is $2+1=3$. There is another factor in (3.28) that contributes to the overall performance: the Hamming weight of the sub-codewords

(d_b, d_r). As long as d_b, d_r are not zeros, diversity order of 3 is achieved. Since that, it is natural to think that if more than two sub-codes are separated from the base code and transmitted by independent channels, higher diversity order can be achieved without the help of space-time code. The second type of coded cooperation is proposed based on these principles.

4.1.1 Code Structure and System Model

The main idea of CP-coded cooperation is to let every sub-codeword of a base codeword transmitted through independent channels. Thus the number of sub-codes depends on the number of independent channels we have.

For 2-user case, we partition the base code into three sub-codes, one is for the source and the other two are for the relays (See Fig. 4-1). The sub-codeword for the source is still called broadcast sub-codeword, but for the other two sub-codewords for the relays, we tag them with numbers (ex. 1st, 2nd relay sub-codeword, etc.), each relay sub-codewords are transmitted by different relays. The code structure is shown in Fig. 4-2.

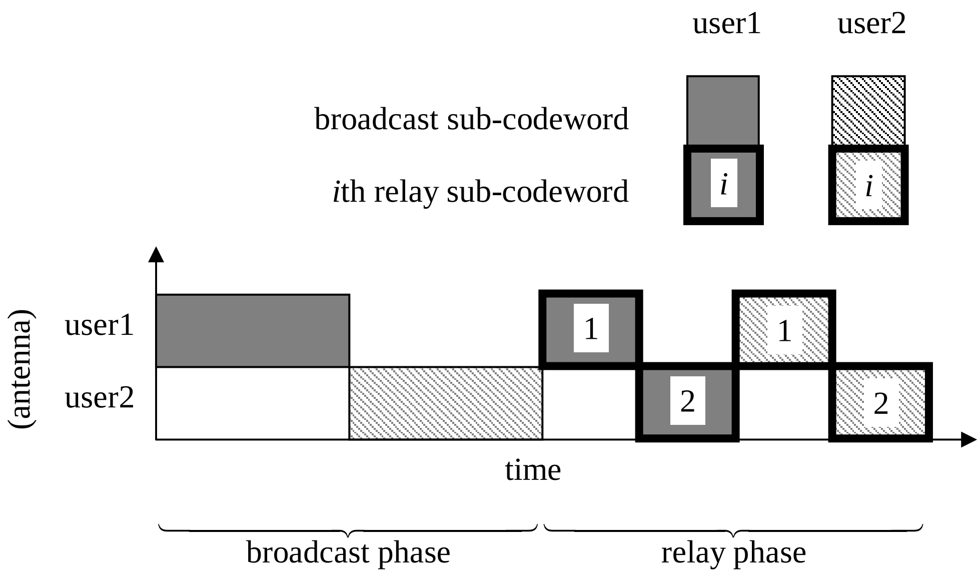

Fig. 4-1. Channel use of CP-coded cooperation with two users (TDMA)

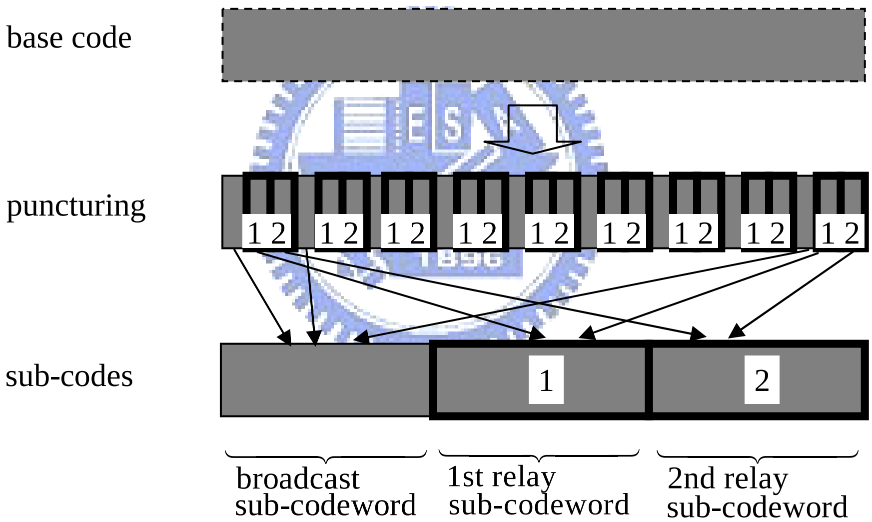

Fig. 4-2. Structure of codeword in CP-coded cooperation (2-user case)

Note that the code-structure is different from the one in Chapter 3 and [18]. We expect a diversity order of 3 is gained for each user data since the data is partitioned into three parts. The analysis will be given in Section 4.2.

Assume quasi-static fading channel similar to the channel used in ST-coded cooperation, the received signal at the destination in broadcast phase is

$$\begin{cases} \mathbf{r}_1^{(b)} = h_1^{(b)} \sqrt{E_s} \mathbf{c}_1^{(b)} + \mathbf{n}_1^{(b)} \\ \mathbf{r}_2^{(b)} = h_2^{(b)} \sqrt{E_s} \mathbf{c}_2^{(b)} + \mathbf{n}_2^{(b)} \end{cases} \tag{4.1}$$

where $\mathbf{c}_1^{(b)}, \mathbf{c}_2^{(b)}$ denotes the broadcast sub-codewords of user U_1 and U_2, respectively.

At relay phase, the received signal can be written as

$$\begin{cases} \mathbf{r}_1^{(ri)} = h_i^{(r)} \sqrt{E_s} \mathbf{c}_1^{(ri)} + \mathbf{n}_1^{(ri)} \\ \mathbf{r}_2^{(ri)} = h_i^{(r)} \sqrt{E_s} \mathbf{c}_2^{(ri)} + \mathbf{n}_2^{(ri)} \end{cases} \tag{4.2}$$

where $i = \{1, 2\}$, $\mathbf{c}_1^{(ri)}, \mathbf{c}_2^{(ri)}$ denotes the *i*th relay sub-codeword of user U_1 and U_2, respectively. Assuming ML detection, the detected signal for user U_1 is

$$\begin{cases} \mathbf{z}_1^{(b)} = \sqrt{E_s} \left| h_1^{(b)} \right|^2 \mathbf{c}_1^{(b)} + \tilde{\mathbf{n}}_1^{(b)} \\ \mathbf{z}_1^{(r1)} = \sqrt{E_s} \left| h_1^{(r)} \right|^2 \mathbf{c}_1^{(r1)} + \tilde{\mathbf{n}}_1^{(r1)} \\ \mathbf{z}_1^{(r2)} = \sqrt{E_s} \left| h_2^{(r)} \right|^2 \mathbf{c}_1^{(r2)} + \tilde{\mathbf{n}}_1^{(r2)} \end{cases} \tag{4.3}$$

where $\tilde{\mathbf{n}}_1^{(b)} = \left(h_1^{(b)}\right)^* \mathbf{n}_1^{(b)}, \tilde{\mathbf{n}}_1^{(r1)} = \left(h_1^{(r)}\right)^* \mathbf{n}_1^{(r1)}$ and $\tilde{\mathbf{n}}_1^{(r2)} = \left(h_1^{(r)}\right)^* \mathbf{n}_1^{(r2)}$. These detected signals are combined at the destination to rebuild the base codeword. Then the destination decodes the base codeword using Viterbi decoder.

This protocol can be easily extended to cases with more users, for example, a 4-user protocol is shown below.

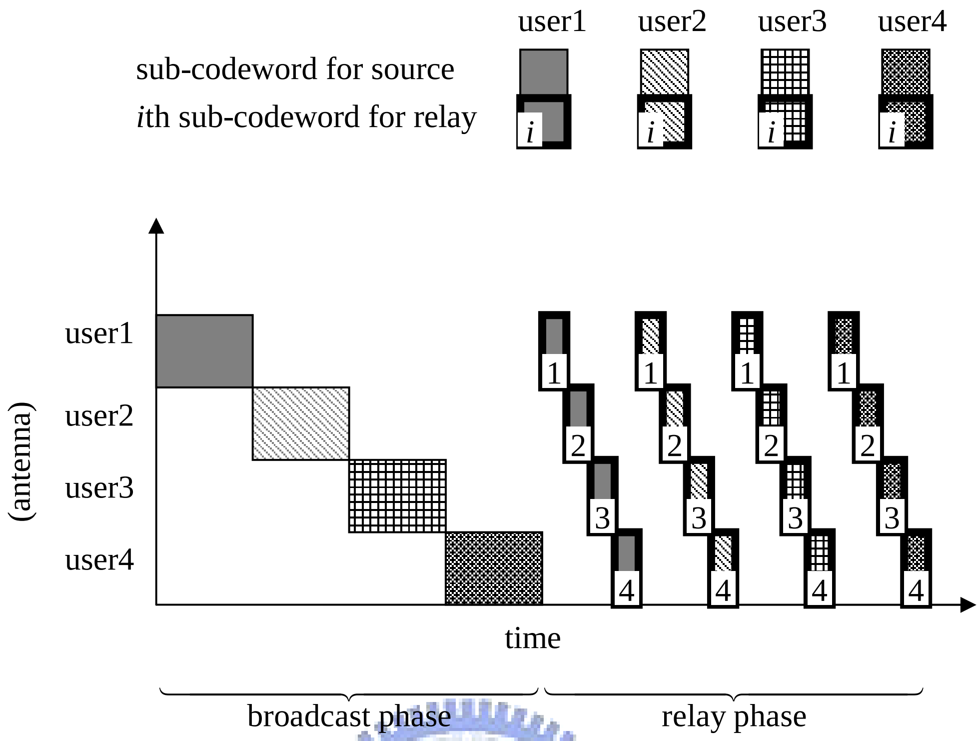

Fig. 4-3. Channel use of CP-coded cooperation with four users (TDMA)

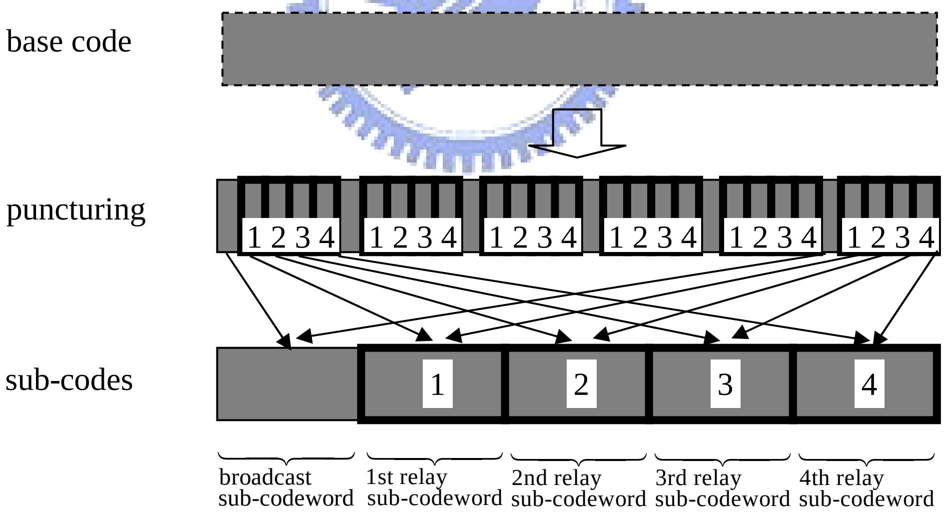

Fig. 4-4. Structure of codeword in CP-coded cooperation (4-user case)

This time the base code is partitioned into five sub-codes, the first one is for broadcasting and the other four is for relaying (four relays). Note that the four relay sub-codewords are still transmitted by different users.

The main difference between ST-coded cooperation and CP-coded cooperation is the number of sub-codes used. In ST-coded cooperation, the number is fixed to two sub-codes: one for broadcasting and one for relaying. It is the space-time code to change with different number of users. However, CP-coded cooperation uses different number of sub-codes to match the number of users participated in the cooperation.

4.1.2 Case of Information Exchange Failure

CP-coded cooperation is more flexible than ST-coded cooperation to the case of data exchange failure; the latter needs all users to exchange data successfully to apply space-time code, while the former doesn't. For example, in a 4-user scheme, if error occurs in the exchange of data between user U_1 and U_2, they can't cooperate with each other so the diversity gain will be lower due to the loss of one independent channel. However, they can still cooperate with U_3 and U_4, so it has no effect to the error rates of U_3 and U_4 data.

There are two kinds of reactions for a user when decoding error occurs: notify other users (method 1) or do nothing (method 2). More specific description will be given in the following paragraph.

Method 1

The first method is the similar to the method used in ST-coded cooperation. Say, in a 4-user scheme, if user U_1 fails to decode the information from U_2, it will send a notification signal to all other users. However, there is no need for all users to go back to no cooperation mode (like we do in 3.1.2), instead, only U_2 takes this notification signal. After receiving the notification, U_1 and U_2 will go back to no cooperation mode and transmit the corresponding sub-codewords by themselves. A figure plotted the channel use under above case is given in Fig. 4-5. From the figure we can see that the sub-codewords (2nd sub-codeword of U_1 and 1st sub-codeword of U_2) which should be sent by other users are now sent by its owners themselves (U_1,U_2, respectively). However, there's no change for U_3 and U_4 data. Thus full diversity order can still be achieved for these two users, while diversity of order one will be lost for U_1 and U_2 data.

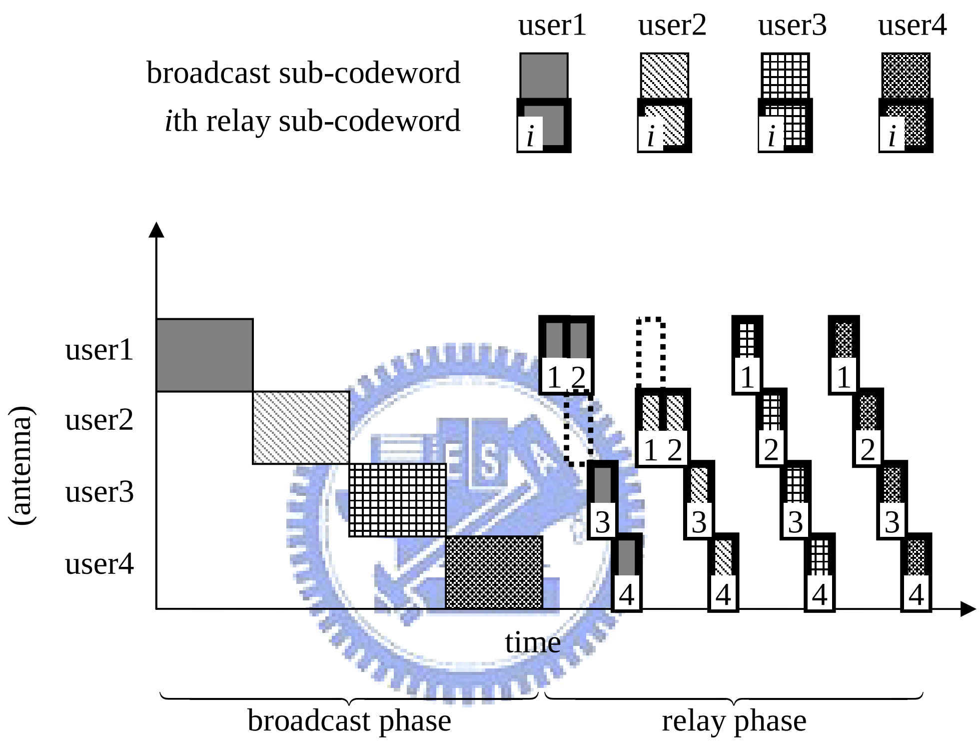

Fig. 4-5. Channel use of 4-user CP-coded cooperation with bad user1 – user2 link, method 1

Method 2

The second method makes slightly modification to the first one. Since a long base codeword is partitioned into several sub-codewords, it may still be okay for the destination to decode if some of the sub-codewords are lost. That is, if a user fails to decode the information from other user in broadcast phase, it does nothing in the corresponding relay phase. The destination will hence lose that part of the base codeword, but data can still be decoded from the remaining parts. This method will lead to some loss in not only diversity gain, but also coding gain; however, the major advantage is that users don't need to notify others if decoding error occurs, hence the system complexity is less than the first method. Fig. 4-6 gives the example when data exchange failure occurs between U_1 and U_2. Note that compared with Fig. 4-5, some sub-codewords are discarded instead of transmitted by their owner.

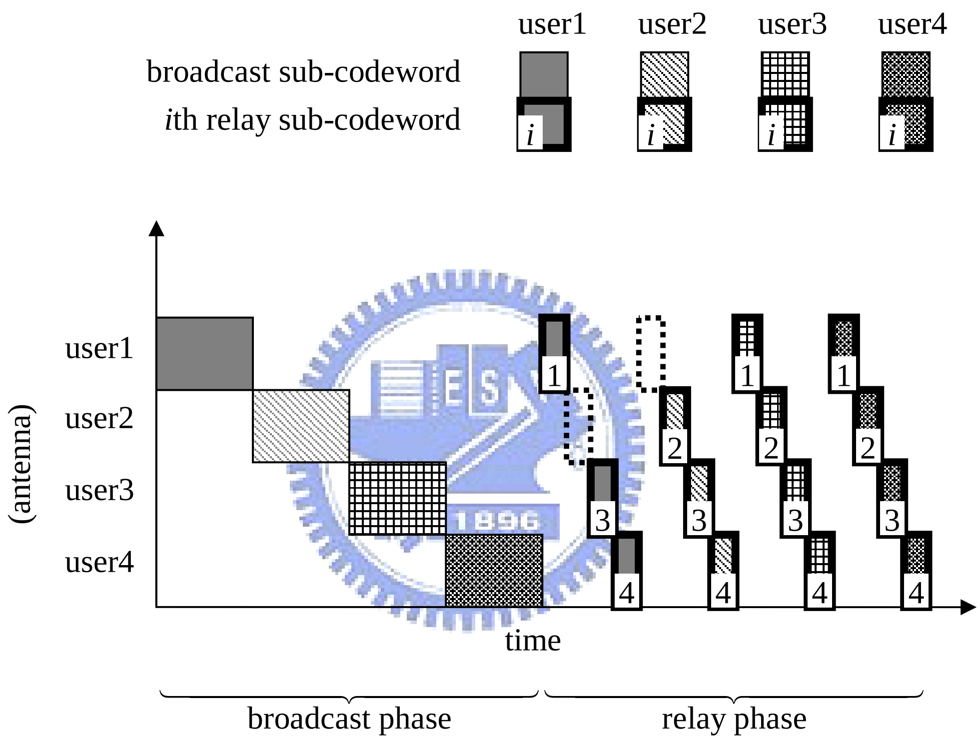

Fig. 4-6. Channel use of 4-user CP-coded cooperation with bad user1 – user2 link, method 2

4.2 Performance Bounds of CP-Coded Cooperation

Using the same technique in Section 3.2, we find the diversity gain through the evaluation of the pairwise error probability of 2-user CP-coded cooperation. Let

N_b, N_{r1}, N_{r2} denote the length of the broadcast, 1st relay and 2nd relay sub-codewords in Fig. 4-2. From (4.3) we can rewrite the detected signal for user U_1 in symbol-wise form:

$$\begin{cases} z_i^{(b)} = \sqrt{E_s}\left|h_1^{(b)}\right|^2 c_i^{(b)} + \tilde{n}_i^{(b)}, & i = 1,\cdots,N_b \\ z_i^{(r1)} = \sqrt{E_s}\left|h_1^{(r)}\right|^2 c_i^{(r1)} + \tilde{n}_i^{(r1)}, & i = 1,\cdots,N_{r1} \\ z_i^{(r2)} = \sqrt{E_s}\left|h_2^{(r)}\right|^2 c_i^{(r2)} + \tilde{n}_i^{(r2)}, & i = 1,\cdots,N_{r2} \end{cases} \tag{4.4}$$

where the index *i* denotes the *i*th element of the corresponding vector in (4.3). Thus the base codeword after combining is

$$\mathbf{z} = \begin{cases} \mathbf{z}^{(b)}, & \text{when } i \in \chi_b \\ \mathbf{z}^{(r1)}, & \text{when } i \in \chi_{r1} \\ \mathbf{z}^{(r2)}, & \text{when } i \in \chi_{r2} \end{cases} \tag{4.5}$$

Note that $\chi_b, \chi_{r1}, \chi_{r2}$ are the sets of indexes which belong to the broadcast, 1st relay and 2nd relay sub-codewords, respectively.

The received SNR in the three phases are

$$\begin{cases} \gamma_b = \dfrac{E_s\left|h_1^{(b)}\right|^2}{N_0} & \text{broadcast sub-codeword} \\ \gamma_{r1} = \dfrac{E_s\left|h_1^{(r)}\right|^2}{N_0} & \text{1st relay sub-codeword} \\ \gamma_{r2} = \dfrac{E_s\left|h_2^{(r)}\right|^2}{N_0} & \text{2nd relay sub-codeword} \end{cases} \tag{4.6}$$

Hence the error probability of each symbol in (4.5) can be expressed as

$$P_s(i) \approx \begin{cases} \tilde{N}_e Q\left(\sqrt{\dfrac{\gamma_b d_{\min}^2}{2}}\right), & \text{when } i \in \chi_b \\ \tilde{N}_e Q\left(\sqrt{\dfrac{\gamma_{r1} d_{\min}^2}{2}}\right), & \text{when } i \in \chi_{r1} \\ \tilde{N}_e Q\left(\sqrt{\dfrac{\gamma_{r2} d_{\min}^2}{2}}\right), & \text{when } i \in \chi_{r2} \end{cases} \tag{4.7}$$

Again we assume BPSK modulation, thus $\widetilde{N}_e = 1$ and $d_{\min}^2 = 4$. Define user U_1's codeword as $\mathbf{x} = \begin{bmatrix} x_1 & x_2 & \cdots & x_N \end{bmatrix}$. The symbol error probability that z_i is decided as an erroneous symbol $\widehat{x}_i \neq x_i$ conditioned on known channel is

$$P\left(z_i \to \widehat{x}_i, \widehat{x}_i \neq x_i \,\middle|\, \mathbf{h}, x_i\right) \approx \begin{cases} Q\left(\sqrt{\dfrac{2E_s}{N_0}\left|h_1^{(b)}\right|^2}\right) & \text{when } i \in \chi_b \\ Q\left(\sqrt{\dfrac{2E_s}{N_0}\left|h_1^{(r)}\right|^2}\right) & \text{when } i \in \chi_{r1} \\ Q\left(\sqrt{\dfrac{2E_s}{N_0}\left|h_2^{(r)}\right|^2}\right) & \text{when } i \in \chi_{r2} \end{cases} \quad (4.8)$$

where the channel vector is defined as $\mathbf{h} = \begin{bmatrix} h_1^{(b)} & h_1^{(r)} & h_2^{(r)} \end{bmatrix}^T$.

In the end, we can write the pairwise error probability of the received codeword conditioned on known channel in the form similar to (3.15)

$$P\left(\hat{\mathbf{x}} \neq \mathbf{x} \,\middle|\, \mathbf{h}, \mathbf{x}\right) = Q\left(\sqrt{\frac{2E_s}{N_0}\left(\sum_{i \in \eta_b}\left|h_1^{(b)}\right|^2 + \sum_{i \in \eta_{r1}}\left|h_1^{(r)}\right|^2 + \sum_{i \in \eta_{r2}}\left|h_2^{(r)}\right|^2\right)}\right) \quad (4.9)$$

where η_b, η_{r1} and η_{r2} are the sub sets of $i \in \chi_b$, $i \in \chi_{r1}$ and $i \in \chi_{r2}$, respectively, that $\widehat{x}_i \neq x_i$. Thus we can remove the summations in (4.9) to simplify it as we have done to (3.15):

$$P\left(\hat{\mathbf{x}} \neq \mathbf{x} \,\middle|\, \mathbf{h}, \mathbf{x}\right) = Q\left(\sqrt{\frac{2E_s}{N_0}\left(d_b\left|h_1^{(b)}\right|^2 + d_{r1}\left|h_1^{(r)}\right|^2 + d_{r2}\left|h_2^{(r)}\right|^2\right)}\right) \quad (4.10)$$

where d_b, d_{r1}, d_{r2} are the sizes of η_b, η_{r1} and η_{r2}, respectively.

Eq. (4.10) is in similar form to (3.16), so we can apply the same simplifying technique used in Section 3.2.1 to evaluate the pairwise error probability. Thus we have

$$P\left(\hat{\mathbf{x}} \neq \mathbf{x} | \mathbf{x}\right) = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \left(1+\frac{d_b \overline{\gamma}_1^{(b)}}{\sin^2\theta}\right)^{-1} \left(1+\frac{d_{r1} \overline{\gamma}_1^{(r)}}{\sin^2\theta}\right)^{-1} \left(1+\frac{d_{r2} \overline{\gamma}_2^{(r)}}{\sin^2\theta}\right)^{-1} d\theta \qquad (4.11)$$

where $\overline{\gamma}_1^{(b)}, \overline{\gamma}_1^{(r)}$ and $\overline{\gamma}_2^{(r)}$ represent the average SNR between user U_1 and destination at broadcast phase and between user U_1, U_2 and destination at relay phase, respectively. It can be seen that when all of the distance parameters d_b, d_{r1} and d_{r2} are not zeros, diversity order of 3 can be achieved.

Diversity of more user case can be easily proved by the same method in this Section. For example, a 4-user scheme with base code partitioned into five parts has diversity order of 5 since it utilized the independent channels $h_1^{(b)}, h_1^{(r)}, h_2^{(r)}, h_3^{(r)}$ and $h_4^{(r)}$ (for U_1).

Union bounds for BER and FER of these protocols can be calculated by the same method in Section 3.2.2, so we skip this part and look directly to the case of data exchange failure.

4.2.1 Impact of Data Exchange Failure

We analyze the impact of imperfect inter-user channel to the overall performance of user U_1 data. Both method 1 and method 2 in Section 4.1.2 will be considered.

Method 1

Denote $P_{f,u}$ as the rate of data exchange failure between users. In 2-user case, the probability of successful cooperation is $\left(1-P_{f,u}\right)$ and the probability of going back to no cooperation mode is $P_{f,u}$. Thus the overall performance can be written as

$$P_f = P_{f,2user}\left(1-P_{f,u}\right) + P_{f,nocoop}P_{f,u} \qquad (4.12)$$

where $P_{f,2user}$ is the FER of 2-user CP-coded cooperation, which can be calculated from (4.11); $P_{f,nocoop}$ is the FER when no cooperation is performed, it can be calculated by replacing $h_2^{(r)}$ in (4.10) with $h_1^{(r)}$, since the sub-codeword is now sent by U_1 itself. Evaluating the pairwise error probability under above situation, we have

$$P_{2,nocoop}\left(\hat{\mathbf{x}} \neq \mathbf{x} \middle| \mathbf{x}\right) = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \left(1 + \frac{d_b \overline{\gamma}_1^{(b)}}{\sin^2\theta}\right)^{-1} \left(1 + \frac{(d_{r1} + d_{r2})\overline{\gamma}_1^{(r)}}{\sin^2\theta}\right)^{-1} d\theta \qquad (4.13)$$

Note that the diversity is lost comparing with (4.11). Also note that (4.12) is in similar form to (3.39), thus loss of diversity to the overall FER is expected at high SNR.

In 4-user case, full diversity can be achieved for user U_1 data if and only if the links between it and other three users are available. Thus the probability of 4-user cooperation is $\left(1 - P_{f,u}\right)^3$. If one of the users doesn't decode successfully, the situation becomes 3-user cooperation scheme and the probability of this condition is $3P_{f,u}\left(1 - P_{f,u}\right)^2$. Also, the probability of 2-user cooperation and no cooperation is $3P_{f,u}^2\left(1 - P_{f,u}\right)$ and $P_{f,u}^3$, respectively. We denote the FER under 3-user, 2-user and no cooperation as $P_{f,3user}$, $P_{f,2user}$ and $P_{f,nocoop}$, respectively. It can be calculated in the same way as $P_{f,nocoop}$ in 2-user case described above. The overall FER at the destination is

$$\begin{aligned} P_f &= P_{f,4user}\left(1 - P_{f,u}\right)^3 + 3P_{f,3user}P_{f,u}\left(1 - P_{f,u}\right)^2 \\ &\quad + 3P_{f,2user}P_{f,u}^2\left(1 - P_{f,u}\right) + P_{f,nocoop}P_{f,u}^3 \end{aligned} \qquad (4.14)$$

Method 2

The only difference between method 1 and method 2 is the value of

$P_{f,3user}, P_{f,2user}$ and $P_{f,nocoop}$; since some of the codewords are discarded in method 2, it will lose some coding gain. The overall FER is calculated using the same equations as method 1, that is, (4.12) for 2-user case and (4.14) for 4-user case.

Consider 2-user case for instance, $P_{f,nocoop}$ is calculated simply by setting the term $\left(1+\frac{d_{r2}\overline{\gamma}_2^{(r)}}{\sin^2\theta}\right)^{-1}$ in (4.11) to 1, because the 2nd sub-codeword is discarded. Thus we have

$$P_{2,nocoop}\left(\mathbf{x}\rightarrow\hat{\mathbf{x}}\right)=\frac{1}{\pi}\int_0^{\frac{\pi}{2}}\left(1+\frac{d_b\overline{\gamma}_1^{(b)}}{\sin^2\theta}\right)^{-1}\left(1+\frac{d_{r1}\overline{\gamma}_1^{(r)}}{\sin^2\theta}\right)^{-1}d\theta \qquad (4.15)$$

Note the difference between (4.13) and (4.15). The distance value in the second term is changed from $\left(d_{r1}+d_{r2}\right)$ to d_{r1}, this is because the sub-codeword for U_2 is discarded instead of transmitted by U_1.

The difference between (4.13) and (4.15) implies loss of coding gain when method 2 is used, but the diversity gain is preserved.

4.3 Computer Simulations

We now simulate the proposed CP-coded cooperation protocols and compare them with the performance bounds. Two base codes will be used: [15 17 13 15] (rate-1/4) and [15 17 13 15 13 17] (rate-1/6). The constraint length is 4 and the frame size is 260 bits. 2-user and 4-user cases will be considered; all users are equipped with single antenna and are communicating with the same destination. To isolate the diversity gain from the cooperation, the destination is equipped with only one antenna. However, more antennas can be used to further enhance the system reliability.

Fig. 4-7 shows the FER of CP-coded cooperation using 1/6 code. The puncturing pattern is [1 1 0 0 0 0] for the broadcast sub-code. For 2-user case, the puncturing patterns for the 1st and 2nd relay sub-codes are [0 0 1 1 0 0], [0 0 0 0 1 1], respectively; for 4-user case, they are [0 0 1 0 0 0], [0 0 0 1 0 0], [0 0 0 0 1 0], [0 0 0 0 0 1], respectively.

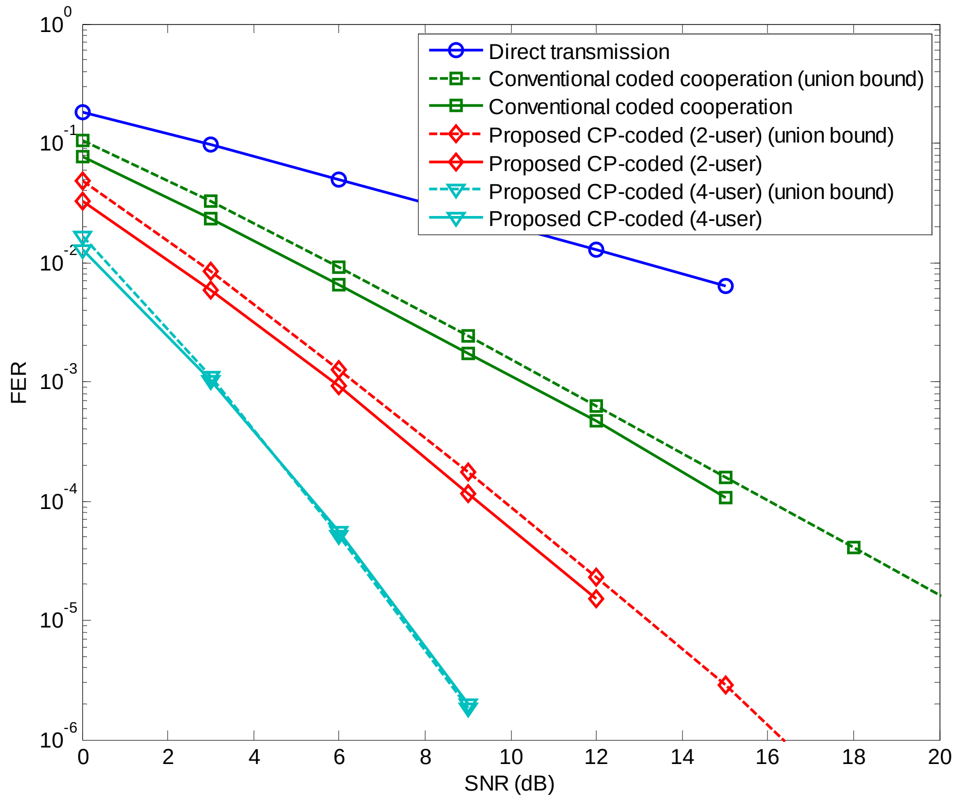

Fig. 4-7. Simulations and bounds of frame error rate (FER) in CP-coded cooperation. Equal uplink SNR, base code [15 17 13 15 13 17]

Comparing with the performance bounds evaluated in Section 4.2 (dotted line with diamonds), we can see that 2-user CP-coded cooperation is consistent with the bound and achieves diversity of order 3. It also holds for 4-user case (line and dotted line with down-triangles). At FER of 10^{-3}, 2-user case (line with diamonds) has nearly 4dB margin comparing to conventional coded cooperation (line with squares) proposed by [18], and 3dB more is gained by using 4-user code partition. Note that all protocols in Fig. 4-7 have equal data rate, equal spectral efficiency and equal power consumption to single user case (line with circles).

Fig. 4-8 uses the same cooperative protocols as Fig. 4-7, but with a shorter base code: [15 17 13 15]. The puncturing pattern of the broadcast sub-code is [1 1 0 0]. For 2-user case, they are [0 0 1 0] and [0 0 0 1] for 1st relay and 2nd relay sub-code; for 4-user case, they are [0 0 1 0 0 0 0 0], [0 0 0 1 0 0 0 0], [0 0 0 0 0 0 1 0] and [0 0 0 0 0 0 0 1].

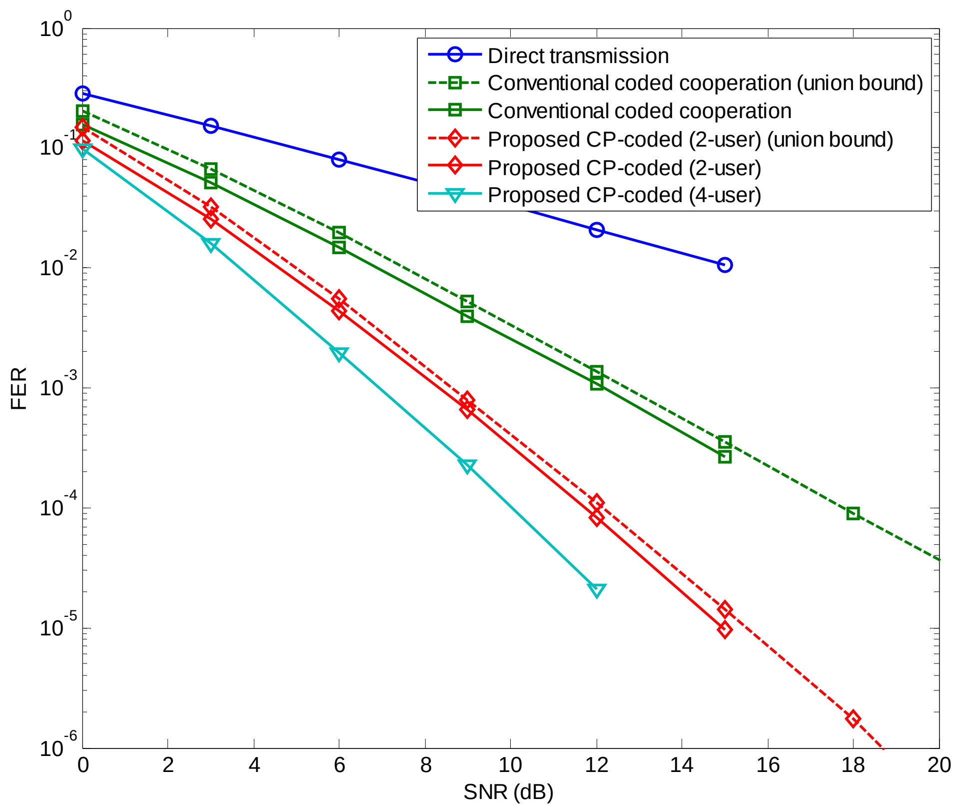

Fig. 4-8. Simulations and bounds of frame error rate (FER) in CP-coded cooperation. Equal uplink SNR, base code [15 17 13 15]

Note that the base code is equal to the base code used in Chapter 3, so this figure gives a comparison of performance between CP- and ST-coded cooperation (Fig. 3-3).

From the figure we can see that in 2-user case (line with diamonds), both protocols have similar performances. But in 4-user case (line with down triangles), the performance gain using CP-coded cooperation is not that significant compared to ST-coded cooperation. It is due to the fact that applying code partitioning on a short base code will generate sub-codes with very short code length, which leads to small distances between codewords (Note that the period of the puncturing pattern in 4-user case is made twice longer to separate the four relay sub-codes). Look closer to the pairwise error probability in (4.11), small distance parameters d_b, d_{r1}, d_{r2} will widen the low-SNR-effect region of the resulting FER expressions, that means although full diversity can still be gained, it is only at higher SNR region.

Fig. 4-9 shows the performance degradation in case of data exchange failure. Rate-1/6 base code is used. The rate of data exchange failure is set to 0.1 and we use method 1 for the relay reaction. From the figure it is clear that simulation result of both 2-user (diamonds) and 4-user (down triangles) cases matches its union bounds (dash line and dash line with dots), which is calculated using the formulas in Section 4.2.1. Comparing the simulation result with perfect cooperation case (line with diamonds for 2-user; line with down triangles for 4-user), we can see the performance degradation due to the diversity loss, but even with high failure rate, CP-coded cooperation still has approximately 4dB and 7dB margin for 2-user and 4-user case, respectively, comparing to the conventional coded cooperation. (at overall FER of 10^{-3})

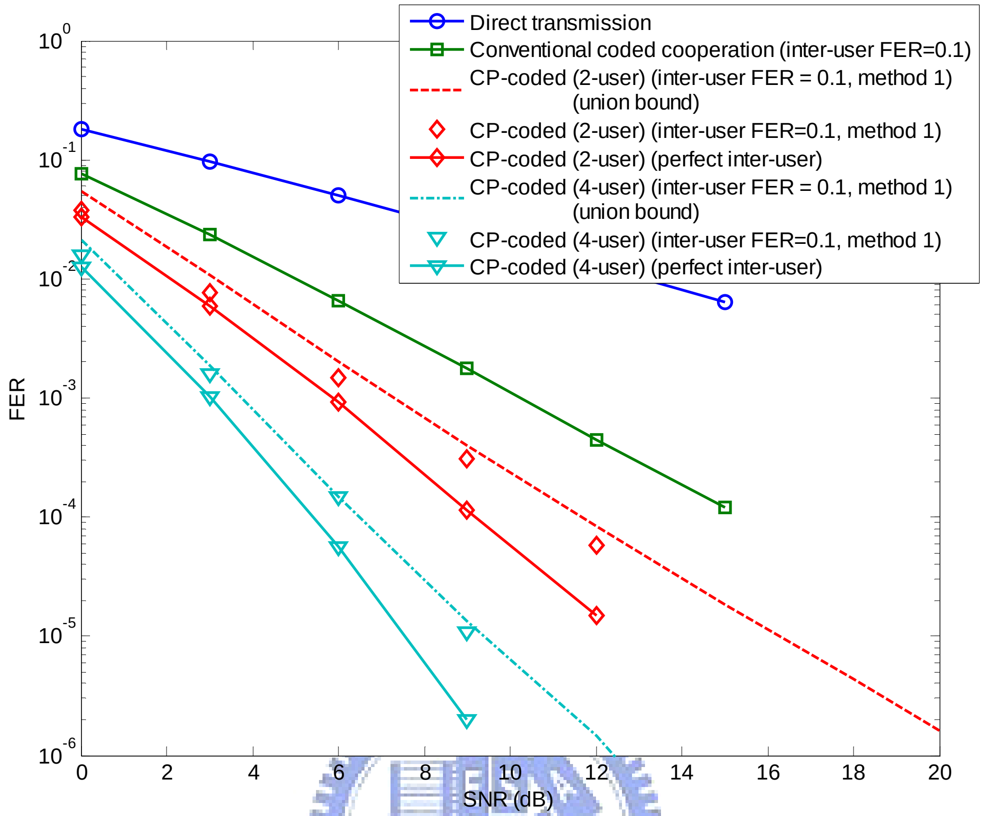

Fig. 4-9. Frame error rate (FER) with imperfect inter-user channels. Equal uplink SNR, generator [15 17 13 15 13 17], inter-user FER=0.1, method 1

Fig. 4-10 simulates in the same condition as Fig. 4-9, except that we use method 2 for the relay reaction. The dash line and the dash line with dots demonstrate the union bounds for 2-user and 4-user cooperation under method 2, respectively. The simulation results (diamonds and down triangles) are in consistent with the analyzed bounds. Comparing Fig. 4-10 with Fig. 4-9, it can be found that there is about 0.7dB loss in 2-user cooperation when method 2 is used; it is 0.8dB in 4-user case. The loss is due to the fact that some sub-codewords are discarded when error occurs in data exchange. As mentioned in the end of Section 4.1.2, this is the tradeoff for lower system complexity.

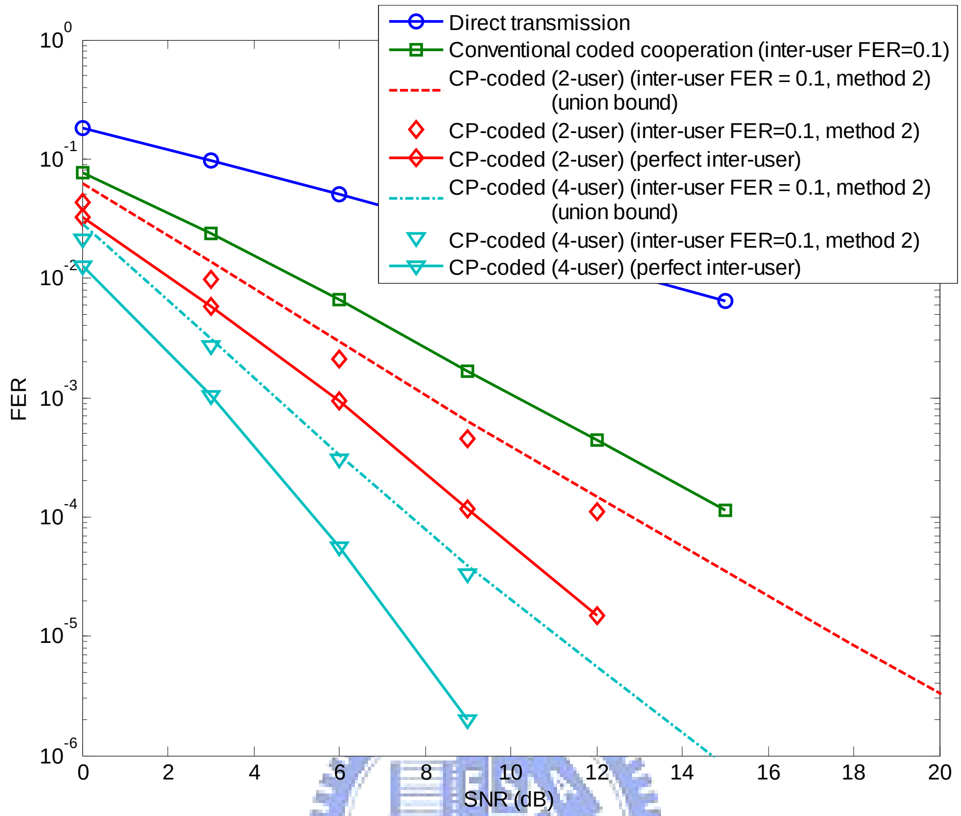

Fig. 4-10. Frame error rate (FER) with imperfect inter-user channels. Equal uplink SNR, generator [15 17 13 15 13 17], inter-user FER=0.1, method 2

4.4 Summary

In this Chapter we demonstrate the protocols and performances of Code Partition (CP) coded cooperation. It achieves diversity gain by partitioning a long base code into several short sub-codes and sending them by different users (independent channels). It has similar performance compared to the ST-coded cooperation in Chapter 3 (equal spectral efficiency, equal diversity gain for a given number of users), but has lower system complexity since no space-time code is used. Besides, it has lower requirements for the inter-user channels; full cooperation can still be achieved for other user when some of the users failed to exchange information. An alternative way for the relays to react to data exchange failure is presented to further simplify the system with reasonable loss of coding gain.

Chapter 5

Conclusions and Future Works

In this thesis, we develop two modified protocols of coded cooperation that enable the users, each equipped with a single antenna, to fully exploit the spatial diversity in the channel without losing spectral efficiency. Channel coding is assumed available for protecting the transmitted data. These protocols separate the codeword into two parts. Each user broadcast the first part (broadcast sub-codeword) to all other users, including the destination. After acquiring the data of other users by decoding the received broadcast sub-codeword, they use it to generate the second part (relay sub-codeword). The second codeword is transmitted with the help of all users using a space-time code or by partitioning it into several parts for each user. These sub-codewords are thus received at the destination through independent channels. For 2-user cooperation, we analyze the performance of the proposed protocols by evaluating the pairwise error probabilities and the BERs as well as FERs.

In Chapter 2, we review the concept of conventional cooperation and coded cooperation. There is spectral efficiency loss for conventional cooperative protocols due to the half-duplex hardware limitation, and we demonstrate how coded cooperation solves these problems by separating the transmit codeword for different purposes. In addition, the potential benefits inherent in coded cooperation is pointed out: all users know each other's data after the broadcast phase, which implies a virtual MISO system with the number of transmit antennas equal to the number of users.

In Chapter 3, we propose the first protocol that utilizes the potential benefits of coded cooperation, which is called space-time (ST) coded cooperation. It uses space-time code at the relay phase to enhance the reliability of relay sub-codewords, thereby enhancing the reliability of the transmitted data. Various space-time codes are chosen according to the number of users. For example, Alamouti code can be used in a 2-user scheme and 4×4 orthogonal space-time block code can be used in a 4-user scheme. It is shown in Section 3.2 that the diversity gain in relay sub-codeword reflects on the overall performance. Since the lengths of sub-codes remain unchanged regardless of the number of users in cooperation, the spectral efficiency is preserved while higher diversity order can be achieved with more users. However, the ST-coded cooperation has harsh requirements for inter-user channels because it requires all users to exchange data successfully to apply space-time code. Analyses and simulations reveal the performance degradation due to this factor. The degradation is rather large in the 4-user case, thus we propose an adaptive algorithm to compensate for it.

The second protocol, which is called code partitioning (CP) coded cooperation, is proposed in Chapter 4. It partitions the relay sub-codeword into several parts based on the number of users and makes each of them transmitted by a different user. Thus every part of the base codeword will be transmitted through an independent channel. We prove it by analysis that CP-coded cooperation achieves same the diversity order as the ST-coded one. In case of data exchange failure, it is also more robust. We have shown in Section 4.1.2 that its diversity order is maintained even when some of the links between users are broken. A modified method for data exchange failure is proposed to further simplify the system complexity with a reasonable tradeoff in coding gain. By this method, users do not need to know if other users receive their messages well, so feedback information is not needed.

The main contributions of this thesis are that two protocols based on coded cooperation are proposed to effectively exploit the benefits of cooperative transmissions. We introduce the concept of applying space-time code and code partitioning to the relay sub-codeword. The two protocols achieve higher system reliability but use equal channel resources as the direct transmission scheme. Both of them are highly flexible for different numbers of users. The larger the number of users that join cooperation, the higher the diversity gain that can be achieved. The code structure is also flexible to choose; it may be implemented using block or convolutional codes, or various methods of partitioning the codewords (puncturing, product codes, parallel and serial concatenation, etc.). Considering the case of data exchange failure in real wireless communications, we have proposed an adaptive algorithm to enhance the robustness of ST-coded cooperation. For CP-coded cooperation, we demonstrate its robustness against data exchange failure. Moreover, we exploit the advantages of CP-coded cooperation by using "blind cooperation" to make the scheme extremely simple.

Some issues that are not considered in this thesis may have considerable effect to the coded cooperative system and are worth future research. The first one is the spectral efficiency loss caused by feedback messages between users. In some of the proposed protocols, a user needs to notify others in case of data exchange failure, thus additional channel resource must be allocated. Although the resource needed is small compared to conventional cooperation, it still causes some loss in spectral efficiency. The second one is the synchronization problem, which is critical in systems using space-time codes. The third one is the choice of cooperative partners. Although there are already many research efforts toward this topic, detailed algorithms may need further investigation according to the specific protocols used. The final one is power allocation, which is not possible in ST-coded cooperation but may be helpful in CP-coded cooperation.

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