國 立 交 通 大 學

電信工程學系

碩 士 論 文

粒子濾波算法應用在多輸入多輸出天線正交分頻多工的訊號檢測之研究

A Study on Data Detection using Particle Filtering in MIMO-OFDM System

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摘　　要

在多輸入輸出正交分頻多工的系統上，由於訊號之間的干擾，因此在接收端要做訊號的檢測的複雜度比單接收天線時的正交分頻多工系統來得複雜。特別是利用最大概似法(ML)作訊號檢測時，接收機的複雜度將會隨着天線的數量的增加或調變的不同呈現指數的增加。這篇論文主要是利用粒子濾波算法應用在多輸入多輸出正交分頻多工的訊號。粒子濾波算法利用統計的原理，造出相對應的事後機率用以作訊號檢測，來達到接近最大概似法的效能的同時減低複雜度。我們再使用一些方法合併粒子濾波算法去得到接近最大概似法的效果，在模擬顯示出粒子濾波演算法的結效能和我們提出的一些改良方法的效能比一般的 VBLAST MMSE OSIC 更接近最大概似法的效能，而且其複雜度遠少於最大概似法。

Data detection using Particle filtering in MIMO-OFDM system

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ABSTRACT

In multiple-input multiple output orthogonal frequency division multiplexing (MIMO-OFDM) system, data detection become more complicated than single input single output (SISO) system especially for Maximum likelihood (ML) detection scheme. The complexity for ML detection scheme will increase exponentially as either the number of transmitting antennas or modulation order increases. In this thesis, we introduce the use of particle filtering to approximate a posteriori distribution so that we can use Maximum a posteriori (MAP) detection scheme to detect signals. We also present some new methods combined with particle filtering for data detection to mitigate the error propagation problem in either spatial multiplexing system or MIMO-OFDM system with space frequency block code system. These proposed methods have an improvement as compared with V-BLAST MMSE OSIC receiver in both systems. Simulations show that the BER performance for our proposed methods will approach to the ML decision algorithm as compared with VBLAST MMSE OSIC and the complexity is lower than ML decision algorithm.

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Chapter 1

Introduction

Multiple-input-multiple-output (MIMO) system gets a great interest in communication system because of its ability to increase the throughput under the same total amount of transmitting power compare with single-input-single-output (SISO) system. The main idea is to transmit signals using multiple transmitting antennas and receiving signals using multiple receiving antennas. The bandwidth efficient can be increased by using this technique.

1.1 MIMO system

MIMO technique is mainly divided into three categories. First category is called spatial multiplexing. Spatial multiplexing is a transmission technique in MIMO system to transmit data signals independent and separately from each of the multiple transmit antennas. Therefore, the space dimension is reused more than once. The capacity can be increased by this technique if the channel matrix is full rank. In [1], 'BLAST (Bell Laboratories -Layered -Space-Time)', is a typical technique for spatial multiplexing.

Second, known as beamforming system, is to form a beam pattern by designing the arrangement of antenna array. It has an improvement as compared with omni-directional transmission because it can select directional transmission so it has directivity gain. Power can be focused on a particular direction and can be diminished the inference of other signals or other users.

Final system is pre-coding system. This system utilizes coding technique that called space time block code to increase diversity. Space time block code are normally presents as orthogonal. This means that each column in the equivalent channel matrix is orthogonal to other columns in the equivalent channel matrix. The decoding scheme for orthogonal space time block code is very simple, and easy to decode at the receiver side. Its disadvantage is that this system decreases the data rate as compared with spatial multiplexing in order to get diversity gain. Another scheme is proposed in space time block code is that such code is not orthogonal but it can achieve a higher data rate. Chapter three is focus on this scheme in order to get higher data rate.

1.2 MIMO-OFDM system

MIMO system can be used to increase the throughput in flat fading channel. Flat fading channel is a good condition for MIMO system. However, in MIMO system, channel may not be flat fading. Orthogonal frequency division multiplexing (OFDM) system can provide a flat fading condition for MIMO system and against ISI effect. Hence, MIMO system combining with OFDM system is frequently proposed for high data rate transmission scheme recently. On the other hand, especially in spatial multiplexing system, interference in MIMO-OFDM is more severe than in single input single output (SISO) OFDM system and the complexity of data detection in MIMO-OFDM system is higher than the complexity in SISO OFDM system. In BLAST system (spatial multiplexing), as proposed in [3], the system called VBLAST (Vertical-Bell Laboratories -Layered -Space-Time) system, is widely used in spatial multiplexing system for rich scattering communication environment because it has better performance and spectral efficiency as compared with spatial multiplexing system using conventional nulling method.

1.3 About the thesis

This thesis is organized as following. Chapter 2 describes the use of particle filtering method in data detection in spatial multiplexing system. Then we describe two modified methods to mitigate the error propagation problem in particle filtering. Chapter 3 presents the use of particle filtering for data detection in MIMO-OFDM with space frequency block code system. Then also presents a method to mitigate the error propagation. Chapter 4 shows all the simulations for each detection scheme in both spatial multiplexing in MIMO-OFDM and MIMO-OFDM with space frequency block code system. Finally, conclusions are introduced in the last chapter.

Chapter 2

Data detection in MIMO-OFDM system with particle filtering method

2.1 Spatial multiplexing system description:

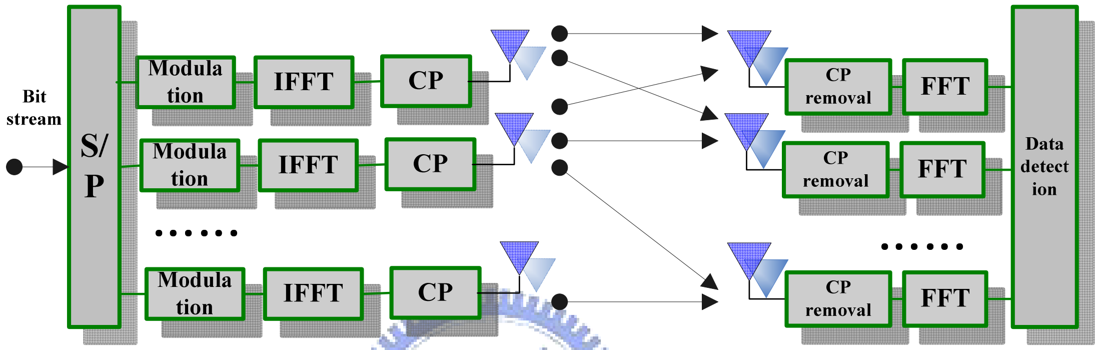


Figure 2.1 Spatial multiplexing system

In MIMO-OFDM spatial multiplexing system, we consider the system shown in figure 1. We assume that there are M transmitting antennas and N receiving antennas. At the transmitter side, bit stream is divided into M data layers and mapped each data layer to be M modulated signal streams. M modulated signal streams in M layer pass through IFFT, add cyclic prefix and then transmit parallel through M transmitting antennas. At the receiver side, there are N receiving antennas, after cyclic prefix removal and pass through FFT, the received signal vector **X** can be expressed as

$$\mathbf{X} = \mathbf{H}\mathbf{S}_{Tx} + \mathbf{N} \tag{2.1}$$

Where **X** is an N by 1 received signal vector, **H** is a N by M channel matrix, $\mathbf{S}_{Tx}$ is a M by 1 transmitted signal vector and **N** is a N by 1 noise vector. The channel matrix **H** is assumed to be full rank. The received signal vector is passed through the data detection scheme as shown in figure 1. There are several schemes for data detection in MIMO-OFDM BLAST system.

One of them is VBLAST- OSIC.

V-BLAST Zero-forcing OSIC [3] scheme is widely used in spatial multiplexing system. The procedure of the V-BLAST can be mainly divided into following steps: first, ordering the received signal according to signal to noise (SNR) ratio in descending order, then detects the first signal that belongs to the highest order of SNR. After detecting the first signal, then treats this signal as interference and cancelled out from the received signal vector, then starts to detect the second highest SNR signal. This process keeps moving until all the data are detected. In spatial multiplexing system, the optimum solution is to use Maximum likelihood (ML) detection. However, ML detection is an exhaustive search, the complexity increases either the number of transmitting antennas or order of modulation increases.

On the other hand, Maximum a posteriori (MAP) detection also give an optimum solution, therefore, if we can obtain the posteriori pdf (probability density function) or pmf(probability mass function), then MAP detection can be used for data detection. MAP approach is as same as ML approach. As describes above, the received signals can be expressed as

$$\mathbf{X} = \mathbf{H}\mathbf{S}_{\mathbf{Tx}} + \mathbf{N}, \tag{2.2}$$

All the elements in vector **X, H,** $\mathbf{S}_{\mathbf{Tx}}$ and **N** are complex number. In this thesis, we only consider the case that the number of transmitting antennas is equal to or less than the number of receiving antennas.

Assuming that the channel matrix **H** is full rank such that it can be decomposed using QR decomposition as shown below

$$\mathbf{H} = \mathbf{QR}, \tag{2.3}$$

where **R** is an upper triangular matrix and **Q** is an orthogonal matrix.

Multiply $\mathbf{Q}^{\mathbf{H}}$ (where $()^H$ denoted as Hermitian of a matrix) to $\mathbf{X}$ and the system model can be expressed as

$$\tilde{\mathbf{X}} = \mathbf{Q}^{\mathbf{H}}\mathbf{X} = \mathbf{R}\mathbf{S}_{\mathbf{Tx}} + \tilde{\mathbf{N}} \tag{2.4}$$

We consider the case that the number of transmitted antennas are equal to the received antennas(M=N), so that

$$\begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \\ . \\ . \\ \hat{x}_M \end{bmatrix} = \begin{bmatrix} R_{11} & R_{12} & \cdots & R_{1M} \\ 0 & R_{22}\ .. & \cdots & R_{2M} \\ 0 & 0\ .. & \cdots & \cdots \\ 0 & 0\ .. & 0 & R_{MM} \end{bmatrix} \begin{bmatrix} s_{tx1} \\ s_{tx2} \\ \cdots \\ \cdots \\ s_{txM} \end{bmatrix} + \begin{bmatrix} \tilde{n}_1 \\ \tilde{n}_2 \\ \cdots \\ \cdots \\ \tilde{n}_M \end{bmatrix} \tag{2.5}$$

We re-define some parameters, first of all, we define three vectors **Y, S and n** as the reverse order of $\tilde{\mathbf{X}}$, $\mathbf{S}_{\mathbf{Tx}}$ and $\tilde{\mathbf{N}}$ where

$\mathbf{Y} = [y_M, y_{M-1}, \ldots. y_2, y_1]^T = [\hat{x}_1, \hat{x}_2, \ldots. \hat{x}_{M-1}, \hat{x}_M]^T$,

$\mathbf{S} = [s_M, s_{M-1}, \ldots. s_1]^T = [s_{tx1}, \ldots., s_{txM-1}, s_{txM}]^T$ and

$\mathbf{n} = [n_M, n_{M-1}, \ldots. n_1] = [\hat{n}_1, \hat{n}_2, \ldots. \hat{n}_{M-1}, \hat{n}_M]^T$

The new expression can be shown as

$$\begin{bmatrix} y_M \\ y_{M-1} \\ . \\ . \\ y_1 \end{bmatrix} = \begin{bmatrix} R_{11} & R_{12} & \cdots & R_{1M} \\ 0 & R_{22}\ .. & \cdots & R_{2M} \\ 0 & 0\ .. & \cdots & \cdots \\ 0 & 0\ .. & 0 & R_{MM} \end{bmatrix} \begin{bmatrix} s_M \\ s_{M-1} \\ \cdots \\ \cdots \\ s_1 \end{bmatrix} + \begin{bmatrix} n_M \\ n_{M-1} \\ \cdots \\ \cdots \\ n_1 \end{bmatrix} \tag{2.6}$$

Where y_k, s_k and n_k represent k^{th} element of vector **Y**, **S** and **n**.

The relationship between y_k and vector **S** is $y_k = h_k(s_{1:k}) + n_k$ where $s_{1:k} = [s_1\ s_2 \ldots\ldots s_k]$.

Our goal is to find a scheme to detect the data sequence $s_{1:M} = [s_1\ s_2 \ldots\ldots s_M]$.

2.2 MAP decision:

Assume that there are M transmitting signal layers from M transmitting antennas, if we obtain the posteriori distribution and assume that each entry in the noise vector is independent Gaussian distribution, zero mean and variance σ^2.

The posteriori distribution will be expressed as:

$$p(s_{1:M} \mid y_{1:M}) = \frac{1}{(2\pi)^{M/2}(\sigma^2)^{M/2}} \exp\left(-\frac{1}{2\sigma^2}(\mathbf{Y}-\mathbf{HS})^{\mathbf{H}}(\mathbf{Y}-\mathbf{HS})\right) \qquad (2.7)$$

Where **S =** $s_{1:M}$ and **Y** = $y_{1:M}$, **H** is the M by M channel matrix.

The MAP decision becomes

$$p(s_{1:M} \mid y_{1:M}) = \frac{1}{(2\pi)^{M/2}(\sigma^2)^{M/2}} \exp\left(-\frac{1}{2\sigma^2}(\mathbf{Y}-\mathbf{HS})^{\mathbf{H}}(\mathbf{Y}-\mathbf{HS})\right) \qquad (2.8)$$

$$\Rightarrow \arg\min_{\mathrm{S}\in\mathrm{A}} (\mathbf{Y}-\mathbf{HS})^{\mathbf{H}}(\mathbf{Y}-\mathbf{HS}) \qquad (2.9)$$

$$\Rightarrow \arg\min_{\mathrm{S}\in\mathrm{A}} \left\|\mathbf{Y}-\mathbf{HS}\right\|^2 \qquad (2.10)$$

From equation (2.10), MAP decision needs to test all the possible combinations and choose the minimum distances. The complexity is related to two factors: first, the modulation scheme, for example, QPSK, 16QAM, and second, the number of transmitting antennas. The complexity increases exponentially as one of the factors increases. So that the complexity is O(A^M), where M is the number of transmitting antennas and A is the modulation scheme. For example, QPSK with 4 transmitting antennas, number of trials will become $4^4 = 256$. Furthermore, if modulation change to 16QAM, number of trials will become $16^4 = 65536$. MAP decision is not practical in this case.

2.3 Monte Carlo method

Before we mention the detail of particle filtering or called sequential Monte Carlo method algorithm, first we take a look on how a posteriori distribution can be approximated by a set of random samples.

$$p(s_{1:M} = \bar{s}_{1:M} \mid y_{1:M}) = \int p(s_{1:M} \mid y_{1:M}) \delta(s_{1:M} - \bar{s}_{1:M}) ds_{1:M} \tag{2.11}$$

If Np is large and then the desired posteriori distribution can be approximated as:

$$p(s_{1:M} = \bar{s}_{1:M} \mid y_{1:M}) \approx \frac{1}{Np} \sum_{i=1}^{Np} \delta(\bar{s}_{1:M} - s_{1:M}^{(i)}) \tag{2.12}$$

$$where\ \ \delta(\bar{s}_{1:M} - s_{1:M}^{(i)}) = \begin{cases} 1 & \text{when } s_{1:M}^{(i)} = \bar{s}_{1:M} \\ 0 & \text{when } s_{1:M}^{(i)} \neq \bar{s}_{1:M} \end{cases} \tag{2.13}$$

As the equation mentioned above, $\{s_{1:k}^{i}\}_{i=1}^{Np}$ denoted a set of samples drawn from a desired posteriori distribution function, the posteriori distribution function can be approximated by

$$p(s_{1:M} \mid y_{1:M}) \approx \frac{1}{Np} \sum_{i=1}^{Np} \delta(s_{1:M} - s_{1:M}^{(i)}) \tag{2.14}$$

Monte Carlo approach is one of the methods to construct the approximation of high dimensional posteriori distribution. If we can draw samples directly from the desired posteriori distribution $p(\mathbf{s}_{1:M} \mid \mathbf{y}_{1:M})$, so that the posteriori distribution can be constructed by all the samples $\{s_{1:M}^{(i)}\}_{i=1}^{Np}$ (where Np represents the number of samples) drawn from the desired posteriori distribution and this approximation will converge to the true posteriori distribution as there are infinite number of samples.

2.4 Importance sampling

Importance sampling is a method to approximate the desired posteriori distribution by drawing samples $\{s^{(i)}\}_{i=1}^{Np}$ from a trial function called importance distribution $q(\mathbf{s}_{1:M} \mid \mathbf{y}_{1:M})$ if the desired posteriori distribution cannot be drawn directly. This importance distribution is tractable for sampling. The different between Monte Carlo method and importance sampling is that importance sampling needs to compute the weights of the corresponding i th sample using $w^{(i)} = \dfrac{p(\mathbf{s}_{1:k}^{(i)} \mid \mathbf{y})}{q(\mathbf{s}_{1:k}^{(i)} \mid \mathbf{y})}$.

The approximation of the posteriori distribution can be derived as :

$$p(s_{1:M} = \bar{s}_{1:M} \mid y_{1:M}) = \int p(s_{1:M} \mid y_{1:M})\delta(s_{1:M} - \bar{s}_{1:M})ds_{1:M} \tag{2.15}$$

$$= \int \frac{p(s_{1:M} \mid y_{1:M})}{q(s_{1:M} \mid y_{1:M})} q(s_{1:M} \mid y_{1:M})\delta(s_{1:M} - \bar{s}_{1:M})ds_{1:M} \quad (define\ \mathrm{w}(s_{1:M}) = \frac{p(s_{1:M} \mid y_{1:M})}{q(s_{1:M} \mid y_{1:M})}) \tag{2.16}$$

$$= \frac{\int \mathrm{w}(s_{1:M})q(s_{1:M} \mid y_{1:M})\delta(s_{1:M} - \bar{s}_{1:M})ds_{1:M}}{\int \mathrm{w}(s_{1:M})q(s_{1:M} \mid y_{1:M})ds_{1:M}} \quad (\text{Since} \int \mathrm{w}(s_{1:M})q(s_{1:M} \mid y_{1:M})ds_{1:M} = 1) \tag{2.17}$$

If Np is large and then the posteriori distribution can be approximated as :

$$\approx \frac{\cancel{(1/Np)} \sum_{i=1}^{Np} w_M^{(i)} \delta(s_{1:M}^{(i)} - \bar{s}_{1:M})}{\cancel{(1/Np)} \sum_{i=1}^{Np} w_M^{(i)}} = (1/W)\sum_{i=1}^{Np} w_M^{(i)} \delta(s_{1:M}^{(i)} - \bar{s}_{1:M}) \quad (\text{where } \mathrm{W} = \sum_{i=1}^{N} w_M^{i}) \tag{2.18}$$

$$where\ \ \delta(s_{1:M}^{(i)} - \bar{s}_{1:M}) = \begin{cases} 1 & \text{when } s_{1:M}^{(i)} = \bar{s}_{1:M} \\ 0 & \text{when } s_{1:M}^{(i)} \neq \bar{s}_{1:M} \end{cases} \text{and } w_M^{(i)} = \frac{p(s_{1:M}^{(i)} \mid y_{1:M})}{q(s_{1:M}^{(i)} \mid y_{1:M})} \tag{2.19}$$

Defined $\tilde{w}^{(i)} = \dfrac{w^{(i)}}{\sum_{i=1}^{Np} w^{(i)}}$ as normalized weight corresponding to i th sample, then the posteriori distribution can be approximated as

$$p(s_{1:M} \mid y_{1:M}) \approx \sum_{i=1}^{Np} \tilde{w}^{(i)} \delta(s_{1:M} - s_{1:M}^{(i)}) . \tag{2.20}$$

The importance distribution can be chosen freely, however, the variance will be increased if the importance function is not highly related to the true posteriori distribution.

2.5 Particle filtering Methods [4]

If we need to draw samples directly from the posteriori distribution, we need to know the joint posteriori distribution first. The complexity is same as or higher than MAP decision. Now, if we do not have any information about the desired posteriori distribution, however, we have the conditional probability distribution $\mathrm{p}(\mathrm{y}_k \mid \mathrm{s}_{1:k})$, particle filtering or called sequential monte carlo method described in [5] and [6] provides a new method to obtain the posteriori distribution with low complexity by using the idea of importance sampling. The main idea is to estimate the desired posteriori distribution by drawing a set of random samples from importance distribution and to update the corresponding weights recursively.

Let's take a look on how all the samples can be drawn recursively. After finishing $k-1^{th}$ tracking, we have samples $\left\{\mathbf{s}_{1:k-1}^{(i)}\right\}_{i=1}^{Np}$ drawn from $q(\mathbf{s}_{1:k-1} \mid \mathbf{y}_{1:k-1})$ and weights $\mathrm{w}_{\mathrm{k-1}}^{(\mathrm{i})} = p(s_{1:k-1}^{(i)} \mid y_{1:k-1}) / q(s_{1:k-1}^{(i)} \mid y_{1:k-1})$,where I = 1 : Np. Furthermore, the importance distribution $q(\mathbf{s}_{1:k} \mid \mathbf{y}_{1:k})$ can be factorized as two components such that

$$q(\mathbf{s}_{1:k} \mid \mathbf{y}_{1:k}) = q(\mathbf{s}_k \mid \mathbf{s}_{1:k-1}, \mathbf{y}_{1:k}) q(\mathbf{s}_{1:k-1} \mid \mathbf{y}_{1:k-1}) \tag{2.21}$$

Which means that we can obtain Np sampled sequences(from 1 to k) $\left\{\mathbf{s}_{1:k}^{(i)}\right\}_{i=1}^{Np}$ from importance distribution $q(\mathbf{s}_{1:k} \mid \mathbf{y}_{1:k})$ by sampling Np sampled sequences with length k-1(from 1 to k-1) $\left\{\mathbf{s}_{1:k-1}^{(i)}\right\}_{i=1}^{Np}$ from $q(\mathbf{s}_{1:k-1} \mid \mathbf{y}_{1:k-1})$ and by sampling a new set of samples $\left\{\mathbf{s}_{k}^{(i)}\right\}_{i=1}^{Np}$ from $q(\mathbf{s}_k \mid \mathbf{s}_{1:k-1}, \mathbf{y}_{1:k})$. The weight update equation is

$$w_k^{(i)} = \frac{p(s_{1:k}^{(i)} \mid y_{1:k})}{q(s_{1:k}^{(i)} \mid y_{1:k})} \tag{2.22}$$

$$= \frac{p(y_k \mid s_{1:k}^{(i)}, y_{1:k-1}) p(s_{1:k}^{(i)} \mid y_{1:k-1}) p(y_{1:k-1})}{p(y_{1:k}) q(s_k^{(i)} \mid s_{1:k-1}^{(i)}, y_{1:k}) q(s_{1:k-1}^{(i)} \mid y_{1:k-1})} \tag{2.23}$$

$$= \frac{p(y_k \mid s_{1:k}^{(i)}) p(s_k^{(i)}) p(s_{1:k-1}^{(i)} \mid y_{1:k-1}) \cancel{p(y_{1:k-1})}}{p(y_k \mid y_{1:k-1}) \cancel{p(y_{1:k-1})} q(s_k \mid s_{1:k-1}^{(i)}, y_{1:k}) q(s_{1:k-1}^{(i)} \mid y_{1:k-1})} \tag{2.24}$$

$$\propto \frac{p(s_{1:k-1}^{(i)} \mid y_{1:k-1})}{q(s_{1:k-1}^{(i)} \mid y_{1:k-1})} * \frac{p(y_k \mid s_{1:k}^{(i)}) p(s_k^{(i)})}{q(s_k^{(i)} \mid s_{1:k-1}^{(i)}, y_{1:k})} \tag{2.25}$$

$$= w_{k-1}^{(i)} \frac{p(y_k \mid s_{1:k}^{(i)}) p(s_k^{(i)})}{q(s_k^{(i)} \mid s_{1:k-1}^{(i)}, y_{1:k})} \tag{2.26}$$

The posteriori distribution can be approximated using $\left\{\mathbf{s}_{1:k}^{(i)}\right\}_{i=1}^{Np}$ recursively as equation (2.21) and updated weights $\{w_{k-1}^{(i)}\}_{i=1}^{Np}$ recursively using equation (2.26), then normalize all the weights, the posteriori distribution can be approximated as

$$p(s_{1:k} \mid y_{1:k}) \approx \sum_{i=1}^{Np} \tilde{w}_k^{(i)} \ \delta(s_{1:k} - s_{1:k}^{(i)}) \tag{2.27}$$

Since it is an approximation method, increasing the number of samples will increase the accuracy of the approximation. In the jargon of particle filtering, these samples in each tracking are called particles.

The problem is how to choose the importance distribution $q(\mathbf{s}_k \mid \mathbf{s}_{1:k-1}, \mathbf{y}_{1:k})$. In [7], it is mentioned, in order to minimize the variance of the approximation, the importance function is chosen as:

$$q(s_k \mid s_{1:k-1}^{(i)}, y_{1:k}) = p(s_k \mid s_{1:k-1}^{(i)}, y_{1:k}) \tag{2.28}$$

If we choose the importance function as equation (2.28), $p(s_k^{(i)} \mid s_{1:k-1}^{(i)}, y_{1:k})$ can be factorize as

$$p(s_k^{(i)} \mid s_{1:k-1}^{(i)}, y_{1:k}) = \frac{p(y_{1:k} \mid s_k^{(i)}, s_{1:k-1}^{(i)})\, p(s_k^{(i)} \mid s_{1:k-1}^{(i)})\, p(s_{1:k-1}^{(i)})}{p(y_{1:k} \mid s_{1:k-1}^{(i)})\, p(s_{1:k-1}^{(i)})} \tag{2.29}$$

$$= \frac{p(y_k \mid s_k^{(i)}, s_{1:k-1}^{(i)})\ \cancel{p(y_{1:k-1} \mid s_{1:k-1}^{(i)})}\ p(s_k^{(i)})\ \cancel{p(s_{1:k-1}^{(i)})}}{p(y_k \mid s_{1:k-1}^{(i)})\ \cancel{p(y_{1:k-1} \mid s_{1:k-1}^{(i)})}\ \cancel{p(s_{1:k-1}^{(i)})}} \tag{2.30}$$

$$= \frac{p(y_k \mid s_{1:k}^{(i)})\, p(s_k^{(i)})}{p(y_k \mid s_{1:k-1}^{(i)})} \tag{2.31}$$

Substitute $q(s_k \mid s_{1:k-1}^{(i)}, y_{1:k}) = p(s_k \mid s_{1:k-1}^{(i)}, y_{1:k}) = \dfrac{p(y_k \mid s_{1:k}^{(i)})\, p(s_k^{(i)})}{p(y_k \mid s_{1:k-1}^{(i)})}$ into equation (2.26), the weight updated equation is

$$w_k^{(i)} \propto w_{k-1}^{(i)} \frac{p(y_k \mid s_{1:k}^{(i)})\, p(s_k^{(i)})}{q(s_k^{(i)} \mid s_{1:k-1}^{(i)}, y_{1:k})} \tag{2.32}$$

$$\propto w_{k-1}^{(i)} \frac{\cancel{p(y_k \mid s_{1:k}^{(i)})}\ \cancel{p(s_k^{(i)})}\, p(y_k \mid s_{1:k-1}^{(i)})}{\cancel{p(y_k \mid s_{1:k}^{(i)})}\ \cancel{p(s_k^{(i)})}} \tag{2.33}$$

$$= w_{k-1}^{(i)} p(y_k \mid s_{1:k-1}^{(i)}) \tag{2.34}$$

$$= w_{k-1}^{(i)} \sum_{s_k} p(y_k \mid s_k, s_{1:k-1}^{(i)})\, p(s_k) \tag{2.35}$$

The term $p(y_k \mid y_{1:k-1})$ can be ignored because it is not affected the approximation of k th tracking after normalization. From the deviation of equation (2.35), we can observe that the weight in i th particle at k th tracking depends on two factors: The previous weights of i th particle at k-1 th tracking and a new term $\sum_{s_k} p(y_k \mid s_k, s_{1:k-1}^{(i)}) p(s_k)$. Suppose that we have Np particles from 1 : k-1 which denoted as $\{s_{1:k-1}^{(i)}\}_{i=1}^{Np}$ and Np weights from k-1 which denoted as $\{w_{k-1}^{(i)}\}_{i=1}^{Np}$.Then the new particles can be drawn from the importance distribution $q(s_k \mid s_{1:k-1}^{(i)}, y_{1:k}) = p(s_k \mid s_{1:k-1}^{(i)}, y_{1:k})$ and then update the corresponding weight using equation (2.35). After that normalize all the weights at M th tracking by $\tilde{w}_M^{(i)} = \frac{w_M^{(i)}}{\sum_{i=1}^{Np} w_M^{(i)}}$. In the jargon of particle filtering, this procedure is called Sequential importance sampling (SIS) scheme.

The procedure of k th tracking is summarized as following:

－For i = 1 to Np

- ◆ Draw a particle from the importance distribution (2.28)
- ◆ Calculate the weight by using equation(2.35)
- ◆ Store the new particle $s_k^{(i)}$ to $s_{1:k-1}^{(i)}$

－End For

- ◆ Normalized all the weights

2.6 Degeneracy

After several tracking, the variance of the estimator will increases as shown in [7], since some of the particles have negligible weights and do not have any contribution to the process. This problem is called degeneracy. In [10], resampling algorithm is used to overcome this problem. The main idea is to replace some small weighted samples by some large weighted samples. In [8] and [9]. Both papers mention that one of the methods to measure degeneracy is to calculate the effective sample size N_{eff}. N_{eff} can be obtained by

$$N_{eff} = \frac{1}{\sum_{i=1}^{Np} (\tilde{w}_k^{(i)})^2}. \qquad (2.36)$$

So that we can set a threshold sample size called Ns. Ns is set as 60% of Np in our simulation. If N_{eff} <Ns, resampling algorithm is needed.

Algorithm for resampling

■ For i = 1 to Np

Generate a random variable U with uniform distribution from [0 1]

For j = 1 to Np

$$w\_new_k^{(i)} = w\_new_k^{(i)} + w_k^{(j)} \qquad (2.37)$$

$$\text{If } w\_new_k^{(i)} > \text{U}, \text{ then } s\_new_k^{(i)} = \hat{s}_k^{(j)} \qquad (2.38)$$

Break;

End For

$$w_k^{(i)} = \frac{1}{Np} \qquad (2.39)$$

End For

After resampling, new set of particles are obtained, the connection with previous samples is broken and their weights at k th tracking are all equal. In the jargon of particle filtering, this procedure is called Sequential importance sampling (SIS) with resampling scheme.

The procedure of k th tracking is summarized as following:

－For i = 1 to Np

- ◆ Draw a particle from the importance distribution from equation (2.28)
- ◆ Calculate the weight by using equation (2.35)
- ◆ Store the new particle $s_k^{(i)}$ to $s_{1:k-1}^{(i)}$

－End For

- ◆ Normalized all the weights by using $\tilde{w}_k^{(i)} = \frac{w_k^{(i)}}{\sum_{i=1}^{Np} w_k^{(i)}}$

- Calculate the effective sample size N_{eff} using (2.36)
- If $N_{eff} < \text{Ns}$, then do the resampling scheme.

2.7 Data detection scheme in MIMO-OFDM BLAST system with particle filtering:

In MIMO-OFDM spatial multiplexing system, we assume that the channel matrix **H** is full rank such that it can be decomposed using QR decomposition as shown $\mathbf{H} = \mathbf{QR}$ and

$$\tilde{\mathbf{X}} = \mathbf{Q}^{\mathbf{H}}\mathbf{X} = \mathbf{R}\mathbf{S}_{\mathbf{Tx}} + \tilde{\mathbf{N}} \tag{2.40}$$

Since **R** is an upper triangular matrix, one of the methods for data detection is to use decision feedback method that detects signals from the bottom to the top.

First, compute the probability of $p(s_{txM} \mid y_M)$. For example, the distribution of noise in each entry is complex Gaussian distribution then detection s_{txM} using minimum distance. The next step is to compute $p(s_{txM-1} \mid s_{txM}, y_{M-1})$ and detect s_{txM-1}. The process keeps moving until all the signals are detected. However, this method has error propagation problem and the SNR of each signal mainly depends on the diagonal. On the other hand, since $\mathbf{Q}^H$ is an orthogonal matrix, so that after multiplying $\mathbf{Q}^H$ to the initial noise vector, the new noise vector is also independent white noise vector. As mentioned in section 2.1, we define three new vectors **Y, S** and **n,** and the relationship between y_k is also dependent on $s_{1:k}$ and n_k which is $y_k = R_{k,k}s_k + R_{k,k+1}s_{k-1} + ...R_{k,M}s_1 + n_k$. We assume that the noise before multiplying $\mathbf{Q}^H$ to the received signal vector is white noise. Hence, after multiplying $\mathbf{Q}^H$ to the received signal vector, the noise vector is still a white noise vector. We treat each noise entry n_k as an independent white noise. A particle $s_k = a_k$ is drawn from the importance function

$p(s_k = a_k \mid s_{1:k-1}^{(i)}, y_{1:k})$ which can be factorized as

$$p(s_k = a_k \mid s_{1:k-1}^{(i)}, y_{1:k}) = \frac{p(y_{1:k} \mid s_k = a_k, s_{1:k-1}^{(i)})\, p(s_k = a_k \mid s_{1:k-1}^{(i)})\, p(s_{1:k-1}^{(i)})}{p(y_{1:k} \mid s_{1:k-1}^{(i)})\, p(s_{1:k-1}^{(i)})} \quad (2.41)$$

$$= \frac{p(y_k \mid s_k = a_k, s_{1:k-1}^{(i)})\, p(s_k)}{p(y_k \mid s_{1:k-1}^{(i)})} = \frac{p(y_k \mid s_k = a_k, s_{1:k-1}^{(i)})\, \cancel{p(s_k)}}{\sum p(y_k \mid s_k, s_{1:k-1}^{(i)})\, \cancel{p(s_k)}} \quad (2.42)$$

We can observe that the first term in numerator $p(y_k \mid s_k, s_{1:k-1}^{(i)})$ is a Gaussian distribution which mean is equal to $y_k - R_{k,k}a_k - R_{k,k+1}s_{k-1} - \ldots R_{k,M}s_1$ (where a_k is one of the signal points in signal constellation) and variance is equal to σ^2 and the second term in numerator is assumed to be equally likely. Finally, we can draw samples from $p(s_k \mid s_{1:k-1}^{(i)}, y_{1:k})$ which is equal to equation (2.36). For example, in QPSK modulation, the set of $\hat{a}_M$ is $\{\frac{1}{\sqrt{2}}(1+j)$,-$\frac{1}{\sqrt{2}}(-1+j)$, $\frac{1}{\sqrt{2}}(-1-j)$, $\frac{1}{\sqrt{2}}(1-j)\}$ also there is 16 combinations for 16-QAM modulation. For example, for the QPSK modulation, the particle filtering (SIS) is shown below:

In k-th tracking:

—For i = 1 to Np

- Draw samples from importance distribution $p(s_k \mid s_{1:k-1}^{(i)}, y_{1:k})$

  Where $p(s_k \mid s_{1:k-1}^{(i)}, y_{1:k}) = \dfrac{p(y_k \mid s_k = a_k, s_{1:k-1}^{(i)})}{\sum_{s_k} p(y_k \mid s_k = a_k, s_{1:k-1}^{(i)})}$

  since $p(y_k \mid s_k = a_k, s_{1:k-1}^{(i)}) = N(y_k - R_{k,k}a_k - R_{k,k+1}s_{k-1} - \ldots R_{k,M}s_1, \sigma^2)$

  $p(y_k \mid s_k = \frac{1}{\sqrt{2}}(1+j), s_{1:k-1}^{(i)}) = \beta_1$, $p(y_k \mid s_k = \frac{1}{\sqrt{2}}(-1+j), s_{1:k-1}^{(i)}) = \beta_2$,

  $py_k \mid s_k = \frac{1}{\sqrt{2}}(-1-j), s_{1:k-1}^{(i)}) = \beta_3$, $p(y_k \mid s_k = \frac{1}{\sqrt{2}}(1-j), s_{1:k-1}^{(i)}) = \beta_4$

And

$$\alpha_1 = p(s_k = \frac{1}{\sqrt{2}}(1+j) \mid s_{1:k-1}^{(i)}, y_{1:k}) = \frac{\beta_1}{\beta_1 + \beta_2 + \beta_3 + \beta_4} \quad ,$$

$$\alpha_2 = p(s_k = \frac{1}{\sqrt{2}}(-1+j) \mid s_{1:k-1}^{(i)}, y_{1:k}) = \frac{\beta_2}{\beta_1+\beta_2+\beta_3+\beta_4}$$

$$\alpha_3 = p(s_k = \frac{1}{\sqrt{2}}(-1-j) \mid s_{1:k-1}^{(i)}, y_{1:k}) = \frac{\beta_3}{\beta_1+\beta_2+\beta_3+\beta_4} ,$$

$$\alpha_4 = p(s_k = \frac{1}{\sqrt{2}}(1-j) \mid s_{1:k-1}^{(i)}, y_{1:k}) = \frac{\beta_4}{\beta_1+\beta_2+\beta_3+\beta_4}$$

Generate a uniform distribution U between [0 ,1]

If $\alpha_1 > U > 0$, then $s_k^{(i)} = \frac{1}{\sqrt{2}}(1+j)$,

If $\alpha_1 + \alpha_2 > U > \alpha_1$, then $s_k^{(i)} = \frac{1}{\sqrt{2}}(-1+j)$

If $\alpha_1 + \alpha_2 + \alpha_3 > U > \alpha_1 + \alpha_2$, then $s_k^{(i)} = \frac{1}{\sqrt{2}}(-1-j)$,

If $1 > U > \alpha_1 + \alpha_2 + \alpha_3$ then $s_k^{(i)} = \frac{1}{\sqrt{2}}(1-j)$

- Update the weight using $w_k^{(i)} = w_{k-1}^{(i)} \sum_{s_k} p(y_k \mid s_k^{(i)}, s_{1:k-1}^{(i)}) p(s_k)$

Since $p(y_k \mid s_k = a_M, s_{1:k-1}^{(i)}) = \beta_M$, hence $w_k^{(i)} = w_{k-1}^{(i)} \frac{(\beta_1+\beta_2+\beta_3+\beta_4)}{4}$.

- Store the new particle $s_k^{(i)}$ to $s_{1:k-1}^{(i)}$

－End For

If k = M

- Normalized all the weights by using $\tilde{w}_M^{(i)} = \frac{w_M^{(i)}}{\sum_{i=1}^{Np} w_M^{(i)}}$.

Example : For MIMO-OFDM 4X4 system with BPSK modulation.
After QR decomposition, the signal model become

$$\begin{bmatrix} y_4 \\ y_3 \\ y_2 \\ y_1 \end{bmatrix} = \begin{bmatrix} R_{11} & R_{12} & R_{13} & R_{14} \\ 0 & R_{22} & R_{23} & R_{24} \\ 0 & 0 & R_{33} & R_{34} \\ 0 & 0 & 0 & R_{44} \end{bmatrix} \begin{bmatrix} s_4 \\ s_3 \\ s_2 \\ s_1 \end{bmatrix} + \begin{bmatrix} n_4 \\ n_3 \\ n_2 \\ n_1 \end{bmatrix} \quad (2.43)$$

In order to draw particles for s_1 , first of all, calculate the probability for

$\frac{p(y_1 \mid s_1 = 1)}{p(y_1 \mid s_1 = 1) + p(y_1 \mid s_1 = -1)}$ and $\frac{p(y_1 \mid s_1 = -1)}{p(y_1 \mid s_1 = 1) + p(y_1 \mid s_1 = -1)}$, draw particles from this two probabilities. For example, assume that $\frac{p(y_1 \mid s_1 = 1)}{p(y_1 \mid s_1 = 1) + p(y_1 \mid s_1 = -1)} = 0.6$,

$\frac{p(y_1 \mid s_1 = -1)}{p(y_1 \mid s_1 = 1) + p(y_1 \mid s_1 = -1)} = 0.4$ and draw 5 particles, we generate 5 random variables with uniform distribution(U1 to U5) between [0 1]. For particle i, if Ui <0.6, $s_1^{(i)}$ is 1,otherwise, $s_1^{(i)}$ is -1. The normalized weight for all particles i = 1 to 5 are $w_1^{(i)}$ = 1/5 for the first tracking. Assuming that the five particles are {1 1 1 -1 -1}. In order to draw particles for the second tracking $s_2^{(i)}$, first calculate the $\frac{p(y_2 \mid s_2 = 1, s_1^{(i)})}{p(y_2 \mid s_2 = 1, s_1^{(i)}) + p(y_2 \mid s_2 = -1, s_1^{(i)})}$ and

$\frac{p(y_2 \mid s_2 = -1, s_1^{(i)})}{p(y_2 \mid s_2 = 1, s_1^{(i)}) + p(y_2 \mid s_2 = -1, s_1^{(i)})}$, i.e., for 2nd particle in 2nd tracking, since $s_1^{(2)}$ = 1, calculate $\frac{p(y_2 \mid s_2 = 1, s_1^{(2)} = 1)}{p(y_2 \mid s_2 = 1, s_1^{(2)} = 1) + p(y_2 \mid s_2 = -1, s_1^{(2)} = 1)}$(assume it is 0.3) and

$\frac{p(y_2 \mid s_2 = -1, s_1^{(2)} = 1)}{p(y_2 \mid s_2 = 1, s_1^{(2)} = 1) + p(y_2 \mid s_2 = -1, s_1^{(2)} = 1)}$(assume it is 0.7) and generate a uniform random variable U, if U<0.3, then $s_2^{(2)}$ =1 ,otherwise $s_2^{(2)}$ =-1 ,and the corresponding weight for 2nd particle for 2nd tracking is

$$w_2^{(2)} = w_1^{(2)} \frac{(p(y_2 \mid s_2 = 1, s_1^{(2)} = 1) + p(y_2 \mid s_2 = -1, s_1^{(2)} = 1))}{2},$$

From the equation shown above, we observe that the i th particle at 2 th tracking is related to the previous i th particle and weight. After getting five new particles and update five corresponding weights for i th particle at 2nd tracking. Assume that they are {-1 -1 -1 -1 1}, attach these five particles to the first five particles, we can get

$\begin{Bmatrix} 1 & 1 & 1 & -1 & -1 \\ -1 & -1 & -1 & -1 & 1 \end{Bmatrix}$,the first row represents the first tracking particles($s_1^{(i)}$) and second row represents the second tracking particles($s_2^{(i)}$). Assume that the corresponding weights at

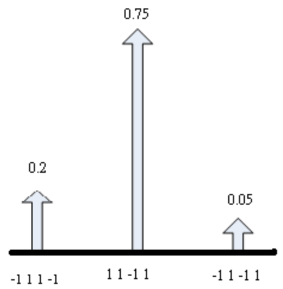
2^{nd} tracking is $w_2^{(1:5)} = \{0.3\ 0.3\ 0.3\ 0.05\ 0.05\}$. Keep moving until 4^{th} tracking is done. We can get

$$\begin{Bmatrix} 1 & 1 & 1 & -1 & -1 \\ -1 & -1 & -1 & -1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & -1 & -1 \end{Bmatrix} \quad \text{and} \quad w_4^{(1:5)} = \{0.25,\ 0.25,\ 0.25\ 0.05\ 0.2\ \}$$

First, consider the first three columns, we discover that the first three columns are identical to each other which is $s_{1:4}^{(i)} = \{1\ \text{-}1\ 1\ 1\}$, the 4^{th} and 5^{th} column are different to the first 3 columns, they are $s_{1:4}^{(4)} = \{\text{-}1\ 1\ \text{-}1\ 1\}$ and $s_{1:4}^{(5)} = \{\text{-}1\ 1\ 1\ \text{-}1\}$. The posteriori distribution can be approximated as:

$$p(s_{1:4} \mid y_{1:4}) \approx 0.75 * \delta(s_{1:4} - \{1\ \text{-}1\ 1\ 1\}) + 0.05 * \delta(s_{1:4} - \{\text{-}1\ 1\ \text{-}1\ 1\}) + 0.2 * \delta(s_{1:4} - \{\text{-}1\ 1\ 1\ \text{-}1\})$$

The data is the reverse order of $s_{tx1:4} = [s_4\ s_3\ s_2\ s_1]$ and the histogram can be plotted as



2.8 Detection Scheme

Approach I : **Sequence detection**

$$\hat{s}_{1:M} = \arg\max_{s_{1:M}} p(s_{1:M} \mid y_{1:M}) \tag{2.44}$$

This process needs to find all the same sequences and adds all the weights which belong to the same sequence This process will increase the complexity if the sequence is too long, which means that if the number of tracking increases, the complexity will increases.

Approach II : Detect directly from the marginal posteriori probability

$$p(s_k \mid y_{1:k}) \approx \sum_{i=1}^{Np} \tilde{w}_k^{(i)} \delta(s_k - s_k^{(i)}) \tag{2.45}$$

the detection scheme needs to find one dimension only. The searching process is to sort all the signals which belong to the same constellation and to add all the weights which belong to the same signal. The detection scheme after sorting and adding all the weights is shown as

$$\hat{s}_k = \arg\max_{s_k} p(s_k \mid y_{1:k}) \tag{2.46}$$

Approach III : Find the expectation value $\hat{s}_k$ from the marginal posteriori distribution

$$\hat{s}_k = E[s_k \mid y_{1:k}] \approx \sum_{i=1}^{Np} \tilde{w}_k^{(i)} s_k^{(i)} \tag{2.47}$$

As the equation shown above, no sorting is needed. However, Multiplications are needed for this approach. The performance will have same degradation for using approach II and III for data detection.

2.9 Error mitigation method

For approach II and III, one of the problems using particle filtering for data detection in spatial multiplexing is the error propagation problem. If the particles in previous tracings did not draw well, the estimated posterior distribution will be affected by error sampling. We can see that the top signal will be affected by all the other signals. Data detection using approach II and III for the top signal will has the worst performance as compared with other signals. We proposed a modified method for data detection in spatial multiplexing system with particle filtering. First, we consider the channel matrix and review the complex value problem of Gram-Schmidt algorithm for QR decomposition. Assume that all the entries in channel **H** are complex and consider the case that the number of transmitting antennas M is equal to the number of received antennas **N** (Assume that M=N), the channel matrix is shown as

$$\mathbf{H} = [\mathbf{h}_1 \quad \mathbf{h}_2 \quad .... \quad .... \quad \mathbf{h}_\mathbf{M}] \tag{2.48}$$

Gram Schmidt process is

$$\text{Step 1 : } \mathbf{q_1} = \frac{\mathbf{h_1}}{\|\mathbf{h_1}\|} \tag{2.49}$$

Step 2 : For n = 2 : M

$$\mathbf{q_n} = (\mathbf{h_n} - \sum_{i=1}^{n-1}(\mathbf{q_i}^{\mathbf{H}}\mathbf{h_n})\mathbf{q_i}) / \left\|(\mathbf{h_n} - \sum_{i=1}^{n-1}(\mathbf{q_i}^{\mathbf{H}}\mathbf{h_n})\mathbf{q_i})\right\| \tag{2.50}$$

End For

$$\text{Step 3 : } \mathbf{H} = [\mathbf{q_1} \quad \mathbf{q}_2 \quad .... \quad ... \quad \mathbf{q_M}] \begin{bmatrix} \mathbf{q}_1^{\mathbf{H}}\mathbf{h}_1 & \mathbf{q}_1^{\mathbf{H}}\mathbf{h}_2 & .... & ... & \mathbf{q}_1^{\mathbf{H}}\mathbf{h_M} \\ 0 & \mathbf{q}_2^{\mathbf{H}}\mathbf{h}_2 & ... & ... & \mathbf{q}_2^{\mathbf{H}}\mathbf{h_M} \\ 0 & 0 & \mathbf{q}_3^{\mathbf{H}}\mathbf{h}_3 & .... & . \\ 0 & 0 & 0 & .... & . \\ 0 & 0 & 0 & 0 & \mathbf{q_n}^{\mathbf{H}}\mathbf{h_M} \end{bmatrix}, \tag{2.51}$$

On the other hand, we implement the Gram-Schmidt QR decomposition in reverse order as:

$$\text{Step 1 : } \mathbf{q_1} = \frac{\mathbf{h_M}}{\|\mathbf{h_M}\|} \tag{2.52}$$

Step 2 : For n = 2 : M

$$\hat{\mathbf{q}}_{\mathbf{n}} = (\mathbf{h_{M\text{-}n+1}} - \sum_{i=1}^{n-1}(\hat{\mathbf{q}}_{\mathbf{i}}^{\mathbf{H}} * \mathbf{h_{M\text{-}n+1}})\,\hat{\mathbf{q}}_{\mathbf{i}}) / \left\|(\mathbf{h_{M\text{-}n+1}} - \sum_{i=1}^{n-1}(\hat{\mathbf{q}}_{\mathbf{i}}^{\mathbf{H}} * \mathbf{h_{M\text{-}n+1}})\,\hat{\mathbf{q}}_{\mathbf{i}})\right\| \tag{2.53}$$

End For

A new orthogonal matrix is obtained which is $\mathbf{Q_2} = [\hat{\mathbf{q}}_{\mathbf{1}} \quad \hat{\mathbf{q}}_2 \quad .... \quad ... \quad \hat{\mathbf{q}}_{\mathbf{M}}]$. All the column vectors in channel matrix can be expressed as following:

The new QR expression is

$$[\mathbf{h}_1 \quad ... \quad .... \quad \mathbf{h_{M-1}} \quad \mathbf{h_M}] = [\hat{\mathbf{q}}_{\mathbf{1}} \quad \hat{\mathbf{q}}_2 \quad .. \quad .. \quad \hat{\mathbf{q}}_{\mathbf{M}}] \begin{bmatrix} (\hat{\mathbf{q}}_1^{\mathbf{H}}\mathbf{h}_1) & (\hat{\mathbf{q}}_1^{\mathbf{H}}\mathbf{h}_2) & ... & ... & (\hat{\mathbf{q}}_1^{\mathbf{H}}\mathbf{h_M}) \\ (\hat{\mathbf{q}}_2^{\mathbf{H}}\mathbf{h}_1) & (\hat{\mathbf{q}}_2^{\mathbf{H}}\mathbf{h}_2) & ... & ... & 0 \\ ... & .... & ... & 0 & 0 \\ ... & (\hat{\mathbf{q}}_{M-1}^{\mathbf{H}}\mathbf{h}_2) & 0 & 0 & 0 \\ (\hat{\mathbf{q}}_M^{\mathbf{H}}\mathbf{h}_1) & 0 & 0 & 0 & 0 \end{bmatrix} \tag{2.54}$$

$$\mathbf{R}_2 = \begin{bmatrix} (\hat{\mathbf{q}}_1^{\mathbf{H}}\mathbf{h}_1) & (\hat{\mathbf{q}}_1^{\mathbf{H}}\mathbf{h}_2) & \cdots & \cdots & (\hat{\mathbf{q}}_1^{\mathbf{H}}\mathbf{h}_{\mathbf{M}}) \\ (\hat{\mathbf{q}}_2^{\mathbf{H}}\mathbf{h}_1) & (\hat{\mathbf{q}}_2^{\mathbf{H}}\mathbf{h}_2) & \cdots & \cdots & 0 \\ \cdots & \cdots\cdot & \cdots & 0 & 0 \\ \cdots & (\hat{\mathbf{q}}_{M-1}^{\mathbf{H}}\mathbf{h}_2) & 0 & 0 & 0 \\ (\hat{\mathbf{q}}_M^{\mathbf{H}}\mathbf{h}_1) & 0 & 0 & 0 & 0 \end{bmatrix}$$

, so that channel matrix can be expressed as another form of QR decomposition.

From the discussion above, we get two forms of QR decomposition which are

$$\mathbf{H} = \mathbf{Q}_1\mathbf{R}_1 = [\mathbf{q}_1 \quad \mathbf{q}_2 \quad \cdots\cdot \quad \cdots \quad \mathbf{q}_{\mathbf{N}}] \begin{bmatrix} \mathbf{q}_1^{\mathbf{H}}\mathbf{h}_1 & \mathbf{q}_1^{\mathbf{H}}\mathbf{h}_2 & \cdots\cdot & \cdots & \mathbf{q}_1^{\mathbf{H}}\mathbf{h}_{\mathbf{M}} \\ 0 & \mathbf{q}_2^{\mathbf{H}}\mathbf{h}_2 & \cdots & \cdots & \mathbf{q}_2^{\mathbf{H}}\mathbf{h}_{\mathbf{M}} \\ 0 & 0 & \mathbf{q}_3^{\mathbf{H}}\mathbf{h}_3 & \cdots\cdot & . \\ 0 & 0 & 0 & \cdots\cdot & . \\ 0 & 0 & 0 & 0 & \mathbf{q}_{\mathbf{N}}^{\mathbf{H}}\mathbf{h}_{\mathbf{M}} \end{bmatrix}, \qquad (2.55)$$

and

$$\mathbf{H} = \mathbf{Q}_2\mathbf{R}_2 = [\hat{\mathbf{q}}_1 \quad \hat{\mathbf{q}}_2 \quad \cdots\cdot \quad \cdots \quad \hat{\mathbf{q}}_{\mathbf{N}}] \begin{bmatrix} (\hat{\mathbf{q}}_1^{\mathbf{H}}\mathbf{h}_1) & (\hat{\mathbf{q}}_1^{\mathbf{H}}\mathbf{h}_2) & \cdots & \cdots & (\hat{\mathbf{q}}_1^{\mathbf{H}}\mathbf{h}_{\mathbf{M}}) \\ (\hat{\mathbf{q}}_2^{\mathbf{H}}\mathbf{h}_1) & (\hat{\mathbf{q}}_2^{\mathbf{H}}\mathbf{h}_2) & \cdots & \cdots & 0 \\ \cdots & \cdots\cdot & \cdots & 0 & 0 \\ \cdots & (\hat{\mathbf{q}}_{M-1}^{\mathbf{H}}\mathbf{h}_2) & 0 & 0 & 0 \\ (\hat{\mathbf{q}}_M^{\mathbf{H}}\mathbf{h}_1) & 0 & 0 & 0 & 0 \end{bmatrix} \qquad (2.56)$$

The received vector passes through $\mathbf{Q}_1^{\mathbf{H}}$ and $\mathbf{Q}_2^{\mathbf{H}}$ matrix are

$$\mathbf{Y}_1 = \mathbf{Q}_1^{\mathbf{H}}\mathbf{X} = \mathbf{R}_1\mathbf{S}_{\mathbf{Tx}} + \tilde{\mathbf{N}}_1 = \begin{bmatrix} (\hat{\mathbf{q}}_1^{\mathbf{H}}\hat{\mathbf{h}}_1) & (\hat{\mathbf{q}}_1^{\mathbf{H}}\hat{\mathbf{h}}_2) & \cdots\cdot & \cdots & (\hat{\mathbf{q}}_1^{\mathbf{H}}\hat{\mathbf{h}}_{\mathbf{N}}) \\ 0 & (\hat{\mathbf{q}}_2^{\mathbf{H}}\hat{\mathbf{h}}_2) & \cdots & \cdots & (\hat{\mathbf{q}}_2^{\mathbf{H}}\hat{\mathbf{h}}_{\mathbf{N}}) \\ 0 & 0 & (\hat{\mathbf{q}}_3^{\mathbf{H}}\hat{\mathbf{h}}_3) & \cdots\cdot & . \\ 0 & 0 & 0 & \cdots\cdot & . \\ 0 & 0 & 0 & 0 & (\hat{\mathbf{q}}_{\mathbf{N}}^{\mathbf{H}}\hat{\mathbf{h}}_{\mathbf{N}}) \end{bmatrix} \begin{bmatrix} s_{tx1} \\ s_{tx2} \\ .. \\ .. \\ s_{txM} \end{bmatrix} + \tilde{\mathbf{N}}_1 \qquad (2.57)$$

and

$$\mathbf{Y}_2 = \mathbf{Q}_2^{\mathbf{H}}\mathbf{X} = \mathbf{R}_2\mathbf{S}_{\mathbf{Tx}} + \tilde{\mathbf{N}}_2 = \begin{bmatrix} (\hat{\mathbf{q}}_1^{\mathbf{H}}\hat{\mathbf{h}}_{\mathrm{N}}) & (\hat{\mathbf{q}}_1^{\mathbf{H}}\hat{\mathbf{h}}_{\mathrm{N}-1}) & \dots & \dots & (\hat{\mathbf{q}}_1^{\mathbf{H}}\hat{\mathbf{h}}_1) \\ (\hat{\mathbf{q}}_2^{\mathbf{H}}\hat{\mathbf{h}}_{\mathrm{N}}) & (\hat{\mathbf{q}}_2^{\mathbf{H}}\hat{\mathbf{h}}_{\mathrm{N}-1}) & \dots & \dots & 0 \\ \dots & \dots. & \dots & 0 & 0 \\ \dots & \dots. & 0 & 0 & 0 \\ (\hat{\mathbf{q}}_{\mathbf{N}}^{\mathbf{H}}\hat{\mathbf{h}}_{\mathrm{N}}) & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} s_{tx1} \\ s_{tx2} \\ .. \\ .. \\ s_{txM} \end{bmatrix} + \tilde{\mathbf{N}}_2 \qquad (2.58)$$

We observe from two equations shown above. In equation (2.57), we can use particle filtering, draw particles from the bottom signal to the top and use approach III to find the expectation value for each entry in the signal vector. On the other hand, in equation (2.58), we can use particle filtering method, draw particles from top to bottom and use approach III to find the expectation value for each entry in signal vector. Finally, we average two results, error propagation can be mitigated.

Block diagram for error propagation mitigation method

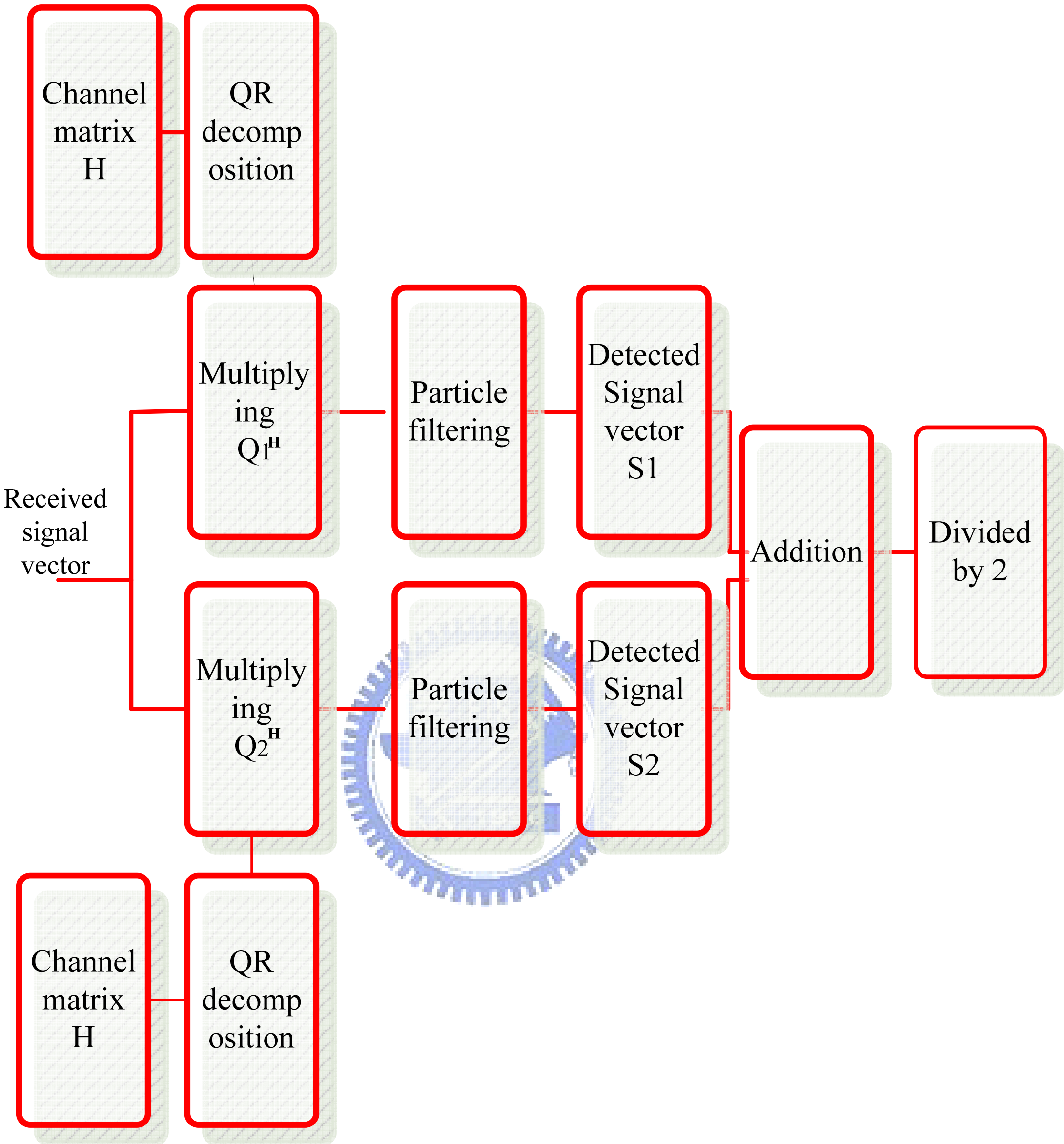


Figure 2.2 Block diagram for error propagation mitigation method

2.10 Sorted QR decomposition method

In [11], it is mentioned a method for sorted QR decomposition, which is similar to Gram-Schmidt algorithm. The idea of this method is to re-order the columns of channel matrix **H** for each orthogonal base searching. For Gram- Schmidt QR decomposition, we decompose the channel matrix **H** as shown in equation (2.55). Data detection by QR decomposition using particle filtering with approach II or III, as described before, the top signal will be affected by all the other signals. If particles in the previous stages did not draw well, the next stage signal samples will be affected by the previous stage samples. So that we need a large number of samples in order to obtain a much reliable posteriori probability. Sorted QR decomposition can improve such situation. The sorted QR decomposition combine with particle filtering use fewer particles to obtain a better performance compare with ordinary Gram Schmidt decomposition as shown in simulations. The idea of sorted QR decomposition is to maximize the diagonal entry of channel matrix **H** from M to 1 by using a permutation vector **p** (where M is the number of transmitting antennas), such that minimizing the diagonal elements in each decomposition step in order to maximize the diagonal element in the subsequent steps. The algorithm is shown as:

Step 1 : Let **R** = **0**; **Q** = **H** ; **p** = 1 ,2,..M

Step 2 : For i = 1 to M

$$k = \text{column of } (\arg\min_{k=1,..M} |\mathbf{q}_k|^2) \tag{2.59}$$

Exchange columns i to k for **Q** , **R** and **p**

$$r_{i,i} = |\mathbf{q}_i| \tag{2.60}$$

$$\mathbf{q}_i = \mathbf{q}_i / r_{i,i} \tag{2.61}$$

For j = i+1 to M

$$r_{i,j} = q_i^H * q_j \tag{2.62}$$

$$\mathbf{q}_j = \mathbf{q}_j - r_{i,j}\mathbf{q}_i \tag{2.63}$$

End

End

Where (M is the number of transmitting antenna , $\mathbf{q}_l$ is the lth column of orthogonal matrix **Q**, $r_{i,j}$ is the (i,j) entry of the upper triangular matrix R)

The procedure of MIMO-OFDM system with particle filtering and SQR decomposition

Step 1 : Using sorted QR algorithm to obtain matrix **Q, R** and **p.**

Step 2 : Multiply $\mathbf{Q}^{\mathbf{H}}$ to the received signal vector.

Step 3:

For k = 1 to M (Where M is the number of transmitting antenna)

For i = 1 to Np (Where Np is number of particles)

- Draw a particle from the importance distribution $p(s_k \mid s_{1:k-1}^{(i)}, y_{1:k})$
- Calculate the weight by using equation (2.35)
- Store the new particle $s_k^{(i)}$ to $s_{1:k-1}^{(i)}$

End For

- Normalized all the weights $\tilde{w}_k^{(i)} = \dfrac{w_k^{(i)}}{\sum_{i=1}^{Np} w_k^{(i)}}$
- Calculate the effective sample size N_{eff} using (2.36)
- If N_{eff} < Ns , then do the re-sampling scheme.

$$p(s_{1:k} \mid y_{1:k}) \approx \sum_{i=1}^{Np} \tilde{w}_k^{(i)} \delta(s_k - s_k^{(i)})$$

End For

Step 4 : Detect signal using $\hat{s}_{1:M} = \arg\max_{s_{1:k}} p(s_{1:M} \mid y_{1:M})$

Step 5 : Reordering all the signals using permutation vector **p**

Chapter 3

Data detection in MIMO-OFDM with space frequency block code with particle filtering

3.1 System model:

We consider the system which has M transmitting antennas and N receiving antennas. The transmitter architecture for MIMO-OFDM with space frequency block code system is shown in figure 3.1.

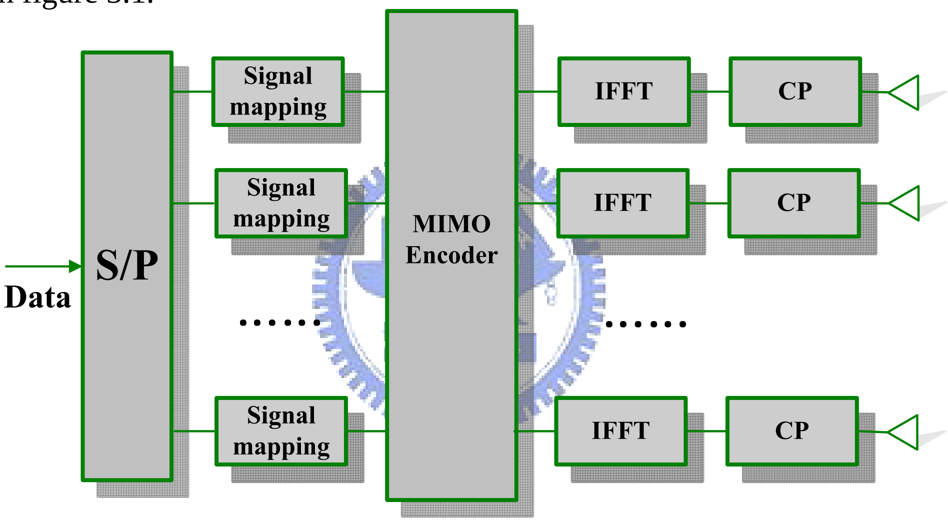


Figure 3.1 Transmitter structure for MIMO-OFDM with space frequency block code

The data stream is mapped first, then these mapped signals are encoded by M/2 pairs of Alamouti code as shown in equation (3.1). For 4 transmitting antennas, 2 pairs of Alamouti code is called Double space time transmitting diversity (DSTTD) code as described in [12].

$$\mathbf{S} = \begin{bmatrix} S_1 \\ S_2 \\ \cdot \\ \cdot \\ S_{M-1} \\ S_M \end{bmatrix} \rightarrow \mathbf{S}_i = \begin{bmatrix} S_1 & -S_2^* \\ S_2 & S_1^* \\ \cdot & \cdot \\ \cdot & \cdot \\ S_{M-1} & -S_M^* \\ S_M & S_{M-1}^* \end{bmatrix} \tag{3.1}$$

The encoding process is shown as below :

$$S_1, S_2, \ldots\ldots, S_{M(FFT\_len/2)} \rightarrow \begin{bmatrix} S_1 & -S_2^* & \ldots & S_{M(FFT\_len/2)-M+1} & -S_{M(FFT\_len/2)-M+2}^* \\ S_2 & S_1^* & \ldots & S_{M(FFT\_len/2)-M+2} & S_{M(FFT\_len/2)-M+1}^* \\ \cdot & \cdot & \cdot & & \\ \cdot & \cdot & \cdot & & \\ S_{M-1} & -S_M^* & \ldots & S_{M(FFT\_len/2)-1} & -S_{M(FFT\_len/2)}^* \\ S_M & S_{M-1}^* & \ldots & S_{M(FFT\_len/2)} & S_{M(FFT\_len/2)-1}^* \end{bmatrix} = \mathbf{S} \quad (3.2)$$

(where FFT_len is the length of a OFDM symbol),

The modulated signal S_1 to $S_{M(fft\_len/2)}$ are encoded as equation (3.2). Each column vector in matrix **S** represents an encoded signal vector allocated in a particular sub-carrier and each row vector in matrix **S** represents an encoded signal vector allocated in a particular antenna. As the graph shown below:

$$\begin{array}{r} \\ 1^{st}\ \text{Tx antenna} \rightarrow \\ 2^{nd}\ \text{Tx antenna} \rightarrow \\ .. \\ .. \\ .. \\ M^{th}\ \text{Tx antenna} \rightarrow \end{array} \begin{array}{c} \begin{array}{ccccc} \text{Sub 1} & \text{Sub 2} & \ldots\ldots\ldots\ldots & & \text{Sub FFT\_len} \end{array} \\ \begin{bmatrix} S_1 & -S_2^* & \ldots & S_{M(FFT\_len/2)-M+1} & -S_{M(FFT\_len/2)-M+2}^* \\ S_2 & S_1^* & \ldots & S_{M(FFT\_len/2)-M+2} & S_{M(FFT\_len/2)-M+1}^* \\ \cdot & \cdot & \cdot & & \\ \cdot & \cdot & \cdot & & \\ S_{M-1} & -S_M^* & \ldots & S_{M(FFT\_len/2)-1} & -S_{M(FFT\_len/2)}^* \\ S_M & S_{M-1}^* & \ldots & S_{M(FFT\_len/2)} & S_{M(FFT\_len/2)-1}^* \end{bmatrix} \end{array}$$

Then converts each row of matrix **S** by using Inverse Fast Fourier transform to time domain signal expressed in the next page.

$$
\begin{bmatrix}
S_1 & -S_2^* & \cdots & S_{M(FFT\_len/2)-M+1} & -S^*_{M(FFT\_len/2)-M+2} \\
S_2 & S_1^* & \cdots & S_{M(FFT\_len/2)-M+2} & S^*_{M(FFT\_len/2)-M+1} \\
. & . & . & & \\
. & . & . & & \\
S_{M-1} & -S_M^* & \cdots & S_{M(FFT\_len/2)-1} & -S^*_{M(FFT\_len/2)} \\
S_M & S_{M-1}^* & \cdots & S_{M(FFT\_len/2)} & S^*_{M(FFT\_len/2)-1}
\end{bmatrix}
\xrightarrow{\text{IFFT}}
\begin{bmatrix}
s_{1,1} & s_{1,2} & \cdots & s_{1,FFT\_len-1} & s_{1,FFT\_len} \\
s_{2,1} & s_{2,2} & \cdots & s_{2,FFT\_len-1} & s_{2,FFT\_len} \\
. & . & . & & \\
. & . & . & & \\
s_{M-1,1} & s_{M-1,2} & \cdots & s_{M-1,FFT\_len-1} & s_{M-1,FFT\_len} \\
s_{M,1} & s_{M,2} & \cdots & s_{M,FFT\_len-1} & s_{M,FFT\_len}
\end{bmatrix}
\tag{3.3}
$$

Adding Guard interval for each row vector, then signals in each row are transmitted from different antenna. Since the encode process is implemented in frequency domain (subcarrier). We treat this type of code as space frequency block code.

3.2 MIMO-decoder

In receiver side, After guard interval removal and Fast Fourier transform, the received signals at n[th] received antenna over subcarrier 1 and 2 are expressed as

$$
\begin{aligned}
Y_n(1) &= H_{1,n}(1)S_1 + H_{2,n}(1)S_2 + \ldots\ldots + H_{(M-1),n}(1)S_{M-1} + H_{M,n}(1)S_M + n_n(1) \\
Y_n^*(2) &= H^*_{2,n}(2)S_1 - H^*_{1,n}(2)S_2 + \ldots\ldots + H^*_{(M),n}(2)S_{M-1} - H^*_{(M-1),n}(2)S_M + n_n^*(2)
\end{aligned}
\tag{3.4}
$$

$Y_n(k)$: Received signal of n th received antenna at k th sub-carrier

$\mathrm{H}_{mn}(k)$: Channel response in frequency domain for m th transmitting antenna and n th receiving antenna

S_m : m th mapped data

$\mathrm{n}_n(k)$: Noise at n th receiving antenna for k th subcarrier

The matrix form representation for MIMO-OFDM 4X2 with space frequency block code system (for subcarrier 1 and 2) can be expressed as

$$\mathbf{Y} = \mathbf{HS} + \mathbf{N} \tag{3.5}$$

$$\begin{bmatrix} Y_1(1) \\ Y_1^*(2) \\ Y_2(1) \\ Y_2^*(2) \end{bmatrix} = \begin{bmatrix} H_{11}(1) & H_{21}(1) & H_{31}(1) & H_{41}(1) \\ H_{21}^*(2) & -H_{11}^*(2) & H_{41}^*(2) & -H_{31}^*(2) \\ H_{12}(1) & H_{22}(1) & H_{32}(1) & H_{42}(1) \\ H_{22}^*(2) & -H_{12}^*(2) & H_{42}^*(2) & -H_{32}^*(2) \end{bmatrix} \begin{bmatrix} S_1 \\ S_2 \\ S_3 \\ S_4 \end{bmatrix} + \begin{bmatrix} n_1(1) \\ n_1^*(2) \\ n_2(1) \\ n_2^*(2) \end{bmatrix} \tag{3.6}$$

Where **H** is the equivalent channel matrix, **S** is the original symbol vector which is one of the columns in equation (3.2) and **N** is the additive complex white Gaussian noise with variance σ^2.Assuming that $H_{mn}(1) \approx H_{mn}(2)$ and define

$$\mathbf{H_{eq}} = \begin{bmatrix} H_{11}(1) & H_{21}(1) & H_{31}(1) & H_{41}(1) \\ H_{21}^*(1) & -H_{11}^*(1) & H_{41}^*(1) & -H_{31}^*(1) \\ H_{12}(1) & H_{22}(1) & H_{32}(1) & H_{42}(1) \\ H_{22}^*(1) & -H_{12}^*(1) & H_{42}^*(1) & -H_{32}^*(1) \end{bmatrix} \tag{3.7}$$

Multiplying $\mathbf{H_{eq}^H}$ (where $()^H$ represents Hermitian of a matrix) to the received vector we obtain

$$\tilde{\mathbf{Y}} = \mathbf{H_{eq}^H Y} = \mathbf{H_{eq}^H HS} + \mathbf{H_{eq}^H N} \tag{3.8},$$

Since we assume $H_{mn}(1) \approx H_{mn}(2)$, for 4 transmitting antennas, the equivalent channel matrix **H** will almost equal to $\mathbf{H_{eq}}$ as shown

$$\mathbf{H_{eq}} = \begin{bmatrix} H_{11}(1) & H_{21}(1) & H_{31}(1) & H_{41}(1) \\ H_{21}^*(1) & -H_{11}^*(1) & H_{41}^*(1) & -H_{31}^*(1) \\ H_{12}(1) & H_{22}(1) & H_{32}(1) & H_{42}(1) \\ H_{22}^*(1) & -H_{12}^*(1) & H_{42}^*(1) & -H_{32}^*(1) \end{bmatrix} \approx \begin{bmatrix} H_{11}(1) & H_{21}(1) & H_{31}(1) & H_{41}(1) \\ H_{21}^*(2) & -H_{11}^*(2) & H_{41}^*(2) & -H_{31}^*(2) \\ H_{12}(1) & H_{22}(1) & H_{32}(1) & H_{42}(1) \\ H_{22}^*(2) & -H_{12}^*(2) & H_{42}^*(2) & -H_{32}^*(2) \end{bmatrix} = \mathbf{H} \tag{3.9}$$

So that
$$\mathbf{H_{eq}}^{\mathbf{H}}\mathbf{H} \approx \mathbf{H_{eq}}^{\mathbf{H}}\mathbf{H_{eq}} = \begin{bmatrix} \rho_1 & 0 & \alpha & \beta \\ 0 & \rho_1 & -\beta^* & \alpha^* \\ \alpha & -\beta & \rho_2 & 0 \\ \beta^* & \alpha^* & 0 & \rho_2 \end{bmatrix} \tag{3.10}$$

where

$$\rho_1 = \sum_{m=1}^{2}\sum_{n=1}^{N}\left|H_{mn}(k)\right|^2 \qquad \text{N : Number of RX antenna} \tag{3.11}$$

$$\rho_2 = \sum_{m=3}^{4}\sum_{n=1}^{N}\left|H_{mn}(k)\right|^2 \tag{3.12}$$

$$\alpha = \sum_{i=1}^{N}(H_{1i}^*(k)H_{3i}(k) + H_{2i}(k)H_{4i}^*(k)) \tag{3.13}$$

$$\beta = \sum_{i=1}^{nr}(H_{1i}^*(k)H_{4i}(k) - H_{2i}(k)H_{3i}^*(k)) \tag{3.14}$$

First, observing the matrix form shown above, we discover that $\mathbf{H_{eq}^H H_{eq}}$ is complex symmetric as shown below

$$\mathbf{H_{eq}^H H_{eq}} = \begin{bmatrix} \rho_1 \mathbf{I} & \mathbf{D} \\ \mathbf{D^H} & \rho_2 \mathbf{I} \end{bmatrix} \tag{3.15}$$

$$where\ \mathbf{D} = \begin{bmatrix} \alpha & \beta \\ -\beta^* & \alpha^* \end{bmatrix}, \mathbf{I} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{3.16}$$

Second, noise vector $\mathbf{H_{eq}^H N}$ is not white noise any more.

On the other hand, if the channel delay spread is large, $\mathbf{H_{eq}}$ which is the average of the k th and k+1 th channel is used for data detection.

$$\mathbf{H_{eq}} =$$

$$\frac{1}{2}\left(\begin{bmatrix} H_{1,1}(k) & H_{2,1}(k) & \dots & \dots & H_{(M-1),1}(k) & H_{M,1}(k) \\ H_{2,1}^*(k) & -H_{1,1}^*(k) & \dots & \dots & H_{M,1}^*(k) & -H_{(M-1),1}^*(k) \\ . & . & \dots & \dots & \dots & \dots \\ . & . & \dots & \dots & \dots & \dots \\ H_{1,(N)}(k) & H_{2,(N)}(k) & \dots & \dots & H_{(M-1),(N-1)}(k) & H_{M,N}(k) \\ H_{2,N}^*(k) & -H_{1,N}^*(k) & \dots. & \dots & H_{M,N}^*(k) & -H_{(M-1),N}^*(k) \end{bmatrix} + \begin{bmatrix} H_{1,1}(k+1) & H_{2,1}(k+1) & \dots & \dots & H_{(M-1),1}(k+1) & H_{M,1}(k+1) \\ H_{2,1}^*(k+1) & -H_{1,1}^*(k+1) & \dots & \dots & H_{M,1}^*(k+1) & -H_{(M-1),1}^*(k+1) \\ . & . & \dots & \dots & \dots & \dots \\ . & . & \dots & \dots & \dots & \dots \\ H_{1,(N)}(k+1) & H_{2,(N)}(k+1) & \dots & \dots & H_{(M-1),(N-1)}(k+1) & H_{M,N}(k+1) \\ H_{2,N}^*(k+1) & -H_{1,N}^*(k+1) & \dots. & \dots & H_{M,N}^*(k+1) & -H_{(M-1),N}^*(k+1) \end{bmatrix}\right) \tag{3.17}$$

After multiplying $\mathbf{H_{eq}^H}$ to received vector **Y,** the new expression can be shown as

$$\tilde{\mathbf{Y}} = \mathbf{H}_{eq}{}^{H}\mathbf{Y} = \mathbf{H}_{eq}{}^{H}\mathbf{HS} + \mathbf{H}_{eq}{}^{H}\mathbf{N} = \mathbf{H}_{eq}{}^{H}(\mathbf{H}_{eq} + \mathbf{E})\mathbf{S} + \mathbf{H}_{eq}{}^{H}\mathbf{N} \quad (3.18)$$

$$= \mathbf{H}_{eq}{}^{H}\mathbf{H}_{eq}\mathbf{S} + \mathbf{H}_{eq}{}^{H}\mathbf{N} + \mathbf{H}_{eq}{}^{H}\mathbf{ES} \quad (3.19)$$

From the equation shown above, there are three terms in equation (3.19). The third term is the error term, since we averaging the equivalent channel matrix, the error term is assumed to be small, so that we can ignore this term. This term will affect the performance if the error term is large.

The matrix $\mathbf{H}_{eq}^{H}\mathbf{H}_{eq}$ is positive definite, so that we can use Cholesky decomposition to decompose such matrix. Cholesky decomposition is a method to separate a matrix to a upper triangular matrix and its hermitian such that $\mathbf{H}_{eq}^{H}\mathbf{H}_{eq} = \mathbf{U}^{H}\mathbf{U}$,where **U** is a upper triangular matrix. We multiply $(\mathbf{U}^{H})^{-1}$ to $\tilde{\mathbf{Y}}$, then

$$\bar{\mathbf{Y}} = (\mathbf{U}^{H})^{-1}\tilde{\mathbf{Y}} \approx (\mathbf{U}^{H})^{-1}\mathbf{H}_{eq}^{H}\mathbf{H}_{eq}\mathbf{S} + (\mathbf{U}^{H})^{-1}\mathbf{H}_{eq}^{H}\mathbf{N} \quad (3.20)$$

$$= (\mathbf{U}^{H})^{-1}\mathbf{U}^{H}\mathbf{US} + (\mathbf{U}^{H})^{-1}\mathbf{H}_{eq}^{H}\mathbf{N} \quad (3.21)$$

The signal after pass through multiplied $(\mathbf{U}^{H})^{-1}$ will become:

$$\bar{\mathbf{Y}} = \mathbf{US} + (\mathbf{U}^{H})^{-1}\mathbf{H}_{eq}^{H}\mathbf{N} \quad (3.22)$$

Where **S** is the symbol vector and $(\mathbf{U}^{H})^{-1}\mathbf{H}_{eq}^{H}\mathbf{N}$ is a new noise vector. There are two properties from the equation(3.22) written above. First of all, the upper triangular matrix is obtained which accompanies with signal vector. Second, we consider the noise vector $(\mathbf{U}^{H})^{-1}\mathbf{H}_{eq}^{H}\mathbf{N}$, the covariance matrix of the new noise vector is

$$\mathbf{E}[((\mathbf{U}^{H})^{-1}\mathbf{H}_{eq}^{H}\mathbf{N})((\mathbf{U}^{H})^{-1}\mathbf{H}_{eq}^{H}\mathbf{N})^{H}] = \mathbf{E}[(\mathbf{U}^{H})^{-1}\mathbf{H}_{eq}^{H}\mathbf{NN}^{H}\mathbf{H}_{eq}\mathbf{U}^{-1}] \quad (3.23)$$

$$= (\mathbf{U}^{H})^{-1}\mathbf{H}_{eq}^{H}\mathbf{E}[\mathbf{NN}^{H}]\mathbf{H}_{eq}\mathbf{U}^{-1} \quad (3.24)$$

$$= (\mathbf{U}^{H})^{-1}\mathbf{H}_{eq}^{H}\sigma^{2}\mathbf{I}\,\mathbf{H}_{eq}\mathbf{U}^{-1} \quad (3.25)$$

$$= (\mathbf{U^H})^{-1}\,\mathbf{H_{eq}^H}\boldsymbol{\sigma}^2\mathbf{I}\,\mathbf{H_{eq}}\mathbf{U}^{-1} \tag{3.26}$$

$$= \boldsymbol{\sigma}^2(\mathbf{U^H})^{-1}\,\mathbf{H_{eq}^H}\mathbf{H_{eq}}\mathbf{U}^{-1} \tag{3.27}$$

$$= \boldsymbol{\sigma}^2(\mathbf{U^H})^{-1}(\mathbf{U^H}\mathbf{U})\mathbf{U}^{-1} = \boldsymbol{\sigma}^2\mathbf{I} \tag{3.28}$$

After multiplying $(\mathbf{U^H})^{-1}$ to the received signal vector, the new noise vector will become an independent white noise vector again. This method is also called whitening filter.

The matrix form is shown as

$$\begin{bmatrix} \hat{y}_1 \\ \hat{y}_2 \\ . \\ . \\ \hat{y}_M \end{bmatrix} = \begin{bmatrix} U_{11} & U_{12} & \dots & U_{1M} \\ 0 & 0\,.. & \dots & U_{2M} \\ 0 & 0\,.. & \dots & \dots \\ 0 & 0\,.. & 0 & U_{MM} \end{bmatrix} \begin{bmatrix} S_1 \\ S_2 \\ \dots \\ \dots \\ S_M \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \\ \dots \\ \dots \\ n_M \end{bmatrix}, \tag{3.29}$$

So that we can use particle filtering method, drawing particles from the bottom signal to the top signal to detect the signal vector **S**.

Block diagram of receiver structure

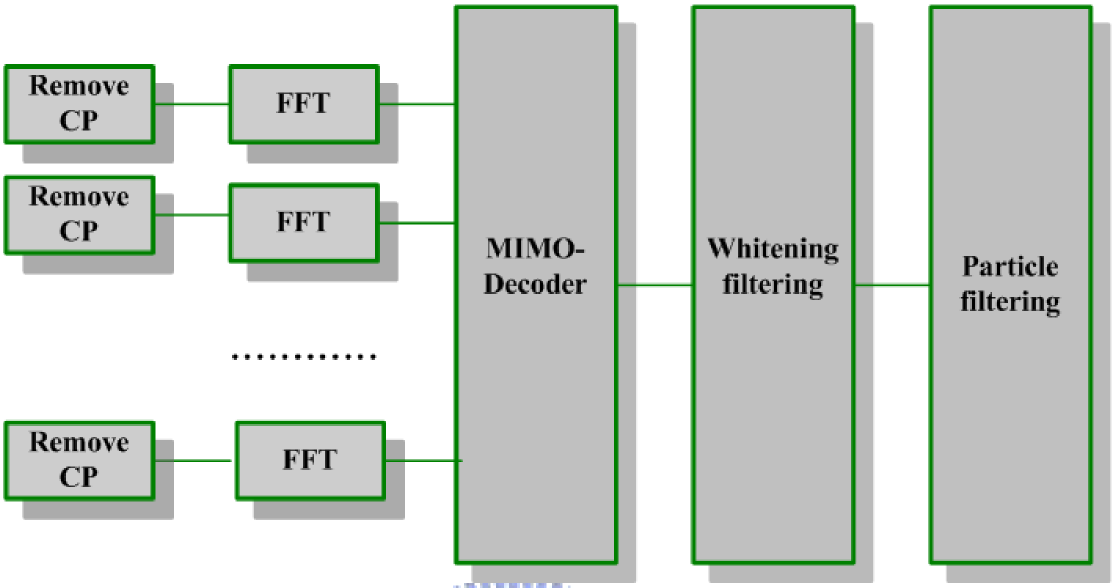


Figure 3.2 Receiver structure for MIMO-OFDM system with space frequency block code

3. 3 Error propagation mitigation method

In the previous section, we use Gram Schmidt decomposition to obtain two upper triangular matrixes. BER performance will be improved using particle filtering using such method in spatial multiplexing system. On the other hand, in MIMO-OFDM with space frequency block code system, this method can be implemented similar to spatial multiplexing system. In the previous section, we decompose channel matrix **H** into $\mathbf{Q_1R_1}$ and $\mathbf{Q_2R_2}$. Now, after Cholesky decomposition, we obtain an upper triangular matrix **U** and the received vector is

$$\tilde{\mathbf{Y}} = \mathbf{US} + (\mathbf{U}^H)^{-1}\mathbf{H}_{eq}^H\mathbf{N} \tag{3.30}$$

The upper triangular matrix can be written as $\mathbf{U} = [\mathbf{U}_1 \quad \mathbf{U}_2 \quad .... \quad .... \quad \mathbf{U}_M]$,where $\mathbf{U}_K$ is the k^{th} column vector in **U**. The upper triangular matrix **U** can be decomposed by

Gram-Schmidt QR decomposition and written as

$$\mathbf{U}=\mathbf{Q}\ \mathbf{R}_2=[\hat{\mathbf{q}}_1\quad \hat{\mathbf{q}}_2\quad ....\quad ...\quad \hat{\mathbf{q}}_{\mathbf{M}}]\begin{bmatrix} (\hat{\mathbf{q}}_1^{\mathbf{H}}\mathbf{U}_1) & (\hat{\mathbf{q}}_1^{\mathbf{H}}\mathbf{U}_2) & \dots & \dots & (\hat{\mathbf{q}}_1^{\mathbf{H}}\mathbf{U}_{\mathbf{M}}) \\ (\hat{\mathbf{q}}_2^{\mathbf{H}}\mathbf{U}_1) & (\hat{\mathbf{q}}_2^{\mathbf{H}}\mathbf{U}_2) & \dots & \dots & 0 \\ \dots & .... & \dots & 0 & 0 \\ \dots & .... & 0 & 0 & 0 \\ (\hat{\mathbf{q}}_{\mathbf{M}}^{\mathbf{H}}\mathbf{U}_1) & 0 & 0 & 0 & 0 \end{bmatrix} \tag{3.31}$$

Where the form of $\mathbf{R}_2$ is another form of upper triangular matrix as same as in the previous section. Multiplying $\mathbf{Q}^{\mathbf{H}}$ to $\mathbf{R}$ and obtain

$$\overline{\mathbf{Y}}=\mathbf{Q}^{\mathbf{H}}\tilde{\mathbf{Y}}=\mathbf{R}_2\mathbf{S}+\mathbf{n} \tag{3.32}$$

Since $\mathbf{Q}$ is an orthogonal matrix, the noise vector is still a white noise.

From the discussion above, we get two matrix forms

$$\tilde{\mathbf{Y}}=\begin{bmatrix} U_{11} & U_{12} & \dots & U_{1M} \\ 0 & 0 \,.. & \dots & U_{2M} \\ 0 & 0 \,.. & \dots & \dots \\ 0 & 0 \,.. & 0 & U_{MM} \end{bmatrix}\begin{bmatrix} S_1 \\ S_2 \\ \dots \\ \dots \\ S_M \end{bmatrix}+\begin{bmatrix} n_1 \\ n_2 \\ \dots \\ \dots \\ n_M \end{bmatrix}, \tag{3.33}$$

Now we define the upper triangular matrix

$$\overline{\mathbf{Y}}=\begin{bmatrix} (\hat{\mathbf{q}}_1^{\mathbf{H}}\mathbf{U}_1) & (\hat{\mathbf{q}}_1^{\mathbf{H}}\mathbf{U}_2) & \dots & \dots & (\hat{\mathbf{q}}_1^{\mathbf{H}}\mathbf{U}_{\mathbf{M}}) \\ (\hat{\mathbf{q}}_2^{\mathbf{H}}\mathbf{U}_1) & (\hat{\mathbf{q}}_2^{\mathbf{H}}\mathbf{U}_2) & \dots & \dots & 0 \\ \dots & .... & \dots & 0 & 0 \\ \dots & .... & 0 & 0 & 0 \\ (\hat{\mathbf{q}}_{\mathbf{M}}^{\mathbf{H}}\mathbf{U}_1) & 0 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} S_1 \\ S_2 \\ \cdots \\ \cdots \\ S_M \end{bmatrix}+\begin{bmatrix} \bar{n}_1 \\ \bar{n}_2 \\ \cdots \\ \cdots \\ \bar{n}_M \end{bmatrix} \tag{3.34}$$

We observe from two equations shown above. In (3.33), we can use particle filtering, drawing particles from the bottom signal to the top one and using approach III to obtain the expectation value for each entry in signal vector. Interference will be severe in S_1.

On the other hand, in equation (3.34), draw particles from the top signal to the bottom one and use approach III to obtain the expectation value, interference will be severe in S_M .

Finally, we average these two sets of soft information and make the decision of each symbol by searching the shortest distance for each entry in signal vector.

Block diagram for error mitigation method in MIMO-OFDM with space frequency block code system

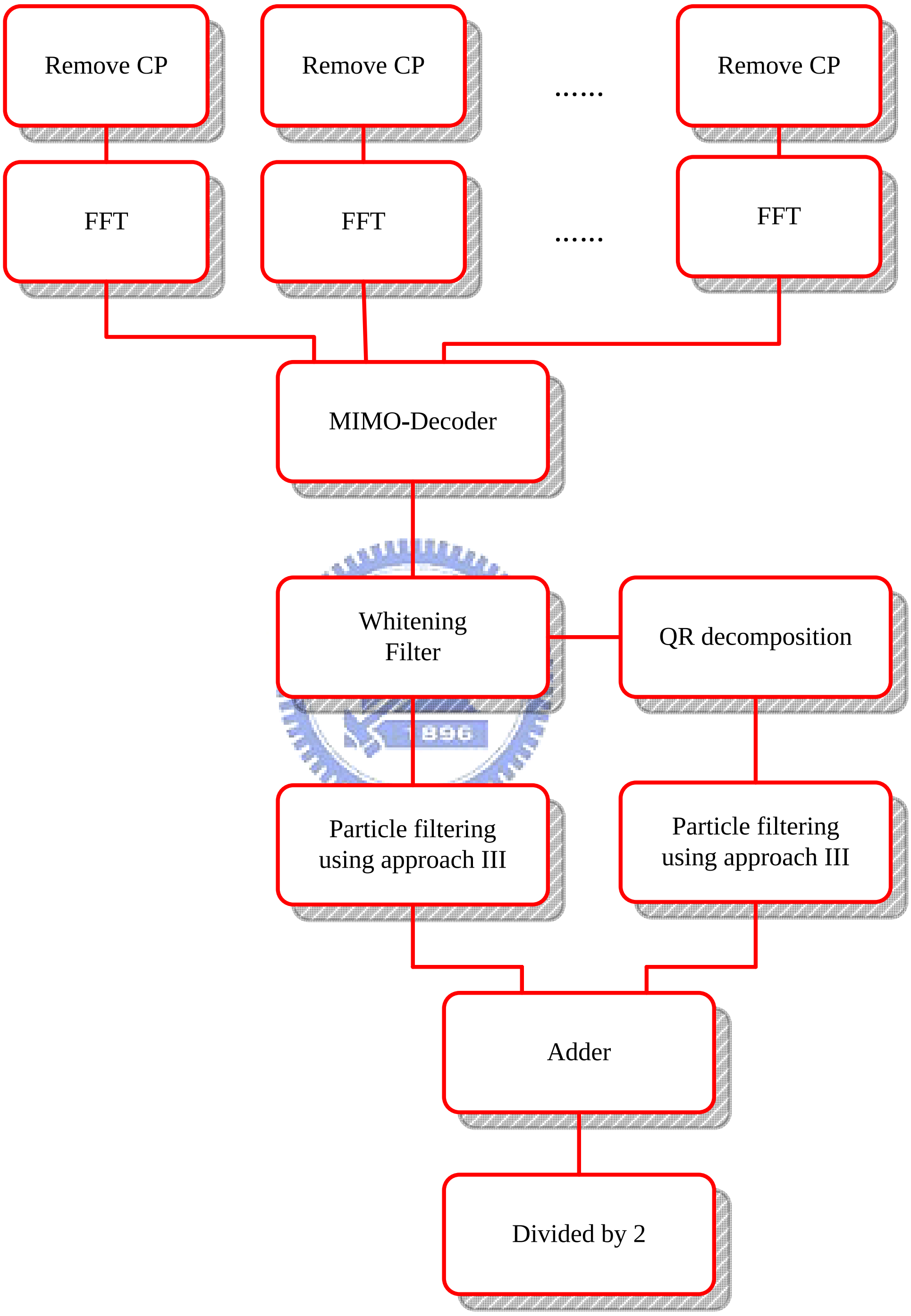


Figure 3.3 Block diagram for error propagation mitigation method in MIMO-OFDM with space frequency block code system

Chapter 4

Simulation results

4.1 Parameters for MIMO-OFDM spatial multiplexing system

Perfect channel information	
Number of subcarriers	256
Length of CP	64
Channel	Two paths model with (0,0)dB
Particles	100 (if not mentioned in the figure)
Approach	I (if not mentioned in the figure)

Table 4.1 Parameters for MIMO-OFDM system

Figure 4.1 shows the BER performance for different approach for perfect CSI in 4X4 spatial multiplexing system for QPSK modulation. As can be observed from figure 4.1, Approach II and III have almost the same performance. For sorted QR decomposition using approach II, performance has 2 dB improvements as compared with unsorted QR using approach II. Approach I has the best performance as compared with approach II and approach III. However, the complexity for approach I is higher than the complexity for approach II and III.

Figure 4.2 shows the BER performance for QPSK modulation in 6X6 spatial multiplexing system for sorted QR decomposition for approach I, QR decomposition with approach II and III with and without sorting. The result shows that the performance for sorted QR decomposition with approach I also has the best BER performance as compared with approach II and III. Moreover, sorted QR decomposition using approach II has 2dB improvement better than without sorting QR decomposition in BER equal to 10^-2.

Figure 4.3 shows the performance of different detection scheme. Approach II is nearly to VBLAST ZF OSIC performance. Performance using sorted QR decomposition and approach II is nearly to the performance for VBLAST MMSE OSIC detection scheme. Moreover, the BER performance of iterative QR decomposition method has 3-4 dB improvement compare with VBLAST MMSE OSIC system and 1dB better than the BER performance of particle filtering using approach I.

Figure 4.4 shows the BER performance for 16QAM modulation with perfect CSI under MIMO-OFDM 4X4 system. As we can see from the figure shown, the performance for the iterative QR decomposition using approach II has 4dB improvement as compared with VBLAST MMSE OSIC. Sorted QR decomposition using approach I has better performance in this system than error propagation mitigation method using approach III.

Figure 4.5 shows the comparison between QR decomposition with and without sorting using approach I in 6X6 system, as shown in figure, the performance has 1 dB improvement under 50 particles as compared with unsorted QR decomposition method using approach I under 50 particles.

Figure 4.6 shows the performance for sorted QR decomposition using approach I in 6X6 system using 50 and 75 particles. There is a little improvement for 75 particles.

Figure 4.7 shows the comparison between the sorted QR decomposition method using approach I in 6X6 spatial multiplexing system with 16 QAM modulation, unsorted QR decomposition method using approach I in 6X6 MIMO-OFDM system and VBLAST MMSE OSIC. Sorted QR decomposition has 3-4 improvement compare with VBLAST MMSE OSIC and 1-2 dB improvement better than unsorted QR decomposition.

Figure 4.8 shows the performance for sorted QR decomposition using approach I in 6X6 system using 50 ,75 and 150 particles for 16 QAM modulation. There is a little improvement for 150 particles and no different between 50 and 75 particles.

4.2 Parameters for MIMO-OFDM with Space frequency block code system

Perfect channel information	
Number of subcarriers	256
Length of CP	64
Channel	Two paths model with (0,0)dB
Particles	100 (if not mentioned in the figure)
Approach	I (if not mentioned in the figure)
MIMO encoder	2 pairs of Alamouti code(DSTTD)

Table 4.2 Parameters for MIMO-OFDM with space frequency block code system

Figure 4.9 and figure 4.10 show the BER performance of 4X2 MIMO-OFDM for QPSK and 16 QAM modulation with space frequency block(2 pairs of Alamouti code) code system for five different detection schemes which are VBLAST MMSE OSIC, Cholesky decomposition with decision feedback, particle filtering using approach I and error propagation mitigation using approach III and ML. First of all, BER performance of particle filtering and error propagation mitigation method have 4dB better than the performance of VBLAST MMSE OSIC and 5dB better than performance of Cholesky decomposition with decision feedback. There are only 2dB worse than the performance of ML decision. For the same system using 16QAM modulation, as shown in figure 4.10, particle filtering has almost the same performance as error propagation mitigation method. Both of them have a 2dB improvement better than VBLAST performance.

In figure 4.11, for 4X4 MIMO-OFDM system with space frequency block code with QPSK modulation, the performance of particle filtering is better than the performance of VBLAST MMSE OSIC and almost the same as ML decision.

In figure 4.12, for 4X4 16QAM modulation system with space frequency block code, the performance of particle filtering is better than the performance of VBLAST MMSE OSIC and Cholesky decomposition with decision feedback.

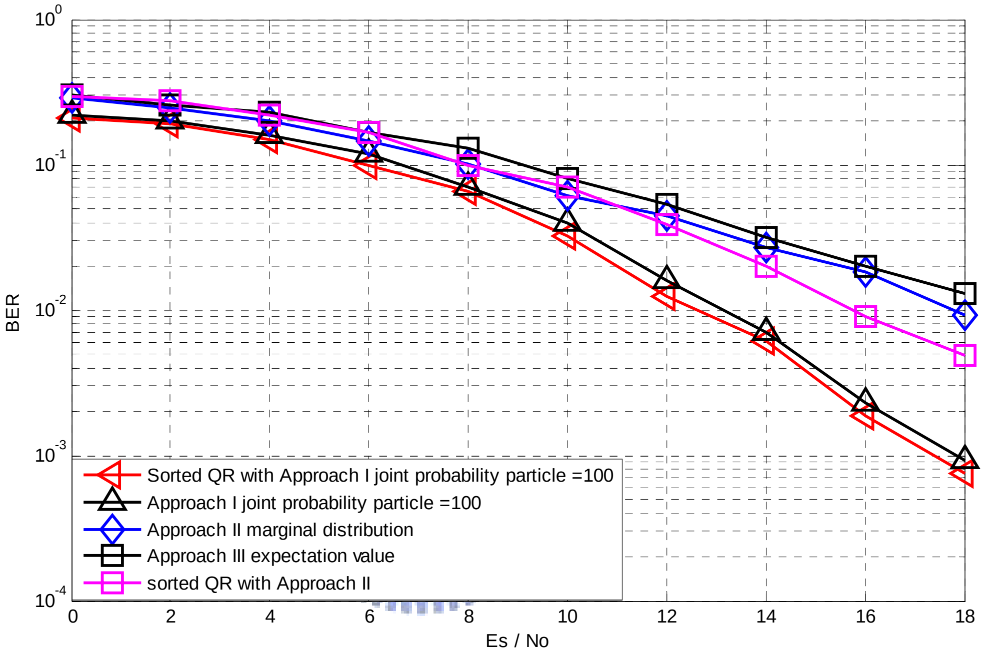


Figure 4.1 Figure 4.1 MIMO-OFDM 4X4 QPSK modulation for different approaches

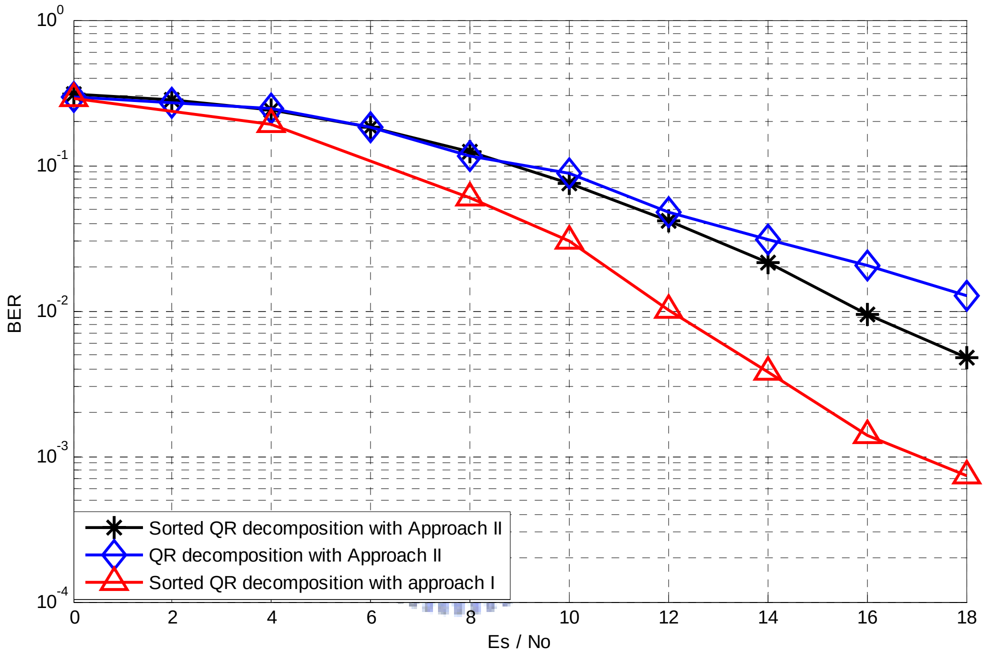


Figure 4.2 MIMO-OFDM 6X6 QPSK modulation for different approaches

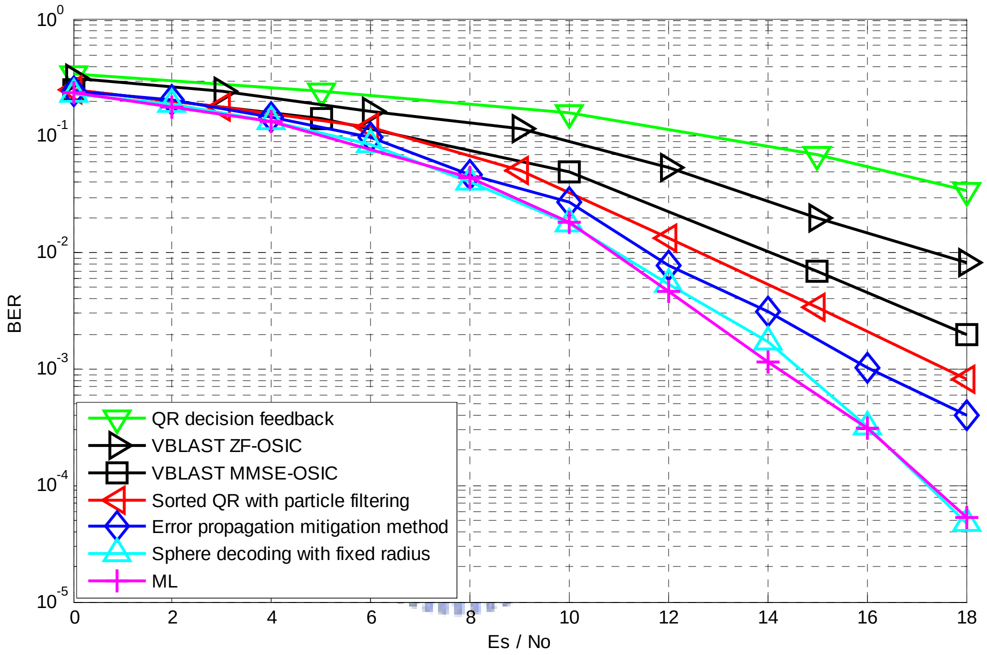


Figure 4.3 MIMO-OFDM 4X4 QPSK for different detection schemes

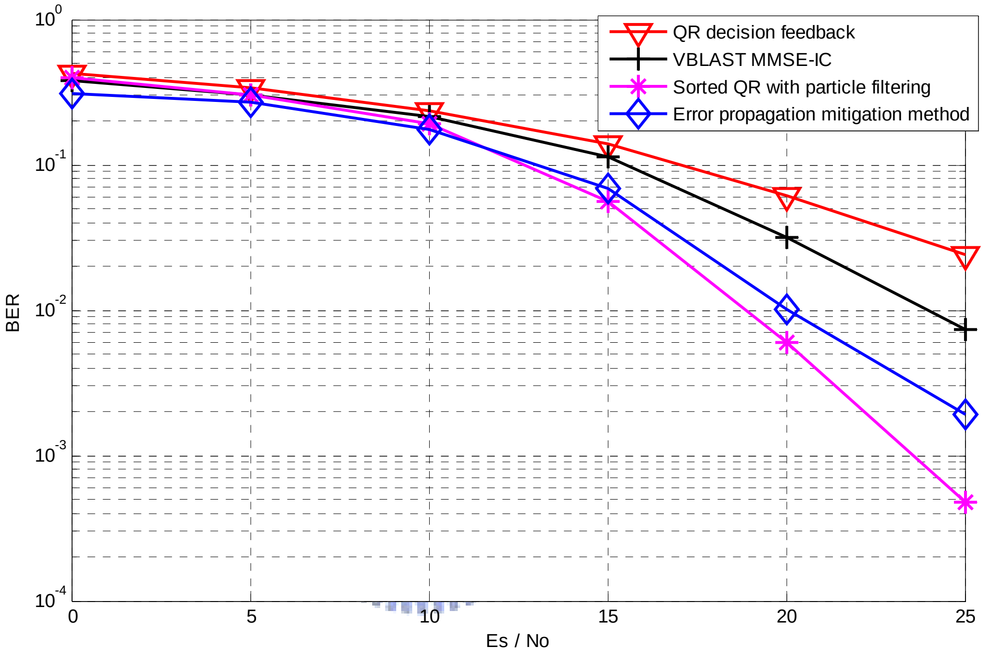


Figure 4.4 MIMO-OFDM 4X4 16 QAM for different detection schemes

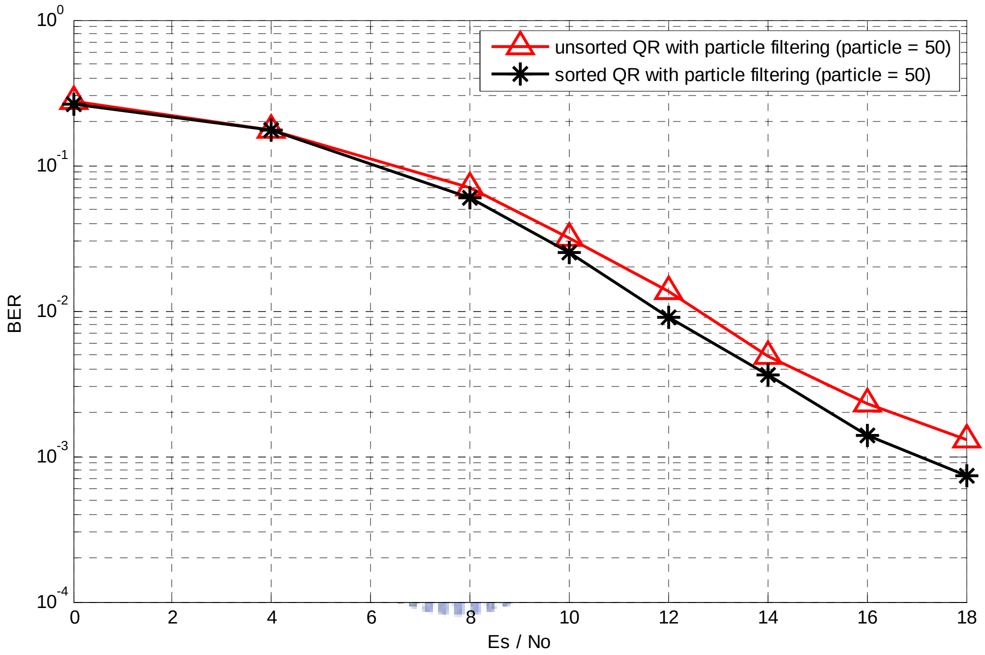


Figure 4.5 MIMO-OFDM 6X6 QPSK modulation with and without sorted QR decomposition

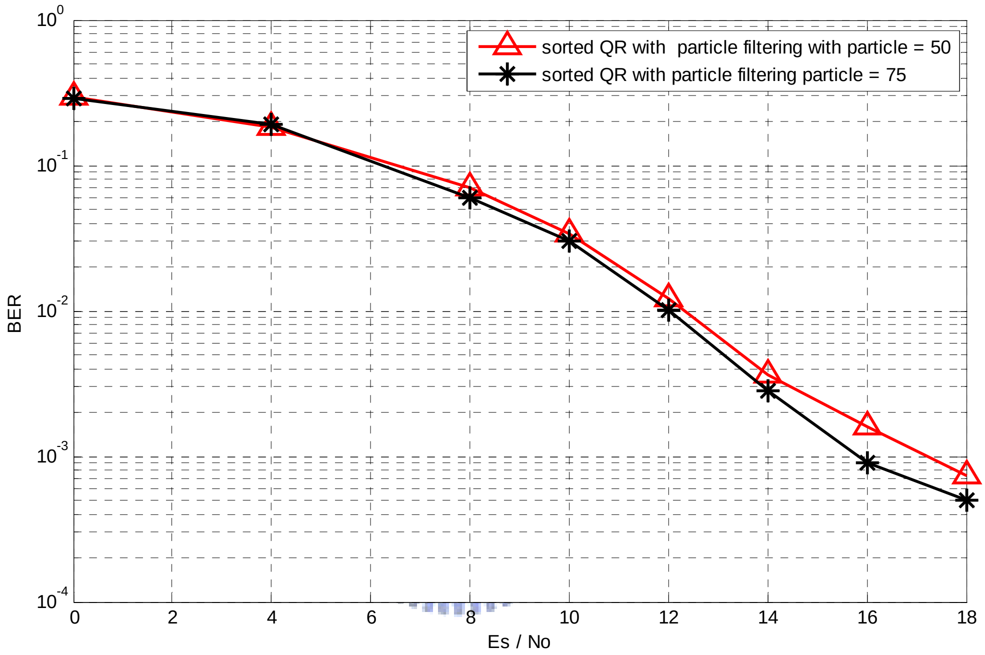


Figure 4.6 MIMO-OFDM 6X6 QPSK with particles equal to 50 and 75

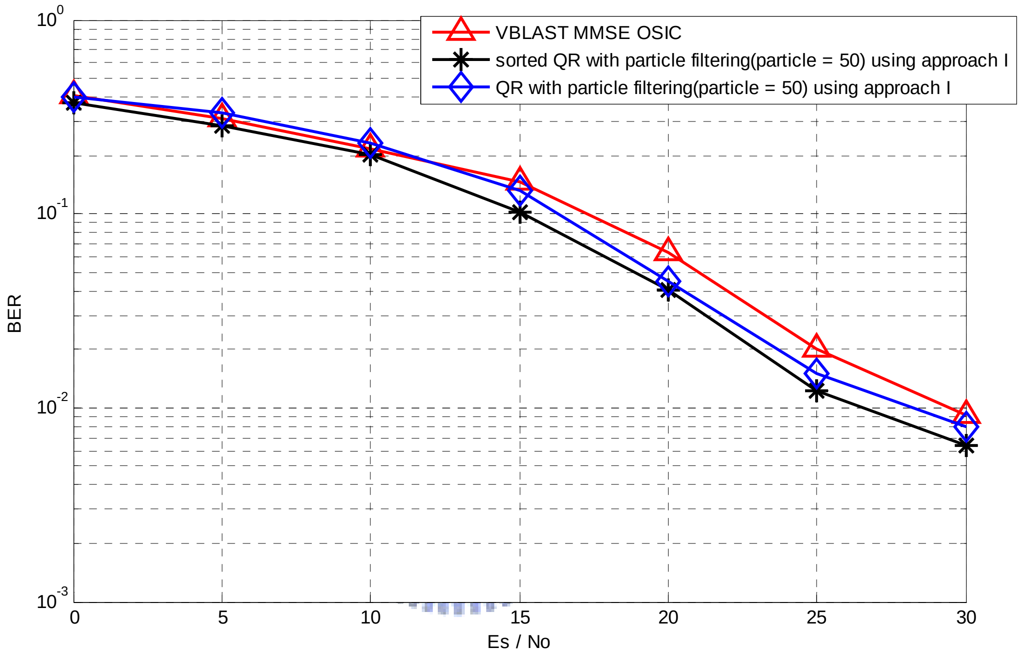


Figure 4.7 MIMO OFDM 6X6 16QAM modulation with and without sorted QR decomposition

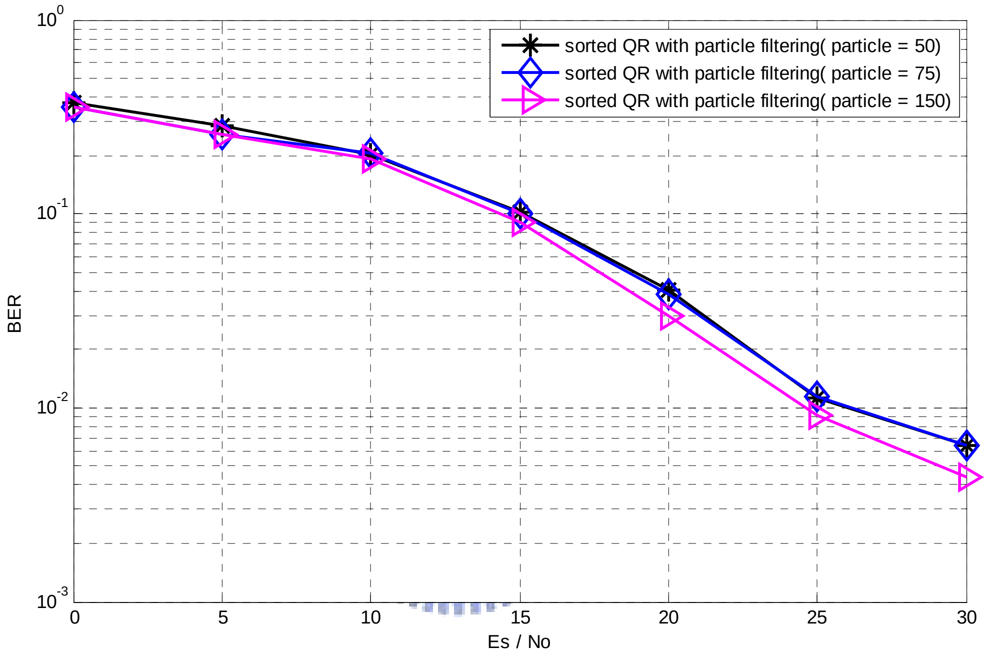


Figure 4.8 MIMO-OFDM 6X6 16QAM modulation particles equal to 50,75 and 200

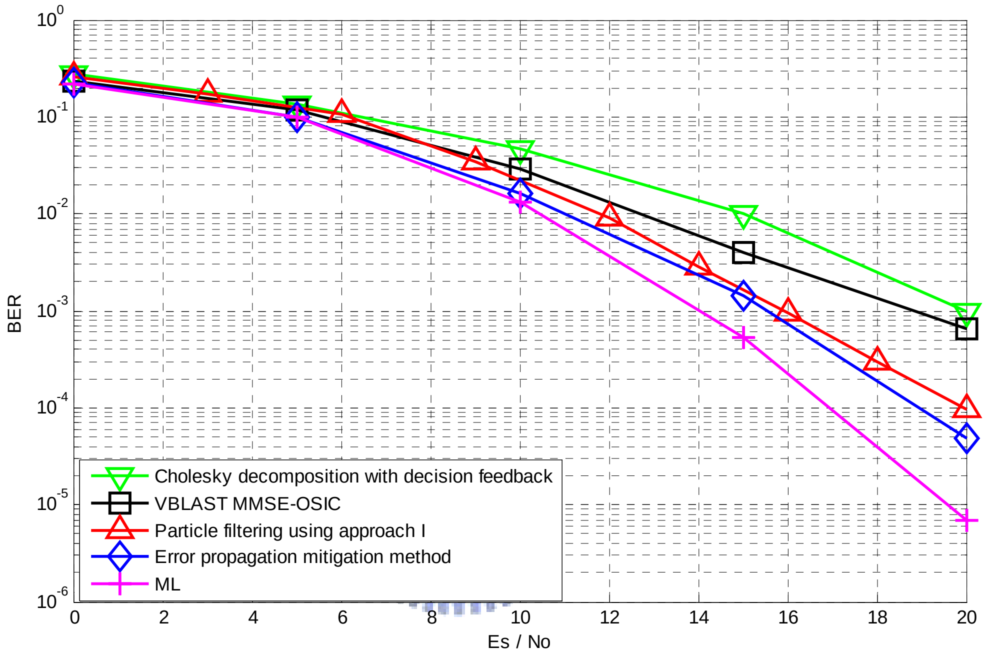


Figure 4.9 MIMO-OFDM 4X2 QPSK with space frequency block code for different detection scheme

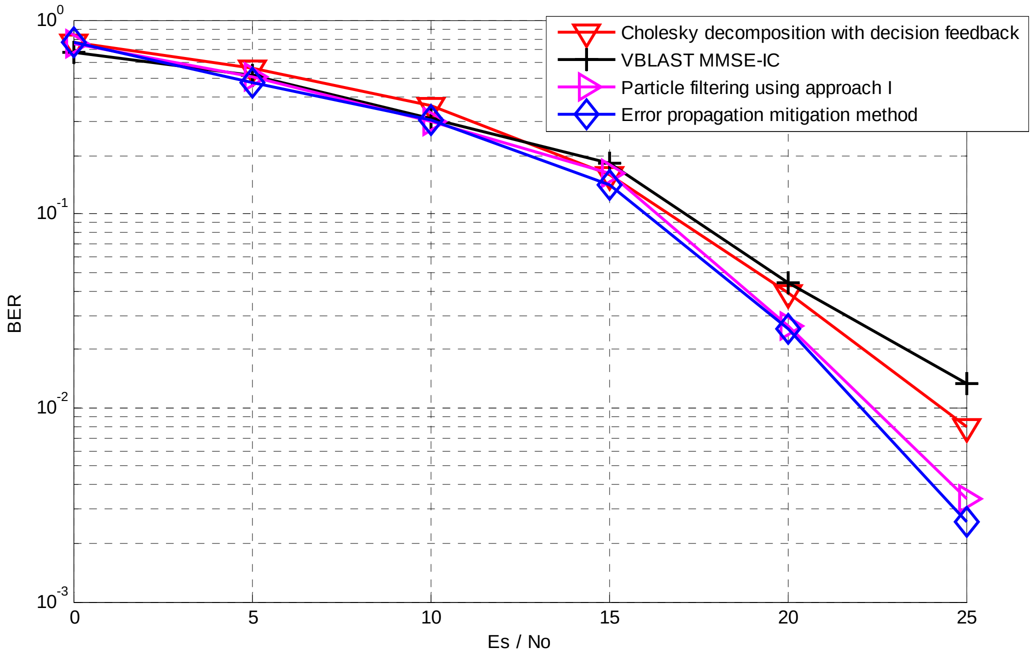


Figure 4.10 MIMO-OFDM 4X2 16QAM with space frequency block code

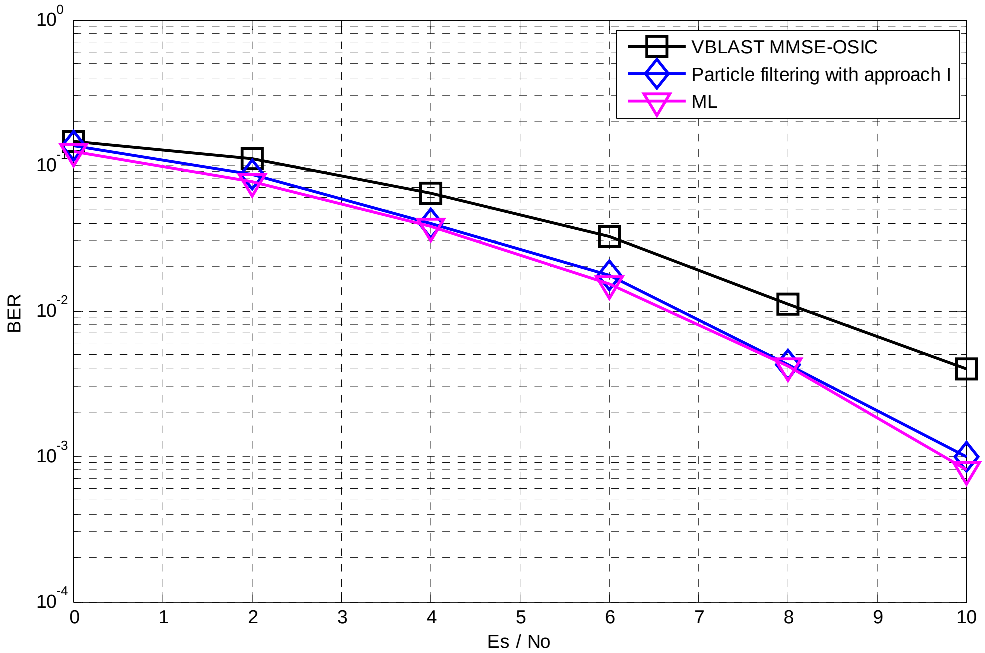


Figure 4.11 MIMO-OFDM 4X4 QPSK with space frequency block code

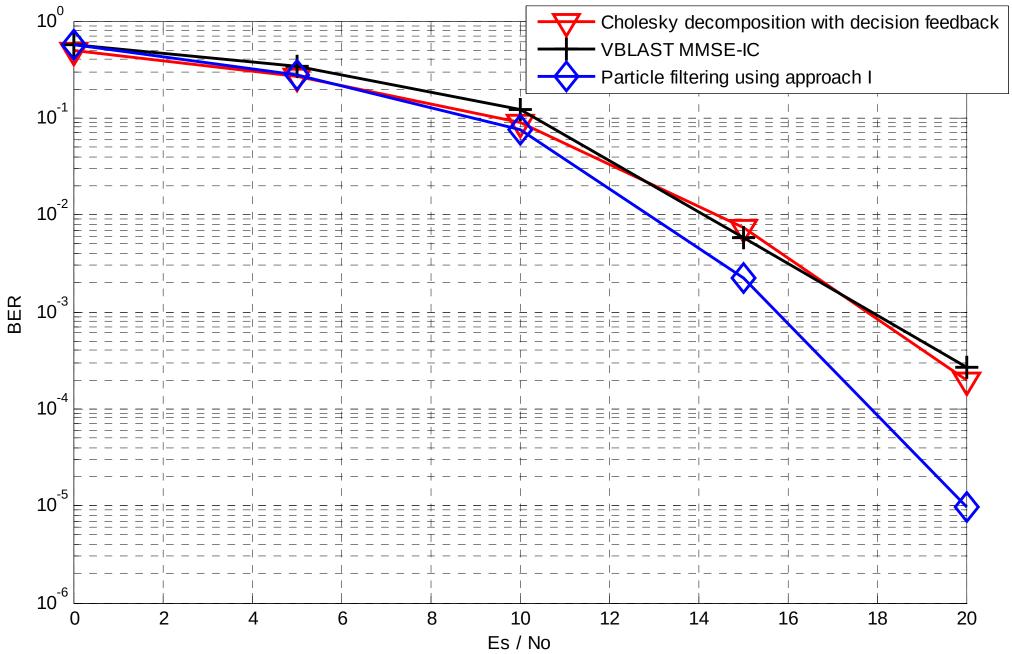


Figure 4.12 MIMO-OFDM 4X4 16QAM with space frequency block code

Chapter 5

Conclusion

Merits and drawbacks of particle filtering algorithm

Except for the complexity of QR decomposition and searching process mentioned in section 2.8, the complexity of particle filtering is directly proportional to three components, the scheme of modulation, the number of transmitting antennas and the number of particles. The complexity for particle filtering is O(A*M*Np), where A is the modulation scheme, eg QPSK, 16QAM. M is number of transmitting antennas and Np is number of particles. The complexity of ML scheme exponentially increases either the number of transmitting antenna or the number of order of modulation increases. The complexity for ML decision is O(A^M). Particle filtering is a practical approach for data detection. As the simulation shown before, the performance of our proposed methods using particle filtering are close to ML decision either in spatial multiplexing system or with space frequency block code system. The complexity for QPSK modulation for 6X6 MIMO-OFDM BLAST system is only 4*4*50 = 800 trials, however, for ML decision method, number of trial is $4^6 = 4096$ trials. The complexity with particle filtering is 5 times lower than the complexity with ML decision, the BER performance of sorted QR decomposition with particle filtering using approach I is only 2 dB worse than ML decision. Moreover, for high order modulation, for example, 16 QAM for 4X4 MIMO-OFDM BLAST system, the complexity for ML will be $16^4 = 65536$ trials, however, particle filtering method only deal with 4*4*100 = 1600 trials. In conclusion, Particle filtering is a suitable approach for high modulation order and large amount of transmitting antenna system in MIMO-OFDM BLAST system.

One of the drawbacks of particle filtering is that the noise distribution is known at the receiver side. For example, if the noise distribution is white Gaussian noise, receiver need to be estimate the variance of noise first, after that pass this variance information to particle filtering.

The second drawback is that particle filtering need the process of QR decomposition or Cholesky decomposition, the complexity will increase for when the number of transmitting antenna increases.

The third drawback is that the searching process mentioned in section 2.8 for approach I and approach II. Especially for approach I, the complexity will increase either the number of transmitting antennas or number of particles increase.

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