國 立 交 通 大 學

電信工程學系碩士班
碩士論文

於通道誤差下空時區塊編碼多用戶多輸入多輸出系統之強健式接收機設計
Design of Robust Receiver for Space-Time Block Coded Multi-user MIMO System under Channel Estimation Errors

研 究 生：林倩羽　　Student: Chien-Yu Lin

指導教授：李大嵩 博士　　Advisor: Dr. Ta-Sung Lee

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中 華 民 國 九 十 七 年 六 月

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摘要

空時區塊編碼（Space-Time Block Code, STBC）在多輸入多輸出（Multiple-Input Multiple-Output, MIMO）的無線通訊系統中是一項被廣泛使用與討論的技術，特別是針對於多用戶的狀況下。目前已有許多空時區塊編碼多用戶多輸入多輸出系統線性接收機演算法被提出，以因應該系統中之多重進接干擾（Multi-Access Inference, MAI）。然而，這些接收機設計有一個共同的問題：它們皆基於完善通道估計的假設下。在實際的傳輸狀況，有限的訓練符元或是嚴重的通道衰退現象時常會導致通道估計錯誤發生。在本論文中，吾人基於通道估計不完善的假設前題，提出一種綜合解碼和消除干擾的強健式接收機設計。此強健式接收機設計是基於限制最佳化（Constrained Optimization）線性等化器，其最佳化問題可藉由廣義旁波帶消除（GSC）技巧被轉換為等效的無限制（Unconstrained）問題而求解。吾人利用擾動 (Perturbation)技巧將通道估計錯誤之效應一併考慮進接收機設計中。吾人同時使用連續干擾消除(successive interference cancellation ,SIC)機制進一步增進接收機效能。由模擬結果可以看出，和其他現存的接收機技術相較，吾人的方法確實能提供較強健之表現。

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Abstract

It is well known that space-time block coding (STBC) has emerged as a popular technique in multiple-input multiple-output (MIMO) wireless communication, especially for multi-user cases. Several linear receiver algorithms have been developed for such a system where multi-access inference (MAI) causes a problem. A common shortcoming of all these techniques, however, is that they are all developed based on the perfect channel assumption. Channel estimation errors do happen in practical situations due to the limited training symbols or severe fading channels. In this thesis, new robust linear receivers for joint space-time decoding and interference rejection in orthogonal space-time block coded multi-user MIMO systems are proposed for combating imperfect channel estimation. The proposed receivers are developed based on the constrained optimization design which can be transformed to an equivalent unconstrained one by the generalized sidelobe canceller (GSC) technique. By using the perturbation techniques, the channel estimation error term can be incorporated into the receiver design. We also apply the successive interference cancellation (SIC) mechanism for further performance enhancement. Numerical simulations confirm the robustness of the proposed receiver when compared with the other existing techniques.

Acknowledgement

First, I would like to express my deepest thanks to my advisors, Dr. Ta-Sung Lee and Dr. Jwo-Yuh Wu for their enthusiastic guidance in completion and improvement of this thesis. I was deeply inspired by their positive attitude in many areas. Heartfelt thanks are also offered to all members in the Communication System Design and Signal Processing (CSDSP) Lab for their constant encouragement. Last but not least, I would like to show my sincere thanks to my parents for their support and love.

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Acronym Glossary

AWGN	additive white Gaussian noise
BER	bit error rate
CSI	channel state information
DL	diagonal loading
GSC	generalized side-lobe canceller
MAI	multi-access interference
MIMO	multiple-input multiple-output
ML	maximum likelihood
MMSE	minimum mean square error
MRC	maximal ratio combining
MV	minimum variance
OFDM	orthogonal frequency division multiplexing
OSIC	ordered successive interference cancellation
OSTBC	orthogonal space-time block codes
QPSK	quaternary phase shift keying
PIC	parallel interference cancellation
SDMA	space division multiple access
SIC	successive interference cancellation
SINR	signal-to- interference-plus-noise ratio
SNR	signal-to-noise ratio
STC	space-time coding
STBC	space-time block codes
SVD	singular value decomposition
TX	transmitter
ZF	zero forcing

Notations

B	blocking matrix
D	desired signal's space-time signature
H	channel matrix between the transmitter and receiver
K	size of transmit symbol vector
M	number of receive antenna
N	number of transmit antenna
Q	numbers of transmitters
R	rate of space-time block codes
s	information symbol vector
T	block length
U	adaptive matrix
V	channel noise
W	coefficient weighting matrix
X	transmission matrix
Y	received signal matrix
σ_D^2	channel estimation error power
σ_v^2	noise power
γ	loading factor

Chapter 1

Introduction

Space-time coding is famous for its ability to exploit spatial diversity and combat fading in multiple-input multiple-output (MIMO) wireless communication systems [1]-[3]. On the other hand, orthogonal space-time block codes (OSTBC) provides low complexity decoding scheme as a result of its special coding structure [2]-[3]. Although there is the optimal maximum likelihood (ML) detector [4] for point-to-point MIMO communications, this simple linear receiver cannot handle the problems of co-channel user interference when it comes to the multi-user case. In order to recover the desired signal, one may in general resort to the joint maximum likelihood (Joint ML) detection but it usually suffers from intensive computational efforts and complicated implementation. Therefore, we here seek for a suboptimal but simple solution for space-time block coded multi-user MIMO system.

Nowadays several alternative approaches have been developed for space-time coded multi-user MIMO systems. Some typical proposals include the Stamoulis's decoupled detection method [5] and the Naguib's parallel interference cancellation (PIC) approaches [6]. The former uses decoupling to perform interference cancellation; the latter designs a two stage procedure that combines the ML and MMSE schemes. However, the Stamoulis's and Naguib's approaches are restricted by the transmitters that consist of two antennas only and the usage of the Alamouti's OSTBC scheme.

Other methods such as MV linear receivers have been proposed in [7]. In contrast to [5]–[6], these techniques are applicable to the general case of arbitrary OSTBCs and multiple interferers. But like the receivers mentioned previously, these proposals are also based on the perfect channel estimation assumption.

Nevertheless, in practical wireless communications, there are several factors which can affect the accuracy of the channel estimation at the receiver. For example, the channel estimation error can be caused by the limited duration/outdating of the training sequence. When it comes to channel mismatch, the performance of above mentioned receivers will degrade seriously. As a result, the robustness of the receiver against imperfect channel estimation becomes an important issue. The robust generalization of the MV techniques was proposed in [8] and developed based on the worst-case performance optimization approach. Later the less conservative receiver was suggested in [9]. It guarantees robustness against CSI errors with a certain selected (high) probability in probability-constrained stochastic optimization format. Yet the above two methods do not provide a linear close form solution such that a built-in convex optimization software may be needed. In this thesis, we first address the case with perfect channel estimation. We resort to the generalized sidelobe canceller (GSC) principle [10]-[12] to transform the original constrained optimization problem in the multi-user MIMO system into an equivalent unconstrained framework. Then we apply the successive cancellation (SIC) mechanism [13]-[14] to GSC-based equalizer for a further performance enhancement. Motivated by the robust design for MIMO-OFDM systems in [15], we propose an efficient approach to the design of robust linear receiver under imperfect channel estimation. The receiver is an extended version of the GSC/SIC-based equalizer and has an elegant close form solution.

The remainder of this thesis is organized as follows. In Chapter 2, the overview of space-time coding is first introduced and then the model of the space-time coded single-user MIMO system is built. In addition, the single user case is extended to the multi-user case. In Chapter 3, under the assumption of perfect channel estimation, a GSC/SIC-based equalizer is proposed to combat multi-user interference and noise at the same time. In Chapter 4, we develop a new robust GSC/SIC-based equalizer based on perturbation analysis. The main results are presented and computer simulations of the proposed scheme are illustrated respectively at the ends of the chapters. Finally, we conclude this thesis and propose some potential future works in Chapter 5

Chapter 2

Models of Space-Time Block Coded MIMO System

This chapter presents the overview of space-time block coded multiple-in multiple-out (MIMO) wireless communication systems. At first, a brief introduction about space-time block code (STBC) is given. We focus on orthogonal space-time block code (OSTBC), which is an attractive class of space-time coding techniques, because of its full diversity gain and low decoding complexity. Since nowadays OSTBC has emerged as an efficient approach to exploiting spatial diversity and combating fading in MIMO system, the model for point-to-point MIMO systems with OSTBC is developed. Finally, we extend the above point-to-point model to a multi-user one. This system model is helpful for us to analyze and design the receiver scheme in the following works.

2.1 Overview of Space-Time Block Code

Space-time coding (STC) is first proposed by Vahid Tarokh *et al.* [1]. The original space-time codes are based on trellis codes which achieve significant performance improvements over single receive antenna systems. Later the simpler block code version which is called space–time block code (STBC) [2] is proposed. Since transmit diversity has been an important factor which combating fading channels especially under the power constraint and bandwidth efficiency considerations, STBC involves temporal and spatial correlations to provide diversity or coding gain at the receiver without sacrificing the bandwidth. Also the properties of data transmission which transmit multiple redundant encoded signal copies among spaced antennas and across time make multiple receiver antennas are not necessary in link end. As a result, it can reduce the complexity of hardware implement at receiver and achieve diversity goal at the same time. Nevertheless due to the number of antennas increasing, the decoding complexity is increasing too. Making space-time block code matrices orthogonal seems to be an attractive approach, which is called orthogonal space-time code (OSTBC). The advantages of OSTBC lead to full diversity gain and convenience of a simple receiver with the maximum-likelihood decoding algorithm. Here we specially address the Alamouti code [3] as a very famous example of OSTBCs. The Alamouti scheme is not only able to provide full diversity gain but also full data rate with two transmit antennas. What follow up are the details about OSTBC and Alamouti code scheme.

2.1.1 Orthogonal Space-Time Block Code

Although STBC [2] is firstly introduced and usually studied as orthogonal, for convenience, orthogonal space-time block code (OSTBC) is defined to be the form that any pair of columns taken from the coding matrix is orthogonal. In order to present the details of OSTBC, we first start form the construction of STBC design. A STBC is usually presented by a matrix. Each row represents a time slot and each column represents one antenna's transmissions over time. At first we assume there are km information bits and M-ary modulation scheme is used. Then we divide the km information bits into k signal groups $x_1, x_2, \ldots x_k$ where each group includes m bits. Now every group is modulated to selects a constellation signal from 2^m points, where $m = \log_2 M$. At the encoding part of STBC, the encoder encodes the k modulated signals to generate n_T parallel signal sequences of length p to compose the transmission matrix $\mathbf{X}$. We define the lth row of $\mathbf{X}$ as a space-time symbol transmitted at time l and nth column of $\mathbf{X}$ as a space-time symbol transmitted from nth transmit antenna. So during the transmission time, there are p space-time symbols transmitted from each antennas for each block of k input symbols. Since the code rate measures the ration of how many modulated symbols to the number of space-time coded symbols transmitted from each antenna. It is defined as

$$R = \frac{k}{p}. \tag{2.1}$$

In order to construct an OSTBC, the transmission matrix $\mathbf{X} \in C^{NT \times P}$ must satisfy the following constraint

$$\mathbf{X} \cdot \mathbf{X}^H = \left(|x_1|^2 + |x_2|^2 + \cdots + |x_{NT}|^2 \right) \mathbf{I}_{NT}, \tag{2.2}$$

where x_i , for $1 \le i \le NT$, is the column taken from the transmission matrix. Note that every column of transmission matrix X is orthogonal with each other. Due to the code orthogonal nature shown above, the decoding process is a simple, linear and optimal scheme at the receiver. However, the most serious disadvantage of OSTBC is that all but one of the codes which satisfy the orthogonal criterion must sacrifice some proportion of their data rate at the same time. Here to present the scheme, the space-time block codes can be divided into two groups based on the type of the signal constellations. One group is the space-time block codes with real signals and the other one is with complex signals. In theorem, it is shown that full rate OSTBC only exists for restrictions of some specific antenna/modulation configurations. Although, STBC exists for any number of transmit antennas and any arbitrary real signal constellation, Tarokh showed that full rate OSTBC for complex constellations exists only for two transmit antennas. This scheme called "Alamouti space-time block codes". The Alamouti is famous for offering the full transmitted diversity and the number of input symbols the encoder taken is equal to the number of transmission symbol periods required. Therefore the Alamouti scheme doesn't require any additional bandwidth expansion.

In other words, any OSTBC with complex signal constellation except for Alamouti scheme can only achieve a data rate under than 1 for any given number of transmit antennas. For example, in the four transmit antennas case, there are four symbols transmitted and the transmission matrix OSTBC is given by

$$\mathbf{X}_4^C = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 \\ -x_2 & x_1 & -x_4 & x_3 \\ -x_3 & x_4 & x_1 & -x_2 \\ -x_4 & -x_3 & x_2 & x_1 \\ x_1^* & x_2^* & x_3^* & x_4^* \\ -x_2^* & x_1^* & -x_4^* & x_3^* \\ -x_3^* & x_4^* & x_1^* & -x_2^* \\ -x_4^* & -x_3^* & x_2^* & x_1^* \end{bmatrix}, \tag{2.3}$$

where it is obvious that the orthogonal nature lead the inner product of any two columns of these matrices is zero. Also the block diagram of the OSTBC transmitter for four transmit antennas and one receive antenna is shown in Figure 2.1. The data stream is separated into four sub-streams which are converted from serial to parallel and mapped in the OSTBC encoder.

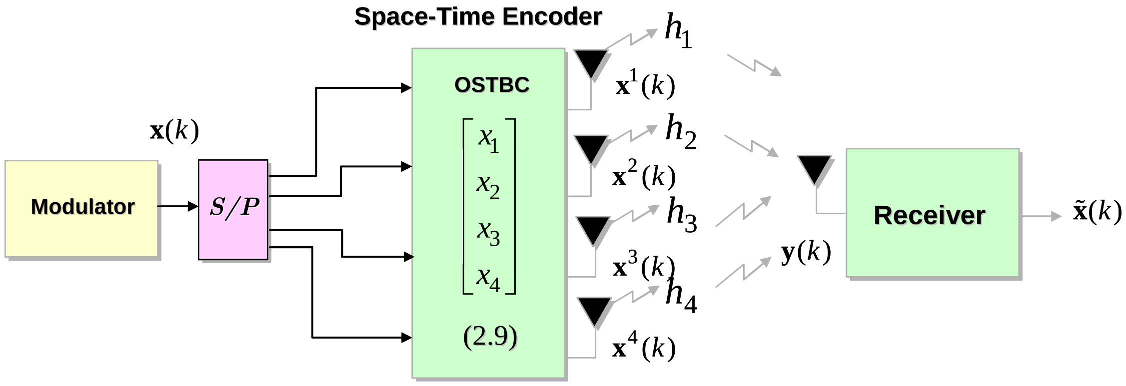

Figure 2.1: A block diagram of the orthogonal space-time block coded system for four transmit antennas and single receive antenna

2.1.2 Alamouti Space-Time Block Code

The Alamouti space-time block code is unique in the sense that it is only one space-time block code which provides the full diversity without any loss of transmission rate for complex signal constellations. The uniqueness has been proved in Tarokh's orthogonal designs associated with Radon-Hurwitz Theorem. As a result, expect for Alamouti scheme, a complex OSTBC which has ability to provide full diversity and full transmission rate is not possible to transmit in more than two antennas.

Since Alamouti STBC is a special case of OSTBC which transmit data symbols in two antennas, the Alamouti space-time encoder is similar to the OSTBC one mentioned in last section. First, an M-ary modulation scheme is assumed to be used in each group of m information bits, where $m = \log_2 M$. Due to the number of transmit antennas is two, the modulated input symbols to the space-time encoder are divided into groups of two symbols in encoding operation. It means during any symbol period, the two symbols in each group $\{x_1, x_2\}$ are transmitted simultaneously from the two antennas. The transmission matrix of Alamouti scheme is defined as

$$\mathbf{X} = \begin{bmatrix} x_1 & x_2 \\ x_2^* & -x_1^* \end{bmatrix}, \tag{2.4}$$

where the coding matrix $\mathbf{X}$ presents the modulated signal symbol x_1 transmitted from antenna 1 and the modulated signal symbol transmitted from antenna 2 is x_2 in the first symbol period. Then in the second symbol period, x_2^* is transmitted from antenna 1 and $-x_1^*$ is transmitted from antenna 2. Superscript $(\bullet)^T$, $(\bullet)^*$ and $(\bullet)^H$ denote transpose, complex conjugate, and Hermitian operation, respectively. In fair comparison, the transmit power must be normalized. In summary, the data rate of the Alamouti code is equal to one.

The transmission matrix $\mathbf{X}$ of Alamouti code also demonstrates the orthogonal property and must satisfy the following constraint

$$\mathbf{X} \cdot \mathbf{X}^H = \begin{bmatrix} |x_1|^2 + |x_2|^2 & 0 \\ 0 & |x_1|^2 + |x_2|^2 \end{bmatrix} = \left(|x_1|^2 + |x_2|^2\right)\mathbf{I}_2, \tag{2.5}$$

where $\mathbf{I}_2$ is a 2×2 identity matrix. It is shown that the transmit sequences from the two transmit antennas are orthogonal since the inner product of the sequences is zero. For real transmission case, the channel assumption remains constant over the two symbol periods. Denote h_1 and h_2 be the fading channel coefficients from antenna 1 and antenna 2. The real part and imaginary part of channel coefficients are same modeled as Gaussian random variables with zero mean and variance 0.5. In the case of only one antenna used at receiver, the received data over two continued symbol periods as y_1 and y_2. The received signals are expressed as

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} x_1 & x_2 \\ x_2^* & -x_1^* \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}, \tag{2.6}$$

where the noise samples n_1 and n_2 are independent complex Gaussian random variables with zero mean. Additionally, the real part and imaginary part of noise have the same variance $n_T/(2\text{SNR})$. Figure 2.2 displays the block diagram including modulator, serial to parallel structure and Alamouti encoder. Like the structure of OSTBC in Figure 2.1, the data stream is de-multiplexed into two sub-streams converted from serial to parallel and mapped to Alamouti encoder.

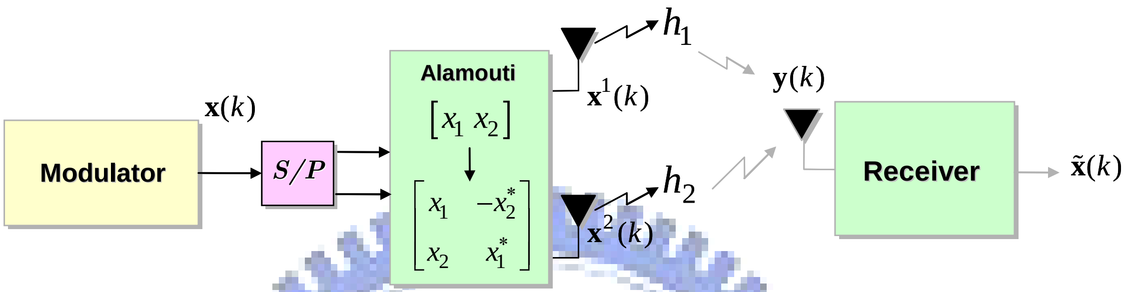

Figure 2.2: A block diagram of the Alamouti space-time coded system

2.2 MIMO Systems with Space-Time Block Code

Multiple-input multiple-output (MIMO) wireless communication systems (see Figure 2.3) are the communication systems which implement multiple antennas respectively in transmit and receive end. These antennas are capable of offering a linear increase in the capacity and additional antenna gain for the same bandwidth and same power consumption. In practical, recent theoretical results has shown that a point-to-point MIMO system capacity can linearly increase with the gain equal to minimum number of transmit and receive antennas. If we apply an antenna array to the base station of a multi-user system, user terminals can lead lower system interference level since multi-antenna array can provide a much larger degrees-of-freedom for interference suppression. Therefore the output system performance equipped with multi-antenna is better than the one with single antenna.

There are two important applicable techniques of MIMO system. One is space-time coding (STC) scheme as a well-known diversity technique; the other one is spatial multiplexing. STC, as mentioned previously, makes the transmitted symbols appropriately map into multiple transmit antennas according to specific coding matrix. Then the receiver exploits the artificially induced signal redundancy to obtain the diversity gain. As a result, STC emerged as a powerful approach to exploit diversity and resist fading in MIMO communication system. Moreover, OSTBC as one class of STC is well known for simple decoding scheme and full diversity. On the other hand, spatial multiplexing scheme is that multiple independent data streams are simultaneously transmitted via different antenna branches at the transmitter and are detected at the receiver based on their unique spatial signatures. By knowing the channel state information, the receiver is able to differentiate among the co-channel signals and extract all signals. And after demodulating those signals, the receiver can obtain the original sub-streams and combine them to give the original bit stream. Spatial multiplexing can also be applied to a multi-user format (MU-MIMO, also known as space division multiple access or SDMA). In this section, a point-to-point STBC MIMO system model is first given. Then the simple decoding scheme is following. At the end of this section, the original point-to-point system model extends to a multi-user one for spatial multiplexing scheme.

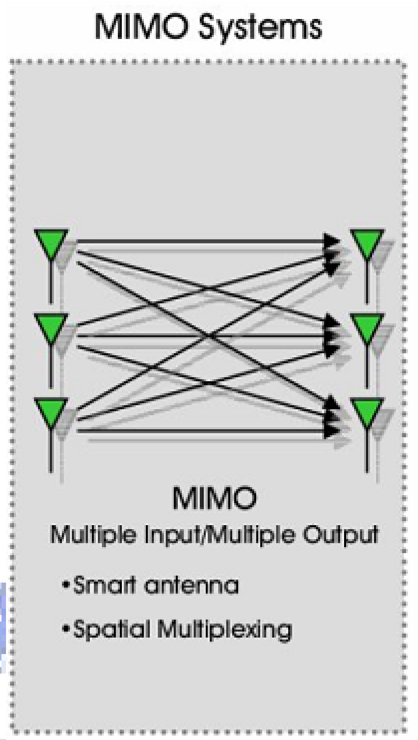

Figure 2.3: Illustration of MIMO system

2.2.1 Point-to-Point MIMO Model

A point-to-point (single user access) MIMO system with N transmit and M receive antennas is shown in Figure 2.4.

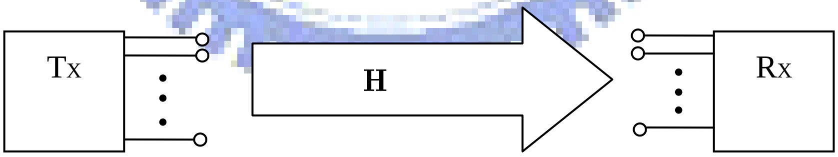

Figure 2.4: A point-to-point MIMO system

Under the assumption that channel is flat block-fading, the relationship between transmitter and receiver can be expressed as follows [1],[2]:

$$\mathbf{Y} = \mathbf{H}\mathbf{X} + \mathbf{V}\,, \tag{2.7}$$

where $\mathbf{H}$ is the $M \times N$ complex channel matrix known at the receiver. The entries of $\mathbf{H}$ are independent identically distributed (i.i.d.) complex zero-mean Gaussian variables so the channel is a Rayleigh flat fading channel. $\mathbf{Y}$, $\mathbf{X}$, and $\mathbf{V}$ denote the matrices of the received signals, transmitted signals, and channel noise, respectively. It is defined as

$$\mathbf{Y} \triangleq \begin{bmatrix} \mathbf{y}^T(1) \\ \mathbf{y}^T(2) \\ \cdot \\ \cdot \\ \cdot \\ \mathbf{y}^T(T) \end{bmatrix}, \mathbf{X} \triangleq \begin{bmatrix} \mathbf{x}^T(1) \\ \mathbf{x}^T(2) \\ \cdot \\ \cdot \\ \cdot \\ \mathbf{x}^T(T) \end{bmatrix}, \mathbf{V} \triangleq \begin{bmatrix} \mathbf{v}^T(1) \\ \mathbf{v}^T(2) \\ \cdot \\ \cdot \\ \cdot \\ \mathbf{v}^T(T) \end{bmatrix}^T, \tag{2.8}$$

where

$$\mathbf{y}(t) = [y_1(t)....y_{\mathrm{M}}(t)],$$
$$\mathbf{x}(t) = [x_1(t)....x_{\mathrm{N}}(t)],$$
$$\mathbf{v}(t) = [v_1(t)....v_{\mathrm{M}}(t)], \tag{2.9}$$

are complex row vectors of the received signals, transmitted signals, and noise respectively. Here the channel noise is spatially and temporally i.i.d. additive white Gaussian noise (AWGN) and $1 \le t \le T$, T is the block length presented the transmit time.

Assume there are K complex information symbols prior to space-time encoding which are denoted as $s_1, s_2, \ldots, s_K$. The information symbol vector is introduced as

$$\mathbf{s} \triangleq [s_1 \ldots s_K]^T, \tag{2.10}$$

noted $\mathbf{s} \in S$. $S = \left\{ s^{(1)} \ldots s^{(L)} \right\}$ is the set of all possible symbol vectors and L is the cardinality of the set since a $L \times K$ space-time codeword matrix is used in this system. The $N \times T$ transmission matrix $\mathbf{X}$ is called OSTBC if it satisfies the following conditions [2]:

- Each element of $\mathbf{X}$ is all linear function of K complex variables $s_1, s_2, \ldots, s_K$ and their complex conjugates;
- For any arbitrary $\mathbf{s}$, the matrix $\mathbf{X}$ has following property:

$$\mathbf{X}\mathbf{X}^H = \|\mathbf{s}\|^2 \mathbf{I}_N \tag{2.11}$$

where $\mathbf{I}_N$ is the identity matrix, and $\|.\|$ denotes the Euclidean norm of a vector or the Frobenius norm of a matrix.

For the considerations of analysis, we first split the user's information vector $\mathbf{s}$, the receive data vector $\mathbf{Y}$ and the noise vector $\mathbf{V}$ into respective real and imaginary part in order to obtain the following vectors:

$$\underline{\mathbf{s}} := \left[\mathrm{Re}\left\{\mathbf{s}^T\right\}\ \mathrm{Im}\left\{\mathbf{s}^T\right\}\right]^T \in R^{2K}, \tag{2.12}$$

$$\underline{\mathbf{Y}} := \left[\mathrm{Re}\left\{\mathbf{Y}^T\right\}\ \mathrm{Im}\left\{\mathbf{Y}^T\right\}\right]^T \in R^{2MT}, \tag{2.13}$$

$$\underline{\mathbf{V}} := \left[\mathrm{Re}\left\{\mathbf{V}^T\right\}\ \mathrm{Im}\left\{\mathbf{V}^T\right\}\right]^T \in R^{2MT}. \tag{2.14}$$

In summary, the original system can be rewritten as

$$\underline{\mathbf{Y}} = \mathbf{A}(\mathbf{H})\,\underline{\mathbf{s}} + \underline{\mathbf{V}}, \tag{2.15}$$

here the $2MT \times 2K$ real matrix $\mathbf{A}(\mathbf{H})$ is called space-time signature matrix [16]. Noting by resorting of the transmitted symbol vector, $\mathbf{A}(\mathbf{H})$ captures and combines both of the effects coming from the space-time codeword matrix and the channel matrix. What important is that $\mathbf{A}(\mathbf{H})$ inherits the same poverty from $\mathbf{X}(\mathbf{s})$ [17]. If $\mathbf{X}(\mathbf{s})$ is the transmission matrix of OSTBC, each column of $\mathbf{A}(\mathbf{H})$ is also orthogonal to each other. The poverty is shown as

$$\mathbf{A}(\mathbf{H})\mathbf{A}^H(\mathbf{H}) = \|\mathbf{A}(\mathbf{H})\|^2 \mathbf{I}_{2MT}. \tag{2.16}$$

2.2.2 Decoding in Point-to-Point MIMO Model

Having the exact channel knowledge at the receiver, maximum likelihood (ML) technique is the optimal space-time decoder [2] for the system which is defined as pervious section. It uses the channel state information to search the closet point to the received signal in the noise-free observation space $\Upsilon = \left\{ \mathbf{Y}^{(1)}, \mathbf{Y}^{(2)}, \ldots, \mathbf{Y}^{(L)} \right\}$. Here $\mathbf{Y}^{(l)}$ denotes the l th noise free received signal matrix correspond to the l th vector of information symbol vector $\mathrm{s}^{(l)}$. So we obtain the optimization

$$l_{opt} = \arg \min_{l=\{1,\ldots,L\}} \| \mathbf{Y} - \mathbf{Y}^{(l)} \|_F , \tag{2.17}$$

to find the index to decode the transmitted bit. The ML receiver can also be regarded as a matched filter whose output SNR is maximized [4]. It has been shown that is equivalent to the MF linear receiver in [7]:

$$\underline{\hat{s}} = \frac{1}{\| \mathbf{H} \|^2} \mathbf{A}^T (\mathbf{H}) \underline{\mathbf{Y}} . \tag{2.18}$$

2.2.3 Multi-user MIMO Model

To consider spatial multiplexing scheme, a multi-user MIMO communication system is illustrated in Figure 2.5.

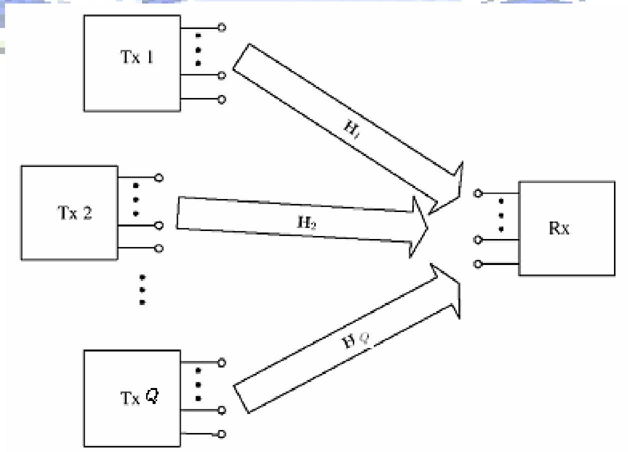

Figure 2.5: Multi-user MIMO model

As Figure 2.5 shown, there are multiple synchronous multi-antenna transmitters communicating with a single multi-antenna receiver. For simplification of notation, we assume each transmitter has the same number of transmit antennas and encode the information symbol by the same STBC. The received signal is a combination of the all signals transmitted by different transmitters. It is given as

$$\mathbf{Y} = \sum_{q=1}^{Q} \mathbf{H}_q \mathbf{X}_q + \mathbf{V}, \tag{2.19}$$

where $\mathbf{X}_q$ is the coding matrix of transmitted signals of the q th transmitter. And $\mathbf{H}_q$ is the channel matrix between the q th transmitter and the receiver. There are totally Q transmitters.

Following the same analysis procedure as used in the point-to-point MIMO model, we separate the q th user's information vector $\mathbf{s}_q \triangleq \left[s_{q1} \ldots s_{qK}\right]^T$, the received data vector $\mathbf{Y}_q$ and the noise vector $\mathbf{V}_q$ into respective real and imaginary part in order to obtain following vectors

$$\underline{\mathbf{s}}_q := \left[\mathrm{Re}\left\{\mathbf{s}_q^{\ T}\right\} \ \ \mathrm{Im}\left\{\mathbf{s}_q^{\ T}\right\}\right]^T \in R^{2K}, \tag{2.20}$$

$$\underline{\mathbf{Y}}_q := \left[\mathrm{Re}\left\{\mathbf{Y}_q^{\ T}\right\} \ \ \mathrm{Im}\left\{\mathbf{Y}_q^{\ T}\right\}\right]^T \in R^{2MT}, \tag{2.21}$$

$$\underline{\mathbf{V}}_q := \left[\mathrm{Re}\left\{\mathbf{V}_q^{\ T}\right\} \ \ \mathrm{Im}\left\{\mathbf{V}_q^{\ T}\right\}\right]^T \in R^{2MT} \tag{2.22}$$

As a result, the original system is rewritten as

$$\underline{\mathbf{Y}} = \sum_{q=1}^{Q} \mathbf{A}\left(\mathbf{H}_q\right) \underline{\mathbf{s}}_q + \underline{\mathbf{V}} \tag{2.23}$$

where the $2MT \times 2K$ real matrix $\mathbf{A}\left(\mathbf{H}_q\right)$ presents the space-time signature of q th transmitter and is useful for the joint decoding plus interference suppression receiver design in the following chapters.

2.3 Summary

First of all, we stress the advantages of STBC and give the details of OSTBC and Alamouti STBC scheme in this Chapter. Because the space-time block coded MIMO communication system has proven capable of achieving high spectral efficiency and high link quality, it is believed to play an important rule in next generation wireless communication systems. Therefore, we here put our emphasis on a point-to-point space-time block coded MIMO system model. In the point-to-point MIMO communication case, the optimal ML detector provides a simple but linear receiver which maximizes the output SNR performance. Extending the above result, a system model for the multi-user space-time block coded MIMO is given in Section 2.2.3. Nevertheless, for multi-user case, ML detector in Section 2.2.2 becomes highly non-optimal since the co-channel interference dominates the receiver performance instead of the channel noise term. And it has a much more complicated structure and higher complexity for multi-user case. Therefore, in the following chapters, we will search for a simple receiver scheme for the multi-user space-time block coded MIMO systems.

Chapter 3

Linear Receivers for Space-time Block Coded Multi-user MIMO System with Perfect Channel Estimation

In Chapter 3, we consider the general case of space-time block coded multi-user MIMO wireless communication systems. The scheme can be presented as that both the receiver and the multiple transmitters are equipped with multiple antennas. In addition, OSTBC is used to send the data simultaneously from each transmitter to the receiver. Here an optimal receiver scheme is provided for this system. It is able to suppress multi-access interference (MAI) and noise while decoding the data which is sent from the transmitter-of-interest by leverage of the spatial resource at the same time. The spatial resource comes from the array gain provided by multiple antennas at the transmitter and the receiver [18]. In other words, the proposed receiver is designed to minimize the filtered interference average power subject to the constraints that all received symbol gains of the transmitter-of-interest are maximal. Given that the exact channel estimation is available at the receiver, the equalizer scheme is regard as an associated reduced complexity implementation in comparison with joint ML detector.

We also applied the successive interference cancellation (SIC) mechanism to utilize the in-built structure of the space-time signature matrix [13]-[14]. Finally, the simulation results of the proposed receiver are provided at the end of this chapter.

3.1 Problem Formulation and System Model

In the multi-user MIMO case, the receiver performance is dominated by the signal-to-interference-plus-noise ratio (SINR) instead of the signal-to-noise ratio (SNR). Therefore, in order to design the receiver for a space-time block coded multi-user MIMO system, we have to define the interference term in the received signal first. Based on the system model proposed in Section 2.2.3, the received signal can be represented as

$$\mathbf{Y} = \tilde{\mathbf{H}}\tilde{\mathbf{X}} + \mathbf{V}, \quad (3.1)$$

where $\tilde{\mathbf{H}} = \left[\mathbf{H}_1\ \mathbf{H}_2 \ldots \mathbf{H}_q \ldots \mathbf{H}_Q\right]$ denotes the whole channel matrix of the system and $\mathbf{H}_q$ denotes the channel matrix between the qth transmitter and the receiver. Q is the total number of transmitters. $\tilde{\mathbf{X}} = \left[\mathbf{X}_1\ \mathbf{X}_2 \ldots \mathbf{X}_q \ldots \mathbf{X}_Q\right]$ is the total transmission matrix of all users, $\mathbf{V}$ is the channel noise vector. The following assumptions are made in the sequel:

1. The transmitted symbol vector $\mathbf{s}_q = \left[s_{q1}\ s_{q2} \ldots s_{qK}\right]$, $1 \le q \le Q$, of qth user is zero mean and uncorrelated to each other. In the other words, it means the expectation of any two transmitted symbol vectors are

$$E\left\{s_q s_p^*\right\} = \delta(q-p), \quad (3.2)$$

where $E\{y\}$ denotes the expectation of the random variable y, and $\delta(.)$ is the Kronecker delta function.

2. For a Rayleigh fading channel, the elements of q th channel matrix $\mathbf{H}_q$, $1 \le q \le Q$, are modeled as i.i.d zero-mean complex Guassian random variables with its variance equal to 0.5.

3. Each element of the noise vector $\mathbf{V}$ is i.i.d zero-mean complex Guassian random variables with variance σ_V^2 in order to model the AWGN channel noise.

For the advantages over joint design of interference suppression and decoding, we rewrite (3.1) into the space-time signature form which is similar to (2.23) to combine the effect of channel and space-time coding upon the transmitted symbol vector

$$\underline{\mathbf{Y}} = \tilde{\mathbf{A}}\tilde{\mathbf{s}} + \underline{\mathbf{V}}, \tag{3.3}$$

where $\tilde{\mathbf{s}} = \left[\underline{s}_1 \underline{s}_2 \ldots \underline{s}_Q\right]$ is a presented vector of all users' transmitted symbol vectors and $\widetilde{\mathbf{A}} = \left[\mathbf{A}(\mathbf{H}_1)\ \mathbf{A}(\mathbf{H}_2) \ldots \mathbf{A}(\mathbf{H}_q) \ldots \mathbf{A}(\mathbf{H}_Q)\right]$ with $1 \le q \le Q$ is denoted as the space-time signature matrix of all transmitters. Then we can use this representation form to characterize the MAI term in the system. From (3.3), we can see the received signal is the combination of each transmitter's signal through its own space-time signature as

$$\underline{\mathbf{Y}} = \mathbf{A}(\mathbf{H}_1)\underline{\mathbf{s}}_1 + \mathbf{A}(\mathbf{H}_2)\underline{\mathbf{s}}_2 + \ldots + \mathbf{A}(\mathbf{H}_Q)\underline{\mathbf{s}}_Q + \underline{\mathbf{V}}. \tag{3.4}$$

Now assuming without any loss of generality that the first transmitter is the transmitter-of-interest, we can observe that the first term on the right hand side of (3.4) comes from the desired signal. And the other terms in (3.4) denote the interference comes from non-desired signal including MAI and channel noise. The separated structure of signal, multi-user interference and noise suggests us to obtain the corresponding description of (3.4)

$$\underline{\mathbf{Y}} = \mathbf{D}\underline{\mathbf{s}} + \mathbf{H}_{\mathbf{I}}\underline{\mathbf{s}}_{\mathbf{I}} + \underline{\mathbf{V}}, \qquad (3.5)$$

where $\mathbf{D} := \mathbf{A}(\mathbf{H}_1)$ denotes the space-time signature matrix mapping to the desired symbol vector $\underline{\mathbf{s}} := \underline{\mathbf{s}}_1$ in the receiver, and $\mathbf{H}_{\mathbf{I}} := \left[\mathbf{A}(\mathbf{H}_2)\mathbf{A}(\mathbf{H}_3)\ldots\mathbf{A}(\mathbf{H}_Q)\right]$ denotes the space-time signature matrix mapping to the other users' symbol vectors $\underline{\mathbf{s}}_{\mathbf{I}} := \left[\underline{\mathbf{s}}_2\ \underline{\mathbf{s}}_3\ldots\underline{\mathbf{s}}_Q\right]$ in the system. In the following procedure of receiver design, we assume that the perfect channel estimation is available at receiver which means that we know the exact information of $\mathbf{D}$ and $\mathbf{H}_{\mathbf{I}}$.

3.2 GSC-Based Interference Suppression

Based on the system model proposed in the previous section, there are several existing linear equalization methods such as zero-forcing (ZF) equalizer or minimum mean square error (MMSE) equalizer...etc[19]-[20]. These linear equalizers are capable of proving simple recovery solution to the desired signal but having limited performance. On the contrary, other nonlinear equalizers can provide the additional performance gain at the expense of higher computation complexity, for example, joint maximum likelihood (ML) equalizer. In this section, we first provide constrained optimization as a typical nonlinear approach of signal recovering. Then we transform the constrained optimization problem into an unconstrained one by generalized sidelobe canceller (GSC) technique. As a result, we proposed a low complexity but optimal solution to the multi-user system in this section.

3.2.1 Constrained Optimization

The diversity and array gain provided by STBC in the multi-user MIMO system not only can resist the channel fading effect but also can reject interference comes from the other users. To fully use the extra degrees-of-freedom in system model, one typical solution for signal recovering is the constrained optimization [21]-[23]. The first step in developing the constrained algorithm is to define the optimum weighting matrix $\mathbf{W}$. Here we follow the same assumption in Section 3.1 that the first transmitter is the user-of-interest without any generality loss. Using the system model (3.5), we can express the output vector of this equalizer as

$$\hat{\mathbf{s}}_1 = \vartheta\left(\mathbf{W}^H \mathbf{Y}\right), \tag{3.6}$$

where $\mathbf{W} = \left[\mathbf{w}_1 \mathbf{w}_2 .... \mathbf{w}_K\right] \in C^{MT \times K}$ is the coefficient weighting matrix of the receiver, and $\hat{\mathbf{s}}_1 \in C^{K \times 1}$ is the estimate of the desired symbol vector $\mathbf{s}_1$. $\vartheta(.)$ denotes the decision slice. Additionally, the column $\mathbf{w}_k$, $1 \leq k \leq K$, in $\mathbf{W}$ can be interpreted as the receiver weight vector for the k th symbol of desired signal.

The second step is to determine the constraints in the receiver. We have two main goals to be achieved by the constraints at the same time: one is able to maintain an undistorted response to the transmitter-of- interest's signal. In other words, we try to seek a weighting matrix $\mathbf{W}$ to linearly combine the desired signal in the maximum ratio sense. As results, the linear weighting matrix must satisfy

$$\mathbf{W}^H \mathbf{Y} \cong \mathbf{D}^H \mathbf{D} \mathbf{s}_1; \tag{3.7}$$

the other goal is to suppress the MAI and the channel noise under the condition that the first goal is set up. It can be implemented via minimizing the total filtered output power of interference-plus-noise as much as possible while keeping the desired signal maximal. The goal can be expressed as the followed mathematic form:

$$\min_{\mathbf{W}} E\left\{ \| \mathbf{W}^H \left(\mathbf{H}_I \mathbf{s}_I + \mathbf{V}\right) \|^2 \right\}. \tag{3.8}$$

Commonly, the closed-form optimal solution to satisfy the above two constraints are solved by Lagrange multiplier method [24]. Although it does represent the solutions to the constrained optimization problems, it is computationally complex in the sense that a correlation matrix of the received signal must be estimated regularly and then inverted in order to arrive at the solutions.

3.2.2 GSC-based Equalizer

Here we use GSC as an efficient tool to solve the above constrained optimization problem instead of Lagrange multiplier method. The advantage over GSC principle is its ability to transform a constrained problem into an unconstrained one. The main idea of GSC is depicted in Figure 3.1.

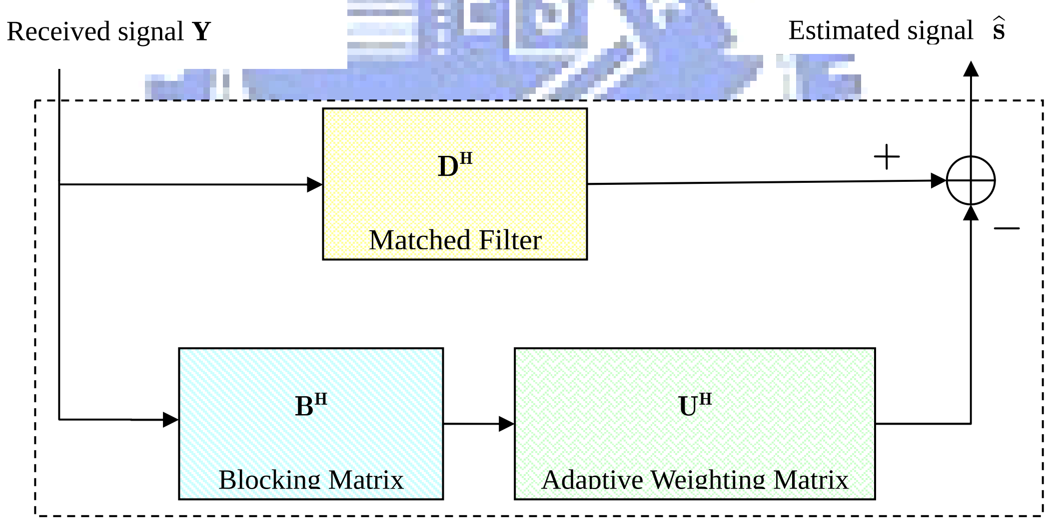

Figure 3.1. Structure of GSC equalizer under the perfect channel estimation.

We can see the linear weighting matrix $\mathbf{W}$ is decomposed into three parts:

$$\mathbf{W} = \mathbf{D} - \mathbf{B}\mathbf{U}, \tag{3.9}$$

where $\mathbf{D} \in C^{MT \times K}$ forms a conventional match filter matrix along the upper path to satisfies the first constraint (3.7). The superscript H is the complex conjugate (Hermitian) transpose. Therefore it is designed to equal the transmitter-of-interest's space-time signature. As the upper path's output is reduced by the lower path where the amount subtracted is the least-squares estimation of the noise and interference. To guard from subtracting the desired signal, the lower path is allocated with a blocking matrix $\mathbf{B}$ that nulls desired signal components. The blocking matrix $\mathbf{B} \in C^{(MT-K)\times MT}$ has the property that $\mathbf{B}^H\mathbf{D} = 0$, so that any component of the desired signal arriving at the lower path will be blocked. The adaptive weighting matrix $\mathbf{U} \in C^{MT-K\times MT}$ is designed to use the remaining degrees of freedom to suppress the noise and other user's interference power.

Following the procedure illustrated in Figure 3.1, the output power of GSC equalizer becomes

$$\mathbf{W}^H\mathbf{Y} = (\mathbf{D} - \mathbf{B}\mathbf{U})^H\,\mathbf{Y} = \mathbf{y}_d - \mathbf{U}^H\mathbf{y}_b\,, \tag{3.10}$$

where the output power of upper path can be represented as

$$\mathbf{y}_d := \mathbf{D}^H\left(\mathbf{D}\mathbf{s}_1 + \mathbf{H}_I\mathbf{s}_I + \mathbf{V}\right), \tag{3.11}$$

and the output power of lower path can be represented as

$$\mathbf{y}_b := \mathbf{B}^H\left(\mathbf{D}\mathbf{s}_1 + \mathbf{H}_I\mathbf{s}_I + \mathbf{V}\right). \tag{3.12}$$

We observe that contaminated term in the upper path output is

$$\mathbf{i} := \mathbf{D}^H\mathbf{H}_I\mathbf{s}_I + \mathbf{D}^H\mathbf{V}. \tag{3.13}$$

Therefore if we want to minimize the filtered noise power and interference power, the adaptive weighting matrix $\mathbf{U}$ must satisfy the following cost function:

$$\min_{\mathbf{U}} J := E\left\{\|\, \mathbf{i} - \mathbf{U}^H\mathbf{y}_b \,\|^2\right\}. \tag{3.14}$$

Simply following the standard procedures of GSC technique[25], the optimal weighting matrix $\mathbf{U}_{opt}$ has to satisfy the linear equation

$$\left[\mathbf{B}^H\mathbf{R}_I\mathbf{B}\right]\mathbf{U}_{opt} = \mathbf{B}^H\mathbf{R}_I\mathbf{D}\,, \tag{3.15}$$

where

$$\mathbf{R}_I := \mathbf{H}_I\mathbf{H}_I^H + \sigma_v^2\mathbf{I}_{MT} \in C^{MT\times MT}. \tag{3.16}$$

So the $\mathbf{U}_{opt}$ can be rewritten as

$$\mathbf{U}_{opt} = \left(\mathbf{B}^H\mathbf{R}_I\mathbf{B}\right)^{-1}\mathbf{B}^H\mathbf{R}_I\mathbf{D}, \tag{3.17}$$

and we can get the optimal GSC weighting matrix

$$\mathbf{W}_{opt} = \mathbf{D} - \mathbf{B}\mathbf{U}_{opt} \tag{3.18}$$

3.3 GSC/SIC-Based Interference Suppression

In the above section we provide a GSC-based receiver to reject multi-user interference and noise at the same time. However by observing the algebraic structure of system model (3.3), we discover that there exists a in-built group partition at the transmit symbol vector $\tilde{\mathrm{s}}$. The partition is according to the transmitter and each group transmits its symbol vector through its own space-time signature matrix. This basic structure gives rise to the multilayered space–time architecture. Here we use the SIC mechanism [13]-[14] to do the multistage detection and cancellation in the extension of proposed GSC-based receiver in Section 3.2. The basic idea is supposed that the first user's symbol vector s_1 is recovering successfully by the GSC-based receiver. After decoding s_1, we subtract the contribution of this signal from the received signals at all receive antennas. In other words, the communication system now are with less transmit antennas and the same number of receive antennas in comparison with the original one. We next use GSC-based receiver to recover the second user's signal then subtract its contribution from the received signals at all receive antennas. Proceeding in this manner, we observe that by subtracting the contribution of previously recovered user's signal from the received signals at receive antennas the space–time code affords an extra diversity gain [13]. This scheme can lead a straightforward power allocation application. In fact, powers at the different layers could be allocated based on the diversity gains. For example, the allocated powers can be decreased geometrically in terms of the diversity gains. In other words, it can also solve the near-far problem commonly happened in the multi-user case. The detailed procedures of the proposed receiver are described in the following algorithm:

GSC/SIC-Based Interference Suppression Receiver Algorithm

Initialization:

$$\mathbf{D}_1 = \mathbf{A}\left(\mathbf{H}_1\right), \mathbf{H}_I = \left[\mathbf{A}\left(\mathbf{H}_2\right)......\mathbf{A}\left(\mathbf{H}_p\right)\right], \mathbf{Y}_1 = \mathbf{Y}$$

Recursive: For $1 \leq i \leq Q$

Step 1) $\mathbf{B}_i$ is the blocking matrix of $\mathbf{D}_i$

Step 2) $\mathbf{R}_{I,i} := \left(\mathbf{H}_{I,i}\mathbf{H}_{I,i}^H\right) + \sigma_v^2 \mathbf{I}_{(MT-K(i-1))}$

Step 3) $\mathbf{U}_{q,i} = \left(\mathbf{B}_i^H \mathbf{R}_{I,i} \mathbf{B}_i\right)^{-1} \mathbf{B}_i^H \mathbf{R}_{I,i} \mathbf{D}_i$

Step 4) $\mathbf{W}_{q,i} = \mathbf{D}_i - \mathbf{B}_i \mathbf{U}_{q,i}$

Step 5) $\widehat{s}_i = \vartheta\left(\mathbf{W}_i^T \mathbf{y}_i\right), \vartheta(.)$ is the decision slice

Step 6) $\mathbf{y}_{i+1} = \mathbf{y}_i - \left(\mathbf{D}_i \hat{s}_i\right)$

Step 7) $\mathbf{D}_{i+1} = \mathbf{A}\left(\mathbf{H}_{i+1}\right), \mathbf{H}_{I,i+1} = \left[\mathbf{A}\left(\mathbf{H}_{2+i}\right)......\mathbf{A}\left(\mathbf{H}_Q\right)\right]$

3.4 Low Computational Complexity Scheme: Alamouti Code

The computational complexity of the above proposed receiver mainly comes from the two places: one is in solving for the blocking matrix via $\mathbf{B}^H\mathbf{D} = 0$; the other one is in multiplying of two large matrix $\mathbf{H}_I$ in order to get $\mathbf{R}_{in} = \mathbf{H}_I\mathbf{H}_I^H + \sigma_v^2\mathbf{I}_{MT}$. In this section, we provide a low complexity scheme with Alamouti code employed in the system. Because of the peculiar structure of Alamouti code block, we can obtain the blocking matrix $\mathbf{B}$ and the multiplication $\mathbf{R}_I$ through much simpler calculations.

The typical approach for obtaining $\mathbf{B} \in C^{QK\times K}$ is through the singular value decomposition (SVD) of the match filter matrix $\mathbf{D} \in C^{QK\times K}$. Since $\mathbf{B}$ is constructed as a basis of the left null space of $\mathbf{D}$, the computational complexity cannot be reduced due to the large user number Q. In the Alamoti code case, the matrix $\mathbf{D}$ is composed of Q Alamouti code blocks as

$$\mathbf{D} = \begin{bmatrix} \left.\begin{matrix} h_{11} & h_{12} \\ h_{12}^* & -h_{11}^* \end{matrix}\right\} block1 \\ \begin{matrix} \cdot & \cdot \\ \cdot & \cdot \end{matrix} \\ \left.\begin{matrix} h_{Q1} & h_{Q2} \\ h_{Q2}^* & -h_{Q1}^* \end{matrix}\right\} blockQ \end{bmatrix} = [\mathbf{d}_1 \ \mathbf{d}_2] \in C^{QK\times K}, \tag{3.19}$$

where $K = 2$. In consequence of this distinctive structure of $\mathbf{D}$, we discover that the blocking matrix $\mathbf{B}$ has the similar structure which is also composed of Q code blocks as

$$\mathbf{B} = \begin{bmatrix} \left.\begin{matrix} a_1 & b_1 \\ -b_1^* & a_1^* \end{matrix}\right\} block1 \\ \begin{matrix} \cdot & \cdot \\ \cdot & \cdot \end{matrix} \\ \left.\begin{matrix} a_Q & b_Q \\ -b_Q^* & a_Q^* \end{matrix}\right\} blockQ \end{bmatrix} = [\mathbf{b}_1 \ \mathbf{b}_2] \in C^{QK \times K}, \tag{3.20}$$

where $K = 2$ in Alamouti case. We observe an important fact that the two columns of $\mathbf{B}$ are composed of the same elements in the different order. It means that as long as we know any one column's information of $\mathbf{B}$, we can find the other one and complete the matrix $\mathbf{B}$. It inspires us that we only need to find a column $\mathbf{b} \in C^{QK \times 1}$ which is satisfied

$$\mathbf{b}^H \mathbf{D} = 0, \tag{3.21}$$

then we can find the matrix $\mathbf{B}$ to satisfied $\mathbf{B}^H \mathbf{D} = 0$ by copying and reordering the column $\mathbf{b}$. Therefore we use the Gram–Schmidt process to find the desired column. It is a method for orthogonalizing a set of vectors in an inner product space, commonly the Euclidean space $\mathbf{R}^n$. The Gram–Schmidt process takes a finite, linearly independent set $S = \{v_1, \ldots, v_n\}$ and generates an orthogonal set $S' = \{u_1, \ldots, u_n\}$ that spans the same subspace as S. To do the process, we firstly define the projection operator by

$$\mathrm{proj}_{\mathbf{u}} \mathbf{v} = \frac{< \mathbf{u}, \mathbf{v} >}{< \mathbf{u}, \mathbf{u} >} \mathbf{u} = < \mathbf{u}, \mathbf{v} > \frac{\mathbf{u}}{< \mathbf{u}, \mathbf{u} >}, \tag{3.22}$$

where <u, v> denotes the inner product of the vectors $\mathbf{u}$ and $\mathbf{v}$. Since the two columns of $\mathbf{D}$ are orthogonal to each other in nature, we directly assign that $\mathbf{u}_1 = \mathbf{v}_1 = \mathbf{d}_1$, $\mathbf{u}_2 = \mathbf{v}_2 = \mathbf{d}_2$. Then $\mathbf{v}_3 \in C^{KQ \times 1}$ is randomly assigned any vector which is orthogonal to $\mathbf{v}_1$. The Gram–Schmidt process works as follows:

$$\mathbf{u}_1 = \mathbf{v}_1 = \mathbf{d}_1 \,, \tag{3.23}$$

$$\mathbf{u}_2 = \mathbf{v}_2 - \text{proj}_{\mathbf{u}1}\mathbf{v}_2 = \mathbf{d}_2 \,, \tag{3.24}$$

$$\mathbf{u}_3 = \mathbf{v}_3 - \text{proj}_{\mathbf{u}1}\mathbf{v}_3 - \text{proj}_{\mathbf{u}2}\mathbf{v}_3 \,, \tag{3.25}$$

where $\mathbf{u}_3$ is the desired column b which is orthogonal to any column in $\mathbf{D}$. Since two pairs of columns $\mathbf{d}_1, \mathbf{d}_2$ and $\mathbf{d}_1, \mathbf{v}_3$ are orthogonal to each other, the above process only has one term $\text{proj}_{u_2} u_3$ in (3.25) which is needed to be computed. It greatly reduced the flop counts from

$$CM_{SVD} = 4K^3Q^2 + \left(8K^3 + 9K^2\right)Q \,, \tag{3.26}$$

to

$$CM_{Gram-Schmidt} = 3QK \tag{3.27}$$

The scheme of Alamouti code also gives some additional computational advantage besides the calculation of the blocking matrix in GSC-based equalizer: the multiplying of the large matrix $\mathbf{H}_I \in C^{QK \times QK}$ is presented as a symmetric form as follows:

$$\mathbf{H}_I * \mathbf{H}_I^H = \begin{bmatrix} a_{11} & a_{12} & a_{13} \cdots\cdots a_{1,Q} \\ a_{12}^* & a_{22} & a_{23} \cdots\cdots a_{2,Q-1} \\ a_{13}^* & a_{23}^* & a_{33} \cdots\cdots a_{3,Q-2} \\ . & . & \\ . & & \\ a_{1Q}^* & a_{2Q-1}^* \cdots\cdots\cdots & a_{Q,Q} \end{bmatrix}, \tag{3.28}$$

where $a_{ij} \in C^{K \times K}, K = 2$ is the multiplication of any two Alamouti block codes. It also has a symmetric form

$$a_{ij} = \begin{cases} \begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix}, \text{ for } i = j \\ \begin{bmatrix} n & p \\ -p^* & n^* \end{bmatrix}, \text{ for } i \neq j \end{cases}, \tag{3.29}$$

where $m, n,$ and p are scalars. As a result, the total number of flops for computing $\mathbf{H}_I * \mathbf{H}_I^H$ is determined as $KQ^3 - KQ^2$ comparing to original $K^3Q^3 - K^3Q^2$ flops. Here the total approximate flop counts which are required to compute the regular OSTBC code scheme and the Alamouti code scheme are respectively summarized as below:

$$\begin{aligned} CM_{regularOSTBC} &= KQ^3 + \left(2K^3 - K\right)Q^2 + \left(3K^2 + 3K\right)Q \\ &\quad + \left(2K^3 - 2K^2 + K\right) \end{aligned}, \qquad (3.30)$$

and

$$\begin{aligned} CM_{Alamouti} &= K^3Q^3 + \left(5K^3 + K^2\right)Q^2 \\ &\quad + \left(8K^3 + 12K^2\right)Q + \left(2K^3 - 2K^2 + K\right) \end{aligned}. \qquad (3.31)$$

From (3.30) and (3.31), it can be seen that the Alamoti scheme can save about K^2 times of computational load at the highest order.

3.5 Computer Simulations

Throughout the simulations in this section, $Q = 2$ transmitters are assumed. Each transmitter is with $N = 2$ transmit antennas and the full-rate Alamouti's OSTBC ($T = 2,\ K = 2$) is used. In addition, we assume a single receiver of $M = 2$ receive antennas. For simplicity, the interfering transmitter uses the same OSTBC as the transmitter-of-interest. Here channel model is assumed to be independent Rayleigh fading channel and the perfect channel estimation is available in receiver end. QPSK modulation is used. All plots are averaged over at least 1000 independent simulation runs.

In the first example, two transmitters are under the assumption of equal power. Figure 3.2 shows the bit error rates (BERs) of the GSC-based receiver and the

GSC/SIC-based receiver versus SNR. We can observe that the GSC-based receiver with the SIC mechanism offers about 1~2 dB gain. This would benefit from the increased received diversity obtained by SIC mechanism. In Figure 3.3, the BERs of all the receivers tested are displayed versus SNR. The simulations compare the proposed GSC/SIC-based with several existing methods: the Stamoulis's method [5], the Naguib's approaches [6], and the minimum variance (MV) receiver [7]. As we can see, the proposed receiver provides better performance over the whole tested SNR range as compared to the other receivers. As expected, the performance of MV is limited by the finite sample effect. And although the Stamoulis's decoupled based detector is free from error-propagation problem but its diversity gain is fixed. Moreover both the Stamoulis's method and the Naguib's approaches have the limitation of the Alamouti code usage and two transmit antennas. In contrast to these restrictions, the proposed GSC/SIC-based receiver is free for any OSTBC and any number of transmit antennas.

In the second example, two transmitters are assumed with unequal power to model the near-far problem in a multi-user's system. In this case, we assume that the power of transmitter is known at the receiver end. The decoding order is based on the amount of power. Figure 3.4 shows the BERs of the GSC-based receiver with and without SIC mechanism versus SNR and Figure 3.5 shows the BERs of all the receivers tested and are displayed versus SNR. Compared to Figure 3.2 and Figure 3.3, the improvement of performance confirm the advantage of SIC mechanism in the unequal power case.

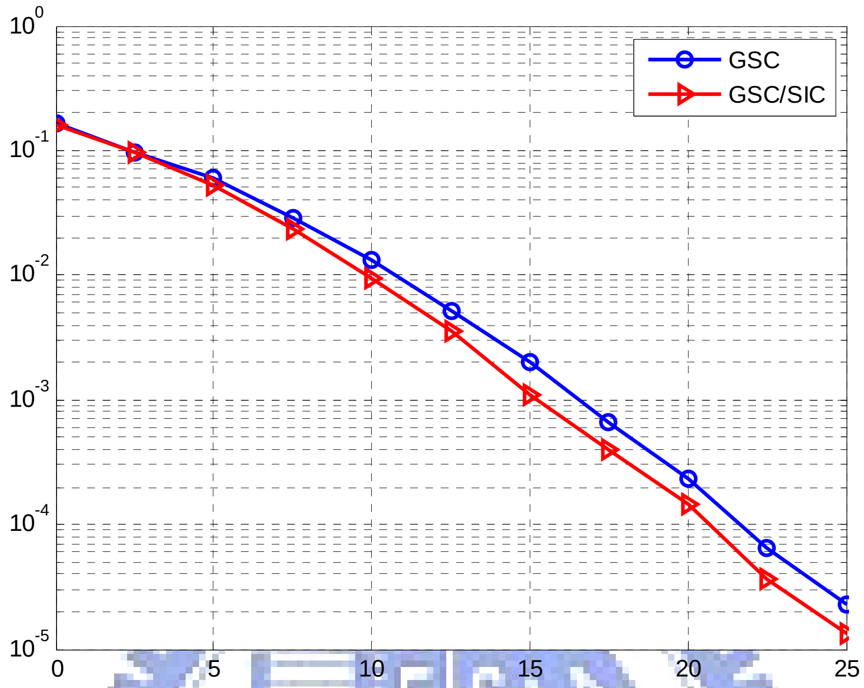

Figure 3.2. BER performances of the GSC-based receiver with and without the SIC mechanism (equal-power case)

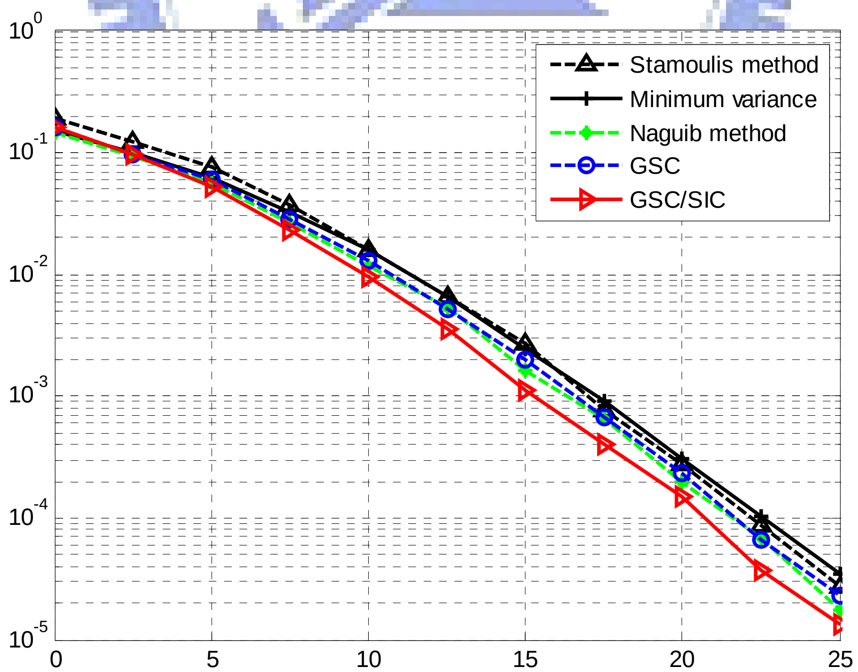

Figure 3.3. BER performances of the proposed receiver and other existing methods (equal-power case)

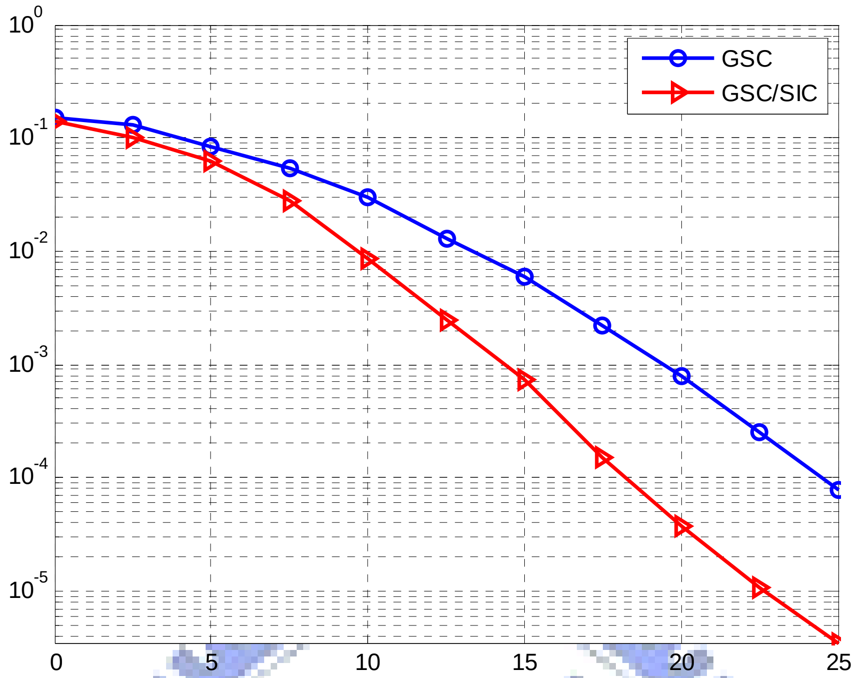

Figure 3.4. BER performances of the GSC-based receiver with and without the SIC mechanism (unequal-power case)

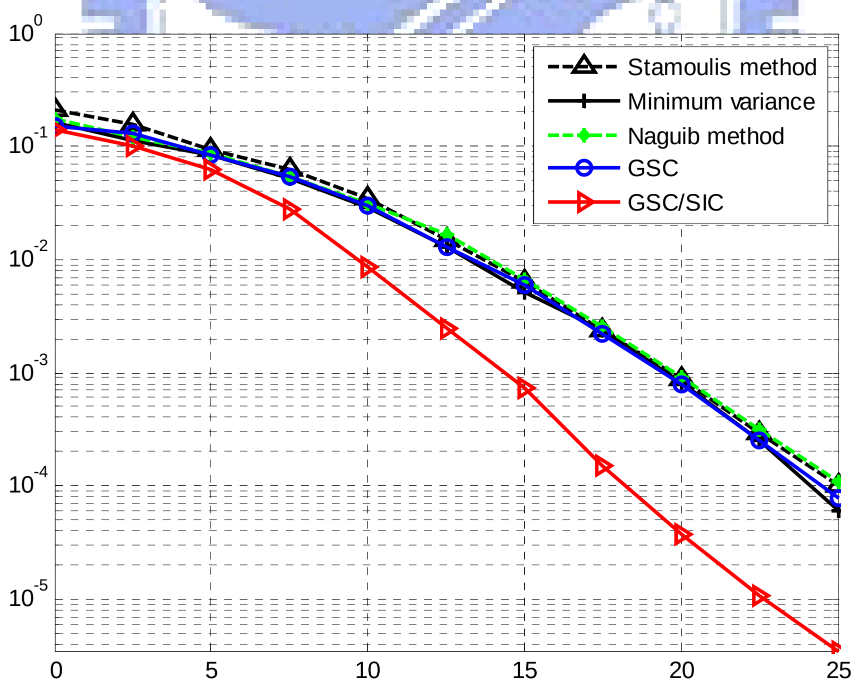

Figure 3.5. BER performances of the proposed receiver and other existing methods (unequal-power case)

3.6 Summary

In Section 3.1, we define the interference term and the system model of a space time block coded multi-user MIMO system. Under the assumption that perfect channel estimation is known at the receiver end, we use the system model to derive the optimal constrained equalizer to reject MAI and noise in Section 3.2. A GSC-based equalizer is also provided to transform the constrained problem into an unconstrained one in the same section. In addition, due to the multi-group structure in the system model, we apply the SIC mechanism to implement the multi-stage detection and interference cancellation in Section 3.3. Since Alamouti code is famous for its full rate and full diversity and commonly used in OSTBC, in Section 3.4 we also derive a low computational complexity scheme for Alamouti case. Finally computer simulation results are available in Section 3.5. It shows the SIC mechanism do improve the performance of GSC-based receiver and this proposed GSC/SIC-based receiver do have the comparable performance with other existing methods for the space time block coded multi-user MIMO system.

Chapter 4

Robust Linear Receivers for Space-time Block Coded Multi-user MIMO System with Imperfect Channel Estimation

The proposed receiver in Chapter 3 and other existing receivers for space-time block coded multi-user MIMO system have a major shortcoming: they use the assumption that the channel estimate is perfect at the receiver. When channel estimation error occurs, the performance of these receivers will degrade since their design does not take the channel estimation error into account. In practice, channel estimation error does happen. It comes from limited or outdating training symbols and leads to serious performance problems as do the effects of MAI and channel noise. In this chapter, we propose a robust receiver which is able to combat the imperfect channel estimation. By modeling the channel estimation error as a random variable and finding its characteristics, we derive an extended version of the GSC/SIC-based receiver in Chapter 3 to fix the performance problem. Here the perturbation technique leads to a very natural cost function for joint interference, noise and channel estimation error mitigation at the same time. Finally we compare the proposed solution with other existing robust solutions for space-time block coded multi-user MIMO system.

4.1 Problem Formulation for Imperfect Channel Estimation Case

When the perfect channel estimation is no longer available at receiver end, it means that the optimal weighting matrix $\mathbf{W}$ in Chapter 3 is obtained only by the estimated version of channel parameter. It causes serious system performance problem of the GSC/SIC-based receiver since its design is crucial to perfect channel estimation assumption. By observing the design procedure of GSC equalizer in Figure 4.1 , we discover the problems which result in performance degradation. As was mentioned above, the weighting matrix is decomposed into

$$\widehat{\mathbf{W}} = \widehat{\mathbf{D}} - \widehat{\mathbf{B}}\mathbf{U} \tag{4.1}$$

where $\widehat{\mathbf{X}}$ denotes the estimated version of $\mathbf{X}$.

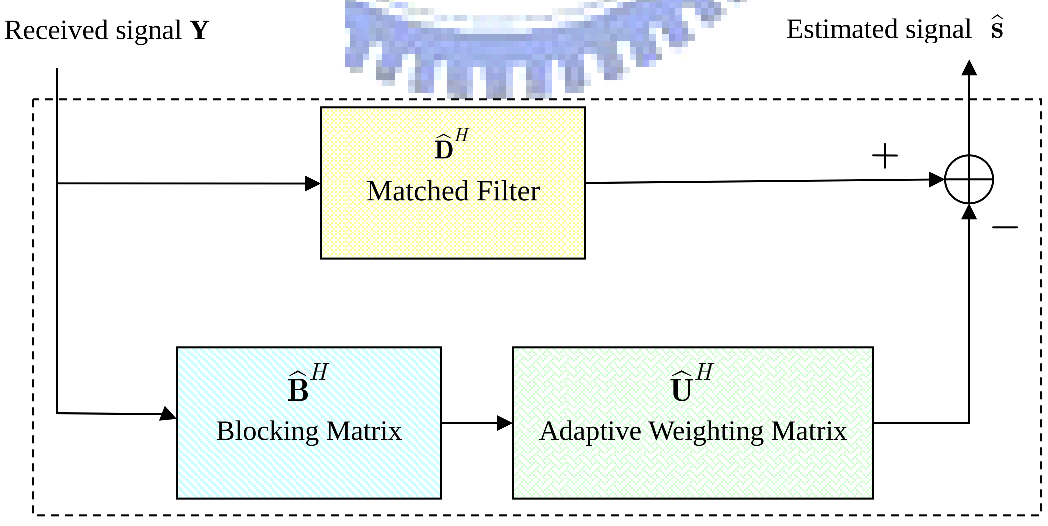

Figure 4.1: Structure of GSC equalizer under imperfect channel estimation

The first problem comes from the upper path: the maximal-ratio combing of the desired signal is impossible because there is only estimated $\widehat{\mathbf{D}} \neq \mathbf{D}$ available, where $\mathbf{D}$ equals the desired signal's space-time signature; the second problem comes from the lower path: since the blocking matrix is designed for $\widehat{\mathbf{D}}$ instead of $\mathbf{D}$, signal leakage happens. In summary, the output power of GSC equalizer under the imperfect channel estimation case becomes

$$\widehat{\mathbf{W}}^H \mathbf{Y} = \left(\widehat{\mathbf{D}} - \widehat{\mathbf{B}}\widehat{\mathbf{U}}\right)^H \mathbf{Y} = \overline{\mathbf{y}}_d - \widehat{\mathbf{U}}^H \overline{\mathbf{y}}_b, \tag{4.2}$$

where the output of the upper path can be represented as

$$\overline{\mathbf{y}}_d := \widehat{\mathbf{D}}^H \left(\mathbf{D}\mathbf{s}_1 + \mathbf{H}_I \mathbf{s}_I + \mathbf{V}\right), \tag{4.3}$$

and the output of the lower path can be represented as

$$\overline{\mathbf{y}}_b := \widehat{\mathbf{B}}^H \left(\mathbf{D}\mathbf{s}_1 + \mathbf{H}_I \mathbf{s}_I + \mathbf{V}\right). \tag{4.4}$$

Note that $\widehat{\mathbf{B}}^H \mathbf{D} \neq 0$ brings signal leakage into $\overline{\mathbf{y}}_b$ term. Now the contaminated term in the upper path output is defined as

$$\overline{\mathbf{i}} := \widehat{\mathbf{D}}^H \mathbf{H}_I \mathbf{s}_I + \widehat{\mathbf{D}}^H \mathbf{V}. \tag{4.5}$$

As a result, we have to define the new cost function in considerations of channel estimation error

$$\min_{\mathbf{U}} \overline{J} := E\left\{ \| \overline{\mathbf{i}} - \widehat{\mathbf{U}}^H \overline{\mathbf{y}_b} \|^2 \right\}, \tag{4.6}$$

where the expectation term is a combination of the source signal, channel estimation error, and AWGN noise. Based on (4.4), (4.5), and (4.6), and by averaging the cost function over the source signal and noise, we have

$$\begin{aligned} \overline{J} := & Tr\left(\widehat{\mathbf{U}}^H E\left\{\widehat{\mathbf{B}}^H \left(\mathbf{D}\mathbf{D}^H + \mathbf{R}_I\right)\widehat{\mathbf{B}}\right\}\widehat{\mathbf{U}}\right) \\ & - Tr\left(\widehat{\mathbf{U}}^H E\left\{\widehat{\mathbf{B}}^H \mathbf{R}_I \widehat{\mathbf{D}}\right\}\right) \\ & - Tr\left(E\left\{\widehat{\mathbf{D}}^H \mathbf{R}_I \widehat{\mathbf{B}}\right\}\widehat{\mathbf{U}}\right) + Tr\left(E\left\{\widehat{\mathbf{D}}^H \mathbf{R}_I \widehat{\mathbf{D}}\right\}\right) \end{aligned} . \tag{4.7}$$

We derive the derivative of $\bar{J}$ with respect to $\widehat{\mathbf{U}}$. By setting $\partial\bar{\mathbf{J}}/\partial\widehat{\mathbf{U}} = 0$, the first order necessary condition satisfies

$$E\left\{\widehat{\mathbf{B}}\left(\widehat{\mathbf{R}}_I + \mathbf{D}\mathbf{D}^H\right)\widehat{\mathbf{B}}\right\}\widehat{\mathbf{U}} = E\left\{\widehat{\mathbf{B}}\widehat{\mathbf{R}}_I\widehat{\mathbf{D}}\right\} \tag{4.8}$$

Note that when $\widehat{\mathbf{B}} = \mathbf{B}$ and $\widehat{\mathbf{D}} = \mathbf{D}$, (4.8) reduce to (3.15) in the perfect channel estimation case. The only way to determine the optimal $\widehat{\mathbf{U}}$ is to know precisely every expectation term in this first order necessary condition. In order to do the evaluation, we will establish a relationship between the estimated blocking matrix $\widehat{\mathbf{B}}$ and estimation error $\Delta\mathbf{D}$ to in next section and derive the optimal weighting matrix $\mathbf{W}$.

4.2 Derivation of Robust Optimal Solution

Although the channel parameters are not exactly known at receiver end, the characteristic of the channel estimation error $\Delta\mathbf{D} = \widehat{\mathbf{D}} - \mathbf{D}$ still can be obtained by some commonly used channel estimation techniques. As a consequence, we can calculate the expectation terms which involve $\widehat{\mathbf{D}}$. In addition, if we can establish a relationship between the estimated blocking matrix $\widehat{\mathbf{B}}$ and the channel estimation error $\Delta\mathbf{D}$, we can calculate all expectation terms in (4.8) and derive the robust optimal solution for imperfect channel estimation case.

4.2.1 Error in Estimated Blocking Matrix

By observation, an error-free version $\mathbf{D}$ and an estimated version $\widehat{\mathbf{D}}$ both can be written as the following singular value decomposition (SVD) form:

$$\mathbf{D} = [\mathbf{U}_D \ \mathbf{U}_B] \begin{bmatrix} \Sigma_D & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \mathbf{V}_D^H \\ \mathbf{V}_D^H \end{bmatrix}, \tag{4.9}$$

and

$$\hat{\mathbf{D}} = [\hat{\mathbf{U}}_D \ \hat{\mathbf{U}}_B] \begin{bmatrix} \hat{\Sigma}_D & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \hat{\mathbf{V}}_D^H \\ \hat{\mathbf{V}}_D^H \end{bmatrix}. \tag{4.10}$$

Since $\mathbf{B}$ and $\hat{\mathbf{B}}$ are constructed as the left null space of $\mathbf{D}$ and $\hat{\mathbf{D}}$ respectively, we can find

$$\mathbf{B} = \mathbf{U}_B \in C^{MT\times(MT-K)}, \tag{4.11}$$

and

$$\hat{\mathbf{B}} = \hat{\mathbf{U}}_B \in C^{MT\times(MT-K)}. \tag{4.12}$$

So we can see that there is a certain relationship between $\hat{\mathbf{B}}$ and $\Delta\mathbf{D}$. By perturbation analysis process in [15],[26], a closed-form expression of $\hat{\mathbf{B}}$ linear in channel estimation error $\Delta\mathbf{D}$ is as follows:

$$\hat{\mathbf{B}} = \mathbf{B} \underbrace{- \mathbf{U}_D \Sigma_D^{-1} \mathbf{V}_D^H \Delta\mathbf{D}^H \mathbf{B}}_{:=\Delta\mathbf{B}}, \tag{4.13}$$

when $\|\Delta\mathbf{D}\|$ is small. Also the estimated channel matrix can be presented as

$$\hat{\mathbf{D}} = \mathbf{D} + \Delta\mathbf{D}, \tag{4.14}$$

where $\Delta\mathbf{D} \in C^{MT\times K}$ models the estimation error and has the similar structure as $\mathbf{D}$. Since $\mathbf{D}$ equals to the space-time signature of desired signal and OSTBC is used to transmit signal, $\Delta\mathbf{D}$ satisfies the following property

$$\Delta\mathbf{D}^H \cdot \Delta\mathbf{D} = \left(|\Delta d_1|^2 + |\Delta d_2|^2 + \cdots + |\Delta d_{MT}|^2\right)\mathbf{I}_K, \tag{4.15}$$

where Δd_i, $1 \le i \le MT$, denotes the estimation error of the ith channel gain. Here we assume least square (LS) channel estimation technique which produces a white channel estimation error. Therefore we can see

$$R_D^{(m_1,m_2)} := E\left\{\Delta D^{(m_1)H} \Delta D^{(m_2)}\right\}$$
$$:= \begin{cases} 0, & for \;\; m_1 \neq m_2 \\ Q\sigma_D^2 I_{MT\times MT}, & for \;\; m_1 = m_2 \end{cases}, \tag{4.16}$$

where σ_D^2 denotes the variance of channel estimation error.

4.2.2 Algorithm Summary of Robust Solution

Based on the characteristics of $\Delta\mathbf{D}$ in (4.15), (4.16) and the relationship between $\mathbf{B}$ and $\Delta\mathbf{D}$ in (4.13), we can determine the expected values in (4.8) which are summarized in the following lemma (see Appendix A for a proof):

***Lemma 4.1*:** the following results hold.

1. $$E\left\{\widehat{\mathbf{B}}^H \widehat{\mathbf{R}}_I \widehat{\mathbf{D}}\right\} = \mathbf{B}^H \widehat{\mathbf{R}}_I \mathbf{D} \tag{4.17}$$

2. $$E\left\{\widehat{\mathbf{B}}^H \mathbf{D}\mathbf{D}\widehat{\mathbf{B}}\right\} = \sigma_D^2 * I_{MT-K} \tag{4.18}$$

3. $$E\left\{\widehat{\mathbf{B}}^H \widehat{\mathbf{R}}_I \widehat{\mathbf{B}}\right\} = \mathbf{B}^H \widehat{\mathbf{R}}_I \mathbf{B} + \frac{\sigma_D^2}{K} trace(\mathbf{J}) I_{MT-K} \tag{4.19}$$

where $\mathbf{J} := \left(\mathbf{D}^H\mathbf{D}\right)^{-1} \mathbf{D}^H \widehat{\mathbf{R}}_I \mathbf{D} \left(\mathbf{D}^H\mathbf{D}\right)^{-1}$

□

By Lemma 4.1 and (4.8), the optimal $\mathbf{U}$ can be obtained by

$$\mathbf{U}_{opt} = \left(\sigma_D^2 \mathbf{I}_{(MT-K)} + \mathbf{B}^H \widehat{\mathbf{R}}_I \mathbf{B} + \frac{\sigma_D^2}{K} trace(\mathbf{J}) \mathbf{I}_{(MT-K)}\right)^{-1} \mathbf{B}^H \widehat{\mathbf{R}}_I \mathbf{D} \tag{4.20}$$

where σ_D^2 denotes the variance of the channel estimation error. And $\mathbf{R}_I$ is defined as

$$\widehat{\mathbf{R}}_I = \widehat{\mathbf{H}}_I \widehat{\mathbf{H}}_I^H + \sigma_v^2 I_{MT-K} \,. \tag{4.21}$$

In practical situation, only an estimated sample version is available at the receiver end. So the weighting matrix of proposed robust receiver is presented as

$$\begin{aligned}\widehat{\mathbf{W}}_{opt} &= \widehat{\mathbf{D}} - \widehat{\mathbf{B}}\widehat{\mathbf{U}}_{opt} \\ &= \widehat{\mathbf{D}} - \widehat{\mathbf{B}}\left(\sigma_D^2 \mathbf{I}_{(MT-K)} + \widehat{\mathbf{B}}^H \widehat{\mathbf{R}}_I \widehat{\mathbf{B}} + \frac{\sigma_D^2}{K} trace\left(\hat{\mathbf{J}}\right)\mathbf{I}_{(MT-K)}\right)^{-1} \widehat{\mathbf{B}}^H \widehat{\mathbf{R}}_I \widehat{\mathbf{D}}\end{aligned} \tag{4.22}$$

Comparing to the original weight matrix $\mathbf{W}$ in Chapter 3, the new robust weighting matrix $\widehat{\mathbf{W}}_{opt}$ has two more terms in the adaptive matrix $\widehat{\mathbf{U}}_{opt}$ in consideration of accounting the signal leakage and parameter perturbation effect. Noting that with different channel estimation technique, (4.22) will be modified in corresponding to different characteristics of the channel estimation error. Nevertheless as long as the channel estimation error remaining independent to the source signal and channel noise, there still exists a closed-form solution similar to (4.22).

Since the SIC mechanism has proven the ability of performance enhancement in Chapter 3, here we also combine this mechanism to the proposed robust receiver in order to gain more received diversity. The algorithm is summarized in Table 4.1:

Table 4.1. Proposed robust receiver algorithm summary

INITIALIZATION:

$\widehat{\mathbf{D}}_1 = \mathbf{A}\left(\widehat{\mathbf{H}}_1\right), \widehat{\mathbf{H}}_I = \left[\mathbf{A}\left(\widehat{\mathbf{H}}_2\right)......\mathbf{A}\left(\widehat{\mathbf{H}}_q\right)\right], \mathbf{Y}_1 = \mathbf{Y}$

RECURSIVE: FOR $1 \le i \le Q$

Step 1) $\widehat{\mathbf{B}}_i$ is the blocking matrix of $\widehat{\mathbf{D}}_i$

Step 2) $\widehat{\mathbf{R}}_{I,i} := \left(\widehat{\mathbf{H}}_{I,i}\widehat{\mathbf{H}}_{I,i}^H\right) + \sigma_v^2 \mathbf{I}_{(MT-K(i-1))}$

where σ_v^2 denotes the variance of channel noise

Step 3) $\mathbf{J}_i = \left(\widehat{\mathbf{D}}_i^H \widehat{\mathbf{D}}_i\right)^{-1} \widehat{\mathbf{D}}_i^H \widehat{\mathbf{R}}_{I,i} \widehat{\mathbf{D}}_i \left(\widehat{\mathbf{D}}_i^H \widehat{\mathbf{D}}_i\right)^{-1}$

Step 4) $\widehat{\mathbf{U}}_{q,i} = \left\{\sigma_D^2 \mathbf{I}_{(MT-K(i-1))} + \widehat{\mathbf{B}}_i^H \widehat{\mathbf{R}}_{I,i} \widehat{\mathbf{B}}_i + \frac{\sigma_D^2}{K} trace\left(\mathbf{J}_i\right) \mathbf{I}_{(MT-K(i-1))}\right\} \widehat{\mathbf{B}}_i^H \widehat{\mathbf{R}}_{I,i} \widehat{\mathbf{D}}_i$

where σ_D^2 denotes the variance of channel estimation error

Step 5) $\widehat{\mathbf{W}}_i = \widehat{\mathbf{D}}_i - \widehat{\mathbf{B}}_i \widehat{\mathbf{U}}_i$

Step 6) $\widehat{s}_i = \vartheta\left(\widehat{\mathbf{W}}_i^T \mathbf{Y}_i\right)$, $\vartheta(.)$ is the decision slice

Step 7) $\mathbf{Y}_{i+1} = \mathbf{Y}_i - \left(\widehat{\mathbf{D}}_i \hat{s}_i\right)$

Step 8) $\widehat{\mathbf{D}}_{i+1} = \mathbf{A}\left(\widehat{\mathbf{H}}_{i+1}\right), \widehat{\mathbf{H}}_{I,i+1} = \left[\mathbf{A}\left(\widehat{\mathbf{H}}_{2+i}\right)......\mathbf{A}\left(\widehat{\mathbf{H}}_Q\right)\right]$

4.2.3 Associated Discussions

As observation, we can rewrite the optimal $\mathbf{U}$ in (4.20) as the following form:

$$\widehat{\mathbf{U}}_{opt} = \left(\sigma_D^2\left(1+\frac{trace(\mathbf{J})}{K}\right)\mathbf{I}_{(MT-K)} + \widehat{\mathbf{B}}^H\widehat{\mathbf{R}}_I\widehat{\mathbf{B}}\right)^{-1}\widehat{\mathbf{B}}^H\widehat{\mathbf{R}}_I\widehat{\mathbf{D}}, \qquad (4.23)$$

We can see (4.23) is in a form which is similar to a commonly used robust approach: the diagonal loading (DL) approach [7][27]. The key idea of DL technique is to regularize the original solution for the weight vector by adding a quadratic term to the objective function. As a result, in the situation of the proposed receiver case, we obtain the DL adaptive matrix is defined as

$$\widehat{\mathbf{U}}_{DL} = \left(\gamma\mathbf{I}_{(MT-K)} + \widehat{\mathbf{B}}^H\widehat{\mathbf{R}}_I\widehat{\mathbf{B}}\right)^{-1}\widehat{\mathbf{B}}^H\widehat{\mathbf{R}}_I\widehat{\mathbf{D}}, \qquad (4.24)$$

where $\gamma \geq 0$ is the loading vector. This operation corresponds to injecting an artificial amount of white noise into the main diagonal of $\widehat{\mathbf{B}}^H\widehat{\mathbf{R}}_I\widehat{\mathbf{B}}$. It has been shown that the diagonal loading will guarantee the abilities of inversion of the matrix $\gamma\mathbf{I}_{(MT-K)} + \widehat{\mathbf{B}}^H\widehat{\mathbf{R}}_I\widehat{\mathbf{B}}$, even in the case $\widehat{\mathbf{B}}^H\widehat{\mathbf{R}}_I\widehat{\mathbf{B}}$ is a singular matrix. Another meaningful interpretation of the DL is that it is the appropriate tool for combating the unexpected interference like channel mismatch, finite sample effect…etc [7]. Unfortunately, it is not a clear from in reference for what the proper choice of γ is and how it depends on the norm of the channel estimation errors. Until now, the optimal choice of the DL factor is dependent case by case which leads its limited robustness [7], [27], [28]. However, by comparing (4.23) and (4.24), we discover that the optimal γ can be set up as

$$\gamma := \sigma_D^2\left(1+\frac{trace(\mathbf{J})}{K}\right). \qquad (4.25)$$

Our simulation results will confirm this scenario.

4.3 Computer Simulations

In this section, we present the computer simulations for the space-time block coded multi-user MIMO system with the imperfect channel estimation. In all of them, we assume there are $Q=4$ transmitters. Here the channel model is assumed as independent Rayleigh fading channel so each element of the channel matrices are independently drawn from a zero-mean complex Guassian distribution. The full-rate Alamouti's OSTBC ($T=2,\ K=2$) is used, in other words, the transmitter is with $N=2$ transmit antennas. A single receiver of $M=4$ receive antennas is assumed. Since the imperfect channel estimation case is assumed, the receivers in this system use the presumed (erroneous) channel matrix $\widehat{\mathbf{H}}=\mathbf{H}+\Delta\mathbf{H}$ rather than the true channel matrix $\mathbf{H}$. In each simulation run, each element of the channel estimation error matrix $\Delta\mathbf{H}$ is generated by independently drawn from a zero-mean complex Guassian variable with the variance σ_D^2 due to LS channel estimation assumption. Then the presumed channel matrix $\widehat{\mathbf{H}}$ is coming from the addition of every element in $\Delta\mathbf{H}$ to the corresponding element in $\mathbf{H}$.

At first, Figure 4.2 shows the simulated bit-error-rate (BER) of the GSC-based receiver (3.18) and the robust GSC-based receiver (4.22) versus SNR with imperfect channel estimation. Here the channel estimation error variance $\sigma_D^2=0.01$ is simulated. As expected, the robust solution substantially outperforms the non-robust one especially in high SNR region since the optimal robust weighting matrix is designed to handle the channel mismatch. Because of the fixed power of the channel estimation error, the performance is dominated by the channel estimation error rather than channel noise and multi-user interference at high SNR. As a result, the improvement of the robust receiver becomes more obvious with SNR increasing.

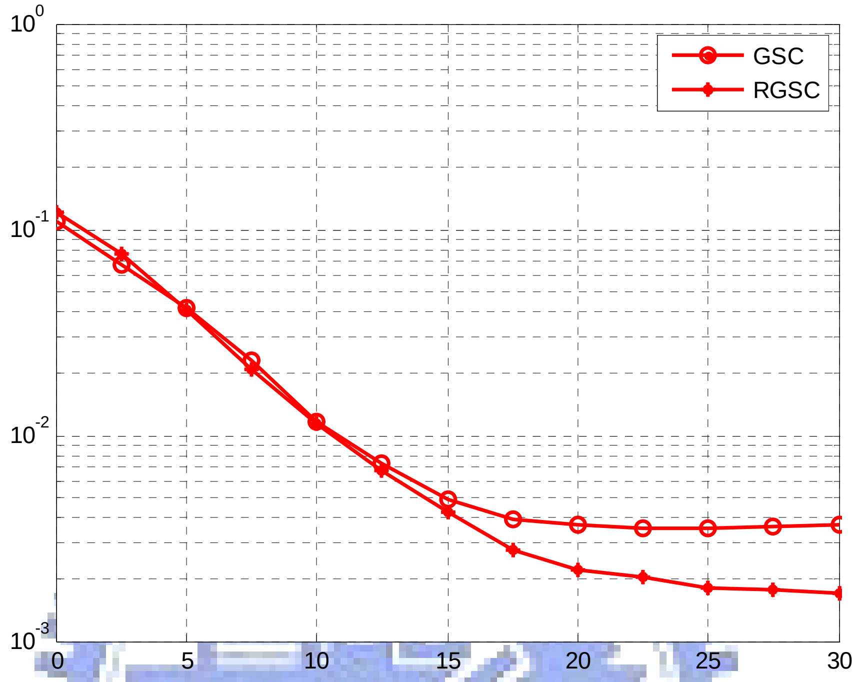

Figure 4.2. BER performance of the GSC-based receiver and the robust GSC-based receiver with imperfect channel estimation

Figure 4.3 illustrates the BER decreasing trend of the GSC-based receiver and the proposed (robust GSC/SIC-based) receiver under the different power (in decreasing form) of the channel estimation error at SNR =35 dB. It can be observed that the proposed receiver is able to combat the different level of channel mismatch from $\sigma_D^2 = 0.1 \sim 0$. And when the channel information is perfectly known ($\sigma_D^2 = 0$), the proposed robust solution reduces to act as the non-robust one. The result can also be confirmed by the derivation of (4.8).

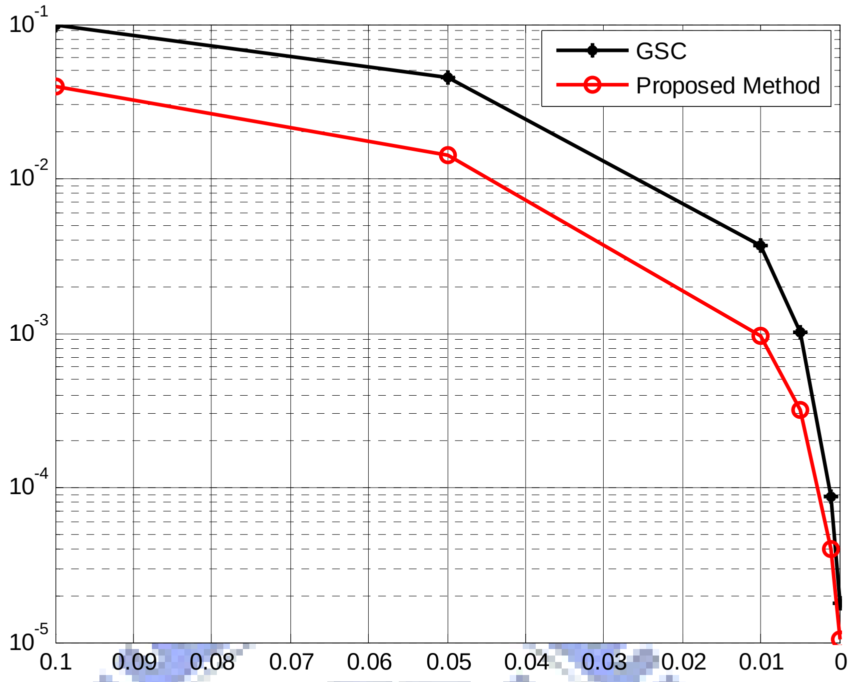

Figure 4.3. BER performances of the GSC-based receiver and the proposed receiver under different channel estimation error variance.

Throughout the second part of simulations, the following techniques are examined:

- The proposed (robust GSC/SIC-based) receiver
- The Stamoulis's method [5]
- The Naguib's approaches [6]
- The minimum variance (MV) receiver [7]
- The DL-based MV receiver [7], where $\gamma = 5\sigma_D^2$ is chosen. Note that this is a popular *ad hoc* choice of γ [7]-[9].

In Figure 4.4, the BERs of all the examined receivers are displayed versus SNR. Here σ_D^2 is also assumed as 0.01 to test the robustness against imperfect channel state information of the proposed receiver and other techniques. And we assume all transmitters are equal power. As expected, all tested receivers expect for the proposed one are sensitive to the channel estimation error since they are designed under the perfect channel assumption. Compared to the simulation result in Chapter 3, the stamoulis, the naguib, and the MV approaches all fail to resist the channel mismatch especially in high SNR region. Note that the DL-based MV receiver is a robust version of MV receiver and performs better than non-DL counterparts. However it suffers the finite sample effect of MV receiver and has limited performance. With Figure 4.2, it can be seen the improvement between robust GSC-based receiver and non-robust GSC-based receiver is more manifest than the one between DL-based MV receiver and MV receiver toward same channel estimation error variance. The result also illustrates the derivation of the optimal choice γ in (4.25) do perform better than the popular choice of $\gamma = 5\sigma_D^2$. Figure 4.5 shows the un-equal power case to confirm the advantage of the SIC mechanism toward the near-far effect.

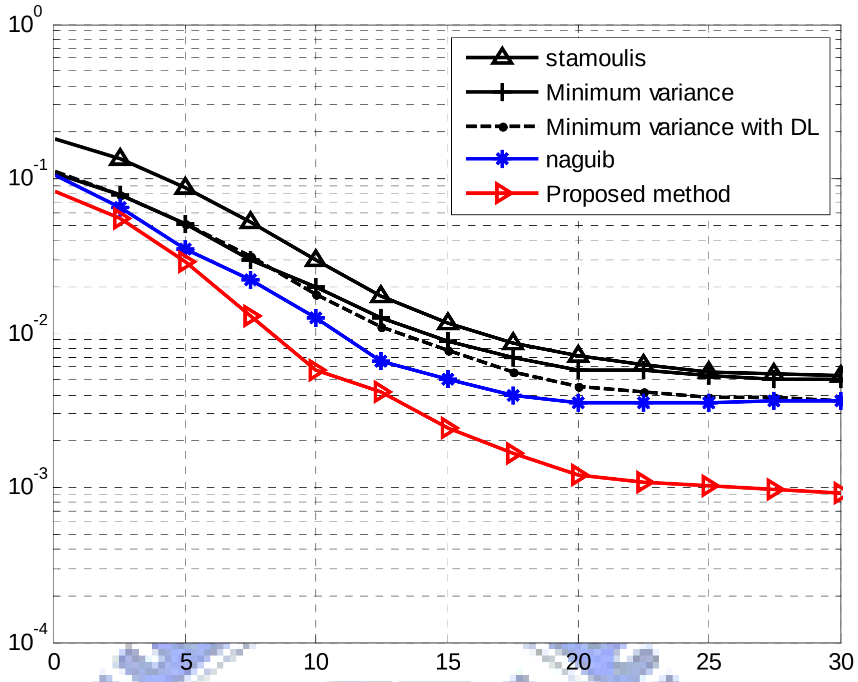

Figure 4.4. BER performances of the proposed receiver and other existing methods (equal-power case)

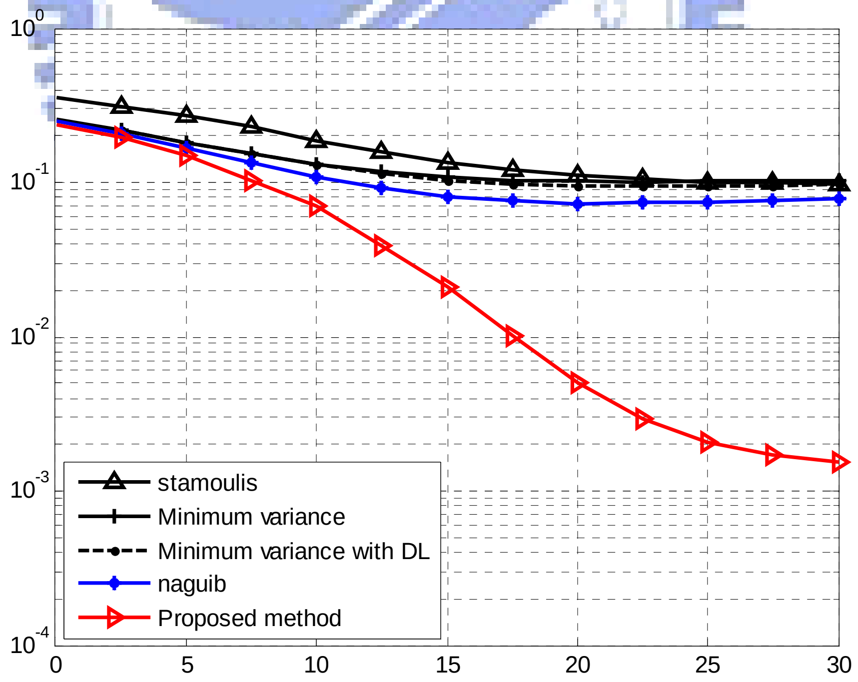

Figure 4.5. BER performances of the proposed receiver and other existing methods (unequal-power case)

4.4 Summary

In this chapter, we solve the receiver performance degradation problem under imperfect channel estimation in the space-time coded multi-user MIMO system. We analyze the effect and the potential problems due to channel mismatch in the GSC-based receiver (3.18) in Section 4.1. And Section 4.2 is shown that by the perturbation techniques we can estimate the error amount of the estimated blocking matrix. Combining the perturbation analysis result and the distinctive structure of STBC, we can derive the robust solution in a simple closed-form. We also give some associated discussions about the proposed robust solution comparing with other existing robust solutions. Finally the computer simulation results are shown to confirm the robustness of the proposed method in Section 4.3.

Chapter 5

Conclusions and Future Works

In this thesis, receivers for the space-time block coded multi-user MIMO system are proposed. The receiver design process can be divided into two parts. The first part emphasizes on the assumption that channel estimation is perfect. Since the channel state information is not often exactly known at the receiver end due to some limitation of training symbols or channel conditions, the second part relaxes the original assumption and considers the imperfect channel estimation case in order to obtain robust solutions for the system. In Chapter 2, we first give a brief review of STBCs. Due to the special structure and simple decoding scheme advantage, we focus on the OSTBCs and Alamouti code. Then we incorporate them into MIMO systems to provide a point-to-point and a multi-user space-time block coded MIMO signal model in the space-time signature form. We use this signal model to design the receivers with joint decoding and interference rejection in the following chapters.

In Chapter 3, the interference terms of the multi-user system model is defined first. Under the goals of combating multi-user interference and noise at the same time, we derive the optimal constrained equalizer to recover the signals-of-interest in the maximum ratio combining sense. In order to avoid the computational efforts in solving the optimization problem, GSC is used to transform the constrained problem into an unconstrained one. For further performance enhancement, we apply the SIC mechanism to implement the multi-stage detection and interference cancellation by using the multi-group nature of the system. Since Alamouti code is commonly used in OSTBC, we also provide a low computational complexity scheme of the proposed GSC/SIC-based receiver with Alamouti coded employed. Finally, the simulation results shows that the SIC mechanism do improve the performance of the GSC-based receiver and the proposed (GSC/SIC-based) receiver do possess comparable performance with other existing methods for the multi-user system. Note that all the works of this chapter are under the assumption that channel estimation is perfect at the receiver end.

In Chapter 4, we try to seek a solution to ease the receiver performance degradation problem under imperfect channel estimation assumption. Firstly, we analyze the channel mismatch effect and potential problems in the GSC/SIC-based receiver proposed in the previous chapter. By exploiting the perturbation techniques, we can analyze the error effect of the estimated blocking matrix as long as the statistical characteristics of estimation error are known. As a result, a simple closed-form robust solution is obtained thanks to the perturbation analysis and distinctive structure of STBC. We compared it with other existing robust solutions and confirm its robustness by the BER performance with channel estimation error at the end of the chapter.

The study presented in the thesis has addressed the robust GSC/SIC-based equalizer for STBC multi-user MIMO systems. Since the OSIC approach has been a well-recognized solution for STBC systems, we may apply the ordered SIC (OSIC) mechanism instead of the non-ordered one to yield a better performance. By deriving an approximated expression for SNR, the detection order could be accordingly determined.

Appendix A

Proof of Lemma 4.1

Proof of (1): With the linear relationship of $\widehat{\mathbf{B}}$ and $\Delta\mathbf{D}$ defined in (4.13), we have

$$
\begin{aligned}
&E\left\{\widehat{\mathbf{B}}^{H}\mathbf{R}_{I}\widehat{\mathbf{D}}\right\} \\
&= E\left\{\left(\mathbf{B}-\mathbf{U}_{D}\Sigma_{D}^{-1}\mathbf{V}_{D}^{H}\Delta\mathbf{D}^{H}\mathbf{B}\right)^{H}\mathbf{R}_{I}\widehat{\mathbf{D}}\right\} \\
&= E\left\{\mathbf{B}^{H}\mathbf{R}_{I}\widehat{\mathbf{D}}\right\} - E\left\{\mathbf{B}^{H}\Delta\mathbf{D}\mathbf{V}_{D}\Sigma_{D}^{-1}\mathbf{U}_{D}^{H}\mathbf{R}_{I}\widehat{\mathbf{D}}\right\} \\
&= E\left\{\mathbf{B}^{H}\mathbf{R}_{I}\left(\mathbf{D}+\Delta\mathbf{D}\right)\right\} - E\left\{\mathbf{B}^{H}\Delta\mathbf{D}\mathbf{V}_{D}\Sigma_{D}^{-1}\mathbf{U}_{D}^{H}\mathbf{R}_{I}\left(\mathbf{D}+\Delta\mathbf{D}\right)\right\} \\
&= E\left\{\mathbf{B}^{H}\mathbf{R}_{I}\mathbf{D}\right\} + E\left\{\mathbf{B}^{H}\mathbf{R}_{I}\Delta\mathbf{D}\right\} \\
&\quad - E\left\{\mathbf{B}^{H}\Delta\mathbf{D}\mathbf{V}_{D}\Sigma_{D}^{-1}\mathbf{U}_{D}^{H}\mathbf{R}_{I}\mathbf{D}\right\} - E\left\{\mathbf{B}^{H}\Delta\mathbf{D}\mathbf{V}_{D}\Sigma_{D}^{-1}\mathbf{U}_{D}^{H}\mathbf{R}_{I}\Delta\mathbf{D}\right\}.
\end{aligned}
\tag{A.1}
$$

By the characteristics of the estimation error $\Delta\mathbf{D}$ is defined in (4.16), the last three terms is equal to zero. The equation (A.1) thus reduces to

$$
E\left\{\widehat{\mathbf{B}}^{H}\mathbf{R}_{I}\widehat{\mathbf{D}}\right\} = E\left\{\mathbf{B}^{H}\mathbf{R}_{I}\mathbf{D}\right\} = \mathbf{B}^{H}\mathbf{R}_{I}\mathbf{D}. \tag{A.2}
$$

Proof of (2): Again with the linear relationship of $\widehat{\mathbf{B}}$ and $\Delta\mathbf{D}$ defined in (4.13), we have

$$
\begin{aligned}
&E\left\{\widehat{\mathbf{B}}^{H}\mathbf{D}\mathbf{D}^{H}\widehat{\mathbf{B}}\right\} \\
&= E\left\{\left(\mathbf{B}^{H}+\Delta\mathbf{B}^{H}\right)\mathbf{D}\mathbf{D}^{H}\left(\mathbf{B}+\Delta\mathbf{B}\right)\right\} \\
&= \mathbf{B}^{H}\mathbf{D}\,\mathbf{D}^{H}\mathbf{B} + \mathbf{B}^{H}\mathbf{D}\,\mathbf{D}^{H}E\left\{\Delta\mathbf{B}\right\} \\
&\quad + E\left\{\Delta\mathbf{B}^{H}\right\}\mathbf{D}\,\mathbf{D}^{H}\mathbf{B} + E\left\{\Delta\mathbf{B}^{H}\mathbf{D}\,\mathbf{D}^{H}\Delta\mathbf{B}\right\}.
\end{aligned}
\tag{A.3}
$$

Since the property $\mathbf{B}^H\mathbf{D} = 0$ and $E\{\Delta\mathbf{B}\} = 0$, the equation (A.3) can be reduced as

$$
\begin{aligned}
& E\left\{\widehat{\mathbf{B}}^H \mathbf{D}\mathbf{D}^H \widehat{\mathbf{B}}\right\} \\
& = E\left\{\Delta\mathbf{B}^H \mathbf{D}\mathbf{D}^H \Delta\mathbf{B}\right\} \\
& = E\left\{\left(-\mathbf{U}_D \Sigma_D^{-1} \mathbf{V}_D^H \Delta\mathbf{D}^H \mathbf{B}\right)^H \mathbf{D}\,\mathbf{D}^H \left(-\mathbf{U}_D \Sigma_D^{-1} \mathbf{V}_D^H \Delta\mathbf{D}^H \mathbf{B}\right)\right\} \\
& = E\left\{\mathbf{B}^H \Delta\mathbf{D}\,\Delta\mathbf{D}^H \mathbf{B}\right\} \\
& = \sigma_D^2 * I_{MT-K},
\end{aligned}
\tag{A.4}
$$

where $\mathbf{B}^H\mathbf{B} = I_{MT-K}$.

Proof of (3): The same as previous proof with the linear relationship of $\widehat{\mathbf{B}}$ and $\Delta\mathbf{D}$ defined in (4.13), we have

$$
\begin{aligned}
& E\left\{\widehat{\mathbf{B}}^H \mathbf{R}_I \widehat{\mathbf{B}}\right\} \\
& = E\left\{(\mathbf{B} + \Delta\mathbf{B})^H \mathbf{R}_I (\mathbf{B} + \Delta\mathbf{B})\right\} \\
& = \mathbf{B}^H \mathbf{R}_I \mathbf{B} + E\left\{\Delta\mathbf{B}^H \mathbf{R}_I \Delta\mathbf{B}\right\} \\
& = \mathbf{B}^H \mathbf{R}_I \mathbf{B} + E\left\{\mathbf{B}^H \Delta\mathbf{D}\, \underbrace{\mathbf{V}_D \Sigma_D^{-1} \mathbf{U}_D^H \mathbf{R}_I \mathbf{U}_D \Sigma_D^{-1} \mathbf{V}_D}_{\mathbf{J}} \Delta\mathbf{D}^H \Delta\mathbf{B}\right\},
\end{aligned}
\tag{A.5}
$$

where the matrix J defined as

$$
\mathbf{J} = \mathbf{V}_D \Sigma_D^{-1} \mathbf{U}_D^H \mathbf{R}_I \mathbf{U}_D \Sigma_D^{-1} \mathbf{V}_D . \tag{A.6}
$$

Then the equation (A.5) is reduced as

$$
\begin{aligned}
& E\left\{\widehat{\mathbf{B}}^H \mathbf{R}_I \widehat{\mathbf{B}}\right\} \\
& = \mathbf{B}^H \mathbf{R}_I \mathbf{B} + \mathbf{B}^H E\left\{\Delta\mathbf{D}\mathbf{J}\Delta\mathbf{D}^H\right\} \mathbf{B} \\
& = \mathbf{B}^H \mathbf{R}_I \mathbf{B} + \frac{\sigma_D^2}{K} trace(J) I_{MT-K}.
\end{aligned}
\tag{A.7}
$$

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