國 立 交 通 大 學

電信工程學系碩士班

碩士論文

適用於具非理想鏈結之無線感測網路之能量效率化分散式最佳線性不偏估計法

Energy-efficient Decentralized BLUE for Wireless Sensor Networks with Non-ideal Links

研 究 生：陳秋如　　　Student: Chiu-Ju Chen

指導教授：李大嵩 博士　　　Advisor: Dr. Ta-Sung Lee

中 華 民 國 九 十 七 年 六 月

適用於具非理想鏈結之無線感測網路之能量效率化分散式最佳線性不偏估計法

Energy-efficient Decentralized BLUE for Wireless Sensor Networks with Non-ideal Links

研 究 生：陳秋如　　　Student: Chiu-Ju Chen
指導教授：李大嵩 博士　　　Advisor: Dr. Ta-Sung Lee

國立交通大學
電信工程學系碩士班
碩士論文

A Thesis
Submitted to Institute of Communication Engineering
College of Electrical Engineering and Computer Science
National Chiao Tung University
in Partial Fulfillment of the Requirements
for the Degree of
Master of Science
in
Communication Engineering
June 2008
Hsinchu, Taiwan, Republic of China

中 華 民 國 九 十 七 年 六 月

適用於具非理想鏈結之無線感測網路之能量效率化分散式最佳線性不偏估計法

學生：陳秋如　　　　　　　　　　指導教授：李大嵩 博士

國立交通大學電信工程學系碩士班

摘要

有鑒於低傳送功率對無線感測網路(wireless sensor network)中之感測器而言為主要需求，如何在有限的總傳送能量下使效能最佳化的設計就愈顯重要。吾人考慮在一個異質性無線感測網路中利用最佳能量分配策略來對一個非隨機信號(deterministic signal)做分散式估計。感測器先將其觀測到之信號取樣成離散訊息後經過瑞利衰減通道(rayleigh fading channel)傳送至融合中心(fusion center)。接著此融合中心利用最佳不偏估計(best linear unbiased estimator)融合規則產生最終估計參數。本論文中提出數種只需得知長期雜訊變異量的統計特性即可求出最佳解之能量分配策略。於前半部，吾人提出的最佳能量分配策略建議對通道環境不佳或觀測品質不佳的感測器降低其所傳送訊息之量化解析度或進而將其關掉來節省能量。每個動態感測器的傳送位元數則由各自的通道衰減、路徑衰減(path loss)、局部觀測雜訊變異量(local observation noise variance)及能量限制來共同決定。於後半部，吾人提出兩個疊代式感測器殘餘能量配置演算法，來進一步提升估計準確度。根據模擬結果，於異質性感測環境中，相較於均衡式能量分配，吾人所提出的最佳能量分配策略可有顯著的效能改善。

Energy-efficient Decentralized BLUE for Wireless Sensor Networks with Non-ideal Links

Student: Chiu-Ju Chen Advisor: Dr. Ta-Sung Lee

Department of Communication Engineering

National Chiao Tung University

Abstract

As low transmitting power of sensors is a major requirement in wireless sensor networks (WSNs), optimizing their design under energy constraints is of primary importance. We consider an optimal power scheduling problem for the decentralized estimation of a deterministic signal in an inhomogeneous WSN. Sensors quantize their observations into discrete messages, which are transmitted to the fusion center (FC) over rayleigh fading channels. The FC which adopts the best-linear-unbiased-estimator (BLUE) fusion rule generates a final estimate. In this thesis, the optimal power allocation strategies which simply rely on long-term noise variance statistics are presented. In the first part, the proposed power scheduling scheme suggests that the sensors with bad channels or poor observation qualities should decrease their quantization resolutions or simply be shut off to save power. The bit load of each active sensor is determined jointly by individual channel fading gain, path loss, local observation noise variance, and the energy constraint. In the second part, two iterative allocation algorithms of residual energy at sensors are proposed to further enhance estimation accuracy. Numerical results show that in inhomogeneous sensing environments, significant performance improvement is possible when compared to the uniform quantization strategy.

Acknowledgement

I would like to express my deepest gratitude to my advisors, Dr. Ta-Sung Lee and Dr. Jwo Yuh Wu, for their enthusiastic guidance and great patience. I learn a lot from their positive attitude in many areas. Heartfelt thanks are also offered to all members in the Communication System Design and Signal Processing (CSDSP) Lab for their constant encouragement. Finally, I would like to show my sincere thanks to my parents for their invaluable love.

Contents

List of Figures

List of Table

Acronym Glossary

AWGN	additive white Gaussian noise
BLUE	best linear unbiased estimator
BSC	binary symmetric channel
CSI	channel state information
DES	decentralized estimation scheme
FC	fusion center
i.i.d.	independent and identically distributed
KKT	Karush-Kuhn-Tucker
MSE	mean square error
PDF	probability density function
QAM	quadrature amplitude modulation
SNR	signal-to-noise ratio
WSN	wireless sensor network

Notations

α	measurement noise variance variation
b_i	number of bits transmitted from the ith sensor
d_i	distance between sensor i and the FC
$d_i^{-\kappa}$	path loss at sensor i
δ	measurement noise variance threshold
E_i	consumed energy for reliable bit decoding at sensor i
E_T	total energy budget
h_i	channel gain between the ith sensor and the FC
κ	path loss exponent
m_i	output message at the ith sensor
N	number of sensors
n_i	measurement noise at the ith sensor
q_i	quantization noise at the ith sensor
$\sigma_{n_i}^2$	measurement noise variance at the ith sensor
$\sigma_{q_i}^2$	quantization noise variance at the ith sensor
σ_v^2	channel noise variance
θ	deterministic source signal
v_i	additive white channel noise between the ith sensor and the FC
x_i	local observation at the ith sensor

y_i received signal at the FC from sensor i

z_i central Chi-Square distributed random variables with degrees-of-freedom equal to one

Chapter 1

Introduction

The wireless sensor network (WSN) is an emerging technology that has many current and future envisioned applications such as environmental monitoring (air, water, and soil), military surveillance, smart factory instrumentation, space exploration, and so on [1]-[3]. A typical WSN architecture comprises a fusion center (FC) and a large number of geographically distributed sensor nodes. The FC can be either a standard base station or a mobile access point such as an unmanned aerial vehicle hovering over the sensor field. Since sensors are only equipped with small size batteries whose replacement can be rather costly, each sensor node can only transmit a compressed version of its raw measurement to the FC owing to bandwidth and power limitations.

Recently, several decentralized estimation schemes (DESs) [4]-[6] have been proposed for parameter estimation in the presence of additive sensor noise, that is, the local measurement noise. These DESs demand that each sensor node can only transmit a few bits to the FC, with message length determined by each sensor node’s local signal-to-noise ratio (SNR). Performance of the resulting estimator is shown to be within a constant factor of the best linear unbiased estimator (BLUE) performance. The studies in [7] and [8] present optimal coded and uncoded transmission strategies for WSNs which can minimize the energy consumption per transmitted bit, though the effect of quantization and the accuracy of final estimate are not considered.

Energy efficiency is a critical concern for contemporary sensor network design [9]-[11]. In practical systems, the probability density function (PDF) of the observation noise is hard to characterize, especially for enormous scale sensor networks. In consideration of this limitation, some signal processing algorithms that do not require the knowledge of sensor noise PDF are proposed in [10] and [11]. Seeing that most of the existing related studies require the knowledge of instantaneous noise variance for energy allocation, the proposed approaches instead simply depend on an associated statistical model. In order to enhance the estimation performance against the variation of sensing environments, repeatedly updating the noise profile would be necessary. This comes inevitably at the cost of more training overhead and extra energy consumption. If the sensing environment is harsh, the sensor noise will change expeditiously. Therefore signal processing algorithms proposed in this thesis are required to depend on an associated sensing noise variance model.

In practical WSNs, the wireless channels from sensor nodes to the FC might have different qualities, depending on the sensor nodes' locations relative to the FC. Intuitively, the local transmitted message length should not only depend on the quality of each sensor node's observation, that is, local SNR, but also depend on the quality of its wireless channel to the FC. The study in [12] models the channel between each sensor node and the FC as a Rayleigh fading channel. A more inhomogeneous transmitting environment is considered in [13]. The channel impairments, namely path loss and fading, limit the performance of sensor networks. Although a sensor node has a high quality of observation, it should not perform any local quantization or transmission as long as its channel quality to the FC is not robust enough in order to conserve energy. Moreover, in order to further improve the estimation performance, two advanced energy allocation approaches are also proposed in this thesis. The performance enhancement is conspicuous, though the energy consumption of these strategies is more significant than the proposed initial allocation strategy.

The thesis is organized as follows. In Chapter 2, the system model of WSNs that experiences Rayleigh fading and path loss with BLUE adopted in the FC is introduced. A minimal mean square error (MSE) decentralized estimation scheme based on long-term noise variance knowledge is proposed in Chapter 3. Chapter 4 describes two procedures that iteratively allocate the remaining energy among sensor nodes. The main results are presented and the numerical performance of the proposed approaches are illustrated in this chapter. Finally, the conclusions of this thesis and some potential future works are given in Chapter 5.

Chapter 2

System Model of Wireless Sensor Network

A common wireless sensor network (WSN) architecture comprises of a fusion center (FC) and a enormous number of geographically distributed sensor nodes which collect observations. Subject to severe energy and bandwidth limitations, each sensor node possesses only limited computation and communication capabilities. It is difficult for sensor nodes to transmit their entire real-valued observations to the FC. In a more practical decentralized estimation scheme (DES), each sensor node preprocesses and extracts information from its raw observations. The information is then aggregated via wireless transmissions at the FC where the received sensor signals are combined to produce a final estimate of the observed quantity. The message lengths are influenced by the power constraint, bandwidth limitation and sensor noise characteristics.

In a practical WSN, the wireless channels from sensor nodes to the FC may have different qualities. The local message length of each sensor node should depend not only on the quality of its local observation, that is, local SNR, but also on the quality of its wireless channel to the FC. The transmitted message length of each sensor node is determined jointly by the local signal-to-noise ratio (SNR) and the quality of its wireless channel to the FC, which is influenced by the fading gain, the path loss, and the noise power. In Section 2.1, the received signal at the FC from each sensor node is discussed. The best linear unbiased estimator (BLUE) is introduced in Section 2.2. Section 2.3 discusses a related previous study. A summary of Chapter 2 is given in Section 2.4.

2.1 Received Signal Model at Sensor Nodes

Consider a wireless sensor network as depicted in Figure 2.1, in which N distributed sensor nodes make observations on the deterministic source signal, θ. The observations of sensor nodes are corrupted by additive noise. The local observation at

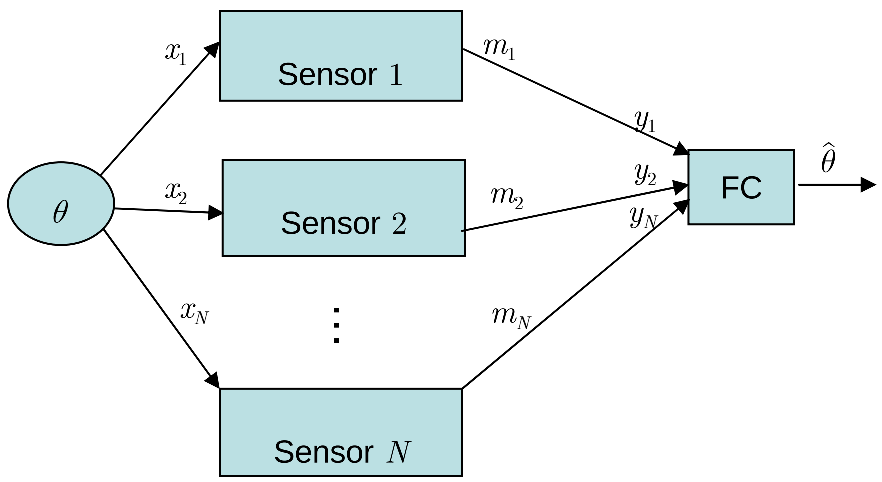

Figure 2.1: System Model of Wireless Sensor Network

the ith node is

$$x_i = \theta + n_i, \quad 1 \le i \le N, \tag{2.1}$$

where n_i is independent and identically distributed (i.i.d.) measurement noise with zero-mean and variance $\sigma_{n_i}^2$. The sensor noise variance $\sigma_{n_i}^2$ is assumed to follow the subsequent relation [9], [10]

$$\sigma_{n_i}^2 = \delta + \alpha z_i, \quad 1 \le i \le N, \tag{2.2}$$

where $\delta > 0$ models the network-wide noise variance threshold, $\alpha > 0$ controls the underlying variation from the nominal minimum, and $\{z_i : i = 1, 2, \ldots, N\}$ are i.i.d. central Chi-Square distributed random variables each with degrees-of-freedom equal to one, i.e., $z_i \sim \chi_1^2$ [14].

Owing to the bandwidth limitation and the power constraint, each sensor node ought to quantize its observation into a b_i-bit message. Afterward each sensor node transmits this locally processed data to the FC to generate a final estimate of θ. In this thesis, the uniform quantization scheme with nearest rounding [15], [16] is adopted. The produced message at the ith sensor node can consequently be modeled as

$$m_i = x_i + q_i, \quad 1 \le i \le N, \tag{2.3}$$

where q_i is the quantization noise which is uniformly distributed with zero mean and variance $\sigma_{q_i}^2 = R^2 4^{-b_i} / 12$ [15], [16]. With the constraint that $R > 0$, $\left[-R/2, R/2\right]$ denotes the available signal amplitude range common to all sensor nodes.

Under the flat-fading channel assumption, the received signal at the FC from the ith sensor node is thus

$$
\begin{aligned}
y_i &= d_i^{-\kappa/2} h_i m_i + v_i \\
&= d_i^{-\kappa/2} h_i \left(\theta + q_i + n_i\right) + v_i \\
&= d_i^{-\kappa/2} h_i \theta + d_i^{-\kappa/2} h_i q_i + d_i^{-\kappa/2} h_i n_i + v_i, \quad 1 \le i \le N,
\end{aligned} \tag{2.4}
$$

where h_i is the channel gain of the ith communication channel, and v_i is the zero-mean additive channel noise with variance σ_v^2, which is modeled as additive white Gaussian noise (AWGN) . The signal power received at the FC is assumed to be proportional to $d_i^{-\kappa}$ where d_i is the distance between sensor i and the FC, and κ is the path loss exponent common to all sensor-to-FC links. By collecting all the received data in (2.3) into a vector we have

$$
\begin{aligned}
\underbrace{\left[y_1,\ldots,y_N\right]^T}_{:=\mathbf{y}} = \underbrace{\left[\tilde{h}_1,\ldots,\tilde{h}_N\right]^T}_{:=\tilde{\mathbf{h}}} \theta + \widetilde{\mathbf{H}} \underbrace{\left[n_1,\ldots,n_N\right]^T}_{:=\mathbf{n}} \\
+ \widetilde{\mathbf{H}} \underbrace{\left[q_1,\ldots,q_N\right]^T}_{:=\mathbf{q}} + \underbrace{\left[v_1,\ldots,v_N\right]^T}_{:=\mathbf{v}},
\end{aligned} \tag{2.5}
$$

with $\tilde{h}_i := d_i^{-\kappa/2} \cdot h_i$ and $\widetilde{\mathbf{H}} := diag\left\{\left[\tilde{h}_1, \ldots, \tilde{h}_N\right]^T\right\}$. Assume that the noise terms $\{\mathbf{n}, \mathbf{q}, \mathbf{v}\}$ in (2.5) are mutually independent and the respective samples n_i's, q_i's, and v_i's are also independent across sensors.

2.2 Best Linear Unbiased Estimator at Fusion Center

The parameter θ is retrieved via BLUE [17] in the FC. This estimator simply requires the knowledge of the first and the second moments of the PDF. Assume that the channel state information (CSI) and the local sensor-to-FC path loss can be observed. Upon receiving sensor messages y_i 's, the FC combines them into an estimator $\hat{\theta}$ given by

$$\hat{\theta} = \frac{\mathbf{1}^T \mathbf{C}^{-1} \mathbf{y}}{\mathbf{1}^T \mathbf{C}^{-1} \mathbf{1}}, \tag{2.6}$$

where $\mathbf{1} = [1,\ldots\ldots,1]^{\mathrm{T}}$ is a vector with each element equal to 1 and $\mathbf{C}$ is the covariance matrix of the associated noise term: $\widetilde{\mathbf{H}}\mathbf{n} + \widetilde{\mathbf{H}}\mathbf{q} + \mathbf{v}$. The covariance matrix of the noise term can therefore be expressed as

$$\mathbf{C} = \begin{bmatrix} \tilde{h}_1^2\left(\sigma_1^2\right) + \sigma_v^2 & 0 & \cdots & 0 \\ 0 & \tilde{h}_2^2\left(\sigma_2^2\right) + \sigma_v^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \tilde{h}_N^2\left(\sigma_N^2\right) + \sigma_v^2 \end{bmatrix}. \tag{2.7}$$

where $\sigma_i^2 = \sigma_{n_1}^2 + \sigma_{q_1}^2$. Thus the final estimate is obtained by

$$\hat{\theta} = \left(\sum_{i=1}^{N} \frac{\tilde{h}_i y_i}{\tilde{h}_i^2 \sigma_{n_i}^2 + \sigma_v^2 + \tilde{h}_i^2 \beta 4^{-b_i}} \right) \left(\sum_{i=1}^{N} \frac{1}{\sigma_{n_i}^2 + \left(\sigma_v^2 / \tilde{h}_i^2\right) + \beta 4^{-b_i}} \right)^{-1}, \tag{2.8}$$

where $\beta := R^2/12$ for notation simplicity. The incurred mean square error (MSE) is thus [16]

$$\begin{aligned} \mathrm{MSE}\left(\hat{\theta}\right) &= E\left|\hat{\theta} - \theta\right|^2 \\ &= \left(\mathbf{1}^{\mathrm{T}} \mathbf{C}^{-1} \mathbf{1}\right)^{-1} \\ &= \left(\sum_{i=1}^{N} \frac{1}{\sigma_{n_i}^2 + \left(\sigma_v^2 / \tilde{h}_i^2\right) + \beta 4^{-b_i}} \right)^{-1}. \end{aligned} \tag{2.9}$$

2.3 Review of Previous Work

Adopting the centralized BLUE at the FC was originally proposed in [9]. Thus the FC can perform linear combination of sensor observations to recover θ. Then the study in [9] proposes an adaptive quantization scheme of sensor observations and discusses its impact on energy saving. The optimal bit loading is obtained via

$$\begin{aligned} &\text{minimize } \|\mathbf{P}\|_2, \\ &\text{subject to } D' \le D_0. \end{aligned} \tag{2.10}$$

where $\|\mathbf{P}\|_2 = \sqrt{\sum_{i=1}^{N} P_i^2}$ with P_i denoting the transmit power consumption of sensor i, D' is the achieved MSE performance, and D_0 is the target MSE performance. Therefore the following optimization problem based on instantaneous noise characteristics is obtained

$$\begin{aligned} &\underset{b_i \in \mathbb{Z}}{\text{minimize}} \ \sum_{i=1}^{N} a_i^2 \left(2^{b_i} - 1\right)^2, \\ &\text{subject to } D' = (1+p_0)^2 \left(\sum_{i=1}^{N} \frac{1}{\sigma_{n_i}^2 + \left(\sigma_v^2 / \tilde{h}_i^2\right) + \beta 4^{-b_i}} \right)^{-1} \le D_0, \\ &\qquad b_i \in \mathbb{Z}_0^+, \quad i = 1, \ldots, N, \end{aligned} \tag{2.11}$$

where $a_i = d_i^{\kappa}$. To facilitate the consequent analyses, the condition $b_i \in \mathbb{Z}_0^+$ is relaxed to $b_i \in \mathbb{R}$. Since the problem is still non-convex, we define

$$f_i = \frac{1}{\sigma_{n_i}^2 + \left(\sigma_v^2 / \tilde{h}_i^2\right) + \beta 4^{-b_i}}, \quad i = 1, \ldots, N. \tag{2.12}$$

Accordingly, the problem (2.11) can be transformed into

$$\text{minimize } \sum_{i=1}^{N} a_i^2 \left\{ \frac{R^2 f_i}{4\left[1 - f_i \sigma_{n_i}^2 - f_i \left(\sigma_v^2 / \tilde{h}_i^2\right)\right]} \right\}^2,$$

$$\text{subject to } \sum_{i=1}^{N} f_i \geq \frac{1}{D_0^{'}},$$

$$0 \leq f_i < \frac{1}{\sigma_{n_i}^2 + \left(\sigma_v^2 / \tilde{h}_i^2\right)}, \quad i = 1, \ldots, N, \tag{2.13}$$

where $D_0^{'} = D_0 \big/ (1 + p_0)^2$.

Since the problem becomes convex, the Lagrangian function can be obtained

$$L(f_1, \ldots, f_N, \lambda, \mu_1, \ldots, \mu_N)$$
$$= \sum_{i=1}^{N} a_i^2 \left\{ \frac{R^2 f_i}{4\left[1 - f_i \sigma_{n_i}^2 - f_i \left(\sigma_v^2 / \tilde{h}_i^2\right)\right]} \right\} + \lambda \left(\frac{1}{D_0^{'}} - \sum_{i=1}^{N} f_i \right) - \sum_{i=1}^{N} \mu_i f_i. \tag{2.14}$$

Equation (2.14) gives the following Karush-Kuhn-Tucker (KKT) conditions

$$\frac{R^2 a_i^2}{4\left[1 - f_i \sigma_{n_i}^2 - f_i \left(\sigma_v^2 / \tilde{h}_i^2\right)\right]^2} - \lambda - \mu_i = 0, \tag{2.15}$$

$$\lambda \left(\sum_{i=1}^{N} f_i - \frac{1}{D_0^{'}} \right) = 0, \tag{2.16}$$

$$\sum_{i=1}^{N} f_i - \frac{1}{D_0^{'}} \geq 0, \tag{2.17}$$

$$\lambda \geq 0 \tag{2.18}$$

$$\mu_i f_i = 0,\ \mu_i \geq 0,\ 0 \leq f_i < \frac{1}{\sigma_{n_i}^2 + \left(\sigma_v^2 / \tilde{h}_i^2\right)}, \quad i = 1, \ldots, N. \tag{2.19}$$

Assume that $a_1 \leq a_2 \leq \ldots \leq a_N$, and define

$$G(M) = a_M \left(\sum_{i=1}^{M} \frac{a_i}{\sigma_{n_i}^2 + \left(\sigma_v^2 / \tilde{h}_i^2\right)} \right)^{-1} \left(\sum_{i=1}^{M} \frac{1}{\sigma_{n_i}^2 + \left(\sigma_v^2 / \tilde{h}_i^2\right)} - \frac{1}{D_0^{'}} \right),$$
$$1 \leq M \leq N. \tag{2.20}$$

Let K_1 be the unique sensor number such that $f(K_1) < 1$ and $f(K_1 + 1) \geq 1$.

Applying simple manipulations of the KKT system leads to

$$\lambda = \left(\frac{R \sum_{i=1}^{M_1} \frac{a_i}{\sigma_{n_i}^2 + \left(\sigma_v^2 / \tilde{h}_i^2 \right)}}{2 \sum_{i=1}^{M_1} \frac{1}{\sigma_{n_i}^2 + \left(\sigma_v^2 / \tilde{h}_i^2 \right)} - \frac{1}{D_0^{'}}} \right)^2 , \tag{2.21}$$

and

$$f_i^{\text{opt}} = \frac{1}{\sigma_{n_i}^2 + \left(\sigma_v^2 / \tilde{h}_i^2 \right)} \left(1 - \frac{a_i R}{2\sqrt{\lambda}} \right)^+ , \quad i = 1, \ldots, N, \tag{2.22}$$

where $(x)^+ = 0$ for $x < 0$ and $(x)^+ = x$ for $x \geq 0$. Equation (2.22) implies that the optimal value of b_i is

$$\begin{aligned} b_i^{\text{opt}} &= \log_2 \left(1 + \sqrt{\frac{R^2}{4\left[\frac{1}{f_i^{\text{opt}}} - \sigma_i^2 \right]}} \right) \\ &= \begin{cases} 0 & \text{for } k \geq M_1 + 1 \\ \log \left(1 + \frac{R}{2\sigma_{n_i}^2 + \left(\sigma_v^2 / \tilde{h}_i^2 \right)} \sqrt{\frac{\xi_0}{a_i} - 1} \right) & \text{for } k \geq M_1 \end{cases} \end{aligned} \tag{2.23}$$

where $\xi_0 = \frac{2\sqrt{\lambda}}{R} = \left(\sum_{i=1}^{M_1} \frac{1}{\sigma_{n_i}^2 + \left(\sigma_v^2 / \tilde{h}_i^2 \right)} - \frac{1}{D_0^{'}} \right)^{-1} \sum_{i=1}^{M_1} \frac{a_i}{\sigma_{n_i}^2 + \left(\sigma_v^2 / \tilde{h}_i^2 \right)}.$

Motivated by this, bit allocation according to long term noise characteristics is proposed in the following chapters.

2.4 Summary

The decentralized estimation of a noise-corrupted deterministic parameter is discussed in this chapter. Each sensor node in the WSN collects an observation, computes a local message, and sends it to the FC. Sensor nodes do not communicate with each other. Local quantization at each sensor node is required to reduce the communication cost. An optimal design of the discrete local message functions and the final fusion function depend on the underlying sensor noise distributions. The sensor noises are assumed to be additive, zero-mean, spatially uncorrelated, but otherwise unknown and possibly different among sensor nodes due to varying sensor quality and inhomogeneous sensing environment. The classical BLUE linearly combines the real-valued sensor observations to minimize the MSE. Unfortunately, transmitting real-valued messages is impractical to implement. Finally, a previous study is discussed. In the next chapter, we construct a DES where each sensor node compresses its observation into a small number of bits with length proportional to the local SNR and the channel quality. The resulting compressed bits from different sensor nodes are then collected and combined by the FC to estimate the unknown parameter.

Chapter 3

Minimal Mean Square Error Decentralized Estimation over Rayleigh Fading Channel Based on Sensor Noise Variance Statistics

In this chapter, a problem similar to the problem recently considered [18]: how to find the optimal bit load which minimizes the average distortion under a fixed total energy budget is studied. A key feature common to the existing related studies [4], [9], [10], [19] is that error free transmission is assumed, that is, perfect wireless channels are considered in these studies. The study in [9] formulates the convex optimization problem to derive an optimal bit loading scheme under the mean square error (MSE) constraint with perfect channel. The study in [20] models the noisy channel between each sensor node and the fusion center (FC) as a binary symmetric channel (BSC) with crossover probability controlled by the transmitted bit energy. However, it requires the instantaneous local sensor noise characteristics to formulate the optimization problem.

This chapter contributes a solution to the minimal MSE decentralized estimation with rayleigh fading channel between each sensor node and the FC by exploiting long term noise variance information. The analytic results reveal that under limited energy budget, sensor nodes with unfavorable channel quality or low local SNR should be shut off. The energy distributed to those active sensor nodes should be proportional to the individual channel gain of each active sensor node. In Section 3.1, the average mean square error (MSE) is analyzed. The optimal closed-form solution is provided in Section 3.2. Section 3.3 and Section 3.4 contain discussions and numerical results of the proposed optimal solution. Section 3.5 summarizes this chapter.

3.1 Average Mean Square Error of Decentralized Estimation

Assume that the ith sensor node transmits its locally processed b_i-bit message, m_i, via applying quadrature amplitude modulation (QAM) with a constellation size 2^{b_i}. According to [21], the consumed energy for reliable bit decoding is thus

$$E_i = w_i \left(2^{b_i} - 1\right), \quad 1 \le i \le N, \tag{3.1}$$

where $w_i := \sigma_v^2 / \tilde{h}_i^2 = d_i^{\kappa} \cdot \sigma_v^2 / h_i^2$ and E_i / w_i is the local signal-to-noise ratio (SNR) received by the FC from the ith sensor node. For a fixed set of the measurement noise variances, that is, $\sigma_{n_i}^2$'s, the problem of decentralized best linear unbiased estimator

(BLUE), under an permissible total energy budget E_T, can be formulated as

$$\begin{aligned}
&\text{minimize} \left(\sum_{i=1}^{N} \frac{1}{\sigma_{n_i}^2 + w_i + \beta 4^{-b_i}} \right)^{-1}, \\
&\text{subject to} \sum_{i=1}^{N} w_i \left(2^{b_i} - 1\right) \le E_T, \\
&\qquad b_i \in \mathbb{Z}_0^+, \\
&\qquad 1 \le i \le N,
\end{aligned} \tag{3.2}$$

or equivalently,

$$\begin{aligned}
&\text{maximize} \sum_{i=1}^{N} \frac{1}{\sigma_{n_i}^2 + w_i + \beta 4^{-b_i}}, \\
&\text{subject to} \sum_{i=1}^{N} w_i \left(2^{b_i} - 1\right) \le E_T, \\
&\qquad b_i \in \mathbb{Z}_0^+, \\
&\qquad 1 \le i \le N,
\end{aligned} \tag{3.3}$$

where $\mathbb{Z}_0^+$ denotes the set of all nonnegative integers.

Toward a solution independent of instantaneous noise conditions, as in [18] and [19], we evaluate the following optimization problem, in which the equivalent distortion cost function in (3.3) is averaged over the noise variance statistic characterized in (2.2):

$$\begin{aligned}
&\text{maximize} \underbrace{\int_{\mathbf{z}} \sum_{i=1}^{N} \frac{1}{\delta + \alpha z_i + w_i + \beta 4^{-b_i}} p(\mathbf{z})\, d\mathbf{z}}_{:=J_0}, \\
&\text{subject to} \sum_{i=1}^{N} w_i \left(2^{b_i} - 1\right) \le E_T, \\
&\qquad b_i \in \mathbb{Z}_0^+, \\
&\qquad 1 \le i \le N,
\end{aligned} \tag{3.4}$$

where $\mathbf{z} = [z_1, z_2, \ldots, z_N]^T$ with $p(\mathbf{z})$ denoting the associated distribution. Since $z_i \sim \chi_1^2$ is a central i.i.d. Chi-Square distributed random variable with degrees-of-freedom equal to one [14]

$$p_{\chi_1^2}(z) = \begin{cases} \frac{1}{\sqrt{2\pi z}} e^{-z/2}, & z \geq 0, \\ 0, & z < 0. \end{cases} \tag{3.5}$$

The averaged MSE performance can be calculated as

$$\begin{aligned} \overline{\text{MSE}} &= \int_{\mathbf{z}} \sum_{i=1}^{N} \frac{1}{\delta + \alpha z_i + w_i + \beta 4^{-b_i}} p(\mathbf{z}) \, d\mathbf{z} \\ &= \sum_{i=1}^{N} \int_0^{\infty} \frac{1}{\delta + \alpha z_i + w_i + \beta 4^{-b_i}} \cdot \frac{e^{-z_i/2}}{\sqrt{2\pi z_i}} dz_i \\ &= \frac{1}{\sqrt{2\pi}} \sum_{i=1}^{N} \int_0^{\infty} \frac{e^{-z_i/2}}{\left[\delta + \alpha z_i + w_i + \beta 4^{-b_i}\right]\sqrt{z_i}} dz_i, \end{aligned} \tag{3.6}$$

A similar closed-form expression of the cost function J_0 in (3.4) can be found in [18] and [19]. The following lemma, with proof presented in Appendix A, provides a closed-form expression of the integral involved in the summation in (3.6).

Lemma 3.1: The following result holds:

$$J_0 = \sqrt{2\pi} \cdot \sum_{i=1}^{N} \frac{e^{\left[\delta + w_i + \beta 4^{-b_i}\right]/2\alpha} \cdot Q\left(\sqrt{\left[\delta + w_i + \beta 4^{-b_i}\right]/\alpha}\right)}{\sqrt{\alpha\left[\delta + w_i + \beta 4^{-b_i}\right]}}, \tag{3.7}$$

where $Q(x) := \int_x^{\infty} \frac{e^{-t^2/2}}{\sqrt{2\pi}} dt$ is the Gaussian tail function.

From (3.5) and Lemma 3.1, the optimization problem in (3.4) can be equivalently expressed as

$$\begin{aligned} &\text{maximize } \sqrt{2\pi} \cdot \sum_{i=1}^{N} \frac{e^{\left(\delta + w_i + \beta 4^{-b_i}\right)/2\alpha} \cdot Q\left(\sqrt{\frac{\left[\delta + w_i + \beta 4^{-b_i}\right]}{\alpha}}\right)}{\sqrt{\alpha\left[\delta + w_i + \beta 4^{-b_i}\right]}}, \\ &\text{subject to } \sum_{i=1}^{N} w_i \left(2^{b_i} - 1\right) \leq E_T, \\ &\qquad b_i \in \mathbb{Z}_0^+, \\ &\qquad 1 \leq i \leq N. \end{aligned} \tag{3.8}$$

Equation (3.8) shows that J_0 is unfortunately a highly non-linear (and non-convex) function of the sensor's bit load b_i. The problem therefore becomes intractable if we stick on direct maximization of J_0. An alternative formulation which is more tractable is proposed and an analytic solution can be obtained. By the following approximation to the Gaussian tail function [22]

$$Q(x) \approx \frac{1}{\sqrt{2\pi}}\left[\frac{e^{-x^2/2}}{\left(1-\pi^{-1}\right)x+\pi^{-1}\sqrt{x^2+2\pi}}\right], \tag{3.9}$$

and some straightforward manipulations, the cost function can thus be approximated by

$$\begin{aligned}&\sqrt{2\pi}\cdot\sum_{i=1}^{N}\frac{e^{\left[g_i+\beta 4^{-b_i}\right]/2\alpha}\cdot Q\left(\sqrt{\left[g_i+\beta 4^{-b_i}\right]/\alpha}\right)}{\sqrt{\alpha\left[g_i+\beta 4^{-b_i}\right]}}\\ &\approx \sum_{i=1}^{N}\frac{1}{\left(1-\pi^{-1}\right)\left(g_i+\beta 4^{-b_i}\right)+\pi^{-1}\sqrt{\left(g_i+\beta 4^{-b_i}\right)^2+2\pi\alpha\left(g_i+\beta 4^{-b_i}\right)}},\end{aligned} \tag{3.10}$$

where $g_i := \delta + w_i$. The main advantage of (3.10) is that it leads to an associated lower bound on the reciprocal MSE, J_0, in a more tractable form.

Through further modification of variable, the problem can be formulated in the form of convex optimization problem which then yields a closed-form solution. By the inequality equation:

$$\sqrt{\left(g_i+\beta 4^{-b_i}\right)^2+2\pi\alpha\left(g_i+\beta 4^{-b_i}\right)} \le \left(g_i+\beta 4^{-b_i}\right)+\pi\alpha, \tag{3.11}$$

the approximated cost function in (3.10) is lower bounded by

$$
\begin{aligned}
J_0 &= \sum_{i=1}^{N} \frac{1}{\left(1-\pi^{-1}\right)\left(g_i + \frac{\beta}{4^{b_i}}\right) + \pi^{-1}\sqrt{\left(g_i + \frac{\beta}{4^{b_i}}\right)^2 + 2\pi\alpha\left(g_i + \frac{\beta}{4^{b_i}}\right)}} \\
&\geq \sum_{i=1}^{N} \frac{1}{\left(1-\pi^{-1}\right)\left(g_i + \beta 4^{-b_i}\right) + \pi^{-1}\left[\left(g_i + \beta 4^{-b_i}\right) + \pi\alpha\right]} \\
&= \sum_{i=1}^{N} \frac{1}{\left(g_i + \beta 4^{-b_i}\right) + \alpha} \\
&= \sum_{i=1}^{N} \frac{4^{b_i}}{\gamma_i 4^{b_i} + \beta},
\end{aligned}
\tag{3.12}
$$

where $\gamma_i := \alpha + g_i = \alpha + \delta + w_i$. According to (3.12), the optimization problem is thus

$$
\begin{aligned}
&\text{maximize } \sum_{i=1}^{N} \frac{4^{b_i}}{\gamma_i 4^{b_i} + \beta}, \\
&\text{subject to } \sum_{i=1}^{N} w_i\left(2^{b_i} - 1\right) \leq E_T, \\
&\qquad b_i \in \mathbb{Z}_0^+, \\
&\qquad 1 \leq i \leq N.
\end{aligned}
\tag{3.13}
$$

To facilitate analysis we first observe that, since $b_i \in \mathbb{Z}_0^+$, it follows that $\sum_{i=1}^{N} w_i\left(2^{b_i} - 1\right) \leq \sum_{i=1}^{N} w_i\left(4^{b_i} - 1\right)$. This statement implies that the total energy constraint in (3.13) can be substituted by the following one without violating the overall energy budget requirement:

$$
\sum_{i=1}^{N} w_i\left(4^{b_i} - 1\right) \leq E_T, \tag{3.14}
$$

With the aid of (3.14) and by performing a modification of variable with $B_i := 4^{b_i} - 1$, the optimization problem finally becomes

$$\begin{aligned}
&\text{maximize } \sum_{i=1}^{N} \frac{B_i + 1}{\gamma_i (B_i + 1) + \beta}, \\
&\text{subject to } \sum_{i=1}^{N} w_i B_i \le E_T, \\
&B_i \ge 0, \\
&1 \le i \le N.
\end{aligned} \tag{3.15}$$

In (3.15), the intermediate variable B_i is relaxed to be a non-negative real number so as to render the optimization problem tractable. AS long as the optimal real-valued B_i (and hence b_i) is evaluated, the associated bit loads can therefore be obtained through nearest-integer rounding. The major contribution of the alternative problem formulation (3.15) is that it admits the form of convex optimization and can moreover lead to a closed-form solution, as shown below.

3.2 Optimal Closed-form Solution for Sensor Node Resource Allocation

In order to solve the problem in (3.15), let us form the associated Lagrangian as

$$\begin{aligned}
&L(B_1, \ldots, B_N, \lambda, \mu_1, \ldots, \mu_N) \\
&= \sum_{i=1}^{N} \frac{B_i + 1}{\gamma_i (B_i + 1) + \beta} - \lambda \left(\sum_{i=1}^{N} w_i B_i - E_T \right) + \sum_{i=1}^{N} \mu_i B_i.
\end{aligned} \tag{3.16}$$

The associated set of Karush-Kuhn-Tucker (KKT) conditions [23] are listed as follows:

$$\frac{\beta}{\left[\gamma_i (B_i + 1) + \beta\right]^2} - \lambda w_i + \mu_i = 0, \quad i = 1, \ldots, N, \tag{3.17}$$

$$\lambda \left(\sum_{i=1}^{N} w_i B_i - E_T \right) = 0, \tag{3.18}$$

$$\lambda \ge 0, \tag{3.19}$$

$$\mu_i \geq 0, \quad \mu_i B_i = 0, \quad B_i \geq 0, \quad 1 \leq i \leq N. \tag{3.20}$$

Condition (3.17) leads to

$$B_i = \frac{\sqrt{\beta}}{\gamma_i \sqrt{\lambda w_i - \mu_i}} - \frac{\beta}{\gamma_i} - 1, \tag{3.21}$$

According to (3.21), λ and μ_i 's should be determined to fulfill the desired constraints. If $\lambda = 0$, (3.17) implies $\mu_i > 0$ for all $1 \leq i \leq N$ and hence $B_i = 0, \ 1 \leq i \leq N$. Since all sensor nodes are turned off under this circumstance, it should be precluded.

The assumption $w_1 \geq w_2 \geq \cdots \geq w_N$ is made without loss of generality. Corresponding to the previous assumption: $\gamma_i = \alpha + \delta + w_i$, $\gamma_1 \geq \gamma_2 \geq \cdots \geq \gamma_N$ is demonstrated. Let us define the function

$$f(K) := \frac{E_T + \sum_{j_1=K}^{N} \left(\frac{\beta w_{j_1}}{\gamma_{j_1}} + w_{j_1} \right)}{(\gamma_K + \beta)\sqrt{w_K} \sum_{j_2=K}^{N} \frac{\sqrt{w_{j_2}}}{\gamma_{j_2}}}, \quad 1 \leq K \leq N. \tag{3.22}$$

Let $1 \leq K_1 \leq N$ be the unique integer such that $f(K_1 - 1) < 1$ and $f(K_1) \geq 1$. If $f(K) \geq 1$ for all $1 \leq K \leq N$, we simply set $K_1 = 1$. The existence and uniqueness of such K_1 is shown in Lemma 3.2 with proof given in Appendix B

Lemma 3.2: For the function $f(K)$ defined in (3.22), if there exist K_1 such that $f(K_1) \geq 1$, then $f(K_1 + 1) \geq 1$. Furthermore, $f(N) \geq 1$ always holds.

The optimal λ_{opt} and $B_{i,opt}$ can be obtained by

$$\sqrt{\lambda_{opt}} = \frac{\sqrt{\beta} \sum_{j_1=K_1}^{N} \frac{\sqrt{w_{j_1}}}{\gamma_{j_1}}}{E_T + \sum_{j_2=K_1}^{N} \frac{(\beta + \gamma_{j_2}) w_{j_2}}{\gamma_{j_2}}}, \tag{3.23}$$

and

$$B_{i,opt} = \begin{cases} 0, & 1 \le i \le K_1 - 1, \\ \dfrac{E_T + \sum\limits_{j_1=K_1}^{N} \left(\dfrac{\beta w_{j_1}}{\gamma_{j_1}} + w_{j_1} \right)}{\gamma_i \sqrt{w_i} \sum\limits_{j_2=K_1}^{N} \dfrac{\sqrt{w_{j_2}}}{\gamma_{j_2}}} - \dfrac{\beta}{\gamma_i} - 1, & K_1 \le i \le N. \end{cases} \tag{3.24}$$

Since $B_i = 4^{b_i} - 1$, the optimal bit load is thus

$$b_{i,opt} = \begin{cases} 0, & 1 \le i \le K_1 - 1, \\ \dfrac{1}{2}\log_2\left(B_{i,opt} + 1\right), & K_1 \le i \le N. \end{cases} \tag{3.25}$$

The practical bit load transmitted from the ith sensor node, that is, b_i, can be evaluated via applying nearest-integer rounding to the optimal bit load, $b_{i,opt}$. The bit allocation strategy described in this chapter can be illustrated by Figure 3.1. Sensor nodes are initially sorted according to w_i. Then applying the bit distribution strategy described in Section 3.2 we obtain the bit loading topology.

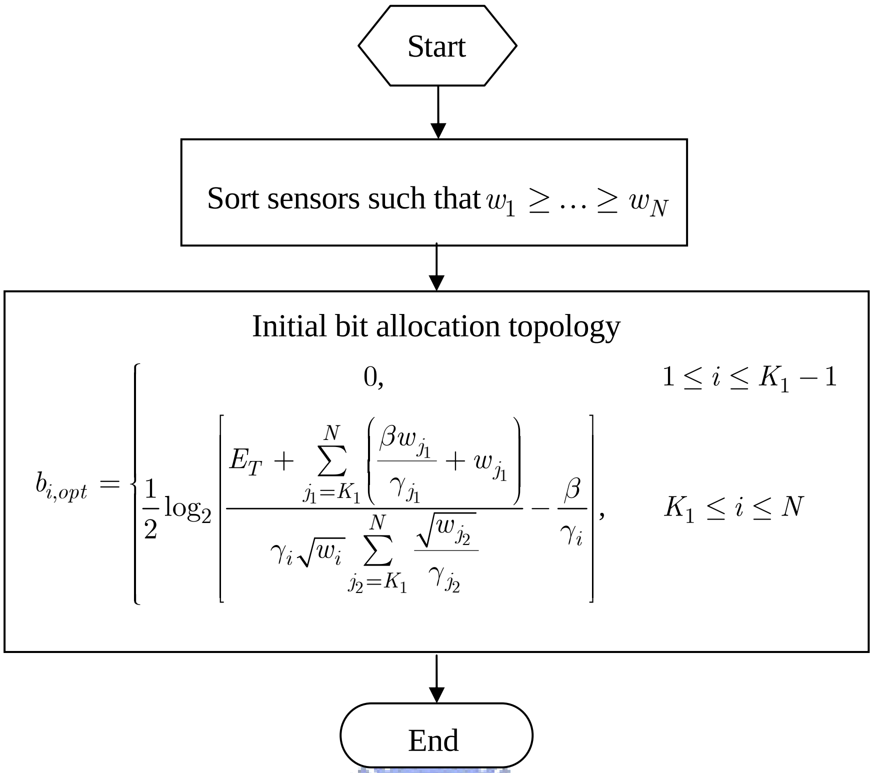

Figure 3.1: Flow chart of initial allocation method

3.3 Further Discussions on Optimal Solution

1. Since the path loss is normally much larger than the rayleigh fading gain, sensor i which is deployed remote to the FC usually corresponds to the poor background channel gain and therefore large value of w_i. If there are sensor nodes suffer from path losses which vary negligibly, the channel quality w_i is mainly influenced by the rayleigh fading gain. If the sensor nodes are sorted according to w_i, The first $K_1 - 1$ sensor nodes are shut off to conserve energy. Similar energy conservation strategies via turning off sensor nodes with poor channel links can be identified in [9], [10], [18], and [19]. From (3.25), the

assigned message length for each active sensor node is inversely proportional to w_i. This is intuitively reasonable while sensor nodes with better channel conditions should be distributed with more bits (and thus more energy) to enhance the estimation accuracy.

2. The optimization problem described in this chapter merely takes the effect of limited total energy into consideration. In order to prevent sensor nodes from exhausting energy quickly, one intuitive solution is to impose an additional peak energy constraint

$$w_i\left(2^{b_i}-1\right) \le E_P, \quad 1 \le i \le N, \tag{3.26}$$

where E_P is the energy budget per sensor node. By defining a node index set: $\Gamma := \left\{ i \middle| w_i\left(2^{b_{i,opt}}-1\right) > E_P, \quad 1 \le i \le N \right\}$, the bit load is thus

$$b_{i,opt} = \begin{cases} 0, & 1 \le i \le K_1 - 1, \\ \frac{1}{2}\log_2\left(B_{i,opt}+1\right), & K_1 \le i \le N \text{ and } i \notin \Gamma, \\ \log_2\left(1 + {E_P}/{w_i}\right), & K_1 \le i \le N \text{ and } i \in \Gamma. \end{cases} \tag{3.27}$$

This concept can also be applied to the strategies which are described in the next chapter.

3. The practical bit load is evaluated via rounding the optimal real bit load to the nearest integer. Compared to upper-integer rounding and lower-integer rounding, nearest-integer rounding is more closed to the optimal bit load. Thus energy is used more efficiently.

3.4 Computer Simulations

In Section 3.2, an optimal bit allocation scheme is proposed to minimize the reconstruction MSE. The simulated performance of the proposed solution in (3.25) is compared with the scheme of uniform energy allocation with bit load determined via the following equation:

$$w_i\left(2^{b_i}-1\right)=\frac{E_T}{N},\quad 1\le i\le N. \tag{3.28}$$

In (3.28), b_i is evaluated via applying lower integer rounding so that the resultant total energy can be kept beneath E_T. Therefore it leads to

$$b_{i,uni}=\left\lfloor \log_2\left(\frac{E_T}{Nw_i}+1\right)\right\rfloor, \tag{3.29}$$

where $\lfloor x\rfloor$ returns the highest integer that is less than or equal to x. In each independent simulation we simply choose $\kappa=2$ and $d_i\in[1,10]$ which is a uniform distributed random variable with possible values $1, 2, \dots, 10$. The total number of trials is 100000 and the number of sensor nodes is set to be 150 in the consequent experiments. The available total energy, that is, energy constraint, is thus

$$E_T=\rho\sum_{i=1}^{N}w_i, \tag{3.30}$$

where ρ is defined as the level of total energy.

With fixed $\delta=2$ and $\alpha=4$, Figure 3.2 displays the computed average MSE as ρ varies from 0.1 to 3. As ρ increases, that is, E_T increases, the estimation performance increases. The proposed solution (3.25) outperforms the uniform energy allocation strategy described in (3.29), especially when E_T is small. Moreover, the average total energy consumption of the proposed method is less than the uniform energy allocation strategy. The MSE of each scheme saturates as ρ (E_T) is larger than a certain value. Under this circumstance, the estimation performance is limited due to the limited number of sensors. This phenomenon will be discussed in Section 4.3. Even though the estimation performance can not be enhanced further, the proposed method still outperforms the uniform energy allocation strategy described in (3.29).

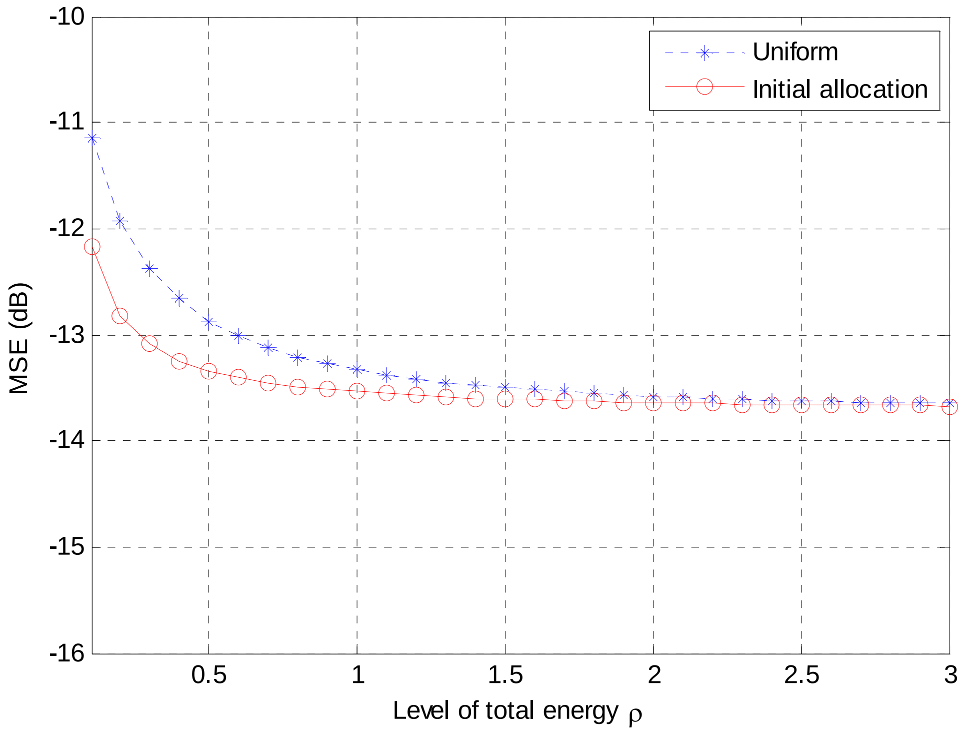

Figure 3.2: Average MSE vs. varying level of total energy

With $\delta = 2$, Figure 3.3 displays the computed average MSE as α varies from 0.5 to 8. Three different levels of total available energy are considered in Figure 3.3. The performance enhancement of the proposed method becomes more significant as the noise variance variation (α) gets larger, which means a more inhomogeneous sensing environment. The estimation performance of each strategy degrades while the noise variance variation becomes larger. Moreover, as observed in Figure 3.2, the estimation performance improves as the total available energy E_T increases. The proposed strategy outperforms the uniform energy allocation strategy more significantly under low energy constraint since the proposed strategy allocates energy more efficiently.

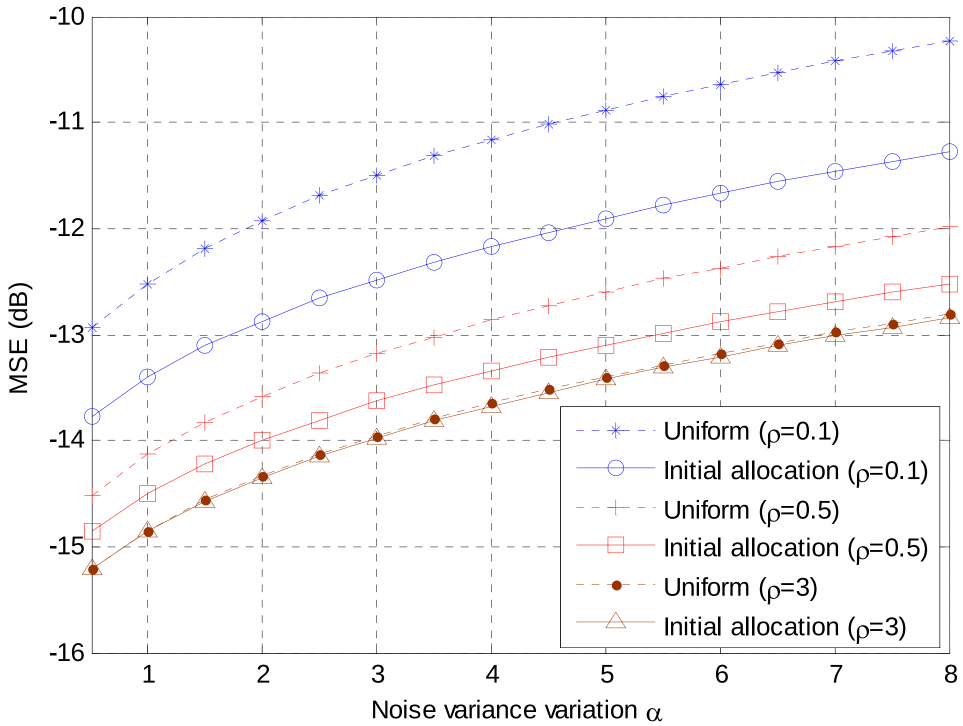

Figure 3.3: Average MSE vs. varying noise variance variation factor

The percentage of energy consumption is defined as

$$\text{percentage of energy consumption} = \frac{\sum_{i=1}^{N} w_i \left(2^{b_i} - 1\right)}{E_T}, \tag{3.31}$$

Where b_i is the bit load allocated to sensor i. With $\delta = 2$, Figure 3.4 and Figure 3.5 display the average percentage of energy consumption for α varying from 0.5 to 8. When the energy level is low, the energy consumed by the proposed method is lower than the uniform energy allocation strategy. A similar phenomenon can be observed for high energy level as depicted in Figure 3.5. Although the energy consumption of the proposed method is lower than the uniform energy allocation strategy, the estimation performance of the former is better.

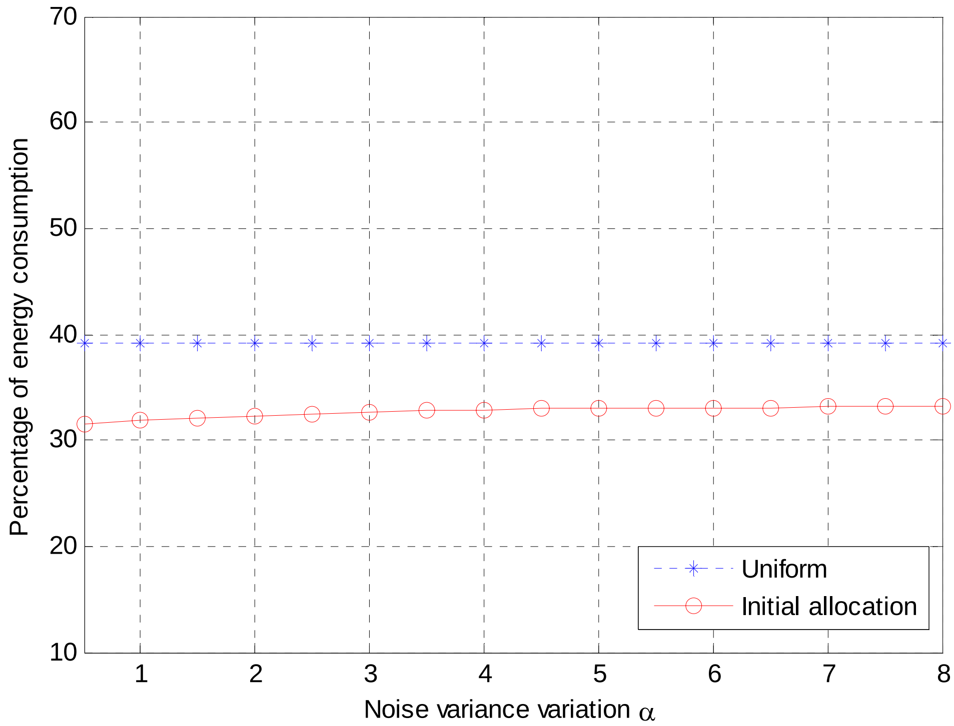

Figure 3.4: Percentage of energy consumption vs. varying noise variance variation factor with low energy budget

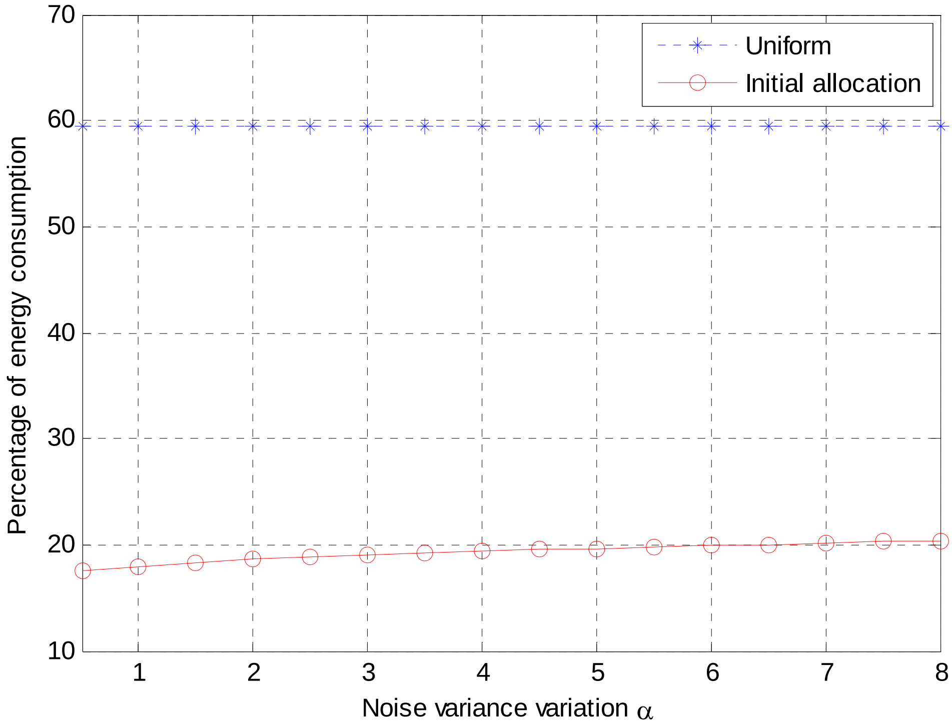

Figure 3.5: Percentage of energy consumption vs. varying noise variance variation factor with high energy budget

With $\alpha = 4$, Figure 3.6 displays the computed average MSE as δ varies from 0.5 to 8. Three different levels of total available energy are considered in Figure 3.6. The performance enhancement of the proposed method becomes more significant as the minimal noise variance (δ) gets larger, which means that the local SNR degrades. While the minimal noise variance, that is, δ, increases, the estimation accuracy degrades obviously. The performance degradation is corresponding to lower SNR since the noise variance increases. However, the estimation performance enhancement is more significant as the minimal noise variance increases. As observed in Figure 3.2, the estimation accuracy improves as the total available energy E_T increases. The proposed strategy outperforms the uniform energy allocation strategy more significantly when the energy constraint E_T is extremely small. This phenomenon is reasonable since the proposed method focus on effectively distributing energy to sensor nodes under stern environments.

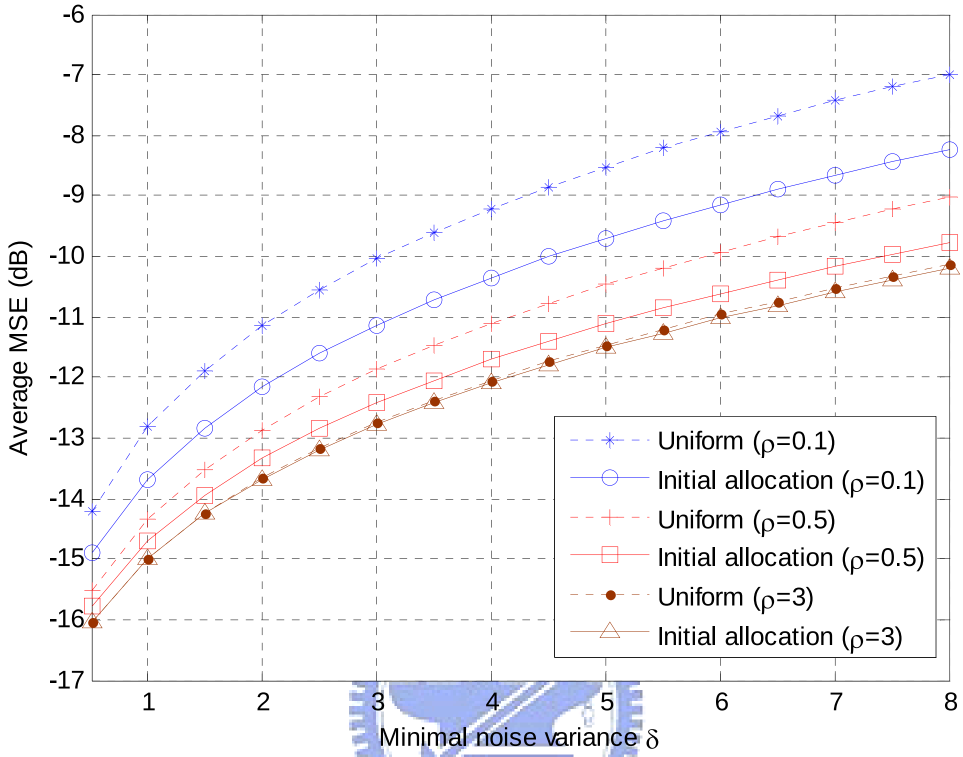

Figure 3.6: Average MSE vs. varying noise variance threshold

3.5 Summary

In this chapter, a closed-form solution to the minimal-MSE decentralized estimation problem is provided by exploiting a statistical noise variance model. Based on the closed-form expression of the performance measure averaged over the noise variance distribution, MSE minimization becomes a convex optimization problem. The analytic closed-form solution presents the energy saving policy. The proposed solution simply allocates energies to sensor nodes with large channel gains and shut off those suffering from poor link quality. Numerical simulations reveal that the estimation accuracy is upgraded as total energy increases. The proposed solution outperforms the uniform energy allocation strategy especially when the total available energy is extremely low. Thus the proposed strategy is more effective in an energy-limited environment, which approaches practical wireless sensor network environments.

Chapter 4

Iterative Allocation of Remaining Energy at Sensor Nodes

In the previous chapter, a tighter energy constraint (3.14) is adopted in order to facilitate the derivation of the closed-form solution (3.25). However, applying the tighter energy constraint might lead to inefficient usage of the available energy resource since the genuine energy consumed by the bit allocation (3.25) could be substantially beneath the energy budget E_T. To remedy this drawback, one approach is to contrive a mechanism for distributing the remaining energy over a certain sensor nodes, thereby the estimation performance can be enhanced further.

Two procedures for recursively distributing remaining energy are presented in this chapter. The core idea of the first approach proposed is to maximize a certain measure of the incremental increase in the average reciprocal MSE lower bound (3.12) as long as more bits are distributed to sensor nodes. Instead of distributing remaining energy to whole sensor nodes, the second approach proposed mainly focuses on activating sensor nodes which are turned off at present. The first method, namely method of allocation to all sensor nodes, is introduced in Section 4.1. Then the second method, that is, method of allocation to unused sensor nodes only, is presented in Section 4.2. Section 4.3 makes comparison between these two methods proposed in this thesis. Numerical simulations are exhibited in Section 4.4. Finally, Section 4.5 summarizes this chapter

4.1 Method I: Allocation to All Sensor Nodes

By setting $b_i^{(0)} = b_i$, the ith summand in the summation (3.12), namely,

$$I_i^{(0)} := \frac{4^{b_i^{(0)}}}{\gamma_i 4^{b_i^{(0)}} + \beta}, \tag{4.1}$$

can be regarded as the amount of average MSE reduction contributed by the ith sensor node with $b_i^{(0)}$ quantization bits. If extra $b_i^{(1)}$ bits are allocated to the ith sensor node after the first iteration (hence there are totally $b_i^{(0)} + b_i^{(1)}$ bits assigned to this sensor node), $I_i^{(0)}$ in (4.1) is increased to

$$I_i^{(1)} := \frac{4^{b_i^{(0)}+b_i^{(1)}}}{\gamma_i 4^{b_i^{(0)}+b_i^{(1)}} + \beta} = \frac{4^{b_i^{(0)}}}{\gamma_i 4^{b_i^{(0)}} + \beta 4^{-b_i^{(1)}}}. \tag{4.2}$$

$I_i^{(1)} \geq I_i^{(0)}$ can be verified since $4^{b_i^{(1)}} \geq 1$. A conceivable criterion for distributing the remaining energy resource would be directly maximizing the summation, $\sum_{i=1}^{N} I_i^{(1)}$.

Motivated by this observation, the problem of allocating the remaining energy after $l-1$ times iterations can be formulated as:

$$
\begin{aligned}
&\text{maximize} \sum_{i=1}^{N} \frac{4^{t_i^{(l-1)}+b_i^{(l)}}}{\gamma_i 4^{t_i^{(l-1)}+b_i^{(l)}} + \beta}, \\
&\text{subject to} \sum_{i=1}^{N} w_i \left(2^{t_i^{(l-1)}+b_i^{(l)}} - 1\right) \le E_T, \\
&\qquad t_i^{(l-1)}, b_i^{(l)} \in \mathbb{Z}_0^+, \\
&\qquad 1 \le i \le N,
\end{aligned} \tag{4.3}
$$

where $t_i^{(l-1)} = b_i^{(0)} + \cdots + b_i^{(l-1)}$ denotes the total bits assigned to the ith sensor node before the lth iteration begins. The optimization problem in (4.3) is similar to (3.13), with b_i replaced by $t_i^{(l-1)} + b_i^{(l)}$. Since $b_i^{(l)} \in \mathbb{Z}_0^+$, it follows

$$
\begin{aligned}
\sum_{i=1}^{N} w_i \left(2^{t_i^{(l-1)}+b_i^{(l)}} - 1\right) &\le \sum_{i=1}^{N} w_i 2^{t_i^{(l-1)}} 2^{b_i^{(l)}} \\
&\le \sum_{i=1}^{N} w_i 2^{t_i^{(l-1)}} 4^{b_i^{(l)}}.
\end{aligned} \tag{4.4}
$$

Therefore the total energy constraint in (4.3) can be substituted by the subsequent equation without violating the overall energy budget requirement:

$$
\sum_{i=1}^{N} w_i 2^{t_i^{(l-1)}} 4^{b_i^{(l)}} \le E_T. \tag{4.5}
$$

With the aid of (4.5), the optimization problem becomes

$$
\begin{aligned}
&\text{maximize} \sum_{i=1}^{N} \frac{T_i^2 \widetilde{B}_i}{\gamma_i T_i^2 \widetilde{B}_i + \beta}, \\
&\text{subject to} \sum_{i=1}^{N} w_i T_i \widetilde{B}_i \le E_T, \\
&\qquad \widetilde{B}_i \ge 1, \\
&\qquad 1 \le i \le N,
\end{aligned} \tag{4.6}
$$

where $T_i = 2^{t_i^{(l-1)}}$ and $\widetilde{B}_i = 4^{b_i^{(l)}}$ for notation simplicity. The iteration index, l, is omitted in the formulations below. In order to render the problem tractable, $\widetilde{B}_i$ is relaxed to be a non-negative real number.

In order to solve problem (4.6), we can form the Lagrangian

$$
\begin{aligned}
& L\left(B_1,\ldots,B_N,\lambda,\mu_1,\ldots,\mu_N\right) \\
& = \sum_{i=1}^{N} \frac{T_i^2 \widetilde{B}_i}{\gamma_i T_i^2 \widetilde{B}_i + \beta} - \lambda\left(\sum_{i=1}^{N} w_i T_i B_i - E_T\right) + \sum_{i=1}^{N} \mu_i \left(B_i - 1\right).
\end{aligned} \tag{4.7}
$$

The set of KKT conditions [23] then yields:

$$
\frac{T_i^2 \beta}{\left(\gamma_i T_i^2 \widetilde{B}_i + \beta\right)^2} - \lambda w_i T_i + \mu_i = 0, \quad i = 1,\ldots,N, \tag{4.8}
$$

$$
\lambda\left(\sum_{i=1}^{N} w_i T_i \widetilde{B}_i - E_T\right) = 0, \tag{4.9}
$$

$$
\lambda \geq 0, \tag{4.10}
$$

$$
\mu_i \geq 0, \quad \mu_i\left(\widetilde{B}_i - 1\right) = 0, \quad \widetilde{B}_i \geq 1, \quad 1 \leq i \leq N. \tag{4.11}
$$

Solving for (4.8) leads to

$$
\widetilde{B}_i = \frac{\sqrt{\beta T_i^2}}{\gamma_i T_i^2 \sqrt{\lambda w_i T_i - \mu_i}} - \frac{\beta}{\gamma_i T_i^2}. \tag{4.12}
$$

According to (4.12), λ and μ_i 's should be determined to satisfy the desired constraints. If $\lambda = 0$, (4.8) implies $\mu_i > 0$ for all $1 \leq i \leq N$ and thus $\widetilde{B}_i - 1 = 0, \ 1 \leq i \leq N$. This circumstance should be precluded since all sensor nodes are turned off. By sorting sensor nodes according to the term: $\left[\left(\beta + \gamma_i T_i^2\right)\sqrt{w_i}\right] \big/ \sqrt{T_i}$ and by arranging new indexes to the sensor nodes, the following relationship is obtained:

$$
\left[\left(\beta + \gamma_1 T_1^2\right)\sqrt{w_1}\right] \Big/ \sqrt{T_1} \geq \cdots \geq \left[\left(\beta + \gamma_N T_N^2\right)\sqrt{w_N}\right] \Big/ \sqrt{T_N}\,. \tag{4.13}
$$

Then we define the function

$$f_1(K) = \frac{\sqrt{T_K}\left(E_T + \beta \sum_{j_1=K}^{N} \frac{w_{j_1}}{\gamma_{j_1} T_{j_1}}\right)}{\left(\beta + \gamma_K T_K^2\right)\sqrt{w_K} \sum_{j_2=K}^{N} \frac{\sqrt{w_{j_2}}}{\gamma_{j_2}\sqrt{T_{j_2}}}}, \quad 1 \le K \le N. \tag{4.14}$$

Let $1 \le K_2 \le N$ be the unique integer that satisfies $f_1(K_2 - 1) < 1$ and $f_1(K_2) \ge 1$. If $f_1(K) \ge 1$ for all $1 \le K \le N$, then we simply set $K_2 = 1$. The existence and uniqueness of such K_2 is shown in Lemma 4.1 with proof given in Appendix C.

Lemma 4.1: For the function $f_1(K)$ defined in (4.14), if there exist K_2 such that $f_1(K_2) \ge 1$, then $f_1(K_2 + 1) \ge 1$. Moreover, $f_1(N) \ge 1$ holds all the time.

The optimal solution pair $\left(\lambda_{opt}, B_{i,opt}\right)$ can be obtained by

$$\sqrt{\lambda_{opt}} = \frac{\sqrt{\beta} \sum_{j_1=K_2}^{N} \frac{\sqrt{w_{j_1}}}{\gamma_{j_1}\sqrt{T_{j_1}}}}{E_T + \beta \sum_{j_2=K_2}^{N} \frac{w_{j_2}}{\gamma_{j_2} T_{j_2}}}, \tag{4.15}$$

and

$$\widetilde{B}_{i,opt} = \begin{cases} 0, & 1 \le i \le K_2 - 1, \\ \dfrac{E_T + \beta \sum_{j_1=K_2}^{N} \frac{w_{j_1}}{\gamma_{j_1} T_{j_1}}}{\gamma_i \sqrt{w_i T_i^3} \sum_{j_2=K_2}^{N} \frac{\sqrt{w_{j_2}}}{\gamma_{j_2}\sqrt{T_{j_2}}}} - \dfrac{\beta}{\gamma_i T_i^2}, & K_2 \le i \le N. \end{cases} \tag{4.16}$$

Since $\widetilde{B}_i = 4^{b_i^{(l)}}$, the optimal bit load is thus

$$b_{i,opt}^{(l)} = \begin{cases} 0, & 1 \le i \le K_2 - 1, \\ \frac{1}{2}\log_2\left(\widetilde{B}_{i,opt}\right), & K_2 \le i \le N. \end{cases} \tag{4.17}$$

The extra bit load transmitted from the ith sensor node after the lth iteration, that is,

$b_i^{(l)}$, can be evaluated via rounding the optimal bit load, $b_{i,opt}^{(l)}$, to the nearest integer. The reason that we do not adopt the actual bit load is owing to practical system's limitation. Finally, the iteration terminates in the l_1th iteration as soon as $\sum_{i=1}^{N} w_i \left(2^{t_i^{(l)}} - 1\right) \le E_T$ for $l = 1, \ldots, l_1$ and $\sum_{i=1}^{N} w_i \left(2^{t_i^{(l)}} - 1\right) > E_T$ for $l > l_1$, iteration terminates. By taking the l_1-times iterations into consideration, the total bits allocated to the ith sensor node is

$$t_i^{(l_1)} = b_i^{(0)} + \cdots + b_i^{(l_1)}. \tag{4.18}$$

The method for further bit allocation described in this section can be illustrated by Figure 4.1. Sensor nodes are initially sorted. Then applying the bit distribution strategy described in this section we obtain the bit loading topology for the lth iteration. If there is still remaining energy, we sort sensor nodes and adopt the same procedure repeatedly. When the energy allocated to sensor nodes becomes larger than available energy, we discard the lth bit allocation and the procedure terminates.

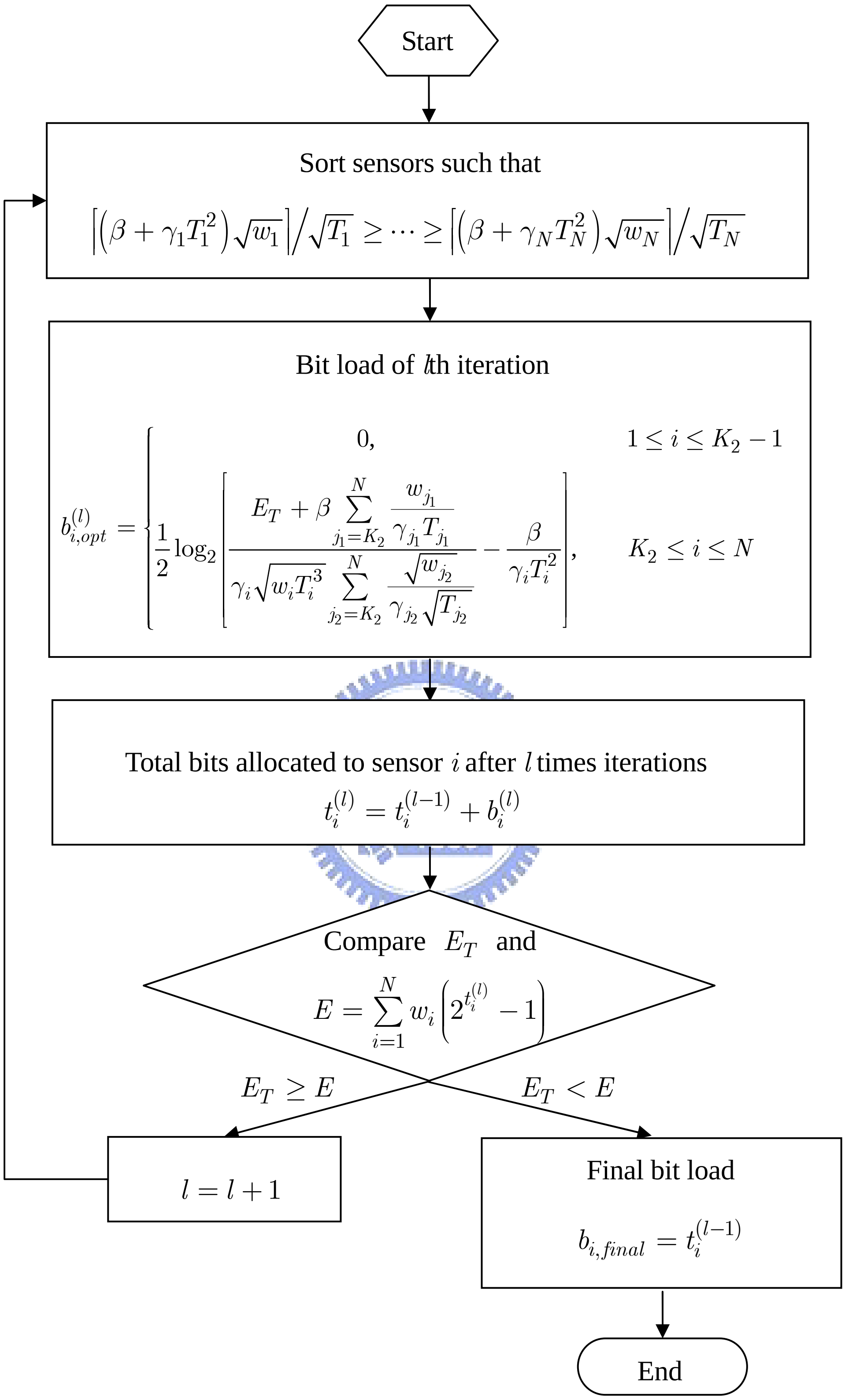

Figure 4.1: Flow chart of method of allocation to all sensor nodes

4.2 Method II: Allocation to Unused Sensor Nodes Only

Assume that the first $N^{(1)}$ sensor nodes are inactive after initial bit allocation described in Chapter 3. Owing to the practical system's limitation, the optimal bit load of each sensor node is rounded to the nearest integer. Therefore there might be more than $K_1 - 1$ sensor nodes shut off due to the rounding topology applied. The remaining energy is distributed to these inactive sensor nodes to further improve the estimation performance. The optimization problem which is similar to the one described in Chapter 3, can then be formulated as

$$\begin{aligned}
&\text{maximize} \sum_{i=1}^{N^{(1)}} \frac{B_i + 1}{\gamma_i (B_i + 1) + \beta}, \\
&\text{subject to} \sum_{i=1}^{N^{(1)}} w_i B_i \le E_T - \sum_{i=N^{(1)}+1}^{N} w_i B_i, \\
&\qquad B_i \ge 0, \\
&\qquad 1 \le i \le N^{(1)}.
\end{aligned} \tag{4.19}$$

where $B_i = 4^{b_i} - 1$ is the same as the one defined in Chapter 3 with $B_i,\ \ i = N^{(1)} + 1, \ldots, N$ fixed and $B_i,\ \ i = 1, \ldots, N^{(1)}$ able to be modified while the first iteration completes.

Assuming that the first $N^{(l)}$ sensor nodes are turned off after $l-1$ times iterations, the optimization problem of distributing the residual energy can be formulated as:

$$
\begin{aligned}
&\text{maximize } \sum_{i=1}^{N^{(l)}} \frac{B_i + 1}{\gamma_i (B_i + 1) + \beta}, \\
&\text{subject to } \sum_{i=1}^{N^{(l)}} w_i B_i \le \widetilde{E}_T^{(l)}, \\
&B_i \ge 0, \\
&1 \le i \le N^{(l)},
\end{aligned}
\tag{4.20}
$$

where $\widetilde{E}_T^{(l)} := E_T - \sum_{i=N^{(l)}+1}^{N} w_i B_i$ is the residual energy after $l-1$ times iterations. Moreover, B_i, $i = N^{(l)}+1, \ldots, N$ is fixed and B_i, $i = 1, \ldots, N^{(l)}$ can be modified while the lth iteration terminates. By relaxing the bit loads under consideration, that is, B_i, $i = 1, \ldots, N^{(l)}$, to be a non-negative real number so as to render the optimization problem tractable, the associated Largrangian is as follows:

$$
\begin{aligned}
&L\left(B_1, \ldots, B_{N^{(l)}}, \lambda, \mu_1, \ldots, \mu_{N^{(l)}}\right) \\
&= \sum_{i=1}^{N^{(l)}} \frac{B_i + 1}{\gamma_i (B_i + 1) + \beta} - \lambda \left(\sum_{i=1}^{N^{(l)}} w_i B_i - E_T \right) + \sum_{i=1}^{N^{(l)}} \mu_i B_i.
\end{aligned}
\tag{4.21}
$$

The associated set of KKT conditions [23] is described below:

$$
\frac{\beta}{\left[\gamma_i (B_i + 1) + \beta\right]^2} - \lambda w_i + \mu_i = 0, \quad i = 1, \ldots, N^{(l)}, \tag{4.22}
$$

$$
\lambda \left(\sum_{i=1}^{N^{(l)}} w_i B_i - E_T \right) = 0, \tag{4.23}
$$

$$
\lambda \ge 0, \tag{4.24}
$$

$$
\mu_i \ge 0, \quad \mu_i B_i = 0, \quad B_i \ge 0, \quad 1 \le i \le N^{(l)}. \tag{4.25}
$$

The condition (4.22) leads to

$$
B_i = \frac{\sqrt{\beta}}{\gamma_i \sqrt{\lambda w_i - \mu_i}} - \frac{\beta}{\gamma_i} - 1. \tag{4.26}
$$

According to (4.26), λ and μ_i 's should be determined to fulfill the desired constraints. If $\lambda = 0$, (4.22) implies $\mu_i > 0$ for all $1 \le i \le N^{(l)}$ and hence $B_i = 0,\ 1 \le i \le N^{(l)}$. Since all sensor nodes are shut off under this circumstance, it should be precluded.

From the assumptions in Chapter 3: $w_1 \ge \cdots \ge w_N$ and $\gamma_1 \ge \cdots \ge \gamma_N$, the following statements can be demonstrated naturally

$$w_1 \ge w_2 \ge \cdots \ge w_{N^{(l)}}, \tag{4.27}$$

and

$$\gamma_1 \ge \gamma_2 \ge \cdots \ge \gamma_{N^{(l)}}. \tag{4.28}$$

Let us define the function

$$f_2(K) := \frac{\widetilde{E}_T^{(l)} + \sum_{j_1=K}^{N^{(l)}} \left(\frac{\beta w_{j_1}}{\gamma_{j_1}} + w_{j_1} \right)}{(\gamma_K + \beta)\sqrt{w_K} \sum_{j_2=K}^{N^{(l)}} \frac{\sqrt{w_{j_2}}}{\gamma_{j_2}}}, \quad 1 \le K \le N^{(l)}. \tag{4.29}$$

Let $1 \le K_3 \le N^{(l)}$ be the unique integer that satisfies $f_2(K_3 - 1) < 1$ and $f_2(K_3) \ge 1$. If $f_2(K) \ge 1$ for all $1 \le K \le N^{(l)}$, we simply set $K_3 = 1$. Since (4.29) is similar to (3.22) with E_T replaced by $\widetilde{E}_T^{(l)}$ and N replaced by $N^{(l)}$, the existence and uniqueness of K_3 can be shown in Lemma 3.2.

Eventually, the optimal λ_{opt} and $B_{i,opt}$, $i = 1, \ldots, N^{(l)}$ of the lth iteration can be obtained by

$$\sqrt{\lambda_{opt}} = \frac{\sqrt{\beta}\sum_{j_1=K_3}^{N^{(l)}}\frac{\sqrt{w_{j_1}}}{\gamma_{j_1}}}{\widetilde{E}_T^{(l)} + \sum_{j_2=K_3}^{N^{(l)}}\frac{\left(\beta+\gamma_{j_2}\right)w_{j_2}}{\gamma_{j_2}}}, \tag{4.30}$$

and

$$B_{i,opt} = \begin{cases} 0, & 1 \le i \le K_3 - 1, \\ \dfrac{\widetilde{E}_T^{(l)} + \sum_{j_1=K_3}^{N^{(l)}}\left(\dfrac{\beta w_{j_1}}{\gamma_{j_1}} + w_{j_1}\right)}{\gamma_i\sqrt{w_i}\sum_{j_2=K_3}^{N^{(l)}}\dfrac{\sqrt{w_{j_2}}}{\gamma_{j_2}}} - \dfrac{\beta}{\gamma_i} - 1, & K_3 \le i \le N^{(l)}. \end{cases} \tag{4.31}$$

Since $B_i = 4^{b_i} - 1$, the optimal bit load is thus

$$b_{i,opt} = \begin{cases} 0, & 1 \le i \le K_3 - 1, \\ \dfrac{1}{2}\log_2\left(B_{i,opt}+1\right), & K_3 \le i \le N^{(l)}. \end{cases} \tag{4.32}$$

The practical bit load transmitted from the ith sensor node, that is, b_i, for $i = 1, \ldots, N^{(l)}$ can be evaluated via applying nearest-integer rounding to the optimal bit load, $b_{i,opt}$. If $\sum_{i=1}^{N} w_i\left(2^{b_i}-1\right) \le E_T$ for $l = 1, \ldots, l_2$ and $\sum_{i=1}^{N} w_i\left(2^{b_i}-1\right) > E_T$ for $l > l_2$, iteration stops. Otherwise, if there is only one sensor node left inactive, the residual energy is allocated to this sensor node and the iterative procedure terminates.

The method for further bit allocation described in this section can be illustrated by Figure 4.2. Sensor nodes are initially sorted. Then applying the bit distribution strategy described in this section we obtain the bit loading topology for the lth iteration. If there are still remaining inactive sensors, we adopt the same procedure repeatedly without sorting them again. When all sensor nodes become active, the procedure terminates.

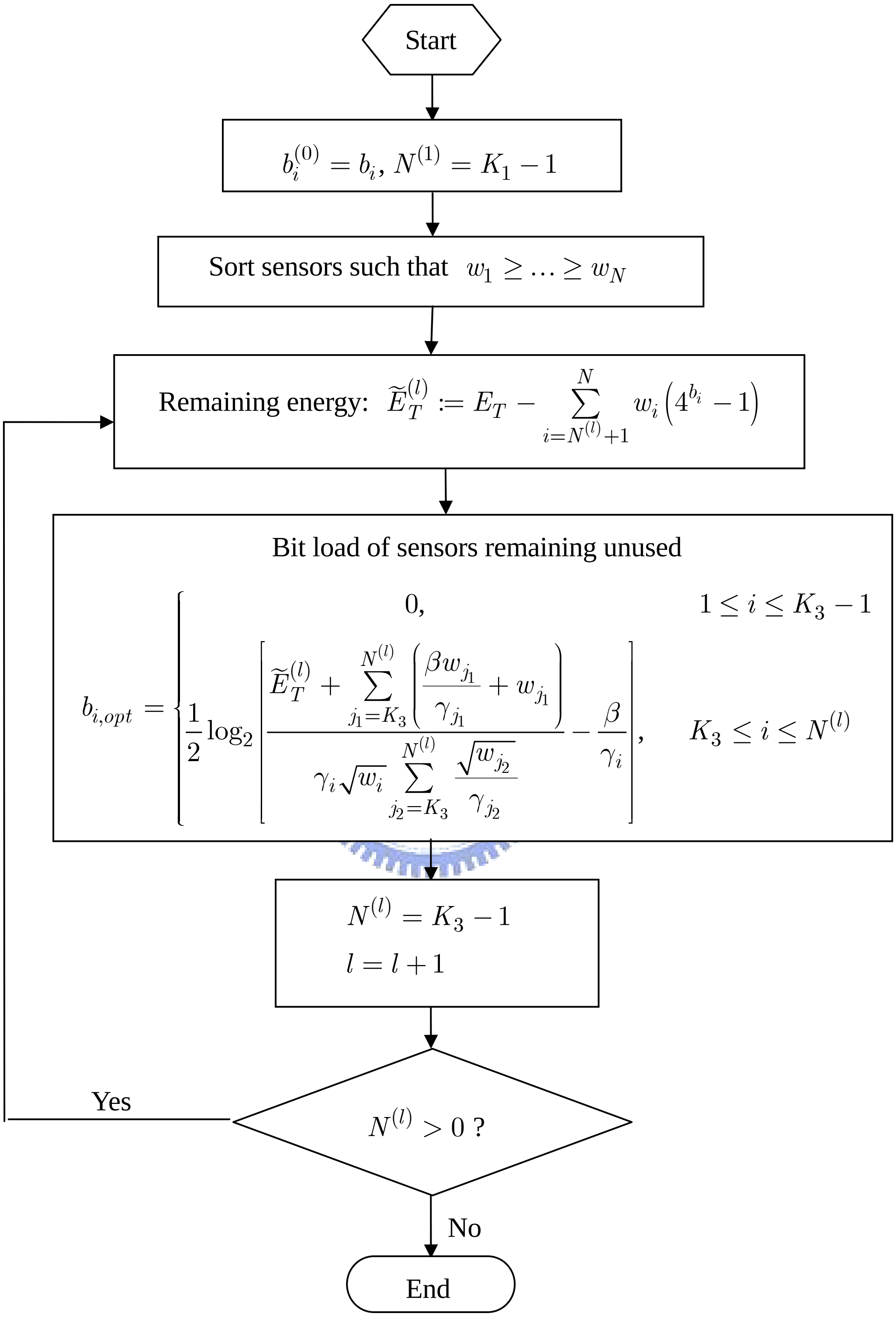

Figure 4.2: Flow chart of method of allocation to unused sensor nodes only

4.3 Discussions on Proposed Methods

In Section 4.1, the proposed iterative strategy distributes the remaining energy over all sensor nodes regardless of sensor nodes' activation. Depending on both the channel conditions and the bit loads already allocated to sensor nodes, this strategy might turn on inactive sensor nodes or assign more bits to active sensor nodes. The ith summand after l times iterations is

$$I_i^{(l)} = \frac{4^{t_i^{(l)}}}{\gamma_i 4^{t_i^{(l)}} + \beta}. \tag{4.33}$$

Observing (4.33), by letting Λ denote the set of activate sensor nodes' indexes, the contribution of the ith summand in the summation, that is,

$$\sum_{i\in\Lambda} \frac{4^{t_i^{(l-1)}+b_i^{(l)}}}{\gamma_i 4^{t_i^{(l-1)}+b_i^{(l)}} + \beta} = \sum_{i\in\Lambda} I_i^{(l)}, \tag{4.34}$$

is limited since

$$\lim_{t_i^{(l)}\to\infty} I_i^{(l)} = \frac{1}{\gamma_i}. \tag{4.35}$$

In other words, while a sensor node's bit load exceeds a certain threshold, allocating more bits to this sensor node does not facilitate the performance enhancement.

However, since the energy consumption of each sensor node is exponentially proportional to the bits it transmitting, assigning additional bits to active sensor nodes leads to significant increase on energy consumption. Thus the remaining energy is mainly consumed by these active sensor nodes and the improvement of estimation accuracy is limited. Since a upper bound of energy consumption is adopted in the optimization problem, the first iterative scheme should terminate while the rounded bit load of each sensor node is equal to zero.

An alternative iterative method is proposed in Section 4.2 as a remedy. Instead of distributing residual energy to all sensor nodes in spite of these nodes' activity, this method mainly focuses on the allocation to unused sensor nodes. This strategy is similar to the initial allocation strategy except that the energy constraint changes into the residual available energy and we merely need to consider the sensor nodes which are still turned off after previous allocations. Compared to the method proposed in Section 4.1, increasing the number of active sensor nodes contributes to the performance enhancement significantly, especially when the available energy is low. This phenomenon is more evident when the bits transmitted from a certain active sensor nodes exceed the threshold. Since the estimation accuracy improvement contributed by one sensor saturates when it transmits over the threshold. The concept of the proposed method in Section 4.2 is similar to water-filling which is mentioned in [25].

There are two situations that the method of allocation to unused sensors only completes. The first is that, it should finish as long as the rounded bit load of each sensor is equal to zero. Moreover, the second situation is that, since the entire residual energy is allocated to one sensor if there is only one sensor node left inactive, applying nearest rounding might lead to energy overflow. To prevent energy consumption from outstripping the energy constraint, the iterative procedure should stop as soon as energy overflows. For example, if energy consumption outstrips the energy budget after the $(l+1)$th iteration, we simply adopt the previous l times iterations' result.

Computer simulations in the next section show that both proposed iterative procedures outperform the performance of initial allocation. Furthermore, the proposed method of allocation to unused sensor nodes only has more estimation accuracy enhancement than the first one while the energy budget is low.

Table 4.1: Comparison of energy distributing methods

Method	Uniform energy	Initial allocation	Iterative method I	Iterative method II
Allocation to	All sensors	All sensors	All sensors	Unused sensors
Performance under low energy budget	Poor	Fair	Good	Best
Performance under high energy budget	Poor	Fair	Best	Good

4.4 Computer Simulations

In Section 4.1 and 4.2, two iterative schemes of allocating remaining energy are proposed to further minimize the reconstruction MSE. The simulated performance of these two strategies are compared to the bit allocating strategy as described in Section 3.2 and the uniform energy allocation strategy as described in Section 3.4. In each independent simulation we simply choose $\kappa = 2$ and $d_i \in [1, 10]$ which is a uniformly distributed random variable with possible values $1, 2, \ldots, 10$. The total number of trials is 100000 and the number of sensors is set to be 150 in the following experiments. The available total energy is $E_T = \rho \sum_{i=1}^{N} w_i$ where ρ can be chosen by us.

With fixed $\delta = 2$ and $\alpha = 4$, Figure 4.3 displays the computed average MSE as ρ varies from 0.1 to 3. As ρ increases, i.e., E_T increases, the estimation accuracy increases. Both the proposed iterative methods outperform not only uniform energy allocation strategy in (3.29), but also the strategy described in (3.25), since these two iterative methods perform further bit load allocation after initial allocation. The performance enhancement is obvious, especially when E_T is small. However, when the available energy budget is extremely small, the performance improvement of the method of allocation to all sensor nodes is not significant. As discussed in Section 4.3, since the available energy is too small, allocating additional bits to active sensors might easily lead to enormous extra energy consumption which leads to energy overflow. Thus the additional bit load allocated to each sensor node at this iteration is not adopted and the bit load finally distributed to each sensor is almost identical to the initial allocation. Moreover, the method of allocation to unused sensor nodes only performs the best under low energy budget as observed, which is consistent with the discussions in the previous section.

As the total available energy increases, the method of allocation to all sensor nodes becomes slightly better than the method of allocation to unused sensor nodes only. The reason for this phenomenon is that the method of allocation to unused sensor nodes only allocates the whole remaining energy to the only sensor which is still unused after previous iterations without optimal evaluation while the method of allocation to all sensor nodes can still apply the optimal bit allocation strategy. We also observe that the MSE of each scheme saturates as ρ (or E_T) is higher than some value. Under this circumstance, the performance is limited due to sensor number constraint. However, even though the performance of the method of allocation to unused sensor nodes only can not be improved further, the proposed methods still outperforms the uniform energy allocation strategy in (3.29).

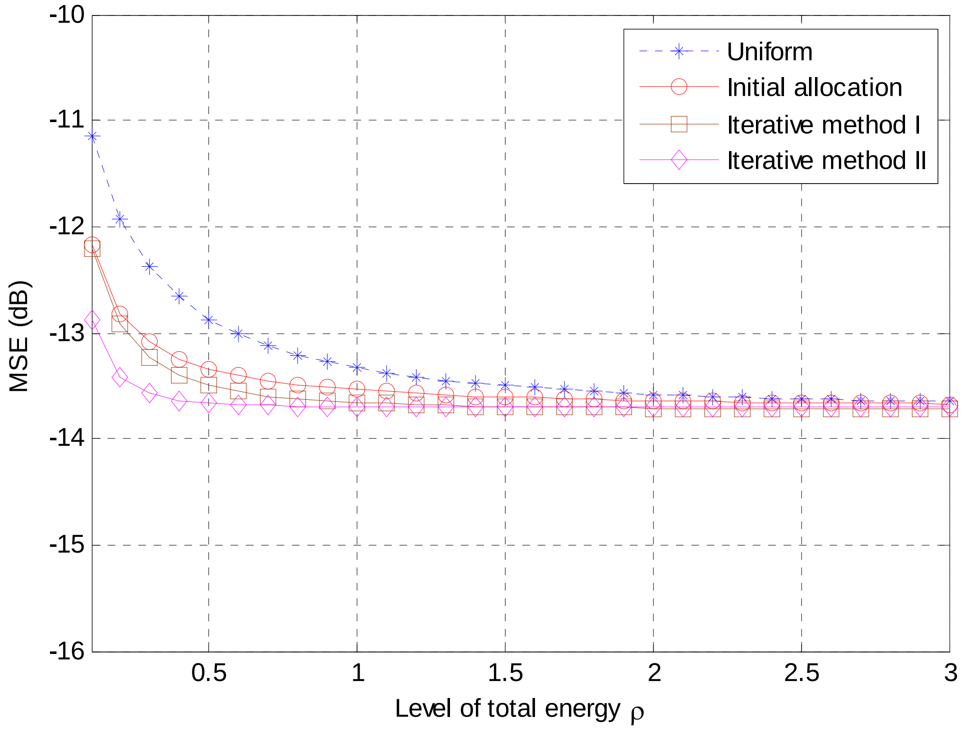

Figure 4.3: Average MSE vs. varying level of total energy

With $\delta = 2$, Figures 4.4-4.6 display the computed average MSE as α varies from 0.5 to 8. Three different levels of total available energy are considered in these figures. The performance enhancement of the proposed method becomes more significant as the noise variance variation (α) gets larger, which corresponds to a more inhomogeneous sensing environment. The estimation accuracy degrades as the noise variance variation (α) increases, which means that the sensing environment becomes more inhomogeneous. Furthermore, the estimation accuracy of all methods improves as the total available energy E_T increases. When E_T is extremely small, both the proposed iterative strategies outperform the uniform energy allocation strategy and the strategy presented in (3.25) significantly as depicted in Figure 4.4. Furthermore, the method which turns on the unused sensor nodes performs the best among these bit allocation strategies under low energy constraint. The energy consumption is exponentially proportional to the bit load. Thus the method of allocation to unused sensor nodes can only distribute the remaining energy to silent sensors without causing energy overflow while the method of allocation to all sensor nodes might easily lead to energy overflow since it distributes energy to active sensors. When E_T is high, the proposed iterative strategies still outperform the uniform energy allocation strategy. However, Figure 4.6 demonstrates that the method of allocation to all sensor nodes becomes the most robust method among these methods for bit allocation. The method described in Section 4.2, which turns on the unused sensor nodes, performs slightly inferior to the method of allocation to all sensor nodes because the second proposed method allocates the whole remaining energy to the only sensor which is still unused after previous iterations without optimal evaluation while the first proposed method still can apply the optimal bit allocation strategy.

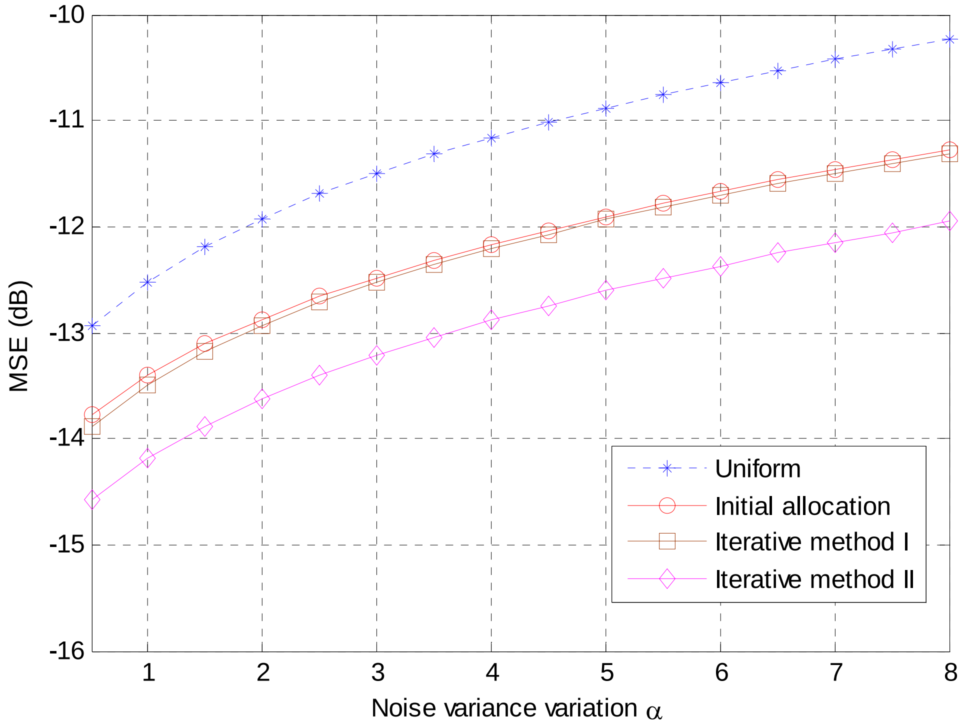

Figure 4.4: Average MSE vs. varying noise variance variation factor with low energy budget

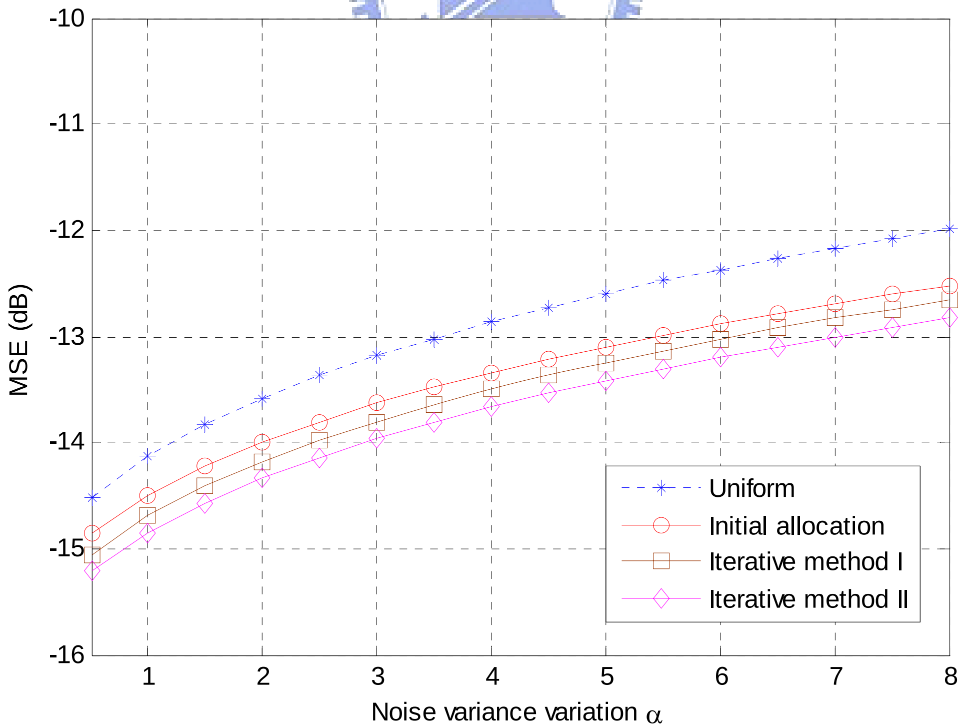

Figure 4.5: Average MSE vs. varying noise variance variation factor with medium energy budget

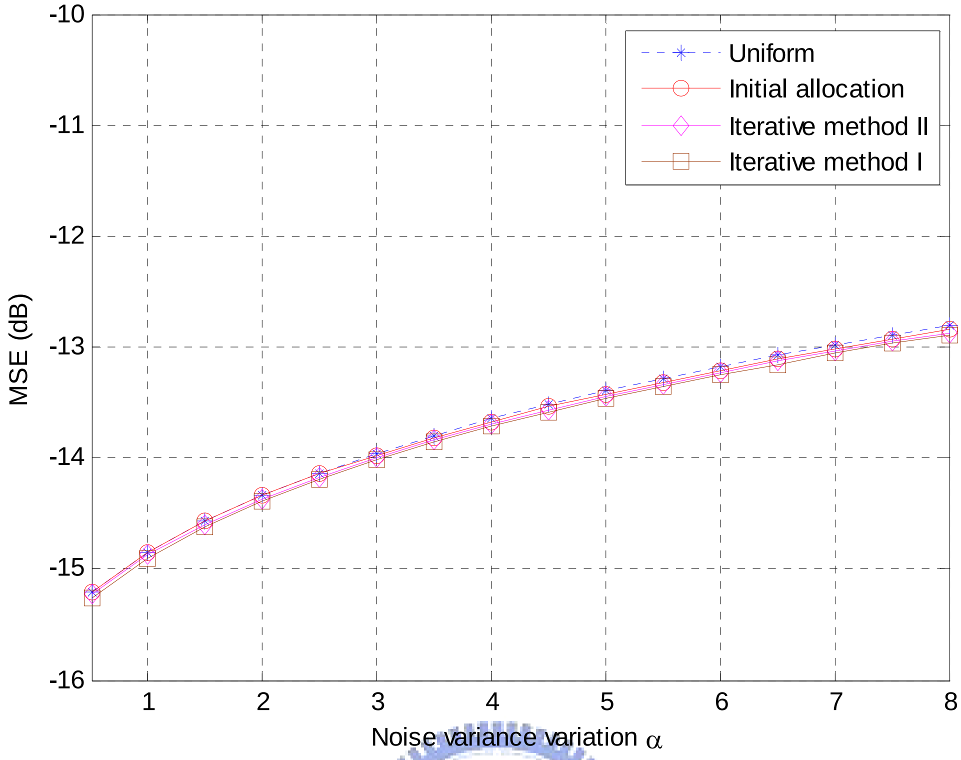

Figure 4.6: Average MSE vs. varying noise variance variation factor with high energy budget

With $\delta = 2$, Figures 4.7-4.9 display the average percentage of energy consumption for α varying from 0.5 to 8. As the percentage of energy consumption upgrades, the estimation accuracy is also enhanced for the proposed methods. When the energy level is low and medium, the energy consumed by the proposed iterative methods is higher than the uniform energy allocation strategy and the initial allocation as depicted in Figure 4.7 and Figure 4.8. It is necessary since further bit distribution is performed. However, as the available energy increases, Figure 4.9 shows that the percentage of energy consumption of the proposed method of allocation to unused sensor nodes only becomes even lower than the uniform energy allocation strategy.

This phenomenon is owing to the rounding scheme applied. Since the whole remaining energy is assigned to the last silent sensor, applying nearest rounding might caused energy overflow. To avoid energy overflow, the ultimate iteration is not adopted, and therefore the residual energy is considerable. Although the energy consumption of the method of allocation to unused sensor nodes only is lower than the uniform energy allocation strategy, the estimation performance of the former is better.

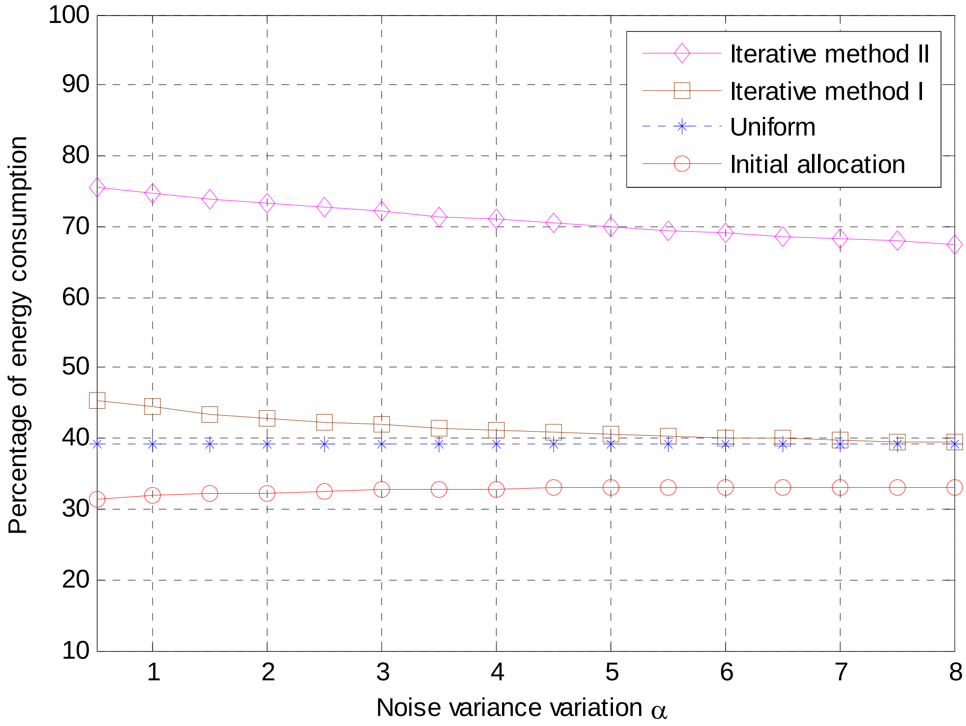

Figure 4.7: Percentage of energy consumption vs. varying noise variance variation factor with low energy budget

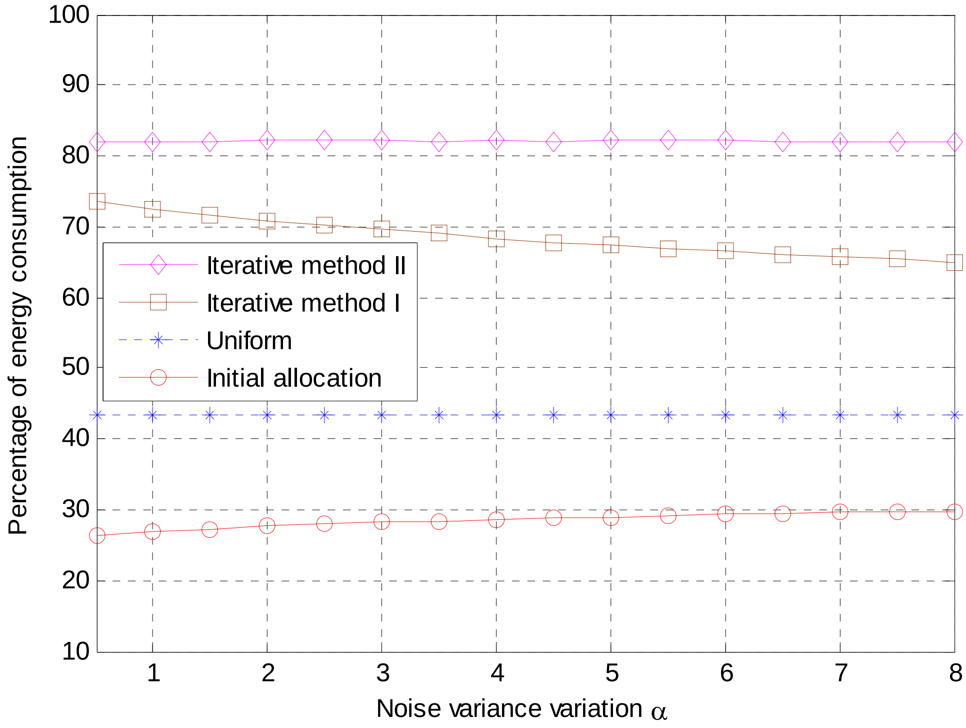

Figure 4.8: Percentage of energy consumption vs. varying noise variance variation factor with medium energy budget

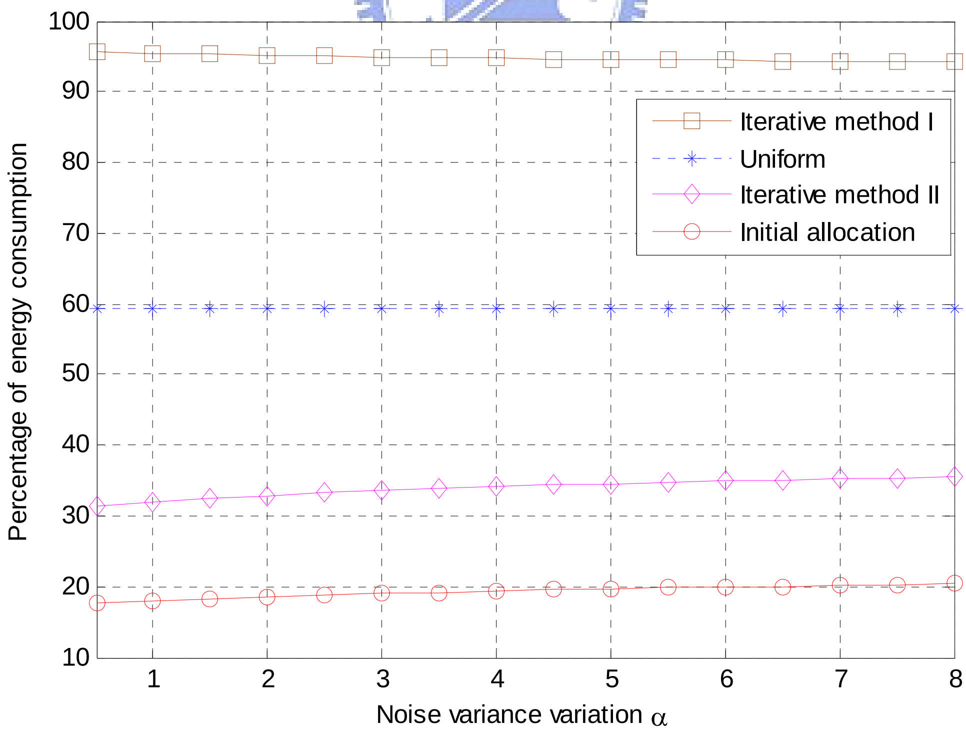

Figure 4.9: Percentage of energy consumption vs. varying noise variance variation factor with high energy budget

With $\alpha = 4$, Figure 4.10 displays the computed average MSE as δ varies from 0.5 to 8. Two different levels of total available energy are considered as in Figure 4.10. The performance enhancement of the proposed method becomes more significant as the minimal noise variance (δ) gets larger, which means that the local SNR degrades. While the minimal noise variance increases, the estimation accuracy degrades more significantly. The performance degradation is due to lower SNR since the noise variance increases. As in Figure 4.3, the estimation accuracy improves as the total available energy E_T increases. The iterative strategies described in this chapter outperform the uniform energy allocation strategy more significantly when the energy constraint E_T is extremely small. The method proposed in Section 4.2 performs the best among these bit allocation strategies.

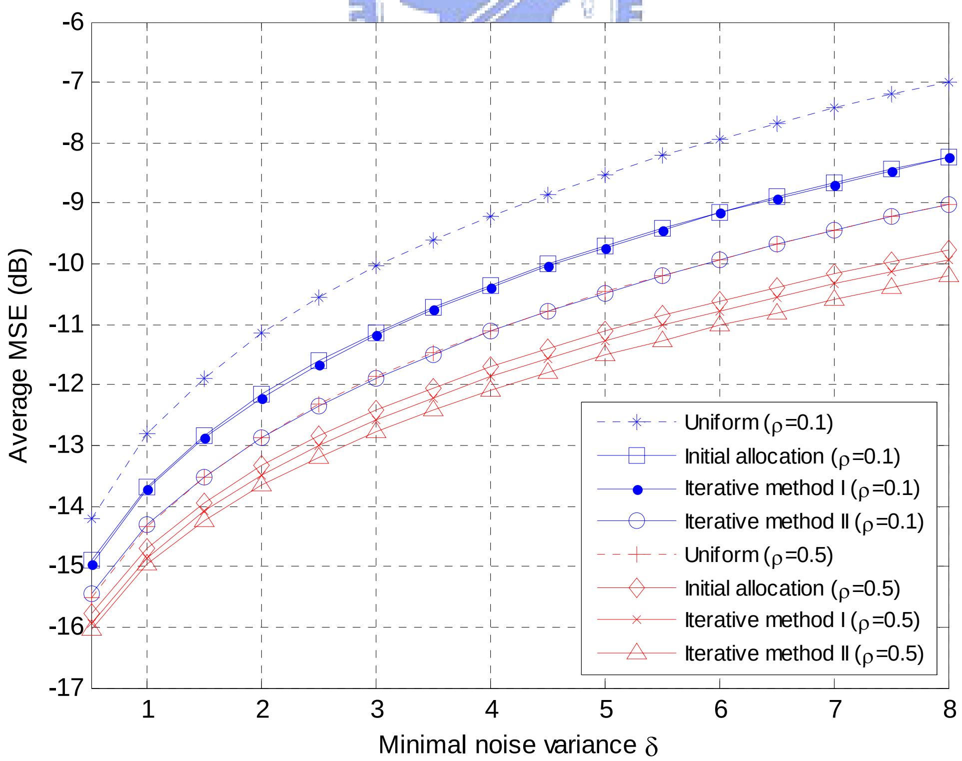

Figure 4.10: Average MSE vs. varying noise variance threshold

4.5 Summary

Two iterative allocation strategies are described in this chapter. Both strategies provide further performance enhancement after initial bit allocation. The first proposed method presented in Section 4.1 takes the entire sensor nodes under consideration. The method of allocation to unused sensor nodes only simply distributes the remaining energy to inactive sensor nodes. Computer simulations show that the estimation accuracy increases as the energy budget increases. The method of allocation to unused sensor nodes only outperforms the method of allocation to all sensor nodes as their energy consumptions are almost the same. Thus the method of allocation to unused sensor nodes only is more effective, especially in an energy-limited inhomogeneous environment.

Chapter 5

Conclusions and Future Works

In this thesis, the wireless sensor network (WSN) which is appropriate for environmental monitoring is considered. A typical WSN architecture consists of a fusion center (FC) and an enormous number of geographically distributed sensor nodes. Sensor nodes in the WSN collect local observations and occasionally transmit local processed messages to the FC via wireless channels. Nevertheless, each sensor node can only perform limited computations and transmissions owing to restrictions of realistic systems such as small sized battery, bandwidth, and cost. Consequently, it is unfeasible for sensor nodes to transmit their entire real-valued local observations to the FC. A more practical decentralized estimation scheme is to let each sensor node quantize its real-valued local measurement into an appropriate length message and then transmit the preprocessed discrete message to the FC. Therefore the FC can combine the whole received messages from sensor nodes to produce a final estimate of the unknown deterministic parameter. Normally, the transmitted message length of each sensor node is determined by power restriction, bandwidth limitation, sensor noise characteristic, and wireless channel condition.

Since energy efficiency is a critical concern for WSN design [9]-[11], the decentralized estimation is formulated as an optimal bit-loading problem. The probability density function (PDF) of the observation noise is difficult to characterize, especially for enormous scale sensor networks. Compared to most of the existing related studies which require the knowledge of instantaneous noise variances to perform power allocation among sensor nodes, the approaches presented instead simply relies on long-term noise variance knowledge. If the sensing environment is harsh, the sensing noise would vary rather rapidly. The proposed signal processing algorithm which depends on an associated sensing noise statistic is thus in demand. Moreover, while the instantaneous noise variance is hard to characterize in the FC, the signal processing algorithm described in this thesis is advantageous.

As error free transmissions are assumed in previous work [18], [19], a more severe channel condition which is closer to the practical sensing environment is considered in this thesis. All sensor nodes transmit their messages to the FC via noisy channels which are modeled as Rayleigh fading channels. The system model and the best linear unbiased estimator (BLUE) scheme adopted in the thesis are described in Chapter 2. According to the system model and the estimation topology presented in Chapter 2, the optimal bit allocation strategies in the consequent chapters are proposed

Chapter 3 of this thesis attempts to provide an optimal solution toward minimizing the average mean square error (MSE) distortion under a stationary energy constraint by exploiting the long-term noise variance information. A commonly used observation noise variance statistical model [9], [10] is adopted and the estimation performance is assessed through an MSE based metric averaged over the considered distribution.

Therefore, the MSE minimization problem can be reformulated in the form of convex optimization problem. A closed form solution of this optimization problem is thus obtained. The method presented in Chapter 3 suggests that each sensor node's behavior should be determined jointly by individual channel fading gain, path loss, local observation noise variance, and energy constraint.

In Chapter 4, two iterative allocation strategies are proposed for further enhancement of performance after initial bit distribution. The first proposed method, namely the method of allocation to all sensor nodes, considers the entire sensor nodes. Thus each sensor node's behavior is determined via individual channel fading gain, path loss, local observation noise variance, energy constraint, and the bit load that is already assigned to the sensor node. The second method, that is, the method of allocation to unused sensor nodes only, simply distributes the remaining energy to inactive sensor nodes by applying a similar distribution energy algorithm as described in Chapter 3. Computer simulations demonstrate that the estimation accuracy increases as the energy constraint increases. The method of allocation to unused sensor nodes only outperforms the method of allocation to all sensor nodes without additional energy consumption while the available total energy is low.?? The approaches proposed in this thesis all outperform the scheme of uniform energy allocation. In an energy-limited inhomogeneous environment, the method of allocation to unused sensor nodes only is the most effective method among the ones proposed in this thesis. As the available total energy increases, the method of allocation to all sensor nodes might become the most effective one.

If the sensor measurement noise is considered as correlated noise, the results might be more appropriate for practical sensing environments because in common sensing environments, the measurement noises of adjacent sensor nodes are highly correlated. A fast-fading channel between each sensor node and the FC can also be considered for realistic applications. These results might be helpful for the applications of mobile sensor networks. However, this would require additional overheads to monitor the channel conditions and is more difficult for the system to derive an optimal power allocation strategy.

Appendices

Appendix A: Proof of Lemma 3.1

Let $\eta_i = \delta + w_i + \beta 4^{-b_i}$ and $u = \alpha z_i + \eta_i$, hence $z_i = (u - \eta_i)/\alpha$ and

$$\int_0^\infty \frac{e^{-z_i/2}}{\left[\delta + \alpha z_i + w_i + \beta 4^{-b_i}\right]\sqrt{z_i}} dz_i = \int_0^\infty \frac{e^{-z_i/2}}{\left[\alpha z_i + \eta_i\right]\sqrt{z_i}} dz_i$$
$$= \frac{1}{\alpha}\int_{\eta_i}^\infty \frac{e^{-(u-\eta_i)/2\alpha}}{u\sqrt{(u-\eta_i)/\alpha}} du$$
$$= \frac{e^{\eta_i/2\alpha}}{\sqrt{\alpha}} \int_{\eta_i}^\infty \frac{e^{-u/2\alpha}}{u\sqrt{u-\eta_i}} du. \tag{A.1}$$

By defining $u = \eta_i \csc^2\theta$, we obtain

$$\int_{\eta_i}^\infty \frac{e^{-u/2\alpha}}{u\sqrt{u-\eta_i}} du = \int_{\pi/2}^0 \frac{e^{-\eta_i \csc^2\theta/2\alpha}}{\eta_i^2 \csc^2\theta \cdot \sqrt{\eta_i}\cot\theta}\left(-2\eta_i^2 \csc^2\theta \cot\theta\right) d\theta$$
$$= \frac{2}{\sqrt{\eta_i}} \int_0^{\pi/2} e^{-\eta_i/2\alpha \sin^2\theta} d\theta. \tag{A.2}$$

Adopting the following alternative expression [26]:

$$\mathrm{Q}(x) = \frac{1}{\pi}\int_0^{\pi/2} e^{-x^2/2\sin^2\theta} d\theta, \tag{A.3}$$

we obtain

$$\int_{\eta_i}^\infty \frac{e^{-u/2\alpha}}{u\sqrt{u-\eta_i}} du = \frac{2\pi}{\sqrt{\eta_i}} Q\left(\sqrt{\frac{\eta_i}{\alpha}}\right). \tag{A.4}$$

Thus

$$J_0 = \sqrt{2\pi} \cdot \sum_{i=1}^{N} \frac{e^{\left[\delta + w_i + \beta 4^{-b_i}\right]/2\alpha} \cdot Q\left(\sqrt{\left[\delta + w_i + \beta 4^{-b_i}\right]/\alpha}\right)}{\sqrt{\alpha\left[\delta + w_i + \beta 4^{-b_i}\right]}}, \tag{A.5}$$

is verified.

Appendix B: Proof of Lemma 3.2

Assume that $f(K_1) \geq 1$. It is straightforward to see that

$$\begin{aligned} f(K_1+1) &= \frac{E_T + \sum\limits_{j_1=K_1+1}^{N}\left(\frac{\beta w_{j_1}}{\gamma_{j_1}} + w_{j_1}\right)}{\left(\gamma_{K_1+1} + \beta\right)\sqrt{w_{K_1+1}} \sum\limits_{j_2=K_1+1}^{N} \frac{\sqrt{w_{j_2}}}{\gamma_{j_2}}} \\ &= \frac{E_T + \sum\limits_{j_1=K_1}^{N}\left(\frac{\beta w_{j_1}}{\gamma_{j_1}} + w_{j_1}\right) - \left(\frac{\beta w_{K_1}}{\gamma_{K_1}} + w_{K_1}\right)}{\left(\gamma_{K_1+1} + \beta\right)\sqrt{w_{K_1+1}}\left(\sum\limits_{j_2=K_1}^{N} \frac{\sqrt{w_{j_2}}}{\gamma_{j_2}} - \frac{\sqrt{w_{K_1}}}{\gamma_{K_1}}\right)}. \end{aligned} \tag{B.1}$$

Adopting the properties $w_{K_1} \geq w_{K_1+1}$ and $\gamma_{K_1} \geq \gamma_{K_1+1}$ yields

$$f(K_1+1) \geq \frac{E_T + \sum\limits_{j_1=K_1}^{N}\left(\frac{\beta w_{j_1}}{\gamma_{j_1}} + w_{j_1}\right) - \left(\frac{\beta w_{K_1}}{\gamma_{K_1}} + w_{K_1}\right)}{\left(\gamma_{K_1} + \beta\right)\sqrt{w_{K_1}} \sum\limits_{j_2=K_1}^{N} \frac{\sqrt{w_{j_2}}}{\gamma_{j_2}} - \left(\frac{\beta w_{K_1}}{\gamma_{K_1}} + w_{K_1}\right)}. \tag{B.2}$$

Let us define

$$\delta = \frac{\dfrac{\beta w_{K_1}}{\gamma_{K_1}} + w_{K_1}}{\left(\gamma_{K_1} + \beta\right)\sqrt{w_{K_1}} \sum\limits_{j_2=K_1}^{N} \dfrac{\sqrt{w_{j_2}}}{\gamma_{j_2}}} > 0. \tag{B.3}$$

From (B.2) and (B.3), the lower bound of $f\left(K_1+1\right)$ can be obtained:

$$f\left(K_1+1\right) \geq \frac{f\left(K_1\right) - \delta}{1 - \delta} \geq 1. \tag{B.4}$$

In addition,

$$\begin{aligned} f(N) &= \frac{E_T + \left(\dfrac{\beta w_N}{\gamma_N} + w_N\right)}{\left(\gamma_N + \beta\right)\sqrt{w_N}\dfrac{\sqrt{w_N}}{\gamma_N}} \\ &= \frac{E_T + w_N + \dfrac{\beta w_N}{\gamma_N}}{w_N + \dfrac{\beta w_N}{\gamma_N}} \\ &\geq 1. \end{aligned} \tag{B.5}$$

Appendix C: Proof of Lemma 4.1

$$\begin{aligned} f_1\left(K_2+1\right) &= \frac{\sqrt{T_{K_2+1}}\left(E_T + \beta \sum\limits_{j_1=K_2+1}^{N} \dfrac{w_{j_1}}{\gamma_{j_1} T_{j_1}}\right)}{\left(\beta + \gamma_{K_2+1} T_{K_2+1}^2\right)\sqrt{w_{K_2+1}} \sum\limits_{j_2=K_2+1}^{N} \dfrac{\sqrt{w_{j_2}}}{\gamma_{j_2}\sqrt{T_{j_2}}}} \\ &= \frac{E_T + \beta \sum\limits_{j_1=K_2}^{N} \dfrac{w_{j_1}}{\gamma_{j_1} T_{j_1}} - \dfrac{\beta w_{K_2}}{\gamma_{K_2} T_{K_2}}}{\left(\beta + \gamma_{K_2+1} T_{K_2+1}^2\right)\sqrt{w_{K_2+1} T_{K_2+1}^{-1}}\left(\sum\limits_{j_2=K_2}^{N} \dfrac{\sqrt{w_{j_2}}}{\gamma_{j_2}\sqrt{T_{j_2}}} - \dfrac{\sqrt{w_{K_2}}}{\gamma_{K_2}\sqrt{T_{K_2}}}\right)} \end{aligned} \tag{C.1}$$

The property $\left(\beta + \gamma_{K_2} T_{K_2}^2\right)\sqrt{w_{K_2} T_{K_2}^{-1}} \geq \left(\beta + \gamma_{K_2+1} T_{K_2+1}^2\right)\sqrt{w_{K_2+1} T_{K_2+1}^{-1}}$ from

(4.13) yields

$$f_1(K_2+1) \geq \frac{E_T + \beta \sum_{j_1=K_2}^{N} \frac{w_{j_1}}{\gamma_{j_1} T_{j_1}} - \frac{\beta w_{K_2}}{\gamma_{K_2} T_{K_2}}}{\left(\beta + \gamma_{K_2} T_{K_2}^2\right)\sqrt{w_{K_2} T_{K_2}^{-1}} \sum_{j_2=K_2}^{N} \frac{\sqrt{w_{j_2}}}{\gamma_{j_2}\sqrt{T_{j_2}}} - \frac{\beta w_{K_2}}{\gamma_{K_2} T_{K_2}}}$$
$$= \frac{f_1(K_2) - \delta_1}{1 - \delta_1}$$
$$\geq 1, \tag{C.2}$$

where $f_1(K_2) \geq 1$ from assumption and

$$\delta_1 = \frac{\frac{\beta w_{K_2}}{\gamma_{K_2} T_{K_2}}}{\left(\beta + \gamma_{K_2} T_{K_2}^2\right)\sqrt{w_{K_2} T_{K_2}^{-1}} \sum_{j_2=K_2}^{N} \frac{\sqrt{w_{j_2}}}{\gamma_{j_2}\sqrt{T_{j_2}}}}. \tag{C.3}$$

Moreover,

$$f_1(N) = \frac{\sqrt{T_N}\left(E_T + \frac{\beta w_N}{\gamma_N T_N}\right)}{\left(\beta + \gamma_N T_N^2\right)\sqrt{w_N}\frac{\sqrt{w_N}}{\gamma_N \sqrt{T_N}}}$$
$$= \frac{E_T + \frac{\beta w_N}{\gamma_N T_N}}{w_N T_N + \frac{\beta w_N}{\gamma_N T_N}} \tag{C.4}$$
$$\geq 1.$$

Bibliography

[1] G. J. Pottie and W. J. Kaiser, "Wireless integrated network sensors," *Commun. ACM*, vol. 43, no. 5, pp. 51-58, May 2000.

[2] D. Estrin, L. Girod, G. J. Pottie, and M. Srivastava, "Instrumenting the world with wireless sensor networks," *Proc. ICASSP 2001*, vol. 4, pp2033-2036, May 2001.

[3] I. F. Akyildiz, W. Su, Y. Sankarsubramaniam, and E. Cayirci, "Wireless sensor networks: A survey," *Comput. Networks*, vol. 38, pp. 393-422, Mar. 2002.

[4] Z.-Q. Luo, "Universal decentralized estimation in a bandwidth constrained sensor network," *IEEE Trans. Inf. Theory*, vol. 51, no. 6, pp. 2210-2219, Jun. 2005.

[5] Z.-Q. Luo, "An isotropic universal decentralized estimation scheme for a bandwidth constrained ad hoc sensor network," *IEEE J. Sel. Areas Commun.*, vol. 23, no. 4, pp. 735-744, Apr. 2005.

[6] Z.-Q. Luo and J.-J. Xiao, "Universal decentralized estimation in an inhomogeneous environment," *IEEE Trans. Inf. Theory*, vol. 51, no. 10, pp. 3564-3575, Oct. 2005.

[7] S. Cui, A. J. Goldsmith, and A. Bahai, "Energy-constrained modulation optimization," *IEEE Trans. Wireless Commun.*, vol. 4, no. 5, pp. 2349-2360, Sep. 2005.

[8] S. Cui, A. J. Goldsmith, and A. Bahai, "Joint modulation and multiple access optimization under energy constraints," *Proc. GLOBECOM*, pp. 151-155, Nov.-Dec. 2004.

[9] J. Xiao, S. Cui, Z.-Q. Luo and A. J. Goldsmith, “Power scheduling of universal decentralized estimation in sensor networks,” *IEEE Trans. Signal Process.*, vol. 54, no. 2, pp. 413-422, Feb. 2006.

[10] A. Krasnopeev, J. J. Xiao, and Z.Q. Luo, “Minimum energy decentralized estimation in a wireless sensor network with correlated sensor noises,” *EURASIP J. Wireless Commun. and Networking*, pp. 473-482, Apr. 2005.

[11] P. Venkitasubramaniam, G. Mergen, L. Tong, and A. Swami, “Quantization for distributed estimation in large scale sensor networks,” *Proc. ICISIP*, pp. 121-127, 2005.

[12] R. Niu, B. Chen, and P. K. Varshney, “Fusion of decisions transmitted over rayleigh fading channels in wireless sensor networks,” *IEEE Trans. Signal Process.*, vol. 54, no. 3, pp. 1018-1027, Mar. 2006.

[13] A. del Coso, U. Spagnolini, and C. Ibars, “Cooperative distributed MIMO channels in wireless sensor networks,” *IEEE J. Sel. Areas Commun.*, vol. 25, no. 2, pp. 402-414, Feb. 2007.

[14] S. M. Kay, *Fundamentals of Statistical Signal Processing: Detection Theory*, Prentice-Hall PTR, 1998.

[15] S. J. Orfanidis, *Introduction to Signal Processing*, Prentice-Hall, Inc., 1996.

[16] A.V. Oppenheim and R. W. Schafer, *Discrete-Time Signal Processing*, Prentice-Hall, Inc., 1989.

[17] S. M. Kay, *Fundamentals of Statistical Signal Processing: Estimation Theory*, Prentice-Hall PTR, 1993.

[18] J. Y. Wu, Q. Z. Huang, and T. S. Lee, “Energy-constrained decentralized best-linear-unbiased estimation via partial sensor noise variance knowledge,” *IEEE Signal Process. Letters* , vol. 15, pp. 33-36, 2008.

[19] J. Y. Wu, Q. Z. Huang, and T. S. Lee, “Minimal energy decentralized estimation based on sensor noise variance statistic,” *Proc. ICASSP 2007*, vol. 2, pp. II-1001-II-1004, Apr. 2007.

[20] X. Luo and G. B. Giannakis, "Energy-constrained optimal quantization for wireless sensor networks," *Proc. 1st Annual IEEE Communications Society Conference on Sensor and Ad Hoc Communications and Networks*, pp. 272-278, Oct. 2004.

[21] Y. Huang and Y. Hua, "Multihop progressive decentralized estimation in wireless sensor networks," *IEEE Signal Process. Letters*, vol. 14, pp. 1004-1007, Dec. 2007.

[22] A. Leon-Garcia, *Probability and Random Process for Electrical Engineering*, 2nd ed., Addison-Wesley, 1994.

[23] E. K. P. Chong and S. H. Zak, *An Introduction to Optimization*, John Wiley & Sons, Inc, 2001.

[24] S. Haykin, *Communication Systems*, 4th ed., John Wiley & Sons, Inc, 2001.

[25] M. K. Simon and M. S. Alouini, *Digital Communication over Fading Channel: A Unified Approach to Performance Analysis*, John Wiley &Sons, Inc., 2000.