國 立 交 通 大 學

電信工程學系

碩 士 論 文

應用於無線干擾環境與多波單音干擾下結合時空編碼技術及跳頻展頻系統之研究

Combined Space-Time Coding with Frequency-Hopping Spread Spectrum for Wireless Channels with Multitone Jammers

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研究生：沈晏麟　　Student: Yan Lin Shen
指導教授：王忠炫　　Advisor: Chung-Hsuan Wang

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摘要

在無線傳輸的環境中，傳輸的信號常遭受到惡意的干擾源及通道衰減效應，導致接收訊號產生嚴重的失真。跳頻展頻系統是一般最常用來抑制干擾效應的技術，而具有分集增益及編碼增益的時空編碼技術可有效的降低通道衰減效應。因此，在本篇論文裡吾人便結合了兩者之優點，提出了時空編碼結合跳頻展頻技術以提升傳輸系統在無線干擾環境中之整體效能。

為了能夠專注於分析時空碼的解碼設計，吾人考慮了兩種較簡單的跳頻方式。第一種定義為所有傳送天線的訊號都跳至相同的頻帶上，稱此為最差跳頻。第二種情形為所有傳送信號皆設計為避免互相發生碰撞，稱此為最佳跳頻。其中最差及最佳跳頻方式分別代表為此系統效能分析的上界與下界。針對上述跳頻方式，吾人推導出在路徑增益已知或未知情況下此系統的最大可能性解碼。吾人亦針對兩種不同的跳頻系統推導出建立適合的空時編碼的準則。此外，吾人亦針對此系統提出了在無線干擾環境下好的時空碼準則。最後經由模擬結果驗證出，在相同的訊雜比及頻寬效益的考量之下，此系統比傳統單進單出編碼效能來的更佳。

Combined Space-Time Coding with Frequency-Hopping Spread Spectrum for Wireless Jamming Channels with Multitone Jammers

Student: Yan Lin Shen Advisor: Chung-Hsuan Wang
Department of Communication Engineering
National Chiao Tung University

Abstract

In wireless jamming environments, the transmitted signals usually suffer from hostile jammers and undesired channel impairments, e.g., multipath fading. Conventionally, frequency-hopping spread spectrum (FHSS) systems are most effective anti-jamming techniques, and space-time coding (STC), which introduces temporal and spatial correlation into the transmitted signals to achieve transmitter diversity without sacrificing the bandwidth, has been shown to provide excellent performance against multipath fading. Therefore, in this thesis, we combine STC with the FHSS to construct a powerful high-rate transmission scheme for wireless jamming channels.

Two cases of FH are considered here to simplify the design of STC. One is the worst-case frequency hopping which hops the symbols from all transmitter antennas into the same frequency band, and the other is the perfect frequency hopping which avoids any possible collision of the transmitted symbols. The actual performance of the combined STC/FHSS system with arbitrary hopping patterns can then be upper and lower bounded by the evaluated performance of the worst case and perfect case, respectively. The maximum likelihood decoding of space-time codes is derived with respect to different reception conditions, and the design criteria for constructing good space-time codes with respect to two kinds of FH are also derived. Verified by the simulation results, the proposed system can provide better performance than the conventional schemes in terms of both bandwidth efficiency and signal-to-noise ratio.

誌謝

時間過的非常的快，回顧這兩年的研究生活，真是讓我人生多了很多歷練，首先要感謝指導老師王忠炫教授，這段時間在研究上給予細心的指導與耐心教誨，令我在研究上及做事的態度上成長許多、獲益匪淺，非常感謝您！也要感謝口試委員：翁詠祿教授與翁芳標教授在口試期間給予我許多指導與建議。除此之外，還要感謝實驗室的同學們：大師兄、力仁學長，感謝你們在我研究遇到難題時，總是很熱心的給予我意見與幫助。同屆的一哥、老菜和小白，因為有你們的陪伴，彼此支持鼓勵，兩年來讓我在遇到困難的時候不會覺得孤獨。學弟郭胖、阿標、和白兔，很開心能夠和你們相處，一同聊天打屁，真的是一群活潑的學弟妹！還有謝老師實驗室的強哥、宏益、小湯、duck、施施、振偉及冠亨，常常一起去吃飯聊天，紓解壓力，很謝謝你們的陪伴。最後，要感謝姿靜和我的家人們，總是默默的陪伴，在我心情低落時給予最大的鼓勵與支持。因為有你們的陪伴與支持，我的研究所生活多了很多色彩，感謝你們！

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Chapter 1

Introduction

Wireless communication systems have been used for a long time and undergone a notable development. Wireless communication technology is moving towards higher mobility and higher data rates. A communication system emploied in wireless channel consists of three main components: transmitter, receiver, and channel. In general, the signals are transmitted through a wireless channel by using electromagnetic wave forms from the transmitter to the receiver. The singals arrived the receiver from different directions with different delays, that causes the variations in the amplitude and the phase of the composite received signals. That phenomenon is called multipath fading. The fading channel might bring significant degradation in the performance of a communication system. In addition, the received signals are also distorted by channel impairments and the intentional or unintentional interference signals, such as, thermal noise and the signals transmitted from other users. We can regard pratial band noise jammer as the unintentional interference, and regard multitone noise jammer as intentional interference. The thermal noise is caused by the random motion of the electrons in conductors at the receiver. These factors make the transmitted signals distort seriously.

Frequency-hopping spread spectrum (FHSS) systems are typically used to against the jammers in wireless channel environments [1]. The M-ary frequency-shift-keying modulation is usually utilized with the FHSS system. The MFSK signals are hopped with a pseudo-random sequence, and the pseudo-random sequence is used to select a set of the carrier frequency. Therefore, the signals are pseudo-randomly hopped over the total bandwidth, and the jammer can not generate the same pseudo-random numbers and frequency hopping bands which are emploied by the FHSS system. That reduces the effect of the jammers. FHSS systems usually combine with ordinary singal-input and singal-output channel codes [2]-[8]. The performance analyses of combining the Reed-Solomon(RS) code scheme and the fast frequecy hopping spread spectrum system are shown in [2]. In [7][8], convolutional codes (CC) are combined with the noise-normalized method in FHSS systems to improve the system performance. However, the overall performance is not as satisfactory as the performance with the fading effect considered.

The design of channel codes for providing high date rate and high quality of communications over fading channels using multiple transmitter antennas have been investigated in recent years. Tarohk, Seshadri, and Calderbank *et al* [9][10], first proposed the space-time coding (STC) scheme, which is an effective way to to make the system data rate closer to the capacity of multiple-input and multiple-output wireless channels. The STC scheme introduces a temproal and spatial correlation into the transmitted signals by using multiple antennas and has been shown to provide excellent performance against multipath fading. It can achieve the transmit diversity as well as a coding gain without sacrificing the bandwidth. Generally, the FHSS system is the most effective anti-jamming communication techniques, and the STC scheme can minimize the effects of multipath fading. Therefore, we combine the FHSS system with the STC scheme to construct a power transmission scheme which can mitgate the effect of the multipath fading and the jamming interferences.

An overview of the FHSS system and the jamming environments are given in Chapter 2. The STC schemes are introduced in Chapter 3, and the design criteria for STC system with MFSK modulation over fading channel is also shown in Chapter 3. The STC/FHSS systems which combine STC scheme with FHSS systems are proposed in Chapter 4. In Chapter 4, we focuse on two kinds of FHSS systems. One is the worst case frequency hopping spread spectrum system which hops the signals from all transmitter antennas into the same M-ary band, and the other is the optimal case frequency hopping spread spectrum system which hops the signals from any transmitter antennas into different M-ary band. The ML decoding schemes with respect to both two STC/FHSS systems are also presented. The design criteria for constructing good space-time codes and the simulation results are alse given in this chapter. The conclusions for this thesis are in Chapter 5.

Chapter 2

Overview of Frequency-Hopping Spread Spectrum Systems and Jamming Environments

In wireless channels, sometimes the signals we transmitted are interfered by the jammers [1][11]. FHSS system is one of the most effective anti-jamming communication techniques. In this chapter, we will discribe FHSS system and the jamming environment.

2.1 FHSS System

Spread spectrum techinques are usually used for anti-jamming [11][12]. For spread spectrum systems, the bit singal-to-jammer noise ratio is define as

$$\frac{E_b}{N_J} = \frac{W_s S}{R_b J} \tag{2.1}$$

where W_s is the total spread spectrum signal bandwidth, S is the singal power, R_b is the data rate for bit per second, $E_b = S/R_b$ is the energy per bit, J is the jamming power, and $N_J = J/W_s$ is the signal-sided jammer noise power spectral density. We can also define the processing gain (PG)

$$PG = \frac{W_s}{R_b}. \tag{2.2}$$

The bit signal-to-jammer noise ratio represented in decibels (dB) is

$$\frac{E_b}{N_J}_{(dB)} = (PG)_{(dB)} - \frac{J}{S}_{(dB)} \tag{2.3}$$

where $\frac{J}{S}$ is the jammer-to-signal power ratio, and we can find that when PG is increasing, the value of E_b/N_J also increases. Figure 2.1 shows the block diagram of the uncoded FH system with MFSK modulation. The binary data are fed into the MFSK modulator, then the modulated signal is hopped pseudo-randomly over the total system bandwidth W_s under the control of pseudonoise (PN) sequence. In FHSS system, the carrier frequency is changed

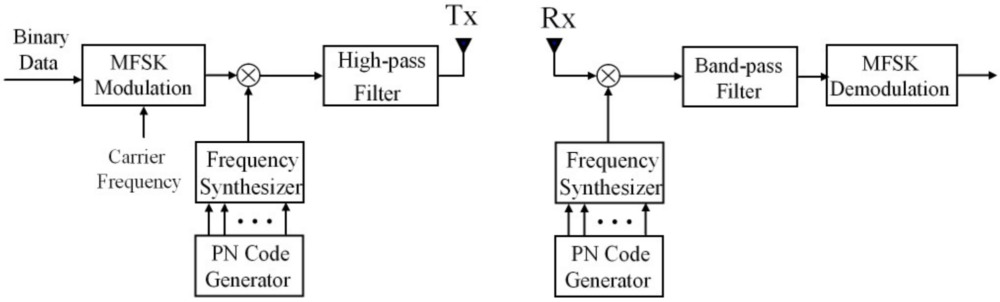

Figure 2.1: FH/MFSK system model.

periodically, such that the jammers do not konw where to jam. FHSS systems are classified into slow frequency hopping (SFH) and fast frequency hopping (FFH) [12]. Hop rate R_h of the FFH system is an multiple of the MFSK symbol rates R_s, and the SFH hops serval symbols each time. Each symbol of FFH system is hopped into serval chips, and each chip is transmitted in distinct M-ary band. Symbol can be demodulated after all the chips of this symbol is being collected and dehopped. Every symbol of SFH system is hopped into only one chip, and each chip is also transmitted in distinct M-ary band. The complexity of receiver of the FFH system is much higher than the receiver of the SFH system.

2.2 Jamming Environments

There are a lot of jamming waveforms that could distort the transmitted signal. A class of jamming waveforms are selected to illustrate in this section, such as broadband noise jammer, partial-band noise jammer, and multitone noise jammer[1][11].

2.2.1 Broaband Noise Jammer

A broadband noise jammer spreads it's total power J over the frequency range of the system bandwidth W_s. The broadband noise jammer can be regard as the additive white Gaussian nosie (AWGN) channel with zero mean shows in Figure 2.2, but the one-sided noise power spectral density (PSD) is

$$N_J = \frac{J}{W_s}. \tag{2.4}$$

A slow frequency hopping with noncoherent MFSK modulation system is used in AWGN channel without any jammers, and the bit error probability is

$$P_s = \frac{1}{M}\exp\left(-\frac{E_s}{2N_0}\right)\sum_{q=2}^{M}\binom{M}{q}(-1)^q\exp\left[\frac{E_s(2-q)}{2N_0 q}\right] \tag{2.5}$$

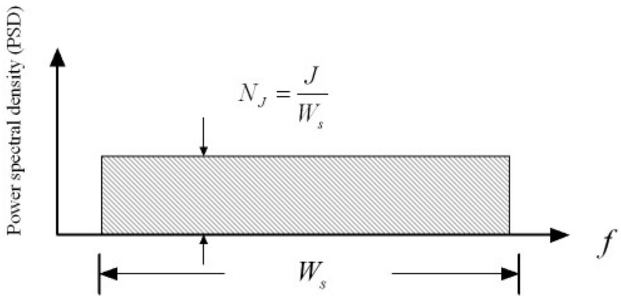

Figure 2.2: Power spectral density of broadband noise jammer.

where N_0 is the one-sided PSD of AWGN and E_s is the energy per symbol. When a symbol error occurs, the error probability can be regarded as the probability of choosing any other $M-1$ orthogonal symbols. Then the number of bit errors corresponding to a symbol error is

$$\frac{1}{M-1}\sum_{i=1}^{1}\binom{M}{q}i = \frac{l2^{l-1}}{M-1} = \frac{M}{2(M-1)}l \tag{2.6}$$

where l is the number of bits per symbol. By (2.5) and (2.6), the bit error probability is

$$\begin{aligned} P_b\frac{E_b}{N_0} &= \left[\frac{M}{2(M-1)}\right] \\ &= \frac{M}{2(M-1)}\exp\left(-\frac{lE_b}{2N_0}\right)\sum_{q=2}^{M}\binom{M}{q}(-1)^q\exp\left[\frac{E_b(2-q)}{2N_0q}\right]. \end{aligned} \tag{2.7}$$

In a AWGM channel with power J broadband jammer, the one-sided PSD is replaced by $N_0 + N_J$.Then the bit error probability could be written

$$P_b = \frac{M}{2(M-1)}\exp\left(-\frac{lE_b}{2(N_0+N_J)}\right)\sum_{q=2}^{M}\binom{M}{q}(-1)^q\exp\left[\frac{E_b(2-q)}{2(N_0+N_J)q}\right] \tag{2.8}$$

and it is defined as $P_b(\frac{E_b}{N_0+N_J})$. For a special case $l=1$, equation (2.8) becomes

$$P_b = \frac{1}{2}\exp\left(-\frac{E_b}{2(N_0+N_J)}\right) \tag{2.9}$$

when N_J decreases, the performance could be better.

2.2.2 Partial-Band Noise Jammer

The partial-band noise jammers can be regarded as the signals which are transmitted by other users and occupy a fraction of the frequency bandwidth. Power of partial-band noise jammer is restricted over the frequency range of bandwidth $W_J = \rho W_s$, which is a fraction ρ $(0 \leq \rho \leq 1)$ of the total system bandwidth W_s. The power spectral density of the partial-band nosie jammer is

$$N_J^{'} = \frac{J}{W_J} = \frac{J}{\rho W_S}. \tag{2.10}$$

Assume that the partial-band noise jammer can be regarded as AWGN then the average probability is

$$\overline{P}_b = (1-\rho)P_b\left(\frac{E_b}{N_0}\right) + \rho P_b\left(\frac{E_b}{N_0 + N_J}\right) \tag{2.11}$$

In general, we assume the power of the partial-band noise jammer is much larger than the power of thermal noise, such that N_J is much larger than N_0. The bit error probability is

$$\begin{aligned}\overline{P}_b &= \rho P_b(\frac{\rho E_b}{N_J}) \\ &= \frac{\rho}{2(M-1)}\sum_{q=2}^{M}\binom{M}{q}(-1)^q \text{exp}\left[\frac{l\rho E_b(1-q)}{N_0 q}\right]\end{aligned} \tag{2.12}$$

The worst case partial-band noise jammer chooses ρ to maximize $\overline{P}_b$ with a given M and $\frac{E_b}{N_J}$, and the average performance can be expressed as

$$(\overline{P}_b)\text{max} = \text{max}_{0<\rho\leq 1}\left[\frac{\rho}{2(M-1)}\sum_{q=2}^{M}\binom{M}{q}(-1)^q\text{exp}\left(\frac{l\rho E_b(1-q)}{N_0 q}\right)\right] \tag{2.13}$$

Let ρ_0 denote the worst case partial-band noise jammer [1][13] and maximize $\overline{P}_b$

$$\rho_0 = \begin{cases} \frac{2}{E_b/N_J}, & \text{for } \frac{E_b}{N_J} > 2 \\ 1, & \text{for } \frac{E_b}{N_J} \geq 2 \end{cases} \tag{2.14}$$

From (2.14), the miximum $\overline{P}_b$ is

$$(\overline{P}_b)_{\text{max}} = \begin{cases} \frac{0.3679}{E_b/N_J}, & \text{for } \frac{E_b}{N_J} > 2 \\ \frac{1}{2}\text{exp}\left(-\frac{E_b}{2N_J}\right), & \text{for } \frac{E_b}{N_J} \geq 2 \end{cases} \tag{2.15}$$

Figure 2.4 shows the performance curves of an FH/BFSK system in partial-band noise jammer environment with different factors ρ. When E_b/N_J is small, partial-band jammer with value of $\rho = 1$ which can be view as broadband noise jammer has the best efficency to interfere with the signal. However, when E_b/N_J exceeds a threshold level, the partial-band jammers with factor ρ $(0 < \rho \leq 1)$ have better efficency.

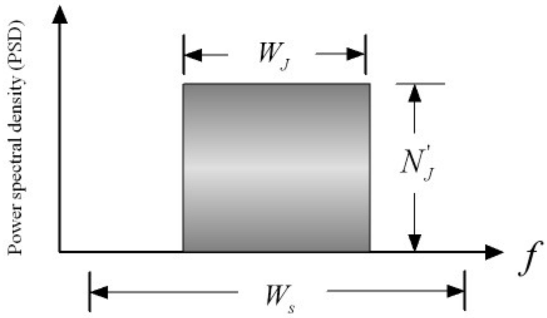

Figure 2.3: Power spectral density of partial-band noise jammer.

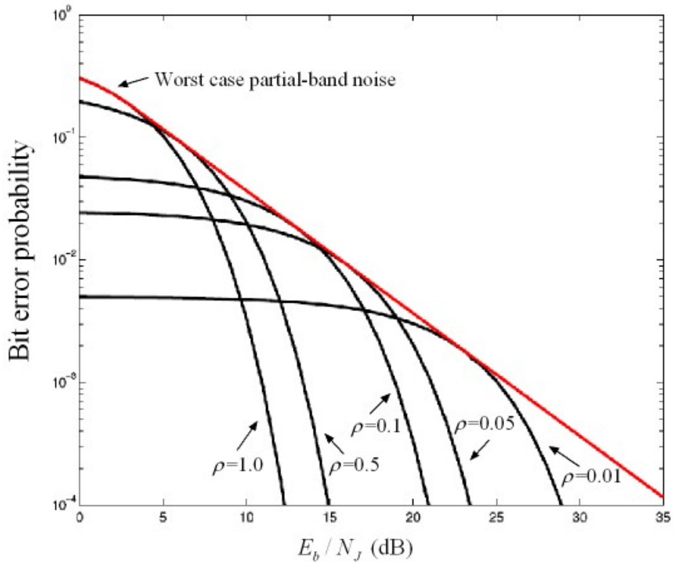

Figure 2.4: Performance of FH/MFSK system in partial-band noise jamming environment.

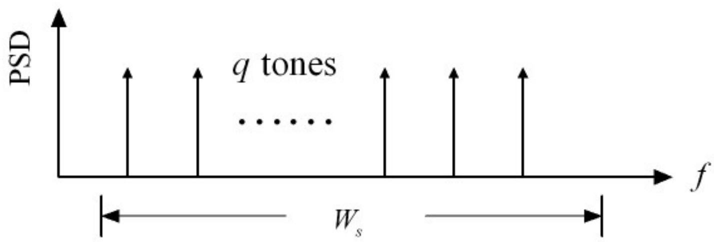

Figure 2.5: Power spectral density of multitone noise jammer.

2.2.3 Multitone Noise Jammer

We can consider multitone noise jammer as the signals transmitted from other users, and the frequencies of the carriers are in the range of the system bandwidth. Multitone noise jammer divides its total power into Q tone jammers with equal power and random continuous wave. The waveform of multitone noise jammer is

$$J(t) = \sum_{l=1}^{Q} \sqrt{\frac{2J}{Q}} \cos\left[\omega_0 t + \phi_l\right] \tag{2.16}$$

where ϕ_l is a random varible in $(0, 2\pi]$ for $\forall l$. Figure 2. illustrates the PSD of the multitone noise jammer.

We assume that there is at most one multitone jammer per frequency slot. In general, there are two kinds of multitone jammers. One is band multitone jammer which places n jamming tones in each jammed M-ary band. The fraction of the jammed FH slots is defined as

$$\rho = \frac{Q}{MN} \tag{2.17}$$

where N is the number of frequency band. The probability of n jamming tones in each jammed M-ary band is

$$\mu = \frac{Q/n}{N}. \tag{2.18}$$

The other one is called independent multitone noise jammer which places Q equal power jamming tones into NM FH frequency slots pseudo-randomly. The jamming noise could be independently hopped over the entire spread-spectrum bandwidth.

Assume the signal power is S, and the fraction of signal power to the power of each jamming tone is

$$\alpha = \frac{S}{J/Q}. \tag{2.19}$$

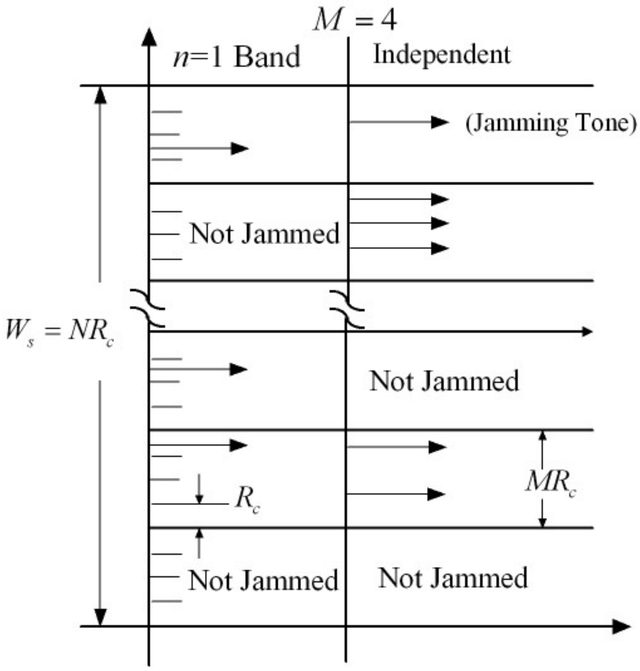

Figure 2.6: Band multitone noise jammer and independent multitone noise jammer strategies.

When the data symbol is not jammed and any of the other slots in this M-ary band is hit by a jamming tone, an error will occur if $\alpha < 1$. In contrast, no error will occur if $\alpha > 1$. Therefore, choosing Q appropariately could determine the worst case of α and seriously degrade the performance of FH/MFSK systems [1][14]. For slow frequency hopping, the bandwidth of a M-ary band is

$$W_b = MR_s = \frac{MR_b}{\log_2 M} = \frac{MR_b}{k} \tag{2.20}$$

where R_b is the bit rate and $R_s = R_b/\log_2$ is the symbol rate. Then the probability of a M-ary band being jamming is

$$\mu = \frac{Q}{W/W_b}. \tag{2.21}$$

By (2.4), (2.19), (2.21), and $E_b = S/R_b$, we can reweite μ as following form

$$\mu = \frac{\alpha M}{nkE_b/N_J}. \tag{2.22}$$

When the data symbol is not hit and any other frequency slots in this M-ary band are jammed, the symbol error probability for $\alpha < 1$ is

$$P_s = \mu\frac{M-1}{M}. \tag{2.23}$$

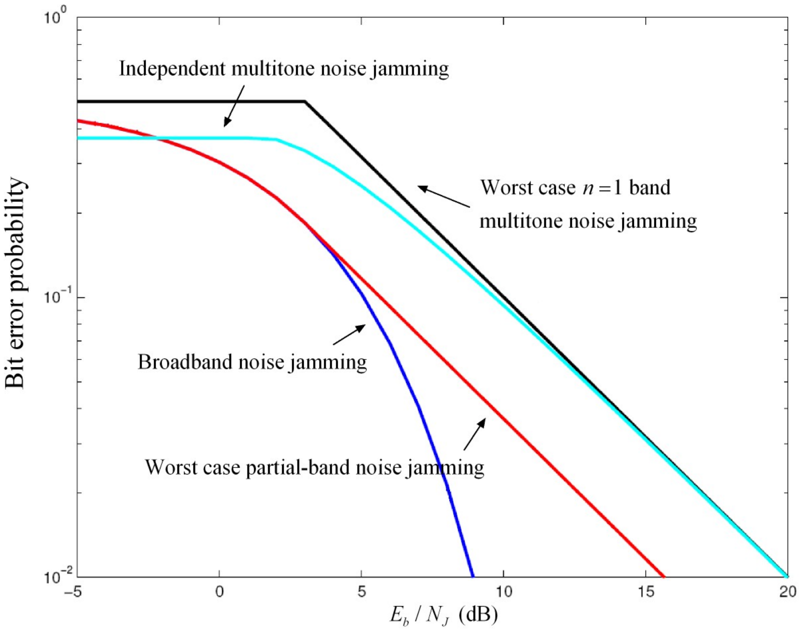

Figure 2.7: Performance of FH/MFSK in several different jamming environment.

The relation between P_s and P_b is

$$P_s = \frac{M}{2(M-1)} P_s, \tag{2.24}$$

so the bit error probability is

$$\begin{aligned} P_b &= \frac{M}{2(M-1)} \mu \frac{M-1}{M} \\ &= \frac{\alpha M}{2nkE_b/N_J}. \end{aligned} \tag{2.25}$$

We can make the system achieve the worst case performance by adjusting α, and we restrict the number of jamming tones to be smaller than the number of M-ary bands. The worst case band multitone jammer sets α_{wc} to be

$$\alpha_{wc} = \begin{cases} \frac{kE_b}{MN_J}, & \text{for } \frac{E_b}{N_J} < \frac{M}{k} \\ 1, & \text{for } \frac{E_b}{N_J} \leq \frac{M}{k} \end{cases} \tag{2.26}$$

Figure 2.7 shows that the partial-band and multitone jammers are both significantly more effective than the broadband noise jammer to against the FH/MFSK system. And the $n = 1$ band multitone is the most effective to against the FH/MFSK system.

Chapter 3

Review of Space-Time Coding

The multiple transmitter antennas system can be used to increase the transmitted data rate and against the multipath fading [15]. Tarohk, Seshadri, and Calderbank *et al* proposed the space-time coding scheme in 1998. Space-time coding scheme is an effective way to make the system data rate closer to the capacity of multiple-input and multiple-output wireless channels [9]. Temporal and spatial correlation are introduced into transmitted signals to achieve transmit diversity and coding gain without sacrificing system bandwith. This chapter introduces the encoding scheme, the decoding scheme, and the design criteria over fading channels of space-time coding system.

3.1 STC System Model

A space-time coding system with n transmitter antennas and m receiver antennas is shown in Figure 3.1. First, the information bits fed into the space-time encoder. After encoding, the encoded data is divided into n codeword symbols, and the symbols are passed into the modulator and transmitted by n transmitter antennas. The signal are degraded by multipath fading at the each m receiver antenna. The received signal is a superposition of the signals from n transmitter antennas with noise. Assume the wireless channels are a quasic-static flat fading and memoryless channels. Let S_t^i with energy E_s be the symbol

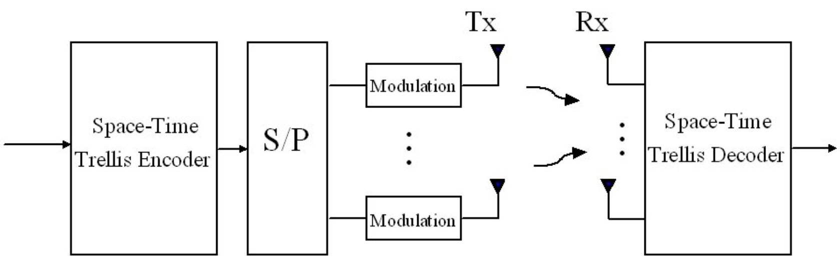

Figure 3.1: STC system model.

which is transmitted by the ith antenna at time t. The received signal r_t^q of the qth receiver antenna at time t fot all $0 \leq q \leq m$ and $0 \geq t \leq L$ is given by

$$r_t^q = \sum_{i=1}^{n} \alpha_{i,q} S_t^i + \eta_t^q \tag{3.1}$$

where $\alpha_{i,q}$ is the fading gain of the multipath from the ith transmitter antenna to qth receiver antenna and η_t^q is the thermal noise of the qth receiver antenna at time t. Assume $\alpha_{i,q}$ is a constant during a frame L of information sequences and vary from one frame to another. Assume η_t^q are independent Gaussian distribution with zero mean and one-sided power spectral density N_0 for $\forall q$ and $\forall t$.

STC systems differ with respect to distinct coding schemes, such as space-time block coding [16][17], space- time trellis coding [18][19], unitary space-time modulation [20][21], space-time turbo trellis coding [22], differential space-time coding [23][24], layered space-time coding [25][26], and space-time frequency coding [27][28], etc. The following section focuses on space-time trellis coding scheme (STTC).

3.2 Encoder Structure and Maximum Likelihood Decoding for STTC

Space-time trellis codes are provided by Tarohk, Seshadri, and Calderbank *et al.* STTC scheme combines the modulation and the trellis coding scheme to transmit data over multiple antennas. The generator sequences of the system are shown in Figure 3.2

$$(x_1^t, x_2^t) = b_{t-1}(1,1) \oplus_4 a_{t-1}(2,2) \oplus_4 b_t(2,1) \oplus_4 a_t(3,2) \tag{3.2}$$

where (x_1^t, x_2^t) stand for 2 coded QPSK symbols transmitted through the first antenna and the second antenna. a_t and b_t represent a pair of input data bits at time t,and $\oplus_4$ is an operation to take added module 4. For example, assume $(a_t, b_t) = (1,1)$ and $(a_{t-1}, b_{t-1}) = (0,1)$ then the output sequence generated by (3.2) at time t is $(x_1^t, x_2^t) = (2,0)$.

Let the received signals $\boldsymbol{r} = (r_t^q\ \forall q, t)$, the fading gain $\boldsymbol{\alpha} = (\alpha_{i,q}\ \forall q, t)$, and the estimated symbols $\hat{\boldsymbol{S}} = (\hat{S_t^i})\ \forall i, t$. Assume $\boldsymbol{\alpha}$ is available at the receiver, and then the ML decoding is given by

$$\begin{aligned} f\left(\boldsymbol{r}|\boldsymbol{\alpha}, \hat{\boldsymbol{S}}\right) &= \prod_{t=1}^{L}\prod_{q=1}^{m} f\left(\eta_t^q = r_t^q - \sum_{i=1}^{n} \alpha_{i,q} S_t^i | \alpha_{i,q}, S_t^i\ \forall i, q, t\right) \\ &= \prod_{t=1}^{L}\prod_{q=1}^{m} \left[\frac{1}{\sqrt{\pi N_0}} \exp\left(-\frac{|r_t^q - \sum_{i=1}^{n} \alpha_{i,q} S_t^i|^2}{N_0}\right)\right]. \end{aligned} \tag{3.3}$$

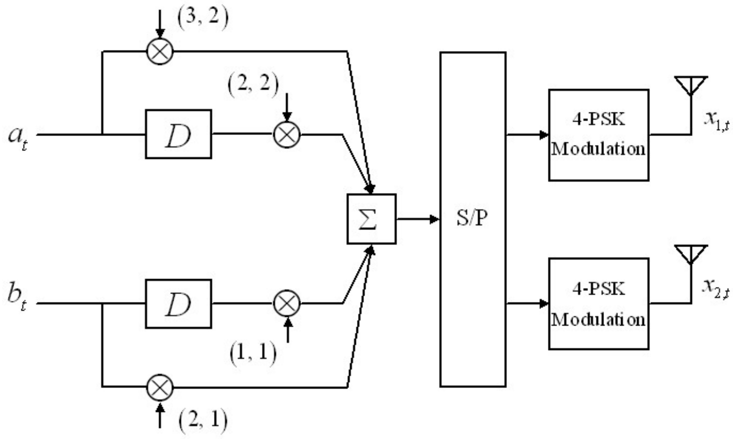

Figure 3.2: The encoder of STTC system for two transmitter antennas.

Drop the factors of $\frac{1}{\sqrt{\pi N_0}}$ and $\frac{1}{N_0}$ in (3.4), and apply the log-domain metric:

$$\min_{\hat{\boldsymbol{S}}} \sum_{t=1}^{L} \sum_{q=1}^{m} \left| r_t^q - \sum_{i=1}^{n} \alpha_{i,q} S_t^i \right|^2 . \tag{3.4}$$

Use the Viterbi algorithm to select the minimum path metric as the decoding sequence when this ML decoding is used.

3.3 Design Criteria for Constructing Good Space-Time Codes

Conisder the coded communication system with ML decoding shown in (3.5) [29]. A block of transmitted symbols is denoted by

$$\boldsymbol{S} = \left(S_t^i | \ \forall\, i, 1 \leq t \leq L \right) \tag{3.5}$$

and an erroneous sequence selected by the decoder is denoted by

$$\hat{\boldsymbol{S}} = \left(\hat{S}_t^i | \ \forall\, i, 1 \leq t \leq L \right). \tag{3.6}$$

Assume $\alpha_{i,q}$ is available at the receiver for $\forall i, q$, and then the pairwise error probability is derived as following

$$
\begin{aligned}
&\Pr\left(\boldsymbol{S} \to \hat{\boldsymbol{S}} | \alpha_{i,q}, \forall i, q\right) \\
&= \Pr\left[\sum_{t=1}^{L}\sum_{q=1}^{m}\left| r_t^j - \sum_{i=1}^{n}\alpha_{i,q}\sqrt{E_s}S_t^i\right|^2 \geq \sum_{t=1}^{L}\sum_{q=1}^{m}\left| r_t^j - \sum_{i=1}^{n}\alpha_{i,q}\sqrt{E_s}\hat{S}_t^i\right|^2\right] \\
&= \left[\sum_{t=1}^{L}\sum_{q=1}^{m} 2\mathrm{Re}\left\{\eta_t^j \sum_{i=1}^{n}\alpha_{i,q}\sqrt{E_s}\left(S_t^i - \hat{S}_t^i\right)\right\} \geq \sum_{t=1}^{L}\sum_{q=1}^{m}\left|\sum_{i=1}^{n}\alpha_{i,q}\sqrt{E_s}\left(s_t^i - \hat{S}_t^i\right)\right|^2\right] \\
&= \mathrm{Q}\left(\sqrt{d^2\left(\boldsymbol{S}, \hat{\boldsymbol{S}}\right)\frac{E_s}{2N_0}}\right)
\end{aligned} \tag{3.7}
$$

where

$$
d^2\left(\boldsymbol{S}, \hat{\boldsymbol{S}}\right) = \sum_{t=1}^{L}\sum_{q=1}^{m}\left|\sum_{i=1}^{n}\alpha_{i,q}\sqrt{E_s}\left(S_t^i - \hat{S}_t^i\right)\right|^2 \tag{3.8}
$$

and $Q(x)$ is the complementary error function defined by

$$
Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^{\infty}\exp\left(-x^2/2\right)dx. \tag{3.9}
$$

Use the Chernoff Bound inequality

$$
Q(x) \leq \frac{1}{2}e^{-x^2/2} \tag{3.10}
$$

and then the conditional pairwise error probability can be upper bounded by

$$
\Pr\left(\boldsymbol{S} \to \hat{\boldsymbol{S}} | \alpha_{i,q}\ \forall i, q\right) \leq \frac{1}{2}\mathrm{exp}\left(-d^2\left(\boldsymbol{S}, \hat{\boldsymbol{S}}\right)\frac{E_s}{4N_0}\right) \tag{3.11}
$$

Assume the fading coeficients $\alpha_{i,q}$ are independent Gaussian random variables with zero mean and varance 1/2. Let "$*$" denote the operator of taking complex conjugate, and H denotes the operator of taking Hermirian, and $\Omega_j = (\alpha_{1,q}, \alpha_{2,q}, \ldots, \alpha_{n,q})$. Then we can rewrite equation (3.8) as

$$
\begin{aligned}
d^2\left(\boldsymbol{S}, \hat{\boldsymbol{S}}\right) &= \sum_{q=1}^{m}\sum_{i=1}^{n}\sum_{l=1}^{n}\alpha_{i,q}\alpha_{i,q}^{*}\sum_{t=1}^{L}\left(s_t^i - \hat{s}_t^i\right)\left(s_t^l - \hat{s}_t^l\right)^* \\
&= \sum_{q=1}^{m}\Omega_q B\left(\boldsymbol{S}, \hat{\boldsymbol{S}}\right)B^H\left(\boldsymbol{S}, \hat{\boldsymbol{S}}\right)\Omega_q^H \\
&= \sum_{q=1}^{m}\Omega_q A\left(\boldsymbol{S}, \hat{\boldsymbol{S}}\right)\Omega_q^H
\end{aligned} \tag{3.12}
$$

where

$$B\left(\boldsymbol{S},\hat{\boldsymbol{S}}\right)=\begin{bmatrix} S_1^1-\hat{S}_1^1 & S_2^1-\hat{S}_2^1 & \dots & S_L^1-\hat{S}_L^1 \\ S_1^1-\hat{S}_1^2 & S_2^2-\hat{S}_2^2 & \dots & S_L^2-\hat{S}_L^2 \\ \vdots & \vdots\ddots & \vdots & \\ S_1^1-\hat{S}_1^n & S_2^n-\hat{S}_2^n & \dots & S_L^n-\hat{S}_L^n \end{bmatrix}$$

and $A\left(\boldsymbol{S},\hat{\boldsymbol{S}}\right)=B\left(\boldsymbol{S},\hat{\boldsymbol{S}}\right)B^H\left(\boldsymbol{S},\hat{\boldsymbol{S}}\right)$. $A\left(\boldsymbol{S},\hat{\boldsymbol{S}}\right)$ is nonnegative definite and Hermitian, and the eigenvalues of $A\left(\boldsymbol{S},\hat{\boldsymbol{S}}\right)$ are real numbers. Then we have

$$A\left(\boldsymbol{S},\hat{\boldsymbol{S}}\right)=\boldsymbol{V}\boldsymbol{D}\boldsymbol{V}^H \tag{3.13}$$

where $\boldsymbol{V}=(v_1,v_2,\dots,v_n)$ is a unitary matrix and D is a diagonal matrix, where v_i's are the eigenvectors of $A\left(\boldsymbol{S},\hat{\boldsymbol{S}}\right)$. Let λ_i be the diagonal elements of $\boldsymbol{D}$, where $1\le i\le n$, and

$$\Omega_q\boldsymbol{V}^H=(\beta_{1,q},\dots,\beta_{n,q})\,. \tag{3.14}$$

From (3.13) and (3.14), we can rewrite the equation (3.8) as following

$$d^2\left(\boldsymbol{S},\hat{\boldsymbol{S}}\right)=\sum_{q=1}^{m}\sum_{i=1}^{n}\lambda_i\left|\beta_{i,q}\right|^2. \tag{3.15}$$

Use equation (3.15) to replace $d^2\left(\boldsymbol{S},\hat{\boldsymbol{S}}\right)$ in (3.11), then we have

$$\Pr\left(\boldsymbol{S}\to\hat{\boldsymbol{S}}|\alpha_{i,q}\ \forall i,q\right)\le\frac{1}{2}\exp\left(-\frac{E_s}{4N_0}\sum_{q=1}^{m}\sum_{i=1}^{n}\lambda_i\left|\beta_{i,q}\right|^2\right) \tag{3.16}$$

Obviously, all of $\beta_{i,q}$ are independent complex Gaussian random variables with mean $\mu_{i,q}$ and variance 1/2 per dimension. The $\mu_{i,q}$ is given by

$$\begin{aligned}\mu_{i,q}&=E\left[\Omega_q v_i\right]\\&=[\alpha_{1,q},\alpha_{2,q},\dots,\alpha_{n,q}]v_i\end{aligned} \tag{3.17}$$

where $E[\,]$ denotes the expectation. $|\beta_{i,q}|$ is a Rician distribution demonstrated by following probability density function

$$p\left(|\beta_{i,q}|\right)=2\left|\beta_{i,q}\right|\exp\left(-\left|\beta_{i,q}\right|^2-\left|\mu_{i,q}\right|^2\right)\boldsymbol{I}_0\left(2\left|\beta_{i,q}\right|\left|\mu_{i,q}\right|\right) \tag{3.18}$$

where $\boldsymbol{I}_0$ represents the zero-order modified Bessel function of the first kind. The pairwise error probability is derived by averaging $|\beta_{i,q}|$, then the pairwise error probability is

$$\begin{aligned}
\Pr\left(\boldsymbol{S} \to \hat{\boldsymbol{S}}\right) &= \int_0^\infty \cdots \int_0^\infty \Pr\left(\boldsymbol{S} \to \hat{\boldsymbol{S}} | \alpha_{i,q}\ \forall i, q\right) p(\alpha_{1,1}) p(\alpha_{1,2}) \ldots p(\alpha_{n,m}) d\alpha_{1,1} \\
&\quad d\alpha_{1,2} \ldots d\alpha_{n,m} \\
&= \int_0^\infty \cdots \int_0^\infty \Pr\left(\boldsymbol{S} \to \hat{\boldsymbol{S}} |\, |\beta_{i,q}|\, \forall i, q\right) p\left(|\beta_{1,1}|\right) p\left(|\beta_{1,2}|\right) \ldots p\left(|\beta_{n,m}|\right) \\
&\quad d\left|\beta_{1,1}\right| d\left|\beta_{1,2}\right| \ldots d\left|\beta_{n,m}\right| \\
&\leq \int_0^\infty \cdots \int_0^\infty \frac{1}{2} \exp\left(-\frac{E_s}{4N_0} \sum_{q=1}^{m} \sum_{i=1}^{n} \lambda_i \left|\beta_{i,q}\right|^2\right) p\left(|\beta_{1,1}|\right) p\left(|\beta_{1,2}|\right) \ldots \\
&\quad p\left(|\beta_{n,m}|\right) d\left|\beta_{1,1}\right| d\left|\beta_{1,2}\right| \ldots d\left|\beta_{n,m}\right| \\
&\leq \frac{1}{2} \left(\prod_{i=1}^{n} \frac{1}{1 + \frac{E_s}{4N_0}\lambda_i} \exp\left(-\frac{\left|\beta_{i,q}\right|^2 \frac{E_s}{4N_0}\lambda_i}{1 + \frac{E_s}{4N_0}\lambda}\right)\right). \qquad (3.19)
\end{aligned}$$

Assume $\mu_{i,q} = 0$, then $\beta_{i,q}$ become a Rayleigh distribution random variable and the probability density function is

$$\Pr\left(\boldsymbol{S} \to \hat{\boldsymbol{S}}\right) \leq \frac{1}{2} \left(\prod_{i=1}^{n} \frac{1}{1 + \frac{E_s}{4N_0}\lambda_i}\right)^m. \qquad (3.20)$$

When SNR is a big number, (3.20) can be expressed as

$$\Pr\left(\boldsymbol{S} \to \hat{\boldsymbol{S}}\right) \leq \frac{1}{2} \left(\prod_{i=1}^{r} \lambda_i\right)^{-m} \left(\frac{E_s}{4N_0}\right)^{-rm}. \qquad (3.21)$$

where r is the rank of $A\left(\boldsymbol{S}, \hat{\boldsymbol{S}}\right)$. The exponent of SNR term, rm, is called the diversity gain, and the product of eigenvalues is called the coding gain. In order to minimize the error probability, to make the diversity gain and the coding gain as large as posible is necessary. These are the two criteria which are called rank cirteria and determinant criteria.

Chapter 4

Design of Space-Time Coding with FHSS Technique in Wireless Channels

The transmitted signals are commonly distorted by some intentional or unintentional jamming noise in wireless channels. As disscussed in Chapter 2 and Chapter 3, we know that spread spectrum systems are the most effective anti-jamming communication techniques, and the space-time coding schemes effectively minimize the effects of multipath fading. So, we propose the design schemes combin with space- time coding scheme and the spread spectrum system. Two kinds of FHSS systems are discussed in this chapter, one is the worst case frequency hopping spread spectrum (WFHSS) system which hops the symbols from all transmitter antennas into the same M-ary band. Another is the optimum case frequency hopping spread spectrum (OFHSS) system which hops the symbols from any transmitter antennas into different M-ary band. The two system are called STC/WFHSS and STC/OFHSS systems.

The detailed description of the STC/FHSS system model and the ML decoding are given in this chapter. The criteria for constructing good space-time codes are also proposed. Some simulation results are also presented in the last section.

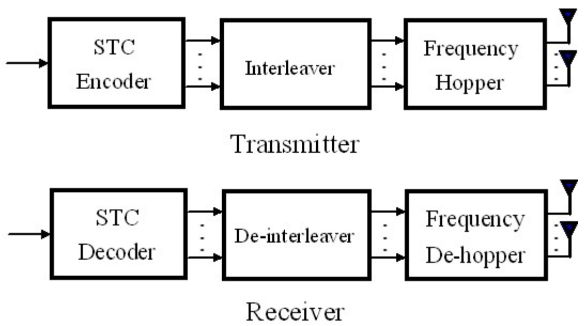

Figure 4.1: The proposed STC/FHSS system.

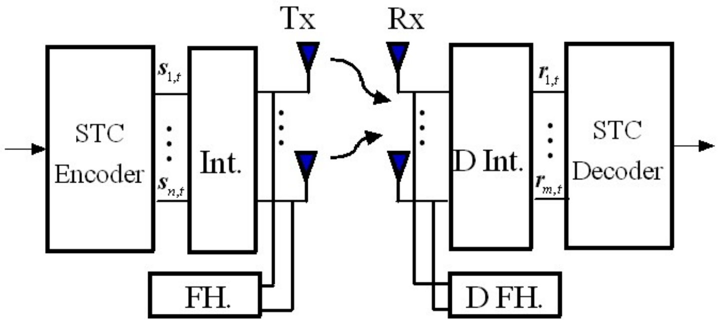

Figure 4.2: The STC/WFHSS system model.

4.1 STC/FHSS System Model

The STC/FHSS system model is shown in Figure 4.1. There are n transmitter antennas and m receiver antennas. Interleaver is inserted to break bust channel errors to guarantee memoryless channels, and the MFSK modulation is utilized with the FHSS system. The slow frequency hopping with one hop per symbol is assumed for simplicity, and the hopping patterns generated from the transmitter are available to the receiver.

The STC/WFHSS system is shown in Figure 4.2. Let the signal of the qth receiver antenna be

$$r_q(t) = \sum_{i=1}^{n} A_{i,q}(t)\, s_i(t) + B_q(t)\, n_J(t) + n(t) \tag{4.1}$$

where $A_{i,q}(t)$ is the fading gain of the multipath from the ith transmitter antenna to qth receiver antenna, $B_q(t)$ is the fading gain of the multipath from the jamming transmitter to the qth receiver antenna, $n_J(t)$ is the jammer , and $n(t)$ is statistically independent low pass white Gaussian noise process with one-side spectral density N_0. For the slow fading channel, assume the fading coefficients are the same during a frame L and vary from one to frame another. Due to the system has no perfect synchronization, the received signal which is dehopped and demodulated is composed of cos part and sin part. The cos part of the recived signal of the qth receiver antenna in the kth frequency slot at time t $r^k_{R,q,t}$ is

$$r^k_{R,q,t} = \int_t^{t+T_s} \left[\sum_{k'=1}^{M} \left(\sum_{i=1}^{n} \left(A_{i,q} \sqrt{\frac{2}{T_s}} s^{k'}_{i,t'} \cos\left((\omega_b + \omega_{k'}) t' + \theta \right) \right) + x^{k'}_t B_q \right.\right.$$
$$\left.\left. \sqrt{\frac{2J}{Q}} \cos\left((\omega_b + \omega_{k'}) t' + \phi_{q,t} \right) \right) + n(t') \right] \sqrt{\frac{2}{T_s}} \cos\left((\omega_b + \omega_{k'}) t' \right) dt'$$

$$
\begin{aligned}
&= \sum_{i=1}^{n} \left(A_{i,q}\cos(\theta)\right) s_{i,t}^{k} + x_t^k B_q \cos(\phi_{q,t}) \sqrt{\frac{2J}{Q}} + n_{R,q,t}^{k} \\
&= \sum_{i=1}^{n} \alpha_{R,i,q} s_{i,t}^{k} + x_t^k B_q \cos(\phi_{q,t}) \sqrt{\frac{2J}{Q}} + n_{R,q,t}^{k} \\
&= \sum_{i=1}^{n} \alpha_{R,i,q} s_{i,t}^{k} + n_{J,R,q,t}^{k} + n_{R,q,t}^{k}
\end{aligned} \tag{4.2}
$$

where $s_{i,t}^k$ is the symbol transmitted by the ith antenna in kth slot at time t, θ is the phase error of signal, ϕ is the phase error of the jammer, T_s is the bit interval, Q is the number of tone jammer, and J is the total jamming power, ω_k is the particular carrier frequency for modulation, ω_b is the particular carrier frequency for hopping, and x_t^k is the jamming state information (JSI) of the multitone noise jammer (MTNJ) taking value from 1 and 0 with probability MQ/N_t and $1 - MQ/N_t$. $x_t^k = 1$ means the kth slot of the band which the signal is transmitted in is jammed at time t. The sin part of the recived signal of the qth receiver antenna in the kth frequency slot at time t $r_{I,q,t}^k$ is

$$
\begin{aligned}
r_{I,q,t}^{k} = \int_{t}^{t+T_s} &\left[\sum_{k'=1}^{M}\left(\sum_{i=1}^{n}\left(A_{i,q}\sqrt{\frac{2}{T_s}} s_{i,t'}^{k'} \cos\left((\omega_b+\omega_{k'})\,t'+\theta\right)\right) + x_t^{k'} B_q \right.\right. \\
&\left.\left. \sqrt{\frac{2J}{Q}} \cos\left((\omega_b+\omega_{k'})\,t'+\phi_{q,t}\right)\right) + n(t')\right] \sqrt{\frac{2}{T_s}} \sin\left((\omega_b+\omega_{k'})\,t'\right) dt' \\
= \sum_{i=1}^{n} &\left(A_{i,q}\sin(\theta)\right) s_{i,t}^{k} + x_t^k B_q \sin(\phi_{q,t}) \sqrt{\frac{2J}{Q}} + n_{I,q,t}^{k} \\
= \sum_{i=1}^{n} &\alpha_{I,i,q} s_{i,t}^{k} + x_t^k B_q \sin(\phi_{q,t}) \sqrt{\frac{2J}{Q}} + n_{I,q,t}^{k} \\
= \sum_{i=1}^{n} &\alpha_{I,i,q} s_{i,t}^{k} + n_{J,I,q,t}^{k} + n_{I,q,t}^{k}.
\end{aligned} \tag{4.3}
$$

Therefore the received signal can be expressed as following

$$
\begin{aligned}
r_{q,t}^{k} &= r_{R,q,t}^{k} + j r_{I,q,t}^{k} \\
&= \sum_{i=1}^{n} \alpha_{i,q} s_{i,t}^{k} + x_t^k n_{J,q,t}^{k} + n_{q,t}^{k} \\
&= \sum_{i=1}^{n} \alpha_{i,q} s_{i,t}^{k} + \eta_{q,t}^{k}
\end{aligned} \tag{4.4}
$$

The noise $\eta_{q,t}^k$ includes the AWGN $n_{q,t}^k$ and the MTNJ $n_{J,q,t}^k$. By observing (4.2), (4.3), and

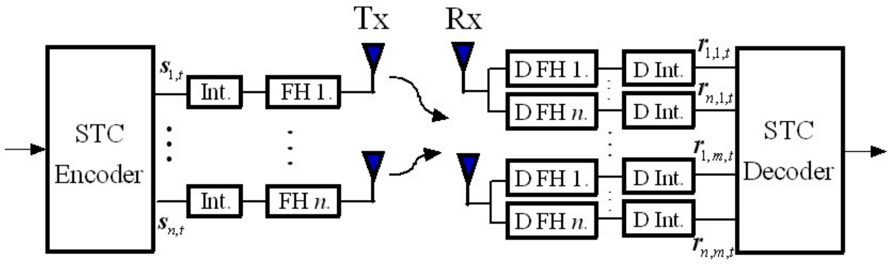

Figure 4.3: The STC/OFHSS system model.

(4.4), the MTNJ $n^k_{J,q,t}$ can be written as

$$\begin{aligned} n^k_{J,q,t} &= n^k_{J,R,q,t} + jn^k_{J,I,q,t} \\ &= x^k_t \left(B_q \cos(\phi_{q,t}) + jB_q \sin(\phi_{q,t})\right) \sqrt{\frac{2J}{Q}}. \end{aligned} \tag{4.5}$$

In Rayleigh fading channel, B_q is a Rayleigh random variable. Then $n^k_{J,q,t}$ is a complex Gaussian random variable with zero mean and variance $\frac{JT_s}{Q}\sigma^2_{J,q}$. Assume AWGN $n^k_{q,t}$ and MTNJ $n^k_{J,q,t}$ are independent for $\forall\ q,t,k$. The probability of $\eta^k_{q,t}$ conditioned on x^k_t is

$$f\left(\eta^k_{q,t}|x^k_t\right) = \frac{1}{\sqrt{\pi\left(N_0 + x^k_t 2\frac{J_T s}{Q}\sigma^2_{J,q}\right)}} \exp\left(-\frac{\left|\eta^k_{q,t}\right|}{\left(N_0 + x^k_t 2\frac{JT_s}{Q}\sigma^2_{J,q}\right)}\right). \tag{4.6}$$

The equation (4.6) can be used to derive the likelihood function of the decoding scheme with respect to the STC/WFHSS system.

The STC/OFHSS is shown in Figure 4.3. The received signal $r^k_{i,q,t}$ of the qth receiver antenna and from the ith transmitter antenna in kth slot at time t is

$$r^k_{i,q,t} = \alpha_{i,q} s^k_{i,t} + \eta^k_{i,q,t} \tag{4.7}$$

where

$$\eta^k_{i,q,t} = n^k_{i,q,t} + x^k_{i,t} n^k_{J,i,q,t} \tag{4.8}$$

where $x^k_{i,t}$ is the jamming state information (JSI) of the MJNJ taking value from 1 and 0 with probability MQ/N_t and $1 - MQ/N_t$, but $x^k_{i,t}$ and $x^k_{i,t'}$ are not independent for $t \neq t'$. The probability density function of $\eta^k_{i,q,t}$ conditioned on $x^k_{i,t}$ is

$$f\left(\eta^k_{i,q,t}|x^k_{i,t}\right) = \frac{1}{\sqrt{\pi\left(N_0 + x^k_{i,t} 2\frac{JT_s}{Q}\sigma^2_{J,q}\right)}} \exp\left(-\frac{\left|\eta^k_{i,q,t}\right|}{\left(N_0 + x^k_{i,t} 2\frac{JT_s}{Q}\sigma^2_{J,q}\right)}\right). \tag{4.9}$$

The likelihood function of the decoding scheme for the STC/OFHSS system can be derived by the equation (4.9).

Space-time codes achieve the transmit diversity as well as a coding gain. In addition, the signal transmitted by the frequency hopping system avoid the multitone jammers effectively. Therefore, the STC/FHSS combines with temporal, frequency, and spatial domain to against the multipath fadding and the multitone jamming interferences. With respect to the two types of STC/FHSS systems, the performance variation is observed for comparison.

4.2 STC Combined with the Worst Case Frequency Hopping

For STC/WFHSS system which is shown in Figure 4.2, the encoded codewords form all transmitter antennas are hopped into the same M-ary band at time t. The received symbols form any receiver antennas are dehopped with the same hopping pattern. Assume the symbols are transmited in the slow fading channel, and the fading coefficients $\alpha_{i,q}$ are complexe Gaussian random variable with zero mean and variance $\sigma_{i,q}^2$.

4.2.1 Decoing with CSI and JSI Available

The ML decoding scheme will be derived in this section. The derived result is shown here for discussion and compairson with respect to the proposed system. Let the received signals $\boldsymbol{r} = \left(r_{q,t}^k \middle| \forall q, k,\ 1 \le t \le L\right)$, the jamming state infromation $\boldsymbol{x} = \left(x_t^k \middle| \forall k,\ 1 \le t \le L\right)$, the fading coefficients $\boldsymbol{\alpha} = (\alpha_{i,q} | \forall i, q)$, and the estimated symbols $\hat{\boldsymbol{s}} = \left(s_{i,t}^k \middle| \forall i,\ 1 \le t \le L\right)$. Assume the fading coefficients $\alpha_{i,q}$ and the jamming state information x_t^k are available at the receiver, the likelihood of $\boldsymbol{r}$ given $\hat{\boldsymbol{s}}$, $\boldsymbol{x}$, and $\boldsymbol{\alpha}$ can be express as

$$\begin{aligned} f\{\boldsymbol{r}|\hat{\boldsymbol{s}}, \boldsymbol{x}, \boldsymbol{\alpha}\} &= \prod_{t=1}^{L}\prod_{k=1}^{M}\prod_{q=1}^{m} f\left\{\eta_{q,t}^k = r_{q,t}^k - \sum_{i=1}^{n}\alpha_{i,q}\hat{s}_{i,t}^k \middle| \hat{s}_{i,t}^k, \alpha_{i,q}, x_t^k\right\} \\ &= \prod_{t=1}^{L}\prod_{k=1}^{M}\prod_{q=1}^{m} \frac{1}{\sqrt{\pi\left(N_0 + x_t^k 2\frac{JT_s}{Q}\sigma_{J,q}^2\right)}} \exp\left(-\frac{\left|r_{q,t}^k - \sum_{i=1}^{n}\alpha_{i,q}\hat{s}_{i,t}^k\right|^2}{\left(N_0 + x_t^k 2\frac{JT_s}{Q}\sigma_{J,q}^2\right)}\right). \end{aligned} \tag{4.10}$$

We can decode the codeword in ML decoding sense by minimizing the following metric

$$\sum_{t=1}^{L}\sum_{k=1}^{M}\sum_{q=1}^{m} \frac{\left|r_{q,t}^k - \sum_{i=1}^{n}\alpha_{i,q}\hat{s}_{i,t}^k\right|}{\left(N_0 + x_t^k 2\frac{JT_s}{Q}\sigma_{J,q}^2\right)}. \tag{4.11}$$

4.2.2 Decoing with CSI but without JSI

Assume the fading coefficients $\alpha_{i,q}$ are available at the receiver, but the jamming state information x_t^k are not available at the receiver. The likelihood function of $\boldsymbol{r}$ given $\hat{\boldsymbol{s}}$ and $\boldsymbol{\alpha}$ can be obtained by averaging (4.10) with respect to $\boldsymbol{x}$.

$$\begin{aligned} & f\left(\boldsymbol{r}|\hat{\boldsymbol{s}},\boldsymbol{\alpha}\right) \\ & = E_{\boldsymbol{x}}\left[f\left(\boldsymbol{r}|\hat{\boldsymbol{s}},\boldsymbol{x},\boldsymbol{\alpha}\right)\right] \\ & = E_{\boldsymbol{x}}\left[\prod_{t=1}^{L}\prod_{k=1}^{M}\prod_{q=1}^{m}\frac{1}{\sqrt{\pi\left(N_0+x_t^k 2\frac{JT_s}{Q}\sigma_{J,q}^2\right)}}\exp\left(-\frac{\left|r_{q,t}^k-\sum_{i=1}^{n}\alpha_{i,q}\hat{s}_{i,t}^k\right|^2}{\left(N_0+x_t^k 2\frac{JT_s}{Q}\sigma_{J,q}^2\right)}\right)\right] \end{aligned} \tag{4.12}$$

where x_t^k takes value from 1 and 0 with probability MQ/N_t and $1-MQ/N_t$. x_t^k are independent for different t , but are not independent for different k. Assume there are $n=1$ band multitone jammers in the channel, the probability density function of $\boldsymbol{x}_t = \left(x_t^k|1\leq k\leq M\right)$ is

$$\begin{aligned} \Pr\left(x_t^1=1,x_t^2=0,\cdots,x_t^M=0\right) &= \frac{Q}{N_t} \\ \Pr\left(x_t^1=0,x_t^2=1,\cdots,x_t^M=0\right) &= \frac{Q}{N_t} \\ &\vdots \\ \Pr\left(x_t^1=0,x_t^2=0,\cdots,x_t^M=1\right) &= \frac{Q}{N_t} \\ \Pr\left(x_t^1=0,x_t^2=0,\cdots,x_t^M=0\right) &= 1-\frac{MQ}{N_t}. \end{aligned} \tag{4.13}$$

After averaging (4.10) with respect to $\boldsymbol{x}$, the likelihood function of $\boldsymbol{r}$ given $\hat{\boldsymbol{s}}$ and $\boldsymbol{\alpha}$ is derived in Appendix A and can be express as

$$\begin{aligned} & f\left(\boldsymbol{r}|\hat{\boldsymbol{s}},\boldsymbol{\alpha}\right) \\ & = \prod_{t=1}^{L}\prod_{q=1}^{m}\left\{\exp\left[-\sum_{i=1}^{M}\frac{\left|r_{q,t}^k-\sum_{i=1}^{n}\alpha_{i,q}\hat{s}_{i,t}^k\right|^2}{N_0}\right]\left\{\sum_{i=1}^{M}\frac{Q}{N_t}\frac{1}{\sqrt{\pi a}}\left(\frac{1}{\sqrt{\pi N_0}}\right)^{M-1}\right.\right. \\ & \left.\left.\cdot\ \exp\left[\frac{(a-N_0)\left|r_{q,t}^k-\sum_{i=1}^{n}\alpha_{i,q}\hat{s}_{i,t}^k\right|^2}{N_0 a}\right]+\left(1-\frac{MQ}{N_t}\right)\left(\frac{1}{\sqrt{\pi N_0}}\right)^2\right\}\right\} \end{aligned} \tag{4.14}$$

where $a = N_0 + 2\frac{JT_s}{Q}\sigma_{J,q}^2$. By taking logarithm on that likelihood function, codewords can be decoded in the ML decoding sense by maximizing the following metric

$$\sum_{t=1}^{L}\sum_{q=1}^{m}\left\{-\sum_{k=1}^{M}\frac{\left|r_{q,t}^k-\sum_{i=1}^{n}\alpha_{i,q}\hat{s}_{i,t}^k\right|^2}{N_0}+\ln\left\{\sum_{k=1}^{M}\frac{Q}{N_t}\frac{1}{\sqrt{\pi a}}\left(\frac{1}{\sqrt{\pi N_0}}\right)^{M-1}\right.\right.$$
$$\left.\left.\cdot\ \exp\left[\frac{(N_0-a)\left|r_{q,t}^k-\sum_{i=1}^{n}\alpha_{i,q}\hat{s}_{i,t}^k\right|^2}{N_0 a}\right]+\left(1-\frac{MQ}{N_t}\right)\left(\frac{1}{\sqrt{\pi N_0}}\right)^2\right\}\right\} \tag{4.15}$$

4.2.3 Decoing with JSI but without CSI

Suppose the fading coefficients $\alpha_{i,q}$ are not available at the receiver, and the fading coefficients are modeled as independent complex Gaussian random variables with zero mean and variance $\sigma_{i,q}^2$ per dimension with respect to Rayleigh fading channels. In order to simplify mathematics, we assume $\sigma_{i,q}^2 = 1/2$ and $\sigma_{J,q}^2 = 1/2$ for $\forall i, q$ in this section.

Let $a_t^k = N_0 + x_t^k\frac{JT_s}{Q}$, then the likelihood function $f(\boldsymbol{r}|\hat{\boldsymbol{s}},\boldsymbol{\alpha},\boldsymbol{x})$ can be rewritten as

$$f(\boldsymbol{r}|\hat{\boldsymbol{s}},\boldsymbol{\alpha},\boldsymbol{x})$$
$$=\prod_{t=1}^{L}\prod_{k=1}^{M}\prod_{q=1}^{m}\frac{1}{\sqrt{\pi a_t^k}}\exp\left\{-\frac{1}{a_t^k}\left|r_{q,t}^k-\sum_{i=1}^{n}\alpha_{i,q}\hat{s}_{i,t}^k\right|^2\right\}$$
$$=\prod_{t=1}^{L}\prod_{k=1}^{M}\prod_{q=1}^{m}\frac{1}{\sqrt{\pi a_t^k}}\exp\left\{-\sum_{t=1}^{L}\sum_{k=1}^{M}\sum_{q=1}^{m}\frac{1}{a_t^k}\left[\left|r_{q,t}^k\right|^2-2\mathrm{Re}\left(r_{q,t}^k\sum_{i=1}^{n}\alpha_{i,q}\hat{s}_{i,t}^k\right)\right.\right.$$
$$\left.\left.+\sum_{i=1}^{n}\alpha_{i,q}\hat{s}_{i,t}^k\sum_{l=1}^{n}\alpha_{l,q}^*\hat{s}_{l,t}^{k*}\right]\right\} \tag{4.16}$$

and the fading coefficients $\alpha_{i,q}$ can be presented as

$$\alpha_{i,q} = \alpha_{R,i,q} + j\alpha_{I,i,q} \tag{4.17}$$

where $\alpha_{R,i,q}$ and $\alpha_{I,i,q}$ are statistically independent Gaussian random variables with zero mean and variance $\sigma_{i,q} = 1/2$. Rewrite $\mathrm{Re}\left(r_{q,t}^k\sum_{i=1}^{n}\alpha_{i,q}\hat{s}_{i,t}^k\right)$ and $\sum_{i=1}^{n}\alpha_{i,q}\hat{s}_{i,t}^k\sum_{l=1}^{n}\alpha_{l,q}^*\hat{s}_{l,t}^{k*}$ in (4.16) as

$$\mathrm{Re}\left(r_{q,t}^k\sum_{i=1}^{n}\alpha_{i,q}\hat{s}_{i,t}^k\right)=\mathrm{Re}\left(r_{q,t}^k\sum_{i=1}^{n}\left(\alpha_{R,i,q}+j\alpha_{I,i,q}\right)\hat{s}_{i,t}^k\right)$$
$$=\mathrm{Re}\left(r_{q,t}^k\sum_{i=1}^{n}\alpha_{R,i,q}\hat{s}_{i,t}^k\right)+\mathrm{Re}\left(r_{q,t}^k\sum_{i=1}^{n}j\alpha_{I,i,q}\hat{s}_{i,t}^k\right)$$
$$=\mathrm{Re}\left(r_{q,t}^k\sum_{i=1}^{n}\alpha_{R,i,q}\hat{s}_{i,t}^k\right)+\mathrm{Im}\left(r_{q,t}^k\sum_{i=1}^{n}\alpha_{I,i,q}\hat{s}_{i,t}^k\right) \tag{4.18}$$

and

$$
\begin{aligned}
\sum_{i=1}^{n} \alpha_{i,q} \hat{s}_{i,t}^{k} \sum_{l=1}^{n} \alpha_{l,q}^{*} \hat{s}_{l,t}^{k*} &= \sum_{i=1}^{n} \sum_{l=1}^{n} \left(\alpha_{R,i,q} + j\alpha_{I,i,q}\right) \hat{s}_{i,t}^{k} \left(\alpha_{R,l,q} + j\alpha_{I,l,q}\right) \hat{s}_{l,t}^{k} \\
&= \sum_{i=1}^{n} \sum_{l=1}^{n} \left(\alpha_{R,i,q}\alpha_{R,l,q}^{*} + \alpha_{I,i,q}\alpha_{I,l,q}\right) \hat{s}_{i,t}^{k} \hat{s}_{l,t}^{k} \\
&= \sum_{i=1}^{n} \alpha_{R,i,q}^{2} \left|\hat{s}_{i,t}^{k}\right|^{2} + \sum_{i=1}^{n} \alpha_{I,i,q}^{2} \left|\hat{s}_{i,t}^{k}\right|^{2} \\
&\quad + \sum_{\substack{i=1 \\ i \neq l}}^{n} \sum_{\substack{l=1 \\ l \neq i}}^{n} \alpha_{R,i,q}\alpha_{R,l,q} \hat{s}_{i,t}^{k} \hat{s}_{l,t}^{k} + \sum_{\substack{i=1 \\ i \neq l}}^{n} \sum_{\substack{l=1 \\ l \neq i}}^{n} \alpha_{I,i,q}\alpha_{I,l,q} \hat{s}_{i,t}^{k} \hat{s}_{l,t}^{k}. \qquad (4.19)
\end{aligned}
$$

Then (4.15) can be expressed as

$$
\begin{aligned}
&f\left(\boldsymbol{r} | \hat{\boldsymbol{s}}, \boldsymbol{\alpha}, \boldsymbol{x}\right) \\
&= \prod_{t=1}^{L} \prod_{k=1}^{M} \prod_{q=1}^{m} \frac{1}{\sqrt{\pi a_{t}^{k}}} \exp\left\{ - \sum_{t=1}^{L} \sum_{k=1}^{M} \sum_{q=1}^{m} \frac{1}{a_{t}^{k}} \left[\left|r_{q,t}^{k}\right|^{2} - \operatorname{Re}\left(r_{q,t}^{k} \sum_{i=1}^{n} \alpha_{R,i,q} \hat{s}_{i,t}^{k} \right) \right.\right. \\
&\quad - \operatorname{Im}\left(r_{q,t}^{k} \sum_{i=1}^{n} \alpha_{I,i,q} \hat{s}_{i,t}^{k} \right) + \sum_{i=1}^{n} \alpha_{R,i,q}^{2} \left|\hat{s}_{i,t}^{k}\right|^{2} + \sum_{i=1}^{n} \alpha_{I,i,q}^{2} \left|\hat{s}_{i,t}^{k}\right|^{2} \\
&\quad \left.\left. + \sum_{\substack{i=1 \\ i \neq l}}^{n} \sum_{\substack{l=1 \\ l \neq i}}^{n} \alpha_{R,i,q}\alpha_{R,l,q} \hat{s}_{i,t}^{k} \hat{s}_{l,t}^{k} + \sum_{\substack{i=1 \\ i \neq l}}^{n} \sum_{\substack{l=1 \\ l \neq i}}^{n} \alpha_{I,i,q}\alpha_{I,l,q} \hat{s}_{i,t}^{k} \hat{s}_{l,t}^{k} \right] \right\}. \qquad (4.20)
\end{aligned}
$$

We can get the likelihood function of $\boldsymbol{r}$ given $\hat{\boldsymbol{s}}$ and $\boldsymbol{x}$ by averaging (4.20) with respect to the probability density function of $\alpha_{R,i,q}$ and $\alpha_{I,i,q}$. $f\left(\boldsymbol{r} | \hat{\boldsymbol{s}}, \boldsymbol{x}\right)$ can be written as

$$
\begin{aligned}
&f\left(\boldsymbol{r} | \hat{\boldsymbol{s}}, \boldsymbol{x}\right) \\
&= \left[\prod_{i=1}^{n} \prod_{q=1}^{m} \prod_{k=1}^{M} \left(a_{t}^{k}\right)^{-\frac{1}{2}} \exp\left(-\frac{\left|r_{q,t}^{k}\right|^{2}}{a_{t}^{k}} \right) \right] \left[\prod_{q=1}^{m} \prod_{i=1}^{n} \left(\sum_{t=1}^{L} \sum_{k=1}^{M} \frac{1}{a_{t}^{k}} \lambda_{i,t}^{k} + 1 \right) \right]^{-1} \\
&\quad \cdot \exp\left\{ \sum_{q=1}^{m} \sum_{i=1}^{n} \frac{ \left(\sum_{t=1}^{L} \sum_{k=1}^{M} \frac{1}{a_{t}^{k}} z_{i,q,t}^{k} \right)^{2} + \left(\sum_{t=1}^{L} \sum_{k=1}^{M} \frac{1}{a_{t}^{k}} w_{i,q,t}^{k} \right)^{2} }{ 4 \left(\sum_{k=1}^{M} \sum_{i=1}^{n} \frac{1}{a_{t}^{k}} \lambda_{i,t}^{k} + 1 \right) } \right\} \qquad (4.21)
\end{aligned}
$$

where

$$
\begin{aligned}
z_{i,q,t} &= \left[2\operatorname{Re}\left(r_{q,t}^{k} \hat{s}_{1,t}^{k}\right), 2\operatorname{Re}\left(r_{q,t}^{k} \hat{s}_{2,t}^{k}\right), \ldots, 2\operatorname{Re}\left(r_{q,t}^{k} \hat{s}_{n,t}^{k}\right) \right] v_{i,t}^{k} \\
w_{i,q,t} &= \left[2\operatorname{Im}\left(r_{q,t}^{k} \hat{s}_{1,t}^{k}\right), 2\operatorname{Im}\left(r_{q,t}^{k} \hat{s}_{2,t}^{k}\right), \ldots, 2\operatorname{Im}\left(r_{q,t}^{k} \hat{s}_{n,t}^{k}\right) \right] v_{i,t}^{k}
\end{aligned}
$$

$v_{i,t}^k$ and $\lambda_{i,t}^k$ are the eigenvectors and the eigenvalues of the following matrix, respectively:

$$\begin{bmatrix} \left|\hat{s}_{1,t}^k\right|^2 & \hat{s}_{1,t}^k\hat{s}_{2,t}^k & \cdots & \hat{s}_{1,t}^k\hat{s}_{n,t}^k \\ \hat{s}_{2,t}^k\hat{s}_{1,t}^k & \left|\hat{s}_{2,t}^k\right|^2 & \cdots & \hat{s}_{2,t}^k\hat{s}_{n,t}^k \\ \vdots & \vdots & \ddots & \vdots \\ \hat{s}_{n,t}^k\hat{s}_{1,t}^k & \hat{s}_{n,t}^k\hat{s}_{2,t}^k & \cdots & \left|\hat{s}_{n,t}^k\right|^2 \end{bmatrix}. \tag{4.22}$$

The derivation of (4.21) is in Appendix B. The ML decoding makes the decision by maximizing (4.21). Take logarithm on this likelihood function, then the codewords can also be decoded by maximizing the following metric

$$\sum_{q=1}^{m}\sum_{i=1}^{n}\frac{\left(\sum_{t=1}^{L}\sum_{k=1}^{M}\frac{1}{a_t^k}z_{i,q,t}^k\right)^2+\left(\sum_{t=1}^{L}\sum_{k=1}^{M}\frac{1}{a_t^k}w_{i,q,t}^k\right)^2}{4\left(\sum_{k=1}^{M}\sum_{i=1}^{n}\frac{1}{a_t^k}\lambda_{i,t}^k+1\right)} -\sum_{q=1}^{m}\sum_{i=1}^{n}\left(\sum_{t=1}^{L}\sum_{k=1}^{M}\frac{1}{a_t^k}\lambda_{i,t}^k+1\right) \tag{4.23}$$

4.2.4 Decoing without JSI and CSI

Suppose the fading coefficients $\alpha_{i,q}$ and the jamming state information x_t^k are not available at the receiver.

$$\begin{aligned} & f\left(\boldsymbol{r}|\hat{\boldsymbol{s}}\right) \\ &= \prod_{i=1}^{n}\prod_{q=1}^{m}\left\{\sum_{k'=1}^{M}\frac{Q}{N_t}\left[\prod_{t=1}^{L}\left(\prod_{\substack{k=1\\k\neq k'}}^{M}\frac{1}{\sqrt{N_0}}\exp\left(-\frac{\left|r_{q,t}^k\right|^2}{N_0}\right)+\frac{1}{\sqrt{a}}\exp\left(-\frac{\left|r_{q,t}^k\right|^2}{a}\right)\right)\right]\right. \\ & \exp\left\{\frac{1}{4}\left(\sum_{t}^{L}\left(\sum_{\substack{k=1\\k\neq k'}}^{M}\frac{1}{N_0}u_{i,q,t}^k+\frac{1}{a}u_{i,q,t}^{k'}\right)\right)^2-\left(\sum_{t=1}^{L}\left(\sum_{\substack{k=1\\k\neq k'}}^{M}\frac{1}{N_0}u_{i,q,t}^k+\frac{1}{a}u_{i,q,t}^{k'}\right)\right)^2\right. \\ & \left.\cdot\left(\sum_{t=1}^{L}\left(\sum_{\substack{k=1\\k\neq k'}}^{M}\frac{1}{N_0}\lambda_{i,t}^k+\frac{1}{a}\lambda_{i,t}^{k'}\right)\right)\right\}\left(1-\sum_{t=1}^{L}\left(\sum_{\substack{k=1\\k\neq k'}}^{M}\frac{1}{N_0}\lambda_{i,t}^k+\frac{1}{a}\lambda_{i,t}^{k'}\right)\right) \\ & \left(1-\frac{MQ}{N_t}\right)\left[\prod_{t=1}^{L}\prod_{k=1}^{M}\frac{1}{\sqrt{N_0}}\exp\left(-\frac{\left|r_{q,t}^k\right|^2}{N_0}\right)\right]\exp\left\{\frac{1}{4}\left(\sum_{t=1}^{L}\sum_{k=1}^{M}\frac{1}{N_0}u_{i,q,t}^k\right)^2\right. \\ & \left.\left.-\frac{1}{4}\left(\sum_{t=1}^{L}\sum_{k=1}^{M}\frac{1}{N_0}u_{i,q,t}^k\right)^2\left(\sum_{t=1}^{L}\sum_{k=1}^{M}\frac{1}{N_0}\lambda_{k_{i,t}}\right)\right\}\left(1-\sum_{t=1}^{L}\sum_{k=1}^{M}\frac{1}{N_0}\lambda_{k_{i,t}}\right)\right\} \end{aligned} \tag{4.23}$$

where $u_{i,q,t}^k = r_{q,t}^k s_{i,t}^k$. The derivation is the same as Appendix B and Appendix C.

4.2.5 Design Criteria for Constructing Good Space-Time Codes

We propose a design criteria for constructing good space-time codes of the STC/WFHSS system with respect to wireless multitone jamming channels. Let $\boldsymbol{s} = \left(s_{i,t}^k | 1 \leq i \leq n, 1 \leq t \leq L, 1 \leq k \leq M\right)$ be the codeword sequence transmitted from transmitter, and $\tilde{\boldsymbol{s}} = \left(\tilde{s}_{i,t}^k | 1 \leq i \leq n, 1 \leq t \leq L, 1 \leq k \leq M\right)$ be the error codeword sequence decided at the receiver. Assume the perfect estimation of $\alpha_{i,q}$ and x_t^k are available for $\forall\ i, q, t, k$ at the receiver. The conditional pairwise error probability that the decoder decides in favor of $\tilde{\boldsymbol{s}}$ than $\boldsymbol{s}$ is given by

$$
\begin{aligned}
&\Pr\left(\boldsymbol{s} \rightarrow \tilde{\boldsymbol{s}} | \boldsymbol{\alpha}, \boldsymbol{x}\right) \\
&= \Pr\left\{\prod_{t=1}^{L}\prod_{k=1}^{M}\prod_{q=1}^{m} \frac{1}{\sqrt{\pi\left(N_0 + x_t^k \frac{JT_s}{Q}\right)}} \exp\left(-\frac{\left|r_{q,t}^k - \sum_{i=1}^{n} \alpha_{i,q}\hat{s}_{i,t}^k\right|^2}{\left(N_0 + x_t^k \frac{JT_s}{Q}\right)}\right) \leq \right. \\
&\left. \prod_{t=1}^{L}\prod_{k=1}^{M}\prod_{q=1}^{m} \frac{1}{\sqrt{\pi\left(N_0 + x_t^k \frac{JT_s}{Q}\right)}} \exp\left(-\frac{\left|r_{q,t}^k - \sum_{i=1}^{n} \alpha_{i,q}\tilde{s}_{i,t}^k\right|^2}{\left(N_0 + x_t^k \frac{JT_s}{Q}\right)}\right)\right\} \\
&= Q\left(\sqrt{\frac{\sum_{t=1}^{L}\sum_{q=1}^{m}\sum_{k=1}^{M}\left|\sum_{i=1}^{n} \alpha_{i,q}\left(s_{i,t}^k - \tilde{s}_{i,t}^k\right)\right|^2}{2\left(N_0 + x_t^k \frac{JT_s}{Q}\right)}}\right)
\end{aligned}
\tag{4.24}
$$

where $Q(a)$ is the complementary error function defined by

$$
Q(a) = \frac{1}{\sqrt{2\pi}} \int_a^{\infty} e^{-x^2/2} dx. \tag{4.25}
$$

Accroding to the inequality $Q(a) \leq \frac{1}{2}\exp\left(-a^2/2\right)\ \forall a \geq 0$, (4.24) can be upper bounded by

$$
\boldsymbol{Pr}\left(\boldsymbol{s} \rightarrow \tilde{\boldsymbol{s}} | \boldsymbol{\alpha}, \boldsymbol{x}\right) \leq \frac{1}{2}\exp\left(-\sum_{t=1}^{L}\sum_{q=1}^{m}\sum_{k=1}^{M} \frac{1}{b_t^k}\left|\sum_{i=1}^{n} \alpha_{i,q}\left(s_{i,t}^k - \tilde{s}_{i,t}^k\right)\right|^2\right) \tag{4.26}
$$

where $b_t^k = 4\left(N_0 + x_t^k \frac{JT_s}{Q}\right)$. By averaging (4.26) with respect to $\boldsymbol{\alpha}$, the conditional pairwise error probability given $\boldsymbol{x}$ is

$$
\boldsymbol{Pr}\left(\boldsymbol{s} \rightarrow \tilde{\boldsymbol{s}} | \boldsymbol{x}\right) \leq \frac{1}{2}\prod_{i=1}^{n}\prod_{q=1}^{m}\left(1 + \sum_{t=1}^{L}\sum_{k=1}^{M} \frac{\lambda_{i,t}^k}{b_t^k}\right)^{-1}. \tag{4.27}
$$

where $\lambda_{i,t}^{k}$ $\forall i$ are the eigenvalues of the matrix A_t^k, and matrix A_t^k is expressed as

$$\begin{bmatrix} \left|s_{1,t}^{k}-\tilde{s}_{1,t}^{k}\right|^{2} & \left(s_{1,t}^{k}-\tilde{s}_{1,t}^{k}\right)\left(s_{2,t}^{k}-\tilde{s}_{2,t}^{k}\right) & \dots & \left(s_{1,t}^{k}-\tilde{s}_{1,t}^{k}\right)\left(s_{n,t}^{k}-\tilde{s}_{n,t}^{k}\right) \\ \left(s_{2,t}^{k}-\tilde{s}_{2,t}^{k}\right)\left(s_{1,t}^{k}-\tilde{s}_{1,t}^{k}\right) & \left|s_{2,t}^{k}-\tilde{s}_{2,t}^{k}\right|^{2} & \dots & \left(s_{2,t}^{k}-\tilde{s}_{2,t}^{k}\right)\left(s_{n,t}^{k}-\tilde{s}_{n,t}^{k}\right) \\ \vdots & \vdots & \ddots & \vdots \\ \left(s_{n,t}^{k}-\tilde{s}_{n,t}^{k}\right)\left(s_{1,t}^{k}-\tilde{s}_{1,t}^{k}\right) & \left(s_{n,t}^{k}-\tilde{s}_{n,t}^{k}\right)\left(s_{2,t}^{k}-\tilde{s}_{2,t}^{k}\right) & \dots & \left|s_{n,t}^{k}-\tilde{s}_{n,t}^{k}\right|^{2} \end{bmatrix}. \tag{4.28}$$

Then averaging (4.27) with respect to $\boldsymbol{x}$, the pairwise error probability is approxmated as

$$\begin{aligned} &\boldsymbol{Pr}\left(\boldsymbol{s}\rightarrow\tilde{\boldsymbol{s}}\right) \\ &\leq \frac{1}{2}\prod_{i=1}^{n}\prod_{q=1}^{n}\left\{1-\sum_{t-1}^{L}\left[\left(1-\frac{Q}{N_t}\right)\sum_{k=1}^{M}\frac{1}{4N_0}\lambda_{i,t}^{k}+\frac{Q}{N_t}\sum_{k=1}^{M}\frac{1}{4\left(N_0+\frac{JT_s}{Q}\right)}\lambda_{i,t}^{k}\right]\right\}. \end{aligned} \tag{4.29}$$

From (4.29), we konw that we would construct different good space-time codes with different SNR and SJR. In order to simplify (4.29), assume there are only two transmitter antennas, the pairwise error probability is

$$\begin{aligned} &\boldsymbol{Pr}\left(\boldsymbol{s}\rightarrow\tilde{\boldsymbol{s}}\right) \\ &\leq \frac{1}{2}\prod_{q=1}^{n}\left\{1+\left(\left(1-\frac{Q}{N_t}\right)\frac{1}{4N_0}+\frac{Q}{N_t}\frac{1}{4\left(N_0+\frac{JT_s}{Q}\right)}\right)^{2}\prod_{i=1}^{2}\left(\sum_{t=1}^{L}\sum_{k=1}^{M}\lambda_{i,t}^{k}\right)\right. \\ &\qquad\left.-\left(\left(1-\frac{Q}{N_t}\right)\frac{1}{4N_0}+\frac{Q}{N_t}\frac{1}{4\left(N_0+\frac{JT_s}{Q}\right)}\right)\sum_{t=1}^{L}\sum_{i=1}^{2}\sum_{k=1}^{M}\lambda_{i,t}^{k}\right\} \end{aligned} \tag{4.30}$$

Let

$$\begin{aligned} v1 &= \left(\left(1-\frac{Q}{N_t}\right)\frac{1}{4N_0}+\frac{Q}{N_t}\frac{1}{4\left(N_0+\frac{JT_s}{Q}\right)}\right)^{2}\prod_{i=1}^{2}\left(\sum_{t=1}^{L}\sum_{k=1}^{M}\lambda_{i,t}^{k}\right) \\ v2 &= 1-\left(\left(1-\frac{Q}{N_t}\right)\frac{1}{4N_0}+\frac{Q}{N_t}\frac{1}{4\left(N_0+\frac{JT_s}{Q}\right)}\right)\sum_{t=1}^{L}\sum_{i=1}^{2}\sum_{k=1}^{M}\lambda_{i,t}^{k}. \end{aligned} \tag{4.31}$$

Assume the multitone jamming power is much larger than the thermal noise power, then $N_0+\frac{JT_S}{Q}\gg N_0$ and $\frac{1}{N_0+\frac{JT_S}{Q}}\ll\frac{1}{N_0}$ and we also have following inequality functions

$$\begin{aligned} &\prod_{i=1}^{2}\left(\sum_{t=1}^{L}\sum_{k=1}^{M}\lambda_{i,t}^{k}\right)\leq\left(LME_s\right)^{2} \\ &\sum_{t=1}^{L}\sum_{i=1}^{2}\sum_{k=1}^{M}\lambda_{i,t}^{k}\leq 2LME_s. \end{aligned} \tag{4.32}$$

Base on the above inequality functions, $v1$ and $v2$ can be upper bounded by

$$v1 \le \left(\left(1-\frac{Q}{N_t}\right)\frac{LM}{4}\text{SNR}\right)^2$$
$$v2 \le 1-\left(1-\frac{Q}{N_t}\right)\frac{LM}{2}\text{SNR}. \tag{4.33}$$

After doing the simulation, we konw that $|v1| \gg |v2|$. Then good codes could be constructed by maximizing $m(\boldsymbol{s}, \tilde{\boldsymbol{s}})$ for all possiable $\boldsymbol{s}$ and $\tilde{\boldsymbol{s}}$, and $m(\boldsymbol{s}, \tilde{\boldsymbol{s}})$ can be expressed as

$$m(\boldsymbol{s}, \tilde{\boldsymbol{s}}) = \prod_{i=1}^{2}\left(\sum_{t=1}^{L}\sum_{k=1}^{M}\lambda_{i,t}^{k}\right). \tag{4.34}$$

According to the design criteria, good space-time codes are searched by the computer are given in table (4.1)

Table 4.1: Optimal Space-time codes of the STC/WFHSS system with 4FSK and 2 transmitter antennas for wireless jamming channels.

Memory	**Generator Sequences**
2	$(x_1^t, x_2^t) = b_{t-1}(1,0) \oplus_4 a_{t-1}(3,0) \oplus_4 b_t(1,3) \oplus_4 a_t(2,2)$
3	$(x_1^t, x_2^t) = a_{t-2}(1,2) \oplus_4 b_{t-1}(1,1) \oplus_4 a_{t-1}(1,0) \oplus_4 b_t(2,1)$ $\oplus_4 a_t(3,2)$
4	$(x_1^t, x_2^t) = b_{t-2}(1,1) \oplus_4 a_{t-2}(2,2) \oplus_4 b_{t-1}(0,2) \oplus_4 a_{t-1}(3,0)$ $\oplus_4 b_t(2,0) \oplus_4 a_t(3,0)$

4.3 STC Combined with the Optimum Case Frequency Hopping

The other system is STC/OFHSS system is shown in Figure4.3. The encoded codewords from any transmitter antennas are hopped into distinct M-ary bands. The ML decoding schemes with respect to STC/OFHSS system are derived in this section, and the criteria of constructing good space-time codes for OFHSS system is also proposed.

4.3.1 Decoing with CSI and JSI Available

The ML decoding of STC/OFHSS system is derived as follow, and the system are assumed to transmit the signal in slow fading channel with n=1 band multitone jammers. Let the received signals $\boldsymbol{r} = \left(r_{i,q,t}^{k} \middle| \forall q, k,\ 1 \le t \le L\right)$, the jamming state infromation $\boldsymbol{x} = \left(x_{i,t}^{k} \middle| \forall i, k,\ 1 \le t \le L\right)$, the fading coefficients $\boldsymbol{\alpha} = (\alpha_{i,q} | \forall i, q)$, and the estimated symbols $\hat{\boldsymbol{s}} = \left(s_{i,t}^{k} \middle| \forall i,\ 1 \le t \le L\right)$. Assume the fading coefficients $\alpha_{i,q}$ and the jamming state information x_t^k are available at the receiver, the likelihood of $\boldsymbol{r}$ given $\hat{\boldsymbol{s}}$, $\boldsymbol{x}$, and $\boldsymbol{\alpha}$ can be express as

$$
\begin{aligned}
& f\left\{\boldsymbol{r}|\hat{\boldsymbol{s}},\boldsymbol{x},\boldsymbol{\alpha}\right\} \\
&= \prod_{t=1}^{L}\prod_{k=1}^{M}\prod_{q=1}^{m}\prod_{i=1}^{n} f\left\{\eta_{i,q,t}^k = r_{i,q,t}^k - \alpha_{i,q}\hat{s}_{i,t}^k|\hat{s}_{i,t}^k,\alpha_{i,q},x_t^k\right\} \\
&= \prod_{t=1}^{L}\prod_{k=1}^{M}\prod_{q=1}^{m}\prod_{i=1}^{n} \frac{1}{\sqrt{\pi\left(N_0 + x_{i,t}^k 2\frac{JT_s}{Q}\sigma_{J,q}^2\right)}}\exp\left(-\frac{\left|r_{i,q,t}^k - \alpha_{i,q}\hat{s}_{i,t}^k\right|^2}{\left(N_0 + x_{i,t}^k 2\frac{JT_s}{Q}\sigma_{J,q}^2\right)}\right).
\end{aligned}
\tag{4.36}
$$

The codeword can also be decoded in ML decoding sense by minimizing the following metric

$$
\sum_{t=1}^{L}\sum_{k=1}^{M}\sum_{q=1}^{m}\sum_{i=1}^{n}\frac{\left|r_{i,q,t}^k - \alpha_{i,q}\hat{s}_{i,t}^k\right|}{\left(N_0 + x_{i,t}^k 2\frac{JT_s}{Q}\sigma_{J,q}^2\right)}. \tag{4.37}
$$

4.3.2 Decoing with CSI but without JSI

Assume the fading coefficients $\alpha_{i,q}$ are available at the receiver, but the jamming state information $x_{i,t}^k$ are not available at the receiver. The likelihood function of $\boldsymbol{r}$ given $\hat{\boldsymbol{s}}$ and $\boldsymbol{\alpha}$ can be obtained by averaging (4.36) with respect to $\boldsymbol{x}$.

$$
\begin{aligned}
& f\left(\boldsymbol{r}|\hat{\boldsymbol{s}},\boldsymbol{\alpha}\right) \\
&= E_{\boldsymbol{x}}\left[f\left(\boldsymbol{r}|\hat{\boldsymbol{s}},\boldsymbol{x},\boldsymbol{\alpha}\right)\right] \\
&= E_{\boldsymbol{x}}\left[\prod_{t=1}^{L}\prod_{k=1}^{M}\prod_{q=1}^{m}\prod_{i=1}^{n} \frac{1}{\sqrt{\pi\left(N_0 + x_{i,t}^k 2\frac{JT_s}{Q}\sigma_{J,q}^2\right)}}\exp\left(-\frac{\left|r_{q,t}^k - \alpha_{i,q}\hat{s}_{i,t}^k\right|^2}{\left(N_0 + x_{i,t}^k 2\frac{JT_s}{Q}\sigma_{J,q}^2\right)}\right)\right].
\end{aligned}
\tag{4.38}
$$

The probability of $\boldsymbol{x}$ with respect to two transmitter antennas is

$$
\begin{aligned}
\boldsymbol{Pr}\left(\boldsymbol{x}_{1,t} = (1,0,\ldots,0), \boldsymbol{x}_{2,t} = (1,0,\ldots,0)\right) &= \frac{\phi_1}{M^2} \\
\boldsymbol{Pr}\left(\boldsymbol{x}_{1,t} = (1,0,\ldots,0), \boldsymbol{x}_{2,t} = (0,1,\ldots,0)\right) &= \frac{\phi_1}{M^2} \\
&\vdots \\
\boldsymbol{Pr}\left(\boldsymbol{x}_{1,t} = (0,0,\ldots,1), \boldsymbol{x}_{2,t} = (0,0,\ldots,1)\right) &= \frac{\phi_1}{M^2} \\
\boldsymbol{Pr}\left(\boldsymbol{x}_{1,t} = (0,0,\ldots,0), \boldsymbol{x}_{2,t} = (1,0,\ldots,0)\right) &= \frac{\phi_2}{M} \\
\boldsymbol{Pr}\left(\boldsymbol{x}_{1,t} = (0,0,\ldots,0), \boldsymbol{x}_{2,t} = (0,1,\ldots,0)\right) &= \frac{\phi_2}{M}
\end{aligned}
$$

$$
\begin{aligned}
&\vdots \\
&\boldsymbol{Pr}\left(\boldsymbol{x}_{1,t}=(0,0,\ldots,0),\boldsymbol{x}_{2,t}=(0,0,\ldots,1)\right)=\frac{\phi_2}{M} \\
&\boldsymbol{Pr}\left(\boldsymbol{x}_{1,t}=(1,0,\ldots,0),\boldsymbol{x}_{2,t}=(0,0,\ldots,0)\right)=\frac{\phi_3}{M} \\
&\boldsymbol{Pr}\left(\boldsymbol{x}_{1,t}=(0,1,\ldots,0),\boldsymbol{x}_{2,t}=(0,0,\ldots,0)\right)=\frac{\phi_3}{M} \\
&\vdots \\
&\boldsymbol{Pr}\left(\boldsymbol{x}_{1,t}=(0,0,\ldots,1),\boldsymbol{x}_{2,t}=(0,0,\ldots,0)\right)=\frac{\phi_3}{M} \\
&\boldsymbol{Pr}\left(\boldsymbol{x}_{1,t}=(0,0,\ldots,0),\boldsymbol{x}_{2,t}=(0,0,\ldots,0)\right)=\phi_4
\end{aligned}
\tag{4.39}
$$

where $\boldsymbol{x}_{i,t}=\left(x_{i,t}^1,x_{i,t}^2,\ldots,x_{i,t}^M\right)$, $\phi_1=\frac{Q}{N_t/M}\frac{Q-1}{N_t/M-1}$, $\phi_2=\frac{Q}{N_t/M}\left(1-\frac{Q-1}{N_t/M-1}\right)$, $\phi_3=\left(1-\frac{Q}{N_t/M}\right)$ $\cdot\frac{Q}{N_t/M-1}$, and $\phi_4=\left(1-\frac{Q}{N_t/M}\right)\left(1-\frac{Q}{N_t/M-1}\right)$. Therefore, a close-form expression of $f\left(\boldsymbol{r}|\hat{\boldsymbol{s}},\boldsymbol{\alpha}\right)$ with respect to two transmitter antennas is derived in Appendix D

$$
\begin{aligned}
&f\left(\boldsymbol{r}|\hat{\boldsymbol{s}},\boldsymbol{\alpha}\right) \\
&=\prod_{t=1}^{L}\prod_{q=1}^{m}\left\{\exp\left[-\sum_{i=1}^{n}\sum_{k=1}^{M}\frac{\left|r_{i,q,t}^k-\alpha_{i,q}\hat{s}_{i,t}^k\right|^2}{N_0}\right]\left\{\frac{\phi_1}{M^2}\left(\frac{1}{\sqrt{\pi N_0}}\right)^{2M-2}\left(\frac{1}{\sqrt{\pi a}}\right)^2\right.\right. \\
&\quad\cdot\sum_{k=1}^{M}\sum_{k'=1}^{M}\exp\left[\frac{a-N_0}{aN_0}\left(\left|r_{1,q,t}^k-\alpha_{1,q}\hat{s}_{1,t}^k\right|^2+\left|r_{2,q,t}^k-\alpha_{2,q}\hat{s}_{2,t}^k\right|^2\right)\right]+\frac{\phi_2}{M} \\
&\quad\cdot\left(\frac{1}{\sqrt{\pi N_0}}\right)^{2M-1}\left(\frac{1}{\sqrt{\pi a}}\right)\sum_{k=1}^{M}\exp\left[\frac{a-N_0}{aN_0}\left(\left|r_{1,q,t}^k-\alpha_{1,q}\hat{s}_{1,t}^k\right|^2\right)\right]+\frac{\phi_3}{M} \\
&\quad\left.\left.\cdot\left(\frac{1}{\sqrt{\pi N_0}}\right)^{2M-1}\left(\frac{1}{\sqrt{\pi a}}\right)\sum_{k=1}^{M}\exp\left[\frac{a-N_0}{aN_0}\left(\left|r_{2,q,t}^k-\alpha_{2,q}\hat{s}_{2,t}^k\right|^2\right)\right]+\phi_4\left(\frac{1}{\sqrt{\pi N_0}}\right)^{2M}\right\}\right\}
\end{aligned}
\tag{4.40}
$$

where $a=N_0+2\frac{JT_s}{Q}\sigma_{J,q}^2$. By taking logarithm on that likelihood function, codewords can be decoded in the ML decoding sense by maximizing the following metric :

$$
\begin{aligned}
&\sum_{t=1}^{L}\sum_{q=1}^{m}\left\{-\sum_{i=1}^{n}\sum_{k=1}^{M}\frac{\left|r_{i,q,t}^k-\alpha_{i,q}\hat{s}_{i,t}^k\right|^2}{N_0}+\ln\left\{\frac{\phi_1}{M^2}\left(\frac{1}{\sqrt{\pi N_0}}\right)^{2M-2}\left(\frac{1}{\sqrt{\pi a}}\right)^2\right.\right. \\
&\cdot\sum_{k=1}^{M}\sum_{k'=1}^{M}\exp\left[\frac{a-N_0}{aN_0}\left(\left|r_{1,q,t}^k-\alpha_{1,q}\hat{s}_{1,t}^k\right|^2+\left|r_{2,q,t}^k-\alpha_{2,q}\hat{s}_{2,t}^k\right|^2\right)\right]+\frac{\phi_2}{M} \\
&\cdot\left(\frac{1}{\sqrt{\pi N_0}}\right)^{2M-1}\left(\frac{1}{\sqrt{\pi a}}\right)\sum_{k=1}^{M}\exp\left[\frac{a-N_0}{aN_0}\left(\left|r_{1,q,t}^k-\alpha_{1,q}\hat{s}_{1,t}^k\right|^2\right)\right]+\frac{\phi_3}{M}
\end{aligned}
$$

$$\cdot \left(\frac{1}{\sqrt{\pi N_0}} \right)^{2M-1} \left(\frac{1}{\sqrt{\pi a}} \right) \sum_{k=1}^{M} \exp \left[\frac{a - N_0}{a N_0} \left(\left| r_{2,q,t}^k - \alpha_{2,q} \hat{s}_{2,t}^k \right|^2 \right) \right] + \phi_4$$

$$\cdot \left(\frac{1}{\sqrt{\pi N_0}} \right)^{2M} \Bigg\} \Bigg\} \tag{4.41}$$

4.3.3 Decoidng with JSI but without CSI

Suppose the fading coefficients $\alpha_{i,q}$'s are not available at the receiver, and the fading coefficients are modeled as independent complex Gaussian random variables with zero mean and variance $\sigma_{i,q}^2$ per dimension with respect to Rayleigh fading channels. In order to simplify mathematics, we assume $\sigma_{i,q}^2 = 1/2$ and $\sigma_{J,q}^2 = 1/2$ for $\forall i, q$ in this section.

Let $a_{i,t}^k = N_0 + x_{i,t}^k \frac{JT_s}{Q}$, then (4.36) can be rewritten as

$$f\left\{ \boldsymbol{r} | \hat{\boldsymbol{s}}, \boldsymbol{x}, \boldsymbol{\alpha} \right\}$$
$$= \left\{ \prod_{t=1}^{L} \prod_{k=1}^{M} \prod_{q=1}^{m} \prod_{i=1}^{n} \frac{1}{\sqrt{\pi a_{i,t}^k}} \right\} \exp \left\{ - \sum_{t=1}^{L} \sum_{k=1}^{M} \sum_{q=1}^{m} \sum_{i=1}^{n} \frac{1}{a_{i,t}^k} \left[\left| r_{i,q,t}^k \right|^2 \right. \right.$$
$$\left. \left. -2\mathrm{Re} \left(r_{i,q,t}^k \alpha_{i,q} \hat{s}_{i,t}^k \right) + \left| \alpha_{i,q} \hat{s}_{i,t}^k \right|^2 \right] \right\} \tag{4.42}$$

and the fading gain $\alpha_{i,q}$ can be presented as

$$\alpha_{i,q} = \alpha_{R,i,q} + j\alpha_{I,i,q}$$

where $\alpha_{R,i,q}$ and $\alpha_{I,i,q}$ are statistically independent Gaussian random variables with zero mean and variance $\sigma_{i,q} = 1/2$. Rewrite $\mathrm{Re}\left(r_{i,q,t}^k \alpha_{i,q} \hat{s}_{i,t}^k \right)$ and $\left| \alpha_{i,q} \hat{s}_{i,t}^k \right|^2$ in (4.42) as

$$\begin{aligned} \mathrm{Re}\left(r_{i,q,t}^k \alpha_{i,q} \hat{s}_{i,t}^k \right) &= \mathrm{Re}\left(r_{i,q,t}^k \left(\alpha_{R,i,q} + j\alpha_{I,i,q} \right) \hat{s}_{i,t}^k \right) \\ &= \mathrm{Re}\left(r_{i,q,t}^k \alpha_{R,i,q} \hat{s}_{i,t}^k \right) + \mathrm{Re}\left(r_{i,q,t}^k j\alpha_{I,i,q} \hat{s}_{i,t}^k \right) \\ &= \mathrm{Re}\left(r_{i,q,t}^k \alpha_{R,i,q} \hat{s}_{i,t}^k \right) + \mathrm{Im}\left(r_{i,q,t}^k \alpha_{I,i,q} \hat{s}_{i,t}^k \right) \end{aligned} \tag{4.43}$$

and

$$\begin{aligned} \left| \alpha_{i,q} \hat{s}_{i,t}^k \right|^2 &= \left(\alpha_{R,i,q} + j\alpha_{I,i,q} \right) \left(\alpha_{R,i,q} - j\alpha_{I,i,q} \right) \left| \hat{s}_{i,t}^k \right|^2 \\ &= \left(\alpha_{R,i,q}^2 + \alpha_{I,i,q}^2 \right) \left| \hat{s}_{i,t}^k \right|^2 . \end{aligned} \tag{4.44}$$

The likelihood of $\boldsymbol{r}$ given $\hat{\boldsymbol{s}}$ and $\boldsymbol{x}$ can be derived by averaging $\boldsymbol{\alpha}$, and can be expressed as

$$
\begin{aligned}
&= f\left\{\boldsymbol{r}|\hat{\boldsymbol{s}},\boldsymbol{x}\right\} \\
&= \int_{-\infty}^{\infty} f\left\{\boldsymbol{r}|\hat{\boldsymbol{s}},\boldsymbol{x},\boldsymbol{\alpha}\right\} d\boldsymbol{\alpha} \\
&= \left\{\prod_{t=1}^{L}\prod_{k=1}^{M}\prod_{q=1}^{m}\prod_{i=1}^{n}\frac{1}{\sqrt{\pi a_{i,t}^{k}}}\exp\left(-\frac{1}{\left|r_{i,q,t}^{k}\right|^{2}}\right)\right\}\left\{\prod_{q=1}^{m}\prod_{i=1}^{n}\left(\sum_{t=1}^{L}\sum_{k=1}^{M}\frac{\left|\hat{s}_{i,t}^{k}\right|^{2}}{a_{i,t}^{k}}+1\right)\right\}^{-1} \\
&\quad \exp\left\{\sum_{q=1}^{m}\sum_{i=1}^{n}\frac{\left|\sum_{t=1}^{L}\sum_{k=1}^{M}\frac{2}{a_{i,t}^{k}}\mathrm{Re}\left(r_{i,q,t}^{k}\hat{s}_{i,t}^{k}\right)\right|^{2}+\left|\sum_{t=1}^{L}\sum_{k=1}^{M}\frac{2}{a_{i,t}^{k}}\mathrm{Im}\left(r_{i,q,t}^{k}\hat{s}_{i,t}^{k}\right)\right|^{2}}{4\left(\sum_{t=1}^{L}\sum_{k=1}^{M}\frac{\left|\hat{s}_{i,t}^{k}\right|^{2}}{a_{i,t}^{k}}+1\right)}\right\}. \qquad (4.45)
\end{aligned}
$$

The derivation of (4.45) is in Appendix E. The codewords can also be decoded by minimizing the followind metric:

$$
\begin{aligned}
&\sum_{q=1}^{m}\sum_{i=1}^{n}\left\{\frac{\left|\sum_{t=1}^{L}\sum_{k=1}^{M}\frac{2}{a_{i,t}^{k}}\mathrm{Re}\left(r_{i,q,t}^{k}\hat{s}_{i,t}^{k}\right)\right|^{2}+\left|\sum_{t=1}^{L}\sum_{k=1}^{M}\frac{2}{a_{i,t}^{k}}\mathrm{Im}\left(r_{i,q,t}^{k}\hat{s}_{i,t}^{k}\right)\right|^{2}}{4\left(\sum_{t=1}^{L}\sum_{k=1}^{M}\frac{\left|\hat{s}_{i,t}^{k}\right|^{2}}{a_{i,t}^{k}}+1\right)}\right\} \\
&-\sum_{q=1}^{m}\sum_{i=1}^{n}\ln\left\{\left(\sum_{t=1}^{L}\sum_{k=1}^{M}\frac{\left|\hat{s}_{i,t}^{k}\right|^{2}}{a_{i,t}^{k}}+1\right)\right\}. \qquad (4.46)
\end{aligned}
$$

4.3.4 Design Criteria for Constructing Good Space-Time Codes

We proposed a design criteria for contructing good space-time codes of the STC/OFHSS system with respect to the wireless channels. To evaluate the performance of the ML decoding, two transmitted sequences $\boldsymbol{s} = \left(s_{i,t}^{k}\forall i,k,1\leq t\leq L\right)$ and $\hat{\boldsymbol{s}} = \hat{s}_{i,t}^{k}\forall i,k,1\leq t\leq L$. Assume perfect estimation of $\alpha_{i,q}$ and $x_{i,t}^{k}$ are available at the receiver. The conditional pairwise error probability that the decoder decides in favor of $\hat{\boldsymbol{s}}$ than $\boldsymbol{s}$ is given by

$$
\begin{aligned}
&\Pr\left\{\boldsymbol{s}\rightarrow\hat{\boldsymbol{s}}|\boldsymbol{x},\boldsymbol{\alpha}\right\} \\
&= \Pr\left\{\sum_{t=1}^{L}\sum_{k=1}^{L}\sum_{q=1}^{m}\sum_{i=1}^{n}\ln\left[\frac{1}{\sqrt{\pi\left(N_0+x_{i,t}^{k}\frac{JT_s}{Q}\right)}}\exp\left(-\frac{\left|r_{i,q,t}^{k}-\alpha_{i,q}\hat{s}_{i,t}^{k}\right|^{2}}{\left(N_0+x_{i,t}^{k}\frac{JT_s}{Q}\right)}\right)\right]\right. \\
&\quad\left.\leq\sum_{t=1}^{L}\sum_{k=1}^{L}\sum_{q=1}^{m}\sum_{i=1}^{n}\ln\left[\frac{1}{\sqrt{\pi\left(N_0+x_{i,t}^{k}\frac{JT_s}{Q}\right)}}\exp\left(-\frac{\left|r_{i,q,t}^{k}-\alpha_{i,q}\hat{s}_{i,t}^{k}\right|^{2}}{\left(N_0+x_{i,t}^{k}\frac{JT_s}{Q}\right)}\right)\right]\right\}
\end{aligned}
$$

$$= Q\left(\sqrt{\frac{\sum_{t=1}^{L}\sum_{k=1}^{M}\sum_{q=1}^{m}\sum_{i=1}^{n}\left|\alpha_{i,q}\left(s_{i,t}^{k}-\hat{s}_{i,t}^{k}\right)\right|^{2}}{2\left(N_0+x_{i,t}^{k}\frac{JT_s}{Q}\right)}}\right). \quad (4.47)$$

Refer to the inequality $Q(a) \leq \frac{1}{2}\exp\left(-a^2/2\right) \forall a \geq 0$, the conditional pairwise error probability (4.47) can be upper bounded by

$$\Pr\{\boldsymbol{s}\rightarrow\hat{\boldsymbol{s}}|\boldsymbol{x},\boldsymbol{\alpha}\} \leq \frac{1}{2}\exp\left(-\frac{\sum_{t=1}^{L}\sum_{k=1}^{M}\sum_{q=1}^{m}\sum_{i=1}^{n}\left|\alpha_{i,q}\left(s_{i,t}^{k}-\hat{s}_{i,t}^{k}\right)\right|^{2}}{4\left(N_0+x_{i,t}^{k}\frac{JT_s}{Q}\right)}\right). \quad (4.48)$$

By averaging (4.48) with respect to $\boldsymbol{\alpha}$, the conditional pairwise error probability given $\boldsymbol{x}$ is approximated as

$$\Pr\{\boldsymbol{s}\rightarrow\hat{\boldsymbol{s}}|\boldsymbol{x}\} \leq \frac{1}{2}\prod_{i=1}^{n}\prod_{q=1}^{m}\left(1+\sum_{t=1}^{L}\sum_{k=1}^{M}a_{i,t}^{k}\left|s_{i,t}^{k}-\hat{s}_{i,t}^{k}\right|^{2}\right)^{-1} \quad (4.49)$$

where $a_{i,t}^{k}=\left(N_0+x_{i,t}^{k}\frac{JT_s}{Q}\right)^{-1}$. Assume there are only two transmitter antennas, then the conditional pairwise error probability can be approximated as

$$\begin{aligned}\Pr\{\boldsymbol{s}\rightarrow\hat{\boldsymbol{s}}|\boldsymbol{x}\} &\leq \frac{1}{2}\prod_{i=1}^{n}\prod_{q=1}^{m}\left(1+\sum_{t=1}^{L}\sum_{k=1}^{M}a_{i,t}^{k}\left|s_{i,t}^{k}-\hat{s}_{i,t}^{k}\right|^{2}\right)^{-1}\\ &=\frac{1}{2}\prod_{q=1}^{m}\prod_{i=1}^{n}\left[\sum_{j=0}^{\infty}\left(-\sum_{t=1}^{L}\sum_{k=1}^{M}a_{i,t}^{k}\left|s_{i,t}^{k}-\hat{s}_{i,t}^{k}\right|^{2}\right)^{j}\right]\\ &\cong\frac{1}{2}\prod_{q=1}^{m}\prod_{i=1}^{n}\left[1-\sum_{t=1}^{L}\sum_{k=1}^{M}a_{i,t}^{k}\left|s_{i,t}^{k}-\hat{s}_{i,t}^{k}\right|^{2}\right] \quad (4.50)\end{aligned}$$

The probability density function of $\boldsymbol{x}$ for two transmitter antennas is shown in (4.39), then the pairwise error probability is derived by averaging (4.50) with respect to $\boldsymbol{x}$. The pairwise error probability for two transmitter antennas can be written as

$$\begin{aligned}&\Pr\{\boldsymbol{s}\rightarrow\hat{\boldsymbol{s}}\}\\ &\leq\frac{1}{2}\prod_{q=1}^{m}\Bigg\{1-\sum_{t=1}^{L}\sum_{k=1}^{M}\left[\left(1-\frac{Q}{N_t}\right)\frac{1}{N_0}\left|s_{1,t}^{k}-\hat{s}_{1,t}^{k}\right|^{2}+\frac{Q}{N_t}\frac{1}{a}\left|s_{1,t}^{k}-\hat{s}_{1,t}^{k}\right|^{2}\right]\\ &-\sum_{t=1}^{L}\sum_{k=1}^{M}\left[\left(1-\frac{Q}{N_t}\right)\frac{1}{N_0}\left|s_{2,t}^{k}-\hat{s}_{2,t}^{k}\right|^{2}+\frac{Q}{N_t}\frac{1}{a}\left|s_{2,t}^{k}-\hat{s}_{2,t}^{k}\right|^{2}\right]\\ &+\tau\sum_{t=1}^{L}\sum_{k=1}^{M}\sum_{t'=1}^{L}\sum_{k'=1}^{M}\left|s_{1,t}^{k}-\hat{s}_{1,t}^{k}\right|^{2}\left|s_{2,t'}^{k'}-\hat{s}_{2,t'}^{k'}\right|^{2}\Bigg\} \quad (4.51)\end{aligned}$$

where

$$\tau = \left(\frac{Q}{N_t - M} \cdot \frac{Q}{N_t}\right)\left(\frac{1}{a}\right)^2 + \left(1 - \frac{Q}{N_t - M}\right)\frac{Q}{N_t}\left(\frac{1}{aN_0}\right)$$
$$+ \left[\frac{Q-1}{N_t - M}\frac{Q}{N_t}(M-1) + \frac{Q}{N_t - M}\left(1 - \frac{MQ}{N_t}\right)\right]\frac{1}{aN_0}$$
$$+ \left[\left(1 - \frac{Q-1}{N_t - M}\right)\frac{Q}{N_t}(M-1) + \left(1 - \frac{Q}{N_t - M}\right)\left(1 - \frac{MQ}{N_t}\right)\right]\left(\frac{1}{N_0}\right)^2.$$

Let

$$W_1 = 1 - \sum_{t=1}^{L}\sum_{k=1}^{M}\left[\left(1 - \frac{Q}{N_t}\right)\frac{1}{N_0}\left|s_{1,t}^k - \hat{s}_{1,t}^k\right|^2 + \frac{Q}{N_t}\frac{1}{a}\left|s_{1,t}^k - \hat{s}_{1,t}^k\right|^2\right]$$
$$- \sum_{t=1}^{L}\sum_{k=1}^{M}\left[\left(1 - \frac{Q}{N_t}\right)\frac{1}{N_0}\left|s_{2,t}^k - \hat{s}_{2,t}^k\right|^2 + \frac{Q}{N_t}\frac{1}{a}\left|s_{2,t}^k - \hat{s}_{2,t}^k\right|^2\right]$$

$$W_2 = \tau \sum_{t=1}^{L}\sum_{k=1}^{M}\sum_{t'=1}^{L}\sum_{k'=1}^{M}\left|s_{1,t}^k - \hat{s}_{1,t}^k\right|^2\left|s_{2,t'}^{k'} - \hat{s}_{2,t'}^{k'}\right|^2. \tag{4.52}$$

Assume the power of the multitone jammer is much larger than the power of the thermal noise, then $a \gg N_0$ and $\frac{1}{N_0} \gg \frac{1}{a}$. Base on the above inequations we have

$$\tau \cong \left[\left(1 - \frac{Q-1}{N_t - M}\right)\frac{Q}{N_t}(M-1) + \left(1 - \frac{Q}{N_t - M}\right)\left(1 - \frac{MQ}{N_t}\right)\right]\left(\frac{1}{N_0}\right)^2 \tag{5.53}$$

and

$$\sum_{t=1}^{L}\sum_{k=1}^{M}\left[\left(1 - \frac{Q}{N_t}\right)\frac{1}{N_0}\left|s_{1,t}^k - \hat{s}_{1,t}^k\right|^2 + \frac{Q}{N_t}\frac{1}{a}\left|s_{1,t}^k - \hat{s}_{1,t}^k\right|^2\right]$$
$$\cong \sum_{t=1}^{L}\sum_{k=1}^{M}\left[\left(1 - \frac{Q}{N_t}\right)\frac{1}{N_0}\left|s_{1,t}^k - \hat{s}_{1,t}^k\right|^2\right]. \tag{5.54}$$

Cause $\left|s_{i,t}^k - \hat{s}_{i,t}^k\right|^2 \in (0, E_s)$, then W_1 and W_2 can be bounded as

$$W_1 \le \left(1 - \frac{Q}{N_t}\right)\frac{1}{N_0}LME_s = \left(1 - \frac{Q}{N_t}\right)LM(SNR)$$
$$W_2 \le \left[\left(1 - \frac{Q-1}{N_t - M}\right)\frac{Q}{N_t}(M-1) + \left(1 - \frac{Q}{N_t - M}\right)\left(1 - \frac{MQ}{N_t}\right)\right]\left(\frac{1}{N_0}\right)^2 2LME_s^2$$
$$= \left[\left(1 - \frac{Q-1}{N_t - M}\right)\frac{Q}{N_t}(M-1) + \left(1 - \frac{Q}{N_t - M}\right)\left(1 - \frac{MQ}{N_t}\right)\right]2LM(SNR)^2 \tag{4.55}$$

After doing the simulation, we konw that $|w1| \gg |w2|$. Then good codes could be constructed by maximizing $m(\boldsymbol{s}, \tilde{\boldsymbol{s}})$ for all possiable $\boldsymbol{s}$ and $\tilde{\boldsymbol{s}}$, and $m(\boldsymbol{s}, \tilde{\boldsymbol{s}})$ can be expressed as

$$m(\boldsymbol{s}, \tilde{\boldsymbol{s}}) = \sum_{t=1}^{L} \sum_{k=1}^{M} \sum_{t'=1}^{L} \sum_{k'=1}^{M} \left| s_{1,t}^{k} - \hat{s}_{1,t}^{k} \right|^2 \left| s_{2,t'}^{k'} - \hat{s}_{2,t'}^{k'} \right|^2 . \tag{4.56}$$

According to the design criteria, good space-time codes are searched by the computer, and are given in following table.

Table 4.2: Optimal Space-time codes of the STC/OFHSS system with 4FSK and 2 transmitter antennas for wireless jamming channels.

Memory	Generator Sequences
2	$(x_1^t, x_2^t) = b_{t-1}(1,0) \oplus_4 a_{t-1}(1,0) \oplus_4 b_t(0,1) \oplus_4 a_t(0,2)$
3	$(x_1^t, x_2^t) = a_{t-2}(1,3) \oplus_4 b_{t-1}(1,1) \oplus_4 a_{t-1}(1,0) \oplus_4 b_t(2,1)$ $\oplus_4 a_t(0,1)$

4.4 Design Criteria for Constructing Good Space-Time Codes with FSK Modulation

Consider a coded communication system with MFSK modulation and ML decoding. A block of transmitted symbols is denoted by

$$\boldsymbol{s} = \left(s_{i,t}^{k} | \forall i, k,\ 1 \le t \le L \right) \tag{4.57}$$

and an erroneous sequence selected by the decoder is

$$\hat{\boldsymbol{s}} = \left(\hat{s}_{i,t}^{k} | \forall i, k,\ 1 \le t \le L \right) . \tag{4.58}$$

We know that the likelihood function can be expressed as

$$\begin{aligned} f(\boldsymbol{r}|\boldsymbol{\alpha}, \boldsymbol{s}) &= \prod_{t=1}^{L} \prod_{q=1}^{m} f\left(\eta_t^q = r_t^q - \sum_{i=1}^{n} \alpha_{i,q} s_{i,t}^{k} | \alpha_{i,q}, s_{i,t}^{k}\ \forall i, q, t \right) \\ &= \prod_{t=1}^{L} \prod_{q=1}^{m} \prod_{q=1}^{m} \left[\frac{1}{\sqrt{\pi N_0}} \exp\left(- \frac{\left| r_t^q - \sum_{i=1}^{n} \alpha_{i,q} s_{i,t}^{k} \right|^2}{N_0} \right) \right] . \end{aligned} \tag{4.59}$$

Assume the fading coefficients are available at the receiver, then the pairwise error probability is

$$
\begin{aligned}
&\Pr\left(\boldsymbol{s} \rightarrow \hat{\boldsymbol{s}} | \alpha_{i,q}, \forall i, q\right) \\
&= \Pr\left[\sum_{t=1}^{L}\sum_{q=1}^{m}\sum_{k=1}^{M}\left|r_t^j - \sum_{i=1}^{n}\alpha_{i,q}\sqrt{E_s}s_{i,t}^k\right|^2 \geq \sum_{t=1}^{L}\sum_{q=1}^{m}\sum_{k=1}^{M}\left|r_t^j - \sum_{i=1}^{n}\alpha_{i,q}\sqrt{E_s}\hat{s}_{i,t}^k\right|^2\right] \\
&\leq \frac{1}{2}\exp\left(-d^2\left(\boldsymbol{s}, \hat{\boldsymbol{s}}\right)\frac{E_s}{4N_0}\right) \qquad (4.60)
\end{aligned}
$$

where

$$
d^2\left(\boldsymbol{s}, \hat{\boldsymbol{s}}\right) = \sum_{t=1}^{L}\sum_{q=1}^{m}\sum_{k=1}^{M}\left|\sum_{i=1}^{n}\alpha_{i,q}\sqrt{E_s}\left(s_{i,t}^k - \hat{s}_{i,t}^k\right)\right|^2. \qquad (4.61)
$$

Assume the fading coeficients $\alpha_{i,q}$ are independent Gaussian random variables with zero mean and varance 1/2. Let "$*$" denote the operator of taking complex conjugate, and H denotes the operator of taking Hermirian, and $\Omega_j = (\alpha_{1,q}, \alpha_{2,q}, \ldots, \alpha_{n,q})$. Then we can rewrite equation (4.61) as

$$
\begin{aligned}
d^2\left(\boldsymbol{s}, \hat{\boldsymbol{s}}\right) &= \sum_{q=1}^{m}\sum_{i=1}^{n}\sum_{l=1}^{n}\alpha_{i,q}\alpha_{i,q}^*\sum_{t=1}^{L}\sum_{k=1}^{M}\left(s_{i,t}^k - \hat{s}_{i,t}^k\right)\left(s_{l,t}^k - \hat{s}_{l,t}^k\right)^* \\
&= \sum_{q=1}^{m}\Omega_q B\left(\boldsymbol{s}, \hat{\boldsymbol{s}}\right)B^H\left(\boldsymbol{s}, \hat{\boldsymbol{s}}\right)\Omega_q^H \\
&= \sum_{q=1}^{m}\Omega_q A\left(\boldsymbol{s}, \hat{\boldsymbol{s}}\right)\Omega_q^H \qquad (4.62)
\end{aligned}
$$

where

$$
\begin{aligned}
B\left(\boldsymbol{s}, \hat{\boldsymbol{s}}\right) &= \begin{bmatrix}
s_{1,1}^1 - \hat{s}_{1,1}^1 & \ldots & s_{1,1}^M - \hat{S}_{1,1}^M & \ldots & s_{1,L}^1 - \hat{s}_{1,L}^1 & \ldots & s_{1,L}^M - \hat{s}_{1,L}^M \\
s_{2,1}^1 - \hat{s}_{2,1}^1 & \ldots & s_{2,1}^M - \hat{S}_{2,1}^M & \ldots & s_{2,L}^1 - \hat{s}_{2,L}^1 & \ldots & s_{2,L}^M - \hat{s}_{2,L}^M \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
s_{n,1}^1 - \hat{s}_{n,1}^1 & \ldots & s_{n,1}^M - \hat{S}_{n,1}^M & \ldots & s_{n,L}^1 - \hat{s}_{n,L}^1 & \ldots & s_{n,L}^M - \hat{s}_{n,L}^M
\end{bmatrix} \\
&= \begin{bmatrix}
\boldsymbol{s}_{1,1} - \hat{\boldsymbol{s}}_{1,1} & \boldsymbol{s}_{1,2} - \hat{\boldsymbol{s}}_{1,2} & \ldots & \boldsymbol{s}_{1,L} - \hat{\boldsymbol{s}}_{1,L} \\
\boldsymbol{s}_{2,1} - \hat{\boldsymbol{s}}_{2,1} & \boldsymbol{s}_{2,2} - \hat{\boldsymbol{s}}_{2,2} & \ldots & \boldsymbol{s}_{2,L} - \hat{\boldsymbol{s}}_{2,L} \\
\vdots & \vdots & \ddots & \vdots \\
\boldsymbol{s}_{n,1} - \hat{\boldsymbol{s}}_{n,1} & \boldsymbol{s}_{n,2} - \hat{\boldsymbol{s}}_{n,2} & \ldots & \boldsymbol{s}_{n,L} - \hat{\boldsymbol{s}}_{n,L}
\end{bmatrix}
\end{aligned}
$$

where $\boldsymbol{s}_{n,1} = \left(s_{i,t}^1, s_{i,t}^2, \cdots, s_{i,t}^M\right)$, $\hat{\boldsymbol{s}}_{n,1} = \left(\hat{s}_{i,t}^1, \hat{s}_{i,t}^2, \cdots, \hat{s}_{i,t}^M\right)$, and $A(\boldsymbol{s}, \hat{\boldsymbol{s}}) = B(\boldsymbol{s}, \hat{\boldsymbol{s}}) B^H(\boldsymbol{s}, \hat{\boldsymbol{s}})$. $A(\boldsymbol{s}, \hat{\boldsymbol{s}})$ is nonnegative definite and Hermitian, and the eigenvalues of $A(\boldsymbol{s}, \hat{\boldsymbol{s}})$ are real numbers. Then we have

$$A(\boldsymbol{s}, \hat{\boldsymbol{s}}) = \boldsymbol{V}\boldsymbol{D}\boldsymbol{V}^H \tag{4.63}$$

where $\boldsymbol{V} = (v_1, v_2, \ldots, v_n)$ is a unitary matrix and D is a diagonal matrix, where v_i's are the eigenvectors of $A(\boldsymbol{s}, \hat{\boldsymbol{s}})$. Let λ_i be the diagonal elements of $\boldsymbol{D}$, where $1 \le i \le n$, and

$$\Omega_q \boldsymbol{V}^H = (\beta_{1,q}, \ldots, \beta_{n,q}) \,. \tag{4.64}$$

From (4.63) and (4.64), we can rewrite the equation (4.61) as following

$$d^2(\boldsymbol{s}, \hat{\boldsymbol{s}}) = \sum_{q=1}^{m} \sum_{i=1}^{n} \lambda_i \left|\beta_{i,q}\right|^2 . \tag{4.65}$$

Use equation (4.65) to replace $d^2(\boldsymbol{s}, \hat{\boldsymbol{s}})$ in (4.60), then we have

$$\Pr\left(\boldsymbol{s} \to \hat{\boldsymbol{s}} | \alpha_{i,q}\ \forall i, q\right) \le \frac{1}{2} \exp\left(-\frac{E_s}{4N_0} \sum_{q=1}^{m} \sum_{i=1}^{n} \lambda_i \left|\beta_{i,q}\right|^2\right) \tag{4.66}$$

By Using the same derivation in (3.19), we have the pairwise error probability

$$\Pr(\boldsymbol{s} \to \hat{\boldsymbol{s}}) \le \frac{1}{2} \left(\prod_{i=1}^{n} \frac{1}{1 + \frac{E_s}{4N_0}\lambda_i}\right)^m . \tag{4.67}$$

When SNR is a big number, (4.67) can be expressed as

$$\Pr(\boldsymbol{s} \to \hat{\boldsymbol{s}}) \le \frac{1}{2} \left(\prod_{i=1}^{r} \lambda_i\right)^{-m} \left(\frac{E_s}{4N_0}\right)^{-rm} . \tag{4.68}$$

where r is the rank of $A(\boldsymbol{s}, \hat{\boldsymbol{s}})$. In order to minimize the error probability, to make rm and the the product of eigenvalues as large as posible is necessary. A good space time code with memory 2 is searched by the computer with these cirteria, and the generator sequence is

Table 4.3: Optimal Space-time codes of the STC/FSK system with 4FSK and 2 transmitter antennas for wireless jamming channels.

Memory	**Generator Sequences**
2	$(x_1^t, x_2^t) = b_{t-1}(1,0) \oplus_4 a_{t-1}(2,0) \oplus_4 b_t(0,1) \oplus_4 a_t(0,2)$

4.5 Simulation Result

In this sectio, we simulate the 4-state space-time code with two transmitter antennas and two receiver antennas, 4FSK modulation, and 1000 Mary bands for used over Rayleigh fading channel with AWGN and $n = 1$ band multitone jammers to explore the performance of STC/FHSS system. In Figure 4.3-4.14, the space-time code of STC/WFHSS system we used for simulation is

$$\left(x_1^t, x_2^t\right) = b_{t-1}(1,0) \oplus a_{t-1}(3,0) \oplus b_t(1,3) \oplus a_t(2,2). \tag{4.69}$$

Observed from the performance curves in Figures 4.3-4 with $E_b/N_0 = 5$dB and ML decoding with CSI and JSI available. Figure 4.3 shows the performance curves with $\mu = 0.2$, $\mu = 0.5$, $\mu = 0.7$, and $\mu = 1$. Figure 4.4 shows the performance curves with $E_b/N_J = 0$dB, $E_b/N_J = 10$dB, $E_b/N_J = 15$dB, and $E_b/N_J = 20$dB. The performance curves shown in Figures 4.5-6 are simulated with ML decoding with JSI available but without JSI available. Figure 4.5 shows the performance curves with $E_b/N_J = 10$dB, $\mu = 0.1$, $\mu = 0.4$, $\mu = 0.7$, and $\mu = 1$. Figure 4.6 shows the performance curves with $E_b/N_0 = 10$dB, $E_b/N_J = 0$dB, $E_b/N_J = 5$dB, $E_b/N_J = 10$dB, and $E_b/N_J = 15$dB. The performance plots of the ML decoding with CSI available without JSI available are shown in Figures 4.7-8. Figure 4.7 show the performance with diferent values of $/mu$, and Figure 4.8 shows the perfromance with different values of E_b/N_J. When the value of E_b/N_J is small, the worst performance is located at $\mu = 1$. When the value of E_b/N_J gets larger, the worst performance is located at lower μ. Figures 4.9-10 show two performance curves with $E_b/N_0 = 12$dB, $\mu = 0.3$, $\mu = 1$. One is simulated with ML decoding with JSI and CSI available, and the other is simulated with CSI available but without JSI available. We can find that the system with JSI available is much better than the system without JSI available.

Finally, two systems are compared with STC/WFHSS. One is the system with the original space-time coding which is designed with FSK modulation. The other one is the system with the convolutional coding and the Alamouti coding, and we combine convolutional code scheme and Alamouti code scheme for this system. The generator equation of the space-time code used in the first system is

$$\left(x_1^t, x_2^t\right) = b_{t-1}(1,0) \oplus a_{t-1}(2,0) \oplus b_t(0,1) \oplus a_t(0,2). \tag{4.70}$$

A $(4,2)$ convolutional code is emploied for the second system with memory 2 and following generator matrix

$$G(D) = \begin{bmatrix} 1+D & D & D & 1+D \\ 1+D & 1+D & 1 & 0 \end{bmatrix}. \tag{4.71}$$

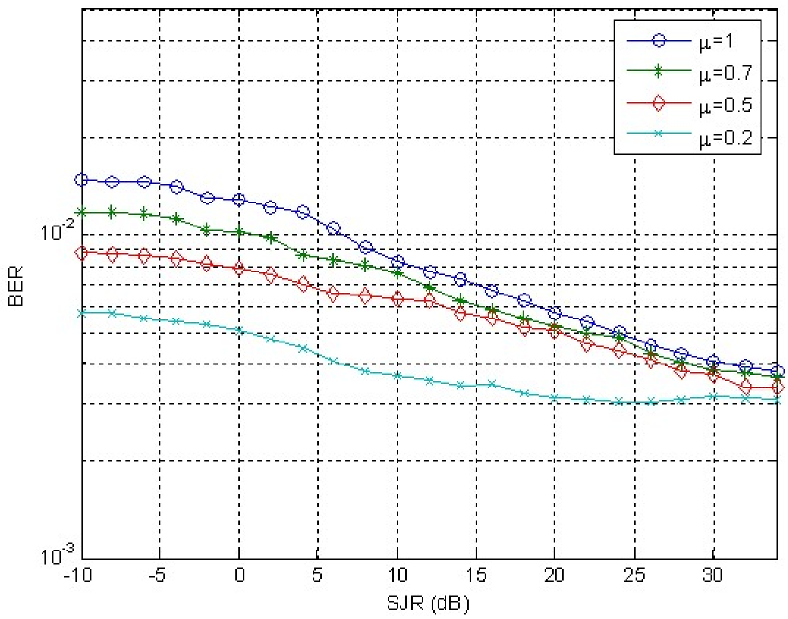

Figure 4.4: Performance plots of STC/WFHSS with CSI and JSI available for $E_b/N_0 = 5$dB.

Perfomance of these two system and STC/WFHSS system with $E_b/N_J = 15$dB, $\mu = 0.1$, $\mu = 0.4$, $\mu = 0.7$, and $\mu = 1$ are shown in Figure 4.11-4.14. We can find that STC/WFHSS system provides better performance than the others.

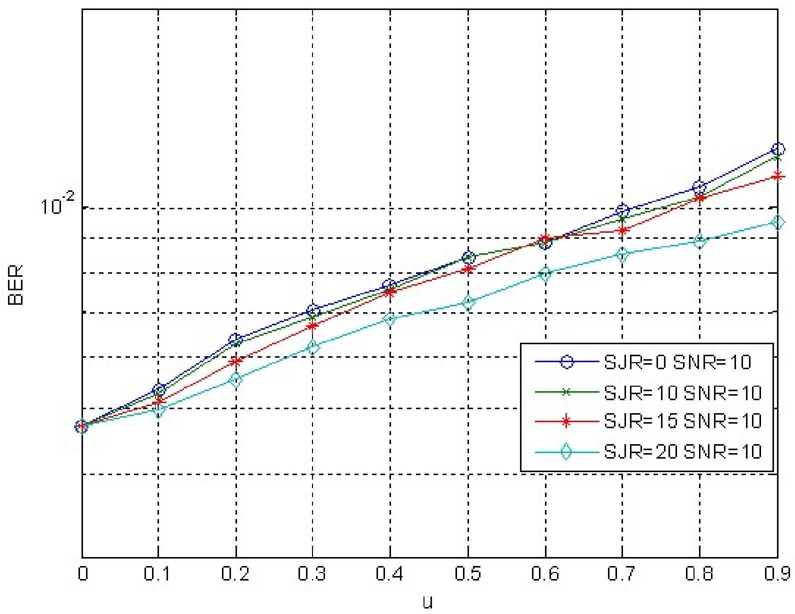

Figure 4.5: Performance plots of STC/WFHSS with CSI and JSI available for $E_b/N_0 = 5$dB.

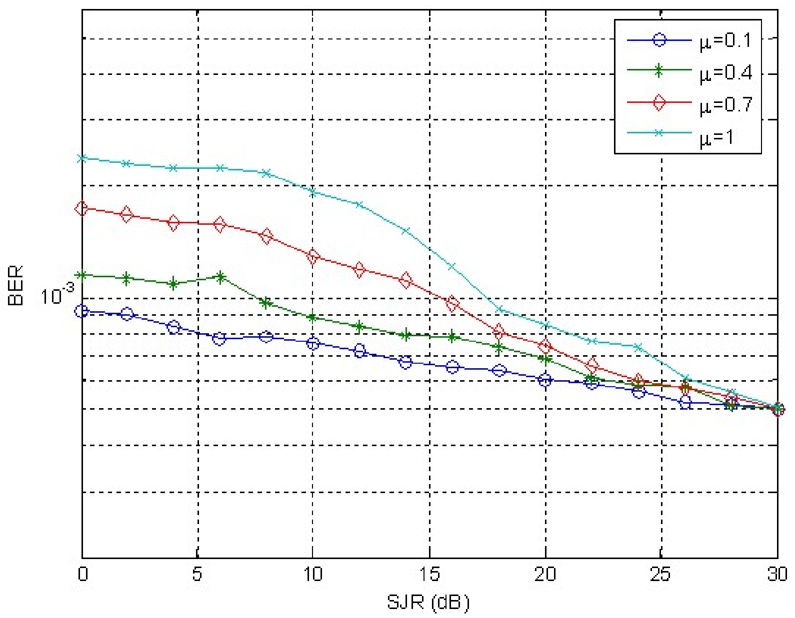

Figure 4.6: Performance plots of STC/WFHSS with JSI available but without CSI available for $E_b/N_0 = 10$dB.

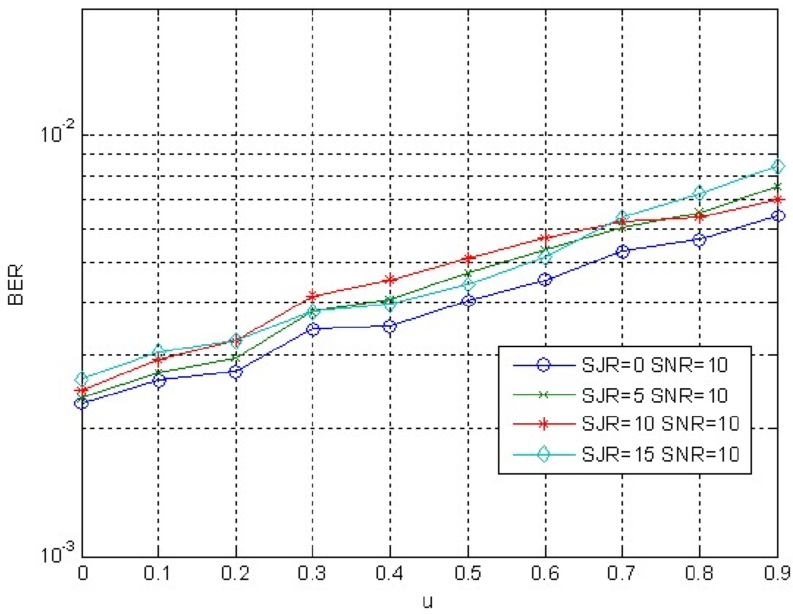

Figure 4.7: Performance plots of STC/WFHSS with JSI available but without CSI available for $E_b/N_0 = 10$dB.

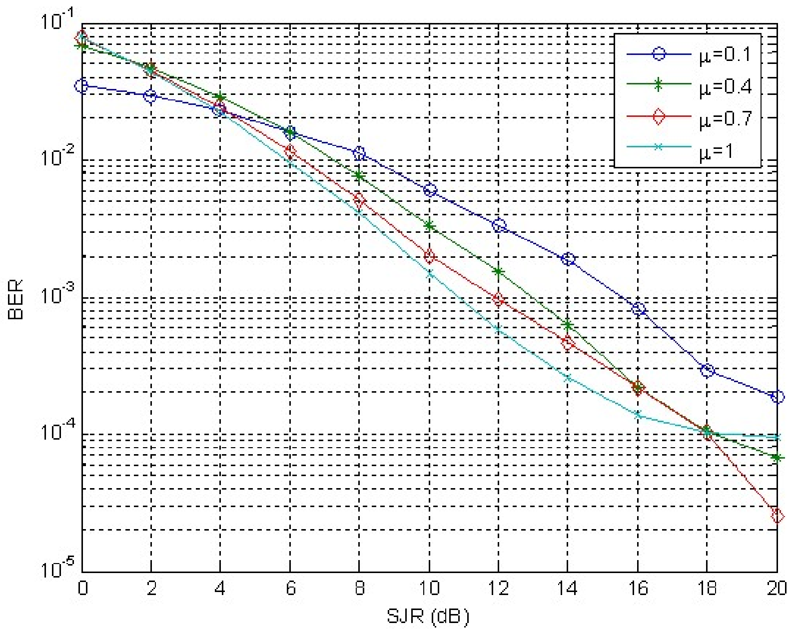

Figure 4.8: Performance plots of STC/WFHSS with CSI available but without JSI available for $E_b/N_0 = 15$dB.

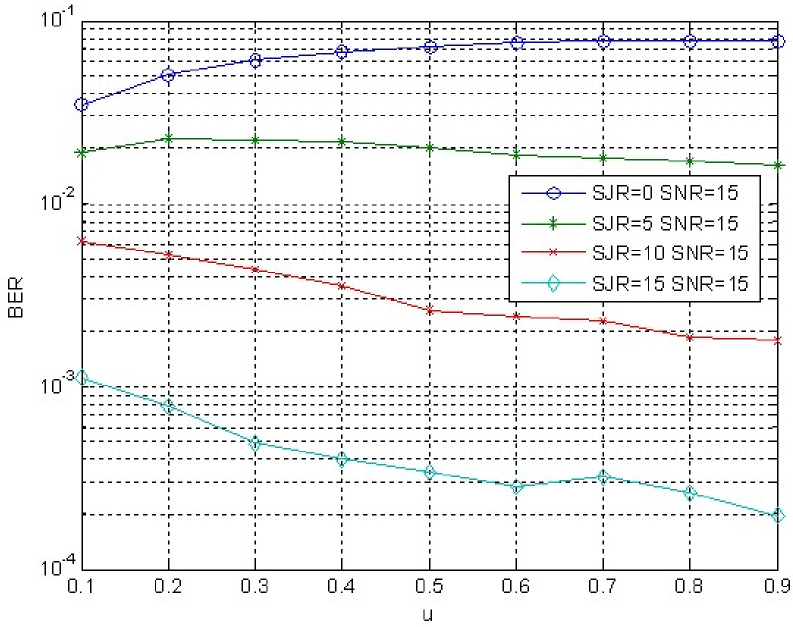

Figure 4.9: Performance plots of STC/WFHSS with CSI available but without JSI available for $E_b/N_0 = 15$dB.

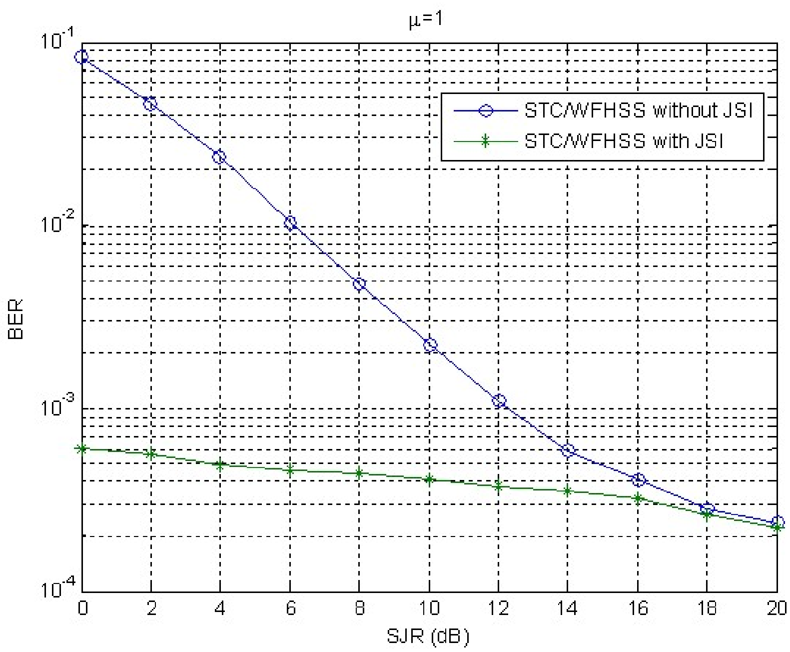

Figure 4.10: Performance of STC/WFHSS with CSI and JSI available and STC/WFHSS with CSI available but without JSI available for $E_b/N_0 = 12$dB and $\mu = 1$.

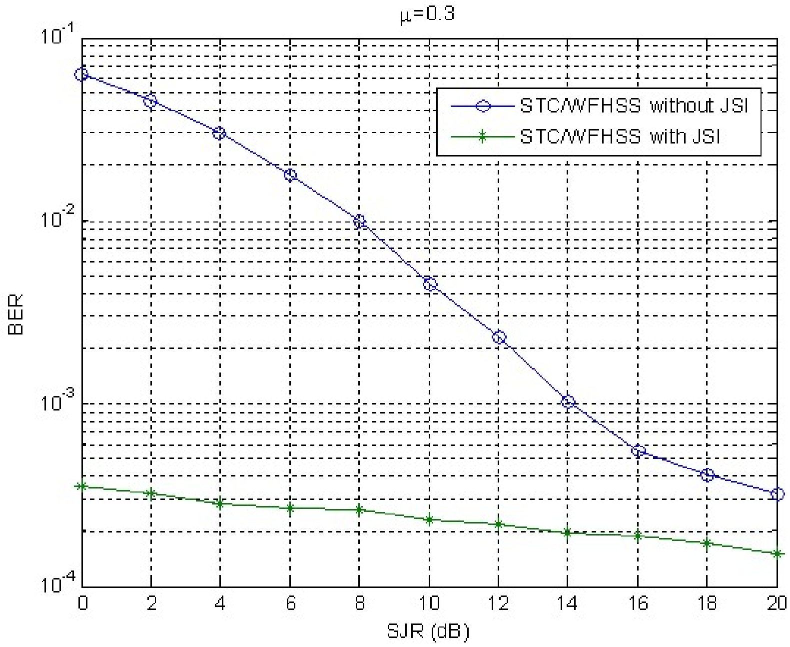

Figure 4.11: Performance of STC/WFHSS with CSI and JSI available and STC/WFHSS with CSI available but without JSI available for $E_b/N_0 = 12$dB and $\mu = 0.3$.

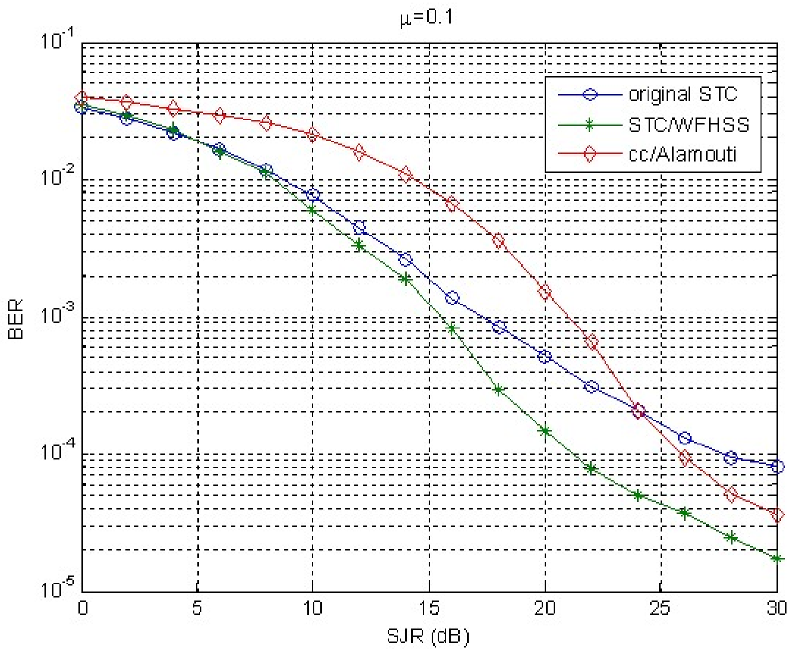

Figure 4.12: Performance of STC/WFHSS, original STC, and CC/Alamouti systems for $E_b/N_0 = 15$dB and $\mu = 0.1$.

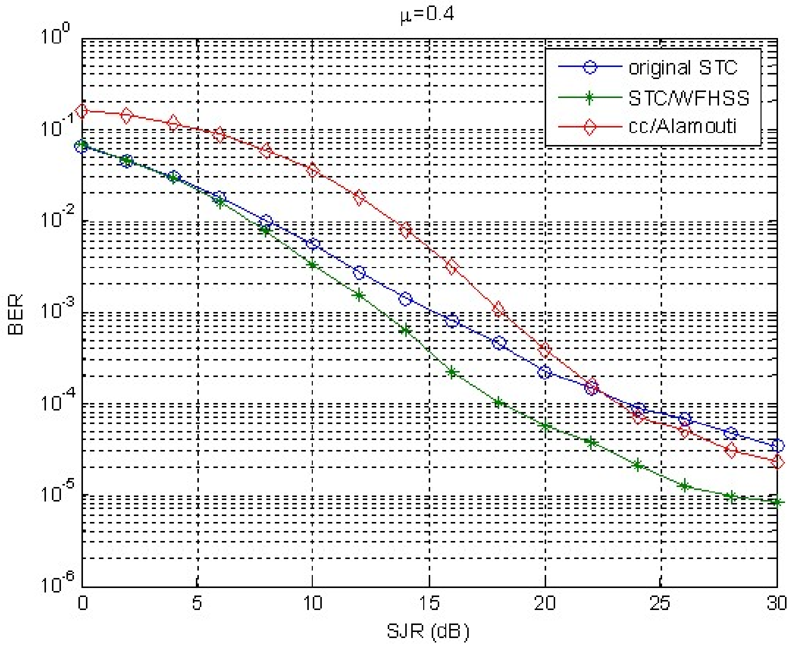

Figure 4.13: Performance of STC/WFHSS, original STC, and CC/Alamouti systems for $E_b/N_0 = 15$dB and $\mu = 0.4$.

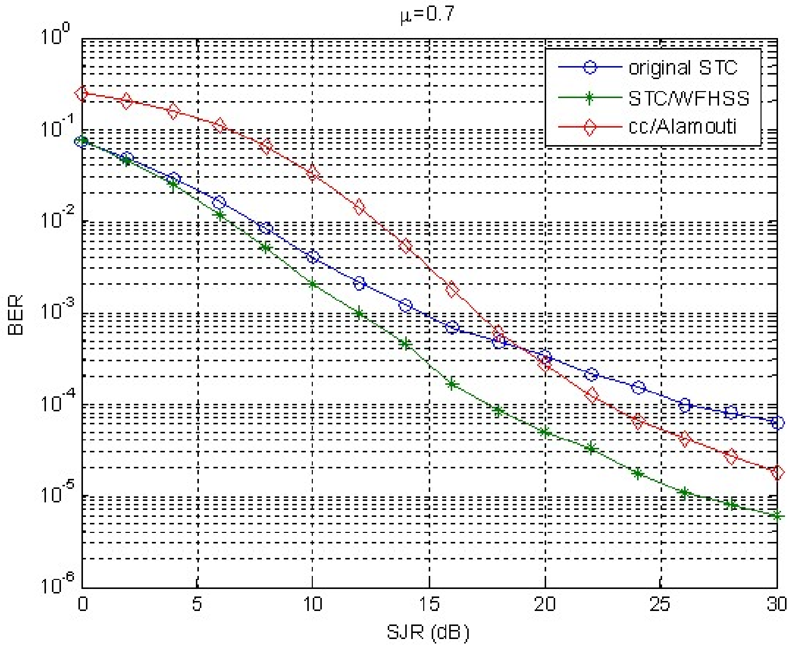

Figure 4.14: Performance of STC/WFHSS, original STC, and CC/Alamouti systems for $E_b/N_0 = 15$dB and $\mu = 0.7$.

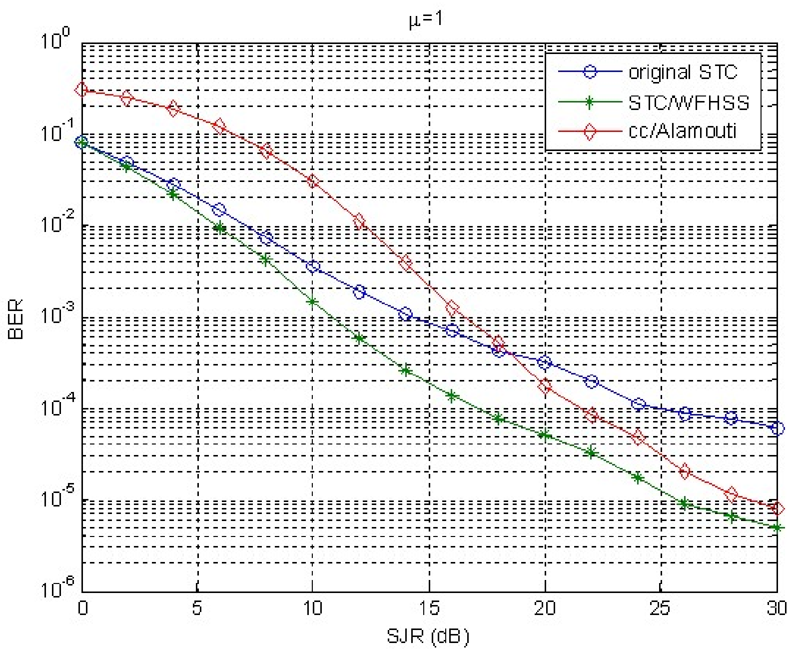

Figure 4.15: Performance of STC/WFHSS, original STC, and CC/Alamouti systems for $E_b/N_0 = 15$dB and $\mu = 1$.

Chapter 5

Conclusion

In this thesis, we investigate issues related to the performance of the STC/FHSS system in wireless MTNJ environments. There are two types of STC/FHSS systems we proposed for performance evaluation: STC/WFHSS system and STC/OFHSS system. The actural performance of STC/FHSS system can be upper bounded by the performance of STC/OFHSS system, and can be lower bounded by the performance of STC/WFHSS system. Based on these proposed system model, the corresponding ML decoding is derived. Although the ML decoding with respect to different conditions has benn derived, however, the decoding complexity of the optimum decodings are too high. Beside, the complicated arithmetic of ML decoding with JSI available but without CSI available not only requires high computational complexity but also excudes the use of the efficient Viterbi algorithm. Therefore, we have to find some suboptimal decoding scheme in the future. We also present two design criteria for constructing good space-time codes with respect to the wireless channel with $n = 1$ band multitone jammers. Good space time codes are also given via a computer search. Verified by the simulation result, the performance of our system is better than the system with space-time coding which is designed with FSK modulation. The performance of our system is also better than the system employs both convolution code and Alamouti code. We can also find that when E_b/N_J is large, the performance of the system with space-time coding is worse than the system uses both convolution code and Alamouti code. That means the system with space-time coding but without any designs which in connection wiht the environments is not good enought to be used for the wireless channels with multitone noise jammers.

Although we have persented two types of STC/FHSS systems for wireless channel with multitone nosie jammers, there are still serveral related issues that remain to be investigated. The coding scheme we consider in the proposed system is the space-time block coding scheme. We can consider other coding scheme, e.g., space-time trellis coding scheme, differential space-time coding scheme, and space-time turbo trellis coding scheme for wireless channel with jammers. The frequency hopping we used in this thesis is slow frequency hopping. Fast frequency hopping could also be used for high frequency dievesity gain. Therefore, we could use the STC/FHSS system with different coding scheme and fast frequency hopping to improve the perfromance in the wireless channels with jamming environments.

Appendix A

Derivation of the ML Decoding of STC/WFHSS System without JSI

The derivation in (4.14) is discussed in this appendix. (4.14) is derived by averaging (4.10) with respect to $\boldsymbol{x}$. Let $\boldsymbol{x}_t = \left(x_t^1, x_t^2, \ldots, x_t^M\right)$, then we have following function

$$
\begin{aligned}
& f\left(\boldsymbol{r}|\hat{\boldsymbol{s}},\boldsymbol{\alpha}\right) \\
& = E_{\boldsymbol{x}}\left[f\left(\boldsymbol{r}|\hat{\boldsymbol{s}},\boldsymbol{x},\boldsymbol{\alpha}\right)\right] \\
& = E_{\boldsymbol{x}}\left[\prod_{t=1}^{L}\prod_{k=1}^{M}\prod_{q=1}^{m}\frac{1}{\sqrt{\pi\left(N_0+x_t^k 2\frac{JT_s}{Q}\sigma_{J,q}^2\right)}}\exp\left(-\frac{\left|r_{q,t}^k-\sum_{i=1}^{n}\alpha_{i,q}\hat{s}_{i,t}^k\right|^2}{\left(N_0+x_t^k 2\frac{JT_s}{Q}\sigma_{J,q}^2\right)}\right)\right] \\
& = \prod_{t=1}^{L}\prod_{q=1}^{m}E_{\boldsymbol{x}_t}\left[\prod_{k=1}^{M}\frac{1}{\sqrt{\pi\left(N_0+x_t^k 2\frac{JT_s}{Q}\sigma_{J,q}^2\right)}}\exp\left(-\frac{\left|r_{q,t}^k-\sum_{i=1}^{n}\alpha_{i,q}\hat{s}_{i,t}^k\right|^2}{\left(N_0+x_t^k 2\frac{JT_s}{Q}\sigma_{J,q}^2\right)}\right)\right] \\
& = \prod_{t=1}^{L}\prod_{q=1}^{m}\left\{\frac{Q}{N_t}\frac{1}{\sqrt{\pi a}}\left(\frac{1}{\sqrt{\pi N_0}}\right)^{M-1}\exp\left[-\sum_{k=2}^{M}\frac{\left|r_{q,t}^k-\sum_{i=1}^{n}\alpha_{i,q}\hat{s}_{i,t}^k\right|^2}{N_0}-\frac{\left|r_{q,t}^1-\sum_{i=1}^{n}\alpha_{i,q}\hat{s}_{i,t}^1\right|^2}{a}\right]\right. \\
& \quad +\cdots+\frac{Q}{N_t}\frac{1}{\sqrt{\pi a}}\left(\frac{1}{\sqrt{\pi N_0}}\right)^{M-1}\exp\left[-\sum_{k=1}^{M-1}\frac{\left|r_{q,t}^k-\sum_{i=1}^{n}\alpha_{i,q}\hat{s}_{i,t}^k\right|^2}{N_0}-\frac{\left|r_{q,t}^M-\sum_{i=1}^{n}\alpha_{i,q}\hat{s}_{i,t}^M\right|^2}{a}\right] \\
& \quad \left.+\left(1-\frac{MQ}{N_t}\right)\left(\frac{1}{\sqrt{\pi N_0}}\right)^{M}\exp\left[-\sum_{k=1}^{M}\frac{\left|r_{q,t}^k-\sum_{i=1}^{n}\alpha_{i,q}\hat{s}_{i,t}^k\right|^2}{N_0}\right]\right\}
\end{aligned}
$$

$$
\begin{aligned}
&= \prod_{t=1}^{L}\prod_{q=1}^{m}\left\{\exp\left[-\sum_{k=1}^{M}\frac{\left|r_{q,t}^{k}-\sum_{i=1}^{n}\alpha_{i,q}\hat{s}_{i,t}^{k}\right|^2}{N_0}\right]\left\{\frac{Q}{N_t}\frac{1}{\sqrt{\pi a}}\left(\frac{1}{\sqrt{\pi N_0}}\right)^{M-1}\right.\right.\\
&\cdot\exp\left[\frac{\left|r_{q,t}^{1}-\sum_{i=1}^{n}\alpha_{i,q}\hat{s}_{i,t}^{1}\right|^2}{N_0}-\frac{\left|r_{q,t}^{1}-\sum_{i=1}^{n}\alpha_{i,q}\hat{s}_{i,t}^{1}\right|^2}{a}\right]+\cdots+\frac{Q}{N_t}\frac{1}{\sqrt{\pi a}}\left(\frac{1}{\sqrt{\pi N_0}}\right)^{M-1}\\
&\left.\left.\exp\left[\frac{\left|r_{q,t}^{M}-\sum_{i=1}^{n}\alpha_{i,q}\hat{s}_{i,t}^{M}\right|^2}{N_0}-\frac{\left|r_{q,t}^{M}-\sum_{i=1}^{n}\alpha_{i,q}\hat{s}_{i,t}^{M}\right|^2}{a}\right]+\left(1-\frac{MQ}{N_t}\right)\left(\frac{1}{\sqrt{\pi N_0}}\right)^{M}\right\}\right\}\\
&= \prod_{t=1}^{L}\prod_{q=1}^{m}\left\{\exp\left[-\sum_{k=1}^{M}\frac{\left|r_{q,t}^{k}-\sum_{i=1}^{n}\alpha_{i,q}\hat{s}_{i,t}^{k}\right|^2}{N_0}\right]\left\{\sum_{k=1}^{M}\frac{Q}{N_t}\frac{1}{\sqrt{\pi a}}\left(\frac{1}{\sqrt{\pi N_0}}\right)^{M-1}\right.\right.\\
&\left.\left.\cdot\exp\left[\frac{\left|r_{q,t}^{k}-\sum_{i=1}^{n}\alpha_{i,q}\hat{s}_{i,t}^{k}\right|^2}{N_0}-\frac{\left|r_{q,t}^{1}-\sum_{i=1}^{n}\alpha_{i,q}\hat{s}_{i,t}^{1}\right|^2}{a}\right]+\left(1-\frac{MQ}{N_t}\right)\left(\frac{1}{\sqrt{\pi N_0}}\right)^{M}\right\}\right\}\\
&= \prod_{t=1}^{L}\prod_{q=1}^{m}\left\{\exp\left[-\sum_{k=1}^{M}\frac{\left|r_{q,t}^{k}-\sum_{i=1}^{n}\alpha_{i,q}\hat{s}_{i,t}^{k}\right|^2}{N_0}\right]\left\{\sum_{k=1}^{M}\frac{Q}{N_t}\frac{1}{\sqrt{\pi a}}\left(\frac{1}{\sqrt{\pi N_0}}\right)^{M-1}\right.\right.\\
&\left.\left.\cdot\exp\left[\frac{a-N_0}{N_0 a}\left|r_{q,t}^{k}-\sum_{i=1}^{n}\alpha_{i,q}\hat{s}_{i,t}^{k}\right|^2\right]+\left(1-\frac{MQ}{N_t}\right)\left(\frac{1}{\sqrt{\pi N_0}}\right)^{M}\right\}\right\} \qquad \text{(A.1)}
\end{aligned}
$$

where $a = N_0 + 2\frac{JT_s}{Q}\sigma_{J,q}^2$.

Appendix B

Derivation of the ML Decoding of STC/WFHSS System with JSI but without CSI

The derivation in (4.21) is discussed in this appendix. (4.21) is derived by averaging (4.20) with respect to $\boldsymbol{\alpha}$. Then we got

$$
\begin{aligned}
& f\left(\boldsymbol{r} \mid \hat{\boldsymbol{s}}, \boldsymbol{\alpha}, \boldsymbol{x}\right) \\
&= \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} \prod_{t=1}^{L} \prod_{k=1}^{M} \prod_{q=1}^{m} \frac{1}{\sqrt{\pi a_t^k}} \exp\left\{ -\sum_{t=1}^{L} \sum_{k=1}^{M} \sum_{q=1}^{m} \frac{1}{a_t^k} \left[\left| r_{q,t}^k \right|^2 - \mathrm{Re}\left(r_{q,t}^k \sum_{i=1}^{n} \alpha_{R,i,q} \hat{s}_{i,t}^k \right) \right.\right. \\
&\quad - \mathrm{Im}\left(r_{q,t}^k \sum_{i=1}^{n} \alpha_{I,i,q} \hat{s}_{i,t}^k \right) + \sum_{i=1}^{n} \alpha_{R,i,q}^2 \left| \hat{s}_{i,t}^k \right|^2 + \sum_{i=1}^{n} \alpha_{I,i,q}^2 \left| \hat{s}_{i,t}^k \right|^2 + \sum_{\substack{i=1 \\ i \neq l}}^{n} \sum_{\substack{l=1 \\ l \neq i}}^{n} \alpha_{R,i,q} \alpha_{R,l,q} \hat{s}_{i,t}^k \hat{s}_{l,t}^k \\
&\quad \left.\left. + \sum_{\substack{i=1 \\ i \neq l}}^{n} \sum_{\substack{l=1 \\ l \neq i}}^{n} \alpha_{I,i,q} \alpha_{I,l,q} \hat{s}_{i,t}^k \hat{s}_{l,t}^k \right] \right\} f\left(\alpha_{R,1,1}\right) f\left(\alpha_{R,1,2}\right) \cdots f\left(\alpha_{R,n,m}\right) f\left(\alpha_{I,1,1}\right) f\left(\alpha_{I,1,2}\right) \cdots \\
&\quad f\left(\alpha_{I,n,m}\right) d\alpha_{R,1,1} d\alpha_{R,1,2} \cdots d\alpha_{R,n,m} d\alpha_{I,1,1} d\alpha_{I,1,2} \cdots d\alpha_{I,n,m} \\
&= \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} \prod_{t=1}^{L} \prod_{k=1}^{M} \prod_{q=1}^{m} \frac{1}{\sqrt{\pi a_t^k}} \exp\left\{ -\sum_{t=1}^{L} \sum_{k=1}^{M} \sum_{q=1}^{m} \frac{1}{a_t^k} \left[\left| r_{q,t}^k \right|^2 - \mathrm{Re}\left(r_{q,t}^k \sum_{i=1}^{n} \alpha_{R,i,q} \hat{s}_{i,t}^k \right) \right.\right. \\
&\quad - \mathrm{Im}\left(r_{q,t}^k \sum_{i=1}^{n} \alpha_{I,i,q} \hat{s}_{i,t}^k \right) + \sum_{i=1}^{n} \alpha_{R,i,q}^2 \left| \hat{s}_{i,t}^k \right|^2 + \sum_{i=1}^{n} \alpha_{I,i,q}^2 \left| \hat{s}_{i,t}^k \right|^2 + \sum_{\substack{i=1 \\ i \neq l}}^{n} \sum_{\substack{l=1 \\ l \neq i}}^{n} \alpha_{R,i,q} \alpha_{R,l,q} \hat{s}_{i,t}^k \hat{s}_{l,t}^k \\
&\quad \left.\left. + \sum_{\substack{i=1 \\ i \neq l}}^{n} \sum_{\substack{l=1 \\ l \neq i}}^{n} \alpha_{I,i,q} \alpha_{I,l,q} \hat{s}_{i,t}^k \hat{s}_{l,t}^k \right] \right\} \frac{1}{\sqrt{2\pi \sigma_{R,1,1}}} \exp\left(-\frac{\alpha_{R,1,1}^2}{2\sigma_{R,1,1}^2} \right) \frac{1}{\sqrt{2\pi \sigma_{R,1,2}}} \exp\left(-\frac{\alpha_{R,1,2}^2}{2\sigma_{R,1,2}^2} \right)
\end{aligned}
$$

$$\cdots \frac{1}{\sqrt{2\pi\sigma_{R,n,m}}}\exp\left(-\frac{\alpha_{R,n,m}^2}{2\sigma_{R,n,m}^2}\right)\frac{1}{\sqrt{2\pi\sigma_{I,1,1}}}\exp\left(-\frac{\alpha_{I,1,1}^2}{2\sigma_{I,1,1}^2}\right)\frac{1}{\sqrt{2\pi\sigma_{I,1,2}}}\exp\left(-\frac{\alpha_{I,1,2}^2}{2\sigma_{I,1,2}^2}\right)$$
$$\cdots \frac{1}{\sqrt{2\pi\sigma_{I,n,m}}}\exp\left(-\frac{\alpha_{I,n,m}^2}{2\sigma_{I,n,m}^2}\right) d\alpha_{R,1,1}d\alpha_{R,1,2}\cdots d\alpha_{R,n,m}d\alpha_{I,1,1}d\alpha_{I,1,2}\cdots d\alpha_{I,n,m}$$
$$= \int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}\left[\prod_{i=1}^{n}\prod_{q=1}^{m}\prod_{k=1}^{M}\left(\pi a_t^k\right)^{-\frac{1}{2}}\exp\left(-\frac{\left|r_{q,t}^k\right|^2}{a_t^k}\right)\right]\exp\left\{\sum_{q=1}^{m}\left[\sum_{t=1}^{L}\sum_{k=1}^{M}\frac{1}{a_t^k}\right.\right.$$
$$\cdot\left[2\mathrm{Re}\left(r_{q,t}^k\sum_{i=1}^{n}\alpha_{R,i,q}\hat{s}_{i,t}^k\right)+2\mathrm{Im}\left(r_{q,t}^k\sum_{i=1}^{n}\alpha_{I,i,q}\hat{s}_{i,t}^k\right)-\sum_{i=1}^{n}\alpha_{R,i,q}^2\left|\hat{s}_{i,t}^k\right|^2-\sum_{i=1}^{n}\alpha_{I,i,q}^2\left|\hat{s}_{i,t}^k\right|^2\right.$$
$$\left.\left.\left.-\sum_{\substack{i=1\\i\neq l}}^{n}\sum_{\substack{l=1\\l\neq i}}^{n}\alpha_{R,i,q}\alpha_{R,l,q}\hat{s}_{i,t}^k\hat{s}_{l,t}^k-\sum_{\substack{i=1\\i\neq l}}^{n}\sum_{\substack{l=1\\l\neq i}}^{n}\alpha_{I,i,q}\alpha_{I,l,q}\hat{s}_{i,t}^k\hat{s}_{l,t}^k\right]-\sum_{i=1}^{n}\frac{\alpha_{R,i,q}^2}{2\sigma_{R,i,q}^2}-\sum_{i=1}^{n}\frac{\alpha_{I,i,q}^2}{2\sigma_{I,i,q}^2}\right\}$$
$$d\alpha_{R,1,1}d\alpha_{R,1,2}\cdots d\alpha_{R,n,m}d\alpha_{I,1,1}d\alpha_{I,1,2}\cdots d\alpha_{I,n,m}. \tag{B.1}$$

For the real part of the exponent could be represented by $R\left(\alpha_{R,1,q},\alpha_{R,2,q},\ldots,\alpha_{R,n,q}\right)$

$$R\left(\alpha_{R,1,q},\alpha_{R,2,q},\ldots,\alpha_{R,n,q}\right)$$
$$=\sum_{t=1}^{L}\sum_{k=1}^{M}\frac{1}{a_t^k}2\mathrm{Re}\left(r_{q,t}^k\sum_{i=1}^{n}\alpha_{R,i,q}\hat{s}_{i,t}^k\right)-\sum_{t=1}^{L}\sum_{k=1}^{M}\frac{1}{a_t^k}\sum_{i=1}^{n}\alpha_{R,i,q}^2\left|\hat{s}_{i,t}^k\right|^2$$
$$-\sum_{t=1}^{L}\sum_{k=1}^{M}\frac{1}{a_t^k}\sum_{\substack{i=1\\i\neq l}}^{n}\sum_{\substack{l=1\\l\neq i}}^{n}\alpha_{R,i,q}\alpha_{R,l,q}\hat{s}_{i,t}^k\hat{s}_{l,t}^k-\sum_{i=1}^{n}\frac{\alpha_{R,i,q}^2}{2\sigma_{R,i,q}^2}$$
$$=\sum_{i=1}^{n}\alpha_{R,1,q}2\mathrm{Re}\left(\sum_{t=1}^{L}\sum_{k=1}^{M}\frac{1}{a_t^k}r_{q,t}^k\hat{s}_{i,t}^k\right)-\sum_{i=1}^{n}\alpha_{R,i,q}^2\left(\sum_{t=1}^{L}\sum_{k=1}^{M}\frac{1}{a_t^k}\left|\hat{s}_{i,t}^k\right|^2-\frac{1}{2\sigma_{R,i,q}^2}\right)$$
$$-\sum_{\substack{i=1\\i\neq l}}^{n}\sum_{\substack{l=1\\l\neq i}}^{n}\alpha_{R,i,q}\alpha_{R,l,q}\sum_{t=1}^{L}\sum_{k=1}^{M}\frac{1}{a_t^k}\hat{s}_{i,t}^k\hat{s}_{l,t}^k$$
$$=B_{R,q}\Lambda_{R,q}^T-\Lambda_{R,q}\left(A+I\right)\Lambda_{R,q}^T \qquad \left(\sigma_{R,i,q}=\frac{1}{2}\forall i,q\right) \tag{B.2}$$

where

$$\Lambda_{R,i,q}=\left(\alpha_{R,1,q},\alpha_{R,2,q},\ldots,\alpha_{R,n,q}\right)$$

$$
\begin{aligned}
B_{R,q} &= \left[2\mathrm{Re}\left(\sum_{t=1}^{L} \sum_{k=1}^{M} \frac{1}{a_t^K} r_{q,t}^k \hat{s}_{1,t}^k \right), 2\mathrm{Re}\left(\sum_{t=1}^{L} \sum_{k=1}^{M} \frac{1}{a_t^K} r_{q,t}^k \hat{s}_{2,t}^k \right), \dots, 2\mathrm{Re}\left(\sum_{t=1}^{L} \sum_{k=1}^{M} \frac{1}{a_t^K} r_{q,t}^k \hat{s}_{n,t}^k \right) \right] \\
&= \sum_{t=1}^{L} \sum_{k=1}^{M} \frac{1}{a_t^K} \left[2\mathrm{Re}\left(r_{q,t}^k \hat{s}_{1,t}^k \right), 2\mathrm{Re}\left(r_{q,t}^k \hat{s}_{2,t}^k \right), \dots, 2\mathrm{Re}\left(r_{q,t}^k \hat{s}_{n,t}^k \right) \right] \\
&= \sum_{t=1}^{L} \sum_{k=1}^{M} \frac{1}{a_t^K} B_{R,q,t}^k
\end{aligned}
$$

$$
\begin{aligned}
A &= \begin{bmatrix}
\sum_{t=1}^{L} \sum_{k=1} \frac{1}{a_t^k} \left| \hat{s}_{1,t}^k \right|^2 & \sum_{t=1}^{L} \sum_{k=1} \frac{1}{a_t^k} \hat{s}_{1,t}^k \hat{s}_{2,t}^k & \dots & \sum_{t=1}^{L} \sum_{k=1} \frac{1}{a_t^k} \hat{s}_{1,t}^k \hat{s}_{n,t}^k \\
\sum_{t=1}^{L} \sum_{k=1} \frac{1}{a_t^k} \hat{s}_{2,t}^k \hat{s}_{1,t}^k & \sum_{t=1}^{L} \sum_{k=1} \frac{1}{a_t^k} \left| \hat{s}_{2,t}^k \right|^2 & \dots & \sum_{t=1}^{L} \sum_{k=1} \frac{1}{a_t^k} \hat{s}_{2,t}^k \hat{s}_{n,t}^k \\
\vdots & \vdots & \ddots & \vdots \\
\sum_{t=1}^{L} \sum_{k=1} \frac{1}{a_t^k} \hat{s}_{n,t}^k \hat{s}_{1,t}^k & \sum_{t=1}^{L} \sum_{k=1} \frac{1}{a_t^k} \hat{s}_{n,t}^k \hat{s}_{2,t}^k & \dots & \sum_{t=1}^{L} \sum_{k=1} \frac{1}{a_t^k} \left| \hat{s}_{n,t}^k \right|^2
\end{bmatrix} \\
&= \sum_{t=1}^{L} \sum_{k=1} \frac{1}{a_t^k} \begin{bmatrix}
\left| \hat{s}_{1,t}^k \right|^2 & \hat{s}_{1,t}^k \hat{s}_{2,t}^k & \dots & \hat{s}_{1,t}^k \hat{s}_{n,t}^k \\
\hat{s}_{2,t}^k \hat{s}_{1,t}^k & \left| \hat{s}_{2,t}^k \right|^2 & \dots & \hat{s}_{2,t}^k \hat{s}_{n,t}^k \\
\vdots & \vdots & \ddots & \vdots \\
\hat{s}_{n,t}^k \hat{s}_{1,t}^k & \hat{s}_{n,t}^k \hat{s}_{2,t}^k & \dots & \left| \hat{s}_{n,t}^k \right|^2
\end{bmatrix} \\
&= \sum_{t=1}^{L} \sum_{k=1}^{M} \frac{1}{a_t^k} A_t^k
\end{aligned}
$$

and I is an identity matrix. $B_{R,q,t}^k$ and A_t^k are defined as

$$
B_{R,q,t}^k = \left[2\mathrm{Re}\left(r_{q,t}^k \hat{s}_{1,t}^k \right), 2\mathrm{Re}\left(r_{q,t}^k \hat{s}_{2,t}^k \right), \dots, 2\mathrm{Re}\left(r_{q,t}^k \hat{s}_{n,t}^k \right) \right] \tag{B.3}
$$

and

$$
A_t^k = \begin{bmatrix}
\left| \hat{s}_{1,t}^k \right|^2 & \hat{s}_{1,t}^k \hat{s}_{2,t}^k & \dots & \hat{s}_{1,t}^k \hat{s}_{n,t}^k \\
\hat{s}_{2,t}^k \hat{s}_{1,t}^k & \left| \hat{s}_{2,t}^k \right|^2 & \dots & \hat{s}_{2,t}^k \hat{s}_{n,t}^k \\
\vdots & \vdots & \ddots & \vdots \\
\hat{s}_{n,t}^k \hat{s}_{1,t}^k & \hat{s}_{n,t}^k \hat{s}_{2,t}^k & \dots & \left| \hat{s}_{n,t}^k \right|^2
\end{bmatrix}. \tag{B.4}
$$

It is clear that A_t^k is nonnegative definite Hermitian, and the eigenvalues of A_t^k are nonnegative real numbers. Therefore, we have

$$
A_t^k = V_t^k D_t^k V_t^{k^H} \tag{B.5}
$$

where V_t^k is a unitary matrix and D_t^k is a real diagonal matrix. The rows of V_t^k, forming a complete orthonormal basis of an N-dimensional vector space, are eigenvectors of A_t^k. The diagonal elements of D_t^k are the eigenvalues $\lambda_{i,t}^k$ of A_t^k. (B.2) can be rewirtten as

$$
\begin{aligned}
& R\left(\alpha_{R,1,q}, \alpha_{R,2,q}, \ldots, \alpha_{R,n,q}\right) \\
&= B_{R,q}\Lambda_{R,q}^T - \Lambda_{R,q}\left(A+I\right)\Lambda_{R,q}^T \\
&= \sum_{t=1}^{L}\sum_{k=1}\frac{1}{a_t^k}B_{R,q,t}^k\Lambda_{R,q}^T - \Lambda_{R,q}\left(\sum_{t=1}^{L}\sum_{k=1}\frac{1}{a_t^k}A_t^k + I\right)\Lambda_{R,q}^T \\
&= \sum_{t=1}^{L}\sum_{k=1}^{M}\frac{1}{a_t^k}B_{R,q,t}^k\Lambda_{R,q}^T - \sum_{t=1}^{L}\sum_{k=1}^{M}\frac{1}{a_t^k}\Lambda_{R,q}\left(V_t^k D_t^k V_t^{k^T} + \frac{LM}{a_t^k}I\right)\Lambda_{R,q}^T \\
&= \sum_{t=1}^{L}\sum_{k=1}^{M}\frac{1}{a_t^k}\left[B_{R,q,t}^k\Lambda_{R,q}^T - \Lambda_{R,q}\left(V_t^k D_t^k V_t^{k^T} + \frac{LM}{a_t^k}I\right)\Lambda_{R,q}^T\right] \\
&= \sum_{t=1}^{L}\sum_{k=1}^{M}\frac{1}{a_t^k}\left[B_{R,q,t}^k V_t^k V_t^{k^T}\Lambda_{R,q}^T - \Lambda_{R,q}V_t^k\left(D_t^k + \frac{LM}{a_t^k}I\right)\left(\Lambda_{R,q}V_t^k\right)^T\right] \\
&= \sum_{t=1}^{L}\sum_{k=1}^{M}\frac{1}{a_t^k}\left[B_{R,q,t}^k V_t^k\left(\Lambda_{R,q}V_t^k\right)^T - \Lambda_{R,q}V_t^k\left(D_t^k + \frac{LM}{a_t^k}I\right)\left(\Lambda_{R,q}V_t^k\right)^T\right] \\
&= \sum_{t=1}^{L}\sum_{k=1}^{M}\frac{1}{a_t^k}\left[Z_{q,t}^k Y_{R,q}^T - Y_{R,q}\left(D_t^k + \frac{LM}{a_t^k}I\right)Y_{R,q}^T\right] \\
&= \sum_{t=1}^{L}\sum_{k=1}^{M}\frac{1}{a_t^k}\left[\sum_{i=1}^{n} z_{i,q,t}^k y_{R,i,q}^T - \sum_{i=1}^{n}\left(\lambda_{i,t}^k + \frac{LM}{a_t^k}\right)y_{R,i,q}^2\right]
\end{aligned}
\tag{B.6}
$$

where

$$Z_{q,t}^k = B_{R,q,t}^k V_t^k = \left[z_{1,q,t}^k, z_{2,q,t}^k, \ldots, z_{n,q,t}^k\right]$$

and

$$Y_{R,q} = \Lambda_{R,i,q}V_t^k = \left[y_{R,1,q}, y_{R,2,q}, \ldots, y_{R,n,q}\right].$$

We can average $\alpha_{R,i,q}$ for the real part of the exponent in (B.1)

$$
\begin{aligned}
& \int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}\exp\left\{\sum_{q=1}^{m}\sum_{i=1}^{n}\sum_{t=1}^{L}\sum_{k=1}^{M}\frac{1}{a_t^k}\left[\sum_{i=1}^{n} z_{i,q,t}^k y_{R,i,q}^T - \sum_{i=1}^{n}\left(\lambda_{i,t}^k + \frac{LM}{a_t^k}\right)y_{R,i,q}^2\right]\right\} \\
& dy_{R,1,q}dy_{R,2,q}\ldots dy_{R,n,q} \\
&= \prod_{q=1}^{m}\prod_{i=1}^{n}\int_{-\infty}^{\infty}\exp\left\{\sum_{t=1}^{L}\sum_{k=1}^{M}\frac{1}{a_t^k}\left[\sum_{i=1}^{n} z_{i,q,t}^k y_{R,i,q}^T - \sum_{i=1}^{n}\left(\lambda_{i,t}^k + \frac{LM}{a_t^k}\right)y_{R,i,q}^2\right]\right\}dy_{R,i,q}
\end{aligned}
$$

$$
= \sqrt{\frac{\pi^{nm}}{\prod_{q=1}^{m}\prod_{i=1}^{n}\left(\sum_{t=1}^{L}\sum_{k=1}^{M}\frac{\lambda_{i,t}^{k}}{a_t^k}+1\right)}}\exp\left[\sum_{q=1}^{m}\sum_{i=1}^{n}\frac{\left(\sum_{t=1}^{L}\sum_{k=1}^{M}\frac{1}{a_t^k}z_{i,q,t}^{k}\right)^2}{4\left(\sum_{t=1}^{L}\sum_{k=1}^{M}\frac{\lambda_{i,t}^{k}}{a_t^k}+1\right)}\right]. \tag{B.7}
$$

The imaginary part of the exponent in (B.1) are defined by $I\left(\alpha_{I,1,q},\alpha_{I,2,q},\ldots,\alpha_{I,n,q}\right)$

$$
\begin{aligned}
& I\left(\alpha_{I,1,q},\alpha_{I,2,q},\ldots,\alpha_{I,n,q}\right) \\
&= \sum_{t=1}^{L}\sum_{k=1}^{M}\frac{1}{a_t^k}2\mathrm{Im}\left(r_{q,t}^{k}\sum_{i=1}^{n}\alpha_{I,i,q}\hat{s}_{i,t}^{k}\right)-\sum_{t=1}^{L}\sum_{k=1}^{M}\frac{1}{a_t^k}\sum_{i=1}^{n}\alpha_{I,i,q}^{2}\left|\hat{s}_{i,t}^{k}\right|^2 \\
&\quad -\sum_{t=1}^{L}\sum_{k=1}^{M}\frac{1}{a_t^k}\sum_{\substack{i=1\\ i\neq l}}^{n}\sum_{\substack{l=1\\ l\neq i}}^{n}\alpha_{I,i,q}\alpha_{I,l,q}\hat{s}_{i,t}^{k}\hat{s}_{l,t}^{k}-\sum_{i=1}^{n}\frac{\alpha_{I,i,q}^{2}}{2\sigma_{I,i,q}^{2}} \\
&= \sum_{i=1}^{n}\alpha_{I,1,q}2\mathrm{Im}\left(\sum_{t=1}^{L}\sum_{k=1}^{M}\frac{1}{a_t^k}r_{q,t}^{k}\hat{s}_{i,t}^{k}\right)-\sum_{i=1}^{n}\alpha_{I,i,q}^{2}\left(\sum_{t=1}^{L}\sum_{k=1}^{M}\frac{1}{a_t^k}\left|\hat{s}_{i,t}^{k}\right|^2-\frac{1}{2\sigma_{I,i,q}^{2}}\right) \\
&\quad -\sum_{\substack{i=1\\ i\neq l}}^{n}\sum_{\substack{l=1\\ l\neq i}}^{n}\alpha_{I,i,q}\alpha_{I,l,q}\sum_{t=1}^{L}\sum_{k=1}^{M}\frac{1}{a_t^k}\hat{s}_{i,t}^{k}\hat{s}_{l,t}^{k} \\
&= B_{I,q}\Lambda_{I,q}^{T}-\Lambda_{I,q}\left(A+I\right)\Lambda_{I,q}^{T}\quad\left(\sigma_{I,i,q}=\frac{1}{2}\forall i,q\right)
\end{aligned} \tag{B.8}
$$

where

$$
\Lambda_{I,i,q}=\left(\alpha_{I,1,q},\alpha_{I,2,q},\ldots,\alpha_{I,n,q}\right)
$$

and

$$
\begin{aligned}
B_{I,q} &= \left[2\mathrm{Im}\left(\sum_{t=1}^{L}\sum_{k=1}^{M}\frac{1}{a_t^K}r_{q,t}^{k}\hat{s}_{1,t}^{k}\right),2\mathrm{Re}\left(\sum_{t=1}^{L}\sum_{k=1}^{M}\frac{1}{a_t^K}r_{q,t}^{k}\hat{s}_{2,t}^{k}\right),\ldots,2\mathrm{Re}\left(\sum_{t=1}^{L}\sum_{k=1}^{M}\frac{1}{a_t^K}r_{q,t}^{k}\hat{s}_{n,t}^{k}\right)\right] \\
&= \sum_{t=1}^{L}\sum_{k=1}^{M}\frac{1}{a_t^K}\left[2\mathrm{Re}\left(r_{q,t}^{k}\hat{s}_{1,t}^{k}\right),2\mathrm{Re}\left(r_{q,t}^{k}\hat{s}_{2,t}^{k}\right),\ldots,2\mathrm{Re}\left(r_{q,t}^{k}\hat{s}_{n,t}^{k}\right)\right] \\
&= \sum_{t=1}^{L}\sum_{k=1}^{M}\frac{1}{a_t^K}B_{I,q,t}^{k}.
\end{aligned}
$$

Then (B.2) can be rewirtten as

$$
\begin{aligned}
& I\left(\alpha_{I,1,q},\alpha_{I,2,q},\ldots,\alpha_{I,n,q}\right) \\
&= B_{I,q}\Lambda_{I,q}^{T} - \Lambda_{I,q}\left(A+I\right)\Lambda_{I,q}^{T} \\
&= \sum_{t=1}^{L}\sum_{k=1}\frac{1}{a_t^k}B_{I,q,t}^{k}\Lambda_{I,q}^{T} - \Lambda_{I,q}\left(\sum_{t=1}^{L}\sum_{k=1}\frac{1}{a_t^k}A_t^k + I\right)\Lambda_{I,q}^{T} \\
&= \sum_{t=1}^{L}\sum_{k=1}^{M}\frac{1}{a_t^k}B_{I,q,t}^{k}\Lambda_{I,q}^{T} - \sum_{t=1}^{L}\sum_{k=1}^{M}\frac{1}{a_t^k}\Lambda_{I,q}\left(V_t^k D_t^k V_t^{k^T} + \frac{LM}{a_t^k}I\right)\Lambda_{I,q}^{T} \\
&= \sum_{t=1}^{L}\sum_{k=1}^{M}\frac{1}{a_t^k}\left[B_{I,q,t}^{k}\Lambda_{I,q}^{T} - \Lambda_{I,q}\left(V_t^k D_t^k V_t^{k^T} + \frac{LM}{a_t^k}I\right)\Lambda_{I,q}^{T}\right] \\
&= \sum_{t=1}^{L}\sum_{k=1}^{M}\frac{1}{a_t^k}\left[B_{I,q,t}^{k}V_t^k V_t^{k^T}\Lambda_{I,q}^{T} - \Lambda_{I,q}V_t^k\left(D_t^k + \frac{LM}{a_t^k}I\right)\left(\Lambda_{I,q}V_t^k\right)^T\right] \\
&= \sum_{t=1}^{L}\sum_{k=1}^{M}\frac{1}{a_t^k}\left[B_{I,q,t}^{k}V_t^k\left(\Lambda_{I,q}V_t^k\right)^T - \Lambda_{I,q}V_t^k\left(D_t^k + \frac{LM}{a_t^k}I\right)\left(\Lambda_{I,q}V_t^k\right)^T\right] \\
&= \sum_{t=1}^{L}\sum_{k=1}^{M}\frac{1}{a_t^k}\left[W_{q,t}^{k}Y_{I,q}^{T} - Y_{I,q}\left(D_t^k + \frac{LM}{a_t^k}I\right)Y_{I,q}^{T}\right] \\
&= \sum_{t=1}^{L}\sum_{k=1}^{M}\frac{1}{a_t^k}\left[\sum_{i=1}^{n}w_{i,q,t}^{k}y_{I,i,q}^{T} - \sum_{i=1}^{n}\left(\lambda_{i,t}^{k} + \frac{LM}{a_t^k}\right)y_{I,i,q}^{2}\right] \qquad \text{(B.9)}
\end{aligned}
$$

where

$$
W_{q,t}^{k} = B_{I,q,t}^{k}V_t^k = \left[w_{1,q,t}^{k}, w_{2,q,t}^{k}, \ldots, w_{n,q,t}^{k}\right]
$$

and

$$
Y_{I,q} = \Lambda_{I,i,q}V_t^k = \left[y_{I,1,q}, y_{I,2,q}, \ldots, y_{I,n,q}\right].
$$

We can average $\alpha_{I,i,q}$ for the imaginary part of the exponent in (B.1)

$$
\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}\exp\left\{\sum_{q=1}^{m}\sum_{i=1}^{n}\sum_{t=1}^{L}\sum_{k=1}^{M}\frac{1}{a_t^k}\left[\sum_{i=1}^{n}w_{i,q,t}^{k}y_{I,i,q}^{T} - \sum_{i=1}^{n}\left(\lambda_{i,t}^{k} + \frac{LM}{a_t^k}\right)y_{I,i,q}^{2}\right]\right\}
$$

$dy_{I,1,q}dy_{I,2,q}\ldots dy_{I,n,q}$

$$
= \sqrt{\frac{\pi^{nm}}{\prod_{q=1}^{m}\prod_{i=1}^{n}\left(\sum_{t=1}^{L}\sum_{k=1}^{M}\frac{\lambda_{i,t}^{k}}{a_t^k}+1\right)}}\exp\left[\sum_{q=1}^{m}\sum_{i=1}^{n}\frac{\left(\sum_{t=1}^{L}\sum_{k=1}^{M}\frac{1}{a_t^k}w_{i,q,t}^{k}\right)^2}{4\left(\sum_{t=1}^{L}\sum_{k=1}^{M}\frac{\lambda_{i,t}^{k}}{a_t^k}+1\right)}\right]. \qquad \text{(B.10)}
$$

Therefore, the likelihood function can be expressed as

$$
\begin{aligned}
& f\left(\boldsymbol{r}|\hat{\boldsymbol{s}},\boldsymbol{x}\right) \\
&= \left[\prod_{i=1}^{n}\prod_{q=1}^{m}\prod_{k=1}^{M}\left(a_t^k\right)^{-\frac{1}{2}}\exp\left(-\frac{\left|r_{q,t}^k\right|^2}{a_t^k}\right)\right]\left[\prod_{q=1}^{m}\prod_{i=1}^{n}\left(\sum_{t=1}^{L}\sum_{k=1}^{M}\frac{1}{a_t^k}\lambda_{i,t}^k+1\right)\right]^{-1} \\
&\quad \cdot \exp\left\{\sum_{q=1}^{m}\sum_{i=1}^{n}\frac{\left(\sum_{t=1}^{L}\sum_{k=1}^{M}\frac{1}{a_t^k}z_{i,q,t}^k\right)^2+\left(\sum_{t=1}^{L}\sum_{k=1}^{M}\frac{1}{a_t^k}w_{i,q,t}^k\right)^2}{4\left(\sum_{k=1}^{M}\sum_{i=1}^{n}\frac{1}{a_t^k}\lambda_{i,t}^k+1\right)}\right\}
\end{aligned}
\tag{B.11}
$$

Appendix C

Derivation of the Design Criteria of STC/WFHSS System

The derivation in (4.29) is discussed in this appendix. (4.29) is derived by averaging (4.26) with respect to $\boldsymbol{x}$ and $\boldsymbol{\alpha}$. Then (4.27) is derived first by averaging (4.26) with respect to $\boldsymbol{\alpha}$.

$$\boldsymbol{Pr}\left(\boldsymbol{s}\rightarrow\tilde{\boldsymbol{s}}|\boldsymbol{x}\right)\leq\int_{-\infty}^{\infty}\frac{1}{2}\exp\left(-\sum_{t=1}^{L}\sum_{q=1}^{m}\sum_{k=1}^{M}\frac{1}{b_t^k}\left|\sum_{i=1}^{n}\alpha_{i,q}\left(s_{i,t}^k-\tilde{s}_{i,t}^k\right)\right|^2\right)f\left(\boldsymbol{\alpha}\right)d\boldsymbol{\alpha}. \tag{C.1}$$

$\left|\sum_{i=1}^{n}\alpha_{i,q}\left(s_{i,t}^k-\tilde{s}_{i,t}^k\right)\right|^2$ in (C.1) can be rewritten as

$$\begin{aligned}
&\left|\sum_{i=1}^{n}\alpha_{i,q}\left(s_{i,t}^k-\tilde{s}_{i,t}^k\right)\right|^2\\
&=\left(\sum_{i=1}^{n}\alpha_{i,q}\left(s_{i,t}^k-\tilde{s}_{i,t}^k\right)\right)\left(\sum_{i=1}^{n}\alpha_{i,q}\left(s_{i,t}^k-\tilde{s}_{i,t}^k\right)\right)^*\\
&=\sum_{i=}^{n}\sum_{i'=1}^{n}\alpha_{i,q}\alpha_{i,q}^*\left(s_{i,t}^k-\tilde{s}_{i,t}^k\right)\left(s_{i',t}^k-\tilde{s}_{i',t}^k\right)\\
&=\Lambda_q A_t^k\Lambda_q^H
\end{aligned} \tag{C.2}$$

where

$$\Lambda_q=\left(\alpha_{1,q},\alpha_{2,q},\ldots,\alpha_{n,q}\right)$$

and

$$A_t^k=\begin{bmatrix}
\left|s_{1,t}^k-\tilde{s}_{1,t}^k\right|^2 & \left(s_{1,t}^k-\tilde{s}_{1,t}^k\right)\left(s_{2,t}^k-\tilde{s}_{2,t}^k\right) & \ldots & \left(s_{1,t}^k-\tilde{s}_{1,t}^k\right)\left(s_{n,t}^k-\tilde{s}_{n,t}^k\right)\\
\left(s_{2,t}^k-\tilde{s}_{2,t}^k\right)\left(s_{1,t}^k-\tilde{s}_{1,t}^k\right) & \left|s_{2,t}^k-\tilde{s}_{2,t}^k\right|^2 & \ldots & \left(s_{2,t}^k-\tilde{s}_{2,t}^k\right)\left(s_{n,t}^k-\tilde{s}_{n,t}^k\right)\\
\vdots & \vdots & \ddots & \vdots\\
\left(s_{n,t}^k-\tilde{s}_{n,t}^k\right)\left(s_{1,t}^k-\tilde{s}_{1,t}^k\right) & \left(s_{n,t}^k-\tilde{s}_{n,t}^k\right)\left(s_{2,t}^k-\tilde{s}_{2,t}^k\right) & \ldots & \left|s_{n,t}^k-\tilde{s}_{n,t}^k\right|^2
\end{bmatrix}.$$

It is clear that A_t^k is nonnegative definite Hermitian. Therefore,we have

$$A_t^k = V_t^k D_t^k V_t^{k^H} \tag{C.3}$$

where V_t^k is a unitary matrix and D_t^k is a diagonal matrix. The diagonal elements of D_t^k are the eigenvalues $\lambda_{i,t}^k$ of A_t^k. Then, (C.2) can be rewritten as

$$\begin{aligned}
&\left|\sum_{i=1}^{n} \alpha_{i,q}\left(s_{i,t}^k - \tilde{s}_{i,t}^k\right)\right|^2 \\
&= \Lambda_q A_t^k \Lambda_q^H \\
&= \Lambda_q V_t^k D_t^k V_t^{k^H} \Lambda_q^H \\
&= Y_q D_t^k Y_q^H \\
&= \sum_{i=1}^{n} \lambda_t^k \left|y_{i,q}\right|^2
\end{aligned} \tag{C.3}$$

where $Y_q = \Lambda_q V_t^k = (y_{1,q}, y_{2,q}, \ldots, y_{n,q})$. $y_{i,q}$ are complex Gaussian random variable with zero mean and variance 1/2 for $\forall i, q$. Then $|y_{i,q}|$ is a Rayleigh distribution random variable and the probability density function is

$$p\left(|y_{i,q}|\right) = 2\left|y_{i,q}\right| \exp\left(-\left|y_{i,q}\right|^2\right). \tag{C.4}$$

The conditional pairwise error probability can be expressed as

$$\begin{aligned}
&\boldsymbol{Pr}\left(\boldsymbol{s} \rightarrow \tilde{\boldsymbol{s}} | \boldsymbol{x}\right) \\
&\leq \int_{-\infty}^{\infty} \frac{1}{2} \exp\left(-\sum_{t=1}^{L}\sum_{q=1}^{m}\sum_{k=1}^{M} \frac{1}{b_t^k}\left|\sum_{i=1}^{n} \alpha_{i,q}\left(s_{i,t}^k - \tilde{s}_{i,t}^k\right)\right|^2\right) df(\boldsymbol{\alpha})\, d\boldsymbol{\alpha} \\
&= \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} \frac{1}{2} \exp\left(-\sum_{t=1}^{L}\sum_{q=1}^{m}\sum_{k=1}^{M}\sum_{i=1}^{n} \frac{\lambda_t^k}{b_t^k}\left|y_{i,q}\right|^2\right) f\left(|y_{1,1}|\right) \cdots f\left(|y_{n,m}|\right) dy_{1,1} \ldots dy_{n,m} \\
&= \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} \frac{1}{2} \prod_{q=1}^{m}\prod_{i=1}^{n} \exp\left(-\sum_{t=1}^{L}\sum_{k=1}^{M} \frac{\lambda_t^k}{b_t^k}\left|y_{i,q}\right|^2\right) 2\left|y_{i,q}\right| \exp\left(-\left|y_{i,q}\right|^2\right) dy_{i,q} \\
&= \frac{1}{2} \prod_{q=1}^{m}\prod_{i=1}^{n} \int_{-\infty}^{\infty} 2\left|y_{i,q}\right| \exp\left(-\sum_{t=1}^{L}\sum_{k=1}^{M}\left(\frac{\lambda_t^k}{b_t^k} + 1\right)\left|y_{i,q}\right|^2\right) dy_{i,q} \\
&= \frac{1}{2} \prod_{q=1}^{m}\prod_{i=1}^{n} \left(\sum_{t=1}^{L}\sum_{k=1}^{M} \frac{\lambda_t^k}{b_t^k} + 1\right)^{-1}.
\end{aligned} \tag{C.5}$$

The pairwise error probability can be derived by averaging (C.5) with respect to $\boldsymbol{x}$

$$
\begin{aligned}
&\boldsymbol{Pr}\left(\boldsymbol{s} \to \tilde{\boldsymbol{s}}\right) \\
&\leq \mathrm{E}_{\boldsymbol{x}}\left[\frac{1}{2}\prod_{q=1}^{m}\prod_{i=1}^{n}\left(\sum_{t=1}^{L}\sum_{k=1}^{M}\frac{\lambda_t^k}{b_t^k}+1\right)^{-1}\right] \\
&= \frac{1}{2}\prod_{q=1}^{m}\prod_{i=1}^{n}\mathrm{E}_{\boldsymbol{x}}\left[\left(\sum_{t=1}^{L}\sum_{k=1}^{M}\frac{\lambda_t^k}{b_t^k}+1\right)^{-1}\right] \\
&= \frac{1}{2}\prod_{q=1}^{m}\prod_{i=1}^{n}\mathrm{E}_{\boldsymbol{x}}\left[\sum_{j=1}^{\infty}\left(-\sum_{t=1}^{L}\sum_{k=1}^{M}\frac{\lambda_t^k}{b_t^k}\right)^{j}\right] \\
&\cong \frac{1}{2}\prod_{q=1}^{m}\prod_{i=1}^{n}\mathrm{E}_{\boldsymbol{x}}\left[\left(1-\sum_{t=1}^{L}\sum_{k=1}^{M}\frac{\lambda_t^k}{b_t^k}\right)\right] \\
&= \frac{1}{2}\prod_{q=1}^{m}\prod_{i=1}^{n}\left\{1-\sum_{t=1}^{L}\mathrm{E}_{\boldsymbol{x}}\left[\sum_{k=1}^{M}\frac{\lambda_t^k}{b_t^k}\right]\right\} \\
&= \frac{1}{2}\prod_{q=1}^{m}\prod_{i=1}^{n}\left\{1-\sum_{t=1}^{L}\left[\frac{Q}{N_t}\left(\sum_{k=1}^{M-1}\frac{1}{4N_0}\lambda_{i,t}^k+\frac{1}{4\left(N_0+\frac{JT_s}{Q}\right)}\lambda_{i,t}^1\right)+\cdots+\frac{Q}{N_t}\right.\right. \\
&\quad \left.\left.\cdot\left(\sum_{K=2}^{M}\frac{1}{4N_0}\lambda_{i,t}^k+\frac{1}{4\left(N_0+\frac{JT_s}{Q}\right)}\lambda_{i,t}^M\right)+\left(1-\frac{MQ}{N_t}\right)\sum_{K=1}^{M}\frac{1}{4N_0}\lambda_{i,t}^k\right]\right\} \\
&= \frac{1}{2}\prod_{q=1}^{m}\prod_{i=1}^{n}\left\{1-\sum_{t=1}^{L}\left[\left((M-1)\frac{Q}{N_t}+\left(1-\frac{MQ}{N_t}\right)\right)\sum_{K=1}^{M}\frac{1}{4N_0}\lambda_{i,t}^k\right.\right. \\
&\quad \left.\left.+\frac{Q}{N_t}\frac{1}{4\left(N_0+\frac{JT_s}{Q}\right)}\sum_{K=1}^{M}\lambda_{i,t}^k\right]\right\} \\
&= \frac{1}{2}\prod_{i=1}^{n}\prod_{q=1}^{n}\left\{1-\sum_{t-1}^{L}\left[\left(1-\frac{Q}{N_t}\right)\sum_{k=1}^{M}\frac{1}{4N_0}\lambda_{i,t}^k+\frac{Q}{N_t}\sum_{k=1}^{M}\frac{1}{4\left(N_0+\frac{JT_s}{Q}\right)}\lambda_{i,t}^k\right]\right. \qquad \text{(C.6)}
\end{aligned}
$$

Appendix D

Derivation of the ML Decoding of STC/OFHSS System without JSI

The derivation in (4.40) is discussed in this appendix. (4.40) is derived by averaging (4.36) with respect to $\boldsymbol{x}$. Let $\boldsymbol{x}_t = \left(x_{i,t}^k | \forall i, k\right)$, Assume there are only two transmitter antennas, then we have following function

$$
\begin{aligned}
& f\left(\boldsymbol{r}|\hat{\boldsymbol{s}}, \boldsymbol{\alpha}\right) \\
&= E_{\boldsymbol{x}}\left[\prod_{t=1}^{L}\prod_{k=1}^{M}\prod_{q=1}^{m}\prod_{i=1}^{2} \frac{1}{\sqrt{\pi\left(N_0 + x_{i,t}^k 2\frac{JT_s}{Q}\sigma_{J,q}^2\right)}} \exp\left(-\frac{\left|r_{q,t}^k - \alpha_{i,q}\hat{s}_{i,t}^k\right|^2}{\left(N_0 + x_{i,t}^k 2\frac{JT_s}{Q}\sigma_{J,q}^2\right)}\right)\right] \\
&= \prod_{t=1}^{L}\prod_{q=1}^{m} E_{\boldsymbol{x}_t}\left[\prod_{k=1}^{M}\prod_{i=1}^{2} \frac{1}{\sqrt{\pi\left(N_0 + x_{i,t}^k 2\frac{JT_s}{Q}\sigma_{J,q}^2\right)}} \exp\left(-\frac{\left|r_{q,t}^k - \alpha_{i,q}\hat{s}_{i,t}^k\right|^2}{\left(N_0 + x_{i,t}^k 2\frac{JT_s}{Q}\sigma_{J,q}^2\right)}\right)\right] \\
&= \prod_{t=1}^{L}\prod_{q=1}^{m}\left\{\frac{\phi_1}{M^2}\left(\frac{1}{\sqrt{\pi N_0}}\right)^{2M-2}\left(\frac{1}{\sqrt{\pi a}}\right)^2 \exp\left[-\sum_{k=2}^{M}\frac{\left|r_{1,q,t}^k - \alpha_{1,q}\hat{s}_{1,t}^k\right|^2}{N_0}\right.\right. \\
&\quad \left. - \frac{\left|r_{1,q,t}^1 - \alpha_{1,q}\hat{s}_{1,t}^1\right|^2}{a} - \sum_{k=2}^{M}\frac{\left|r_{2,q,t}^k - \alpha_{2,q}\hat{s}_{2,t}^k\right|^2}{N_0} - \frac{\left|r_{2,q,t}^1 - \alpha_{2,q}\hat{s}_{2,t}^1\right|^2}{a}\right] + \cdots \\
&\quad + \frac{\phi_1}{M^2}\left(\frac{1}{\sqrt{\pi N_0}}\right)^{2M-2}\left(\frac{1}{\sqrt{\pi a}}\right)^2 \exp\left[-\sum_{k=1}^{M-1}\frac{\left|r_{1,q,t}^k - \alpha_{1,q}\hat{s}_{1,t}^k\right|^2}{N_0} - \frac{\left|r_{1,q,t}^M - \alpha_{1,q}\hat{s}_{1,t}^M\right|^2}{a}\right. \\
&\quad \left. - \sum_{k=1}^{M-1}\frac{\left|r_{2,q,t}^k - \alpha_{2,q}\hat{s}_{2,t}^k\right|^2}{N_0} - \frac{\left|r_{2,q,t}^M - \alpha_{2,q}\hat{s}_{2,t}^M\right|^2}{a}\right] + \frac{\phi_2}{M}\left(\frac{1}{\sqrt{\pi N_0}}\right)^{2M-1}\left(\frac{1}{\sqrt{\pi a}}\right) \\
&\quad \cdot \exp\left[-\sum_{k=2}^{M}\frac{\left|r_{1,q,t}^k - \alpha_{1,q}\hat{s}_{1,t}^k\right|^2}{N_0} - \frac{\left|r_{1,q,t}^1 - \alpha_{1,q}\hat{s}_{1,t}^1\right|^2}{a} - \sum_{k=1}^{M}\frac{\left|r_{2,q,t}^k - \alpha_{2,q}\hat{s}_{2,t}^k\right|^2}{N_0}\right]
\end{aligned}
$$

$$+\cdots+\frac{\phi_2}{M}\left(\frac{1}{\sqrt{\pi N_0}}\right)^{2M-1}\left(\frac{1}{\sqrt{\pi a}}\right)\exp\Bigg[-\sum_{k=1}^{M-1}\frac{\left|r_{1,q,t}^{k}-\alpha_{1,q}\hat{s}_{1,t}^{k}\right|^2}{N_0}-\frac{\left|r_{1,q,t}^{M}-\alpha_{1,q}\hat{s}_{1,t}^{M}\right|^2}{a}$$

$$-\sum_{k=1}^{M}\frac{\left|r_{2,q,t}^{k}-\alpha_{2,q}\hat{s}_{2,t}^{k}\right|^2}{N_0}\Bigg]+\frac{\phi_3}{M}\left(\frac{1}{\sqrt{\pi N_0}}\right)^{2M-1}\left(\frac{1}{\sqrt{\pi a}}\right)\exp\Bigg[-\sum_{k=1}^{M}\frac{\left|r_{1,q,t}^{k}-\alpha_{1,q}\hat{s}_{1,t}^{k}\right|^2}{N_0}$$

$$-\sum_{k=2}^{M}\frac{\left|r_{2,q,t}^{k}-\alpha_{2,q}\hat{s}_{2,t}^{k}\right|^2}{N_0}-\frac{\left|r_{2,q,t}^{1}-\alpha_{2,q}\hat{s}_{2,t}^{1}\right|^2}{a}\Bigg]+\cdots+\frac{\phi_3}{M}\left(\frac{1}{\sqrt{\pi N_0}}\right)^{2M-1}\left(\frac{1}{\sqrt{\pi a}}\right)$$

$$\cdot\exp\Bigg[-\sum_{k=1}^{M}\frac{\left|r_{1,q,t}^{k}-\alpha_{1,q}\hat{s}_{1,t}^{k}\right|^2}{N_0}-\sum_{k=1}^{M-1}\frac{\left|r_{2,q,t}^{k}-\alpha_{2,q}\hat{s}_{2,t}^{k}\right|^2}{N_0}-\frac{\left|r_{2,q,t}^{M}-\alpha_{2,q}\hat{s}_{2,t}^{M}\right|^2}{a}\Bigg]$$

$$+\phi_4\left(\frac{1}{\sqrt{\pi N_0}}\right)^{2M}\exp\Bigg[-\sum_{i=1}^{2}\sum_{k=1}^{M}\frac{\left|r_{1,q,t}^{k}-\alpha_{1,q}\hat{s}_{1,t}^{k}\right|^2}{N_0}\Bigg]\Bigg]\Bigg\}$$

$$=\prod_{t=1}^{L}\prod_{q=1}^{m}\Bigg\{\exp\Bigg[-\sum_{i=1}^{2}\sum_{k=1}^{M}\frac{\left|r_{1,q,t}^{k}-\alpha_{1,q}\hat{s}_{1,t}^{k}\right|^2}{N_0}\Bigg]\Bigg\{\frac{\phi_1}{M^2}\left(\frac{1}{\sqrt{\pi N_0}}\right)^{2M-2}\left(\frac{1}{\sqrt{\pi a}}\right)^2$$

$$\cdot\exp\Bigg[\frac{\left|r_{1,q,t}^{1}-\alpha_{1,q}\hat{s}_{1,t}^{1}\right|^2}{N_0}-\frac{\left|r_{1,q,t}^{1}-\alpha_{1,q}\hat{s}_{1,t}^{1}\right|^2}{a}+\frac{\left|r_{2,q,t}^{1}-\alpha_{2,q}\hat{s}_{2,t}^{1}\right|^2}{N_0}-\frac{\left|r_{2,q,t}^{1}-\alpha_{2,q}\hat{s}_{2,t}^{1}\right|^2}{a}\Bigg]$$

$$+\cdots+\frac{\phi_1}{M^2}\left(\frac{1}{\sqrt{\pi N_0}}\right)^{2M-2}\left(\frac{1}{\sqrt{\pi a}}\right)^2\exp\Bigg[\frac{\left|r_{1,q,t}^{M}-\alpha_{1,q}\hat{s}_{1,t}^{M}\right|^2}{N_0}-\frac{\left|r_{1,q,t}^{M}-\alpha_{1,q}\hat{s}_{1,t}^{M}\right|^2}{a}$$

$$+\frac{\left|r_{2,q,t}^{M}-\alpha_{2,q}\hat{s}_{2,t}^{M}\right|^2}{N_0}-\frac{\left|r_{2,q,t}^{M}-\alpha_{2,q}\hat{s}_{2,t}^{M}\right|^2}{a}\Bigg]+\frac{\phi_2}{M}\left(\frac{1}{\sqrt{\pi N_0}}\right)^{2M-1}\left(\frac{1}{\sqrt{\pi a}}\right)$$

$$\cdot\exp\Bigg[\frac{\left|r_{1,q,t}^{1}-\alpha_{1,q}\hat{s}_{1,t}^{1}\right|^2}{N_0}-\frac{\left|r_{1,q,t}^{1}-\alpha_{1,q}\hat{s}_{1,t}^{1}\right|^2}{a}\Bigg]+\cdots+\frac{\phi_2}{M}\left(\frac{1}{\sqrt{\pi N_0}}\right)^{2M-1}\left(\frac{1}{\sqrt{\pi a}}\right)$$

$$\cdot\exp\Bigg[\frac{\left|r_{1,q,t}^{M}-\alpha_{1,q}\hat{s}_{1,t}^{M}\right|^2}{N_0}-\frac{\left|r_{1,q,t}^{M}-\alpha_{1,q}\hat{s}_{1,t}^{M}\right|^2}{a}\Bigg]+\frac{\phi_3}{M}\left(\frac{1}{\sqrt{\pi N_0}}\right)^{2M-1}\left(\frac{1}{\sqrt{\pi a}}\right)$$

$$\cdot\exp\Bigg[\frac{\left|r_{2,q,t}^{1}-\alpha_{2,q}\hat{s}_{2,t}^{1}\right|^2}{N_0}-\frac{\left|r_{2,q,t}^{1}-\alpha_{2,q}\hat{s}_{2,t}^{1}\right|^2}{a}\Bigg]+\cdots+\frac{\phi_3}{M}\left(\frac{1}{\sqrt{\pi N_0}}\right)^{2M-1}\left(\frac{1}{\sqrt{\pi a}}\right)$$

$$\cdot\exp\Bigg[\frac{\left|r_{2,q,t}^{M}-\alpha_{2,q}\hat{s}_{2,t}^{M}\right|^2}{N_0}-\frac{\left|r_{2,q,t}^{M}-\alpha_{2,q}\hat{s}_{2,t}^{M}\right|^2}{a}\Bigg]+\phi_4\left(\frac{1}{\sqrt{\pi N_0}}\right)^{2M}\Bigg\}\Bigg\}$$

$$=\prod_{t=1}^{L}\prod_{q=1}^{m}\Bigg\{\exp\Bigg[-\sum_{i=1}^{2}\sum_{k=1}^{M}\frac{\left|r_{1,q,t}^{k}-\alpha_{1,q}\hat{s}_{1,t}^{k}\right|^2}{N_0}\Bigg]\Bigg\{\sum_{k=1}^{M}\sum_{k'=1}^{M}\frac{\phi_1}{M^2}\left(\frac{1}{\sqrt{\pi N_0}}\right)^{2M-2}\left(\frac{1}{\sqrt{\pi a}}\right)^2$$

$$\cdot\exp\Bigg[\frac{\left|r_{1,q,t}^{k}-\alpha_{1,q}\hat{s}_{1,t}^{k}\right|^2}{N_0}-\frac{\left|r_{1,q,t}^{k}-\alpha_{1,q}\hat{s}_{1,t}^{k}\right|^2}{a}+\frac{\left|r_{2,q,t}^{k'}-\alpha_{2,q}\hat{s}_{2,t}^{k'}\right|^2}{N_0}-\frac{\left|r_{2,q,t}^{k'}-\alpha_{2,q}\hat{s}_{2,t}^{k'}\right|^2}{a}\Bigg]$$

$$
\begin{aligned}
&+\sum_{k=1}^{M}\frac{\phi_2}{M}\left(\frac{1}{\sqrt{\pi N_0}}\right)^{2M-1}\left(\frac{1}{\sqrt{\pi a}}\right)\exp\left[\frac{\left|r_{1,q,t}^{k}-\alpha_{1,q}\hat{s}_{1,t}^{k}\right|^2}{N_0}-\frac{\left|r_{1,q,t}^{k}-\alpha_{1,q}\hat{s}_{1,t}^{k}\right|^2}{a}\right]\\
&+\sum_{k=1}^{M}\frac{\phi_3}{M}\left(\frac{1}{\sqrt{\pi N_0}}\right)^{2M-1}\left(\frac{1}{\sqrt{\pi a}}\right)\exp\left[\frac{\left|r_{2,q,t}^{k}-\alpha_{2,q}\hat{s}_{2,t}^{k}\right|^2}{N_0}-\frac{\left|r_{2,q,t}^{k}-\alpha_{2,q}\hat{s}_{2,t}^{k}\right|^2}{a}\right]\\
&+\phi_4\left(\frac{1}{\sqrt{\pi N_0}}\right)^{2M}\Bigg\}\Bigg\}\\
=&\prod_{t=1}^{L}\prod_{q=1}^{m}\left\{\exp\left[-\sum_{i=1}^{n}\sum_{k=1}^{M}\frac{\left|r_{i,q,t}^{k}-\alpha_{i,q}\hat{s}_{i,t}^{k}\right|^2}{N_0}\right]\left\{\frac{\phi_1}{M^2}\left(\frac{1}{\sqrt{\pi N_0}}\right)^{2M-2}\left(\frac{1}{\sqrt{\pi a}}\right)^{2}\right.\right.\\
&\cdot\sum_{k=1}^{M}\sum_{k'=1}^{M}\exp\left[\frac{a-N_0}{aN_0}\left(\left|r_{1,q,t}^{k}-\alpha_{1,q}\hat{s}_{1,t}^{k}\right|^2+\left|r_{2,q,t}^{k}-\alpha_{2,q}\hat{s}_{2,t}^{k}\right|^2\right)\right]+\frac{\phi_2}{M}\\
&\cdot\left(\frac{1}{\sqrt{\pi N_0}}\right)^{2M-1}\left(\frac{1}{\sqrt{\pi a}}\right)\sum_{k=1}^{M}\exp\left[\frac{a-N_0}{aN_0}\left(\left|r_{1,q,t}^{k}-\alpha_{1,q}\hat{s}_{1,t}^{k}\right|^2\right)\right]+\frac{\phi_3}{M}\\
&\cdot\left(\frac{1}{\sqrt{\pi N_0}}\right)^{2M-1}\left(\frac{1}{\sqrt{\pi a}}\right)\sum_{k=1}^{M}\exp\left[\frac{a-N_0}{aN_0}\left(\left|r_{2,q,t}^{k}-\alpha_{2,q}\hat{s}_{2,t}^{k}\right|^2\right)\right]+\phi_4\left(\frac{1}{\sqrt{\pi N_0}}\right)^{2M}\Bigg\}\Bigg\}
\end{aligned}
\tag{D.1}
$$

where $a = N_0 + 2\frac{JT_s}{Q}\sigma_{J,q}^2$.

Appendix E

Derivation of the ML Decoding of STC/OFHSS System with JSI but without CSI

The derivation in (4.45) is discussed in this appendix. (4.45) is derived by averaging (4.36) with respect to $\boldsymbol{\alpha}$. Then we got

$$
\begin{aligned}
& f\left\{\boldsymbol{r}|\hat{\boldsymbol{s}},\boldsymbol{x}\right\} \\
&= \int_{-\infty}^{\infty}\prod_{t=1}^{L}\prod_{k=1}^{M}\prod_{q=1}^{m}\prod_{i=1}^{n}\frac{1}{\sqrt{\pi\left(N_0+x_{i,t}^k 2^{\frac{JT_s}{Q}}\sigma_{J,q}^2\right)}}\exp\left(-\frac{\left|r_{i,q,t}^k-\alpha_{i,q}\hat{s}_{i,t}^k\right|^2}{\left(N_0+x_{i,t}^k 2^{\frac{JT_s}{Q}}\sigma_{J,q}^2\right)}\right) f\left(\boldsymbol{\alpha}\right)d\boldsymbol{\alpha} \\
&= \int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}\left\{\prod_{t=1}^{L}\prod_{k=1}^{M}\prod_{q=1}^{m}\prod_{i=1}^{n}\frac{1}{\sqrt{\pi a_{i,t}^k}}\right\}\exp\left\{-\sum_{t=1}^{L}\sum_{k=1}^{M}\sum_{q=1}^{m}\sum_{i=1}^{n}\frac{1}{a_{i,t}^k}\left[\left|r_{i,q,t}^k\right|^2\right.\right. \\
&\quad \left.\left.-2\mathrm{Re}\left(r_{i,q,t}^k\alpha_{R,i,q}\hat{s}_{i,t}^k\right)-2\mathrm{Im}\left(r_{i,q,t}^k\alpha_{I,i,q}\hat{s}_{i,t}^k\right)+\alpha_{R,i,q}^2\left|\hat{s}_{i,t}^k\right|^2+\alpha_{I,i,q}^2\left|\hat{s}_{i,t}^k\right|^2\right]\right\} \\
&\quad \frac{1}{\sqrt{2\pi}\sigma_{1,1}}\exp\left(-\frac{\alpha_{R,1,1}^2}{2\sigma_{1,1}^2}\right)\cdots\frac{1}{\sqrt{2\pi}\sigma_{n,m}}\exp\left(-\frac{\alpha_{R,n,m}^2}{2\sigma_{n,m}^2}\right)\frac{1}{\sqrt{2\pi}\sigma_{1,1}}\exp\left(-\frac{\alpha_{I,1,1}^2}{2\sigma_{1,1}^2}\right) \\
&\quad \cdots\frac{1}{\sqrt{2\pi}\sigma_{n,m}}\exp\left(-\frac{\alpha_{I,n,m}^2}{2\sigma_{n,m}^2}\right)d\alpha_{R,1,1}\cdots d\alpha_{R,n,m}d\alpha_{I,1,1}\cdots d\alpha_{I,n,m} \\
&= \left\{\prod_{t=1}^{L}\prod_{k=1}^{M}\prod_{q=1}^{m}\prod_{i=1}^{n}\frac{1}{\sqrt{\pi a_{i,t}^k}}\exp\left(-\frac{1}{a_{i,t}^k}\left|r_{i,q,t}^k\right|^2\right)\right\}\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}\exp\left\{\sum_{i=1}^{n}\sum_{q=1}^{m}\left[\sum_{t=1}^{L}\sum_{k=1}^{M}\right.\right. \\
&\quad \frac{1}{a_{i,t}^k}\left[2\mathrm{Re}\left(r_{i,q,t}^k\alpha_{R,i,q}\hat{s}_{i,t}^k\right)+2\mathrm{Im}\left(r_{i,q,t}^k\alpha_{I,i,q}\hat{s}_{i,t}^k\right)-\alpha_{R,i,q}^2\left|\hat{s}_{i,t}^k\right|^2-\alpha_{I,i,q}^2\left|\hat{s}_{i,t}^k\right|^2\right] \\
&\quad \left.\left.-\frac{\alpha_{R,i,q}^2}{2\sigma_{i,q}^2}-\frac{\alpha_{I,i,q}^2}{2\sigma_{i,q}^2}\right]\right\}\left(\prod_{i=1}^{n}\prod_{q=1}^{m}\frac{1}{2\pi\sigma_{i,q}}\right)d\alpha_{R,1,1}\cdots d\alpha_{I,n,m}
\end{aligned}
\tag{E.1}
$$

where $a_{i,t^k}=N_0+x_{i,t}^k 2^{\frac{JT_s}{Q}}\sigma_{J,q}^2$. The real part of the exponent could be represented by

$$
\begin{aligned}
&R\left(\alpha_{R,1,q},\alpha_{R,2,q},\cdots,\alpha_{R,n,q}\right)\\
&=\sum_{t=1}^{L}\sum_{k=1}^{M}\frac{2}{a_{i,t}^{k}}\mathrm{Re}\left(r_{i,q,t}^{k}\alpha_{R,i,q}\hat{s}_{i,t}^{k}\right)-\sum_{t=1}^{L}\sum_{k=1}^{M}\frac{1}{a_{i,t}^{k}}\alpha_{R,i,q}^{2}\left|\hat{s}_{i,t}^{k}\right|^{2}-\frac{\alpha_{R,i,q}^{2}}{2\sigma_{i,q}^{2}}\\
&=\alpha_{R,i,q}\sum_{t=1}^{L}\sum_{k=1}^{M}\frac{2}{a_{i,t}^{k}}\mathrm{Re}\left(r_{i,q,t}^{k}\hat{s}_{i,t}^{k}\right)-\alpha_{R,i,q}^{2}\left(\sum_{t=1}^{L}\sum_{k=1}^{M}\frac{1}{a_{i,t}^{k}}\left|\hat{s}_{i,t}^{k}\right|^{2}-\frac{1}{2\sigma_{i,q}^{2}}\right).
\end{aligned}
\tag{E.2}
$$

By equation (E.1) with $\sigma_{i,q}=1/2\ \forall i,q$. The real part of exponent can be averaged with respect to $\alpha_{R,i,q}$, then we got

$$
\begin{aligned}
&\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}\exp\left\{\sum_{i=1}^{n}\sum_{q=1}^{m}\left[\alpha_{R,i,q}\sum_{t=1}^{L}\sum_{k=1}^{M}\frac{2}{a_{i,t}^{k}}\mathrm{Re}\left(r_{i,q,t}^{k}\hat{s}_{i,t}^{k}\right)-\alpha_{R,i,q}^{2}\left(\sum_{t=1}^{L}\sum_{k=1}^{M}\frac{1}{a_{i,t}^{k}}\left|\hat{s}_{i,t}^{k}\right|^{2}\right.\right.\right.\\
&\left.\left.\left.-\frac{1}{2\sigma_{i,q}^{2}}\right)\right]\right\}d\alpha_{R,1,1}\cdots d\alpha_{R,n,m}\\
&=\sqrt{\frac{\pi^{nm}}{\prod_{i=1}^{n}\prod_{q=1}^{m}\left(\sum_{t=1}^{L}\sum_{k=1}^{M}\frac{1}{a_{i,t}^{k}}\left|\hat{s}_{i,t}^{k}\right|^{2}+1\right)}}\exp\left\{\sum_{i=1}^{n}\sum_{q=1}^{m}\frac{\left|\sum_{t=1}^{L}\sum_{k=1}^{M}\frac{2}{a_{i,t}^{k}}\mathrm{Re}\left(r_{i,q,t}^{k}\hat{s}_{i,t}^{k}\right)\right|^{2}}{4\left(\sum_{t=1}^{L}\sum_{k=1}^{M}\frac{1}{a_{i,t}^{k}}\left|\hat{s}_{i,t}^{k}\right|^{2}\right)}\right\}
\end{aligned}
\tag{E.3}
$$

The imaginary part of the exponent could also be represented by

$$
\begin{aligned}
&I\left(\alpha_{I,1,q},\alpha_{I,2,q},\cdots,\alpha_{I,n,q}\right)\\
&=\alpha_{I,i,q}\sum_{t=1}^{L}\sum_{k=1}^{M}\frac{2}{a_{i,t}^{k}}\mathrm{Im}\left(r_{i,q,t}^{k}\hat{s}_{i,t}^{k}\right)-\alpha_{I,i,q}^{2}\left(\sum_{t=1}^{L}\sum_{k=1}^{M}\frac{1}{a_{i,t}^{k}}\left|\hat{s}_{i,t}^{k}\right|^{2}-\frac{1}{2\sigma_{i,q}^{2}}\right).
\end{aligned}
\tag{E.4}
$$

Next, the imaginary part of exponent can be averaged with respect to $\alpha_{I,i,q}$.

$$
\begin{aligned}
&\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}\exp\left\{\sum_{i=1}^{n}\sum_{q=1}^{m}\left[\alpha_{I,i,q}\sum_{t=1}^{L}\sum_{k=1}^{M}\frac{2}{a_{i,t}^{k}}\mathrm{Im}\left(r_{i,q,t}^{k}\hat{s}_{i,t}^{k}\right)-\alpha_{I,i,q}^{2}\left(\sum_{t=1}^{L}\sum_{k=1}^{M}\frac{1}{a_{i,t}^{k}}\left|\hat{s}_{i,t}^{k}\right|^{2}\right.\right.\right.\\
&\left.\left.\left.-\frac{1}{2\sigma_{i,q}^{2}}\right)\right]\right\}d\alpha_{I,1,1}\cdots d\alpha_{I,n,m}\\
&=\sqrt{\frac{\pi^{nm}}{\prod_{i=1}^{n}\prod_{q=1}^{m}\left(\sum_{t=1}^{L}\sum_{k=1}^{M}\frac{1}{a_{i,t}^{k}}\left|\hat{s}_{i,t}^{k}\right|^{2}+1\right)}}\exp\left\{\sum_{i=1}^{n}\sum_{q=1}^{m}\frac{\left|\sum_{t=1}^{L}\sum_{k=1}^{M}\frac{2}{a_{i,t}^{k}}\mathrm{Im}\left(r_{i,q,t}^{k}\hat{s}_{i,t}^{k}\right)\right|^{2}}{4\left(\sum_{t=1}^{L}\sum_{k=1}^{M}\frac{1}{a_{i,t}^{k}}\left|\hat{s}_{i,t}^{k}\right|^{2}\right)}\right\}
\end{aligned}
\tag{E.5}
$$

Hance, the conditional likelihood function $f\left\{\boldsymbol{r}|\hat{\boldsymbol{s}},\boldsymbol{x}\right\}$ can be expressed as

$$
\begin{aligned}
&= f\left\{\boldsymbol{r}|\hat{\boldsymbol{s}},\boldsymbol{x}\right\} \\
&= \left\{\prod_{t=1}^{L}\prod_{k=1}^{M}\prod_{q=1}^{m}\prod_{i=1}^{n}\frac{1}{\sqrt{\pi a_{i,t}^{k}}}\exp\left(-\frac{1}{\left|r_{i,q,t}^{k}\right|^{2}}\right)\right\}\left\{\prod_{q=1}^{m}\prod_{i=1}^{n}\left(\sum_{t=1}^{L}\sum_{k=1}^{M}\frac{\left|\hat{s}_{i,t}^{k}\right|^{2}}{a_{i,t}^{k}}+1\right)\right\}^{-1} \\
&\quad \exp\left\{\sum_{q=1}^{m}\sum_{i=1}^{n}\frac{\left|\sum_{t=1}^{L}\sum_{k=1}^{M}\frac{2}{a_{i,t}^{k}}\mathrm{Re}\left(r_{i,q,t}^{k}\hat{s}_{i,t}^{k}\right)\right|^{2}+\left|\sum_{t=1}^{L}\sum_{k=1}^{M}\frac{2}{a_{i,t}^{k}}\mathrm{Im}\left(r_{i,q,t}^{k}\hat{s}_{i,t}^{k}\right)\right|^{2}}{4\left(\sum_{t=1}^{L}\sum_{k=1}^{M}\frac{\left|\hat{s}_{i,t}^{k}\right|^{2}}{a_{i,t}^{k}}+1\right)}\right\}. \qquad \text{(E.6)}
\end{aligned}
$$

Appendix F

Derivation of the Design Criteria of STC/OFHSS System

The derivation in (4.51) is discussed in this appendix. (4.51) is derived by averaging (4.48) with respect to $\boldsymbol{x}$ and $\boldsymbol{\alpha}$. Then (4.49) is derived first by averaging (4.48) with respect to $\boldsymbol{\alpha}$.

$$\begin{aligned}
&\Pr\{\boldsymbol{s} \to \hat{\boldsymbol{s}} | \boldsymbol{x}\} \\
&\leq \int_{-\infty}^{\infty} \frac{1}{2} \exp\left(- \frac{\sum_{t=1}^{L} \sum_{k=1}^{M} \sum_{q=1}^{m} \sum_{i=1}^{n} \left| \alpha_{i,q} \left(s_{i,t}^{k} - \hat{s}_{i,t}^{k} \right) \right|^2}{4 \left(N_0 + x_{i,t}^{k} \frac{J T_s}{Q} \right)} \right) f(\boldsymbol{\alpha})\, d\boldsymbol{\alpha} \\
&= \frac{1}{2} \prod_{q=1}^{m} \prod_{i=1}^{n} \int_{-\infty}^{\infty} \exp\left(- \sum_{t=1}^{L} \sum_{k=1}^{M} a_{i,t}^{k} \left| \left(s_{i,t}^{k} - \hat{s}_{i,t}^{k} \right) \right|^2 |\alpha_{i,q}|^2 \right) f(|\alpha_{i,q}|)\, d|\alpha_{i,q}|
\end{aligned} \tag{F.1}$$

where $a_{i,t^k} = \left(N_0 + x_{i,t}^{k} \frac{J T_s}{Q} \right)^{-1}$ and $|\alpha_{i,q}|$ is a Rayleigh distribution random variable. The probability density function of $|\alpha_{i,q}|$ is

$$p(|\alpha_{i,q}|) = 2 |\alpha_{i,q}| \exp\left(- |\alpha_{i,q}|^2 \right). \tag{F.2}$$

Then the conditional pairwise error probability can be derived as

$$\begin{aligned}
&\Pr\{\boldsymbol{s} \to \hat{\boldsymbol{s}} | \boldsymbol{x}\} \\
&\leq \frac{1}{2} \prod_{q=1}^{m} \prod_{i=1}^{n} \left(\sum_{t=1}^{L} \sum_{k=1}^{M} a_{i,t}^{k} \left| \left(s_{i,t}^{k} - \hat{s}_{i,t}^{k} \right) \right|^2 + 1 \right)^{-1} \\
&= \frac{1}{2} \prod_{q=1}^{m} \prod_{i=1}^{n} \left[\sum_{j=0}^{\infty} \left(- \sum_{t=1}^{L} \sum_{k=1}^{M} a_{i,t}^{k} \left| \left(s_{i,t}^{k} - \hat{s}_{i,t}^{k} \right) \right|^2 \right)^{j} \right]
\end{aligned}$$

$$\cong \frac{1}{2}\prod_{q=1}^{m}\prod_{i=1}^{n}\left(1-\sum_{t=1}^{L}\sum_{k=1}^{M}a_{i,t}^{k}\left|\left(s_{i,t}^{k}-\hat{s}_{i,t}^{k}\right)\right|^{2}\right). \tag{F.3}$$

Assume there are only two transmitter antennas, the probability density of $\boldsymbol{x}_t = \left(x_{i,t}^{k} | \forall i, k\right)$ is presented in (4.39). Then we have the pairwise error probability by averaging (F.3) with respect to $\boldsymbol{x}$.

$$\begin{aligned}
&\Pr\{\boldsymbol{s}\to\hat{\boldsymbol{s}}\}\\
&\le E_{\boldsymbol{x}}\left[\frac{1}{2}\prod_{q=1}^{m}\prod_{i=1}^{2}\left(1-\sum_{t=1}^{L}\sum_{k=1}^{M}a_{i,t}^{k}\left|\left(s_{i,t}^{k}-\hat{s}_{i,t}^{k}\right)\right|^{2}\right)\right]\\
&=\frac{1}{2}\prod_{q=1}^{m}E_{\boldsymbol{x}_t}\left[\prod_{i=1}^{2}\left(1-\sum_{t=1}^{L}\sum_{k=1}^{M}a_{i,t}^{k}\left|\left(s_{i,t}^{k}-\hat{s}_{i,t}^{k}\right)\right|^{2}\right)\right]\\
&=\frac{1}{2}\prod_{q=1}^{m}E_{\boldsymbol{x}_t}\left\{1-\sum_{t=1}^{L}\sum_{k=1}^{M}a_{1,t}^{k}\left|\left(s_{1,t}^{k}-\hat{s}_{1,t}^{k}\right)\right|^{2}-\sum_{t=1}^{L}\sum_{k=1}^{M}a_{2,t}^{k}\left|\left(s_{2,t}^{k}-\hat{s}_{2,t}^{k}\right)\right|^{2}\right.\\
&\quad\left.+\sum_{t=1}^{L}\sum_{k=1}^{M}a_{1,t}^{k}\left|\left(s_{1,t}^{k}-\hat{s}_{1,t}^{k}\right)\right|^{2}\sum_{t'=1}^{L}\sum_{k'=1}^{M}a_{2,t'}^{k'}\left|\left(s_{2,t'}^{k'}-\hat{s}_{2,t'}^{k'}\right)\right|^{2}\right\}\\
&=\frac{1}{2}\prod_{q=1}^{m}\left\{1-\sum_{t=1}^{L}\sum_{k=1}^{M}E_{\boldsymbol{x}_t}\left[a_{1,t}^{k}\right]\left|\left(s_{1,t}^{k}-\hat{s}_{1,t}^{k}\right)\right|^{2}-\sum_{t=1}^{L}\sum_{k=1}^{M}E_{\boldsymbol{x}_t}\left[a_{2,t}^{k}\right]\left|\left(s_{2,t}^{k}-\hat{s}_{2,t}^{k}\right)\right|^{2}\right.\\
&\quad\left.+\sum_{t=1}^{L}\sum_{k=1}^{M}\sum_{t'=1}^{L}\sum_{k'=1}^{M}E_{\boldsymbol{x}_t}\left[a_{1,t}^{k}a_{2,t'}^{k'}\right]\left|\left(s_{1,t}^{k}-\hat{s}_{1,t}^{k}\right)\right|^{2}\left|\left(s_{2,t'}^{k'}-\hat{s}_{2,t'}^{k'}\right)\right|^{2}\right\}\\
&=\frac{1}{2}\prod_{q=1}^{m}\left\{1-\sum_{t=1}^{L}\sum_{k=1}^{M}\left[\left(1-\frac{Q}{N_t}\right)\frac{1}{N_0}\left|s_{1,t}^{k}-\hat{s}_{1,t}^{k}\right|^{2}+\frac{Q}{N_t}\frac{1}{a}\left|s_{1,t}^{k}-\hat{s}_{1,t}^{k}\right|^{2}\right]\right.\\
&\quad-\sum_{t=1}^{L}\sum_{k=1}^{M}\left[\left(1-\frac{Q}{N_t}\right)\frac{1}{N_0}\left|s_{2,t}^{k}-\hat{s}_{2,t}^{k}\right|^{2}+\frac{Q}{N_t}\frac{1}{a}\left|s_{2,t}^{k}-\hat{s}_{2,t}^{k}\right|^{2}\right]\\
&\quad\left.+\tau\sum_{t=1}^{L}\sum_{k=1}^{M}\sum_{t'=1}^{L}\sum_{k'=1}^{M}\left|s_{1,t}^{k}-\hat{s}_{1,t}^{k}\right|^{2}\left|s_{2,t'}^{k'}-\hat{s}_{2,t'}^{k'}\right|^{2}\right\}
\end{aligned} \tag{F.4}$$

where

$$\begin{aligned}
&E_{\boldsymbol{x}_t}\left[a_{1,t}^{k}\right]=E_{\boldsymbol{x}_t}\left[a_{2,t}^{k}\right]\\
&=\left(1-\frac{Q}{N_t}\right)\frac{1}{N_0}+\frac{Q}{N_t}\frac{1}{a}
\end{aligned}$$

$$
\begin{aligned}
&E_{\boldsymbol{x}_t}\left[a_{1,t}^{k}a_{2,t'}^{k'}\right] \\
&= p\left(a_{1,t}^{k}=\frac{1}{a}, a_{2,t}^{k}=\frac{1}{a}\right)\left(\frac{1}{a}\right)^2 + p\left(a_{1,t}^{k}=\frac{1}{N_0}, a_{2,t}^{k}=\frac{1}{a}\right)\left(\frac{1}{aN_0}\right) \\
&\quad + p\left(a_{1,t}^{k}=\frac{1}{a}, a_{2,t}^{k}=\frac{1}{N_0}\right)\left(\frac{1}{aN_0}\right) + p\left(a_{1,t}^{k}=\frac{1}{N_0}, a_{2,t}^{k}=\frac{1}{N_0}\right)\left(\frac{1}{N_0}\right)^2 \\
&= \left(\frac{Q}{N_t-M}\cdot\frac{Q}{N_t}\right)\left(\frac{1}{a}\right)^2 + \left(1-\frac{Q}{N_t-M}\right)\frac{Q}{N_t}\left(\frac{1}{aN_0}\right) \\
&\quad + \left[\frac{Q-1}{N_t-M}\frac{Q}{N_t}(M-1) + \frac{Q}{N_t-M}\left(1-\frac{MQ}{N_t}\right)\right]\frac{1}{aN_0} \\
&\quad + \left[\left(1-\frac{Q-1}{N_t-M}\right)\frac{Q}{N_t}(M-1) + \left(1-\frac{Q}{N_t-M}\right)\left(1-\frac{MQ}{N_t}\right)\right]\left(\frac{1}{a}\right)^2 \\
&= \tau
\end{aligned}
$$

and $a = N_0 + \frac{JT_s}{Q}$.

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