建構一個編碼率二之差分空時碼
On the Construction of a Rate-Two Differential Space-Time Code

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建構一個編碼率二之差分空時碼

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摘　　要

本篇論文提出了一個編碼率二之差分空時碼，並且可應用於四根與八根傳送端天線數。除此之外，我們也推導出成對錯誤機率(pairwise error probability，簡稱 PEP) 的上界，它提供了一個針對差分空時碼可達到之多樣性增益的理論驗證方法。我們的數學結果顯示出差分空時碼可得到的多樣性增益等同於距離矩陣的秩數與接收端天線個數 M 的乘積。透過使用成對錯誤機率，我們提出一個用來設計在這篇論文所提出之編碼率二，分別使用四根以及八根傳送端天線之差分空時碼之秩準則。從實驗模擬也顯示出我們所提出編碼率二之架構與透過成對錯誤機率所推導出來的分析結果符合，並且這些分別使用四根與八根傳送端天線之編碼率二之差分空時碼皆可得到多樣性增益四。

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ABSTRACT

A rate-two differential space-time code (DSTC) is proposed in this work and is applicable to four and eight transmit antennas. Moreover, the upper bound of the pairwise error probability (PEP) is also investigated herein, providing a theoretical justification for the achievable diversity order of the proposed DSTC scheme. With the assumption of a full rank data matrix, the derivation results show that the diversity order equals to the rank of the distance matrix multiplied by the number of receive antennas M. Based on this PEP expression, we provide a rank criterion on the design of the rate-two DSTC. The simulation results match the analysis obtained from the PEP, and achieve the diversity order of four for four and eight transmit antennas respectively.

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建勝

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Chapter 1

Introduction

With the fast development of wireless communication techniques, the demand for reliable high data rate transmission in fading channel increases significantly. Space-time block code (STBC) provides an effective approach for exploiting the diversity advantage of multi-input and multi-output (MIMO) systems [1], [2], [3]. While, for STBC decoding, channel state information (CSI) is often required at the receiver. In practice, it is estimated with training symbols, and thus may not be accurate enough in fast fading channels. In addition, the training overhead for channel estimation will also sacrifice the effective throughput, especially when the number of transmit antennas becomes large. Based on the reasons, it is helpful to consider a noncoherent and differential space-time coded system which does not require CSI at the receiver.

Various differential space-time codes (DSTC) have been proposed before. For two transmit antennas, a DSTC based on orthogonal designs was proposed in [4] for slow fading channels. It features a simple encoding and decoding algorithms, and the performance is 3dB worse than the coherent STBC at high signal to noise ratio (SNR). In the same year, the results provided in [5] show that the scheme in [4] is optimal among its unitary group codes. In addition, it also provides some optimal unitary group codes for different rates. For more than two transmit antennas, a differential unitary space-time code (DUSTC) was introduced in [6]. Subsequently, a DUSTC based on unitary group codes was presented in [5]. Both schemes have simple structures at the transmitter due to the group codes. Although they can be applied to any number of transmit and receive antennas based on generalized orthogonal designs; however, a high decoding complexity is inevitable. In [7], a DSTC was extended to multiple transmit antennas based on generalized orthogonal designs, but it was limited in STBC structure. Furthermore, the transmission rate is only 1/2, and the decoding complexity is still high. In the subsequent works, based on a simple orthogonal space-time code (OSTC) [8], a DSTC is proposed in [9], which achieves the rate 3/4 for four transmit antennas without the structures of group codes. A rate-one quasi-orthogonal space-time block code (QOSTBC) for four transmit antennas were presented in [10]. The corresponding differential quasi-orthogonal space-time code (DQOSTC) based on the above QOSTBC was introduced in [11]. Compared with the differential orthogonal space-time code (DOSTC) with maximum achievable rate equal to 3/4, it can achieve full rate and full diversity simultaneously by means of constellation rotations (CRs) [12], [13]. Afterward, a single-symbol decodable DSTC, which can provide full transmit diversity was presented in [14]. Particularly, a special idea of dispersion matrices, which can reduce the complexity significantly was introduced. In [15], a rate-2 DSTC with maximin-likelihood (ML) receiver for four transmit antennas was proposed.

It is undeniable that a reliable fixed or mobile wireless transmission at high transmission rates will be an important issue for future communication systems. As mentioned above, DOSTC can provide full transmit diversity as well as lower decoding complexity, but it has a maximum transmission rate 3/4 when more than two transmit antennas are considered. DQOSTC with constellation rotations can achieve full diversity and full rate at the same time. However, these schemes can only provide the transmission rate at most one. Compared with the rich research results for rate less than or equal to one, high-rate DSTC is rather less investigated. On the other hand, spatial multiplexing (SM) can provide the highest possible achievable rate but has no transmit diversity advantage. Furthermore, it requires the number of received antennas greater than or equal to the number of transmit antennas [16], but there is likely to be asymmetry between downlinks and uplinks. Therefore, it is valuable to design a generalized high-rate DSTC with transmission rate greater than one, which is extendable to various number of transmit antennas.

In this thesis, a rate-two DSTC for four transmit antennas is proposed to achieve better diversity order, and it is extendable to eight transmit antennas. On the other hand, based on the approach in [17], the upper bound of the pairwise error probability (PEP) of the DSTC is derived, which provides a theoretical justification for the achievable diversity of the DSTC scheme. Based on the PEP expression, we provide a rank criterion on the design of the proposed rate-two DSTC, and the simulation results coincide with the noncoherent rate-diversity tradeoff [18] for the case of four transmit and two receive antennas. The thesis is organized as follows. In **Chapter 2**, we will briefly describe the basic system model. The differential encoding and decoding algorithm for the DSTC is introduced in **Chapter 3**. In **Chapter 4**, an upper bound of the PEP of the DSTC is derived. Based on the PEP expression, we provide a rank criterion on the design of the rate-two DSTC in this chapter. Based on the results in **Chapter 4**, a rate-one and rate-two DSTCs with more than two transmit antennas is presented in **Chapter 5**.

Chapter 2

System Model

A wireless communication system, with N transmit antennas and M receive antennas over flat Rayleigh fading channels as illustrated in Figure 2.1. Each receive antenna responds to each transmit antenna through a statistically independent fading coefficient. We firstly define k, T as the block index and block length of the system, in which the channel coefficients are fixed during the T symbol periods.

The signal that arrives at the *m-th* receive antenna is a superposition of the fading transmitted signals and noise. At each receive antenna, a demodulator samples the output of the waveform synchronously and produces decision statistics in each symbol interval. Thus, the relationship between the decision statistics and the transmitted signals is given by

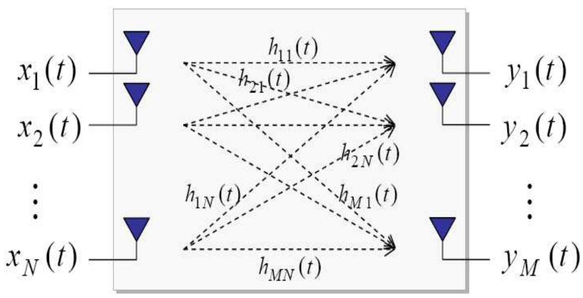

Figure 2.1: Block diagram of a MIMO system

$$y_m(t) = \sum_{n=1}^{N} \sqrt{\rho} h_{mn}(t)\ x_n(t) + n_m(t) \qquad m = 1, ..., M,\ t = 1, ..., T \tag{2.1}$$

where $\rho = Es/\sigma_n^2$ is the ratio of the average received signal energy per symbol period at each antenna to the noise power spectral density (Signal to Noise Ratio, SNR), h_{mn} is the complex fading coefficient from n-th transmit antenna to m-th receive antenna, and n_{mt} is an independent identically distributed (i.i.d.) complex Gaussian noise with zero mean and unit variance with respect to both m and t. The transmit symbols are modulated and differentially space-time coded in blocks. Each DSTC matrix is of dimension $N \times T$ denoted by X_k, and T is the block length of the system in which the channel coefficients are invariant. The $M \times T$ received signal Y_k in block k can be expressed as

$$Y_k = \sqrt{\rho} H_k X_k + N_k \tag{2.2}$$

H_k is an $M \times N$ channel matrix whose entry h_{mn} for $m \in [1, M]$ and $n \in [1, N]$ is complex Gaussian distributed. N_k is an $M \times T$ noise matrix which contains the samples of independent complex Gaussian random variables with zero mean and variance equal to one denoted by $\mathcal{CN}(0,1)$. For the sake of simplicity, at the beginning of the transmission, we assume that the initial DSTC matrix X_0 is an identity matrix I_N. To maintain a constant average power, X_k should meet the power constraint rule:

$$E\ \{\ ||\ X_k\ ||_F^2\ \} = E\ \{ \sum_{n=1}^{N} \sum_{t=1}^{T} |x_{nt}|^2\ \} = T \cdot \frac{L}{T} = L \tag{2.3}$$

where $|| \cdot ||_F$ denotes the Frobenius norm of a matrix, and L denotes the number of modulated symbols to be sent in a block. In other words, we define the transmission rate R of the DSTC as L/T.

Chapter 3

Differential Space-Time Code

3.1 Differential Encoding

One way to communicate with unknown CSI is the DSTC, which can be viewed as a higher-dimensional extension of the conventional differential phase-shift keying (DPSK) mostly applied in the signal-input single-output (SISO) systems. Figure 3.1 shows the block diagram of the differential encoding process. In the *k-th* block, the transmitter determines the transmit matrix X_k by the data matrix S_k generated independently in each block and previously transmit matrix X_{k-1} depending on the following differential

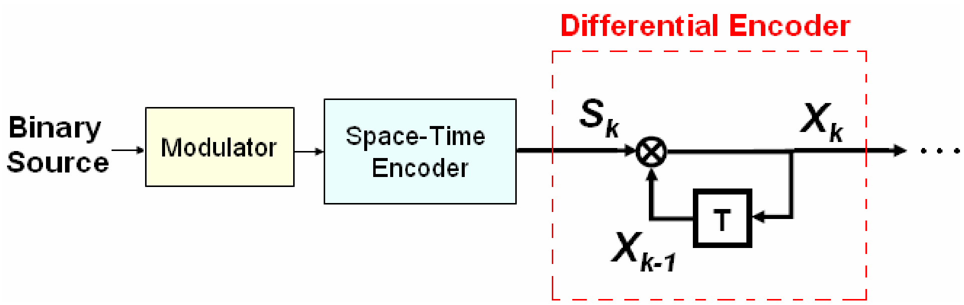

Figure 3.1: Differential Encoding Process

encoding process:

$$X_k = \frac{1}{\alpha_k} X_{k-1} S_k \tag{3.1}$$

We define S_k as a rate-(L/T), $N \times T$ data matrix which contains L modulated symbols during T symbol periods. In other words, the transmission rate R of the whole system is L/T. At the beginning of the transmission (i.e., k=1), we assume that the transmitter sends the initial transmit matrix X_0 to be an identity matrix I_N. To assure that the transmitted signal will not vanish or blow up to infinity, we use the α_k as the normalization factor to keep the mentioned average power constraint.

3.2 Differential Decoding

At the receiver, we assume that the channel coefficients remain fixed at two consecutive time blocks, that is, $H_k \approx H_{k-1} = H$, then according to (2.2), the received matrices at time block k and $k-1$ are respectively given by

$$Y_k = \sqrt{\rho} H X_k + N_k \tag{3.2}$$

$$Y_{k-1} = \sqrt{\rho} H X_{k-1} + N_{k-1} \tag{3.3}$$

From (3.1), (3.3) and (3.4), we can rewrite Y_k in the k-th block as

$$\begin{aligned} Y_k &= \sqrt{\rho} H X_k + N_k \\ &= \sqrt{\rho} H (\frac{1}{\alpha_k} X_{k-1} S_k) + N_k \\ &= \frac{1}{\alpha_k} (\sqrt{\rho} H X_{k-1} + N_{k-1}) S_k + (N_k - \frac{1}{\alpha_k} N_{k-1} S_k) \\ &= \frac{1}{\alpha_k} Y_{k-1} S_k + W_k \end{aligned} \tag{3.4}$$

where $W_k = N_k - \frac{1}{\alpha_k} N_{k-1} S_k$. Note that the channel matrix H disappears in (3.4), which implies that, as long as the channel is approximately constant over two consecutive time blocks, it is possible for DSTC to decode without knowing the CSI at the receiver.

When the data matrix S_k is unitary, the equivalent Gaussian noise W_k in (3.4) is statistically independent of S_k. Thus, the maximum-likelihood receiver for DUSTC can be expressed as

$$\begin{aligned} \hat{S}_k &= arg \min_S ||Y_k - \frac{1}{\alpha_k} Y_{k-1} S||^2 \\ &= arg \min_S tr\{(Y_k - \frac{1}{\alpha_k} Y_{k-1} S)(Y_k - \frac{1}{\alpha_k} Y_{k-1} S)^H\} \end{aligned} \tag{3.5}$$

where the operator $tr\{\cdot\}$ denotes the matrix trace. However, for non-unitary data matrix, the equivalent noise W_k becomes an colored Gaussian noise with covariance matrix C_{ov} as

$$\begin{aligned} C_{ov} &= E[\ W_k^H W_k\ ] \\ &= E[\ (N_k - \frac{1}{\alpha_k} N_{k-1} S_k)^H (N_k - \frac{1}{\alpha_k} N_{k-1} S_k)\ ] \\ &= \sigma_n^2 I_N + (\frac{1}{\alpha_k})^2 S_k^H S_k \end{aligned} \tag{3.6}$$

and thus the ML receiver can be rewritten as

$$\begin{aligned} \hat{S}_k = arg \min_S \{\ tr[-\frac{1}{2}(Y_k - \frac{1}{\alpha_k} Y_{k-1} S) \cdot \\ C_{ov}^{-1} \cdot (Y_k - \frac{1}{\alpha_k} Y_{k-1} S)^H]\ \} \end{aligned} \tag{3.7}$$

Since the ML receiver in (3.7) is complicated to analyze. In the following chapter, the PEP of the DSTC using ML receiver in (3.6) will be applied to analyze the diversity order, and then we will have some rules for the design of the DSTC. We note that for non-unitary condition, the results may not be optimal by using the sub-optimal receiver.

Chapter 4

Diversity Analysis of Differential Space-Time Code

4.1 Diversity

Spatial diversity and spatial multiplexing are the two reasons why MIMO systems offer better performance and higher throughput. Spatial multiplexing involves transmission of several independent data streams over different transmit antennas simultaneously to increase the throughput while spatial diversity sends copies of the same information over different transmit and receive antennas to avoid suffering deep fading simultaneously. Multipath fading causes severe degradation of signals in wireless communication systems. MIMO systems offer a spectacular approach for combating fading due to multipath propagation, scattering, refraction, reflection, etc by means of diversity. Diversity mitigates the effect of fading and hence allows higher level modulation schemes that increase the capacity and greatly reduce bit error rate (BER). We give an example to explain the basic idea of diversity below.

Consider a SISO system, and the output signal can be expressed as

$$y = \sqrt{\rho} h x + n \tag{4.1}$$

where h is the Rayleigh flat-fading channel gain, $\rho = E_s/\sigma_n^2$ is the SNR, and n is the noise at the receiver, which is Gaussian distributed with zero mean and half variance per real dimension. Therefore, the signal to noise ratio (SNR) at the receiver is $\rho|h|^2$. Since h is Rayleigh distributed, $|h|^2$ is exponentially distributed with probability density function (pdf)

$$p(h) = e^{-h} \qquad h > 0 \tag{4.2}$$

The probability that the received SNR is less than a small value ϵ is,

$$p(\rho|h|^2 < \epsilon) = p(|h|^2 < \frac{\epsilon}{\rho}) = 1 - e^{-\frac{\epsilon}{\rho}} \tag{4.3}$$

As the transmit power is very large ($\rho \to \infty$), (4.3) can be approximated to

$$p(\rho|h|^2 < \epsilon) \approx \frac{\epsilon}{\rho} \tag{4.4}$$

which is inversely proportional to the SNR.

Similarly, consider a MIMO system, and the output matrix form with the same transmit power E_s can be written as

$$Y = \sqrt{\rho} H X + N \tag{4.5}$$

where $E\{\ XX^H\ \} = 1$ and the expected received SNR becomes

$$\rho \cdot E\{\ ||HX||^2\ \} = \rho \cdot E\{\ xHH^H x^H\} = \frac{\rho}{N} \sum_{n=1}^{N} \sum_{m=1}^{M} |h_{nm}|^2 \tag{4.6}$$

Note that N, M denote the number of transmit and receive antennas respectively, and $E\{\cdot\}$ denotes the mathematical expectation. Thus, the probability that the received SNR is less than a small value ϵ is

$$\begin{aligned} p(\ \frac{\rho}{N}\sum_{n=1}^{N}\sum_{m=1}^{M}|h_{nm}|^2 < \epsilon\ ) &= p(\ \sum_{n=1}^{N}\sum_{m=1}^{M}|h_{nm}|^2 < \frac{N\epsilon}{\rho}\ ) \\ &< p(\ |h_{11}|^2 < \frac{N\epsilon}{\rho},\ |h_{12}|^2 < \frac{N\epsilon}{\rho}, ...,\ |h_{NM}|^2 < \frac{N\epsilon}{\rho}\ ) \\ &= \prod_{n=1,m=1}^{NM} p(\ |h_{nm}|^2 < \frac{N\epsilon}{\rho}\ ) \\ &= (1 - e^{-\frac{N\epsilon}{\rho}})^{NM} \qquad (4.7) \end{aligned}$$

Also, as the transmit power is very large ($\rho \to \infty$), (4.7) can be approximated to

$$p(\ \frac{\rho}{N}\sum_{n=1}^{N}\sum_{m=1}^{M}|h_{nm}|^2 < \epsilon\ ) \approx (\frac{N\epsilon}{\rho})^{NM} \qquad (4.8)$$

Compared with the results in the SISO system, the probability in (4.8) is inversely proportional to ρ^{NM}. It shows that the MIMO system offer lower error probability than SISO system at high SNR. It is commonly investigated by means of calculating the diversity order. A system which has an average error probability P_e as a function of SNR that behave as

$$\lim_{\rho\to\infty} \frac{\log\ (P_e)}{\log\ (\rho)} = -d \qquad (4.9)$$

is said to have a diversity of order d. In other words, the average error probability in the high SNR region can be expressed as

$$P_e \approx C \cdot \rho^{-d} \qquad (4.10)$$

where C is the coding advantage. We note that the diversity order is a high SNR approximation. However, the exact symbol error probability (PEP) and bit error probability are sometimes too difficult to calculate. In [3], the error probability can be analyzed by calculating the pair-wise error probability PEP). In the following section, we pay more attention to the PEP instead to get a basic idea of the error performance.

4.2 Pairwise Error Probability of Differential Space-Time Code

The general used performance measures for diversity order include the symbol error probability (SEP) and the outage probability. Unfortunately, the SEP is not always analytical depending on the design of techniques. However, the pairwise error probability (PEP), which is commonly used to upper bound the SEP [3], is more feasible. Moreover, an advantage of PEP is that it is not related to the symbol constellations. Therefore, in the following content, we will derive the close form of the PEP based on the approaches in [17] to analyze the diversity order for the design of the proposed DSTC.

Conditioned on y_{k-1} at the receiver, based on (3.5), the receiver will erroneously select $E_{k(i)} = E_i$ when $S_{k(i)} = S_i$ was sent if

$$\begin{aligned}
||Y_k - \frac{1}{\alpha_k} Y_{k-1} E_i||^2 &\leq ||Y_k - \frac{1}{\alpha_k} Y_{k-1} S_i||^2 \\
tr\{(Y_k - \frac{1}{\alpha_k} Y_{k-1} E_i)(Y_k - \frac{1}{\alpha_k} Y_{k-1} E_i)^H\} &\leq tr\{(Y_k - \frac{1}{\alpha_k} Y_{k-1} S_i)(Y_k - \frac{1}{\alpha_k} Y_{k-1} S_i)^H\} \\
\frac{1}{\alpha_k}\, tr\{(Y_{k-1}(S_i - E_i)(S_i - E_i)^H\ Y_{k-1}^H\} &\leq tr\{2Re\{\ W_k (S_i - E_i)^H Y_{k-1}^H\ \}\} \\
\frac{1}{\alpha_k}\, tr\{(Y_{k-1} D D^H\ Y_{k-1}^H\} &\leq tr\{2Re\{\ W_k D^H Y_{k-1}^H\ \}\} \qquad (4.11)
\end{aligned}$$

where $D = S_i - E_i$ is called the error matrix. Concatenating the columns of Y_k into a vector by $vec(Y_k)$, the received signal can be rewritten as

$$\mathbf{y}_k = \sqrt{\rho}\ \mathbf{h}_k \mathcal{X}_k + \mathbf{n}_k \tag{4.12}$$

$$\mathcal{X}_k = \frac{1}{\alpha_k} \mathcal{X}_{k-1} \mathcal{S}_k \tag{4.13}$$

where $vec(\cdot)$ denotes the vectorization operator. $\mathbf{y}_k = vec(Y_k^T)^T$, $\mathbf{h}_k = vec(H_k^T)^T$, $\mathbf{n}_k = vec(N_k^T)^T$, $\mathcal{X}_k = I_M \otimes X_k$ and $\mathcal{S}_k = I_M \otimes S_k$. Thus, $\mathbf{y}_k$ becomes a row vector, and we can simplify the equation in (4.11) for simplicity as

$$\begin{aligned} ||\mathbf{y}_k - \frac{1}{\alpha_k}\mathbf{y}_{k-1}\mathcal{E}_i||^2 \ &\leq \ ||\mathbf{y}_k - \frac{1}{\alpha_k}\mathbf{y}_{k-1}\mathcal{S}_i||^2 \\ \frac{1}{\alpha_k}\ \mathbf{y}_{k-1}\mathcal{D}\mathcal{D}^H\ \mathbf{y}_{k-1}^H \ &\leq \ 2Re\{\ \mathbf{w}_i \mathcal{D}^H\ \mathbf{y}_{k-1}^H\ \} \end{aligned} \tag{4.14}$$

where $\mathcal{D} = \mathcal{S}_i - \mathcal{E}_i = I_M \bigotimes S_i - E_i$. Conditioned on the $\mathbf{y}_{k-1}$, the left hand side of the inequality in (4.13) is a deterministic variable. Since $\mathbf{w}_i$ and $\mathbf{y}_{k-1}$ are Gaussian distributed, the linear combination of Gaussian random variables are still Gaussian distributed. Therefore, the right hand side of (4.13) is a colored Gaussian random variable.

Let $g = 2Re\{\ \mathbf{w}_i\mathcal{D}^H\mathbf{y}_{k-1}^H\ \}$ be Gaussian distributed with conditional mean $m_{g|y_{k-1}}$ and conditional variance $\sigma^2_{g|\mathbf{y}_{k-1}}$ for given $\mathbf{y}_{k-1}$. Conditioned on the received signal $\mathbf{y}_{k-1}$, the conditional mean of g can be defined as

$$\begin{aligned} m_{g|\mathbf{y}_{k-1}} &= E\{\ g \mid \mathbf{y}_{k-1}\ \} \\ &= E\{\ 2Re\{\ \mathbf{w}_i\mathcal{D}^H\mathbf{y}_{k-1}^H\ \} \mid \mathbf{y}_{k-1}\ \} \\ &= 2Re\{\ E\{\ \mathbf{y}_i \mid \mathbf{y}_{k-1}\ \}\ \mathcal{D}^H\ \mathbf{y}_{k-1}^H\ \} \\ &= 2Re\{\ E\{\ \mathbf{n}_k - \mathbf{n}_{k-1}\mathcal{S}_i \mid \mathbf{y}_{k-1}\ \}\ \mathcal{D}^H\ \mathbf{y}_{k-1}^H\ \} \end{aligned} \tag{4.15}$$

Note that $E\{\ \mathbf{n}_k \mid \mathbf{y}_{k-1}\ \} = 0$, and thus (4.14) can be rewritten as

$$\begin{aligned} m_{g|\mathbf{y}_{k-1}} &= 2Re\{\ E\{\ \mathbf{n}_k - \mathbf{n}_{k-1}\mathcal{S}_i \mid \mathbf{y}_{k-1}\ \}\ \mathcal{D}^H\ \mathbf{y}_{k-1}^H\ \} \\ &= -2Re\{\ E\{\ \mathbf{n}_{k-1} \mid \mathbf{y}_{k-1}\ \}\ \mathcal{S}_i\ \mathcal{D}^H\ \mathbf{y}_{k-1}\ \} \\ &= -2Re\{\ m_{\mathbf{n}_{k-1}|\mathbf{y}_{k-1}}\ \mathcal{S}_i\ \mathcal{D}^H\ \mathbf{y}_{k-1}^H\ \} \end{aligned} \tag{4.16}$$

where $m_{\mathbf{n}_{k-1}|\mathbf{y}_{k-1}} = E\{\ \mathbf{n}_{k-1} \mid \mathbf{y}_{k-1}\ \}$. In order to compute $m_{\mathbf{n}_{k-1}|\mathbf{y}_{k-1}}$, we use the theorem in [19] and introduce in ***Appendix A***. By using the results given in ***Appendix A***, $m_{\mathbf{n}_{k-1}|\mathbf{y}_{k-1}}$ can be expressed as

$$m_{\mathbf{n}_{k-1}|\mathbf{y}_{k-1}} = E\{\ \mathbf{n}_{k-1}\ \} + \Sigma_{\mathbf{y}_{k-1},\mathbf{n}_{k-1}}\Sigma^{-1}_{\mathbf{y}_{k-1},\mathbf{y}_{k-1}}(\mathbf{y}_{k-1} - E\{\ \mathbf{y}_{k-1}\ \}) \tag{4.17}$$

where $\Sigma_{\mathbf{y}_{k-1},\mathbf{n}_{k-1}} = E\{\ \mathbf{y}_{k-1}^H\ \mathbf{n}_{k-1}\ \} = \sigma_n^2\ I_{NM}$ and

$$\begin{aligned} \Sigma_{\mathbf{y}_{k-1},\mathbf{y}_{k-1}} &= E\{\ \mathbf{y}_{k-1}^H\ \mathbf{y}_{k-1}\ \} \\ &= E\{\ (\sqrt{\rho}\mathbf{h}\mathcal{X}_{k-1} + \mathbf{n}_{k-1})^H\ (\sqrt{\rho}\mathbf{h}\mathcal{X}_{k-1} + \mathbf{n}_{k-1})\ \} \\ &= E_s(\mathcal{X}_{k-1}^H\mathcal{X}_{k-1}) + \sigma_n^2\ I_{NM} \end{aligned} \tag{4.18}$$

Since $E\{\ \mathbf{n}_{k-1}\ \} = 0$ and $E\{\ \mathbf{y}_{k-1}\ \} = 0$, (4.16) can be derived as

$$\begin{aligned} m_{\mathbf{n}_{k-1}|\mathbf{y}_{k-1}} &= E\{\ \mathbf{n}_{k-1}\ \} + \Sigma_{\mathbf{y}_{k-1},\mathbf{n}_{k-1}}\Sigma^{-1}_{\mathbf{y}_{k-1},\mathbf{y}_{k-1}}(\mathbf{y}_{k-1} - E\{\ \mathbf{y}_{k-1}\ \}) \\ &= \sigma_n^2\ \mathbf{y}_{k-1}\ (E_s(\mathcal{X}_{k-1}^H\mathcal{X}_{k-1}) + \sigma_n^2\ I_{NM})^{-1} \end{aligned} \tag{4.19}$$

Substituting (4.18) for $m_{\mathbf{n}_{k-1}|\mathbf{y}_{k-1}}$ in (4.15) gives the conditional mean $m_{g|\mathbf{y}_{k-1}}$ as

$$\begin{aligned} m_{g|\mathbf{y}_{k-1}} &= -2Re\{\ m_{\mathbf{n}_{k-1}|\mathbf{y}_{k-1}}\ \mathcal{S}_i\ \mathcal{D}^H\ \mathbf{y}_{k-1}^H\ \} \\ &= -2Re\{\ (\ \sigma_n^2\ \mathbf{y}_{k-1}\ (E_s(\mathcal{X}_{k-1}^H\mathcal{X}_{k-1}) + \sigma_n^2\ I_{NM})^{-1}\ )\ \mathcal{S}_i\mathcal{D}^H\mathbf{y}_{k-1}^H\ \} \end{aligned} \tag{4.20}$$

Similarly, given the received signal $\mathbf{y}_{k-1}$, the conditional variance of g can be expressed as

$$\begin{aligned}\sigma^2_{g|\mathbf{y}_{k-1}} &= E\{ \; || \; g - m_{g|\mathbf{y}_{k-1}} \; ||^2 \; | \; \mathbf{y}_{k-1} \; \} \\ &= E\{ \; (\; g - m_{g|\mathbf{y}_{k-1}} \;)^H (\; g - m_{g|\mathbf{y}_{k-1}} \;) \; | \; \mathbf{y}_{k-1} \; \} \qquad (4.21)\end{aligned}$$

where

$$\begin{aligned}g - m_{g|\mathbf{y}_{k-1}} &= 2Re\{ \; \mathbf{w}_i \mathcal{D}^H \mathbf{y}^H_{k-1} \; \} + 2Re\{ \; m_{\mathbf{n}_{k-1}|\mathbf{y}_{k-1}} \mathcal{S}_i \; \mathcal{D}^H \; \mathbf{y}^H_{k-1} \; \} \\ &= 2Re\{ \; \mathbf{w}_i \mathcal{D}^H \mathbf{y}^H_{k-1} + m_{\mathbf{n}_{k-1}|\mathbf{y}_{k-1}} \mathcal{S}_i \; \mathcal{D}^H \; \mathbf{y}^H_{k-1} \; \} \\ &= 2Re\{ \; (\mathbf{n}_k - \mathbf{n}_{k-1} \mathcal{S}_i) \mathcal{D}^H \mathbf{y}^H_{k-1} + m_{\mathbf{n}_{k-1}|\mathbf{y}_{k-1}} \mathcal{S}_i \; \mathcal{D}^H \; \mathbf{y}^H_{k-1} \; \} \\ &= 2Re\{ \; (\mathbf{n}_k - [\mathbf{n}_{k-1} - m_{\mathbf{n}_{k-1}|\mathbf{y}_{k-1}}] \mathcal{S}_i) \mathcal{D}^H \mathbf{y}^H_{k-1} \} \qquad (4.22)\end{aligned}$$

Substituting (4.20) for $g - m_{g|\mathbf{y}_{k-1}}$ in (4.21) gives the conditional variance $\sigma^2_{g|\mathbf{y}_{k-1}}$ as

$$\begin{aligned}\sigma^2_{g|\mathbf{y}_{k-1}} &= E\{ \; (\; g - m_{g|\mathbf{y}_{k-1}} \;)^H (\; g - m_{g|\mathbf{y}_{k-1}} \;) \; | \; \mathbf{y}_{k-1} \; \} \\ &= E\{ \; (\; 2Re\{ \; (\mathbf{n}_k - [\mathbf{n}_{k-1} - m_{\mathbf{n}_{k-1}|\mathbf{y}_{k-1}}] \mathcal{S}_i) \mathcal{D}^H \mathbf{y}^H_{k-1} \; \} \;)^H \\ &\qquad \cdot(\; 2Re\{ \; (\mathbf{n}_k - [\mathbf{n}_{k-1} - m_{\mathbf{n}_{k-1}|\mathbf{y}_{k-1}}] \mathcal{S}_i) \mathcal{D}^H \mathbf{y}^H_{k-1} \; \} \;) \; | \; \mathbf{y}_{k-1} \; \} \\ &= 2 \; \mathbf{y}_{k-1} d \; E\{ \; (\mathbf{n}_k - [\mathbf{n}_{k-1} - m_{\mathbf{n}_{k-1}|\mathbf{y}_{k-1}}] \mathcal{S}_i)^H \\ &\qquad \cdot (\mathbf{n}_k - [\mathbf{n}_{k-1} - m_{\mathbf{n}_{k-1}|\mathbf{y}_{k-1}}] \mathcal{S}_i) \; | \; \mathbf{y}_{k-1} \; \} \; d^H \mathbf{y}^H_{k-1} \\ &= 2 \; \mathbf{y}_{k-1} \mathcal{D} \; [\; \Sigma_{\mathbf{n}_k,\mathbf{n}_k} + \mathcal{S}^H_i \; \Sigma_{\mathbf{n}_{k-1}|\mathbf{y}_{k-1}} \; \mathcal{S}_i \;] \; \mathcal{D}^H \mathbf{y}^H_{k-1} \qquad (4.23)\end{aligned}$$

where $\Sigma_{\mathbf{n}_{k-1},\mathbf{n}_{k-1}} = \sigma^2_n I_{NM}$ and $\Sigma_{\mathbf{n}_{k-1}|\mathbf{y}_{k-1}} = E\{ \; || \; \mathbf{n}_{k-1} - m_{\mathbf{n}_{k-1}|\mathbf{y}_{k-1}} \; ||^2 \; | \; \mathbf{y}_{k-1} \; \}$ is the covariance of the noise vector $\mathbf{n}_{k-1}$ condition on the received vector $\mathbf{y}_{k-1}$. Similarly, using the results obtained in ***Appendix A***, $\Sigma_{\mathbf{n}_{k-1}|\mathbf{y}_{k-1}}$ can be written as

$$
\begin{aligned}
\Sigma_{\mathbf{n}_{k-1}|\mathbf{y}_{k-1}} &= E\{\ ||\ \mathbf{n}_{k-1} - m_{\mathbf{n}_{k-1}|\mathbf{y}_{k-1}}\ ||^2 \mid \mathbf{y}_{k-1}\ \} \\
&= \Sigma_{\mathbf{n}_{k-1},\mathbf{n}_{k-1}} - \Sigma^H_{\mathbf{y}_{k-1},\mathbf{n}_{k-1}}\ \Sigma^{-1}_{\mathbf{y}_{k-1},\mathbf{y}_{k-1}}\ \Sigma_{\mathbf{y}_{k-1},\mathbf{n}_{k-1}} \\
&= \sigma_n^2 I_{NM} - (\sigma_n^2 I_{NM}\ \Sigma^{-1}_{\mathbf{y}_{k-1},\mathbf{y}_{k-1}}\ \sigma_n^2 I_{NM}) \\
&= \sigma_n^2\ [\ I_{NM} - \sigma_n^2\ \Sigma^{-1}_{\mathbf{y}_{k-1},\mathbf{y}_{k-1}}\ ] \qquad (4.24)
\end{aligned}
$$

Substituting (4.22) for $\Sigma_{\mathbf{n}_{k-1}|\mathbf{y}_{k-1}}$ in (4.23) gives the conditional variance $\sigma^2_{g|\mathbf{y}_{k-1}}$ as

$$
\begin{aligned}
\sigma^2_{g|\mathbf{y}_{k-1}} &= E\{\ (\ g - m_{g|\mathbf{y}_{k-1}}\ )^H(\ g - m_{g|\mathbf{y}_{k-1}}\ ) \mid \mathbf{y}_{k-1}\ \} \\
&= 2\ \mathbf{y}_{k-1}\mathcal{D}\ [\ \sigma_n^2 I_{NM} + \mathcal{S}_i^H(\sigma_n^2\ [\ I_{NM} - \sigma_n^2\ \Sigma^{-1}_{\mathbf{y}_{k-1},\mathbf{y}_{k-1}}\ ])\mathcal{S}_i\ ]\ \mathcal{D}^H\mathbf{y}^H_{k-1} \\
&= 2\ \mathbf{y}_{k-1}\mathcal{D}\ [\ \sigma_n^2 I_{NM} + \mathcal{S}_i^H(\sigma_n^2\ [\ I_{NM} - \\
&\qquad \sigma_n^2(\ E_s(\ \mathcal{X}^H_{k-1}\mathcal{X}_{k-1}\ ) + \sigma_n^2\ I_{NM})^{-1}\ ])\mathcal{S}_i\ ]\ \mathcal{D}^H\mathbf{y}^H_{k-1} \qquad (4.25)
\end{aligned}
$$

Summery

Let $g = 2Re\{\ \mathbf{w}_i\mathcal{D}^H\mathbf{y}^H_{k-1}\ \}$, then we have shown that the random variable g which is Gaussian distributed has the conditional mean

$$
\begin{aligned}
m_{g|\mathbf{y}_{k-1}} &= E\{\ g \mid \mathbf{y}_{k-1}\ \} \\
&= -2Re\{\ m_{\mathbf{n}_{k-1}|\mathbf{y}_{k-1}}\ \mathcal{S}_i\ \mathcal{D}^H\ \mathbf{y}^H_{k-1}\ \} \qquad (4.26)
\end{aligned}
$$

where $m_{\mathbf{n}_{k-1}|\mathbf{y}_{k-1}} = \sigma_n^2\ \mathbf{y}_{k-1}\ [\ E_s(\mathcal{X}^H_{k-1}\mathcal{X}_{k-1}) + \sigma_n^2\ I_{NM}\ ]^{-1}$, and the conditional variance

$$
\begin{aligned}
\sigma^2_{g|\mathbf{y}_{k-1}} &= E\{\ ||\ g - m_{g|\mathbf{y}_{k-1}}\ ||^2 \mid \mathbf{y}_{k-1}\ \} \\
&= 2\ \mathbf{y}_{k-1}\mathcal{D}\ [\ \sigma_n^2 I_{NM} + \mathcal{S}_i^H\ \Sigma_{\mathbf{n}_{k-1},\mathbf{y}_{k-1}}\ \mathcal{S}_i\ ]\ \mathcal{D}^H\mathbf{y}^H_{k-1} \qquad (4.27)
\end{aligned}
$$

where $\Sigma_{\mathbf{n}_{k-1}|\mathbf{y}_{k-1}} = \sigma_n^2 \ [\ I_{NM} - \sigma_n^2\ \Sigma^{-1}_{\mathbf{y}_{k-1},\mathbf{y}_{k-1}}\ ]$.

By the chernoff upper bound, the conditional PEP in (4.13) can be expressed as

$$\begin{aligned} p(\ \mathcal{S}_i \rightarrow \mathcal{E}_i \mid \mathbf{y}_{k-1}\ ) &= p(\ ||\mathbf{y}_k - \frac{1}{\alpha_k}\mathbf{y}_{k-1}E_i||^2 \leq ||\mathbf{y}_k - \frac{1}{\alpha_k}\mathbf{y}_{k-1}\mathcal{S}_i||^2\ ) \\ &= p(\ \frac{1}{\alpha_k}\ \mathbf{y}_{k-1}\mathcal{D}\ \mathcal{D}^H\mathbf{y}^H_{k-1} \leq 2Re\{\ \mathbf{w}_i\mathcal{D}^H\ \mathbf{y}^H_{k-1}\ \}\ ) \\ &= Q(\ \frac{\frac{1}{\alpha_k}\ \mathbf{y}_{k-1}\mathcal{D}\ \mathcal{D}^H\mathbf{y}^H_{k-1} - m_{g|\mathbf{y}_{k-1}}}{\sigma_{g|\mathbf{y}_{k-1}}}\ ) \\ &\leq \frac{1}{2}\ exp(-\frac{1}{2}\Omega) \end{aligned} \tag{4.28}$$

where

$$Q(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}\ \sigma}\ exp(-\frac{y^2}{2\sigma^2})\ dy \tag{4.29}$$

$$\Omega = \frac{||\ h - m_{g|\mathbf{y}_{k-1}}\ ||^2}{\sigma^2_{g|\mathbf{y}_{k-1}}} \tag{4.30}$$

In fact, it is difficult to analyze Ω. Moreover, unlike in the coherent STBC, finding the PEP from (4.27) is a hard problem because of the non-zero mean $m_{u|\mathbf{y}_{k-1}}$ and complicated variance $\sigma^2_{g|\mathbf{y}_{k-1}}$. However, in the high SNR region (i.e., keep the SNR $\rho = E_s/\sigma_n^2 \rightarrow \infty$), the conditional mean in (4.25) and the conditional in (4.26) variance of g approach respectively

$$\begin{aligned} m_{g|\mathbf{y}_{k-1}} &= -2Re\{\ m_{\mathbf{n}_{k-1}|\mathbf{y}_{k-1}}\ \mathcal{S}_i\ \mathcal{D}^H\ \mathbf{y}^H_{k-1}\ \} \\ &= -2Re\{\ (\ \sigma_n^2\ \mathbf{y}_{k-1}\ (E_s(\mathcal{X}^H_{k-1}\mathcal{X}_{k-1}) + \sigma_n^2\ I_{NM})^{-1}\ )\ \mathcal{S}_i\mathcal{D}^H\mathbf{y}^H_{k-1}\ \} \\ &= -2Re\{\ (\ \mathbf{y}_{k-1}\ (\ \frac{E_s}{\sigma_n^2}(\mathcal{X}^H_{k-1}\mathcal{X}_{k-1}) + I_{NM}\ )^{-1}\ )\ \mathcal{S}_i d^H\mathbf{y}^H_{k-1}\ \} \\ &\rightarrow 0 \end{aligned} \tag{4.31}$$

and

$$\begin{aligned}
\sigma^2_{g|\mathbf{y}_{k-1}} &= 2\ \mathbf{y}_{k-1}\mathcal{D}\ [\ \sigma_n^2 I_{NM} + \mathcal{S}_i^H\ \Sigma_{\mathbf{n}_{k-1},\mathbf{y}_{k-1}}\ \mathcal{S}_i\ ]\ \mathcal{D}^H\mathbf{y}_{k-1}^H \\
&= 2\ \mathbf{y}_{k-1}\mathcal{D}\ [\ \sigma_n^2 I_{NM} + \mathcal{S}_i^H(\sigma_n^2\ [\ I_{NM} - \\
&\qquad \sigma_n^2(\ E_s(\ \mathcal{X}_{k-1}^H\mathcal{X}_{k-1}\ ) + \sigma_n^2\ I_{NM})^{-1}\ ])\mathcal{S}_i\ ]\ \mathcal{D}^H\mathbf{y}_{k-1}^H \\
&= 2\ \mathbf{y}_{k-1}\mathcal{D}\ [\ \sigma_n^2 I_{NM} + \mathcal{S}_i^H(\sigma_n^2\ [\ I_{NM} - \\
&\qquad (\ \frac{E_s}{\sigma_n^2}(\ \mathcal{X}_{k-1}^H\mathcal{X}_{k-1}\ ) + I_{NM})^{-1}\ ])\mathcal{S}_i\ ]\ \mathcal{D}^H\mathbf{y}_{k-1}^H \\
&\rightarrow 2\ \sigma_n^2\ \mathbf{y}_{k-1}\mathcal{D}\ (\ I_{NM} + \mathcal{S}_i^H\ \mathcal{S}_i\ )\ \mathcal{D}^H\mathbf{y}_{k-1}^H
\end{aligned} \tag{4.32}$$

Based on the forms in (4.30) and (4.31) as SNR approaches to infinity, Ω in (4.29) reduces to

$$\begin{aligned}
\Omega &= \frac{||\ h - m_{g|\mathbf{y}_{k-1}}\ ||^2}{\sigma^2_{g|\mathbf{y}_{k-1}}} \\
&\rightarrow \frac{h^2}{\sigma^2_{g|\mathbf{y}_{k-1}}} \\
&= \frac{||\ \frac{1}{\alpha_k}\ \mathbf{y}_{k-1}\mathcal{D}\ \mathcal{D}^H\mathbf{y}_{k-1}^H\ ||^2}{2\ \sigma_n^2\ \mathbf{y}_{k-1}\mathcal{D}\ (\ I_{NM} + \mathcal{S}_i^H\ \mathcal{S}_i\ )\ \mathcal{D}^H\mathbf{y}_{k-1}^H}
\end{aligned} \tag{4.33}$$

Since $\mathcal{S}_i^H\mathcal{S}_i$ is a hermitian matrix, it can be written as $\mathcal{S}_i^H\mathcal{S}_i = U_i^H\Lambda_i U_i$, where the matrix of eigenvectors U_i obeys $U_i^H U_i = U_i U_i^H = I_{NM}$, and let Λ_i is a matrix of eigenvalues of $\mathcal{S}_i^H\mathcal{S}_i$. We define λ_m as the eigenvalues of $\mathcal{S}_i^H\mathcal{S}_i$ for $m = 1,\ \ldots\ , NM$.

Thus, (4.32) is then given by

$$
\begin{aligned}
\Omega &= \frac{||\ \frac{1}{\alpha_k}\ \mathbf{y}_{k-1}\mathcal{D}\ \mathcal{D}^H \mathbf{y}_{k-1}^H\ ||^2}{2\ \sigma_n^2\ \mathbf{y}_{k-1}\mathcal{D}\ (\ I_{NM} + \mathcal{S}_i^H\ \mathcal{S}_i\ )\ \mathcal{D}^H \mathbf{y}_{k-1}^H} \\
&= \frac{||\ \mathbf{y}_{k-1}\mathcal{D}\ \mathcal{D}^H \mathbf{y}_{k-1}^H\ ||^2}{2\ (\alpha_k)^2\ \sigma_n^2\ \mathbf{y}_{k-1}\mathcal{D}\ (\ U_i^H (I_{NM} + \Lambda_i) U_i\ )\ d^H \mathbf{y}_{k-1}^H} \\
&\geq \frac{||\ \mathbf{y}_{k-1}\mathcal{D}\ \mathcal{D}^H \mathbf{y}_{k-1}^H\ ||^2}{2\ (\alpha_k)^2\ \sigma_n^2\ (1+\mu_{max})\ \mathbf{y}_{k-1}\mathcal{D}\ \mathcal{D}^H \mathbf{y}_{k-1}^H} \\
&= \frac{\mathbf{y}_{k-1}\mathcal{D}\ \mathcal{D}^H \mathbf{y}_{k-1}^H}{2\ (\alpha_k)^2\ \sigma_n^2\ (1+\mu_{max})} \qquad (4.34)
\end{aligned}
$$

where μ_{max} is the maximum eigenvalue of $\mathcal{S}_i^H \mathcal{S}_i$ (i.e., $\mu_{max} = max\ \{\ \mu_1,\ \dots\ , \mu_{NM}\ \}$) and $\mathbf{y}_{k-1}\mathcal{D}\ \mathcal{D}^H \mathbf{y}_{k-1}^H$ is a quadratic form. In order to obtain the exact form of PEP, we need to average (4.27) with respect to the distribution of y_{k-1}. The probability density function (pdf) of y_{k-1} is

$$
p(\mathbf{y}_{k-1}) = \frac{1}{\pi^{NM} |\ \Sigma_{\mathbf{y}_{k-1},\mathbf{y}_{k-1}}\ |}\ exp(-\mathbf{y}_{k-1}\ \Sigma^{-1}_{\mathbf{y}_{k-1},\mathbf{y}_{k-1}}\ \mathbf{y}_{k-1}^H) \qquad (4.35)
$$

where $\Sigma_{\mathbf{y}_{k-1},\mathbf{y}_{k-1}} = E_s(\mathcal{X}_{k-1}^H \mathcal{X}_{k-1}) + \sigma_n^2\ I_{NM}$ and the exact pairwise error probability is given by

$$
\begin{aligned}
p(\ \mathcal{S}_i \rightarrow \mathcal{E}_i\ ) &= E_{y_{k-1}}\{Pr\ (S_{k(i)} \rightarrow E_{k(i)}\ |\ y_{k-1})\} \\
&\leq \int \frac{1}{2}\ exp(-\frac{1}{2}\Omega) \cdot \frac{1}{\pi^{NM} |\ \Sigma_{\mathbf{y}_{k-1},\mathbf{y}_{k-1}}\ |}\ exp(-y\ \Sigma^{-1}_{\mathbf{y}_{k-1},\mathbf{y}_{k-1}}\ y^H)\ dy \\
&= \frac{1}{2} \int \frac{1}{\pi^{NM} |\ \Sigma_{\mathbf{y}_{k-1},\mathbf{y}_{k-1}}\ |}\ exp(-y\ \Sigma^{-1}\ y^H)\ dy \qquad (4.36)
\end{aligned}
$$

where the operator $|\cdot|$ is the matrix determinant and $\Sigma^{-1} = \Sigma^{-1}_{\mathbf{y}_{k-1},\mathbf{y}_{k-1}} + \frac{\mathcal{D}\mathcal{D}^H}{4\ (\alpha_k)^2\ \sigma_n^2\ (1+\mu_{max})}$. By using the normalization property of Gaussian probability density function,

$$
\int \frac{1}{\pi^{NM} |\ \Sigma\ |}\ exp(-y\ \Sigma^{-1}\ y^H)\ dy = 1 \qquad (4.37)
$$

(4.35) can be expressed as

$$
\begin{aligned}
p(\ \mathcal{S}_i \rightarrow \mathcal{E}_i\ ) \ &\leq\ \frac{1}{2} \int \frac{1}{\pi^{NM} |\ \Sigma_{\mathbf{y}_{k-1},\mathbf{y}_{k-1}}\ |}\ exp(-y\ \Sigma^{-1}\ y^H)\ dy \\
&= \frac{|\ \Sigma\ |}{2|\ \Sigma_{\mathbf{y}_{k-1},\mathbf{y}_{k-1}}\ |} \\
&= \frac{1}{2|\ \Sigma^{-1}\Sigma_{\mathbf{y}_{k-1},\mathbf{y}_{k-1}}|} \qquad\qquad (\ |AB| = |A| \cdot |B|\ ) \\
&= \frac{1}{2|\ (\Sigma^{-1}_{\mathbf{y}_{k-1},\mathbf{y}_{k-1}} + \frac{\mathcal{D}\mathcal{D}^H}{4\ (\alpha_k)^2\ \sigma_n^2\ (1+\mu_{max})})\Sigma_{\mathbf{y}_{k-1},\mathbf{y}_{k-1}}|} \\
&= \frac{1}{2|\ (I_{NM} + \frac{1}{4\ (\alpha_k)^2\ \sigma_n^2\ (1+\mu_{max})}\mathcal{D}\mathcal{D}^H(E_s(\mathcal{X}^H_{k-1}\mathcal{X}_{k-1}) + \sigma_n^2\ I_{NM})|} \\
&\rightarrow \frac{1}{2|\ (I_{NM} + \frac{1}{4\ (\alpha_k)^2\ \sigma_n^2\ (1+\mu_{max})}(E_s/\sigma_n^2) \cdot \mathcal{X}_{k-1}\mathcal{D}\mathcal{D}^H\mathcal{X}^H_{k-1}\ |} \\
&= \frac{1}{2} \prod_{m=1}^{NM} (\ 1 + \frac{\rho}{4\ (\alpha_k)^2\ \sigma_n^2\ (1+\mu_{max})}\lambda_m\ )^{-1} \qquad (4.38)
\end{aligned}
$$

where $\rho = E_s/\sigma_n^2$ and λ_m is the eigenvalues of matrix $\mathcal{X}_{k-1}\mathcal{D}\mathcal{D}^H\mathcal{X}^H_{k-1}$. The arrow "→" denotes the approximation as the $SNR \rightarrow \infty$. Equation (4.37) reveals that the error performance of DSTC depends on the distance matrix $\mathcal{D}^2 = \mathcal{D}\mathcal{D}^H = (\mathcal{S}_i - \mathcal{E}_i)(\mathcal{S}_i - \mathcal{E}_i)^H$ and the previously transmit matrix $\mathcal{X}_{k-1}$. In the next section, we will analyze the diversity order of the DSTC based on the proposed analytical expression for the PEP and provide a design criteria to approach the maximum diversity.

4.3 Design Criterion

Recall the general definition of diversity order in (4.9), a system which has an average error probability P_e as a function of SNR (ρ) that behave as

$$\lim_{\rho \rightarrow \infty} \frac{\log\ (P_e)}{\log\ (\rho)} = -d \qquad (4.39)$$

is said to have a diversity of order d. In other words, the average error probability in the high SNR region can be approximated as

$$P_e \approx C \cdot (\rho)^{-d} \tag{4.40}$$

where C is called the coding advantage. We note that the average error probability can be analyzed by the PEP. We define r as the rank of matrix $X_{k-1}DD^H X_{k-1}^H$, then exactly $NM - r$ eigenvalues are zero. Let the nonzero eigenvalues of $X_{k-1}DD^H X_{k-1}^H$ are $\lambda_1,\ \lambda_2,\ ...,\ \lambda_r$, then it gives from inequality (4.37) that

$$\begin{aligned} p(\ \mathcal{S}_i \rightarrow \mathcal{E}_i\ ) &\leq \frac{1}{2} \prod_{m=1}^{NM} (\ 1 + \frac{\rho}{4\ (\alpha_k)^2\ \sigma_n^2\ (1+\lambda_{max})} \lambda_m\ )^{-1} \\ &= \frac{1}{2}\ [\ \prod_{i=1}^{r} (\ \frac{\lambda_i}{4\ (\alpha_k)^2\ \sigma_n^2\ (1+\mu_{max})}\ )\ ]^{-M} \cdot (\rho)^{-RM} \end{aligned} \tag{4.41}$$

Therefore, from (4.39) and (4.40), the diversity order of a differential space-time code is

$$\begin{aligned} d &= -\lim_{\rho\rightarrow\infty} \frac{\log\ (\ p(\mathcal{S}_i \rightarrow \mathcal{E}_i)\ )}{\log\ (\rho)} \\ &= -\lim_{\rho\rightarrow\infty} \frac{\log\ (\ \frac{1}{2}\ [\ \prod_{r=1}^{R} (\ \frac{\lambda_r}{4\ (\alpha_k)^2\ \sigma_n^2\ (1+\mu_{max})}\ )\ ]^{-M} \cdot (\rho)^{-rM}\ )}{\log\ (\rho)} \\ &= -\lim_{\rho\rightarrow\infty} \frac{\log\ (\ (\rho)^{-RM}\ )}{\log\ (\rho)} \\ &= rM \end{aligned} \tag{4.42}$$

and a diversity advantage of rM as well as coding advantage of $\frac{1}{2}\ [\ \prod_{r=1}^{R} (\ \frac{\lambda_r}{4\ (\alpha_k)^2\ \sigma_n^2\ (1+\mu_{max})}\ )\ ]^{-M}$ is achieved. Particularly, we note that the results for DSTC are similar to those for coherent scheme. Therefore, from the above analysis, we have the following design criterions:

Design Criterion of Differential Space-Time Codes

A. The Rank Criterion

From (4.41), it is obvious that a diversity advantage of rM is achieved, which mainly depends on the rank of the previously transmit matrix X_{k-1} as well as distance matrix DD^H. Based on the above analysis, we arrive at the following design criterion:

$$\max_{S_{k,i}} \{ \ rank(D(S_{k,i}, S_{k,j}) \cdot D(S_{k,i}, S_{k,j})^H) \ \} \ , \quad \forall \ i \neq j \tag{4.43}$$

subject to $\det(S_k) \neq 0, \ \ \forall \ k$. Recall the differential encoding process in (3.1), assume that the initial transmit matrix $X_0 = I_N$ at the beginning of the transmission (i.e., $k = 1$). As the block index k increases, we have

$$\begin{aligned}
k = 1 \ , \qquad & X_1 = \frac{1}{\alpha_1} X_0 S_1 = \frac{1}{\alpha_1} S_1 \\
k = 2 \ , \qquad & X_2 = \frac{1}{\alpha_2} X_1 S_2 = \frac{1}{\alpha_2} (\frac{1}{\alpha_1} S_1) \ S_2 \\
\vdots \qquad & \qquad \vdots \\
k = L \ , \qquad & X_L = \frac{1}{\alpha_L} X_{L-1} S_L \\
& \quad = \frac{1}{\alpha_L} \ (\ \frac{1}{\alpha_{L-1}} \cdots (\ \frac{1}{\alpha_1} S_1 \ ) \ S_2 \cdots S_{L-1} \ ) \ S_L \\
& \quad = \prod_{l=1}^{L} \ \frac{1}{\alpha_l} \ S_l
\end{aligned} \tag{4.44}$$

It is easy to show that, at the k-th block, the transmit matrix X_k is the product of the data matrices from S_1 to S_k. Therefore, full rank X_{k-1} implies that S_i must be full rank as well, $\forall i \in [1, k-1]$. By making S_k full rank, $\forall k$, the diversity order becomes related to the rank of $D \cdot D^H$ only and it simplifies the code design of the DSTC. We thus focus on maximizing the rank r of the distance matrix, and have the maximum diversity order of rM.

B. The Determinant Criterion

In (4.40), the coding advantage that we tend to maximize is

$$C = \frac{1}{2} \left[\prod_{i=1}^{r} \left(\frac{\lambda_i}{4 \; (\alpha_k)^2 \; \sigma_n^2 \; (1 + \mu_{max})} \right) \right]^{-M} \tag{4.45}$$

Now we focused on the product of eigenvalues of $X_{k-1}DD^H X_{k-1}^H$. Suppose that a diversity order of rM is the target. The coding advantage mainly depends on the products $\lambda_1 \cdot \lambda_2 \cdot \cdots \cdot \lambda_r$ where r is the rank of $X_{k-1}DD^H X_{k-1}^H$. Our design criteria in this part is making this eigenvalue products as large as possible. Therefore, if a diversity order of rM is gained, the minimum of the products of the eigenvalues taken over all pairs of distinct codewords and must be maximized. Recall that the products of eigenvalues $\lambda_1 \cdot \lambda_2 \cdots \cdot \lambda_r$ is the determinant of $X_{k-1}DD^H X_{k-1}^H$ among the pairs of distinct codewords. Thus, we have the following determinant criteria.

Determinant Criterion:

$$max\{ \min_{\forall \; (S_{i(k)}, E_{i(k)}), \; S_{i(k)} \neq E_{i(k)}} \mid \tilde{D}(S_{i(k)} \rightarrow E_{i(k)}) \mid \} \; , \; \forall \; k$$

We note that $| \cdot |$ is the matrix determinant. However, compared with the results for the coherent STBC in [2], it is more difficult to obtain the determinant value of $X_{k-1}DD^H X_{k-1}^H$ than $D^2 = (S_{k(i)} - S_{k(i)})(S_{k(i)} - S_{k(i)})^H$ since the transmit matrix X_{k-1} are correlated to $S_1, ..., S_{k-1}$. Therefore, we mainly focus on the diversity behavior rather than coding advantage in the following contents. On the other hand, since the PEP is calculated based on the ML receiver in (3.5) rather than that in (3.7) as the noise matrix N_k is not white Gaussian distributed, the design criterion is valid for the ML receiver in (3.5).

Chapter 5

Extendable High-Rate Differential Space-Time Code

The priority of the design of high-rate DSTC is to design the data matrix S_k with the transmission rate R greater than one. Moreover, the code structure must be flexible and simple such that it can be extended from the lower to higher dimensions. This is the point of view when we design the extendable high-rate DSTC.

Alamouti STBC [1] for two transmit and one receive antennas is the only orthogonal space-time block code (OSTBC), which achieves the maximum diversity as well as possible mutual information of a 2×2 MIMO system. Unfortunately, it has been shown that an OSTBC with a transmission rate R equal to one for more than two transmit antennas does not exist [3]. Furthermore, by increasing the number of transmit antennas, the rate of the OSTBC is significantly decreasing, which makes them unattractive for systems with a very high number of transmit antennas. One solution to this problem is to divide the N transmit antennas into groups, where each group employs an OSTBC. Since the rate increases, there is a loss in the diversity order, which results in poor error performance. In this section, our objective is to apply the rate-one Alamouti scheme as our OSTBC and design an extendable high-rate DSTC based on the two rank design criterions obtained form the previous section for more than two transmit antennas.

5.1 Rate-One Differential Space-Time Code

5.1.1 Rate-One for Four Transmit Antennas

Quasi-orthogonal space-time block code (QOSTBC) for four transmit and one receive antennas have been analyzed in [10], [20], [21] and [22]. The basic idea of QOSTBC is to divide the N transmit antennas into groups, where each group employs an OSTBC (e.q., Alamouti STBC). There exist a full-rate QOSTBC which provides without full diversity and the decoder can work on pairs of modulated data symbols instead of single symbols. However, by choosing the suitable signal constellations as done in [12], [23], [13], it is possible to improve the BER performance with ML detection. For the rate-One DSTC, we apply the QOSTBC as the code structure for four transmit antennas. Unfortunately, the complete idea of OSTBC is well understood, but for QOSTBC only some examples have been proposed before without systematic analysis and precise definition. Therefore, we will give a short introduction about QOSTBC and analyze the code construction in the following content.

A QOSTBC is defined by its data matrix S_k, which is a function of the modulated data symbol vector $s = [\ s_1\ ;\ s_2\ ;\ \cdots\ ;\ s_L\ ]^T$. The transmission rate R of a QOSTBC is defined as $R = L/T$ where T is the block length for which the channel coefficients are constant during those T channel uses. Now we focus on the rate-One QOSTBC with block length $T = N$, therefore $L = N$.

Overviews of QOSTBCs

A. ABBA Quasi-Orthogonal Space-Time Block Code

The first class of QOSTBC proposed by Tirkkonen et al. [20], where it applies two

Alamouti STBCs in a block structure resulting in the so called *ABBA QOSTBC* . Starting two Alamouti schemes for $N = 2$ transmit antennas as the building blocks,

$$S_{12} = \begin{bmatrix} s_1 & s_2 \\ -s_2^* & s_1^* \end{bmatrix} \qquad S_{34} = \begin{bmatrix} s_3 & s_4 \\ -s_4^* & s_3^* \end{bmatrix} \tag{5.1}$$

and the data matrix $S_k = S_{4,rate1}^{ABBA}$ of the *ABBA QOSTBC* with $N = 4$ is then given by

$$S_{4,rate1}^{ABBA} = \begin{bmatrix} S_{12} & S_{34} \\ S_{34} & S_{12} \end{bmatrix} = \begin{bmatrix} s_1 & s_2 & s_3 & s_4 \\ -s_2^* & s_1^* & -s_4^* & s_3^* \\ s_3 & s_4 & s_1 & s_2 \\ -s_4^* & s_3^* & -s_2^* & s_1^* \end{bmatrix} \tag{5.2}$$

By multiplying the *ABBA QOSTBC* by its Hermitian, we have the Grammian matrix as

$$S_{4,rate1}^{ABBA} \cdot (S_{4,rate1}^{ABBA})^H = \begin{bmatrix} \alpha & 0 & \beta & 0 \\ 0 & \alpha & 0 & \beta \\ \beta & 0 & \alpha & 0 \\ 0 & \beta & 0 & \alpha \end{bmatrix} = \alpha \cdot \begin{bmatrix} I_2 & Q_{ABBA} \\ Q_{ABBA} & I_2 \end{bmatrix} \tag{5.3}$$

where the Q_{ABBA} is defined as

$$Q_{ABBA} = \begin{bmatrix} \beta/\alpha & 0 \\ 0 & \beta/\alpha \end{bmatrix} \tag{5.4}$$

with

$$\alpha = \sum_{i=1}^{4} \mid s_i \mid^2 \quad \textit{and} \quad \beta = 2Re\{ \ s_1 s_3^* + s_2 s_4^* \ \} \tag{5.5}$$

It can be seen that the $S_{4,rate-1}^{ABBA}$ is not a unitary matrix, since $\alpha \neq 1$ and $\beta \neq 0$. However, from (5.2) it can be seen that the symbols s_1, s_3 and the symbols s_2, s_4 appear in pairs. If conventional memoryless modulation is applied, it is impossible to achieve the unitary property. Therefore, it is apparent that some relationships between the data symbols s_1 and s_3, as well as between s_2 and s_4, are required to let β be zero.

B. Jafarkhani Quasi-Orthogonal Space-Time Block Code

The second class of QOSTBC was proposed by Jafarkhani [10], where it also uses two Alamouti STBCs in a block structure and results in the so called *Jafarkhani QOSTBC*. The data matrix $S_k = S_{4,rate1}^{Jafarkhani}$ generated by two Alamouti STBC S_{12} and S_{34} in (5.1) with $N = 4$ is

$$S_{4,rate-1}^{Jafarkhani} = \begin{bmatrix} S_{12} & S_{34} \\ -S_{34}^* & S_{12}^* \end{bmatrix} = \begin{bmatrix} s_1 & s_2 & s_3 & s_4 \\ -s_2^* & s_1^* & -s_4^* & s_3^* \\ -s_3^* & -s_4^* & s_1^* & s_2^* \\ s_4 & -s_3 & -s_2 & s_1 \end{bmatrix} \tag{5.6}$$

Similar to (5.3), we have the Grammian matrix as follows

$$S_{4,rate-1}^{Jafarkhani} \cdot (\, S_{4,rate-1}^{Jafarkhani}\,)^H = \begin{bmatrix} \alpha & 0 & 0 & \beta \\ 0 & \alpha & -\beta & 0 \\ 0 & -\beta & \alpha & 0 \\ \beta & 0 & 0 & \alpha \end{bmatrix} = \alpha \cdot \begin{bmatrix} I_2 & Q_{Jaf} \\ -Q_{Jaf} & I_2 \end{bmatrix} \tag{5.7}$$

where Q_{Jaf} is defined as

$$Q_{Jaf} = \begin{bmatrix} 0 & \beta/\alpha \\ -\beta/\alpha & 0 \end{bmatrix} \tag{5.8}$$

with

$$\alpha = \sum_{i=1}^{4} \mid s_i \mid^2 \qquad and \qquad \beta = 2Re\{\ s_1 s_4^* - s_2 s_3^*\ \} \tag{5.9}$$

We note that such structure is strongly related to the concept of complex Hadamard matrices. The 4×4 data matrix can be decomposed into four 2×2 submatrices which are Alamouti-like structures. Moreover, the columns of the matrix are not orthogonal to each other, but different Alamouti-like submatrices are orthogonal to each other instead. Based on the ideas, the following are some transformations of *Jafarkhani QOSTBCs*. The first four examples are

$$\begin{bmatrix} -S_{12} & S_{34} \\ S_{34}^* & S_{12}^* \end{bmatrix};\begin{bmatrix} S_{12} & -S_{34} \\ S_{34}^* & S_{12}^* \end{bmatrix};\begin{bmatrix} S_{12} & S_{34} \\ -S_{34}^* & S_{12}^* \end{bmatrix};\begin{bmatrix} S_{12} & S_{34} \\ S_{34}^* & -S_{12}^* \end{bmatrix} \tag{5.10}$$

Inverting the sign of each code matrix we obtain the other four code matrices:

$$\begin{bmatrix} S_{12} & -S_{34} \\ -S_{34}^* & -S_{12}^* \end{bmatrix};\begin{bmatrix} -S_{12} & S_{34} \\ -S_{34}^* & -S_{12}^* \end{bmatrix};\begin{bmatrix} -S_{12} & -S_{34} \\ S_{34}^* & -S_{12}^* \end{bmatrix};\begin{bmatrix} -S_{12} & -S_{34} \\ -S_{34}^* & S_{12}^* \end{bmatrix} \tag{5.11}$$

The above eight code matrices can be complex conjugated and generate the other eight code matrices. We note that these codes have a similar structure of the forms (5.7). Ones in the main diagonal entries and non-zero terms on the off-diagonal entries. Similarly, (5.6) shows the symbols s_1, s_4 and the symbols s_2, s_3 appear in pairs. Some correlations between the symbols s_1 and s_4, as well as between s_2 and s_3, are required.

C. Papadias and Foschini Quasi-Orthogonal Space-Time Block Code

The third class of QOSTBC was presented by Papadias and Foschini [21]. Compared with the above two QOSTBCs, the modulated symbols are arranged in a different way such that it can not be decomposed as a simple combination of two Alamouti-like submatricess , that is, their complex conjugated forms and negative forms. The *P.F. QOSTBC* with $N = 4$ and the data matrix $S_k = S_{4,rate1}^{P.F.}$ can be shown as

$$S_{4,rate-1}^{P.F.} = \begin{bmatrix} s_1 & s_2 & s_3 & s_4 \\ s_2^* & -s_1^* & s_4^* & -s_3^* \\ s_3 & -s_4 & -s_1 & s_2 \\ s_4^* & -s_3^* & -s_2^* & -s_1^* \end{bmatrix} \tag{5.12}$$

Again, by multiplying the data matrix by its Hermitian, the Grammian matrix can be expressed as

$$S_{4,rate-1}^{P.F.} \cdot (\ S_{4,rate-1}^{P.F.}\ )^H = \begin{bmatrix} \alpha & 0 & -\beta & 0 \\ 0 & \alpha & 0 & \beta \\ \beta & 0 & \alpha & 0 \\ 0 & -\beta & 0 & \alpha \end{bmatrix} = \alpha \cdot \begin{bmatrix} I_2 & Q_{P.F.} \\ -Q_{P.F.} & I_2 \end{bmatrix} \tag{5.13}$$

where $Q_{P.F.}$ is defined as

$$Q_{P.F.} = \begin{bmatrix} -\beta/\alpha & 0 \\ 0 & \beta/\alpha \end{bmatrix} \tag{5.14}$$

with

$$\alpha = \sum_{i=1}^{4} \mid s_i \mid^2 \qquad and \qquad \beta = 2jIm\{\ s_1^* s_3 + s_2^* s_4\ \} \tag{5.15}$$

D. STTD-OTD QOSTBC (ABAB Quasi-Orthogonal Space-Time Block Code)

The forth class of QOSTBC proposed by Jalloul et al. [22], which is called the *ABAB QOSTBC*. Using the same Alamouti-like submatrices S_{12} and S_{34}, the generation of $S_k = S_{4,rate-1}^{ABAB}$ as

$$S_{4,rate-1}^{ABAB} = \begin{bmatrix} S_{12} & S_{34} \\ S_{12} & -S_{34} \end{bmatrix} = \begin{bmatrix} s_1 & s_2 & s_3 & s_4 \\ -s_2^* & s_1^* & -s_4^* & s_3^* \\ s_1 & s_2 & -s_3 & -s_4 \\ -s_2^* & s_1^* & s_4^* & -s_3^* \end{bmatrix} \qquad (5.16)$$

and the multiplication of the $S_{4,rate-1}^{ABAB}$ by its Hermitian is

$$S_{4,rate-1}^{ABAB} \cdot (\ S_{4,rate-1}^{ABAB}\ )^H = \begin{bmatrix} \alpha & 0 & \beta & 0 \\ 0 & \alpha & 0 & \beta \\ \beta & 0 & \alpha & 0 \\ 0 & \beta & 0 & \alpha \end{bmatrix} = \alpha \cdot \begin{bmatrix} I_2 & Q_{ABAB} \\ Q_{ABAB} & I_2 \end{bmatrix} \qquad (5.17)$$

where Q_{ABAB} is defined as

$$Q_{ABBA} = \begin{bmatrix} \beta/\alpha & 0 \\ 0 & \beta/\alpha \end{bmatrix} \qquad (5.18)$$

with

$$\alpha = \sum_{i=1}^{4} \mid s_i \mid^2 \qquad \beta = (\ \mid s_1 \mid^2 + \mid s_2 \mid^2\ ) - (\ \mid s_3 \mid^2 \mid + \mid s_4 \mid^2\ ) \qquad (5.19)$$

Apparently, it has diversity order only two since each symbol passes through only two of the four transmit antennas. All symbols must be transmitted over every antenna to achieve full diversity, so it is clear a modification of the code matrix is required. All previously introduced QOSTBCs have the common design criterion that the data matrix is divided into groups where the columns of the code matrix are not orthogonal to each other, but columns of different groups are orthogonal to each other. Moreover, they have another property such that it is called "quasi-orthogonal" space-time code:

Quasi-Orthogonality Property: *A QOSTBC of dimension $N \times N$ is a matrix that satisfies $S_k S_k^H = \sum_{i=1}^{N} |s_i|^2 \cdot Q$ where Q is a sparse matrix with ones on its main diagonal positions and having at least $N^2/2$ zeros at off-diagonal positions.*

Conditions for Full Diversity

In order to achieve the maximum diversity, based on the two rank criterions mentioned in the ***chapter 4***, we not only make S_k become a full rank matrix but also maximize the minimum rank of the distance matrix $D \cdot D^H$ for all distinct codeword pairs $S_{k(i)}$ and $E_{k(i)}$ as possible. Check the determinant of the distance matrix may be used as a test to search for the symbol constellations that allow it to achieve full diversity. Due to the similar code structures of the *ABBA*, *Jafarkhani*, *Papadias and Foschini* and *STTD-OTD QOSTBCs*, we will only take *ABBA* as well as *ABAB QOSTBC* as the examples to show how to achieve the maximum diversity by means of the constellation rotations (CRs).

From (5.2), we can see that it is possible for $S_{4,rate1}^{ABBA}$ to lose rank due to the symmetry of the constellations. Therefore, in order to follow the *design criterion I*, which avoids $S_{4,rate1}^{ABBA}$ losing rank, some constellation rotations (CRs) are required. Since $S_{4,rate1}^{ABBA}$ can be divided into four Alamouti blocks, according to the determinant property, there exists many ways to make it become full rank. For example, we choose s_1 or s_2 (or both of them) from a constellation $\mathcal{C}$ and the others from another distinct constellation $\mathcal{C}_\theta$ rotated by an angle θ. For the sake of simplicity, we have $\theta = \pi/2$ for binary phase shift keying (BPSK) and $\theta = \pi/4$ for quadrature phase shift keying (QPSK) in this paper. Secondly, to achieve the maximum diversity, it is necessary to obey the *design criterion II*, which maximizes the rank of distance matrix. According to the data matrix in (5.2), the error matrix $D^{ABBA}_{4,rate-1} = S_{k(i)} - E_{k(i)}$ can be easily expressed as

$$D^{ABBA}_{4,rate-1} = \begin{bmatrix} \Delta_1 & \Delta_2 & \Delta_3 & \Delta_4 \\ -\Delta_2^* & \Delta_1^* & -\Delta_4^* & \Delta_3^* \\ \Delta_3 & \Delta_4 & \Delta_1 & \Delta_2 \\ -\Delta_4^* & \Delta_3^* & -\Delta_2^* & \Delta_1^* \end{bmatrix} \tag{5.20}$$

where $\Delta_i = s_{k(i)} - e_{k(i)}$ for $s_{k(i)} \neq e_{k(i)}$ in the k-th block. Based on the rank criterion, to achieve full diversity, the error matrix given should be full rank for all possible error matrix pairs. In other words, its determinant must be nonzero. For *ABAB QOSTBC*, the distance matrix $D^{ABBA}_{4,rate-1} \cdot (D^{ABBA}_{4,rate-1})^H$ and its determinant can be expressed respectively as

$$D^{ABBA}_{4,rate-1} \cdot (D^{ABBA}_{4,rate-1})^H = \begin{bmatrix} \Delta a \cdot I_2 & \Delta b \cdot I_2 \\ \Delta b \cdot I_2 & \Delta a \cdot I_2 \end{bmatrix} \tag{5.21}$$

where

$$\Delta a = \Sigma_{i=1}^{4} \mid \Delta_i \mid^2 \tag{5.22}$$

$$\Delta b = \Sigma_{i=1}^{2} [\ (\Delta_i)^* (\Delta_{i+2})\ +\ (\Delta_{i+2})(\Delta_i)^*] \tag{5.23}$$

and the the determinant of $D^{2\ (ABBA)}$ is

$$\begin{aligned} det(\ D^{2\ (ABBA)}\ ) &= det(\begin{bmatrix} \Delta a \cdot I_2 & \Delta b \cdot I_2 \\ \Delta b \cdot I_2 & \Delta a \cdot I_2 \end{bmatrix}) \\ &= (\ \Sigma_{i=1}^{2} \mid (\Delta_i) + (\Delta_{i+2}\ ) \mid^2) \\ &\quad \cdot (\ \Sigma_{i=1}^{2} \mid (\Delta_i) - (\Delta_{i+2}) \mid^2\ ) \end{aligned} \tag{5.24}$$

It is obvious that the determinant in (5.24) will be zero when $\Delta_1 = \Delta_2$ or $\Delta_3 = \Delta_4$ simultaneously. Similarly, based on the data matrix in (5.16), the error matrix $D^{ABAB}_{4,rate-1} =$

$S_{k(i)} - E_{k(i)}$ can be easily expressed as

$$D^{ABAB}_{4,rate-1} = \begin{bmatrix} \Delta_1 & \Delta_2 & \Delta_3 & \Delta_4 \\ -\Delta_2^* & \Delta_1^* & -\Delta_4^* & \Delta_3^* \\ \Delta_1 & \Delta_2 & -\Delta_3 & -\Delta_4 \\ -\Delta_2^* & \Delta_1^* & \Delta_4^* & -\Delta_3^* \end{bmatrix} \tag{5.25}$$

and the distance matrix $D^{ABAB}_{4,rate-1} \cdot (D^{ABAB}_{4,rate-1})^H$ and its determinant are

$$(D^{ABAB}_{4,rate-1})^H D^{ABAB}_{4,rate-1} = \begin{bmatrix} \Delta a \cdot I_2 & \Delta b \cdot I_2 \\ \Delta b \cdot I_2 & \Delta a \cdot I_2 \end{bmatrix} \tag{5.26}$$

where

$$\Delta a = \Sigma_{i=1}^4 \mid \Delta_i \mid^2 \tag{5.27}$$

$$\Delta b = (\mid \Delta_1 \mid^2 + \mid \Delta_2 \mid^2) - (\mid \Delta_3 \mid^2 + \mid \Delta_4 \mid^2) \tag{5.28}$$

and the the determinant is given by

$$det(\begin{bmatrix} \Delta a \cdot I_2 & \Delta b \cdot I_2 \\ \Delta b \cdot I_2 & \Delta a \cdot I_2 \end{bmatrix})$$
$$= (\mid s_1 - e_1 \mid^2 + \mid s_2 - e_2 \mid^2) \cdot (\mid s_3 - e_3 \mid^2 + \mid s_4 - e_4 \mid^2) \tag{5.29}$$

Note that the same problems occur with *Jafarkhani*, *Papadias and Foschini QOSTBCs* when the conventional memoryless modulation are applied. This means that such space-time code structures do not have the full diversity. Therefore, in order to follow the *design criterion I*, which avoids $S^{ABAB}_{4,rate2}$ losing rank, some CRs are required. For example, it can be seen from (5.29) that some correlation between the modulated symbols are required. Note that the use of joint modulation with a specially designed constellation set was proposed in [24]. In this thesis, we make use of some linear transformations of symbols to achieve the full diversity order. Now, we will use a short content to describe this approach. For *ABBA QOSTBC*, we firstly consider m_1, m_2, m_3,and m_4 are four symbols modulated from the original binary sources, and we encode these symbols by the following rules:

$$
\begin{aligned}
s_1 &= \frac{m_1 + m_2}{\sqrt{2}} \quad , \quad s_2 = \frac{m_1 - m_2}{\sqrt{2}} \\
s_3 &= \frac{m_3 + m_4}{\sqrt{2}} \quad , \quad s_4 = \frac{m_3 - m_4}{\sqrt{2}}
\end{aligned}
$$

where s_1, s_2, s_3,and s_4 are the transmit symbols mapped to the data matrix S_k. The coefficient $\sqrt{2}$ here is used to normalize the energy. Based on the transformation above, the determinant of the distance matrix in (5.29) can be reduced to

$$
\begin{aligned}
det(\ D^{2\ (ABBA)}\ ) &= (\ \Sigma_{i=1}^{2} |\ (s_i - e_i) + (s_{i+2} - e_{i+2}\ )\ |^2) \\
&\quad \cdot (\ \Sigma_{i=1}^{2} |\ (s_i - e_i) - (s_{i+2} - e_{i+2})\ |^2\ ) \\
&= (\ |(s_1 - e_1) + (s_3 - e_3)|^2 + |(s_2 - e_2) + (s_4 - e_4)|^2\ ) \\
&\quad \cdot (\ |(s_1 - e_1) - (s_3 - e_3)|^2 + |(s_2 - e_2) - (s_4 - e_4)|^2\ ) \\
&= \frac{1}{2}\{\ (\ |(m_1 - e_1) + (m_2 - e_2) + (m_3 - e_3) + (m_4 - e_4)|^2 \\
&\quad + |(m_1 - e_1) - (m_2 - e_2) + (m_3 - e_3) - (m_4 - e_4)|^2\ ) \\
&\quad \cdot (\ |(m_1 - e_1) + (m_2 - e_2) - (m_3 - e_3) - (m_4 - e_4)|^2 \\
&\quad + |(m_1 - e_1) - (m_2 - e_2) - (m_3 - e_3) + (m_4 - e_4)|^2\ )\}
\end{aligned} \tag{5.30}
$$

To achieve the full diversity, it is equivalent that the determinant of the distance matrix must be nonzero, which implies that the original data symbols $m_1 \sim m_4$ shall

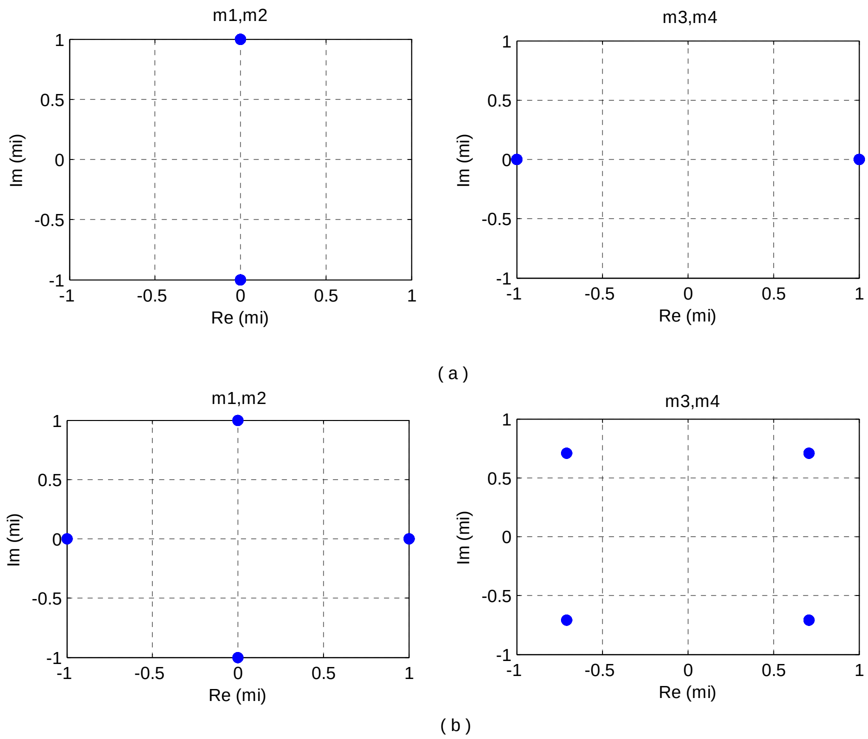

Figure 5.1: (a) BPSK Constellaitons (b) QPSK Constellaitons

follow

$$(m_1 - e_1) + (m_2 - e_2) + (m_3 - e_3) + (m_4 - e_4) \neq 0 \tag{5.31}$$

$$(m_1 - e_1) - (m_2 - e_2) + (m_3 - e_3) - (m_4 - e_4) \neq 0 \tag{5.32}$$

where (5.31) and (5.32) will hold when $m_1,\ m_3 \in \mathcal{C}$ and $m_2,\ m_4 \in \mathcal{C}^{\theta}$. In other words, we choose m_1 and m_2 from $\mathcal{C}$, and the others m_3 and m_4 from $\mathcal{C}^{\theta}$ rotated by an angle θ. For the sake of simplicity, we choose $\theta = \pi/2$ and $= \pi/4$ for BPSK and QPSK and the constellation points are shown in Figure.5.1. By choosing the suitable constellations and phase angle θ,and we can establish a QOSTBC that achieves the full-rate and full-diversity.

All previously introduced QOSTBCs have similar code structures, and the difference between them is the non-diagonal parameter in the corresponding non-orthogonal

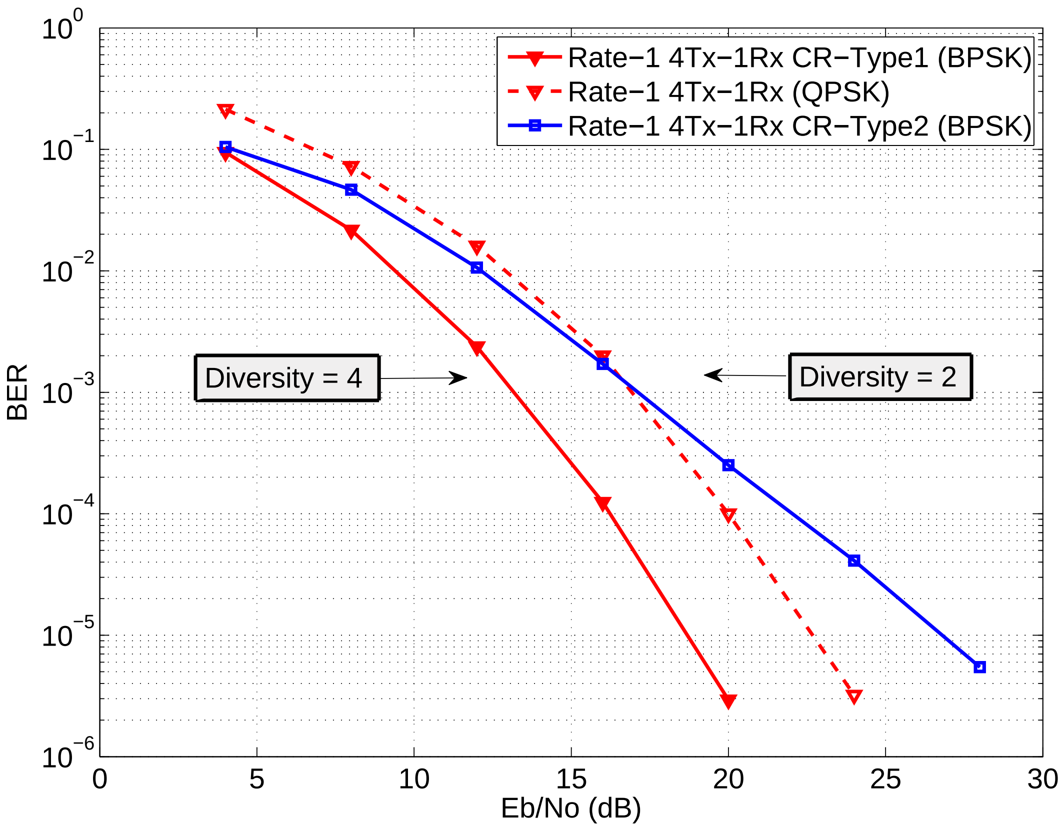

Figure 5.2: Bit error rate (BER) against signal to noise ratio (SNR) for rate-1 (R=1) differential space-time code at 1 and 2 bps/Hz ; four transmit antennas and one receive antenna ; $m_1, m_2 \in \mathcal{C}$ *and* $m_3, m_4 \in \mathcal{C}^{\theta}$; rotation angle $\theta = \pi/2$ and $= \pi/4$ for BPSK and QPSK respectively.

Grammian matrices in (5.3),(5.7),(5.13) and (5.17). Figure 5.2 shows the bit error rate (BER) as a function of the received SNR for four transmit and one receive antennas, rate-one DSTC based on the *ABBA QOSTBC* using two different CRs denoted as CR-Type1 and CR-Type2. For CR-Type1, we choose s_1 and s_2 from a constellation $\mathcal{C}$ and the others from another distinct constellation $\mathcal{C}_\theta$ rotated by an angle θ; for CR-Type2, we choose s_1 from $\mathcal{C}$ and the others from $\mathcal{C}_\theta$. It shows that the diversity order of DSTC is four whatever the constellation rotations are. Furthermore, the performance using QPSK modulation is approximately 3dB worse. The results are similar to the coherent STBC.

5.1.2 Rate-One for Eight Transmit Antennas

So far, we have presented the DSTCs based on quasi-orthogonal structures for four transmit antennas. In this section, we will discuss the case when the number of transmit antennas is greater than four. According to the code structure from *ABBA QOSTBC*, we combine two distinct 4×4 *ABBA QOSTBCs* to have a 8×8 QOSTBC, while keeping the transmission rate unchanged. Similarly, we start with the 4×4 rate-one DSTCs for four transmit antennas mentioned in the previous section as the building blocks,

$$S_{4\ rate-1}^{ABBA}(1:4)=\begin{bmatrix} s_1 & s_2 & s_3 & s_4 \\ -s_2^* & s_1^* & -s_4^* & s_3^* \\ s_3 & s_4 & s_1 & s_2 \\ -s_4^* & s_3^* & -s_2^* & s_1^* \end{bmatrix},\ S_{4\ rate-1}^{ABBA}(5:8)=\begin{bmatrix} s_5 & s_6 & s_7 & s_8 \\ -s_6^* & s_5^* & -s_8^* & s_7^* \\ s_7 & s_8 & s_5 & s_6 \\ -s_8^* & s_7^* & -s_6^* & s_5^* \end{bmatrix}$$

and for eight transmit antennas, the data matrix $S_k = S_{8,rate1}$ of the rate-one DSTC is given by

$$\begin{aligned} S_{8,rate-1} &= \begin{bmatrix} S_{4,rate-1}^{ABBA}(1:4) & S_{4,rate-1}^{ABBA}(5:8) \\ S_{4,rate-1}^{ABBA}(5:8) & S_{4,rate-1}^{ABBA}(1:4) \end{bmatrix} \\ &= \begin{bmatrix} s_1 & s_2 & s_3 & s_4 & s_5 & s_6 & s_7 & s_8 \\ -s_2^* & s_1^* & -s_4^* & s_3^* & -s_6^* & s_5^* & -s_8^* & s_7^* \\ s_3 & s_4 & s_1 & s_2 & s_7 & s_8 & s_5 & s_6 \\ -s_4^* & s_3^* & -s_2^* & s_1^* & -s_8^* & s_7^* & -s_6^* & s_5^* \\ s_5 & s_6 & s_7 & s_8 & s_1 & s_2 & s_3 & s_4 \\ -s_6^* & s_5^* & -s_8^* & s_7^* & -s_2^* & s_1^* & -s_4^* & s_3^* \\ s_7 & s_8 & s_5 & s_6 & s_3 & s_4 & s_1 & s_2 \\ -s_8^* & s_7^* & -s_6^* & s_5^* & -s_4^* & s_3^* & -s_2^* & s_1^* \end{bmatrix} \end{aligned} \tag{5.33}$$

In order to achieve the full diversity, the rank of the data matrix $S_{8\ rate-1}$ must be full rank. According to the following determinant property of block matrices

$$det(\begin{bmatrix} A & B \\ C & D \end{bmatrix}) = det(A) \cdot det(D - CA^{-1}B) \tag{5.34}$$

where A,B,C and D are square matrices respectively. The determinant of the data matrix $S_{8,rate-1}$ thus can be expressed as

$$\begin{aligned} det(S_{8,rate-1}) &= det(S_{4,rate-1}^{ABBA}(1:4)) \\ &\cdot det(S_{4,rate-1}^{ABBA}(1:4) - S_{4,rate-1}^{ABBA}(5:8)S_{4,rate-1}^{ABBA}(1:4)^{-1}S_{4,rate-1}^{ABBA}(5:8)) \end{aligned} \tag{5.35}$$

We note that the form $S_{4,rate-1}^{ABBA}(5:8)S_{4,rate-1}^{ABBA}(1:4)^{-1}S_{4,rate-1}^{ABBA}(5:8)$ has an quasi-orthogonal structure. To maximize the diversity order, it is necessary to follow

$$det(S_{4,rate-1}^{ABBA}(1:4)) \neq 0 \tag{5.36}$$

$$det(S_{4,rate-1}^{ABBA}(1:4) - S_{4,rate-1}^{ABBA}(5:8)S_{4,rate-1}^{ABBA}(1:4)^{-1}S_{4,rate-1}^{ABBA}(5:8)) \neq 0 \tag{5.37}$$

To satisfy the constraint in (5.35), it requires some constellation rotations of the

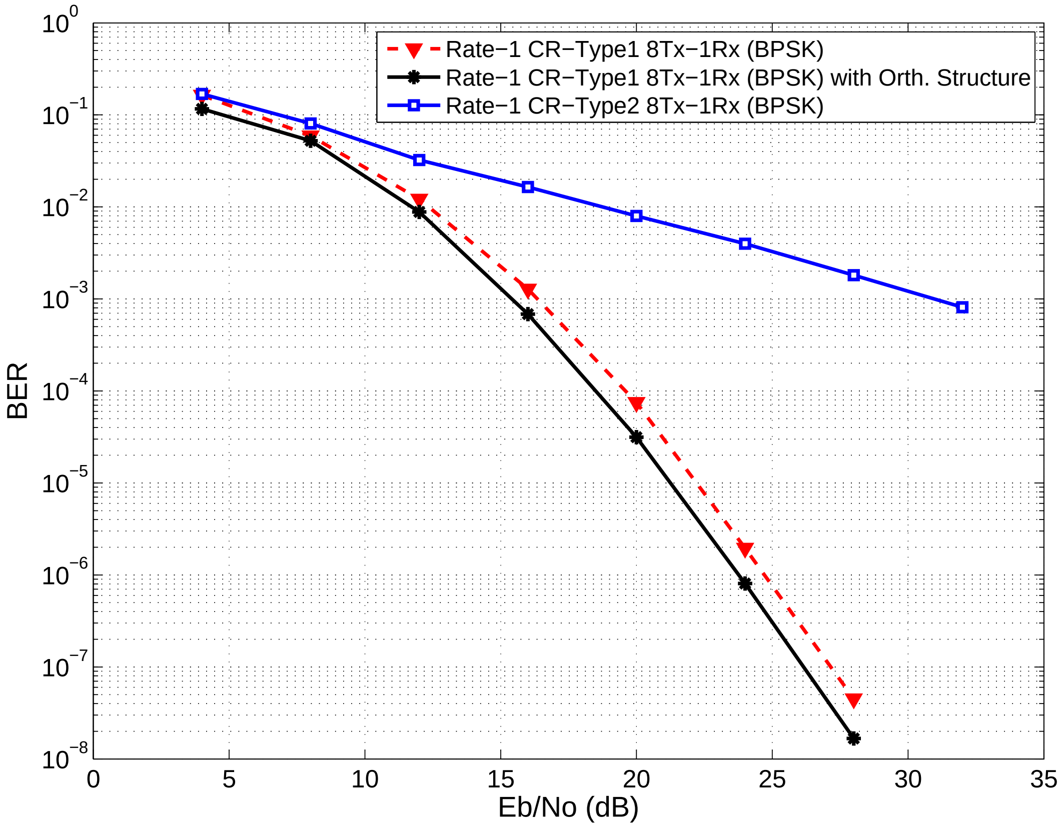

Figure 5.3: BER against SNR for rate-1 (R=1) differential space-time code at 1 bps/Hz ; eight transmit antennas and one receive antenna ; CR-Type1: $s_1, s_2 \in \mathcal{C}$ *and* $s_3 \sim s_8 \in \mathcal{C}^\theta$; CR-Type2: $s_1 \in \mathcal{C}$ *and* $s_2 \sim s_8 \in \mathcal{C}^\theta$ rotation angle $\theta = \pi/2$ and $= \pi/4$ for BPSK and QPSK respectively.

modulated symbols. We consider the determinant of $S_{4\ rate-1}^{ABBA}$ as

$$\begin{aligned} det(S_{4,rate-1}^{ABBA}) &= det(\begin{bmatrix} S_{12} & S_{34} \\ S_{34} & S_{12} \end{bmatrix}) \\ &= det(S_{12}) \\ &\qquad \cdot\, det(S_{12} - S_{34}S_{12}^{-1}S_{34}) \\ &= 2\cdot\ det(\begin{bmatrix} s_3 & s_4 \\ -s_4^* & s_3^* \end{bmatrix} - \frac{1}{2}\begin{bmatrix} p_1 & p_2 \\ -p_2^* & p_1^* \end{bmatrix}) \\ &= 2\ \{\ |\ s_3 - \frac{1}{2}p_1\ |^2 + |\ s_4 - \frac{1}{2}p_2\ |^2\ \} \end{aligned} \tag{5.38}$$

where

$$p_1 = -s_3s_1^*s_3 + s_4s_2^*s_3 - s_3s_2s_4^* - s_4s_1s_4^* \tag{5.39}$$

$$p_2 = -s_3s_1^*s_4 + s_4s_2^*s_4 + s_3s_2s_3^* + s_4s_1s_3^* \tag{5.40}$$

Therefore, $S_{4,rate-1}^{ABBA}$ is full rank ($det(S_{4,rate-1}^{ABBA}) \neq 0$) as long as

$$s_3 \neq \frac{1}{2}\ p_1 \quad and \quad s_4 \neq \frac{1}{2}\ p_2 \tag{5.41}$$

We consider constellations using phase rotations. Conditions (5.41) hold when s_3 or s_4 are rotated by an shift angle θ with respect to s_1 and s_2. On the other hand, we must maximize the rank of the distance matrix $S_{8,rate-1}) \cdot (S_{8,rate-1})^H$ to achieve the full diversity. However, its determinant is hard to derive and analyze. By computer search, the rank of the distance matrix is four, and then the diversity order of the rate-one DSTC for eight transmit antennas and one receive antennas is $M \cdot 4 = 4$. In this case, we assume that s_3 and s_4 are rotated by an shift angle $\theta = \pi/2$ for BPSK and $\theta = \pi/4$

for QPSK with respect to $s_1 \sim s_2$. The constellation points are shown in Figure 5.1. The simulation results are shown in Figure 5.3. We assume that there are two kinds of constellation rotations, which are called constellation rotation type 1 (CR-Type1) and type 2 (CR-Type1) respectively. In CR-Type1, we choose the data symbols s_1, s_2 from a constellation $\mathcal{C}$, and the others $s_3 \sim s_8$ from another constellation $\mathcal{C}^\theta$ rotated by an angle θ. In CR-Type2, we choose s_1 from $\mathcal{C}$ and $s_2 \sim s_8$ from $\mathcal{C}^\theta$. Our simulation shows that the diversity order using CR-Type1 is four, but less than four when using CR-Type2. The reason is the rank of distance matrix is only two in the case of CR-Type2, and thus the diversity order is only $M \cdot 2 = 2$. On the other hand, we denote the orthogonal structure here as another matrix mapping approach for the data matrix $S_{8,rate-1}$ and it can shown as follows

$$S_{8,rate-1} = \begin{bmatrix} S_{4,rate-1}^{ABBA}(1:4) & S_{4,rate-1}^{ABBA}(5:8) \\ -S_{4,rate-1}^{ABBA}(5:8)^H & S_{4,rate-1}^{ABBA}(1:4)^H \end{bmatrix} \tag{5.42}$$

which is Alamouti-like matrix mapping approach. Compared with our proposed *ABBA QOSTBC*, the performance by using this orthogonal structure is 1dB better.

5.2 Rate-Two Differential Space-Time Code

5.2.1 Rate-Two for Four Transmit Antennas

Based on the code structure of the rate-one *ABBA QOSTBC* in [20], we start with the Alamouti schemes for two transmit antennas as the building blocks,

$$S_{12} = \begin{bmatrix} s_1 & s_2 \\ -s_2^* & s_1^* \end{bmatrix} \quad S_{34} = \begin{bmatrix} s_3 & s_4 \\ -s_4^* & s_3^* \end{bmatrix} \quad S_{56} = \begin{bmatrix} s_5 & s_6 \\ -s_6^* & s_5^* \end{bmatrix} \quad S_{78} = \begin{bmatrix} s_7 & s_8 \\ -s_8^* & s_7^* \end{bmatrix}$$

and the rate-two DSTC defined by its data matrix $S_k = S_{4,rate-2}$ is given by

$$S_{4,rate-2} = \begin{bmatrix} S_{12} & S_{34} \\ S_{56} & S_{78} \end{bmatrix} = \begin{bmatrix} s_1 & s_2 & s_3 & s_4 \\ -s_2^* & s_1^* & -s_4^* & s_3^* \\ s_5 & s_6 & s_7 & s_8 \\ -s_6^* & s_5^* & -s_8^* & s_7^* \end{bmatrix} \quad (5.43)$$

By multiplying the data matrix by its Hermitian, we have the Grammian matrix

$$S_{4,rate-2}S_{4,rate-2}^H = \begin{bmatrix} \alpha & 0 & q_1 & q_2 \\ 0 & \alpha & -q_2^* & q_1^* \\ q_1^* & -q_2 & \beta & 0 \\ q_2^* & q_1 & 0 & \beta \end{bmatrix} = \begin{bmatrix} \alpha I_2 & Q_{rate-2} \\ Q_{rate-2}^H & \beta I_2 \end{bmatrix} \quad (5.44)$$

where Q_{rate-2} is an orthogonal matrix (Alamouti-like matrix) with

$$\alpha = \sum_{i=1}^{4} \mid s_i \mid^2 \quad , \quad \beta = \sum_{i=5}^{8} \mid s_i \mid^2 \quad (5.45)$$

and $q_1 = s_1 s_5^* + s_2 s_6^* + s_3 s_7^* + s_4 s_8^*$, $q_2 = -s_1 s_6 + s_2 s_5 - s_3 s_8 + s_4 s_7$. In order to achieve the maximum diversity, according to the rank criterion in the ***chapter 4***, we not only make the data matrix S_k become full rank, but also maximize the minimum of the rank of distance matrix $D_{4,rate-2}(D_{4,rate-2})^H$ for all distinct code matrix pairs $S_{k(i)}$ and $E_{k(i)}$ as possible. At first, we consider the rank of the data matrix $S_k = S_{4,rate2}$. Similarly, due to the data matrix can be divided into four submatrices, and its determinant is given by

$$\begin{aligned} det(S_{4,rate-2}) &= det(\begin{bmatrix} S_{12} & S_{34} \\ S_{56} & S_{78} \end{bmatrix}) \\ &= det(S_{12}) \cdot det(S_{78} - S_{56} S_{12}^{-1} S_{34}) \\ &= 2 \cdot \ det(\begin{bmatrix} s_7 & s_8 \\ -s_8^* & s_7^* \end{bmatrix} - \frac{1}{2} \begin{bmatrix} p_1 & p_2 \\ -p_2^* & p_1^* \end{bmatrix}) \\ &= 2 \, \{ \mid s_7 - \frac{1}{2} p_1 \mid^2 + \mid s_8 - \frac{1}{2} p_2 \mid^2 \} \end{aligned} \quad (5.46)$$

where $p_1 = -s_5 s_1^* s_3 + s_6 s_2^* s_3 - s_5 s_2 s_4^* - s_6 s_1 s_4^*$ and $p_2 = -s_5 s_1^* s_4 + s_6 s_2^* s_4 + s_5 s_2 s_3^* + s_6 s_1 s_3^*$. Therefore, $S_{4,rate-2}$ is full rank (i.e., $det(S_{4,rate-2}) \neq 0$) as long as

$$s_7 \neq \frac{1}{2} p_1 \quad and \quad s_8 \neq \frac{1}{2} p_2 \quad (5.47)$$

We consider constellations using phase rotations. Conditions (5.49) holds when s_7 and s_8 are rotated by an shift angle θ with respect to $s_1 \sim s_6$. For simplicity, we choose $\theta = \pi/2$ and $= \pi/4$ for BPSK and QPSK and the constellation points are shown in Figure.5.1. Secondly, we take the rank of the distance matrix into consideration. The error matrix $D_{4,rate2}$ can be easily expressed as

$$D_{4,rate-2} = \begin{bmatrix} \Delta_1 & \Delta_2 & \Delta_3 & \Delta_4 \\ -\Delta_2^* & \Delta_1^* & -\Delta_4^* & \Delta_3^* \\ \Delta_5 & \Delta_6 & \Delta_7 & \Delta_8 \\ -\Delta_6^* & \Delta_5^* & -\Delta_8^* & \Delta_7^* \end{bmatrix} \quad (5.48)$$

where $\Delta_i = s_{k(i)} - e_{k(i)}$, and the distance matrix can then be expressed as

$$D_{rate-2} \cdot (D_{rate-2})^H = \begin{bmatrix} \Delta\alpha \cdot I_2 & \Delta Q_{rate-2} \\ \Delta Q^H_{rate-2} & \Delta\beta \cdot I_2 \end{bmatrix} \quad (5.49)$$

and its determinant is

$$\begin{aligned} & det(\begin{bmatrix} \Delta\alpha \cdot I_2 & \Delta Q_{rate-2} \\ \Delta Q^H_{rate-2} & \Delta\beta \cdot I_2 \end{bmatrix}) \\ &= (\ \Delta\alpha\Delta\beta\ - \Delta Q_{rate-2}\Delta Q^H_{rate-2}\ ) \\ &= (\ \Delta\alpha\Delta\beta\ - (\ \Delta q_1^2 + \Delta q_2^2\ )) \\ &= (\Delta_1^2 + \Delta_2^2)(\Delta_7^2 + \Delta_8^2) + (\Delta_3^2 + \Delta_4^2)(\Delta_5^2 + \Delta_6^2) \\ &\qquad - 2Re\{\Delta_1\Delta_3^*\Delta g + \Delta_1\Delta_4^*\Delta h\ + \Delta_2\Delta_3^*(-\Delta h)^* + \Delta_2\Delta_4^*(\Delta g)^*\} \end{aligned} \quad (5.50)$$

where $\Delta_i = s_{k(i)} - e_{k(i)}$ for $s_{k(i)} \neq e_{k(i)}$, $\Delta g = \Delta_5^*\Delta_7 + \Delta_6\Delta_8^*$ and $\Delta h = \Delta_5^*\Delta_8 + \Delta_6\Delta_7^*$. Unfortunately, it is obvious that there exists at least one condition that makes the result in (14) become zero whatever the constellation rotations are. For instance, when $\Delta_1 = \Delta_2 = \Delta_3 = \Delta_4 = 0$ or $\Delta_5 = \Delta_6 = \Delta_7 = \Delta_8 = 0$, $det(D_{4,rate-2} \cdot (D_{4,rate-2})^H) = 0$. In other words, it is impossible for rate-two DSTC to achieve full diversity. In general, the performance is always bounded by the worst case of the DSTC. By computer search, the minimum rank of the distance matrix based on the data matrix designed by the *design criterion I* is only two, and the diversity order is then $2 \cdot M$. Intuitively, a higher transmission rate corresponds to a smaller diversity order depending on a general rate-diversity tradeoff. The simulation results are shown in the figure 5.4. We note that there are two kinds of constellation rotation approaches which are named constellation rotation type 1 (CR-Type1) and type 2 (CR-Type1) respectively. In CR-Type1, we choose the data symbols s_1, s_2, s_7, s_8 from a constellation $\mathcal{C}$, and the others $s_3 \sim s_6$ from another constellation $\mathcal{C}^\theta$ rotated by an angle θ. In CR-Type2, we choose s_1, s_7 from $\mathcal{C}$

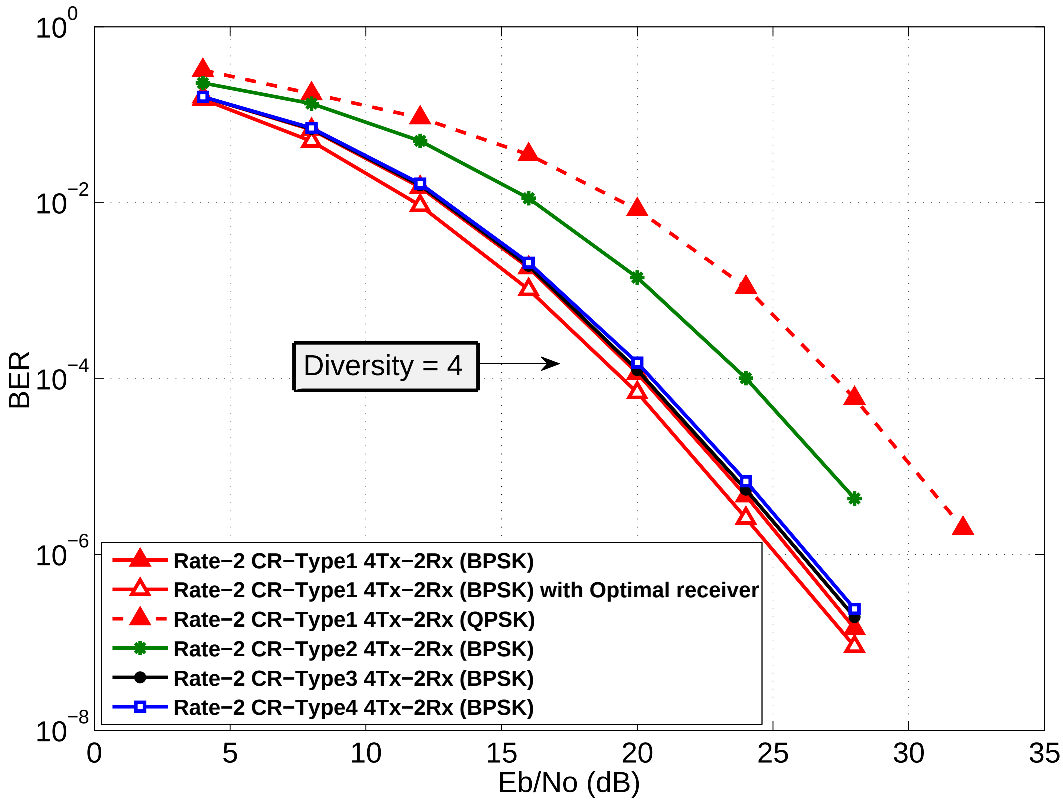

Figure 5.4: BER against SNR for rate-2 (R=2) differential space-time code at 1 and 2 bps/Hz ; four transmit antennas and two receive antennas ; CR-Type1: $s_1, s_2, s_7, s_8 \in \mathcal{C}$ *and* $s_3 \sim s_6 \in \mathcal{C}^\theta$; CR-Type2: $s_1, s_7 \in \mathcal{C}$ *and* $s_2 \sim s_6, s_8 \in \mathcal{C}^\theta$; rotation angle $\theta = \pi/2$ and $= \pi/4$ for BPSK and QPSK respectively.

and $s_2 \sim s_6, s_8$ from $\mathcal{C}^\theta$. The simulation shows that the diversity order of DSTC using CR-Type1 is four which is better than the case using CR-Type2. It implies that different constellation rotation approaches cause different performance. Moreover, our the simulation results match to the analytic results obtained form the derivations of pairwise error probability. Earlier researches show that there is a rate-diversity tradeoff achieved by the coherent MIMO systems [25]. For noncoherent multiple antenna systems, such a rate-diversity tradeoff is still an open question, and there is not a formal mathematical equation to prove this problem. Intuitively, a larger transmission rate corresponds to a smaller diversity order.

5.2.2 Rate-2 for Eight Transmit Antennas

Following by the code construction of rate-one DSTC for eight transmit and one receive antennas, we combine two distinct 4×4 rate-two QOSTBCs obtained in the previous subsection to have a 8×8 rate-two DSTC. Since the number of parallel streams should not exceed min $\{N, M\}$, the number of receive antennas must also be equal two. For eight transmit antennas, the data matrix $S_{8\ rate-2}$ of the rate-two DSTC is

$$
\begin{aligned}
S_{8,rate2} &= \begin{bmatrix} S_{4,rate2}(1:8) & S_{4,rate1}(9:16) \\ S_{4,rate2}(9:16) & S_{4,rate1}(1:8) \end{bmatrix} \\
&= \begin{bmatrix}
s_1 & s_2 & s_3 & s_4 & s_9 & s_{10} & s_{11} & s_{12} \\
-s_2^* & s_1^* & -s_4^* & s_3^* & -s_{11}^* & s_{10}^* & -s_{12}^* & s_{11}^* \\
s_5 & s_6 & s_7 & s_8 & s_{13} & s_{14} & s_{15} & s_{16} \\
-s_6^* & s_5^* & -s_8^* & s_7^* & -s_{14}^* & s_{13}^* & -s_{16}^* & s_{15}^* \\
s_9 & s_{10} & s_{11} & s_{12} & s_1 & s_2 & s_3 & s_4 \\
-s_{10}^* & s_9^* & -s_{12}^* & s_{11}^* & -s_2^* & s_1^* & -s_4^* & s_3^* \\
s_{13} & s_{14} & s_{15} & s_{16} & s_5 & s_6 & s_7 & s_8 \\
-s_{14}^* & s_{13}^* & -s_{16}^* & s_{15}^* & -s_6^* & s_5^* & -s_8^* & s_7^*
\end{bmatrix}
\end{aligned}
\tag{5.51}
$$

where $S_{4,rate2}(1:8)$ and $S_{4,rate2}(9:16)$ are defined as

$$
S_{4,rate2}(1:8) = \begin{bmatrix} s_1 & s_2 & s_3 & s_4 \\ -s_2^* & s_1^* & -s_4^* & s_3^* \\ s_5 & s_6 & s_7 & s_8 \\ -s_6^* & s_5^* & -s_8^* & s_7^* \end{bmatrix}, S_{4,rate2}(9:16) = \begin{bmatrix} s_9 & s_{10} & s_{11} & s_{12} \\ -s_{10}^* & s_9^* & -s_{12}^* & s_{11}^* \\ s_{13} & s_{14} & s_{15} & s_{16} \\ -s_{14}^* & s_{13}^* & -s_{16}^* & s_{15}^* \end{bmatrix}
$$

In order to achieve the full diversity, the rank of the data matrix $S_{8,rate2}$ must be full rank. Similarly, the determinant of $S_{8,rate2}$ is given by

$$\begin{aligned} det(S_{8,rate2}) &= det(S_{4,rate2}(1:8)) \cdot \\ & det(S_{4,rate2}(1:8) - S_{4,rate2}(9:16) S_{4\ rate2}(1:8)^{-1} S_{4,rate2}(9:16)) \end{aligned} \tag{5.52}$$

Due to the quaternion property, $S_{4,rate2}(9:16)S_{4,rate2}(1:8)^{-1}S_{4,rate2}(9:16)$ is still an quasi-orthogonal structure. Therefore, to maximize the diversity order, it is necessary to follow

$$det(S_{4,rate2}(1:8)) \neq 0 \tag{5.53}$$

$$det(S_{4,rate2}(1:8) - S_{4,rate2}(9:16) S_{4,rate2}(1:8)^{-1} S_{4,rate2}(9:16)) \neq 0 \tag{5.54}$$

Based on the results in the rate-two DSTC for four transmit antennas, (5.53) and (5.54) will hold when s_7 and s_8 are rotated by an shift angle $\theta = \pi/2$ for BPSK and $\theta = \pi/4$ for QPSK with respect to $s_1 \sim s_6$. Similarly, since the determinant of the distance matrix is hard to analyze, the rank of the distance matrix is two such that the diversity order is $M \cdot 2 = 4$. The simulation results are shown in Figure 5.5. We only the constellation rotation type 1 (CR-Type1) in this case since it has better performance than the other constellation rotation approaches. In CR-Type1, we choose the data symbols s_1, s_2, s_7, s_8 from a constellation $\mathcal{C}$, and the others $s_3 \sim s_6$ from another constellation $\mathcal{C}^\theta$ rotated by an angle θ. The simulation shows that the diversity order using CR-Type1 is four that matches to the analytical results obtained from PEP. The reason is that the rank of distance matrix is only two, and thus the diversity order is only $M \cdot 2 = 4$. Also, we denote the orthogonal structure here as another matrix mapping approach for the data

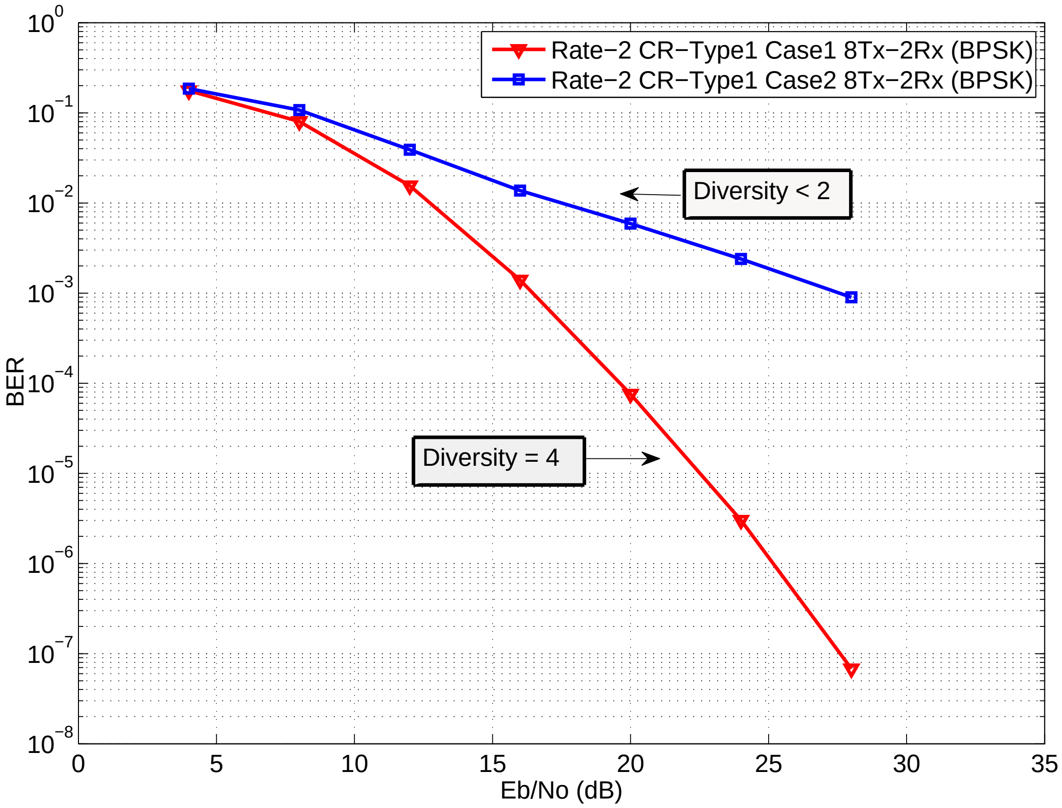

Figure 5.5: BER against SNR for rate-1 (R=1) differential space-time code at 1 bps/Hz ; eight transmit antennas and two receive antennas ; CR-Type1: $s_1, s_2, s_7, s_8 \in \mathcal{C}$ *and* $s_3 \sim s_6 \in \mathcal{C}^\theta$; rotation angle $\theta = \pi/2$ and $= \pi/4$ for BPSK and QPSK respectively

matrix $S_{8,rate2}$ and it can shown as follows

$$S_{8,rate2} = \begin{bmatrix} S_{4,rate2}(1:8) & S_{4,rate1}(9:16) \\ -S_{4,rate2}(9:16)^H & S_{4,rate1}(1:8)^H \end{bmatrix} \tag{5.55}$$

which is Alamouti-like matrix mapping approach. Compared with our proposed *ABBA* structure, the performance by using this orthogonal structure is approximately the same; however, this structure has no quaternion property such that some lower complexity receivers are not feasible under this approach.

Chapter 6

Conclusions

A rate-two DSTC is proposed in this work and is applicable to four and eight transmit antennas. Furthermore, the upper bound of the PEP is also derived, and it gives a theoretical justification for the achievable diversity order of the proposed DSTC scheme. With the assumption of a full rank data matrix S_k, the derivations show that the diversity order is equal to the rank of the distance matrix multiplied by the number of receive antennas M. Based on the PEP expression, we provide a rank design criterion on the construction of the rate-two DSTC for four and eight transmit antennas. The simulation results match the analysis obtained from the PEP, and achieve the diversity order of four for four and eight transmit antennas respectively. Particularly, it coincides with the noncoherent rate-diversity tradeoff in [18] for the case of four transmit and two receive antennas and T=4.

Appendix A

Let X and Y be two random variables. Supposed that before we know that $Y = y$, the random variable X has a probability density function (pdf) $p_X(x)$. Being told that $Y = y$ has the effect of modifying the probability density. The modified probability density function (pdf) is

$$p_{X|Y}(x|y) = \frac{p_{X,Y}(x,y)}{p_Y(y)} \tag{A.1}$$

assuming that $p_Y(y) \neq 0$. Consider a simple communication system as follows

$$y = x + n \tag{A.2}$$

where x, y, n the values of scalar random variables Y, X, and noise.

Let the pair of vectors X and Y be jointly Gaussian, i.e., with $Z = [\ X^T \quad Y^T\ ]^T$; Z is gaussian with mean and covariance

$$m = \begin{bmatrix} E[x] \\ E[y] \end{bmatrix} = \begin{bmatrix} \bar{x} \\ \bar{y} \end{bmatrix} \quad and \quad \Sigma = \begin{bmatrix} \Sigma_{xx} & \Sigma_{xy} \\ \Sigma_{yx} & \Sigma_{yy} \end{bmatrix} \tag{A.3}$$

respectively. Thus, the probability of X conditioned on $Y = y$ is given by

$$\begin{aligned} p_{X|Y}(x|y) &= \frac{p_{X,Y}(x,y)}{p_Y(y)} \\ &= \frac{1}{(2\pi)^{N/2}} \cdot \frac{|\ \Sigma_{yy}\ |^{1/2}}{|\ \Sigma\ |^{1/2}} \cdot \\ & \quad \frac{exp(-\frac{1}{2}[x^T - \bar{x}^T \quad y^T - \bar{y}^T]\ \Sigma^{-1}[x^T - \bar{x}^T \quad y^T - \bar{y}^T]^T)}{exp(-\frac{1}{2}(y-\bar{y})^T\ \Sigma_{yy}^{-1}\ (y-\bar{y}))} \end{aligned} \tag{A.4}$$

where N is the dimension of X. By using the simple check formula

$$\begin{bmatrix} I & -\Sigma_{xy}\Sigma_{yy}^{-1} \\ 0 & I \end{bmatrix} \Sigma \begin{bmatrix} I & 0 \\ -\Sigma_{yy}^{-1}\Sigma_{xy}^T & I \end{bmatrix} = \begin{bmatrix} \Sigma_{xx} - \Sigma_{xy}\Sigma_{yy}^{-1}\Sigma_{yx} & 0 \\ 0 & \Sigma_{yy} \end{bmatrix} \tag{A.5}$$

Σ in (4.20) can be expressed as

$$\Sigma = \begin{bmatrix} I & \Sigma_{xy}\Sigma_{yy}^{-1} \\ 0 & I \end{bmatrix} \begin{bmatrix} \Sigma_{xx} - \Sigma_{xy}\Sigma_{yy}^{-1}\Sigma_{yx} & 0 \\ 0 & \Sigma_{yy} \end{bmatrix} \begin{bmatrix} I & 0 \\ \Sigma_{yy}^{-1}\Sigma_{xy}^T & I \end{bmatrix} \tag{A.6}$$

and the inverse of Σ is

$$\Sigma^{-1} = \begin{bmatrix} I & 0 \\ -\Sigma_{yy}^{-1}\Sigma_{xy}^T & I \end{bmatrix} \begin{bmatrix} (\Sigma_{xx} - \Sigma_{xy}\Sigma_{yy}^{-1}\Sigma_{yx})^{-1} & 0 \\ 0 & \Sigma_{yy}^{-1} \end{bmatrix} \begin{bmatrix} I & -\Sigma_{xy}\Sigma_{yy}^{-1} \\ 0 & I \end{bmatrix} \tag{A.7}$$

Taking the determinants in (4.21) based on the determinant property (i.e., $|ABC| = |A| \cdot |B| \cdot |C|$), we have

$$|\ \Sigma\ | = |\ \Sigma_{xx} - \Sigma_{xy}\Sigma_{yy}^{-1}\Sigma_{yx}\ | \cdot |\ \Sigma_{yy}\ | \tag{A.8}$$

Therefore, it yields

$$[x^T - \bar{x}^T \quad y^T - \bar{y}^T]\ \Sigma^{-1}[x^T - \bar{x}^T \quad y^T - \bar{y}^T]^T$$

$$= [x^T - \bar{x}^T \quad y^T - \bar{y}^T] \begin{bmatrix} I & 0 \\ -\Sigma_{yy}^{-1}\Sigma_{xy}^T & I \end{bmatrix} \begin{bmatrix} (\Sigma_{xx} - \Sigma_{xy}\Sigma_{yy}^{-1}\Sigma_{yx})^{-1} & 0 \\ 0 & \Sigma_{yy}^{-1} \end{bmatrix}$$

$$\times \begin{bmatrix} I & -\Sigma_{xy}\Sigma_{yy}^{-1} \\ 0 & I \end{bmatrix} [x^T - \bar{x}^T \quad y^T - \bar{y}^T]^T \tag{A.9}$$

$$= (\ x^T - \bar{x}^T - \Sigma_{yy}^{-1}\ \Sigma_{xy}^T\ (y^T - \bar{y}^T)\ )(\Sigma_{xx} - \Sigma_{xy}\Sigma_{yy}^{-1}\Sigma_{yx})^{-1}$$

$$\cdot (\ x - \bar{x} - \Sigma_{yy}^{-1}\ \Sigma_{xy}^T\ (y - \bar{y})\ ) + (y^T - \bar{y}^{-1})\ \Sigma_{yy}^{-1}\ (y - \bar{y}) \tag{A.10}$$

$$= (x^T - \hat{x}^T)\ (\Sigma_{xx} - \Sigma_{xy}\Sigma_{yy}^{-1}\Sigma_{yx})^{-1}(X - \hat{X}) + (y^T - \hat{y}^T)\Sigma_{yy}^{-1}(y - \hat{y}) \tag{A.11}$$

where $\hat{x} = \bar{x} + \Sigma_{xy}\Sigma_{yy}^{-1}\ (y - \bar{y})$. Substituting (4.26) in (4.19) gives the conditional probability of X given for $Y = y$

$$p_{X|Y}(x|y) = \frac{p_{X,Y}(x,y)}{p_Y(y)}$$

$$= \frac{1}{(2\pi)^{N/2}} \cdot \frac{|\ \Sigma_{YY}\ |^{1/2}}{|\ \Sigma\ |^{1/2}}$$

$$\cdot \frac{exp(-\frac{1}{2}[x^T - \bar{x}^T \quad y^T - \bar{y}^T]\ \Sigma^{-1}[x^T - \bar{x}^T \quad y^T - \bar{y}^T]^T)}{exp(-\frac{1}{2}(y - \bar{y})^T\ \Sigma_{yy}^{-1}\ (y - \bar{y}))}$$

$$= \frac{1}{(2\pi)^{N/2} \cdot |\ \Sigma_{xx} - \Sigma_{xy}\Sigma_{yy}^{-1}\Sigma_{yx}\ |^{1/2}}$$

$$\cdot\ exp(\ -\frac{1}{2}\ (x^T - \hat{x}^T)\ (\Sigma_{xx} - \Sigma_{xy}\Sigma_{yy}^{-1}\Sigma_{y}x)^{-1}\ (x - \hat{x})\ ) \tag{A.12}$$

As claimed then, X is indeed conditionally Gaussian. In fact, this is true even when Σ and Σ_{yy} are singular. The result shows that the random variable X conditioned on $Y = y$ has conditional mean $\bar{x} + \Sigma_{xy}\Sigma_{yy}^{-1}\ (y - \bar{y})$ and conditional covariance $\Sigma_{xx} - \Sigma_{xy}\Sigma_{yy}^{-1}\Sigma_{y}x$.

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