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結合載波間干擾分集技術於
正交分頻多工系統之編碼技術研究

CP-reduced Coding Techniques with
ICI Diversity Combining for OFDM Systems

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摘要

正交分頻多工是一個引人關注的傳輸技術，此技術可有效地克服多重路徑通道的時間衰褪效應藉由在每個符元前加入循環字首。然而，使用循環字首的需求造成了傳輸上多餘的負擔，並造成通訊系統吞吐量的下降。直接將循環字首縮短可以改善頻譜效益卻會引入符元間干擾和載波間干擾這些不理想的效應，使得整體系統效能受到限制。為了提升縮短循環字首之正交分頻多工系統的效能，吾人因而進行結合載波間干擾分集技術於正交分頻多工系統之編碼技術研究。若將子載波視為虛擬的天線，縮短循環字首之正交分頻多工系統在載波間干擾下的通道可等效地視同一個虛擬的多重輸入多重輸出通道。此外，輔以空頻編碼的觀念，吾人亦發現使用不足的循環字首所引起的載波間干擾其實可當作一種頻率分集增益。本論文中，吾人基於縮短循環字首之正交分頻多工系統所造成的等效多重輸入多重輸出通道下研發虛擬的空頻編碼技術，同時以理論分析來探討此虛擬空頻編碼可達之分集增益上界，並提出相應之設計準則以供發展具良好效能之虛擬空頻編碼。從模擬結果中顯示，相較於傳統的編碼正交分頻多工系統而言，吾人之編碼技術不僅能夠達到較高之分集增益進而提升錯誤率，同時擁有更高之資料傳輸速率。

CP-reduced Coding Techniques with ICI Diversity Combining for OFDM Systems

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Abstract

Orthogonal frequency division multiplexing (OFDM) is an attractive transmission technique, which effectively deals with the delay spread of the multi-path channel by appending a cyclic prefix (CP) in every OFDM symbol. However, the necessary insertion of CP produces a transmission overhead and reduces the throughput of the communication systems. A direct reduction of CP can improve the spectral efficiency but also introduces the undesired inter-symbol interference (ISI) and inter-carrier interference (ICI), hence limiting the overall system performance. To enhance the performance of the CP-shortened OFDM system, the CP-reduced coding techniques with ICI diversity combining for OFDM systems are studied here. Actually the ICI corrupted channel of the CP-reduced OFDM system can be viewed as a virtual MIMO channel when we regard the subcarriers as the virtual antennas. And the ICI induced by insufficient CP can be viewed as a sort of frequency diversity by the concepts of space-frequency coding (SFC). In this thesis, we investigate the virtual space-frequency coding techniques based on the effective MIMO channel of the CP-reduced OFDM system and propose the corresponding design criteria to develop good virtual space-frequency codes. The maximum achievable diversity order of the virtual space-frequency code is determined by the theoretical analysis. Simulation results are also provided to show that the proposed coding schemes not only achieve higher diversity order to enhance the error performance but also have higher data rate compared with the conventional coded OFDM systems.

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回首來時路，感謝指導教授王忠炫博士兩年半以來對於資質努鈍的我，成功地扮演好學術燈塔的角色，給予我充分的自由度去犯錯並從中學習成長，於茫茫學海當中找到自己的方向。除了學術之外，老師亦從日常生活的點滴中，教導我做人做事的道理，使我多所裨益。您的諄諄教誨與提點我都會放在心中，謝謝老師。

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如今碩士論文終於完稿，謹以野人獻曝的心情，將此論文，獻給實質上以及無形中幫助完成論文的你們。

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Chapter 1

Introduction

OFDM is an attractive technique for wireless communication, one of the main reasons is because OFDM effectively deals with the delay spread of the multipath channel by introducing a cyclic prefix (CP) in every OFDM symbol. The length of CP must be longer than the maximum delay spread to assure the inter-symbol interference (ISI) and inter-carrier interference (ICI) are eliminated. However the use of CP result in a lowering of spectral efficiency. Especially when the OFDM systems are encountered to the multipath channels which has a very long delay spread. Increasing the length of the CP to reduce ISI actually has its limitations because it introduces a tremendous bandwidth penalty. In order to make full use of the bandwidth in OFDM systems, several approaches have been proposed to cope with this problem by shortening CP or removing entire CP.

An easy way is to increase the number of subcarrriers or the OFDM symbol duration so that the proportion of the CP can be reduced. But this method narrow the bandwidth of each subcarrier and makes OFDM systems more sensitive to the frequency offset and the time variant channel. The second way is to employ an iterative cancellation method known as the residual ISI cancellation (RISIC) to eliminate interference due to insuffcient CP [1]. The RISIC algorithm firstly removes ISI from a previous decoded OFDM block and restores the circulant structure of the channel iteratively. Many techniques based on RISIC have been proposed, whereas most of them offer good performance only when the length of channel impulse response (CIR) is moderate, which means the interference power is much smaller than the signal power in these circumstances. Additionally, the RISIC is very sensitive to the effects of unideal ISI cancellation. The third way is to utilize time-domain or frequency-domain equalizer to combat the ISI and ICI [2][3][4][5]. The forth way aims to add redundancy by precoding techniques [6][7] or by the successive interference cancellation based on decision feedback structure [8] to mitigate ISI and ICI. Some of these schemes still require CP, and most of them don't outperform the traditional OFDM systems with enough CP. Actually the ICI corrupted channel of the CP-reduced OFDM system can be viewed as a virtual MIMO channel when we regard the subcarriers as the virtual antennas. And the ICI induced by insufficient CP can be viewed as a sort of frequency diversity by the concepts of space-frequency coding (SFC). In other words, instead of eliminating the ICI as the previous studies proposed, we should preserve the ICI and make it a profitable factor for performance enhancement. For above observations, the demand for a carefully design of CP-reduced coding schemes based on OFDM systems to increase both spectral efficiency and error performance is required. In this thesis, we investigate the virtual space-frequency codes based on the effective MIMO channel of the CP-reduced OFDM system, and and propose the corresponding design criteria to develop good virtual space-frequency codes. The maximum achievable diversity order of the virtual space-frequency code is determined by the theoretical analysis. Simulation results are also provided to show that the proposed coding schemes not only achieve higher diversity order to enhance the error performance but also have higher data rate compared with the conventional coded OFDM systems. Note that timing and frequency synchronization are assumed to be perfect throughout this paper.

The remainder of this thesis is organized as follows. In Chapter 2, an overview of the OFDM systems is given, and the mathematical formulations of equivalent ICI channel of CP-reduced OFDM systems after the ISI cancellation are introduced here. In Chapter 3, we give the reviews of the conventional space-time code (STC) and SFC, and we detail the derivation procedures of PEP. The design criteria and maximum achievable diversity order corresponding to STC and SFC are introduced as well. After the brief guild of the background knowledge, we derive the PEP of virtual space-frequency codes based the effective MIMO channel of the CP-reduced OFDM systems in Chapter 4, and determine the corresponding maximum achievable diversity order by mathematical proofs. Before we leave Chapter 4, the design criteria good virtual space-frequency codes are proposed. Besides theoretical interpretation, we also carry out computer simulations in Chapter 5 for performance verifications. In Chapter 6, we summarize our investigations and propose some potential future works for performance optimization.

Chapter 2

Overview of OFDM systems

Orthogonal frequency division multiplexing (OFDM) modulation is a multi-carrier transmission technique that has been recently recognized as an excellent method for high speed bi-directional wireless data communication. OFDM modulation effectively squeezes multiple modulated carriers tightly together, reducing the required bandwidth but keeping the modulated signals orthogonal so they do not interfere with each other. Furthermore, OFDM modulation divides the entire frequency selective fading channel into many narrow band flat fading subchannels in which high bit-rate data are transmitted in parallel and do not undergo inter-symbol interference due to the usage of cyclic prefix and long symbol duration. Therefore, OFDM modulation has been chosen for many standards, including digital audio broadcasting (DAB), High Definition TV (HDTV), and wireless local area network (WLAN), . . . etc. Moreover, it is an important technique for high data-rate transmission over mobile wireless channels. In the following sections of this chapter , we introduce the basic concepts of OFDM modulation, and then give a brief introduction on the usage of cyclic prefix. As we known, OFDM systems with insufficient (length of cyclic prefix is shorter than maximum delay spread) cyclic prefix give raise to inter-symbol interference and inter-carrier interference, thus we introduce the mathematical models of inter-symbol interference and inter-carrier interference induced by using insufficient cyclic prefix before leaving this chapter.

2.1 System Model of OFDM Systems

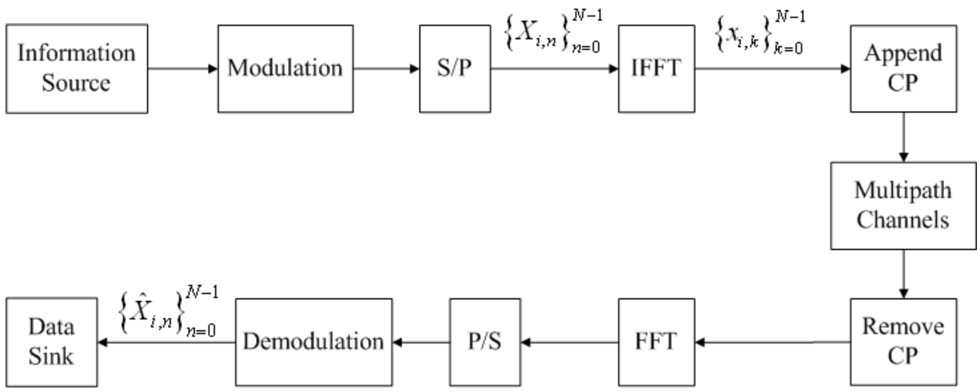


Figure 2.1: System model of OFDM Systems.

Consider the base-band OFDM systems with N subcarriers as shown in Figure 2.1. The serial information data stream is mapped into constellation symbols by employing a general phase shift keying (PSK) modulation or quadrature amplitude modulation (QAM) scheme. The resulting symbol stream is demultiplexed into a vector of N data symbols to become the frequency-domain OFDM symbol $\{X_{i,n}\}_{n=0}^{N-1}$, where $X_{i,n}$ is the modulated symbol on nth subcarrier of ith frequency-domain OFDM block. Next an inverse fast Fourier transform (IFFT)/fast Fourier transform (FFT) is used as a modulator/demodulator. For the ith transmission OFDM block, the N-point IFFT output sequence is

$$x_{i,k} = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X_{i,n} \exp\left\{\frac{j2\pi nk}{N}\right\},\ k = 0, 1, \ldots, N-1, \tag{2.1}$$

where $\{x_{i,k}\}_{k=0}^{N-1}$ is the time-domain OFDM symbol and $x_{i,k}$ stands for the modulated symbol on kth subcarrier of ith OFDM block. After the appending of cyclic prefix, the OFDM symbols are transmitted into the dispersive communication channels. The received OFDM symbols are weighted by the multipath fading gains and perturbed by the complex additive white Gaussian noise (AWGN) induced by the thermal noise. Next the receiver remove the cyclic prefix and employ an N-point FFT to get the frequency-domain received OFDM symbols. The recovered data symbols are demultiplexed in serial order and the demodulator maps received complex signals to the constellation points which has the most likely. The demodulation is performed over all subcarriers of OFDM symbols, then we have the decoded data stream stream of one transmission.

2.2 Cyclic Prefix, Inter-symbol Interference and Inter-carrier Interference

Due to the delay spread nature of the multipath channels, previous OFDM blocks are overlapped to next OFDM blocks and cause inter-symbol interference (ISI) among consecutive OFDM blocks. An easy design for avoiding ISI is to insert a replica of OFDM symbol in front of itself, which is the cyclic prefix (CP) [9][10]. Assumed the subcarrier numbers is N, the length of CP is G, The CP is conventionally chosen as the last sample points of each time-domain OFDM symbol just like the idea depicted in Figure 2.2. And the

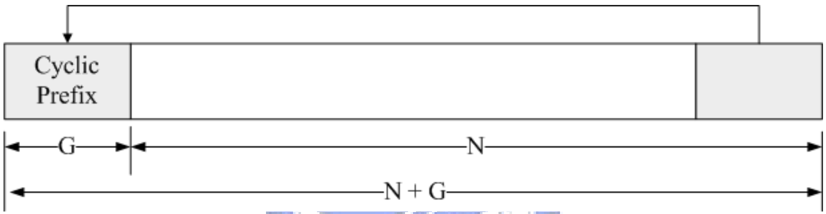
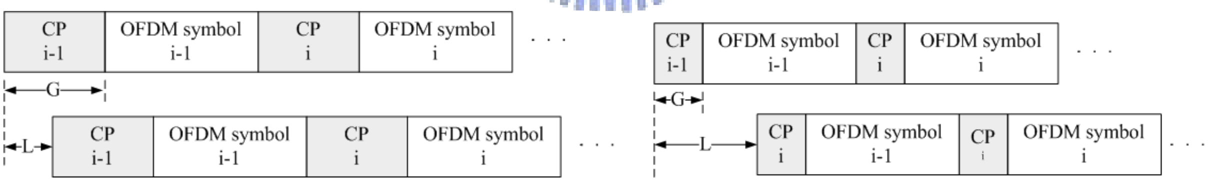


Figure 2.2: Stylized plot of time-domain OFDM symbol with CP of G samples.



(a) CP-OFDM with sufficient CP

(b) CP-OFDM with insufficient CP

Figure 2.3: (a) CP-OFDM with sufficient CP, (b) CP-OFDM with insufficient CP

CP must be longer or at least equals to the order of the maximum delay spread of the multipath channels to guarantee the OFDM system is ISI-free. Later, we assuming the maximum delay spread is no longer than one OFDM duration, and detail the concepts of ISI and ICI induced by insufficient CP. As shown in Figure 2.3(a), since the CP length G is not smaller than maximum delay spread L, the ith received time-domain OFDM block overlaps with two signal components at the receiver, one is the CP part of itself and the other is $(i-1)$th received OFDM block. Due to the orthogonality of the subcarriers, the time-domain OFDM symbols overlap with CP of themself is not harmful. And the receiver discards the CP of ith OFDM block before signal detection, therefore the ISI induced by previous OFDM block is completely deleted. Therefore by appending sufficient length of CP, the ISI problem is simply solved, and the time-domain OFDM symbols are superposed to each other at the CP part with their frequency-domain symbols being perpendicular to each other, just like the stylized plots of CP-sufficient OFDM symbols in time-domain and frequency-domain shown in Figure 2.4. For the case of $L > G$ as shown in Figure 2.3(b),

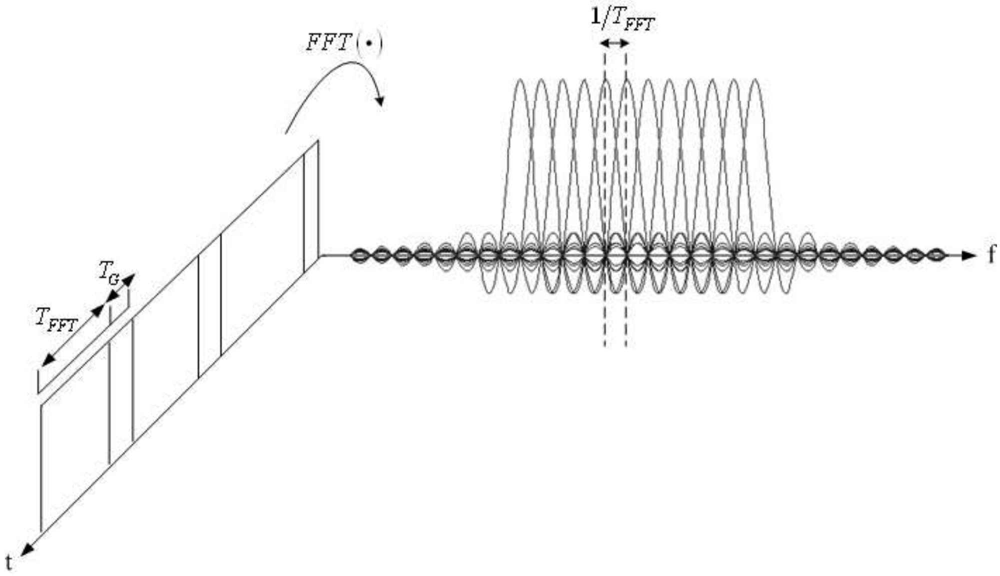


Figure 2.4: Stylized plot of the time-domain and frequency-domain OFDM symbols with sufficient CP.

the ith received OFDM block overlaps with last $L-G$ sample points of $(i-1)$th OFDM block, and the received data on each subcarrier of ith OFDM block is perturbed by received data of $(i-1)$th's, end up causing the degradation of error performance.

After the conceptual guide of CP and ISI, next we introduce the channel matrices of ISI and ICI induced by insufficient CP from analytical point of view. The received signals of CP-reduced OFDM symbols can be always divided into three parts: the received OFDM symbol without ISI and ICI (which is the interference-free components), the ISI contribution from the previous symbol due to insufficient CP, and the ICI contribution caused by insufficient CP. To be more clear, when $G < L$, the OFDM systems undergo ISI and ICI, and the received OFDM symbols can be viewed as an interference-free received OFDM symbol (just like the received OFDM symbol of CP-sufficient case) minus the ICI contributed by the insufficient CP parts of the OFDM symbols and add the ISI contribution from the previous OFDM block [11]. If $G \geq L$, the ISI and ICI contributions are both zero. Now, assuming the channel is static for at least one OFDM duration, and the ith time-domain transmit/receive OFDM block are denoted by $\mathbf{r}^{(i)}$ and $\mathbf{X}^{(i)}$, the mathematical representation of ith received OFDM block is given by

$$\mathbf{r}^{(i)} = \underbrace{\mathbf{H}^{(i)}\mathbf{X}^{(i)}}_{\text{Interference-free}} + \underbrace{\mathbf{H}_{isi}^{(i)}\mathbf{X}^{(i-1)}}_{\text{ISI}} - \underbrace{\mathbf{H}_{ici}^{(i)}\mathbf{X}^{(i)}}_{\text{ICI}}. \tag{2.2}$$

If we denote the multipath channel gains during ith OFDM block as $h_l^{(i)}$, $l = 0, 1, \ldots, L$, the ith received inference-free OFDM block can be formulated into a matrix form of $\mathbf{H}^{(i)}\mathbf{X}^{(i)}$ with $\mathbf{H}^{(i)}$ is given by

$$\mathbf{H}^{(i)} = \begin{bmatrix} h_0^{(i)} & 0 & \ldots & 0 & h_L^{(i)} & \ldots & h_2^{(i)} & h_1^{(i)} \\ h_1^{(i)} & h_0^{(i)} & 0 & \ldots & 0 & h_L^{(i)} & \ldots & h_2^{(i)} \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ h_{L-1}^{(i)} & \ddots & \ddots & h_0^{(i)} & 0 & \ddots & \ddots & h_L^{(i)} \\ h_L^{(i)} & h_{L-1}^{(i)} & \ddots & \ddots & \ddots & \ddots & \ddots & 0 \\ 0 & h_L^{(i)} & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & h_0^{(i)} & 0 \\ 0 & \ldots & 0 & h_L^{(i)} & h_{L-1}^{(i)} & \ldots & h_1^{(i)} & h_0^{(i)} \end{bmatrix}. \tag{2.3}$$

Since $\mathbf{H}^{(i)}$ has a circulant structure in (2.3), $\mathbf{H}^{(i)}$ can be diagonalized by N-point FFT matrix and IFFT matrix, which dramatically simplify the complexity of the channel equalizer. Thus if the length of CP is sufficient, the receiver can use a single weighted frequency-domain equalizer (1-tap FEQ) to compensate the multipath channel response. For the ISI signal component induced by insufficient CP, the time-domain ISI channel matrices are

$$\mathbf{H}_{isi}^{(i)} = \begin{bmatrix} 0 & \dots & 0 & h_L^{(i)} & \dots & h_1^{(i)} \\ \vdots & \dots & \vdots & \ddots & \ddots & \vdots \\ \vdots & \dots & \vdots & \ddots & 0 & h_L^{(i)} \\ \vdots & \dots & \vdots & \ddots & \ddots & 0 \\ 0 & \dots & \vdots & \ddots & \ddots & \vdots \\ 0 & \dots & 0 & \dots & \dots & 0 \end{bmatrix}. \tag{2.4}$$

The time-domain ICI component is induced by subtracting the contribution of the unused CP parts of the OFDM symbols. Therefore the time-domain ICI channel matrix is the column-wise left shift version of the non-zero parts of the time-domain ISI channel matrix by G times. And the time-domain ICI channel matrix is given by

$$\mathbf{H}_{ici}^{(i)} = \mathbf{H}_{isi}^{(i)} \begin{bmatrix} \mathbf{0}_{G\times(N-G)} & \mathbf{I}_{G\times G} \\ \mathbf{I}_{(N-G)\times(N-G)} & \mathbf{0}_{(N-G)\times G} \end{bmatrix} \tag{2.5}$$

where the $\mathbf{0}$ and $\mathbf{I}$ in (2.5) are the all zero matrix and the identity matrix with matrix size specified by their subscripts. We can get the frequency-domain of CP-sufficient, ISI and ICI channel matrix by simply left multiply the IFFT matrix and right multiply the FFT matrix to each of the aforementioned channel matrices (2.3), (2.4), and (2.5).

Chapter 3

Reviews of Space-Time Coding and Space-Frequency Coding

Two of the major impairments of wireless communications systems are fading caused by destructive addition of multipaths in the propagation medium and interference from other users. Severe anttenuation makes it impossible for the receiver to determine the transmitted signal unless some less-attenuated replica of the transmitted signal is provided to the receiver. This resource is called diversity. In 1998, a communication technique called space-time coding (STC) scheme which utilizes diversity to combat fading and interference is firstly proposed by Tarokh, Seshadri,and Calderbank [12][13]. The space-time coding introduces temporal and spatial correlation into the transmitted signals, so as to achieve transmit diversity as well as a coding gain without sacrificing the bandwidth. Moreover, it is an effective way to approach the capacity of multiple-input multiple-output (MIMO) wireless channels. Inspired by STC, space frequency coding (SFC) scheme which extends the diversity technique to multi-carrier systems operating over broadband channels is developed by Bolcskei and Paulraj in 2000 [14]. Bolcskei and Paulraj found out that the STC systems designed to achieve full spatial diversity in the narrowband case is in general not yielding full space-frequency diversity, therefore the space-frequency coding scheme across frequency and spatial domain to achieve not only spatial diversity but frequency diversity was investigated. There are variety of STC systems with respect to distinct coding schemes, such as, space-time block coding [15][16], space-time trellis coding [17][18], space-time frequency coding [19][20], unitary space-time modulation [21][22], space-time turbo trellis coding [23], differential space-time coding [24][25], and layered space-time coding [26][27], ...etc. In the following sections of this chapter, we will focus on the encoding and decoding schemes of STC and SFC along with their design criteria over quasi-static fading channels.

3.1 Space-Time coding

3.1.1 System Model of STC

Consider a MIMO system with M_T transmit antennas and M_R receive antennas as shown in Figure 3.1. After encoding, the M_T antennas simultaneously transmit the encoded data into the communication channel. At the receiver, the transmission signals of different transmit antennas undergo channel fading and then superpose to each other along with the thermal noise at the receive antennas. Assume wireless channels are quasi-static fading and memoryless, the received signal of antenna q at time t is given by

$$r_t^q = \sum_{p=1}^{M_T} h_{q,p}\sqrt{E_s}c_t^p + z_t^q,\ q = 1, 2, \ldots, M_R,\ t = 1, 2, \ldots, L, \tag{3.1}$$

where c_t^p with energy $\sqrt{E_s}$ is the coded symbol transmitted by pth antenna at time t. Channel fading gains for the path from transmit antenna i to receive antenna j are denoted as $h_{q,p}$, and they are assumed to be independent complex Gaussian random variables with mean $m_h^{q,p}$ and variance 0.5 per dimension. z_t^q stands for the thermal noise of qth antenna at time t, and z_t^q are also independent Gaussian distribution with zero mean and one-side power spectral density of N_0.

3.1.2 Design Criteria of STC

The pairwise error probability (PEP), $P(\mathbf{c}, \hat{\mathbf{c}})$ is the probability that decoder selects an erroneous sequence $\hat{\mathbf{c}} = \hat{c_t^q}, (\forall q, t)$, when the transmitted sequence was in fact $\mathbf{c} = c_t^q, (\forall q, t)$. Assume the receiver has perfect channel state information (CSI), and the decoder performs maximum likelihood (ML) decoding to recover signals. The PEP with respect to (3.1) is

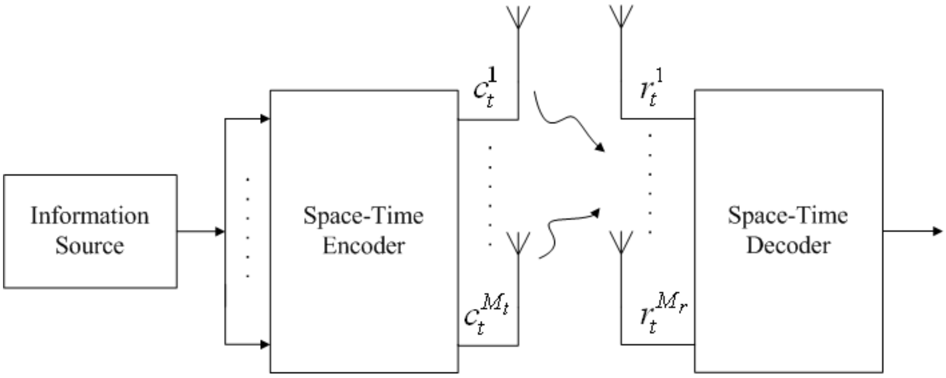


Figure 3.1: System model of STC.

given by

$$
\begin{aligned}
&\mathrm{P_r}\left(\mathbf{c} \rightarrow \hat{\mathbf{c}} | h_{q,p}, \forall\ p, q, t\right) \\
&= \mathrm{P_r}\left[\sum_{t=1}^{L}\sum_{q=1}^{M_R}\left|r_t^q - \sum_{p=1}^{M_T} h_{q,p}\sqrt{E_s}c_t^p\right|^2 \geq \sum_{t=1}^{L}\sum_{q=1}^{M_R}\left|r_t^q - \sum_{p=1}^{M_T} h_{q,p}\sqrt{E_s}\hat{c}_t^p\right|^2\right] \\
&= \mathrm{P_r}\left[\sum_{t=1}^{L}\sum_{q=1}^{M_R} 2\mathrm{Re}\left\{z_t^q \sum_{p=1}^{M_T} h_{q,p}\sqrt{E_s}\left(c_t^p - \hat{c}_t^p\right)\right\} \geq \sum_{t=1}^{L}\sum_{q=1}^{M_R}\left|\sum_{p=1}^{M_T} h_{q,p}\sqrt{E_s}\left(c_t^p - \hat{c}_t^p\right)\right|^2\right] \\
&= Q\left(\sqrt{d^2\left(\mathbf{c}, \hat{\mathbf{c}}\right)\frac{E_s}{2N_0}}\right) \leq \frac{1}{2}\exp\left(-d^2\left(\mathbf{c}, \hat{\mathbf{c}}\right)\frac{E_s}{4N_0}\right) \qquad (3.2)
\end{aligned}
$$

where $Q(x)$ is the complementary error function defined by

$$
Q\left(x\right) = \frac{1}{\sqrt{2\pi}}\int_x^{\infty} \exp\left(-x^2/2\right) dx \qquad (3.3)
$$

and the modified Euclidean distance is denoted as

$$
d^2\left(\mathbf{c}, \hat{\mathbf{c}}\right) = \sum_{t=1}^{L}\sum_{q=1}^{M_R}\left|\sum_{p=1}^{M_T} h_{q,p}\left(c_t^p - \hat{c}_t^p\right)\right|^2. \qquad (3.4)
$$

Define $\mathbf{h}_q = \left(h_{q,1}, h_{q,2}, \ldots, h_{q,M_T}\right)$, we can rewrite (3.4) in matrix form, we have

$$
\begin{aligned}
d^2\left(\mathbf{c}, \hat{\mathbf{c}}\right) &= \sum_{q=1}^{M_R} \mathbf{h}_q \mathbf{B}(\mathbf{c}, \hat{\mathbf{c}}) \mathbf{B}\left(\mathbf{c}, \hat{\mathbf{c}}\right)^H \mathbf{h}_q^H \\
&= \sum_{q=1}^{M_R} \mathbf{h}_q \mathbf{A}(\mathbf{c}, \hat{\mathbf{c}}) \mathbf{h}_q^H \qquad (3.5)
\end{aligned}
$$

where $\mathbf{B}(\mathbf{c},\hat{\mathbf{c}})$ is the codeword difference matrix defined as

$$\mathbf{B}\left(\mathbf{c},\hat{\mathbf{c}}\right)=\begin{bmatrix} c_1^1-\hat{c}_1^1 & c_2^1-\hat{c}_2^1 & \cdots & c_L^1-\hat{c}_L^1 \\ c_1^2-\hat{c}_1^2 & c_2^2-\hat{c}_2^2 & \cdots & c_L^2-\hat{c}_L^2 \\ \vdots & \vdots & \ddots & \vdots \\ c_1^{M_T}-\hat{c}_1^{M_T} & c_2^{M_T}-\hat{c}_2^{M_T} & \cdots & c_L^{M_T}-\hat{c}_L^{M_T} \end{bmatrix} \tag{3.6}$$

and the codeword distance matrix is defined as $\mathbf{A}\left(\mathbf{c},\hat{\mathbf{c}}\right)=\mathbf{B}\left(\mathbf{c},\hat{\mathbf{c}}\right)\mathbf{B}\left(\mathbf{c},\hat{\mathbf{c}}\right)^H$. Note that $\mathbf{A}\left(\mathbf{c},\hat{\mathbf{c}}\right)$ is nonnegative definite matrix, thus we can decompose $\mathbf{A}\left(\mathbf{c},\hat{\mathbf{c}}\right)$ as

$$\mathbf{U}\mathbf{A}\left(\mathbf{c},\hat{\mathbf{c}}\right)\mathbf{U}^H=\mathbf{D} \tag{3.7}$$

where $\mathbf{U}$ is an unitary matrix with its column vectors being eigenvectors of $\mathbf{A}\left(\mathbf{c},\hat{\mathbf{c}}\right)$ and $\mathbf{D}$ is a diagonal matrix with its diagonal elements λ_p, $p=0,1,\ldots,M_T$, being nonnegative eigenvalues of $\mathbf{A}\left(\mathbf{c},\hat{\mathbf{c}}\right)$. Now, (3.5) can be rewritten as

$$d^2\left(\mathbf{c},\hat{\mathbf{c}}\right)=\sum_{q=1}^{M_R}\sum_{p=1}^{M_T}\lambda_p, \left|\beta_{q,p}\right|^2, \tag{3.8}$$

where

$$\beta_{q,p}=\mathbf{h}_q\cdot\mathbf{u}_p \tag{3.9}$$

and $\mathbf{u}_p$ is the pth row of U. Substituting (3.8) into (3.2), we get

$$\mathrm{P_r}\left(\mathbf{c}\rightarrow\hat{\mathbf{c}}|h_{q,p},\ \forall\ p,q\right)\leq\frac{1}{2}\exp\left(-\frac{E_s}{4N_0}\sum_{q=1}^{M_R}\sum_{p=1}^{M_T}\lambda_p\left|\beta_{q,p}\right|^2\right). \tag{3.10}$$

We average the conditional PEP over all channel realization to get pairwise error probability. First of all, $\beta_{q,p}$ are obviously independent complex Gaussian random variables with mean $\mu_{q,p}$ and variance 0.5 per dimension, therefore $|\beta_{q,p}|$ are Rician distribution. The probability density function of Rician random variable is

$$f\left(\left|\beta_{q,p}\right|\right)=2\left|\beta_{q,p}\right|\exp\left(-\left|\beta_{q,p}\right|^2-\kappa_{q,p}\right)I_0\left(2\left|\beta_{q,p}\right|\sqrt{\kappa_{q,p}}\right) \tag{3.11}$$

where $\kappa_{q,p}=\left|\mu_{\beta,p,q}\right|^2$, $\mu_{i,q}=E[\mathbf{h}_q\cdot\mathbf{u}_p]$ and $I_0(\cdot)$ represents the zero-order modified Bessel function of the first kind. With respect to averaging the Rician random variables $|\beta_{q,p}|$, the pairwise error probability can be expressed as

$$
\begin{aligned}
\mathrm{P_r}\left(\mathbf{c} \rightarrow \hat{\mathbf{c}}\right) &= \int_0^{\infty} \cdots \int_0^{\infty} \mathrm{P_r}\left(\mathbf{c} \rightarrow \hat{\mathbf{c}} | \beta_{q,p}, \forall p, q\right) f\left(|\beta_{1,1}|\right) f\left(|\beta_{1,2}|\right) \cdots f\left(|\beta_{M_R,M_T}|\right) \\
& \quad d\left|\beta_{1,1}\right| d\left|\beta_{1,2}\right| \cdots d\left|\beta_{M_R,M_T}\right| \\
&\leq \int_0^{\infty} \cdots \int_0^{\infty} \frac{1}{2} \exp\left(-\frac{E_s}{4N_0} \sum_{q=1}^{M_R} \sum_{p=1}^{M_T} \lambda_p \left|\beta_{q,p}\right|^2\right) f\left(|\beta_{1,1}|\right) f\left(|\beta_{1,2}|\right) \cdots f\left(|\beta_{M_R,M_T}|\right) \\
& \quad d\left|\beta_{1,1}\right| d\left|\beta_{1,2}\right| \cdots d\left|\beta_{M_R,M_T}\right| \\
&\leq \frac{1}{2} \prod_{q=1}^{M_R} \left(\prod_{p=1}^{r} \frac{1}{1+\frac{E_s}{4N_0}\lambda_p} \exp\left(-\frac{\kappa_{q,p}\frac{E_s}{4N_0}\lambda_p}{1+\frac{E_s}{4N_0}\lambda_p}\right)\right) \qquad (3.12)
\end{aligned}
$$

where $r(\cdot)$ stands for rank of $\mathbf{A}\left(\mathbf{c}, \hat{\mathbf{c}}\right)$. Consider the wireless channel to be Rayleigh fading, that is, $\mu_{\beta,q,p} = 0$, the PEP can be upper bounded as

$$
\mathrm{P_r}\left(\mathbf{c} \rightarrow \hat{\mathbf{c}}\right) \leq \frac{1}{2} \left(\prod_{p=1}^{r} \frac{1}{1+\frac{E_s}{4N_0}\lambda_p} \right)^{M_R}. \qquad (3.13)
$$

At high SNR's, (3.13) can be represented as

$$
\mathrm{P_r}\left(\mathbf{c} \rightarrow \hat{\mathbf{c}}\right) \leq \frac{1}{2} \left(\prod_{p=1}^{r} \lambda_p \right)^{-M_R} \left(\frac{E_s}{4N_0} \right)^{-rM_R}. \qquad (3.14)
$$

As a consequence, we have the design criteria of space time coding as follows.

- *Rank (diversity) criterion*: The minimum rank of $\mathbf{A}\left(\mathbf{c}, \hat{\mathbf{c}}\right)$ over all pairs of distinct codewords $\mathbf{c}$ and $\hat{\mathbf{c}}$ should be as large as possible.

- *Determinant criterion*: The minimum product of nonzero eigenvalues $\prod_{p=1}^{r} \lambda_p$ over all pairs of distinct codewords $\mathbf{c}$ and $\hat{\mathbf{c}}$ should also be maximized.

The rank and determinant criteria give us a guild line to develop good space time codes, and the maximum achievable diversity of space time codes is the multiplication of encoding durations and received antenna numbers, i.e. LM_R.

3.2 Space-Frequency coding

The STC systems introduced in previous section is restricted to single-carrier systems operating narrow-band flat fading channels. A strategy which employing STC across OFDM tones over MIMO broad-band channels to provide not only spatial diversity but frequency diversity was investigated, that is, the space-frequency coding. The design criteria of SFC are introduced in the following.

3.2.1 System Model of SFC

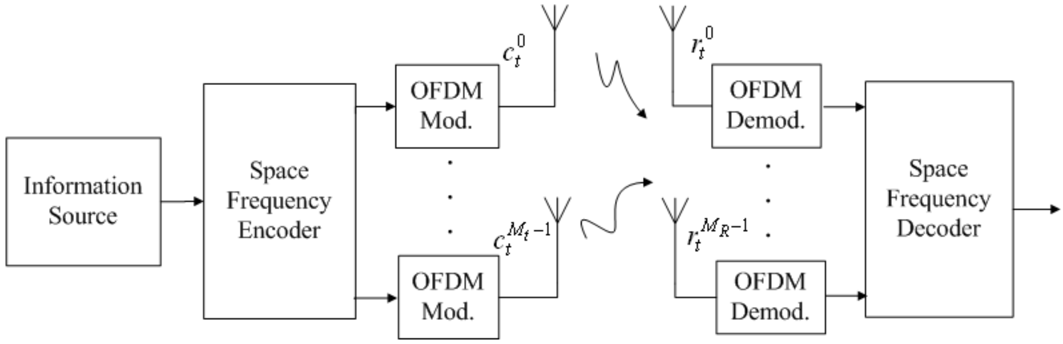


Figure 3.2: System model of SFC.

The MIMO-OFDM system equipped M_T transmit antennas and M_R receive antennas is showned in Figure 3.2. The information bits are encoded by the SFC encoder into blocks of size $M_T \times N$, and the coded symbol is fed to the OFDM modulator to perform IFFT transform and append the cyclic prefix. And then the time-domain OFDM symbols are transmitted into the MIMO channels. At receiver, the transmitted signals of all the transmit antennas are weighted superposition and currupted by AWGN at each receive antenna. Organizing the transmitted data symbols of kth subcarrier into frequency vectors $\mathbf{c_k} = \left(c_k^0,\, c_k^1, \ldots,\, c_k^{M_T-1}\right)^T$ with c_k^i denoting the data symbol transmitted from the ith antenna on the kth tone and c_k^i are taken from a finite complex alphabet such that the average energy of the constellation element is E_s, then the received data vector of the kth tone is given by

$$\mathbf{r}_k = \sqrt{E_s}\mathbf{H}\left(e^{j\frac{2\pi k}{N}}\right)\mathbf{c}_k + \mathbf{n}_k,\ k = 0, 1, \ldots, N-1, \tag{3.15}$$

where $\mathbf{H}\left(e^{j\frac{2\pi k}{N}}\right)$ is the frequency domain channel matrix. And $\mathbf{n}_k$ is independent complex AWGN with mean zero, variance 0.5 per dimension and one sided power spectral density N_0.

3.2.2 Design Criteria of SFC

Assuming the channel is constant over at least one OFDM symbol, and the perfect CSI is available at the receiver. The ML decoder decides the most likely transmitted sequence $\hat{\mathbf{c}}_k$, $k = 0, 1, \ldots, N-1$, over all possible codewords according to

$$\hat{\mathbf{c}}_k = \arg_{\mathbf{C}}\min \sum_{k=0}^{N-1} \left\| \mathbf{r}_k - \sqrt{E_s}\mathbf{H}\left(e^{j\frac{2\pi k}{N}}\right)\mathbf{c}_k \right\|^2 \tag{3.16}$$

where $\mathbf{C} = [\mathbf{c}_0\, \mathbf{c}_1\, \ldots, \mathbf{c}_{N-1}]$. Let $\mathbf{E} = [\mathbf{e}_0\, \mathbf{e}_1\, \ldots, \mathbf{e}_{N-1}]$ be the erroneous decode space-frequency codeword, for a given channel realization, the pairwise error probability is given by

$$\mathrm{P}\left(\mathbf{C} \to \mathbf{E}|\mathbf{H}\left(e^{j\frac{2\pi k}{N}}\right)\right) = \mathrm{Q}\left(\sqrt{\frac{E_s}{2N_0}d^2\left(\mathbf{C}, \mathbf{E}|\mathbf{H}\left(e^{j\frac{2\pi k}{N}}\right)\right)}\right) \tag{3.17}$$

where the squared Euclidean distance between the two codewords $\mathbf{C}$ and $\mathbf{E}$ is dented by

$$\begin{aligned} d^2\left(\mathbf{C}, \mathbf{E}|\mathbf{H}\left(e^{j\frac{2\pi k}{N}}\right)\right) &= \sum_{k=0}^{N-1}\left\|\mathbf{H}\left(e^{j\frac{2\pi k}{N}}\right)(\mathbf{c}_k - \mathbf{e}_k)\right\|^2 \\ &= \|\mathbf{Y}\|^2. \end{aligned} \tag{3.18}$$

Here we define a new matrix $\mathbf{Y} = [\mathbf{y}_0, \mathbf{y}_1, \ldots, \mathbf{y}_{N-1}]^T$ in equation (3.18) for computational convenience, and the definition of $\mathbf{y}_k$, $k = 0, 1, \ldots, N-1$, is given by

$$\mathbf{y}_k = \mathbf{H}\left(e^{j\frac{2\pi k}{N}}\right)(\mathbf{c}_k - \mathbf{e}_k). \tag{3.19}$$

Using the Chernoff bound $\mathrm{Q}(x) \leq e^{-x^2/2}$ into equation (3.17), we get

$$\mathrm{P}\left(\mathbf{C} \to \mathbf{E}|\mathbf{H}\left(e^{j\frac{2\pi k}{N}}\right)\right) \leq e^{\frac{E_s}{4N_0}d^2\left(\mathbf{C},\mathbf{E}|\mathbf{H}\left(e^{j\frac{2\pi k}{N}}\right)\right)}. \tag{3.20}$$

Since the multipath channel were assumed to be i.i.d. complex Gaussian it follows that $\mathbf{H}\left(e^{j\frac{2\pi k}{N}}\right)$ are Gaussian as well and hence the vector $\mathbf{Y}$ is Gaussian. The average over all channel realizations of the right handside in (3.20) is equivalent to solving the characteristic function of $\mathbf{Y}$ and it is fully characterized by the eigenvalues of the covariance matrix of $\mathbf{Y}$ [28]. Therefore the pairwise error probability is given by

$$\mathrm{P}(\mathbf{C} \to \mathbf{E}) \leq \prod_{i=0}^{r(\mathbf{C}_\mathbf{Y})-1}\left(1 + \lambda_i(\mathbf{C}_\mathbf{Y})\frac{E_s}{4N_0}\right)^{-1} \tag{3.21}$$

where $r(\mathbf{C_Y})$ denotes the rank of $\mathbf{C_Y}$, $\lambda_i(\mathbf{C_Y})$, $i = 0, 1, \ldots, r(\mathbf{C_Y})$, are the nonzero eigenvalues of $\mathbf{C_Y}$, the definition of $\mathbf{C_Y}$ is given by

$$\begin{aligned} \mathbf{C_Y} &= E\left[\mathbf{Y}\mathbf{Y}^H\right] \\ &= \sum_{l=0}^{L-1}\left[\mathbf{D}^l(\mathbf{C}-\mathbf{E})^T(\mathbf{C}-\mathbf{E})^*\left(\mathbf{D}^l\right)^H\right]\otimes\mathbf{R}_l \end{aligned} \tag{3.22}$$

where $\mathbf{D} = diag\{e^{-j\frac{2\pi k}{N}}\}_{k=0}^{N-1}$, $\mathbf{R}_l$ denote the correlation matrix of MIMO channel between transmit and receive antennas at time delay l, $\mathbf{A}\otimes\mathbf{B}$ denotes the Kronecker product of the matrices $\mathbf{A}$ and $\mathbf{B}$, and the superscript $*$ stands for complex conjugate operation. Assume the SFC systems are operated at high SNR, the PEP can be upper bounded by

$$\mathrm{P}(\mathbf{C}\rightarrow\mathbf{E}) \leq \left(\prod_{i=0}^{r(\mathbf{C_Y})-1}\lambda_i(\mathbf{C_Y})\right)\left(\frac{E_s}{4N_0}\right)^{-r(\mathbf{C_Y})}. \tag{3.23}$$

The design criteria for space-frequency codes follow from equation (3.23) as the well-known rank and determinant criteria.

- *Rank (diversity) criterion*: The minimum rank of $\mathbf{C_Y}$ over all pairs of distinct codewords $\mathbf{C}$ and $\mathbf{E}$ should be as large as possible.
- *Determinant criterion*: The minimum product of nonzero eigenvalues $\prod_{i=0}^{r(\mathbf{C_Y})}\lambda_i(\mathbf{C_Y})$ over all pairs of distinct codewords $\mathbf{C}$ and $\mathbf{E}$ should also be maximized.

Next, we shortly introduce the maximum achievable diversity of the space-frequency codes by the discussion of $\mathbf{C_Y}$. Assume $N > M_T L$, using the factorizations of $\mathbf{R}_l = \mathbf{R}_l^{\frac{1}{2}}\mathbf{R}_l^{\frac{1}{2}}$, $l = 0, 1, \ldots, L-1$, and the property of Kronecker products

$$(\mathbf{A}\otimes\mathbf{B})\otimes(\mathbf{C}\otimes\mathbf{D}) = (\mathbf{AC})\otimes(\mathbf{BD}) \tag{3.24}$$

the $NM_R \times M_T M_R L$ matrix $\mathbf{C_Y}$ can be decomposed as

$$\mathbf{C_Y} = \mathbf{G}(\mathbf{C},\mathbf{E})\,\mathbf{G}^H(\mathbf{C},\mathbf{E}) \tag{3.25}$$

where $\mathbf{G}(\mathbf{C},\mathbf{E})$ is defined as $\mathbf{G}(\mathbf{C},\mathbf{E}) = [\mathbf{g}_0\ \mathbf{g}_1\ \ldots\ \mathbf{g}_{L-1}]$ and $\mathbf{g}_l = \left[\left(\mathbf{D}^l(\mathbf{C}-\mathbf{E})^T\right)\otimes\mathbf{R}_l^{\frac{1}{2}}\right]$.

If the MIMO channels satisfy the condition of $r(\mathbf{R}_l) = M_R$ and by carefully design of space-frequency codeword to have $r\left(\mathbf{D}^l(\mathbf{C}-\mathbf{E})^T\right) = M_T$ for all l over all distinct codeword pairs, the stacked matrix $\mathbf{G}(\mathbf{C},\mathbf{E})$ would be full rank. As above conditions are fulfilled, we thus have a full rank $\mathbf{C}_\mathbf{Y}$ and the space-frequency codes can achieve the maximum diversity order of $M_T M_R L$.

Chapter 4

CP-reduced Coding Techniques with ICI Diversity Combining for OFDM Systems

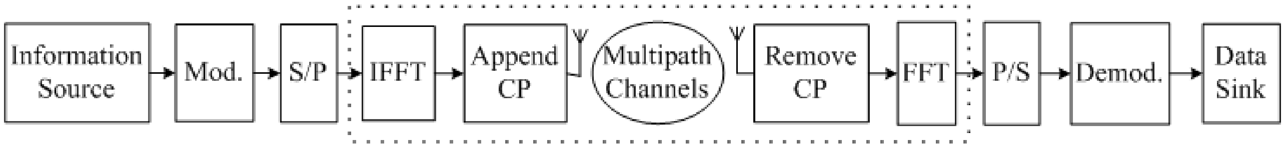
Actually the ICI corrupted channel of the CP-reduced OFDM system can be viewed as a virtual MIMO channel when we regard the subcarriers as the virtual antennas. And the ICI induced by insufficient CP can be viewed as a sort of frequency diversity by the concepts of space-frequency coding (SFC). In this chapter, we study the space-frequency codes under the virtual MIMO channel of CP-reduced OFDM systems, and we term such codes as "*virtual space-frequency codes*". The maximum achievable diversity order and the corresponding design criteria are also given by theoretical analysis of the derived PEP bound.

4.1 System Model of OFDM Systems

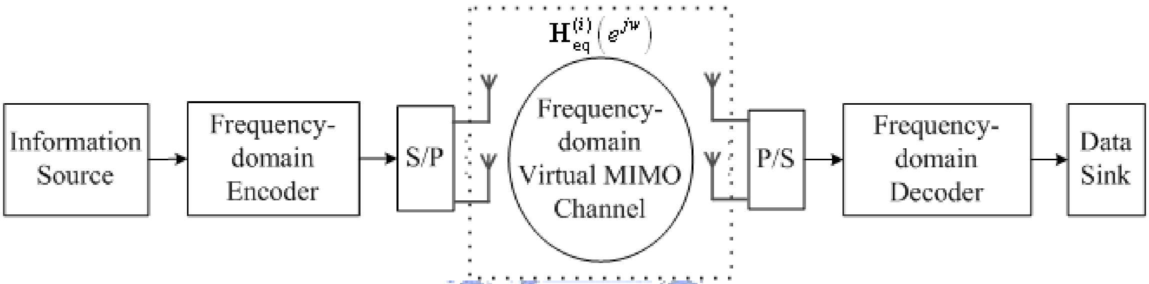
If we view the subcarriers as the virtual antennas, and combine the system blocks circled by dash lines in Figure 4.1(a) into a equivalent frequency-domain MIMO channels as shown in Figure 4.1(b) emphasized by dash lines, then conventional CP-reduced SISO-OFDM systems with perfect ISI canceller can be regarded as virtual MIMO-OFDM systems with their virtual MIMO channels of ith OFDM block index are specified by the frequency-domain ICI channels of

$$\mathbf{H}_{eq}^{(i)}\left(e^{jw}\right) = \mathbf{F}_N\left(\mathbf{H}^{(i)} - \mathbf{H}_{ici}^{(i)}\right)\mathbf{F}_N^H, \tag{4.1}$$

where $\mathbf{F}_N$ is the N-point FFT matrix, $\mathbf{H}^{(i)}$ and $\mathbf{H}_{ici}^{(i)}$ are the time-domain channel matrices in (2.3), and (2.5) which are the channel matrices corresponding to interference-free and ICI signal components. Under the aspects of subcarriers as virtual antennas and following the concepts of SFC, the ICI induced from reduced CP can equivalently be considered as the virtual MIMO channel gains from virtual transmit antennas to virtual received antennas. That is, ICI can be regarded as frequency diversity. After a clear understanding of the virtual spatial-domain, next we derive the PEP based on the virtual MIMO-OFDM systems shown in Figure 4.1(b).



(a) Conventional SISO-OFDM systems



(b) Virtual MIMO-OFDM systems with subcarriers as virtual antennas

Figure 4.1: (a) Conventional SISO-OFDM Systems, (b) Virtual MIMO-OFDM with subcarriers as virtual antennas

4.2 Pairwise Error Probability Analysis

From the viewpoints of regarding subcarriers as virtual antennas, we investigate a coding technique across B OFDM blocks based on the concepts of SFC for the virtual MIMO-OFDM systems with perfect ISI canceller. Assume the channel responses are static at least for one OFDM duration (symbol-wise fading), and the channel gains for different delay are independent. The input bit stream is divided into b bit-long segments, forming 2^b-ary constellation symbols. These symbols are then mapped onto a frequency-domain codeword to be transmitted over the N virtual antennas (subcarriers). Each frequency-domain codeword can be express as an $NB \times 1$ matrix

$$\mathbf{X} = \left[\mathbf{X}_1^T,\ \mathbf{X}_2^T, \ldots,\ \mathbf{X}_B^T\right]^T \tag{4.2}$$

where $\mathbf{X}_i = [x_{i,0},\ x_{i,1},\ ,\cdots,\ x_{i,N-1}]$ is an $N \times 1$ column vector, representing the ith coded OFDM block. At the receiver, the received signal of ith block can be written in matrix form as

$$\mathbf{R}_i = \sqrt{E_s}\mathbf{H}_{eq}^{(i)}\left(e^{jw}\right)\mathbf{X}_i + \mathbf{Z}_i^T, \tag{4.3}$$

where the complex additive white Gaussian noise (AWGN) vector of ith block index is defined as $\mathbf{Z}_i = [z_{i,0},\ z_{i,1}, \ldots, z_{i,N-1}]$, and $z_{i,j} \sim CN(0, N_0)$ is the complex AWGN of ith OFDM block at j subcarrier. Thus the average coded symbol to noise ratio at each subcarrier is given by E_s/N_0. Collect B blocks of received OFDM signals and denote $\mathbf{R} = \left[\mathbf{R}_1^T,\ \mathbf{R}_2^T, \ldots,\ \mathbf{R}_B^T\right]^T$, then $\mathbf{R}$ is an $NB \times 1$ column vector, and is given by

$$\mathbf{R} = \sqrt{E_s}\mathbf{H}_{eq}\left(e^{jw}\right)\mathbf{X} + \mathbf{Z}, \tag{4.4}$$

where the noise vector $\mathbf{Z}$ is formatted as $\mathbf{Z} = \left[\mathbf{Z}_1^T,\ \mathbf{Z}_2^T, \ldots,\ \mathbf{Z}_B^T\right]^T$, and $\mathbf{H}_{eq}\left(e^{jw}\right)$ is the virtual MIMO channel matrix in frequency-domain of consecutive B OFDM blocks, and can be formulated as

$$\mathbf{H}_{eq}\left(e^{jw}\right) = \begin{bmatrix} \left[\mathbf{H}_{eq}^{(1)}\left(e^{jw}\right)\right] & & & \\ & \left[\mathbf{H}_{eq}^{(2)}\left(e^{jw}\right)\right] & & \\ & & \ddots & \\ & & & \left[\mathbf{H}_{eq}^{(B)}\left(e^{jw}\right)\right] \end{bmatrix}. \tag{4.5}$$

Assume perfect channel state information (CSI) is available at the receiver, and the receiver applies a maximum likelihood decoder to detect signals. The PEP of coding across consecutive B OFDM blocks conditioned on the virtual MIMO channels of $\mathbf{H}_{eq}\left(e^{jw}\right)$ is given by

$$P\left(\mathbf{X}, \hat{\mathbf{X}} \left| \mathbf{H}_{eq}\left(e^{jw}\right)\right.\right) = Q\left(\sqrt{\frac{d_H^2\left(\mathbf{X}, \hat{\mathbf{X}}\right)}{2N_0}}\right), \tag{4.6}$$

where $\hat{\mathbf{X}}$ stands for the erroneous decoded codeword, $Q(x)$ is the complementary error function, and $d_H^2(\mathbf{X}, \hat{\mathbf{X}})$ is the modified Euclidean distance with the definition of

$$\begin{aligned} d_H^2\left(\mathbf{X}, \hat{\mathbf{X}}\right) &= \left\|\mathbf{H}_{eq}\left(e^{jw}\right)\left(\mathbf{X}-\hat{\mathbf{X}}\right)\right\|_F^2 \\ &= \sum_{i=1}^{B}\left\|\mathbf{H}_{eq}^{(i)}\left(e^{jw}\right)\left(\mathbf{X}_i-\hat{\mathbf{X}}_i\right)\right\|_F^2. \end{aligned} \tag{4.7}$$

By using the Chernoff bound, the conditional PEP can be upper bounded by

$$P\left(\mathbf{X}, \hat{\mathbf{X}}\left|\mathbf{H}_{eq}\left(e^{jw}\right)\right.\right) \leq \frac{1}{2}\exp\left(-\frac{E_S}{4N_0}d_H^2\left(\mathbf{X}, \hat{\mathbf{X}}\right)\right). \tag{4.8}$$

Next we rewrite $d_H^2(\mathbf{X}, \hat{\mathbf{X}})$ into a more compact form for computational simplicity of channel averaging. By the equalities of $tr\left(\mathbf{A}\mathbf{A}^H\right) = \|\mathbf{A}\|_F^2$ and $tr\left(\mathbf{AB}\right) = vec\left(\mathbf{A}^H\right)^H vec\left(\mathbf{B}\right)$ [29], we can reformulated (4.7) as

$$d_H^2(\mathbf{X}, \hat{\mathbf{X}}) = \sum_{i=1}^{B} vec\left(\mathbf{A}_i\left(\mathbf{X}, \hat{\mathbf{X}}\right)\mathbf{H}_{eq}^{(i)}\left(e^{jw}\right)^H\right)^H vec\left(\mathbf{H}_{eq}^{(i)}\left(e^{jw}\right)^H\right), \tag{4.9}$$

where $\mathbf{A}_i(\mathbf{X}, \hat{\mathbf{X}}) = (\mathbf{X}_i - \hat{\mathbf{X}}_i)(\mathbf{X}_i - \hat{\mathbf{X}}_i)^H$ is the ith codeword distance matrix, and $vec(\cdot)$ is an operator that stacks all the columns of a matrix into a super column matrix orderly according to their column indexes. Apply the equality of $vec\left(\mathbf{ABC}\right) = \left(\mathbf{C}^T \otimes \mathbf{A}\right) vec\left(\mathbf{B}\right)$ [29], (4.9) becomes

$$d_H^2\left(\mathbf{X}, \hat{\mathbf{X}}\right) = \sum_{i=1}^{B} vec\left(\mathbf{H}_{eq}^{(i)}\left(e^{jw}\right)^H\right)^H\left(\mathbf{I}_N \otimes \mathbf{A}_i\left(\mathbf{X}, \hat{\mathbf{X}}\right)\right) vec\left(\mathbf{H}_{eq}^{(i)}\left(e^{jw}\right)^H\right). \tag{4.10}$$

Define $\mathbf{h}_i = vec\left(\mathbf{H}_{eq}^{(i)}\left(e^{jw}\right)^H\right)$ and $\mathbf{K}_i(\mathbf{X}, \hat{\mathbf{X}}) = \mathbf{I}_N \otimes \mathbf{A}_i(\mathbf{X}, \hat{\mathbf{X}})$, the modified Euclidean distance is then given by

$$d_H^2\left(\mathbf{X}, \hat{\mathbf{X}}\right) = \sum_{i=1}^{B} \mathbf{h}_i^H \mathbf{K}_i\left(\mathbf{X}, \hat{\mathbf{X}}\right)\mathbf{h}_i. \tag{4.11}$$

Now we are about to average the conditional PEP over all channel realization, but we can't assure if the random vector $\mathbf{h}_i$ has a full rank covariance matrix for each i (actually $\mathbf{h}_i$ has a rank-deficient covariance matrix for each i, and its proof is shown in the appendix). Therefore, to ease the calculation of integral during the step of channel averaging, we consider a linear transformation to transform $\mathbf{h}_i$ into a random vector with a full rank covariance to simplify the computation of integral. Assume $\mathbf{h}_i$ is a complex Gaussian random vector with its mean denoted by $\boldsymbol{\mu}_{\mathbf{h}}$ and covariance denoted by $\mathbf{R}_{\mathbf{h}}$ with $rank(\mathbf{R}_{\mathbf{h}}) = r \leq N^2$. Consider the linear transformation of the ith channel vector

$$\mathbf{h}_i = \boldsymbol{\mu}_{\mathbf{h}} + \mathbf{R}_{\mathbf{h}}^{\frac{1}{2}} \mathbf{g}_i, \tag{4.12}$$

where $\mathbf{g}_i \sim CN(0, \mathbf{I}_r)$ and $\mathbf{R}_{\mathbf{h}}^{\frac{1}{2}}$ stands for the Cholesky decomposition of the covariance matrix $\mathbf{R}_{\mathbf{h}}$ [30]. By this transformation, we can transform each $\mathbf{h}_i$ to another random vector $\mathbf{g}_i$ which has zero mean and an identity covariance matrix. Substitute (4.12) into (4.11), then the modified Euclidean distance can be finally formulated as

$$\begin{aligned} d_H^2\left(\mathbf{X}, \hat{\mathbf{X}}\right) &= \sum_{i=1}^{B} \left(\boldsymbol{\mu}_{\mathbf{h}} + \mathbf{R}_{\mathbf{h}}^{\frac{1}{2}} \mathbf{g}_i\right)^H \mathbf{K}_i\left(\mathbf{X}, \hat{\mathbf{X}}\right)\left(\boldsymbol{\mu}_{\mathbf{h}} + \mathbf{R}_{\mathbf{h}}^{\frac{1}{2}} \mathbf{g}_i\right) \\ &= \sum_{i=1}^{B} d_{H_i}^2\left(\mathbf{X}, \hat{\mathbf{X}}\right). \end{aligned} \tag{4.13}$$

Now we average the conditional PEP over all channel realizations

$$\begin{aligned} P\left(\mathbf{X}, \hat{\mathbf{X}}\right) &\leq \prod_{i=1}^{B} \int_{\mathbf{g}_i} \left(\tfrac{1}{\pi^r}\right) \exp\left(-\left(\tfrac{E_s}{4N_0}\right) d_{H_i}^2\left(\mathbf{X}, \hat{\mathbf{X}}\right)\right) \exp\left(-\mathbf{g}_i^H \mathbf{g}_i\right) d\mathbf{g}_i \\ &\leq \prod_{i=1}^{B} \left(\det\left(\mathbf{I}_r + \left(\tfrac{E_s}{4N_0}\right)\left(\mathbf{R}_{\mathbf{h}}^{\frac{1}{2}}\right)^H \mathbf{K}_i\left(\mathbf{X}, \hat{\mathbf{X}}\right) \mathbf{R}_{\mathbf{h}}^{\frac{1}{2}}\right)\right)^{-1} \exp\left(-\left(\tfrac{E_s}{4N_0}\right) \boldsymbol{\mu}_{\mathbf{h}}^H \mathbf{K}_i\left(\mathbf{X}, \hat{\mathbf{X}}\right) \boldsymbol{\mu}_{\mathbf{h}}\right) \\ &\quad \cdot \exp\left(\left(\tfrac{E_s}{4N_0}\right)^2 \boldsymbol{\Phi}^H \left[\mathbf{I}_r + \left(\tfrac{E_s}{4N_0}\right)\left(\mathbf{R}_{\mathbf{h}}^{\frac{1}{2}}\right)^H \mathbf{K}_i\left(\mathbf{X}, \hat{\mathbf{X}}\right) \mathbf{R}_{\mathbf{h}}^{\frac{1}{2}}\right]^{-1} \boldsymbol{\Phi}\right), \end{aligned} \tag{4.14}$$

where $\boldsymbol{\Phi} = \left(\mathbf{R}_{\mathbf{h}}^{\frac{1}{2}}\right)^H \mathbf{K}_i(\mathbf{X}, \hat{\mathbf{X}}) \boldsymbol{\mu}_{\mathbf{h}}$. Consider the multipath channels to be Rayleigh fading channels (i.e. $\boldsymbol{\mu}_{\mathbf{h}} = \mathbf{0}$), and after some computations, the PEP can be upper bounded by

$$P\left(\mathbf{X}, \hat{\mathbf{X}}\right) \leq \prod_{i=1}^{B} \left(\det\left(\mathbf{I}_r + \left(\frac{E_s}{4N_0}\right) \mathbf{K}_{eff}^{(i)}\left(\mathbf{X}, \hat{\mathbf{X}}\right)\right)\right)^{-1}, \tag{4.15}$$

where $\mathbf{K}_{eff}^{(i)}(\mathbf{X}, \hat{\mathbf{X}})$ is the *effective* codeword matrix of ith OFDM block, and is defined as

$$\mathbf{K}_{eff}^{(i)}\left(\mathbf{X}, \hat{\mathbf{X}}\right) = \left(\mathbf{R}_{\mathbf{h}}^{\frac{1}{2}}\right)^H \mathbf{K}_i\left(\mathbf{X}, \hat{\mathbf{X}}\right) \mathbf{R}_{\mathbf{h}}^{\frac{1}{2}}. \tag{4.16}$$

Here we use the phase "*effective*" to emphasize $\mathbf{K}_{eff}^{(i)}(\mathbf{X}, \hat{\mathbf{X}})$ is a codeword distance matrix that not only depends on the codeword distance matrix $\mathbf{A}_i(\mathbf{X}, \hat{\mathbf{X}})$, but also depends on the covariance matrix of the multipath channels. Note that $\mathbf{K}_{eff}^{(i)}(\mathbf{X}, \hat{\mathbf{X}})$ is nonnegative definite Hermitian, thus there exists a unitary matrix $\mathbf{U}_i$ and a real diagonal matrix $\mathbf{D}_i$ such that

$$\mathbf{U}_i \left[\left(\mathbf{R}_{\mathbf{h}}^{\frac{1}{2}} \right)^H \mathbf{K}_i \left(\mathbf{X}, \hat{\mathbf{X}} \right) \mathbf{R}_{\mathbf{h}_i}^{\frac{1}{2}} \right] \mathbf{U}_i^H = \mathbf{D}_i, \tag{4.17}$$

where the rows of $\mathbf{U}_i$ are the eigenvectors of $\mathbf{K}_{eff}^{(i)}(\mathbf{X}, \hat{\mathbf{X}})$, forming a complete orthonormal basis of an N^2-dimensional vector space. The diagonal matrix $\mathbf{D}_i$ can be represented as

$$\mathbf{D}_i = \begin{pmatrix} \lambda_1^{(i)} & & \\ & \ddots & \\ & & \lambda_{R_i}^{(i)} \end{pmatrix} \tag{4.18}$$

where $R_i^{(i)}$ stands for the rank of $\mathbf{K}_{eff}^{(i)}(\mathbf{X}, \hat{\mathbf{X}})$, and $\lambda_j^{(i)}$, $j = 0, \ldots, R_i$, $i = 1, \ldots, B$ are the nonzero eigenvalues of $\mathbf{K}_{eff}^{(i)}(\mathbf{X}, \hat{\mathbf{X}})$. At high SNR's, the upper bound of PEP can be simplified as

$$P\left(\mathbf{X}, \hat{\mathbf{X}}\right) \leq \left(\prod_{i=1}^{B} \prod_{j=1}^{R_i} \lambda_j^{(i)} \right)^{-1} \left(\frac{E_s}{4N_0} \right)^{-\sum_{i=1}^{B} R_i} \tag{4.19}$$

or

$$P\left(\mathbf{X}, \hat{\mathbf{X}}\right) \leq \left(\det \left(\mathbf{K}_{eff} \left(\mathbf{X}, \hat{\mathbf{X}} \right) \right) \right)^{-1} \left(\frac{E_s}{4N_0} \right)^{-rank\left(\mathbf{K}_{eff}\left(\mathbf{X}, \hat{\mathbf{X}}\right)\right)}, \tag{4.20}$$

here we further define a total effective codeword distance matrix with its definition is given by

$$\mathbf{K}_{eff}\left(\mathbf{X}, \hat{\mathbf{X}}\right) = \begin{bmatrix} \left[\mathbf{K}_{eff}^{(1)}\left(\mathbf{X}, \hat{\mathbf{X}}\right)\right] & & & \\ & \left[\mathbf{K}_{eff}^{(2)}\left(\mathbf{X}, \hat{\mathbf{X}}\right)\right] & & \\ & & \ddots & \\ & & & \left[\mathbf{K}_{eff}^{(B)}\left(\mathbf{X}, \hat{\mathbf{X}}\right)\right] \end{bmatrix}. \tag{4.21}$$

Based on the upper bounds shown in (4.19) and (4.20), we have the insights on the factors that determine the diversity order and coding gain of system performance, and we can tell that both diversity and coding gains are closely related to the total effective codeword distance matrix. Thus we give a proof of the achievable diversity order along with detail discussions in next section.

4.3 Maximum Achievable Diversity and Design Criteria

We analyze the factors that affect the diversity order of a SF-coded virtual MIMO-OFDM system in this section. First, we derive an upper bound for the maximum achievable diversity for such a system. Second, we propose the design criteria according to the derivation results of the upper bound of maximum achievable diversity. From the PEP in (4.19) and (4.20), we can see that the diversity order is determined by the rank of the total effective distance matrix $\mathbf{K}_{eff}(\mathbf{X}, \hat{\mathbf{X}})$, and $\mathbf{K}_{eff}(\mathbf{X}, \hat{\mathbf{X}})$ depends on $\mathbf{K}_{eff}^{(i)}(\mathbf{X}, \hat{\mathbf{X}})$, for $i = 1, \ldots, B$. Therefore, in order to determine the upper bound of $rank(\mathbf{K}_{eff}(\mathbf{X}, \hat{\mathbf{X}}))$, we should firstly determine the upper bound of $rank\left(\mathbf{K}_{eff}^{(i)}(\mathbf{X}, \hat{\mathbf{X}})\right)$, and then applying the upper bound to determine the rank of $\mathbf{K}_{eff}(\mathbf{X}, \hat{\mathbf{X}})$. The upper bound of $rank\left(\mathbf{K}_{eff}^{(i)}(\mathbf{X}, \hat{\mathbf{X}})\right)$ and $rank(\mathbf{K}_{eff}(\mathbf{X}, \hat{\mathbf{X}}))$ can be calculated in the following theorem.

Theorem 1. Consider a coding scheme based on CP-reduced OFDM systems under symbol-wise Rayleigh fading channels for a given subcarrier number N and channel order $(L + 1)$, the maximum achievable diversity order is given by:

(1.a) If $N \geq (L+1)$ and $\mathbf{K}_{eff}^{(i)}(\mathbf{X}, \hat{\mathbf{X}}) \neq \mathbf{0}$ for $\forall \mathbf{X} \neq \hat{\mathbf{X}}$ in ith OFDM block, then $rank\left(\mathbf{K}_{eff}^{(i)}(\mathbf{X}, \hat{\mathbf{X}})\right) \leq (L + 1)$ and therefore the rank of the coded B OFDM blocks are upper bounded by $rank(\mathbf{K}_{eff}(\mathbf{X}, \hat{\mathbf{X}})) \leq \ N \times (L + 1)$.

(1.b) If $N < (L+1)$ and $\mathbf{K}_{eff}^{(i)}(\mathbf{X}, \hat{\mathbf{X}}) \neq \mathbf{0}$ for $\forall \mathbf{X} \neq \hat{\mathbf{X}}$ in ith OFDM block, then $rank\left(\mathbf{K}_{eff}^{(i)}(\mathbf{X}, \hat{\mathbf{X}})\right) \leq N \times rank(\mathbf{A}_i(\mathbf{X}, \hat{\mathbf{X}}))$ and therefore the the rank of the coded B OFDM blocks are upper bounded by $rank(\mathbf{K}_{eff}(\mathbf{X}, \hat{\mathbf{X}})) \leq N \times \sum_{i=1}^{B} rank(\mathbf{A}_i(\mathbf{X}, \hat{\mathbf{X}}))$.

Proof. By equation (4.16) and the inequality of $rank(\mathbf{AB}) \leq \min\{rank(\mathbf{A}),\ rank(\mathbf{B})\}$, then the upper bound of $rank\left(\mathbf{K}_{eff}^{(i)}(\mathbf{X}, \hat{\mathbf{X}})\right)$ is given by

$$rank\left(\mathbf{K}_{eff}^{(i)}(\mathbf{X}, \hat{\mathbf{X}})\right) \leq \min\left(rank\left(\mathbf{R}_{\mathbf{h}}^{\frac{1}{2}}\right),\ rank\left(\mathbf{K}_i(\mathbf{X}, \hat{\mathbf{X}})\right)\right). \tag{4.22}$$

Therefore we analyze the rank of $\mathbf{R}_{\mathbf{h}}^{\frac{1}{2}}$ and $\mathbf{K}_i(\mathbf{X}, \hat{\mathbf{X}})$ individually to determine which matrix has the minimum rank that upper bounds the diversity order. We deal with $\mathbf{K}_i(\mathbf{X}, \hat{\mathbf{X}})$ first. Using the fact that each eigenvalue of the $N \times N$ matrix $\mathbf{A}_i(\mathbf{X}, \hat{\mathbf{X}})$ is an eigenvalue of the $N^2 \times N^2$ matrix $\mathbf{I}_N \otimes \mathbf{A}_i(\mathbf{X}, \hat{\mathbf{X}})$ with multiplicity N, thus the rank of $\mathbf{K}_i(\mathbf{X}, \hat{\mathbf{X}})$ is given by

$$rank\left(\mathbf{K}^{(i)}\left(\mathbf{X}, \hat{\mathbf{X}}\right)\right) = \begin{cases} N \times rank(\mathbf{A}_i(\mathbf{X}, \hat{\mathbf{X}})) & , \text{if} \mathbf{A}_i(\mathbf{X}, \hat{\mathbf{X}}) \neq \mathbf{0}. \\ 0 & , \text{if} \mathbf{A}_i(\mathbf{X}, \hat{\mathbf{X}}) = \mathbf{0}. \end{cases} \tag{4.23}$$

From (4.23) we can conclude that only if the ith codeword distance matrix $\mathbf{A}_i(\mathbf{X}, \hat{\mathbf{X}})$ is nonzero for all pairs of ith distinct OFDM symbols, then $rank(\mathbf{K}_i(\mathbf{X}, \hat{\mathbf{X}}))$ will be the multiplication of $rank(\mathbf{A}_i(\mathbf{X}, \hat{\mathbf{X}}))$ and subcarrier number N.

The rank of $\mathbf{R}_{\mathbf{h}}^{\frac{1}{2}}$ is equivalent to the rank of $rank\left(\mathbf{R}_{\mathbf{h}}\right)$, therefore we calculate $rank\left(\mathbf{R}_{\mathbf{h}}\right)$ in the following. By the equality of $vec\left(\mathbf{ABC}\right) = \left(\mathbf{C}^T \otimes \mathbf{A}\right) vec\left(\mathbf{B}\right)$, $\mathbf{R}_{\mathbf{h}}$ can be rewritten as

$$\mathbf{R}_{\mathbf{h}} = \left(\mathbf{F}_N^* \otimes \mathbf{F}_N^H\right) \mathbf{\Phi} \left(\mathbf{F}_N^* \otimes \mathbf{F}_N^H\right)^H \tag{4.24}$$

where

$$\mathbf{\Phi} = E\left[\boldsymbol{\eta}\boldsymbol{\eta}^H\right] \tag{4.25}$$

and

$$\boldsymbol{\eta} = vec\left(\left(\mathbf{H}^{(i)} - \mathbf{H}_{ici}^{(i)}\right)^H\right) \tag{4.26}$$

in which $E[\cdot]$ stands for taking expectation and "$*$" stands for the complex conjugate operator. Using the inequality $rank(\mathbf{AB}) \leq \min\left\{rank(\mathbf{A}),\ rank(\mathbf{B})\right\}$ again, then $rank\left(\mathbf{R}_{\mathbf{h}}\right)$ can be upper bounded by

$$rank\left(\mathbf{R}_{\mathbf{h}}\right) \leq \min\left(rank\left(\mathbf{F}_N^* \otimes \mathbf{F}_N^H\right),\ rank\left(\mathbf{\Phi}\right)\right). \tag{4.27}$$

Note that the FFT matrix $\mathbf{F}_N$ is a full rank matrix so we have $rank\left(\mathbf{F}_N^* \otimes \mathbf{F}_N^H\right) = N^2$ by the rank property of kronecker product. Thus $rank\left(\mathbf{R}_{\mathbf{h}}\right)$ is always upper bounded by

$$rank\left(\mathbf{R}_{\mathbf{h}}\right) \leq rank\left(\mathbf{\Phi}\right). \tag{4.28}$$

Now we shall determine $rank\,(\mathbf{\Phi})$ to determine the exact value of the upper bound of $rank\,(\mathbf{R_h})$. Because the channel taps are assumed to be independent, the equality

$$rank\,(\mathbf{\Phi}) = L + 1 \tag{4.29}$$

always holds[†]. Therefore $rank\,(\mathbf{R_h})$ is always upper bounded by the channel order of $(L+1)$, that is,

$$rank\,(\mathbf{R_h}) \leq (L+1)\,. \tag{4.30}$$

The rank of $\mathbf{K}_{eff}^{(i)}(\mathbf{X}, \hat{\mathbf{X}})$ is upper bounded by the minor value of $rank(\mathbf{K}_i(\mathbf{X}, \hat{\mathbf{X}}))$ and $rank\,(\mathbf{R_h})$ which means the upper bound of $rank\left(\mathbf{K}_{eff}^{(i)}(\mathbf{X}, \hat{\mathbf{X}})\right)$ for any one-to-one mapping coding scheme is either $N \times rank(\mathbf{A}_i(\mathbf{X}, \hat{\mathbf{X}}))$ or the channel order $(L+1)$. Therefore we can conclude that $rank\left(\mathbf{K}_{eff}^{(i)}(\mathbf{X}, \hat{\mathbf{X}})\right) \leq (L+1)$ holds for $N \geq (L+1)$, and $rank\left(\mathbf{K}_{eff}^{(i)}(\mathbf{X}, \hat{\mathbf{X}})\right) \leq N \times rank(\mathbf{A}_i(\mathbf{X}, \hat{\mathbf{X}}))$ holds for $N < (L+1)$. Note that the total effective codeword distance matrix $\mathbf{K}_{eff}(\mathbf{X}, \hat{\mathbf{X}})$ is formed by placing 1th to Bth effective distance codeword matrix orderly at its diagonal and by the aforementioned maximum achievable diversity order of any one-to-one mapping coding schemes within one OFDM block, thus the maximum achievable diversity order of any one-to-one mapping coding schemes for consecutive B OFDM blocks is the cumulation of maximum achievable diversity order of each block index, i.e. $rank(\mathbf{K}_{eff}(\mathbf{X}, \hat{\mathbf{X}})) \leq N \times (L+1)$ for $N \geq (L+1)$ and $rank(\mathbf{K}_{eff}(\mathbf{X}, \hat{\mathbf{X}})) \leq N \times \sum_{i=1}^{B} rank(\mathbf{A}_i(\mathbf{X}, \hat{\mathbf{X}}))$ for $N < (L+1)$. □

As shown in Theorem (1.a), when $N \geq (L+1)$, a one-to-one mapping coding scheme within one OFDM block can always achieves $(L+1)$-fold diversity, which means the virtual space-frequency code design based on CP-reduced OFDM systems under symbol-wise Rayleigh fading channels is no longer focusing on maximizing the diversity order since the full diversity can be always achieved only if the codes is designed to be one-to-one mapping. And the maximum achievable diversity order in this setup is consistent with the maximum achievable diversity of conventional SFCs under the case of single transmit and receive antenna. On the other hand, if $N < (L+1)$, the diversity order of a one-to-one mapping

[†] The proof of $rank\,(\mathbf{\Phi}) = (L+1)$ in (4.29) is detailed in Appendix A.

coding scheme within one OFDM block can achieve $N \times rank(\mathbf{A}_i(\mathbf{X}, \hat{\mathbf{X}}))$-fold diversity as shown in Theorem (1.b). From Theorem 1, we propose the design criteria of any one-to-one mapping coding schemes within one OFDM block ($B = 1$, so $\mathbf{K}_{eff}(\mathbf{X}, \hat{\mathbf{X}}) = \mathbf{K}_{eff}^{(1)}(\mathbf{X}, \hat{\mathbf{X}})$) based on CP-reduced OFDM systems under symbol-wise Rayleigh fading channels.

Design Criteria 1: For virtual space-frequency codes within one OFDM symbol based on CP-reduced OFDM systems under symbol-wise Rayleigh fading channels, the design criteria are given as following.

- ► *Rank (diversity) Criterion*:
  - For $N \geq (L+1)$: Design a one-to-one mapping coding scheme, and then such a code can always achieve full diversity of $(L+1)$.
  - For $N < (L+1)$: Maximize the minimum rank of codeword distance matrix $\mathbf{A}_1(\mathbf{X}, \hat{\mathbf{X}})$ over all pairs of distinct OFDM symbols.
- ► *Determinant Criterion*: Maximize th minimum determinant of total effective distance matrix $\mathbf{K}_{eff}(\mathbf{X}, \hat{\mathbf{X}})$ over all pairs of distinct OFDM symbols.

Unlike the diversity criterion of conventional SFCs, Design Criterion 1 raises a different diversity criterion in the case of $N \geq (L+1)$. Since the codes designed to be one-to-one mapping are able to achieved full diversity for $N \geq (L+1)$, the coding redundancy shall be used to enlarge the determinant of $\mathbf{K}_{eff}(\mathbf{X}, \hat{\mathbf{X}})$, rather than the rank of $\mathbf{K}_{eff}(\mathbf{X}, \hat{\mathbf{X}})$, which means the diversity order in this setting is no longer determined by the minimum rank of the codes over all pairs of the distinct OFDM symbols, but determined by the channel order. On the other case of $N < (L+1)$, the rank of $\mathbf{K}_{eff}(\mathbf{X}, \hat{\mathbf{X}})$ is upper bounded by $N \times rank(\mathbf{A}_1(\mathbf{X}, \hat{\mathbf{X}}))$, thus maximize the rank of the codeword distance matrix is still be the main concern of code design which is coincided with the diversity criterion of conventional SFCs. The determinant criterion is consistent in both circumstances, in order to minimize the error probability, the minimum determinant of $\mathbf{K}_{eff}(\mathbf{X}, \hat{\mathbf{X}})$ should be maximized. For a code set of the same achievable diversity order, the code with largest minimum $det(\mathbf{K}_{eff}(\mathbf{X}, \hat{\mathbf{X}}))$

outperforms other codes of the code set, thus we define the effective codeword distance to be the minimum determinant of $\mathbf{K}_{eff}(\mathbf{X}, \hat{\mathbf{X}})$ as a measure to evaluate of the superiority of different coding schemes when they have equivalent achievable diversity. Based on Design Criterion 1, we propose the design critera of any one-to-one mapping virtual space-frequency codes for consecutive B OFDM blocks based on CP-reduced OFDM systems under symbol-wise Rayleigh fading channels.

Design Criteria 2: For virtual space-frequency codes for consecutive B OFDM blocks based on CP-reduced OFDM systems under symbol-wise Rayleigh fading channels, the design criteria are given as following.

▶ *Rank (diversity) Criterion*:

- For $N \geq (L+1)$: Maximize the number of nonzero ith effective codeword distance matrix $\mathbf{K}_{eff}^{(i)}(\mathbf{X}, \hat{\mathbf{X}})$ for each block index over all pairs of distinct consecutive B OFDM blocks.
- For $N < (L+1)$: Maximize the minimum rank of total effective distance matrix $\mathbf{K}_{eff}(\mathbf{X}, \hat{\mathbf{X}})$ over all pairs of distinct consecutive B OFDM symbols.

▶ *Determinant Criterion*: Maximize the minimum determinant of total effective distance matrix $\mathbf{K}_{eff}(\mathbf{X}, \hat{\mathbf{X}})$ over all pairs of distinct consecutive B OFDM symbols.

Because only if the codes have its $\mathbf{K}_{eff}^{(i)}(\mathbf{X}, \hat{\mathbf{X}}) \neq \mathbf{0}$ for $\forall \mathbf{X} \neq \hat{\mathbf{X}}$, the codes achieve full diversity in ith block index. Therefore, we should maximize the number of nonzero effective codeword distance matrix for individual block index in the case of $N \geq (L+1)$, and then the maximum achievable diversity order of the codes is the multiplication of channel order and the number of nonzero ith effective codeword distance matrix out of B OFDM block indexes. For $N > (L+1)$, the diversity gain is maximized by enlarging the rank of $\mathbf{K}_{eff}(\mathbf{X}, \hat{\mathbf{X}})$, to be more exact, it is done by maximizing the rank of $\mathbf{A}_i(\mathbf{X}, \hat{\mathbf{X}})$ for each block index. Moreover, the determinant criterion claims that for the codes own larger effective codeword distance have the larger coding gain under the same diversity order, so the minimum determinant of $\mathbf{K}_{eff}(\mathbf{X}, \hat{\mathbf{X}})$ should be maximized.

Chapter 5

Simulation Results

In addition to theoretical analysis, we also carry out simulations to investigate the performance of virtual space-frequency codes based on CP-reduced OFDM systems by choosing bit-error rate (BER) as our figure of merit, and the BER plots are also used to demonstrate the derived maximum diversity order and design criteria in Chapter 4. The global setting of the simulations and the assumptions are given as following.

Global setting and assumptions:

- Assume the CSI is available at the receiver side, and the ISI component can be canceled perfectly. Timing and frequency synchronizations are assumed to be perfect as well.
- We employ QPSK modulation for all OFDM systems, and use the ML decoder to detect signals.
- There are two types of Rayleigh fading channels used in simulations corresponding to different delay spread, and their power profile are specified as following.
  - ▷ For $L = 1$, the channel power profile is $[\,0.8,\ 0.2\,]$.
  - ▷ For $L = 2$, the channel power profile is $[\,0.642,\ 0.256,\ 0.102\,]$ (SUI-4)[31].

A. Demonstrate the maximum achievable diversity order for $N \geq (L+1)$

From the derivation of achievable diversity bound in Chapter 4, we asserts that the maximum achievable diversity order of virtual space-frequency coding within one OFDM symbol based on CP-reduced OFDM systems for $N \geq (L+1)$ is upper bounded by channel order rather than the rank of effective codeword distance matrix, and this fact disagrees with the common sense of the rank criterion of conventional SFCs. Therefore we apply orthogonal designed SFBCs as the virtual space-frequency codes with their ranks are smaller or larger than the channel order to examine whether the slope of the BER curves is consistent with the channel order or the rank of the SFBCs. There are three orthogonal designed SFBCs applied in the simulations [32][33], and their codeword matrices are given by

$$\mathbf{SFBC}_{4\times 2} = \begin{bmatrix} x_1 & x_2 \\ -x_2^* & x_1 \\ \frac{x_3^*}{\sqrt{2}} & \frac{x_3^*}{\sqrt{2}} \\ \frac{x_3^*}{\sqrt{2}} & \frac{-x_3^*}{\sqrt{2}} \end{bmatrix}, \ \mathbf{SFBC}_{4\times 4} = \begin{bmatrix} x_1 & x_2 & \frac{x_3}{\sqrt{2}} & \frac{x_3}{\sqrt{2}} \\ -x_2^* & x_1 & \frac{x_3}{\sqrt{2}} & \frac{-x_3}{\sqrt{2}} \\ \frac{x_3^*}{\sqrt{2}} & \frac{x_3^*}{\sqrt{2}} & \frac{-x_1-x_1^*+x_2-x_2^*}{\sqrt{2}} & \frac{-x_2-x_2^*+x_1-x_1^*}{\sqrt{2}} \\ \frac{x_3^*}{\sqrt{2}} & \frac{-x_3^*}{\sqrt{2}} & \frac{x_2+x_2^*+x_1-x_1^*}{\sqrt{2}} & \frac{-x_1-x_1^*-x_2+x_2^*}{\sqrt{2}} \end{bmatrix}, \text{ and}$$

$$\mathbf{SFBC}_{8\times 8} = \begin{bmatrix} x_1 & x_2 & x_3 & 0 & x_4 & x_5 & x_6 & 0 \\ -x_2^* & x_1^* & 0 & x_3 & -x_5^* & x_4^* & 0 & x_6 \\ -x_3^* & 0 & x_1^* & -x_2 & -x_6^* & 0 & x_4^* & -x_5 \\ 0 & -x_3^* & x_2^* & x_1 & 0 & -x_6^* & x_5^* & x_4 \\ x_4 & x_5 & x_6 & 0 & x_1 & x_2 & x_3 & 0 \\ -x_5^* & x_4^* & 0 & x_6 & -x_2^* & x_1^* & 0 & x_3 \\ -x_6^* & 0 & x_4^* & -x_5 & -x_3^* & 0 & x_1^* & -x_2 \\ 0 & -x_6^* & x_5^* & x_4 & 0 & -x_3^* & x_2^* & x_1 \end{bmatrix}, \tag{5.1}$$

where x_i, $i = 1, \ldots, 6$, are the constellation points of QPSK modulation. All SFBCs listed in (5.1) can achieve full diversity, so the rank of $\mathbf{SFBC}_{4\times 2}$, $\mathbf{SFBC}_{4\times 4}$, and $\mathbf{SFBC}_{8\times 8}$ are given accordingly as 2, 4, and 8. And the SFBCs used here have one-to-one correspondence, thus applying the SFBCs to the CP-reduced OFDM systems when $N \geq (L+1)$ shall achieve the full diversity of $(L+1)$. Moreover, note that the subcarriers are playing the role of virtual spatial domain, and we assume the channel is static during each SFBC transmission which is quasi-static fading.

For $N = 4$ and $L = 1$, we intentionally apply the SFBCs with rank 2 and 4 based on CP-free OFDM systems with perfect ISI canceler to examine if the achievable diversity order is channel order. As shown in Figure 5.1, the slope of BER curve corresponding to $\mathbf{SFBC}_{4\times 2}$ is consistent to the slope of $\mathbf{SFBC}_{4\times 4}$, and it is about the order of 2. As matter of fact, we can classify the same fact by computing the minimum rank of $\mathbf{K}_{eff}^{(1)}(\mathbf{X}, \hat{\mathbf{X}})$ for all distinct pair of OFDM symbols, and the minimum value of $\mathbf{K}_{eff}^{(1)}(\mathbf{X}, \hat{\mathbf{X}})$ is truly 2 which coincides with the simulation results. Furthermore, the effective codeword distance of $\mathbf{SFBC}_{4\times 4}$ (the minimum $det\left(\mathbf{K}_{eff}^{(1)}(\mathbf{X}, \hat{\mathbf{X}})\right)$ of $\mathbf{SFBC}_{4\times 4}$) is 0.1584 which is larger than the effective codeword distance of $\mathbf{SFBC}_{4\times 2}$ (0.048), consequently, $\mathbf{SFBC}_{4\times 4}$ outperforms $\mathbf{SFBC}_{4\times 2}$ as shown in Figure 5.1. And we note that the uncoded CP-free OFDM systems have better performance than the CP-sufficient (which means the CP length is equal to the maximum delay spread) OFDM systems, only if the CP-free OFDM systems have the perfect ISI canceler and the receiver considers the ICI responses of all subcarriers to decode signals. The similar fact can be observed when the subcarrier number is enlarge to 8 in Figure 5.2, the SFBC applied here has an extremely large rank of 8, but the diversity order of SFBC coded CP-free OFDM systems still follows the channel order of 2.

As given in Figure 5.3, the achievable diversity order when $N = 4$ and $L = 2$ corresponding to $\mathbf{SFBC}_{4\times 2}$ and $\mathbf{SFBC}_{4\times 4}$ are both being the maximum delay spread instead of the rank of SFBCs again. It may be hard to accept that $\mathbf{SFBC}_{4\times 2}$ coded CP-free OFDM systems can achieve diversity order of 3. Once we analyze the upper bound of $rank\left(\mathbf{K}_{eff}^{(1)}(\mathbf{X}, \hat{\mathbf{X}})\right)$, it may be believable. The rank of $\mathbf{SFBC}_{4\times 2}$ is 2, therefore the rank of $\mathbf{K}_i(\mathbf{X}, \hat{\mathbf{X}})$ is $2N$; meanwhile, the rank of $\mathbf{R_h}$ is proved to be channel order. Recall (4.22), as a result, the rank of effective codeword matrix is upper bounded by the channel order which matches the slope of the BER curve of $\mathbf{SFBC}_{4\times 2}$, and this fact also can be proved by means of computing the minimum value of $rank\left(\mathbf{K}_{eff}^{(1)}(\mathbf{X}, \hat{\mathbf{X}})\right)$ over all pairs of OFDM symbols. We enlarge the subcarrier number as 8 to simulate under the same multipath channel setting, and the BER plots in Figure 5.4 confirm that the achievable diversity is the maximum delay spread in this setup, too. At this point, we have verified the achievable diversity order of virtual space-frequency coding within one OFDM block based on CP-free OFDM systems by computer simulations, and also classified that if there are several coding schemes have equivalent achievable diversity, the codes with larger effective codeword distance have the better error performance than the codes with minor effective codeword distance.

B. Apply the virtual space-frequency coding to the convolutional coded CP-reduced OFDM

It is clarified by computer simulations in previous section that only if the the virtual space-frequency codes based on CP-reduced OFDM systems are designed to be one-to-one mapping, the maximum diversity order of the codes are irrelevant to the rank of their codeword different matrices. Therefore once the codes have one-to-one correspondence, in oder to enhance error performance the coding redundancy should be utilized to maximize the effective codeword distance over all pairs of distinct OFDM symbols rather than maximize the rank of the codeword distance matrix $\mathbf{A}_i(\mathbf{X}, \hat{\mathbf{X}})$. Note that **SFBC**s are ineffective in enlarging the effective codeword distance due to diversity gain is the main concern of **SFBC**s. Thus we employ the traditional error control coding as an more effective alternative. For above reasons, we choose the convolutional codes with known d_{free} as an easy way to enlarge the effective codeword distance. In this section, we apply the optimal convolutional codes [34] with the same memory order but different code rates to CP-free OFDM systems, and compare their BER performance to convolutional coded CP-sufficient OFDM systems. The optimal convolutional codes used in simulations with their generator sequences and properties are listed as Compare with the CP appended OFDM systems, the CP-free OFDM systems have extra redundancy saved from the unused guard interval, and we can make the additional rate utilized by more powerful convolutional codes to further enhance the error performance meanwhile preserve original total throughput of the communication systems.

Abbreviation	rate	memory	$G(D)$	d_{free}
CC_1	1/2	6	[117, 155]	10
CC_2	1/3	6	[117, 127, 155]	15

.

Table 5.1: Table of optimal convolutional codes

We also execute computer simulations to verify the aforementioned idea. In addition to the global setting and assumptions, the computer simulation in Simulation B are executed under the extra setting listed as below:

Simulation setting in Simulation B:

- The channel is assumed to be static within each OFDM block but varying among blocks (symbol-wise fading).
- The Viterbi decoder of CP-free OFDM systems perform exhaustive search for each OFDM block to compute $\left\|\mathbf{R}_i - \mathbf{H}_{eq}^{(i)}\left(e^{jw}\right)\right\|_F^2$ as the path metric, and then execute the Viterbi algorithm from block to block.
- There are three convolutional coded OFDM systems with or without CP in every BER plots. We specify the three systems along with their abbreviations, and the abbreviations are used in the BER plots and discussions for the sake of simplicity.
  - ▷ CC_1CP-sufficient OFDM: The convolutional coded CP-sufficient OFDM systems where the convolutional code used here is the CC_1 code of Table 5.1.
  - ▷ CC_1CP-free OFDM with perfect ISI canceler: The coded CP-free OFDM systems equipped with perfect ISI canceler where the convolutional code used here is the CC_1 code of Table 5.1.
  - ▷ CC_2CP-free OFDM with perfect ISI canceler: The coded CP-free OFDM systems equipped with perfect ISI canceler where the convolutional code used here is the CC_2 code of Table 5.1.

Note that all the systems based on CP-sufficient OFDM are lined with blue dash line, and all the systems based on CP-free OFDM are lined with red solid line throughout this section. For $N = 4$, $L = 1$, and $B = 60$, we compare the BER of convolutional coded CP-sufficient and CP-free OFDM systems with perfect ISI canceler by applying the CC_1 code in Table 5.1 to both systems, and it it shown in Figure 5.5 that CC_1CP-free OFDM with perfect ISI canceler outperforms CC_1CP-sufficient OFDM about 4.7dB at the BER of 10^{-5}, yet CC_1CP-free OFDM with perfect ISI canceler even has higher code rate than CC_1CP-sufficient OFDM. To compare the BER performance with CC_1CP-sufficient OFDM under the same rate, we apply the CC_2 code of Table 5.1 to the CP-free OFDM system with perfect ISI canceler (CC_2CP-free OFDM with perfect ISI canceler), and we observe that CC_2CP-free OFDM with perfect ISI canceler is about 7.5dB better than CC_1CP-free OFDM with perfect ISI canceler. Note that CC_2CP-free OFDM with perfect ISI canceler and CC_1CP-sufficient OFDM have the same throughput of rate 1/3, therefore it is worthwhile not to use any guard interval and spend the redundancy saved from CP on a more powerful convolutional code. From the BER comparison of CC_2CP-free OFDM with perfect ISI canceler and CC_1CP-sufficient OFDM provides a method to enhance the BER performance significantly while preserving original system throughput. Furthermore, since the convolutional codes have one-to-one correspondence, the achievable diversity of CC_1CP-free OFDM with perfect ISI canceler and CC_2CP-free OFDM with perfect ISI canceler are supposed to be the channel order. As shown in Figure 5.5, the slope of CC_1CP-free OFDM with perfect ISI canceler and CC_2CP-free OFDM with perfect ISI canceler are about 2, just as the theoretical value.

For the same numbers of subcarrier and encoding OFDM blocks, we also execute the simulations under the multipath channel of channel order 3, and using the channel power profile of SUI-4 to generate the Rayleigh fading gains as shown in Figure 5.6. We observe that CC_2CP-free OFDM with perfect ISI canceler and CC_1CP-free OFDM with perfect ISI canceler have the same diversity gain from Figure 5.6, and the slope of CC_2CP-free OFDM with perfect ISI canceler and CC_1CP-free OFDM with perfect ISI canceler descend much sharper than CC_1CP-sufficient OFDM does at the high SNR regions. It is deserved to mention that CC_2CP-free OFDM with perfect ISI canceler and CC_1CP-free OFDM with perfect ISI canceler both have higher rate than CC_1CP-sufficient OFDM, yet they outperform CC_1CP-sufficient OFDM for all the SNR regions. CC_1CP-free OFDM with perfect ISI canceler has about 6.2dB gain and CC_2CP-free with perfect ISI canceler has about 9.2dB gain better than CC_1CP-sufficient at the BER of 10^{-5}. Furthermore, we observe in Figure 5.5 and Figure 5.6 that the slope of CC_1CP-sufficient OFDM is irrelevant to different maximum delay spread, and both the diversity gain of CC_2CP-free OFDM with perfect ISI canceler and CC_1CP-free OFDM with perfect ISI canceler follow the channel order, which means using convolutional codes as our virtual space-frequency codes for CP-reduced OFDM systems can truly gain the frequency diversity from the ICI.

Since we view the subcarriers as the virtual antennas, the ICI component is increasing with the subcarrier numbers. Therefore we perform all the communication systems once again with the number of subcarrier and encoding OFDM block setting respectively as 8 and 30 to see if the virtual space-frequency coding on CP-reduced OFDM systems equipped with perfect ISI canceler have the better performance than the conventional convolutional coded CP-sufficient OFDM systems. As shown in Figure 5.7 and Figure 5.8, CC_1CP-free OFDM with perfect ISI canceler still outperforms CC_1CP-sufficient OFDM for all SNR regions, meanwhile CC_1CP-free with perfect ISI canceler OFDM has higher throughput than CC_1CP-sufficient OFDM. At this point, the redundancy saved from CP provides a flexibility of performance tradeoff between error performance and throughput, we can either replace the CP-sufficient OFDM systems of conventional convolutional coded OFDM by CP-free OFDM systems to increase the spectral efficiency and BER performance simultaneously, or we can turn the redundancy saved from CP to a more power convolutional codes to improve the error performance remarkably while the communication systems have compareable throughput.

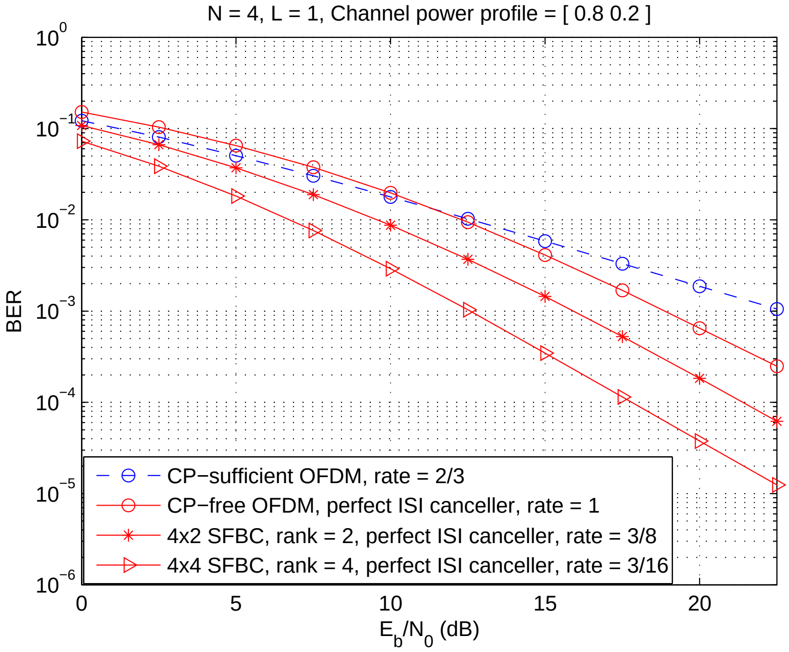


Figure 5.1: BER of CP-sufficient OFDM, CP-free OFDM with perfect ISI canceler, and SFBC coded CP-free OFDM equipped with perfect ISI canceler corresponding to $\mathbf{SFBC}_{4\times 2}$ and $\mathbf{SFBC}_{4\times 4}$ for $N = 4,\ L = 1$.

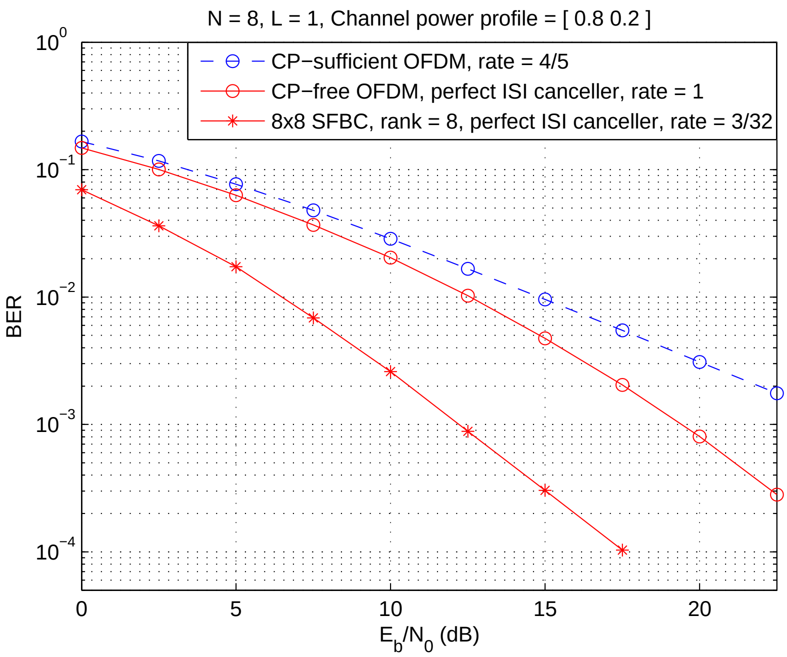


Figure 5.2: BER of CP-sufficient OFDM, CP-free OFDM with perfect ISI canceler, and $\mathbf{SFBC}_{8\times 8}$ coded CP-free OFDM with perfect ISI canceler for $N = 8,\ L = 1$.

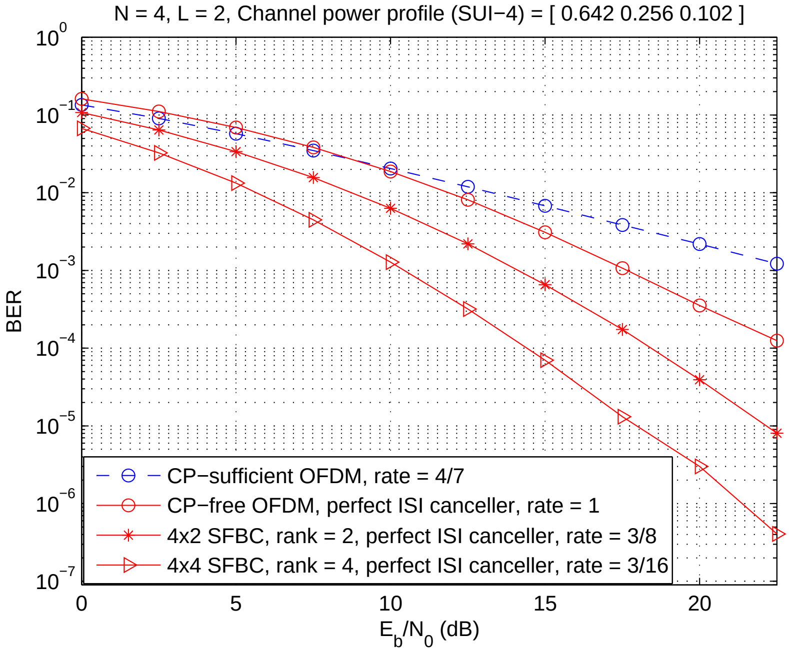


Figure 5.3: BER of CP-sufficient OFDM, CP-free OFDM with perfect ISI canceler, and SFBC coded CP-free OFDM equipped with perfect ISI canceler corresponding to $\mathbf{SFBC}_{4\times2}$ and $\mathbf{SFBC}_{4\times4}$ for $N = 4,\ L = 2$.

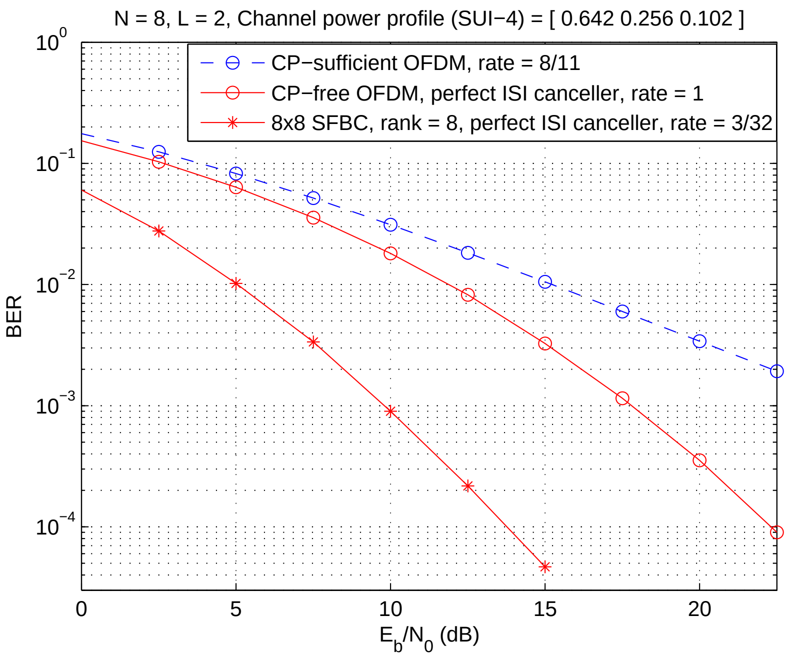


Figure 5.4: BER of CP-sufficient OFDM, CP-free OFDM with perfect ISI canceler, and $\mathbf{SFBC}_{8\times8}$ coded CP-free OFDM with perfect ISI canceler for $N = 8,\ L = 2$.

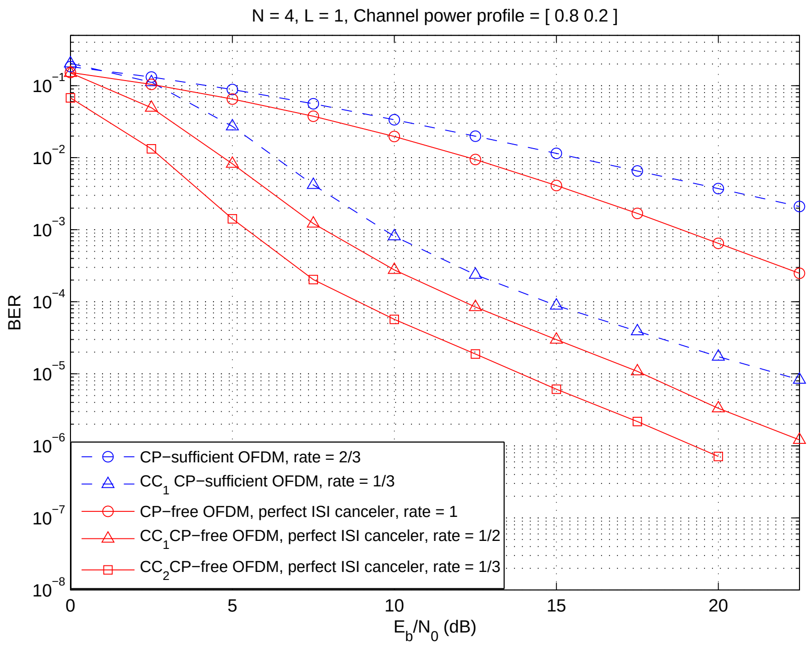


Figure 5.5: BER of CP-sufficient OFDM, CC_1CP-sufficient OFDM, CP-free OFDM, CC_1CP-free OFDM with perfect ISI canceler, and CC_2CP-free OFDM with perfect ISI canceler for N = 4, L = 1.

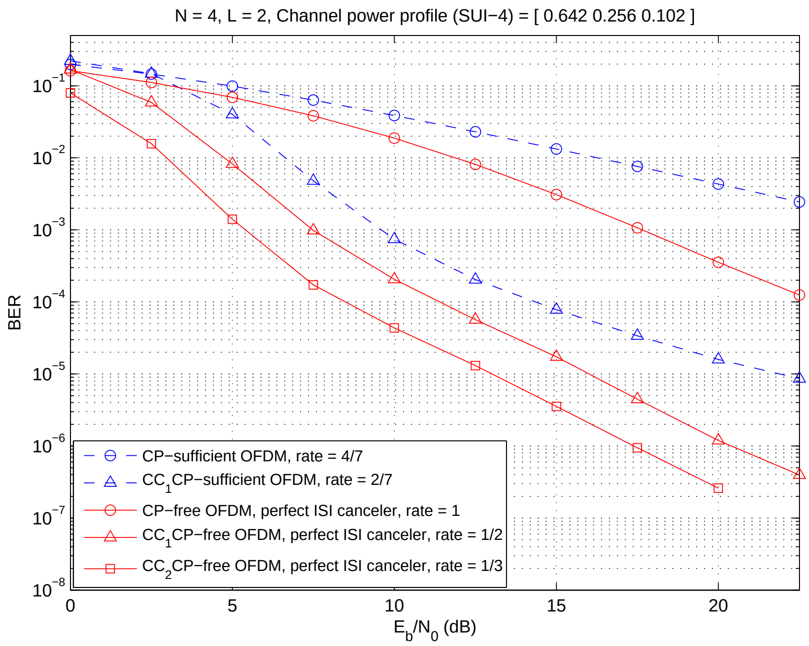


Figure 5.6: BER of CP-sufficient OFDM, CC_1CP-sufficient OFDM, CP-free OFDM, CC_1CP-free OFDM with perfect ISI canceler, and CC_2CP-free OFDM with perfect ISI canceler for N = 4, L = 2.

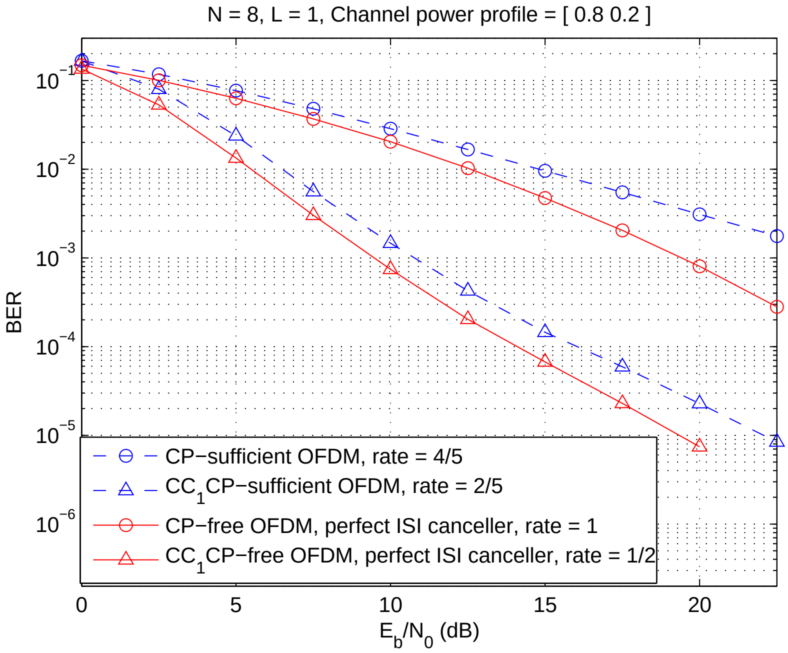


Figure 5.7: BER of CP-sufficient OFDM, CC_1CP-sufficient OFDM, CP-free OFDM, CC_1CP-free OFDM with perfect ISI canceler, and CC_2CP-free OFDM with perfect ISI canceler for N = 8, L = 1.

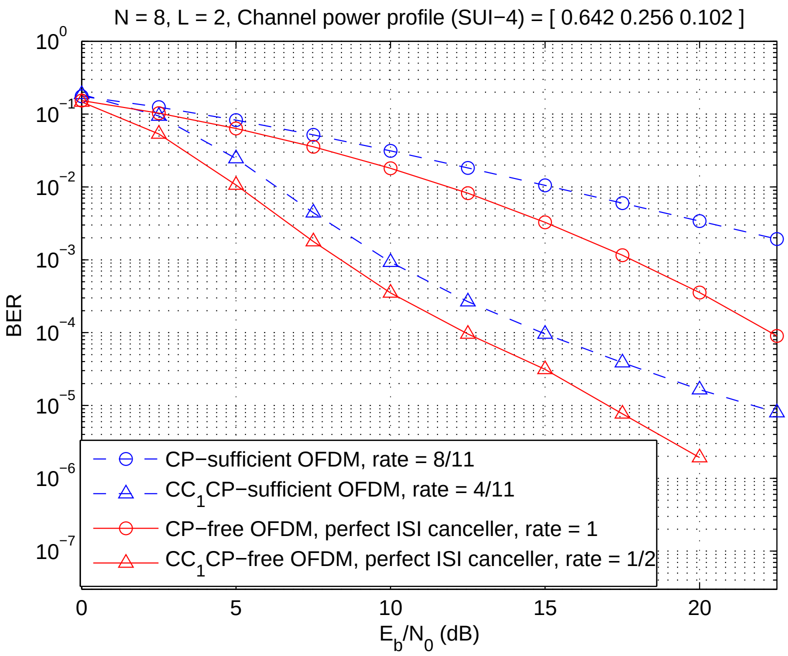


Figure 5.8: BER of CP-sufficient OFDM, CC_1CP-sufficient OFDM, CP-free OFDM, CC_1CP-free OFDM with perfect ISI canceler, and CC_2CP-free OFDM with perfect ISI canceler for N = 8, L = 2.

Chapter 6

Conclusions and Future Works

In this thesis, we built up the mathematical formulations and derived the PEP by the aspects of viewing subcarriers as virtual antennas based on CP-reduced OFDM systems under virtual MIMO channels of symbol-wise Rayleigh fading channels. From the PEP, the maximum achievable diversity order with respect to arbitrary encoded OFDM block numbers are given along with the proofs. The main discover of our study is that only if the virtual space-frequency codes for coding within one OFDM block is designed to have one-to-one correspondence, the maximum achievable diversity order of such codes for $N \geq (L+1)$ will be upper bounded by the maximum delay spread of the channel. The subcarrier number N is chosen to be larger than the channel order $(L+1)$ in conventional use, that is, the virtual space-frequency codes based on CP-reduced OFDM systems can always achieve full diversity for most of the circumstances as long as the designed codes are one-to-one mapping. Based on the maximum achievable diversity bound, we proposed the design criteria for different encoded OFDM block numbers as well. A good virtual space-frequency codes for coding within one OFDM block should be one-to-one mapping and its minimum determinant of effective codeword distance matrix over all distinct OFDM pairs should be maximized. For coding across B OFDM blocks, the virtual space-frequency codes who has the nonzero effective codeword distance matrix for longer time indexes achieve higher diversity order, and the minimum determinant of the total effective distance matrix over all pairs of distinct consecutive B OFDM blocks should be maximized to have better coding gain. In addition to theoretical analysis, the derived diversity bound and the proposed design criteria are verified by the BER simulations along with the calculation of the minimum effective rank and minimum effective distance in Simulation A. We applied the convolutional codes (CC_2 and CC_2 of Table 5.1) as the virtual space-frequency codes to CP-free OFDM in simulation B (CC_2CP-free OFDM with perfect ISI canceler and CC_1CP-free OFDM with perfect ISI canceler), and it not only outperforms the conventional convolutional coded CP-sufficient OFDM (CC_1CP-sufficient OFDM) for all SNR regions, but also has higher rate. It is worthwhile to note that the diversity gain of CC_2CP-free OFDM with perfect ISI canceler and CC_1CP-free OFDM with perfect ISI canceler are both identical and higher than the diversity gain of CC_1CP-sufficient OFDM, which means that the virtual space-frequency coded CP-free OFDM can actually gain frequency diversity from ICI even though the OFDM systems are only equipped with single antenna. Moreover, from the BER comparisons of CC_2CP-free OFDM with perfect ISI canceler and CC_1CP-sufficient OFDM motivates a simple way to enhance the BER performance considerably under the comparable throughput: for a convolutional coded OFDM system, it is more profitable not to use any CP and turn the redundancy saved from unused CP to fill the rate loss from using a more powerful and lower rate convolutional code (e.g. CC_2 code in Simulation B) to improve the BER performance meanwhile preserve the new throughput to be comparable with the original one.

The main disadvantage of virtual space-frequency codes for CP-reduced OFDM is the remarkable complexity. Due to the concepts of viewing subcarriers as virtual antennas, the ML receiver has to consider the frequency channel response of all subcarriers at once to decode signals. Therefore the exhaustive search number of the ML receiver is increasing exponentially by subcarrier number, and that's why the subcarrier numbers used in computer simulations are not so practical. Fortunately, there a suboptimal decoding algorithm to decrease the search number during decoding, and the suboptimal decoding algorithm are introduced as following. Using the power profile of SUI-4, the normalized ICI waveform of the observed subcarrier for $N = 16, 32, 64,$ and128 are plotted in Figure 6.1. We can observe from Figure 6.1 that the ICI components of observed subcarrier only interfere the nearby subcarriers badly, whereas the ICI components have little influence on the subcarriers who are apart from the observed subcarrier. Motivated by the observations, we figure out that whenever the receiver is about to decode a specific subcarrier, say ith subcarrier, the receiver can only consider the ICI components of the neighboring subcarriers of ith subcarrier to be a suboptimal decoding alternative. Compare with the ML decoding algorithm, the suboptimal decoding algorithm can reduce the search number of decoding one OFDM block from 4^N to $N \times 4^P$ where P is the number of considered adjacent subcarriers of a decoded subcarrier. Replacing the ML receiver by the suboptimal decoder and with a careful chosen P, then we can operate the virtual space-frequency coded CP-reduced OFDM systems at a practical subcarrier number. Besides the problems of complexity, the effects of imperfect ISI cancellation on the performance of virtual space-frequency coded CP-reduced OFDM is an important issue to be studied in the following investigations.

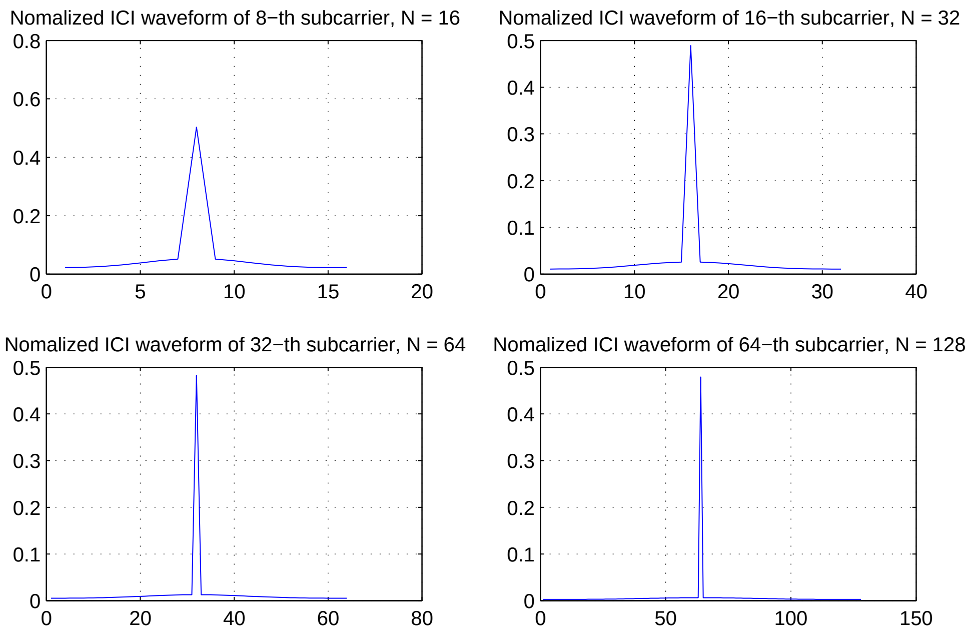


Figure 6.1: The normalized ICI waveform of observed subcarrier for the subcarrier numbers chosen as 16, 32, 64 and 128.

Appendix A: Proof $rank\left(\mathbf{\Phi}\right) = L + 1$

▶ Assume the channel taps are independent to each other, proof

$$rank\left(\mathbf{\Phi}\right) = L + 1 \tag{A.1}$$

where

$$\mathbf{\Phi} = E\left[\boldsymbol{\eta}\boldsymbol{\eta}^{H}\right] \tag{A.2}$$

and

$$\boldsymbol{\eta} = vec\left(\left(\mathbf{H}^{(i)} - \mathbf{H}_{ici}^{(i)}\right)^{H}\right) \tag{A.3}$$

Proof. By the definitions of $\mathbf{H}^{(i)}$ and $\mathbf{H}_{ici}^{(i)}$, the general form of the vector $\boldsymbol{\eta}$ is given by

$$\boldsymbol{\eta} = \left[\left(h_0^{(i)}\ \underbrace{0 \cdots 0}_{N-1}\right)\ \left(h_1^{(i)}\ h_0^{(i)}\ \underbrace{0 \cdots 0}_{N-2}\right) \cdots \left(\underbrace{0 \cdots 0}_{N-L}\ h_L^{(i)}\ h_{L-1}^{(i)} \cdots h_0^{(i)}\right)\ \left(\underbrace{0 \cdots 0}_{N-L+1}\ h_L^{(i)}\ h_{L-1}^{(i)} \cdots h_0^{(i)}\right)\right]^{H}. \tag{A.4}$$

We evaluate $\mathbf{\Phi}$ by applying the independent assumption of channel taps. From the derivation result of $\mathbf{\Phi}$, we observe that $\mathbf{\Phi}$ is a diagonal matrix with only $(L+1)$ linearly independent column vectors[‡]. By the aforementioned observations of $\mathbf{\Phi}$, therefore the rank of $\mathbf{\Phi}$ is $(L+1)$. □

‡ The observations can be seem by a direct computation of $\mathbf{\Phi}$.

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