國　立　交　通　大　學

電信工程學系碩士班

碩士論文

放大傳遞之合作式系統在服務品質限制下聯合前置編碼器設計

Joint Source/Relay Precoders Design with Quality-of-Service (QoS) Constraints in Amplify-and-Forward Cooperative Systems

研　究　生：柯國仁　　　　Student：Kur-Jen Ken

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中文摘要

在放大傳遞協定多輸入多輸出中繼傳輸系統中，傳統前置編碼器設計通常是在總功率限制下討論如何去改善連線品質。在本論文中，我們從不同觀點來看此問題，我們考慮在服務頻質限制下如何讓總傳輸功率最小。因多輸入多輸出中繼傳輸系統包含兩條路徑（中繼路徑和直線路徑）和兩個前置編碼器（發送端前置編碼器和中繼端前置編碼器），要直接導出此問題的最佳解有很大的困難。藉在目的端使用最小方均差接收器，我們首先將此設計的問題轉化成有限制條件的最佳化問題。不過我們發現這最佳化問題的成本函數是前置編碼器的非線性函數直接求取這問題的解仍然困難。我們因此提議一新設計方法來解決這問題。主要想法是使用一最小均方差上限當做成本函數以及使用一限制型的前置編碼器結構。在我們的方法下，這前置編碼器設計問題可以被轉變成一個功率分配的問題，因而能顯著地簡化最佳解的推導。模擬解果顯示相較於傳統非合作式多輸入多輸出傳輸系統我們所提出的方法可以大幅的減少傳輸功率。

Joint Source/Relay Precoders Design with

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Amplify-and-Forward Cooperative Systems

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Abstract

Conventional precoder designs for amplify-and-forward (AF) multiple-input multiple-output (MIMO) relay systems often consider how to improve the link quality under a total power constraint. In this thesis, we consider the design problem from a different perspective by minimizing the total transmission power under a quality of service (QoS) constraint. The problem is difficult since the MIMO relay system involves two links, the relay and direct links, and two precoders, the source and relay precoders. Using the minimum mean-square-error (MMSE) receiver at the destination, we first formulate the design problem as a constrained optimization problem. It is found, however, that the cost function is a highly nonlinear function of the precoders, and it is not feasible to solve the problem directly. We then propose a new design method to remedy the problem. The main idea is to replace the MSE with an upper bound, and apply a constrained structure for the precoders. Using our approach, the precoders design problems can be translated into a power allocation problem, significantly simplify the solution derivation. Simulations show that the proposed methods can dramatically reduce the required transmission power compared to the conventional non-cooperative MIMO systems.

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最後，謹以此文獻給我摯愛的雙親和家人。

Catalog

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Abbreviations and Acronyms

AF	amplify and forward
AWGN	the additive white Gaussian noise
BER	bit error rate
BS	base station
BW	bandwidth
CSI	channel statement information
DF	decode and forward
DFT	discrete Fourier transform
i.i.d.	identical independent distribution
KKT	Karush–Kuhn–Tucker
MIMO	multiple-input and multiple-output
MMSE	minimum mean square error
MRC	maximum ratio combining
MSE	mean square error
SINR	signal to interference plus noise ratio
SISO	single input and single output
SNR	signal to noise ratio
SVD	singular value decomposition
QoS	quality of service

Chapter 1: Introduction

Spatial diversity techniques can effectively mitigate the performance deterioration caused by channel fading, without imposing delays or bandwidth expansion. Spatial diversity can be obtained with multiple transmit/receive antennas. With signals transmitted/received from antennas separated far enough, parallel multiple channels are then generated. Multiple-input multiple-output (MIMO) systems, equipped multiple antennas both in the transmitter and receiver, can further introduce higher degree of freedom allowing the operation of spatial multiplexing to increase the data rate. Recently, user cooperation has been proposed as a mean for further performance enhancement [1]-[3]. With the aid of relay nodes, the system can build a MIMO system which can give higher resistance to fading and shadowing, lower outage probability, higher capacity, less power consumption, less interference power, and more flexible use of the link resource. Cooperative communication has been developed as the key technique for next wireless communication.

In a general cooperative system, each node cannot transmit and receive signal simultaneously, i.e., it is operated in a half-duplex mode. Most systems use a two-phase transmission protocol. Consider a typical three-node system. In the first-phase, the source node broadcasts signal to the destination and the relay node. In the second phase, the relay is forwards processed signal to the destination. The destination then combines the signals received from both nodes to make an estimation of the transmitted data. There are two main cooperative protocols, amplify-and-forward (AF) and decode-and-forward (DF). In AF, the relay only amplifies and retransmits the received signal to the destination. In DF, the relay decodes the received signal, re-encode the detected data, and retransmit the re-encoded signal to the destination. If detection errors occur, DF will degrade the system performance. Also, AF relays usually require a smaller processing delay and has a lower construction cost. Thus, there is also another type of the DF protocol. The relay node will first check if the inter-outage occurs. Outage indicates that the transmit data rate is higher than the source-to-relay channel capacity. Outage also means that the probability that the relay decodes the data correctly cannot be controlled. If the inter-outage occurs, the relay will not forward signal to destination. Other types of the protocol include compress-and-forward (CF) and decode-amplify-forward (DAF) are also discussed in the literature [22]-[23]. In CF, the relay compresses received signal and then forwards the compressed signal to the destination. In DAF, the relay decodes the received signal softly (instead of hard decisions), i.e., calculate log-likelihood ratios (LLRs) of information bits, and then amplifies (scales) and forwards the LLRs to the destination.

In this thesis, we will focus on a three-node MIMO cooperative systems where the relay node employs the AF relaying protocol. The source, the relay and the destination all have multiple antennas. As mentioned, MIMO channels can provide a significant increase in capacity over single-input single-output (SISO) channels [4], [5]. With the MIMO structure at each node, the performance of a cooperative system can be further enhanced. This cooperative structure is referred to as the MIMO relay system. It is well know that precoding can effectively improve the performance of a MIMO system. This is also true for a MIMO relay system. In conventional precoding design, the objective is to maximize the communication quality subject to a transmitter power constraint. In this thesis, however, we will consider a different criterion. We will minimize the required transmission power under a quality of service (QoS) constraint. Since most commercial systems usually provide services with a minimum QoS constraint (e.g., bit error rate), this criterion will maximize the power efficiency of the whole system. Minimizing the transmission power under QoS constraints was considered for conventional MIMO systems [17]. However, it has not been considered for

MIMO relay systems yet.

Based on the individual QoS constraint for each substream, we propose new precoding designs which minimize total transmit power. For single-input-single-output (SISO) systems, we use the maximum ratio combining (MRC) receiver while for MIMO systems, we use the linear minimum mean-squared-error (MMSE) receiver. Unlike conventional designs, we take both the source and the relay precoders into consideration simultaneously. Due to this joint design, the proposed method can effectively improve the performance of MIMO relay systems.

The MIMO relay channel can be viewed as a special type of MIMO channel. As known, a MIMO channel can be decomposed into multiple parallel subchannels. In the thesis, we consider the scenario that the target signal-to-interference-plus-noise ratios (SINRs) for the subchannels are different. This scenario is good for the transmission of different types of data that requires different rates or different SINRs. We also assume that the source, the relay and the destination all have required channel state information (CSI). Using the minimum power criterion and the MSE constraints, we can then formulate a constrained optimization problem. It turns out that the problem is the cost function is a complicated function of precoder matrices. Also, the problem is non-convex and there are many parameters to be optimized (two precoding matrices). As a result, a direct solution for the problem is very difficult to obtain. We propose new design methods to solve the problem. The main idea is to replace the MSE with an upper bound and apply a constrained structure for the precoders. Using our approach, the closed-form solutions of optimum/suboptimum precoders can be obtained by the technique of primal decomposition [20].

The thesis is organized as follows. In Chapter 2, we brief review the basic ideas of the convex optimization method, and the approach in [17]. In Chapter 3, we first consider a SISO relay system, and derive the optimum power loading algorithm. In Chapter 4, we consider the MIMO relay systems and describe the proposed methods in details. In Chapter 5, we report some simulation results demonstrating the effectiveness of the proposed algorithms. Finally, we draw some conclusions and outline some possible future works in Chapter 6.

Chapter 2: Preliminary

The method of convex optimization has been shown to be a useful tool in communications and signal processing. Many problems can either be cast as or be converted into convex optimization problems, which greatly facilitate their analytic and numerical solutions. Convex optimization minimizes an objective function subject to convex constraints. One distinct advantage of the convex optimization problem is that a local optimum is also a global optimum. Since we use the technique throughout this thesis, we give a brief introduction in this chapter.

2.1 Karush-Kuhn-Tucker Condition and Convex Optimization

In order to recognize convex optimization problems in engineering applications, one must first be familiar with the basic concepts of convexity. In the following, we give an overview of convexity, the Lagrange method, and the Karush–Kuhn–Tucker optimality conditions.

2.1.1 Convex Optimization

(A) Convex sets: a set $S \in \Re^n$ is said to be convex if for any two points **x**, $\mathbf{y} \in S$, the line segment joining **x** and **y** also lies in *S*. Mathematically, it is defined by the following property:

$$\delta \mathbf{x} + (1-\delta)\mathbf{y} \in S, \delta \in [0,1], \mathbf{x} \text{ and } \mathbf{y} \in S. \tag{2-1}$$

In general, a convex set must be a solid body, containing no holes, and always curve outward. A simple example for $S \in \Re^2$ is given in Figure 2-1.

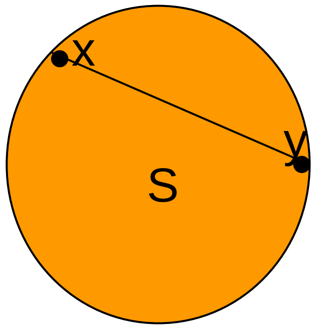

Figure 2-1: A simple example of convex set

(B) Convex functions: a function $f(\mathbf{x}) \in \Re^n \to \Re$ is said to be convex if for any two points $\mathbf{x}$ and $\mathbf{y} \in S$,

$$f(\delta \mathbf{x} + (1-\delta)\mathbf{y}) \le \delta f(\mathbf{x}) + (1-\delta) f(\mathbf{y}), \forall \delta \in [0,1]. \tag{2-2}$$

Geometrically, this means that, when restricted over the line segment joining $\mathbf{x}$ and $\mathbf{y}$, the linear function joining $(\mathbf{x}, f(\mathbf{x}))$ and $(\mathbf{y}, f(\mathbf{y}))$ always dominates the function *f*. A simple example for $S \in \Re$ is given in Figure 2-2. The most important property about convex functions is the fact that they are closed under summation, positive scaling, and point-wise maximum operations. That is if $\{f_i\}_{i=1}^{k}$ are convex functions, then $\sum_{i=1}^{k} f_i(\mathbf{x})$, $\{|f_i|\}_{i=1}^{k}$ and $\max_i f_i(\mathbf{x})$ are also convex functions.

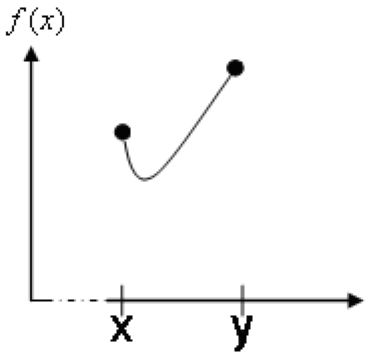

Figure 2-2: A simple example of convex function

(C) Convex optimization problems: consider a generic optimization problem (in the minimization form)

$$\begin{aligned} &\text{minimize} \quad f(\mathbf{x}) \\ &\text{such that } h_i(\mathbf{x}) \le 0, i = 1, 2, \ldots, m, \\ &\qquad\qquad g_j(\mathbf{x}) = 0, \ j = 1, 2, \ldots, k, \\ &\qquad\qquad \mathbf{x} \in S. \end{aligned} \tag{2-3}$$

where is *f* called the objective function (or cost function), $\{h_i\}_{i=1}^m$ and $\{g_j\}_{j=1}^k$ are called the inequality and equality constraint functions, respectively, and *S* is called a constraint set. The optimization variable $\mathbf{x} \in \Re^n$ is said to be feasible if $\mathbf{x} \in S$ and it satisfies all the inequality and equality constraints. A feasible solution $\mathbf{x}_1$ is said to be globally optimal if $f(\mathbf{x}_1) \le f(\mathbf{x})$ for all feasible **x**. In contrast, a feasible vector $\mathbf{x}_2$ is said to be locally optimal if there exists some $\varepsilon > 0$ such that $f(\mathbf{x}_2) \le f(\mathbf{x})$ for all feasible **x** satisfying $\|\mathbf{x}_2 - \mathbf{x}\| \le \varepsilon$. The optimization problem is said to be convex if 1) the functions $\{h_i\}_{i=1}^m$ are convex, 2) $\{g_j\}_{j=1}^k$ are affine functions (i.e., having the form of $\mathbf{a}^T\mathbf{x} + b$ for some $\mathbf{a} \in \Re^n$ and $b \in \Re$), and 3) the set *S* is convex.

2.1.2 Lagrange Duality and Karush-Kuhn-Tucker Condition

Consider (2-3) (not necessarily convex) optimization problem, and let f_{min} denote the global minimum value of *f*(**x**). For the symmetry reason, we will call (2-3) the primal optimization problem, and call **x** the primal variable. Introducing the dual variables $\boldsymbol{\lambda} = [\lambda_{1,} \lambda_2, \cdots, \lambda_m]^T \in \Re^m$ and $\mathbf{v} = [v_1, v_2, \cdots, v_k]^T \in \Re^k$, we can form the Lagrange function as

$$L(\mathbf{x}, \boldsymbol{\lambda}, \mathbf{v}) = f(\mathbf{x}) + \sum_{i=1}^{m} \lambda_i h_i(\mathbf{x}) + \sum_{j=1}^{k} v_j g_j(\mathbf{x}). \tag{2-4}$$

The so-called dual objective function $d(\boldsymbol{\lambda}, \mathbf{v})$ associated with (2-3) is defined as

$$d(\boldsymbol{\lambda}, \mathbf{v}) := \min_{\mathbf{x} \in S} L(\mathbf{x}, \boldsymbol{\lambda}, \mathbf{v}). \tag{2-5}$$

Consider the optimal optimization problem shown below:

$$\begin{aligned} &\text{maximize} \quad d(\boldsymbol{\lambda}, \mathbf{v}) \\ &\text{subject to} \quad \boldsymbol{\lambda} \ge \mathbf{0}, \mathbf{v} \in \Re^{\mathrm{k}}. \end{aligned} \tag{2-6}$$

We will say $(\boldsymbol{\lambda}, \mathbf{v})$ is dual feasible if $\boldsymbol{\lambda} \geq \mathbf{0}$ and the value of $d(\boldsymbol{\lambda}, \mathbf{v})$ is finite. Since $\boldsymbol{\lambda} \geq 0$, $h_i \leq 0$, $g_j = 0$, $d(\boldsymbol{\lambda}, \mathbf{v})$ is a minimum of the linear functions of any primal feasible vector **x,** and any dual feasible vector $(\boldsymbol{\lambda}, \mathbf{v})$, the following relationship between the prime and dual cost functions holds:

$$f(\mathbf{x}) \geq d(\boldsymbol{\lambda}, \mathbf{v}). \tag{2-7}$$

This is the well-known weak duality property [6]. In other words, the dual function value $d(\boldsymbol{\lambda}, \mathbf{v})$ always serves as a lower bound for the primal objective value $f(\mathbf{x})$ for any dual feasible vector $(\boldsymbol{\lambda}, \mathbf{v})$. Notice that $\mathbf{x}$ and $(\boldsymbol{\lambda}, \mathbf{v})$ are chosen independent (so long as they are both feasible). Thus f_{min} is larger than $d(\boldsymbol{\lambda}, \mathbf{v})$ for all dual feasible vectors. The largest lower bound for f_{min} can be found by solving the dual optimization problem shown in (2-6).

When the optimal problem is convex, standard convex optimization results guarantee that the primal problem and the dual problem have the same solution. Note that the lower bound is not always tight, and the difference is called the "duality gap". From [7], we see that if an optimization problem of the form (2-3) satisfies the time-sharing property, it has zero duality gap, i.e. the primal problem and the dual problem have the same solution. The time-sharing property is defined as follow: let $\mathbf{x}_{op1}$ and $\mathbf{x}_{op2}$ be optimal solutions to the problem, then for any $0 \leq \varepsilon \leq 1$ there exists a vector $\mathbf{z}$ such that $h(\mathbf{z}) \leq 0$ and $f(\mathbf{z}) \leq \varepsilon f(\mathbf{x}_{opt1}) + (1-\varepsilon) f(\mathbf{x}_{opt2})$.

Let us denote the maximum value of (2-7) by d_{max}. Then, we have $f_{min} \geq d_{max}$. Interestingly, for most convex optimization problems (satisfying some mild constraint qualification conditions, such as the existence of a strict interior point), we actually have $f_{min} = d_{max}$. This is called strong duality.

In general, the dual function is difficult to compute. However, for some special classes of convex optimization problems, we can derive their duals explicitly by the following conditions. For ease of exposition, let us assume $S \in \Re$. Then, a necessary condition for $\mathbf{x}_2$ to be a local optimal solution of (2-3) is that there exists some $(\boldsymbol{\lambda}^*, \mathbf{v}^*)$ such that

$$h_i(\mathbf{x}_2) \le 0, \ \ \forall \text{i=1,2,...,m}, \tag{2-8}$$

$$g_j(\mathbf{x}_2) = 0, \ \forall j\text{=1,2,...,k}, \tag{2-9}$$

$$\boldsymbol{\lambda}^* \ge 0, \tag{2-10}$$

$$\lambda_i^* h_i(\mathbf{x}_2) = 0, \ \forall \text{i=1,2,...,m}, \tag{2-11}$$

$$\nabla f(\mathbf{x}_2) + \sum_{i=1}^{m} \lambda_i^* \nabla h_i(\mathbf{x}_2) + \sum_{j=1}^{k} v_j^* \nabla g_j(\mathbf{x}_2) = 0. \tag{2-12}$$

Collectively, the conditions (2-8)–(2-12) are called the *Karush–Kuhn–Tucker* (KKT) condition for optimality. Notice that the first two conditions (2-8) and (2-9) represent primal feasibility of $\mathbf{x}_2$, condition (2-10) represents dual feasibility, condition (2-11) signifies the complementary slackness for the primal and dual inequality constraint pairs: $h_i(\mathbf{x}) \le 0$ and $\lambda_i \le 0$, while the last condition (2-12) is equivalent to $\nabla_{\mathbf{x}} L(\mathbf{x}_2, \boldsymbol{\lambda}^*, \mathbf{v}^*) = 0$. In general, the KKT condition is necessary but not sufficient for optimality. However, for convex optimization problems (and under mild constraint qualification conditions), the KKT condition is also sufficient.

2.2 Existing method

The work [17] discusses the precoder design in MIMO systems with a set of QoS constraints. With the aid of majorization theory, the original complicated nonconvex problem with matrix-valued variables was reformulated as a simple convex optimization problem with scalar variables. Then the problem is optimally solved with a multilevel water-filling algorithm. We now describe the approach in the subsection. The block diagram of a MIMO system is shown below

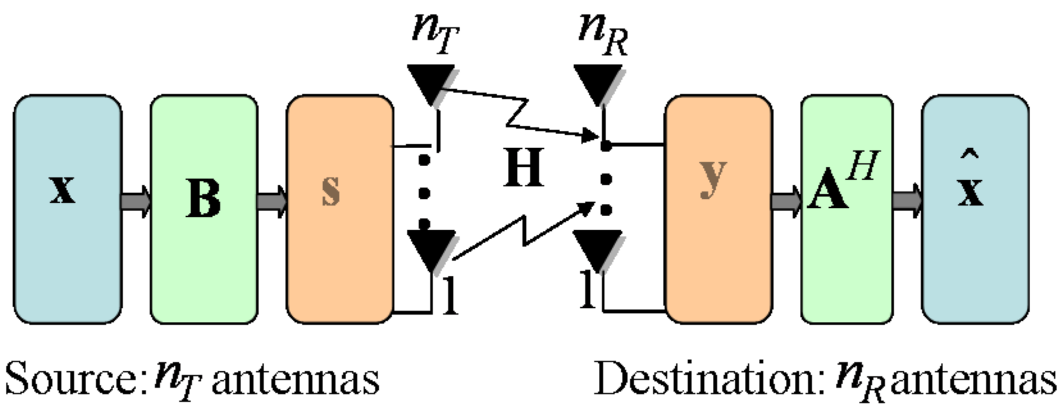

Figure 2-3: Block diagram of the MIMO system in [17]

Considering the MIMO system with n_t transmitting and n_r receiving antenna, we can write the sampled baseband received signal as

$$\mathbf{y} = \mathbf{H}\mathbf{s} + \mathbf{n}, \tag{2-13}$$

where $\mathbf{s} \in \mathbb{C}^{N_t \times 1}$ is the transmit signal vector, and $\mathbf{y} \in \mathbb{C}^{N_r \times 1}$ is the received signal vector, $\mathbf{H} \in \mathbb{C}^{N_r \times N_t}$ is the channel matrix with the (i,j) element denoting the fading coefficient between the jth transmit and ith receive antennas, and $\mathbf{n} \in \mathbb{C}^{N_r \times 1}$ is a zero-mean circularly symmetric complex Gaussian interference-plus-noise vector with a covariance matrix of $\mathbf{R}_n$, i.e. $\mathbf{n} \sim CN(\mathbf{0}, \mathbf{R}_n)$. If the system is precoded with a linear precoding scheme, the transmitted vector can then be written as

$$\mathbf{s}=\mathbf{B}\mathbf{x}, \tag{2-14}$$

where $\mathbf{B} \in \mathbb{C}^{N_t \times L}$ is the precoding matrix, and $\mathbf{x} \in \mathbb{C}^{L \times 1}$ is the symbol vector to be transmitted. Assuming that $E\left[\mathbf{x}\mathbf{x}^H\right] = \mathbf{I}_L$, we can have the average transmission power is

$$P_T = E\left[\|\mathbf{s}\|^2\right] = tr\left\{\mathbf{B}\mathbf{B}^H\right\}. \tag{2-15}$$

If a linear receiver is used in the destination, the estimated symbol vector will

$$\hat{\mathbf{x}} = \mathbf{A}^H \mathbf{y}, \tag{2-16}$$

where $\mathbf{A} \in \mathbb{C}^{L \times N_r}$ is a filtering matrix. Let the QoS constraints be defined in terms of MSE for each of the established links or bitstreams. We then have

$$0 \le \mathrm{MSE}_i = E\left[\left(\hat{x}_i - x_i\right)^2\right] \le \rho_i < 1, \tag{2-17}$$

where MSE_i denotes the MSE of ith bit stream and x_i is the ith element of $\mathbf{x}$. Define a MSE matrix as the covariance matrix of the error vector ($\hat{\mathbf{x}}$-$\mathbf{x}$). Then

$$\begin{aligned} \mathbf{E} &= E\left[(\hat{\mathbf{x}}-\mathbf{x})(\hat{\mathbf{x}}-\mathbf{x})^H\right] \\ &= (\mathbf{A}^H\mathbf{H}\mathbf{B}-\mathbf{I})(\mathbf{B}^H\mathbf{H}^H\mathbf{A}-\mathbf{I}) + \mathbf{A}^H\mathbf{R}_n\mathbf{A}. \end{aligned} \tag{2-18}$$

From the definition, we have $\mathrm{MSE}_i = [\mathbf{E}]_{ii}$. Since the MSE_i is a quadratic function of $\mathbf{a}_i$ (the ith column of $\mathbf{A}$), its minimum value can be found by setting the gradient of (2-18) to zero. The solution is referred to as the linear minimum MSE (LMMSE) filter or *Wiener* filter, i.e.

$$\mathbf{A} = \left(\mathbf{H}\mathbf{B}\mathbf{B}^H\mathbf{H}^H + \mathbf{R}_n\right)^{-1}\mathbf{H}\mathbf{B}. \tag{2-19}$$

By using the matrix inversion lemma and (2-19), the concentrated MSE matrix can be obtained as

$$\begin{aligned} \mathbf{E} &= \mathbf{I} - \mathbf{B}^H\mathbf{H}^H\left(\mathbf{H}\mathbf{B}\mathbf{B}^H\mathbf{H}^H + \mathbf{R}_n\right)^{-1}\mathbf{H}\mathbf{B} \\ &= \left(\mathbf{I} + \mathbf{B}^H\mathbf{R}_H\mathbf{B}\right)^{-1}. \end{aligned} \tag{2-20}$$

where $\mathbf{R}_H = \mathbf{H}^H\mathbf{R}_n^{-1}\mathbf{H}$. Let $\rho_{\mathrm{i}} \ge \rho_{\mathrm{i}+1}$, the optimization problem can now be written as

$$\begin{aligned} &\min_{\mathbf{B}} \ tr\left\{\mathbf{B}\mathbf{B}^H\right\} \\ &\text{s.t.} \quad \left[(\mathbf{I} + \mathbf{B}^H\mathbf{R}_H\mathbf{B})^{-1}\right]_{ii} \le \rho_{i,} \ 1 \le i \le L. \end{aligned} \tag{2-21}$$

From the majorization theory and the derivation in [17], (2-21) can be reformulated as the problem shown below.

$$
\begin{aligned}
&\min_{\{z_i\}} \sum_{i=1}^{\hat{L}} z_i \\
&\text{s.t.} \quad \sum_{i=k}^{\hat{L}} \frac{1}{1+z_i\lambda_{H,i}} \le \sum_{i=k+L_0}^{L} \rho_i \ ,1 \le k \le \hat{L}, \\
&\qquad \sum_{i=1}^{\hat{L}} \frac{1}{1+z_i\lambda_{H,i}} \le \sum_{i=1}^{L} \rho_i - L_0, \\
&\qquad z_k \ge 0, \ \ 1 \le k \le \hat{L},
\end{aligned}
\tag{2-22}
$$

where L is the number of established links, $\hat{L} = \min(L, rank(\mathbf{R}_H))$ is the number of effective channel eigenvalues used, $L_0 = L - \hat{L}$ is the number of links associated to zero eigenvalues, and the set $\{\lambda_{H,i}\}_{i=1}^{\hat{L}}$ contains $\hat{L}$ largest eigenvalues of $\mathbf{R}_H$ in increasing order. And the optimal solution to (2-21) satisfies all QoS constraints with equality. Also, $\mathbf{B}$ is given by $\mathbf{B} = \mathbf{U}_{H,1} \Sigma_{B,1} \mathbf{Q}$ where $\mathbf{U}_{H,1} \in \mathbb{C}^{N_t \times \hat{L}}$ has the eigenvectors of $\mathbf{R}_H$ corresponding to the largest $\hat{L}$ eigenvalues in increasing order as its column vectors, $\Sigma_{B,1} = [0 \ \mathrm{diag}(\{\sigma_{B,i}\})] \in \mathbb{C}^{\hat{L} \times L}$ has zero elements except for the rightmost main diagonal, which are given by $\sigma_{B,i}^2 = z_i$, and $\mathbf{Q}$ is a unitary matrix such that $\left[(\mathbf{I}+\mathbf{B}^H \mathbf{R}_H \mathbf{B})^{-1}\right]_{ii} = \rho_{i,} \ 1 \le k \le \hat{L}$. Equation (2-22) can be rewritten more compactly as

$$
\begin{aligned}
&\min_{\{z_i\}} \sum_{i=1}^{\hat{L}} z_i \\
&\text{s.t.} \quad \sum_{i=k}^{\hat{L}} \frac{1}{1+z_i\lambda_{H,i}} \le \sum_{i=k}^{\hat{L}} \tilde{\rho}_i \ ,1 \le k \le \hat{L}, \\
&\qquad z_k \ge 0, \ \ 1 \le k \le \hat{L},
\end{aligned}
\tag{2-23}
$$

where $\tilde{\rho}_i = \begin{cases} \sum_{k=1}^{L_0+1} \rho_k - L_0, \text{for } i=1 \\ \rho_i + L_0 \qquad \text{for } 1<i \le \hat{L} \end{cases}$.

Using KKT optimality conditions, we can find the optimal solution to (2-23) by the multilevel water-filling solution as

$$
z_i = (\tilde{u}_i^{-1/2} \lambda_i^{-1/2} - \lambda_i^{-1})^+ , \tag{2-24}
$$

where the multiple water-levels $\tilde{u}_i^{1/2}$'s are chosen to satisfy

$$\sum_{i=k}^{\hat{L}} \frac{1}{1+z_i \lambda_{H,i}} \leq \sum_{i=k}^{\hat{L}} \tilde{\rho}_i \quad 1 \leq k \leq \hat{L}, \tag{2-25}$$

$$\sum_{i=1}^{\hat{L}} \frac{1}{1+z_i \lambda_{H,i}} = \sum_{i=1}^{\hat{L}} \tilde{\rho}_i \quad , \tag{2-26}$$

$$\tilde{u}_k \geq \tilde{u}_{k-1} (\tilde{u}_0 = 0), \tag{2-27}$$

$$\left(\tilde{u}_k - \tilde{u}_{k-1}\right) \left(\sum_{i=1}^{\hat{L}} \frac{1}{1+z_i \lambda_{H,i}} - \sum_{i=1}^{\hat{L}} \tilde{\rho}_i\right) = 0 . \tag{2-28}$$

We now give a multilevel waterfilling algorithm that solves the convex optimization problem as follows. The inputs of the algorithm are the number of available positive eigenvalues $\hat{L}$, the set of eigenvalues $\{\lambda_{H,i}\}_{i=1}^{\hat{L}}$, and the set of MSE constraints $\{\tilde{\rho}_i\}_{i=1}^{\hat{L}}$, while the outputs are a set of allocated powers $\{z_i\}_{i=1}^{\hat{L}}$, and a set of waterlevels $\{\tilde{u}_i^{1/2}\}_{i=1}^{\hat{L}}$.

The main algorithm:

0) Set $k_0 = 1$ and $\tilde{L} = \hat{L}$.

1) Solve the QoS constrained problem in $[k_0, \tilde{L}]$ using the waterfilling algorithm shown below with the set of $\tilde{L} - k_0 + 1$ eigenvalues $\{\lambda_{H,i}\}_{i=k_0}^{\tilde{L}}$ and the MSE constraint given by $\tilde{\rho} = \sum_{i=k_0}^{\tilde{L}} \tilde{\rho}_i$.

2) If any intermediate constraint is not satisfied, ($\sum_{i=k}^{\tilde{L}} \left(1+z_i \lambda_{H,i}\right)^{-1} \leq \sum_{i=k}^{\tilde{L}} \tilde{\rho}_i$, $k_0 \leq k \leq \tilde{L}$), then set k_0 equal to the smallest index (whose constraint is not satisfied) and go to Step 1. Otherwise, if $k_0 = 1$, the algorithm stops, or if $k_0 > 1$, set $\tilde{L} = k_0 - 1$, $k_0 = 1$, and go to Step 1.

The inputs of the water-filling algorithm are the number of available positive eigenvalues $\hat{L}$, the set of eigenvalues $\{\lambda_{H,i}\}_{i=1}^{\hat{L}}$, and the set of MSE constraints $\tilde{\rho}$, while the outputs are the set of allocated powers $\{z_i\}_{i=1}^{\hat{L}}$, and the set of waterlevels $\tilde{u}^{1/2}$.

The watering-filling algorithm:

0) Reorder the $\lambda_{H,i}$'s in decreasing order, and set $\tilde{L} = \hat{L}$.

1) Set u = $\lambda_{\tilde{L}}^{-1}$ (if $\lambda_{\tilde{L}} = \lambda_{\tilde{L}+1}$, then set $\tilde{L} = \tilde{L} - 1$ and go to Step 1).

2) If $u^{1/2} \geq \sum_{i=1}^{\tilde{L}} \lambda_i^{-1/2} / \left(\tilde{\rho} - \left(\hat{L} - \tilde{L}\right)\right)$, then set $\tilde{L} = \tilde{L} - 1$ and go to Step 1. Otherwise obtain the definitive water-level $\tilde{u}^{1/2}$ and allocated powers as $u^{1/2} = \sum_{i=1}^{\tilde{L}} \lambda_i^{-1/2} / \left(\tilde{\rho} - \left(\hat{L} - \tilde{L}\right)\right)$ and $z_i = (\tilde{u}^{1/2} \lambda_i^{-1/2} - \lambda_i^{-1})^+$.

It is interesting to consider a suboptimum but very simple solution to the problem. We can impose a diagonality constraint in the MSE matrix. In other words, $\mathbf{E} = \left(\mathbf{I} + \mathbf{B}^H \mathbf{R}_H \mathbf{B}\right)^{-1}$ will have a diagonal structure. Imposing such a structure implies that the transmission is performed in a parallel fashion through the channel eigenmodes. The problem in (2-21) now becomes

$$\begin{aligned} &\min_{\mathbf{B}} \ tr\{\mathbf{B}\mathbf{B}^H\} \\ &\text{s.t.} \ \left[(\mathbf{I} + \mathbf{B}^H \mathbf{R}_H \mathbf{B})^{-1}\right]_{ii} \leq \rho_i, \ 1 \leq i \leq L, \\ &\qquad \mathbf{B}^H \mathbf{R}_H \mathbf{B} \ \text{diagonal.} \end{aligned} \tag{2-29}$$

And the optimal solution is given by $\mathbf{B} = \mathbf{U}_{H,1} \Sigma_{B,1}$ where $\mathbf{U}_{H,1} \in \mathbb{C}^{n_t \times L}$ have its column vectors as the eigenvectors of $\mathbf{R}_H$ corresponding to the largest L eigenvalues in increasing order, and $\Sigma_{B,1} \in \mathbb{C}^{L \times L}$ is a diagonal matrix with squared-diagonal elements given by

$$z_i = \lambda_{H,i}^{-1}(\rho_i^{-1} - 1), \qquad 1 \leq i \leq L. \tag{2-30}$$

The optimal solution under the diagonality constraint becomes the true optimum if and only if

$$\lambda_{H,i}\rho_i^2 \geq \lambda_{H,i+1}\rho_{i+1}^2, \qquad 1 \leq i \leq L. \tag{2-31}$$

This implies the feasibility condition is $L \leq rank(\mathbf{R}_H)$.

Chapter 3: SISO Relay Systems

In this chapter, we consider a power allocation problem in SISO relay systems. For simplicity, we assume that the system consists of 3 nodes: a designated source-destination node pair and one relay node. Each node is equipped with a single transmit/receive antenna. All nodes are operated in the half-duplex mode, so transmission occurs over two time slots via two hops. Among the various possible cooperation strategies, we adopt the simplest type (i.e. the AF method). In the first time slot, the source broadcast the signal to the relay and the destination. The relay node then receives and processes the signal, and retransmits the processed signal to the destination (in the second time slot). The destination finally combines the signals received from the source and the relay and makes a joint decision. Here, we assume perfect synchronization is attained at the destination node. Since only the source will transmit signal at the first time slot, and the relay will transmit signal at the second time slot (i.e. no concurrent transmission), there is no concern for inter-user synchronization, which makes the system simple and practical. Note that in the cooperation scheme, each of transmission time slots is divided into 2 non-overlapping slots, and therefore the transmission duration for each slot is half of that available for the direct transmission scheme. Consequently, if we want to maintain the same total power consumption, the energy available per bit for the cooperative scheme is half of that for the direct transmission scheme.

3.1 Input-Output Relationship

In the AF mode, the relay node simply amplifies the received signal and forwards it to the destination. To simplify the problem, we consider the flat and slowing fading channel, i.e. the channel coefficients for all the bits in a packet are the same. And we assume that the power of the signal retransmitted at the relay node is scaled uniformly with respect to all the bits in a packet, such that the average transmission energy per signal in a packet is constant and satisfies the QoS constraint. Let h_{sd}, h_{sr} and h_{rd} denote the channel path gains between the source and the destination, the source and the relay, and the relay and the destination, respectively. In time slot 1, the signals received at the destination and the relay can be written [24]

$$\mathbf{y}_{sd} = \sqrt{p_s} h_{sd} \mathbf{x} + \mathbf{n}_{sd}, \tag{3-2}$$

and

$$\mathbf{y}_{sr} = \sqrt{p_s} h_{sr} \mathbf{x} + \mathbf{n}_{sr}, \tag{3-3}$$

respectively, where $\mathbf{x}$ is the transmitted signal vector from the source node , $\mathbf{y}_{sr}$ is the received signal vector at the relay nodes, $\mathbf{y}_{sd}$ is the received signal vector at the destination node, $\mathbf{n}_{sd}$ and $\mathbf{n}_{sr}$ are noise vectors. Each component of $\mathbf{n}_{sd}$ and $\mathbf{n}_{sr}$ has an independent complex Gaussian distribution and its variance equals N_0. As defined, h_{sd} and h_{sr} represent the effect of path loss and static fading on transmission channels. Note that the transmission unit considered here is the packet. For simplicity, we ignore the packet index in (3-2)-(3-3).

In the second time slot, the relay amplifies the signal by a gain of $\frac{\sqrt{p_r}}{\sqrt{p_s |h_{sr}|^2 + N_0}}$ and retransmits the amplified signal. The signal received at the destination can then be written as

$$\mathbf{y}_{rd} = \sqrt{p_r} \underbrace{\frac{1}{\sqrt{p_s |h_{sr}|^2 + N_0}}}_{normalized \quad power} h_{rd} \mathbf{y}_{sr} + \mathbf{n}_{rd}, \tag{3-4}$$

where $\mathbf{y}_{rd}$ is the received signal vector at the destination node, $\mathbf{n}_{rd}$ is a zero-mean complex AWGN vector with a variance of N_0, and h_{rd} represents the path loss effect and static fading of the relay channel. Moreover, $\mathbf{n}_{SD}$, $\mathbf{n}_{SR}$ and $\mathbf{n}_{rd}$ are assumed to be independent and mutually uncorrelated with **x**, and h_{sd}, h_{sr} and h_{rd} are modeled as zero mean, mutually independent complex jointly Gaussian random variables with the same variance of σ^2. It is also assume that all the CSI is known at the destination.

3.2 Problem Formulation and Solution

At the destination, the received signals in the first and the second time slots are combined by the maximum ratio combiner (MRC). From [13], we know that to maximum the output SNR, the signal component with a higher SNR should be weighted heavier than that with a lower SNR. It turns out that the weight of a signal component is equal to its received SNR. Furthermore the SNR of the combiner output is the sum of the received SNRs. The output signal of the MRC can be written as

$$\mathbf{y}_d = a_1 \mathbf{y}_{sd} + a_2 \mathbf{y}_{rd} . \tag{3-5}$$

From (3-2) and (3-4), we have the SNRs of the receiver signal in time slot one and two as

$$SNR_1 = \frac{p_s |h_{sd}|^2}{N_0} \quad \text{and} \quad SNR_2 = \frac{1}{N_0} \frac{p_s p_r |h_{sr}|^2 |h_{rd}|^2}{p_s |h_{sr}|^2 + p_r |h_{rd}|^2 + N_0} . \tag{3-6}$$

So, we can have the weights as

$$a_1 = \frac{\sqrt{p_s} h_{sd}^*}{N_0} \tag{3-7}$$

and

$$a_2 = \frac{\sqrt{\frac{p_s p_r}{\left(p_s |h_{sr}|^2 + N_0\right)}} h_{sr}^* h_{rd}^*}{N_0 + N_0 \frac{p_r |h_{rd}|^2}{\left(p_s |h_{sr}|^2 + N_0\right)}}. \tag{3-8}$$

In order to make the signal of the combiner output have a unit power, i.e. $E\{x^2\}=1$, we must normalize the signal of the combiner output. The gain for the normalization can be expressed as

$$g = a_1 \sqrt{p_s} h_{sd} + a_2 \frac{\sqrt{p_s p_r} h_{sr} h_{rd}}{\sqrt{p_s |h_{sr}|^2 + N_0}}. \tag{3-9}$$

And the SNR after the MRC can then be expressed as

$$SNR_1 + SNR_2 = \gamma = p_s a + \frac{p_s p_r bc}{p_s c + p_r b + 1}, \tag{3-10}$$

where

$$a = \frac{|h_{sd}|^2}{N_0}, \tag{3-11}$$

$$b = \frac{|h_{rd}|^2}{N_0}, \tag{3-12}$$

and

$$c = \frac{|h_{sr}|^2}{N_0}. \tag{3-13}$$

From [14], we know that the relationship between the bit-error-rate (BER), denoted as P_{eb}, and SNR for the M-QAM can be approximated by an upper bound as $P_{eb} \approx \frac{1}{5} e^{-\frac{1.5}{M-1} SNR}$. Here, $M \geq 4$ and $0 \leq SNR \leq 30\,(dB)$. Thus, for a given P_{eb} we can then have the required SNR as

$$\gamma = p_s a + \frac{p_s p_r bc}{p_s c + p_r b + 1} = \frac{2}{3}\log\left(\frac{1}{5P_{eb}}\right)(M-1) \triangleq D. \tag{3-14}$$

Multiplying both sides of (3-14) by $\left(p_s c + p_r b + 1\right)$, we have

$$\underbrace{ac}_{:=\alpha}\, p_s^2 + \underbrace{\left(p_r ab + a + p_r bc - cD\right)}_{:=\beta} p_s - \underbrace{D\left(p_r b + 1\right)}_{:=\gamma} = 0. \tag{3-15}$$

Since (3-15) has a quadratic form, we can obtain the solution straightforwardly as

$$p_s = \frac{-\beta + \sqrt{\beta^2 + 4\alpha\gamma}}{2\alpha} > 0, \quad \text{for a given} \quad p_r. \tag{3-16}$$

Note that α and γ are positive, and the solution in (3-16) is always positive, automatically satisfying the positive constraint of p_s. Thus the optimization problem can thus be formulated as

$$\begin{aligned} &\min\ f\left(p_r\right) \triangleq p_s + p_r = p_r + \frac{-\beta + \sqrt{\beta^2 + 4\alpha\gamma}}{2\alpha} \\ &s.t.\quad p_s > 0,\ \ p_r \geq 0,\ \ P_{eb} \leq \rho, \end{aligned} \tag{3-17}$$

where ρ is the BER constraint.

We can see that taking the first and second derivative of (3-17), we have

$$\frac{\partial f\left(p_r\right)}{\partial p_r} = 1 + \frac{1}{2\alpha}\left(-(ab+bc) + \frac{1}{2}\left(\beta^2 + 4\alpha\gamma\right)^{-\frac{1}{2}}\left(2\beta\underbrace{(ab+bc)}_{\triangleq\Delta} + 4\alpha bD\right)\right). \tag{3-18}$$

and

$$\begin{aligned} \frac{\partial^2 f\left(p_r\right)}{\partial {p_r}^2} &= \frac{1}{2\alpha}\left(\frac{-1}{4}\left(\beta^2 + 4\alpha\gamma\right)^{\frac{-3}{2}}\left(2\beta\Delta + 4Db\right)^2 + \frac{1}{2}\left(\beta^2 + 4\alpha\gamma\right)^{\frac{-1}{2}} 2\Delta^2\right) \\ &= \frac{1}{2\alpha}\left(\beta^2 + 4\alpha\gamma\right)^{\frac{-1}{2}}\left(\Delta^2 - \left(\beta^2 + 4\alpha\gamma\right)^{-1}\left(\beta\Delta + 2Db\right)^2\right) \end{aligned}. \tag{3-19}$$

From (3-18) and (3-19), we can see that the optimization problem is not necessarily a convex optimization problem. This implies that a global minimum may be not guaranteed to obtain.

Setting (3-18) to zero, we can have

$$2\alpha-\left(ab+bc\right)=-\left(\beta^2+4\alpha\gamma\right)^{-\frac{1}{2}}\left(\beta\left(ab+bc\right)+2\alpha bD\right). \tag{3-20}$$

Squaring and multiplying both sides of (3-20) by $\left(\beta^2+4\alpha\gamma\right)$, and substituting $\alpha=ac$, $\beta=\left(p_r ab+p_r bc+a-cD\right)$, and $\gamma=D\left(p_r b+1\right)$ into the result, we can have

$$\begin{aligned}&4ac(ac-ab-bc)\left[(ab+bc)^2 {p_r}^2+2(a^2b+abc+abcD-bc^2D)p_r+(a+cD)^2\right]\\&-4ab^2c^2D(a+c+cD)=0.\end{aligned} \tag{3-21}$$

As we can see, the left hand side of (3-21) is a quadratic function, and we can rewrite (3-21) as

$$e{p_r}^2+fp_r+g=0, \tag{3-22}$$

where

$$e=4ab^2c(ac-ab-bc)(a+c)^2, \tag{3-23}$$

$$f=8ac(ac-ab-bc)(a^2b+abc+abcD-bc^2D), \tag{3-24}$$

$$g=4ac(ac-ab-bc)(a+cD)^2-4ab^2c^2D(a+c+cD). \tag{3-25}$$

Thus, using (3-22), we can obtain a solution for p_r straightforwardly. Since the signal power is a positive real number and the solution can be obtained at the point where it's first derivative equal to zero or a boundary point. When $f^2-4eg<0$, a solution with real value cannot be obtained. This indicates that the first derivative will not be equal to zero at the allowable range of p_r. In other words, the optimal solution is observed at a boundary point. When $f^2-4eg\geq 0$, we can obtain two solutions with real values. Since p_r is positive, it is simple to obtain the solution by selecting the positive one with a smaller first derivative value. If none of the solutions is positive, the optimal solution must be located at a boundary point. Substituting the solution of p_r. into (3-16), we can obtain the solution of p_s.

Chapter 4: MIMO Relay Systems

In this Chapter, we will consider the precoders design problem in MIMO relay systems. In this scenario, each node is equipped with multiple antennas such that the whole system can be formulated as a MIMO system. It is well known that precoders can greatly enhance the performance of a MIMO system. This is also true for the MIMO relay system. Since the source and the relay both have multiple antennas, we have two precoders to work with. We will use the MSE as the QoS constraint, and propose a new method to derive the optimum precoders.

4.1 Input-Output Relationship of MIMO Relay Systems

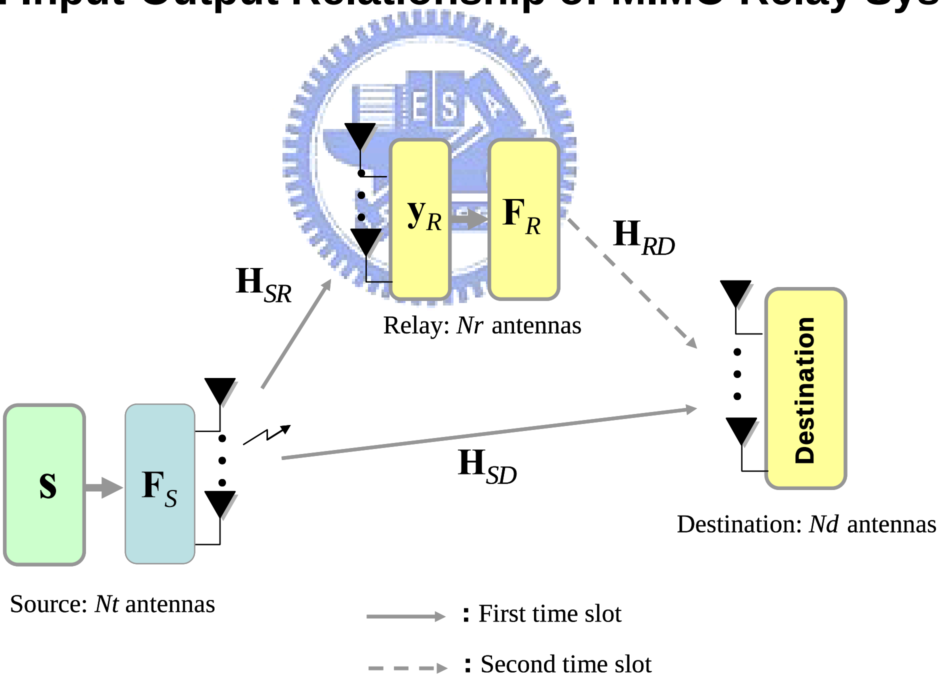

Figure 4-1: Description of an AF MIMO cooperative uplink transmission scheme.

Let N_t, N_r, and N_d denote the number of antennas at the source node, the relay node, and the destination node, respectively. Assume that the number of bit streams transmitted N is less than or equal to $\min(N_t, N_r, N_d)$ such that sufficient degrees of freedom is guaranteed in our design. Let all channels be flat-fading, and we can write the system model as:

$$\mathbf{y}_d = \begin{bmatrix} \mathbf{y}_{d1} \\ \mathbf{y}_{d2} \end{bmatrix} = \underbrace{\begin{bmatrix} \mathbf{H}_{sd} \\ \mathbf{H}_{rd}\mathbf{F}_r\mathbf{H}_{sr} \end{bmatrix}}_{:=\mathbf{H}} \mathbf{F}_s\mathbf{s} + \underbrace{\begin{bmatrix} \mathbf{n}_{d1} \\ \mathbf{H}_{rd}\mathbf{F}_r\mathbf{n}_r + \mathbf{n}_{d2} \end{bmatrix}}_{:=\mathbf{w}}, \tag{4-1}$$

where $\mathbf{s} \in \mathbb{C}^{N\times 1}$ is the transmitted signal vector, $\mathbf{F}_s \in \mathbb{C}^{N_t \times N}$ is the precoding matrix at the source node, $\mathbf{F}_r \in \mathbb{C}^{N_r \times N_r}$ is the precoding matrix at the relay node, $\mathbf{y}_d \in \mathbb{C}^{2N_d \times 1}$ is the received signal vector at the destination node, $\mathbf{y}_{di} \in \mathbb{C}^{N_d \times 1}$ is the received signal vector in the *i*th time slot, $\mathbf{n}_r \in \mathbb{C}^{N_r \times 1}$, $\mathbf{n}_{d1} \in \mathbb{C}^{N_d \times 1}$, and $\mathbf{n}_{d2} \in \mathbb{C}^{N_d \times 1}$ denote the zero-mean complex AWGN vectors received at the relay, at the destination corresponding to the first time-slot, and the destination corresponding to the second time-slot. $\mathbf{H}_{sr} \in \mathbb{C}^{N_r \times N_t}$, $\mathbf{H}_{rd} \in \mathbb{C}^{N_d \times N_r}$, and $\mathbf{H}_{sd} \in \mathbb{C}^{N_d \times N_t}$ denote the channel matrices between the source and the relay, the relay and the destination, and the source and the destination. The elements of $\mathbf{H}_{sd}$, $\mathbf{H}_{sr}$ and $\mathbf{H}_{rd}$ are modeled as zero mean, mutually independent complex Gaussian random variables with the same variance. Also, $\mathbf{H}$ denotes the combined channel matrix, and $\mathbf{w}$ the combined noise vector at destination. Since the noise received at the relay is amplified by the relay-to-destination link, this is different from the precoding design in conventional MIMO systems. Note that the precoder design problem is a joint transceiver problem, that is, a different receiver will yield different precoders.

4.2 Problem Formulation and Solutions

Here, we propose to use the linear MMSE receiver at the destination. Let $\mathbf{G}$ be the filtering matrix at the receiver. The MSE for the estimation of $\mathbf{s}$ (the transmitted signal vector), denoted as J, is given by

$$J = E\left[\left\|\mathbf{G}\mathbf{y}_d - \mathbf{s}\right\|^2\right]. \tag{4-2}$$

Define a MSE matrix as $\mathbf{E}=E\left[(\mathbf{G}\mathbf{y}_d-\mathbf{s})(\mathbf{G}\mathbf{y}_d-\mathbf{s})^H\right]$. Then, we have $J=tr\{\mathbf{E}\}$, where

$$
\begin{aligned}
\mathbf{E} &= E\left[(\mathbf{G}\mathbf{y}_d-\mathbf{s})(\mathbf{G}\mathbf{y}_d-\mathbf{s})^H\right] \\
&= \mathbf{G}\mathbf{H}\mathbf{F}_s\mathbf{R}_s\mathbf{F}_s^H\mathbf{H}^H\mathbf{G}^H-\mathbf{G}\mathbf{H}\mathbf{F}_s\mathbf{R}_s-\mathbf{R}_s\mathbf{F}_s^H\mathbf{H}^H\mathbf{G}^H+\mathbf{G}\mathbf{R}_w\mathbf{G}^H \\
&= \left(\mathbf{G}-\mathbf{R}_s\mathbf{F}_s^H\mathbf{H}^H(\mathbf{H}\mathbf{F}_s\mathbf{R}_s\mathbf{F}_s^H\mathbf{H}^H+\mathbf{R}_w)^{-1}\right)\left(\mathbf{H}\mathbf{F}_s\mathbf{R}_s\mathbf{F}_s^H\mathbf{H}^H+\mathbf{R}_w\right)\times \\
&\quad \left(\mathbf{G}^H-(\mathbf{H}\mathbf{F}_s\mathbf{R}_s\mathbf{F}_s^H\mathbf{H}^H+\mathbf{R}_w)^{-1}\mathbf{H}\mathbf{F}_s\mathbf{R}_s\right)+\mathbf{R}_s-\mathbf{R}_s\mathbf{F}_s^H\mathbf{H}^H(\mathbf{H}\mathbf{F}_s\mathbf{R}_s\mathbf{F}_s^H\mathbf{H}^H+\mathbf{R}_w)^{-1}\mathbf{H}\mathbf{F}_s\mathbf{R}_s \\
&\geq \mathbf{R}_s-\mathbf{R}_s\mathbf{F}_s^H\mathbf{H}^H(\mathbf{H}\mathbf{F}_s\mathbf{R}_s\mathbf{F}_s^H\mathbf{H}^H+\mathbf{R}_w)^{-1}\mathbf{H}\mathbf{F}_s\mathbf{R}_s \\
&= \left(\mathbf{R}_s^{-1}+\mathbf{F}_s^H\mathbf{H}^H\mathbf{R}_w^{-1}\mathbf{H}\mathbf{F}_s\right)^{-1}.
\end{aligned}
\tag{4-3}
$$

Here in the last equality we use the matrix inverse lemma from [16], i.e.,

$$\mathbf{A}^{-1}-\mathbf{A}^{-1}\mathbf{B}\left(\mathbf{D}\mathbf{A}^{-1}\mathbf{B}+\mathbf{C}^{-1}\right)^{-1}\mathbf{D}\mathbf{A}^{-1}=\left(\mathbf{A}+\mathbf{B}\mathbf{C}\mathbf{D}\right)^{-1}. \tag{4-4}$$

It is well known that the optimal receive filter, known as the *Wiener* filter, is

$$\mathbf{G}_{opt}=\mathbf{R}_s\mathbf{F}_s^H\mathbf{H}^H\left(\mathbf{H}\mathbf{F}_s\mathbf{R}_s\mathbf{F}_s^H\mathbf{H}^H+\mathbf{R}_w\right)^{-1}, \tag{4-5}$$

where $\mathbf{R}_w=E\left[\mathbf{w}\mathbf{w}^H\right]$ is the covariance matrix of the combined noise vector $\mathbf{w}$, and $\mathbf{R}_s=E\left[\mathbf{s}\mathbf{s}^H\right]$ is the covariance matrix of the signal vector. So we can have the minimum mean-square-error (MMSE), denoted as J_{min}, as

$$J_{min}=tr\left\{(\mathbf{R}_s^{-1}+\mathbf{F}_s^H\mathbf{H}^H\mathbf{R}_w^{-1}\mathbf{H}\mathbf{F}_s)^{-1}\right\}. \tag{4-6}$$

Invoking assumptions made previously, we have $\mathbf{R}_{n_{d,1}}=E\left[\mathbf{n}_{d,1}\mathbf{n}_{d,1}^H\right]=\sigma_n^2\mathbf{I}_{N_d}$, and $\mathbf{R}_{n_{d,2}}=E\left[\mathbf{n}_{d,2}\mathbf{n}_{d,2}^H\right]=\sigma_n^2\mathbf{I}_{N_d}$, $\mathbf{R}_{n_r}=E\left[\mathbf{n}_r\mathbf{n}_r^H\right]=\sigma_n^2\mathbf{I}_{N_r}$, where σ_n^2 is the noise variance of each vector component. Then we have

$$\mathbf{R}_w=\begin{bmatrix}\sigma_n^2\mathbf{I}_{N_d} & 0 \\ 0 & \sigma_n^2\mathbf{I}_{N_d}+\sigma_n^2\mathbf{H}_{rd}\mathbf{F}_r\mathbf{F}_r^H\mathbf{H}_{rd}^H\end{bmatrix}. \tag{4-7}$$

As assumed, each element of the signal vector is identical independent distributed (i.i.d.), and the covariance matrix of the signal vector can be expressed as $\mathbf{R}_s = E[\mathbf{s}\mathbf{s}^H] = \sigma_s^2 \mathbf{I}_N$, where σ_s^2 denoted the transmitted symbol power. And the MMSE can be further written as

$$J_{min} = tr\left\{\left(\sigma_s^{-2}\mathbf{I}_N + \underbrace{\sigma_n^{-2}\mathbf{F}_s^H\mathbf{H}_{sd}^H\mathbf{H}_{sd}\mathbf{F}_s}_{:=\mathbf{E}_s} + \underbrace{\sigma_n^{-2}\mathbf{F}_s^H\mathbf{H}_{sr}^H\mathbf{F}_r^H\mathbf{H}_{rd}^H(\mathbf{I}_{N_d} + \mathbf{H}_{rd}\mathbf{F}_r\mathbf{F}_r^H\mathbf{H}_{rd}^H)^{-1}\mathbf{H}_{rd}\mathbf{F}_r\mathbf{H}_{sr}\mathbf{F}_s}_{:=\mathbf{E}_r}\right)^{-1}\right\}. \tag{4-8}$$

As we can see from the equation, the MMSE is a function of precoder matrices. We note here that $\mathbf{E}_r$ and $\mathbf{E}_s$ account for the MSE components due to the relay and the direct communication links. Let us define the power consumption at the relay and the source respectively as

$$\begin{aligned} P_r &= tr\left\{E\left[\mathbf{F}_r(\mathbf{H}_{sr}\mathbf{F}_s\mathbf{s} + \mathbf{n}_r)(\mathbf{H}_{sr}\mathbf{F}_s\mathbf{s} + \mathbf{n}_r)^H \mathbf{F}_r^H\right\}\right] \\ &= tr\left\{\mathbf{F}_r\left(\sigma_s^2\mathbf{H}_{sr}\mathbf{F}_s\mathbf{F}_s^H\mathbf{H}_{sr}^H + \sigma_n^2\mathbf{I}_{N_r}\right)\mathbf{F}_r^H\right\}, \end{aligned}$$

and

$$P_s = tr\left(E\left[\mathbf{F}_s\mathbf{s}\mathbf{s}^H\mathbf{F}_s^H\right]\right) = \sigma_s^2 tr\left(\mathbf{F}_s\mathbf{F}_s^H\right),$$

in which we assume that the symbol and noise are uncorrelated, i.e. $E\left[\mathbf{s}\mathbf{n}_r^H\right] = E\left[\mathbf{n}_r\mathbf{s}^H\right] = \mathbf{0}$.

Now, the design problem can be formulated as follows：

$$\begin{aligned} &\min_{\mathbf{F}_s,\mathbf{F}_r} \underbrace{\sigma_s^2 tr\left(\mathbf{F}_s\mathbf{F}_s^H\right)}_{:=P_s} + \underbrace{\sigma_n^2 tr\left(\mathbf{F}_r\mathbf{F}_r^H\right) + \sigma_s^2 tr\left(\mathbf{F}_r\mathbf{H}_{sr}\mathbf{F}_s\mathbf{F}_s^H\mathbf{H}_{sr}^H\mathbf{F}_r^H\right)}_{:=P_r}, \\ &s.t. \quad \left(\mathbf{R}_s^{-1} + \mathbf{F}_s^H\mathbf{H}^H\mathbf{R}_w^{-1}\mathbf{H}\mathbf{F}_s\right)^{-1}_{(i,i)} \le \rho_i, \end{aligned} \tag{4-9}$$

where ρ_i is the QoS constraint, i.e. the MMSE constraint, for the ith bit stream. The problem in (4-9) indicates that the precoders must satisfy the QoS constraint, and at the same time minimize the total transmitted power. From (4-8), we can see that the MSE matrix contains three matrix, i.e. $\sigma_s^{-2}\mathbf{I}_N$, $\mathbf{E}_s$ and $\mathbf{E}_r$. The first term can be ignored since it is a function of the signal power only. The second term indicates the contribution from the direct link and is a function of $\mathbf{F}_s$, while the third term indicates the contribution from the relay link and is a function of $\mathbf{F}_s$ and $\mathbf{F}_r$. If only the relay link is considered, the problem reduces to a two-hop relay system and the MSE becomes

$$J = tr\left\{\left(\sigma_s^{-2}\mathbf{I}_N + \underbrace{\sigma_n^{-2}\mathbf{F}_s^H\mathbf{H}_{sr}^H\mathbf{F}_r^H\mathbf{H}_{rd}^H(\mathbf{I}_{N_d} + \mathbf{H}_{rd}\mathbf{F}_r\mathbf{F}_r^H\mathbf{H}_{rd}^H)^{-1}\mathbf{H}_{rd}\mathbf{F}_r\mathbf{H}_{sr}\mathbf{F}_s}_{:=\mathbf{E}_r}\right)^{-1}\right\}. \quad (4\text{-}10)$$

From [17], we know that there exists a one-to-one mapping function between the SINR (signal to interference-plus-noise ratio) and the MSE :

$$\mathrm{MSE}_i = \frac{1}{1+\mathrm{SINR}_i}. \quad (4\text{-}11)$$

Under the Gaussian assumption, the symbol error probability can be analytically expressed as a function of the SINR [18]

$$P_e = \alpha Q(\sqrt{\beta \mathrm{SINR}}), \quad (4\text{-}12)$$

where α and β are constants that depend on the signal constellation, and $Q(x) = (1/\sqrt{2\pi})\int_x^\infty e^{-\lambda^2/2}d\lambda$. The BER, defined as P_{eb}, can be approximately obtained from the symbol error probability (assuming that a Gray encoding is used to map the bits into the constellation points) as

$$P_{eb} \approx \frac{P_e}{k}, \quad (4\text{-}13)$$

where $k = \log_2 M$ is the number of bits per symbol, and M is the constellation size. This is to say that if the QoS constraints are given in terms of MSE, they can also be equivalently expressed in terms of BER. As we can see, the MSE matrix involves a series of multiplications and inversions, and the MSE in (4-10) is a complicated function of $\mathbf{F}_s$ and $\mathbf{F}_r$. As a result, a direct solution is very difficult to obtain. In the following, we will show how to solve the joint precoder design problem, effectively. We propose a method to simplify the constraint function J such that $\mathbf{E}$ can be expressed as a simple function of unknown parameters. The main idea is to use a constrained precoder structure, derive a MMSE upper bound having a simple expression, and conduct minimization with this upper bound. This method is proposed in [19] to design precoders in MIMO relay systems. The original problem is to minimize the MSE under a total power constraint. Here, we extend its use to minimize the total transmission power under the constraint of a MSE.

If the MSE matrix $\mathbf{E}$ can be diagonal, the trace operation and the MSE of each link (i.e. the diagonal element of $\mathbf{E}$) become easy to conduct, and the whole problem can be greatly simplified, i.e. the matrix operations involved in the cost function can be reduced to scalar operations. To achieve this objective we first consider singular-value-decomposition (SVD) for the channel matrices in all links. By SVD, we have $\mathbf{H}_{sd} = \mathbf{U}_{sd}\mathbf{\Sigma}_{sd}\mathbf{V}_{sd}^H$, $\mathbf{H}_{sr} = \mathbf{U}_{sr}\mathbf{\Sigma}_{sr}\mathbf{V}_{sr}^H$, and $\mathbf{H}_{rd} = \mathbf{U}_{rd}\mathbf{\Sigma}_{rd}\mathbf{V}_{rd}^H$, where $\mathbf{U}_{sd} \in \mathbb{C}^{N_d \times N_d}$, $\mathbf{U}_{sr} \in \mathbb{C}^{N_r \times N_r}$, and $\mathbf{U}_{rd} \in \mathbb{C}^{N_d \times N_d}$ are the left singular matrices of $\mathbf{H}_{sd}$, $\mathbf{H}_{sr}$, and $\mathbf{H}_{rd}$, respectively. $\mathbf{V}_{sd}^H \in \mathbb{C}^{N_t \times N_t}$, $\mathbf{V}_{sr}^H \in \mathbb{C}^{N_t \times N_t}$, and $\mathbf{V}_{rd}^H \in \mathbb{C}^{N_r \times N_r}$ are the right singular matrices of $\mathbf{H}_{sd}$, $\mathbf{H}_{sr}$, and $\mathbf{H}_{rd}$,, respectively. $\mathbf{\Sigma}_{sd} \in \mathbb{R}^{N_d \times N_t}$, $\mathbf{\Sigma}_{sr} \in \mathbb{R}^{N_r \times N_t}$, and $\mathbf{\Sigma}_{rd} \in \mathbb{R}^{N_d \times N_r}$ are the diagonal singular value matrices of $\mathbf{H}_{sd}$, $\mathbf{H}_{sr}$, and $\mathbf{H}_{rd}$, respectively. Then, the MSE matrix in (4-8) can be re-written as follows：

$$\mathbf{E}=\left(\sigma_s^{-2}\mathbf{I}_N+\underbrace{\sigma_n^{-2}\mathbf{F}_s^H\mathbf{V}_{sd}\boldsymbol{\Sigma}_{sd}^H\boldsymbol{\Sigma}_{sd}\mathbf{V}_{sd}^H\mathbf{F}_s}_{:=\mathbf{E}_s}+\left(\underbrace{\sigma_n^{-2}\mathbf{F}_s^H\mathbf{V}_{sr}\boldsymbol{\Sigma}_{sr}^H\mathbf{U}_{sr}^H\mathbf{F}_r^H\mathbf{V}_{rd}\boldsymbol{\Sigma}_{rd}^H\mathbf{U}_{rd}^H\times\left(\mathbf{I}_{N_d}+\mathbf{U}_{rd}\boldsymbol{\Sigma}_{rd}\mathbf{V}_{rd}^H\mathbf{F}_r\mathbf{F}_r^H\mathbf{V}_{rd}\boldsymbol{\Sigma}_{rd}^H\mathbf{U}_{rd}^H\right)^{-1}\times\mathbf{U}_{rd}\boldsymbol{\Sigma}_{rd}\mathbf{V}_{rd}^H\mathbf{F}_r\mathbf{U}_{sr}\boldsymbol{\Sigma}_{sr}\mathbf{V}_{sr}^H\mathbf{F}_s}_{:=\mathbf{E}_r}\right)\right)^{-1}. \tag{4-14}$$

Since $\mathbf{E}_s$ only depends on $\mathbf{F}_s$ while $\mathbf{E}_r$ depends on $\mathbf{F}_s$ and $\mathbf{F}_r$**,** simultaneous diagonalization of $\mathbf{E}_s$ and $\mathbf{E}_r$ appears difficult. So, we choose to diagonalize $\mathbf{E}_r$ first. Let the precoders have a constrained structure shown below

$$\mathbf{F}_s=\mathbf{V}_{sr}\boldsymbol{\Sigma}_s\in\mathbb{C}^{N_t\times N}\ , \tag{4-15}$$

and

$$\mathbf{F}_r=\mathbf{V}_{rd}\boldsymbol{\Sigma}_r\mathbf{U}_{sr}^H\in\mathbb{C}^{N_r\times N_r}, \tag{4-16}$$

where $\boldsymbol{\Sigma}_s\in\mathbb{R}^{N_t\times N}$ and $\boldsymbol{\Sigma}_r\in\mathbb{R}^{N_r\times N_r}$ are the diagonal matrix to be determined. Using the structures, the precoders can be regarded as a special case of shaping matrices with the incorporation of $\mathbf{V}_{sr}$ and $\mathbf{V}_{rd}$. Then the MSE becomes

$$\begin{aligned} J&=tr\left\{\mathbf{E}\right\}\\ &=tr\left\{\left(\underbrace{\sigma_s^{-2}\mathbf{I}_N+\sigma_n^{-2}\boldsymbol{\Sigma}_s^H\boldsymbol{\Sigma}_{sr}^H\boldsymbol{\Sigma}_r^H\boldsymbol{\Sigma}_{rd}^H\left(\boldsymbol{\Sigma}_{rd}\boldsymbol{\Sigma}_r^2\boldsymbol{\Sigma}_{rd}^H+\mathbf{I}_{N_d}\right)^{-1}\boldsymbol{\Sigma}_{rd}\boldsymbol{\Sigma}_r\boldsymbol{\Sigma}_{sr}\boldsymbol{\Sigma}_s}_{\in Diag}\right.\right.\\ &\qquad\left.\left.+\boldsymbol{\Sigma}_s^H\underbrace{\sigma_n^{-2}\mathbf{V}^H\boldsymbol{\Sigma}_{sd}^H\boldsymbol{\Sigma}_{sd}\mathbf{V}}_{:=\mathbf{C}}\boldsymbol{\Sigma}_s\right)^{-1}\right\} \end{aligned} \tag{4-17}$$

where

$$\mathbf{V}=\mathbf{V}_{sd}^H\mathbf{V}_{sr}\in\mathbb{C}^{N_t\times N_t}. \tag{4-18}$$

It is noteworthy that $\mathbf{C}=\sigma_n^{-2}\mathbf{V}^H\boldsymbol{\Sigma}_{sd}^H\boldsymbol{\Sigma}_{sd}\mathbf{V}$ is not a diagonal matrix. Next, we will try to transform the non-diagonal structure into a diagonal structure such that the original optimization problem can become a scalar optimization problem. If we let

$$\mathbf{A} = \sigma_s^{-2}\mathbf{I}_N + \sigma_n^{-2}\boldsymbol{\Sigma}_s^H\boldsymbol{\Sigma}_{sr}^H\boldsymbol{\Sigma}_r^H\boldsymbol{\Sigma}_{rd}^H\left(\mathbf{I}_{N_d} + \boldsymbol{\Sigma}_{rd}\boldsymbol{\Sigma}_r^2\boldsymbol{\Sigma}_{rd}^H\right)\boldsymbol{\Sigma}_{rd}\boldsymbol{\Sigma}_r\boldsymbol{\Sigma}_{sr}\boldsymbol{\Sigma}_s, \tag{4-19}$$

and

$$\mathbf{C} = \sigma_n^{-2}\mathbf{V}\boldsymbol{\Sigma}_{sd}^H\boldsymbol{\Sigma}_{sd}\mathbf{V}. \tag{4-20}$$

Then the MSE for the ith link is

$$\left(\left(\mathbf{A} + \boldsymbol{\Sigma}_s^H\mathbf{C}\boldsymbol{\Sigma}_s\right)^{-1}\right)_{(i,i)} = \left(\mathbf{A}^{-1}\right)_{(i.i)} - \left(\underbrace{\mathbf{A}^{-1}\boldsymbol{\Sigma}_s^H}_{:=\mathbf{D}_1^H}(\underbrace{\mathbf{C}^{-1}}_{:=\mathbf{X}} + \underbrace{\boldsymbol{\Sigma}_s\mathbf{A}^{-1}\boldsymbol{\Sigma}_s^H}_{:=\mathbf{D}_2})^{-1}\underbrace{\boldsymbol{\Sigma}_s^H\mathbf{A}^{-1}}_{:=\mathbf{D}_1}\right)_{(i,i)}, \tag{4-21}$$

where $\mathbf{A}$ and $\boldsymbol{\Sigma}_s^H$ are diagonal matrices, so do $\mathbf{D}_1$, $\mathbf{D}_2$.

Since $\mathbf{X}$ is not diagonal, $\mathbf{E}$ cannot be completely diagonalized. In the following, we will derive a MSE upper bound (4-27) such that using this bound as the constraint will lead to a scalar optimization problem. Let $\mathbf{Z} = (\mathbf{X} + \mathbf{D}_2) = \mathbf{U}\boldsymbol{\Sigma}\mathbf{U}^H$ is a Hermitian matrix where $\mathbf{U}$ is a unitary matrix and

$$\mathbf{Z} = \mathbf{U}\boldsymbol{\Sigma}\mathbf{U}^H = \mathbf{U}\boldsymbol{\Sigma}^{1/2}\mathbf{U}^H\mathbf{U}\boldsymbol{\Sigma}^{1/2}\mathbf{U}^H = \mathbf{Z}^{1/2}\mathbf{Z}^{1/2}, \tag{4-22}$$

and

$$\mathbf{Z}^{-1} = \mathbf{U}\boldsymbol{\Sigma}^{-1}\mathbf{U}^H = \mathbf{U}\boldsymbol{\Sigma}^{-1/2}\mathbf{U}^H\mathbf{U}\boldsymbol{\Sigma}^{-1/2}\mathbf{U}^H = \mathbf{Z}^{-1/2}\mathbf{Z}^{-1/2}. \tag{4-23}$$

Using Cauchy Schwarz inequality, we have

$$1 = \left\|\mathbf{e}_i\mathbf{Z}^{1/2}\mathbf{Z}^{-1/2}\mathbf{e}_i^H\right\|_2^2 \le \left\|\mathbf{e}_i\mathbf{Z}^{1/2}\right\|_2^2\left\|\mathbf{Z}^{-1/2}\mathbf{e}_i^H\right\|_2^2, \tag{4-24}$$

and

$$\left\|\mathbf{e}_i\mathbf{Z}^{1/2}\right\|_2^2 = (\mathbf{e}_i\mathbf{Z}\mathbf{e}_i^H) = \mathbf{Z}_{(i,i)}, \quad \left\|\mathbf{Z}^{-1/2}\mathbf{e}_i^H\right\|_2^2 = (\mathbf{e}_i\mathbf{Z}^{-1}\mathbf{e}_i^H) = \mathbf{Z}_{(i,i)}^{-1}, \tag{4-25}$$

where $\mathbf{e}_i$ is the ith unit standard vector. We then have $\mathbf{Z}^{-1}_{(i,i)} \geq \left(\mathbf{Z}_{(i,i)}\right)^{-1}$. So, we have the following inequality

$$\begin{aligned}
&\left(\underbrace{\mathbf{A}^{-1}\boldsymbol{\Sigma}_s^H}_{\mathbf{D}_1^H} (\underbrace{\mathbf{C}^{-1}}_{\mathbf{X}} + \underbrace{\boldsymbol{\Sigma}_s\mathbf{A}^{-1}\boldsymbol{\Sigma}_s^H}_{\mathbf{D}_2})^{-1} \underbrace{\boldsymbol{\Sigma}_s^H\mathbf{A}^{-1}}_{\mathbf{D}_1} \right)_{(i,i)} \\
&= \left|\mathbf{D}_1(i,i)\right|^2 (\mathbf{X}+\mathbf{D}_2)^{-1}_{(i,i)} \\
&\geq \frac{\left|\mathbf{D}_1(i,i)\right|^2}{(\mathbf{X}+\mathbf{D}_2)_{(i,i)}} \qquad\qquad (4\text{-}26)\\
&= \frac{\left|\mathbf{D}_1(i,i)\right|^2}{(diag\left(\mathbf{X}\right)+\mathbf{D}_2)_{(i,i)}} \\
&= \left(\underbrace{\mathbf{A}^{-1}\boldsymbol{\Sigma}_s^H}_{\mathbf{D}_1^H} (\underbrace{diag\left(\mathbf{C}^{-1}\right)}_{:=diag(\mathbf{X})} + \underbrace{\boldsymbol{\Sigma}_s\mathbf{A}^{-1}\boldsymbol{\Sigma}_s^H}_{\mathbf{D}_2})^{-1} \underbrace{\boldsymbol{\Sigma}_s^H\mathbf{A}^{-1}}_{\mathbf{D}_1} \right)_{(i,i)}
\end{aligned}$$

As a result, the QoS constraint for the ith link has an upper bound as:

$$\begin{aligned}
&\mathbf{E}_{(i,i)} \\
&\leq \left(\mathbf{A}^{-1}\right)_{(i,i)} - \left(\mathbf{A}^{-1}\boldsymbol{\Sigma}_s^H (diag(\mathbf{C}^{-1}) + \boldsymbol{\Sigma}_s\mathbf{A}^{-1}\boldsymbol{\Sigma}_s^H)^{-1}\boldsymbol{\Sigma}_s^H\mathbf{A}^{-1}\right)_{(i,i)} \qquad (4\text{-}27)\\
&= \frac{1}{\sigma_s^{-2} + \sigma_n^{-2}\dfrac{\sigma_{s,i}^2\sigma_{r,i}^2\sigma_{sr,i}^2\sigma_{rd,i}^2}{\sigma_{r,i}^2\sigma_{rd,i}^2+1} + \dfrac{\sigma_{s,i}^2}{\mathbf{C}^{-1}_{(i,i)}}}
\end{aligned}$$

where $\mathbf{C} = \sigma_n^{-2}\mathbf{V}_{sr}^H\mathbf{V}_{sd}\boldsymbol{\Sigma}_{sd}^H\boldsymbol{\Sigma}_{sd}\mathbf{V}_{sd}^H\mathbf{V}_{sr}$. $\sigma_{sd,i}$, $\sigma_{sr,i}$ and $\sigma_{rd,i}$ are the ith singular values of the channel matrices $\mathbf{H}_{sd}$, $\mathbf{H}_{sr}$, and $\mathbf{H}_{rd}$, respectively. $\sigma_{s,i}$ and $\sigma_{r,i}$ are the precoder coefficients, i.e. the diagonal elements of $\boldsymbol{\Sigma}_s \in \mathbb{R}^{N_t \times N}$ and $\boldsymbol{\Sigma}_r \in \mathbb{R}^{N_r \times N_r}$. Note that the equality holds when $\mathbf{V} = \mathbf{V}_{sd}^H\mathbf{V}_{sr} = \mathbf{I}_{N_t}$.

With the method shown above, we can then transfer the precoders design problem into a power allocation problem. Let $p_{s,i} \triangleq \sigma_{s,i}^2$, then $p_{s,i}$ can be seen as the power allocated for the ith transmitted bit stream at the source. Let $p_{r,i} \triangleq \sigma_{r,i}^2$, then $p_{r,i}$ can be seen as the power allocated factor for the ith transmitted bit stream at the relay. Now, the optimization problem becomes as

$$
\begin{aligned}
&\min_{p_{s,i}, p_{r,i}} \ \sigma_s^2 \sum_{i=1}^{L} p_{s,i} + \sum_{i=1}^{L} p_{r,i}(\sigma_n^2 + \sigma_s^2 p_{s,i} \sigma_{sr,i}^2) \\
&s.t. \quad \left(\sigma_s^{-2} + \sigma_n^{-2} \frac{p_{s,i} p_{r,i} \sigma_{sr,i}^2 \sigma_{rd,i}^2}{p_{r,i} \sigma_{rd,i}^2 + 1} + \frac{\sigma_{s,i}^2}{\mathbf{C}^{-1}{}_{(i,i)}} \right)^{-1} \le \rho_i, \\
&\qquad p_{s,i} \ge 0,\ p_{r,i} \ge 0,\ i = 1, \cdots, L,
\end{aligned}
\tag{4-28}
$$

where $\mathbf{C} = \sigma_n^{-2} \mathbf{V}_{sr}^H \mathbf{V}_{sd} \mathbf{\Sigma}_{sd}^H \mathbf{\Sigma}_{sd} \mathbf{V}_{sd}^H \mathbf{V}_{sr}$. Since we use an upper bound instead of the MSE itself, the solution in (4-28) is suboptimal.

4.2.1 Special Case I: Source Precoding

In this scenario, we assume that the relay power has been properly allocated, and use $P_{r,i}$ to represent the power allocated for the ith component. We then solve the optimum power allocation at the source. Now, (4-28) can be simplified as

$$
\begin{aligned}
&\min_{p_{s,i}} \sigma_s^2 \sum_{i=1}^{L} p_{s,i} (1 + P_{r,i} \sigma_{sr,i}^2) + \sigma_n^2 \sum_{i=1}^{L} P_{r,i} \\
&s.t. \quad \left(\sigma_s^{-2} + p_{s,i} \underbrace{\left(\sigma_n^{-2} \frac{P_{r,i} \sigma_{sr,i}^2 \sigma_{rd,i}^2}{P_{r,i} \sigma_{rd,i}^2 + 1} + \frac{1}{\mathbf{C}^{-1}{}_{(i,i)}} \right)}_{:= \alpha_i (P_{r,i})} \right)^{-1} \le \rho_i, \\
&\qquad p_{s,i} \ge 0, \quad \forall i.
\end{aligned}
\tag{4-29}
$$

To find the optimum value, we use the Lagrange multiplier method. The Lagrange function with respect to the (4-29) can be expressed as:

$$
\begin{aligned}
L = {} & \sigma_s^2 \sum_{i=1}^{L} p_{s,i} (1 + P_{r,i} \sigma_{sr,i}^2) + \sigma_n^2 \sum_{i=1}^{L} P_{r,i} \\
& + \sum_{i=1}^{L} \lambda_i \left(\left(\sigma_s^{-2} + p_{s,i} \alpha_i (P_{r,i}) \right)^{-1} - \rho_i \right) - \sum_{i=1}^{L} \mu_{s,i} p_{s,i}.
\end{aligned}
\tag{4-30}
$$

And the associated KKT conditions can be described with the following equations:

$$\frac{\partial L}{\partial p_{s,i}} = 0 \Rightarrow \sigma_s^2(1+P_{r,i}\sigma_{sr,i}^2) = \lambda_i\alpha_i(P_{r,i})\left(\sigma_s^{-2} + p_{s,i}\alpha_i(P_{r,i})\right)^{-2} + \mu_{s,i}, \tag{4-31}$$

$$\lambda_i\left(\left(\sigma_s^{-2} + p_{s,i}\alpha_i(P_{r,i})\right)^{-1} - \rho_i\right) = 0, \tag{4-32}$$

$$\mu_{s,i}p_{s,i} = 0, \tag{4-33}$$

$$\mu_{s,i} \geq 0, \tag{4-34}$$

$$\lambda_i \geq 0, \tag{4-35}$$

$$p_{s,i} \geq 0. \tag{4-36}$$

Since $p_{s,i} \geq 0$, from (4-33) we have

$$\mu_{s,i} = 0. \tag{4-37}$$

Substituting (4-37) into (4-31), we have

$$\lambda_i = \frac{\sigma_s^2(1+P_{r,i}\sigma_{sr,i}^2)\left(\sigma_s^{-2} + p_{s,i}\alpha_i(P_{r,i})\right)^2}{\alpha_i(P_{r,i})} > 0. \tag{4-38}$$

Using (4-32), we have the following solution as

$$\left(\sigma_s^{-2} + p_{s,i}\alpha_i(P_{r,i})\right)^{-1} - \rho_i = 0.$$

Then we have

$$p_{s,i} = \frac{\rho_i^{-1} - \sigma_s^{-2}}{\alpha_i(P_{r,i})} \tag{4-39}$$

Finally, from (4-36) we have

$$p_{s,i} = \left[\frac{\rho_i^{-1} - \sigma_s^{-2}}{\alpha_i(P_{r,i})}\right]^+. \tag{4-40}$$

Because $p_{s,i}$ is always larger than zero and $\alpha_i(P_{r,i})$ is positive, ρ_i^{-1} must be large than σ_s^{-2}. It means that MSE constraint ρ_i must be smaller than the transmitted symbol power σ_s^2.

4.2.2 Special Case II: Relay Precoding

In this scenario, we assume that the source power is properly allocated, and use $P_{s,i}$ to represent the power allocated for the ith component. We then solve the optimum power allocation at the relay. In this case, (4-28) can be simplified as

$$\begin{aligned} &\min_{p_{r,i}} \sum_{i=1}^{L} p_{r,i}(\sigma_n^2 + \sigma_s^2 P_{s,i}\sigma_{sr,i}^2) + \sigma_s^2 \sum_{i=1}^{L} P_{s,i} \\ &s.t.\ \left(\sigma_s^{-2} + P_{s,i}\sigma_n^{-2}\frac{\sigma_{sr,i}^2\sigma_{rd,i}^2}{\sigma_{rd,i}^2 + p_{r,i}^{-1}} + \frac{P_{s,i}}{\mathbf{C}^{-1}{}_{(i,i)}}\right)^{-1} \le \rho_i, \\ &\quad p_{r,i} \ge 0, \quad \forall i, \end{aligned} \tag{4-41}$$

where $\mathbf{C} = \sigma_n^{-2}\mathbf{V}_{sr}^H\mathbf{V}_{sd}\mathbf{\Sigma}_{sd}^H\mathbf{\Sigma}_{sd}\mathbf{V}_{sd}^H\mathbf{V}_{sr}$ .Similar to the previous case, we use the Lagrange multiplier method. The Lagrange function with respect to the (4-41) is:

$$\begin{aligned} L = &\sum_{i=1}^{L} p_{r,i}(\sigma_n^2 + \sigma_s^2 P_{s,i}\sigma_{sr,i}^2) + \sigma_s^2 \sum_{i=1}^{L} P_{s,i} \\ &+ \sum_{i=1}^{L}\lambda_i\left(\left(\sigma_s^{-2} + P_{s,i}\sigma_n^{-2}\frac{\sigma_{sr,i}^2\sigma_{rd,i}^2}{\sigma_{rd,i}^2 + p_{r,i}^{-1}} + \frac{P_{s,i}}{\mathbf{C}^{-1}{}_{(i,i)}}\right)^{-1} - \rho_i\right) - \sum_{i=1}^{L}\mu_{r,i}p_{r,i} \ . \end{aligned} \tag{4-42}$$

The KKT conditions can be described with the following equations:

$$\frac{\partial L}{\partial P_{r,i}} = 0,$$

then we have

$$\mu_{r,i} = (\sigma_n^2 + \sigma_s^2 P_{s,i}\sigma_{sr,i}^2) - \lambda_i \frac{\sigma_n^{-2}P_{s,i}\sigma_{sr,i}^2\sigma_{rd,i}^2 p_{r,i}^{-2}}{\left(\left(\sigma_s^{-2} + \frac{P_{s,i}}{\mathbf{C}^{-1}{}_{(i,i)}}\right)\left(\sigma_{rd,i}^2 + p_{r,i}^{-1}\right) + P_{s,i}\sigma_n^{-2}\sigma_{sr,i}^2\sigma_{rd,i}^2\right)^2}, \tag{4-43}$$

$$\lambda_i\left(\left(\sigma_s^{-2}+\sigma_n^{-2}\frac{P_{s,i}\sigma_{sr,i}^2\sigma_{rd,i}^2}{\sigma_{rd,i}^2+p_{r,i}^{-1}}+\frac{P_{s,i}}{\mathbf{C}^{-1}{}_{(i,i)}}\right)^{-1}-\rho_i\right)=0\,, \tag{4-44}$$

$$\mu_{r,i}p_{r,i}=0\,, \tag{4-45}$$

$$\mu_{r,i}\geq 0\,, \tag{4-46}$$

$$\lambda_i\geq 0\,, \tag{4-47}$$

$$p_{r,i}\geq 0\,. \tag{4-48}$$

Since $p_{r,i}\geq 0$, from (4-45) we have

$$\mu_{r,i}=0\,. \tag{4-49}$$

Substituting (4-49) into (4-43) leads to

$$\frac{p_{r,i}{}^2\sigma_n^2(\sigma_n^2+\sigma_s^2P_{s,i}\sigma_{sr,i}^2)\left(\left(\sigma_s^{-2}+\frac{P_{s,i}}{\mathbf{C}^{-1}{}_{(i,i)}}\right)\left(\sigma_{rd,i}^2+p_{r,i}{}^{-1}\right)+P_{s,i}\sigma_n^{-2}\sigma_{sr,i}^2\sigma_{rd,i}^2\right)^2}{P_{s,i}\sigma_{sr,i}^2\sigma_{rd,i}^2}=\lambda_i>0\,, \tag{4-50}$$

Using (4-44), we have the solution as

$$p_{r,i}=\left(\left(\frac{\sigma_n^{-2}P_{s,i}\sigma_{sr,i}^2}{\rho_i^{-1}-\sigma_s^{-2}-P_{s,i}\left(\mathbf{C}^{-1}{}_{(i,i)}\right)^{-1}}-1\right)\sigma_{rd,i}^2\right)^{-1}, \tag{4-51}$$

From (4-48), we finally have

$$p_{r,i}=\left[\frac{\rho_i^{-1}-\sigma_s^{-2}-P_{s,i}\left(\mathbf{C}^{-1}{}_{(i,i)}\right)^{-1}}{\left(\sigma_n^{-2}P_{s,i}\sigma_{sr,i}^2+\sigma_s^{-2}+P_{s,i}\left(\mathbf{C}^{-1}{}_{(i,i)}\right)^{-1}-\rho_i^{-1}\right)\sigma_{rd,i}^2}\right]^+. \tag{4-52}$$

As that in the previous case, we know ρ_i^{-1} must be large than σ_s^{-2}. Here, $P_{s,i}\left(\mathbf{C}^{-1}{}_{(i,i)}\right)^{-1}$

means the MSE changed by the direct link (the channel from source to destination). So if we want that $p_{r,i}$ is large than zero, ρ_i^{-1} must be larger than $\sigma_s^{-2} + P_{s,i}\left(\mathbf{C}^{-1}{}_{(i,i)}\right)^{-1}$. It means the MSE constraint cannot be satisfied by the direct link. And, ρ_i^{-1} must be smaller than $\sigma_n^{-2} P_{s,i} \sigma_{sr,i}^2 + \sigma_s^{-2} + P_{s,i}\left(\mathbf{C}^{-1}{}_{(i,i)}\right)^{-1}$, it means the MSE constraint can be satisfied by the direct link and the channel from source to relay and then to destination.

The channel effect is obvious. So we draw the following conclusions from (4-40) and (4-52). When $\sigma_{sd,i}$ is large, $\mathbf{C}^{-1}{}_{(i,i)}$ will be small and $\left(C^{-1}{}_{(i,i)}\right)^{-1}$ will be large. So from (4-29) we see when $\sigma_{sd,i}$, $\sigma_{sr,i}$ and $\sigma_{rd,i}$ are large, then $\alpha_i(P_{r,i})$ will be large. On the contrary, $p_{s,i}$ will be small from (4-40), and $p_{r,i}$ will be small from (4-52).

4.2.3 General Case: Joint Source/relay Precoding

Now we solve the general power allocation problem using the results obtained in Section 4.2.1 and 4.2.2. Firstly, we can observe that (4-28) is not a convex optimization. As a result, the optimal value is difficult to derive even if the problem is formulated as a scalar-valued problem. However, by using the primal decomposition method [20], we can reformulate the original problem to obtain two convex problems - the master and the sub-problem optimization problems. Using this approach, the closed-form solutions can be easily obtained by means of KKT conditions. We give the detailed derivation in the following section.

First, we assume that $p_{r,i}$'s are known as a priori and then substitute the optimal solution for $p_{s,i}$ found in Section 4.2.1 into (4-28). Then the original problem in (4-28) becomes the function of $p_{r,i}$. We can then find the optimal solution using the Lagrange multiplier method. For a given $p_{r,i}$, the optimal solution for $p_{s,i}$ can be derived as

$$p_{s,i} = \left[\frac{\rho_i^{-1} - \sigma_s^{-2}}{\alpha_i(P_{r,i})} \right]^+ \tag{4-40}$$

where

$$\alpha_i(P_{r,i}) = \left(\sigma_n^{-2} \frac{P_{r,i}\sigma_{sr,i}^2\sigma_{rd,i}^2}{P_{r,i}\sigma_{rd,i}^2 + 1} + \frac{1}{\left|C^{-1}{}_{(i,i)}\right|} \right). \tag{4-53}$$

Substituting (4-53) into (4-32), we obtain

$$\min_{p_{r,i}} \sigma_s^2 \sum_{i=1}^{L} \frac{\rho_i^{-1} - \sigma_s^{-2}}{\left(\sigma_n^{-2} \frac{p_{r,i}\sigma_{sr,i}^2\sigma_{rd,i}^2}{p_{r,i}\sigma_{rd,i}^2 + 1} + \frac{1}{\left|C^{-1}{}_{(i,i)}\right|} \right)} + \sum_{i=1}^{L} p_{r,i}(\sigma_n^2 + \sigma_s^2 \frac{\rho_i^{-1} - \sigma_s^{-2}}{\left(\sigma_n^{-2} \frac{p_{r,i}\sigma_{sr,i}^2\sigma_{rd,i}^2}{p_{r,i}\sigma_{rd,i}^2 + 1} + \frac{1}{\left|C^{-1}{}_{(i,i)}\right|} \right)} \sigma_{sr,i}^2)$$

$$s.t. \quad p_{r,i} \geq 0, \; i = 1, \cdots, L. \tag{4-54}$$

Note that if $p_{s,i} = 0$, the condition $\left(\sigma_s^{-2}\right)^{-1} \leq \rho_i$ cannot be satisfied. Thus, the Lagrange function with respect to the (4-54) is:

$$\begin{aligned} L = {} & \sigma_s^2 \sum_{i=1}^{L} \frac{\rho_i^{-1} - \sigma_s^{-2}}{\left(\sigma_n^{-2} \frac{p_{r,i}\sigma_{sr,i}^2\sigma_{rd,i}^2}{p_{r,i}\sigma_{rd,i}^2 + 1} + \frac{1}{\left|C^{-1}{}_{(i,i)}\right|} \right)} - \sum_{i=1}^{L} \mu_{r,i} p_{r,i} + \\ & \sum_{i=1}^{L} p_{r,i}(\sigma_n^2 + \sigma_s^2 \frac{\rho_i^{-1} - \sigma_s^{-2}}{\left(\sigma_n^{-2} \frac{p_{r,i}\sigma_{sr,i}^2\sigma_{rd,i}^2}{p_{r,i}\sigma_{rd,i}^2 + 1} + \frac{1}{\left|C^{-1}{}_{(i,i)}\right|} \right)} \sigma_{sr,i}^2) \end{aligned}. \tag{4-55}$$

The KKT conditions can be described by the following equations:

$$\begin{aligned} \frac{\partial L}{\partial p_{r,i}} = {} & \sigma_n^2 + \frac{\sigma_s^2 \rho_i^{-1} - 1}{\left(\sigma_n^{-2} \frac{\sigma_{sr,i}^2\sigma_{rd,i}^2}{\sigma_{rd,i}^2 + p_{r,i}{}^{-1}} + \frac{1}{\left|C^{-1}{}_{(i,i)}\right|} \right)} \sigma_{sr,i}^2 - \mu_{r,i} - \\ & \left(p_{r,i}\sigma_{sr,i}^2 + 1\right) \frac{\left(\sigma_s^2\rho_i^{-1} - 1\right)\sigma_n^{-2}\sigma_{sr,i}^2\sigma_{rd,i}^2 p_{r,i}{}^{-2}}{\left(\sigma_n^{-2} \frac{\sigma_{sr,i}^2\sigma_{rd,i}^2}{\sigma_{rd,i}^2 + p_{r,i}{}^{-1}} + \frac{1}{\left|C^{-1}{}_{(i,i)}\right|} \right)^2 \left(\sigma_{rd,i}^2 + p_{r,i}{}^{-1}\right)^2}, \\ = {} & 0 \end{aligned} \tag{4-56}$$

$$\mu_{r,i} p_{r,i} = 0, \tag{4-57}$$

$$\mu_{r,i} \geq 0, \tag{4-58}$$

$$\lambda_i \geq 0, \tag{4-59}$$

$$p_{r,i} \geq 0, \tag{4-60}$$

If $p_{r,i} > 0$, from (4-57), $\mu_{r,i} = 0$. Then,

$$\begin{aligned}
&\sigma_n^2\left(\sigma_n^{-2}\frac{\sigma_{sr,i}^2\sigma_{rd,i}^2}{\sigma_{rd,i}^2+p_{r,i}{}^{-1}}+\frac{1}{\left|C^{-1}{}_{(i,i)}\right|}\right)^2\left(\sigma_{rd,i}^2+p_{r,i}{}^{-1}\right)^2 \\
&+\left(\sigma_s^2\rho_i^{-1}-1\right)\sigma_{sr,i}^2\left(\sigma_n^{-2}\frac{\sigma_{sr,i}^2\sigma_{rd,i}^2}{\sigma_{rd,i}^2+p_{r,i}{}^{-1}}+\frac{1}{\left|C^{-1}{}_{(i,i)}\right|}\right)\left(\sigma_{rd,i}^2+p_{r,i}{}^{-1}\right)^2 \\
&=\left(p_{r,i}\sigma_{sr,i}^2+1\right)\left(\sigma_s^2\rho_i^{-1}-1\right)\sigma_n^{-2}\sigma_{sr,i}^2\sigma_{rd,i}^2p_{r,i}{}^{-2}.
\end{aligned} \tag{4-61}$$

Dividing both sides of (4-61) by $\left(\sigma_s^2\rho_i^{-1}-1\right)\sigma_{sr,i}^2$, and letting $\alpha=\sigma_n^{-2}\sigma_{sr,i}^2\sigma_{rd,i}^2$, $\beta=\left(\sigma_s^2\rho_i^{-1}-1\right)$, $\gamma=\left(\sigma_{rd,i}^2+p_{r,i}{}^{-1}\right)$, and $\delta=\left(\left|C^{-1}{}_{(i,i)}\right|\right)^{-1}$, we have

$$\begin{aligned}
&\gamma^2\left(\frac{\delta^2\sigma_n^2}{\beta\sigma_{sr,i}^2}+\delta-\sigma_n^{-2}\sigma_{rd,i}^2\right)+\gamma\left(\frac{2\alpha\delta\sigma_n^2}{\beta\sigma_{sr,i}^2}+\underbrace{\alpha-\sigma_n^{-2}\sigma_{rd,i}^2\sigma_{sr,i}^2}_{0}+2\sigma_n^{-2}\sigma_{rd,i}^4\right) \\
&+\left(\frac{\alpha^2\sigma_n^2}{\beta\sigma_{sr,i}^2}+\sigma_n^{-2}\sigma_{sr,i}^2\sigma_{rd,i}^4-\sigma_n^{-2}\sigma_{rd,i}^6\right)=0
\end{aligned} \tag{4-62}$$

Substituting $\gamma=\left(\sigma_{rd,i}^2+p_{r,i}{}^{-1}\right)$ and multiplying both sides of (4-62) by ρ_i^2 leads to

$$\begin{aligned}
&\underbrace{\left(\frac{\delta^2\sigma_n^2\sigma_{rd,i}^4}{\beta\sigma_{sr,i}^2}+\delta\sigma_{rd,i}^4+\frac{2\alpha\delta\sigma_n^2\sigma_{rd,i}^2}{\beta\sigma_{sr,i}^2}+\frac{\alpha^2\sigma_n^2}{\beta\sigma_{sr,i}^2}+\sigma_n^{-2}\sigma_{sr,i}^2\sigma_{rd,i}^4\right)}_{:=a_i}p_{r,i}{}^2 \\
&+\underbrace{\left(\frac{2\sigma_{rd,i}^2\delta^2\sigma_n^2}{\beta\sigma_{sr,i}^2}+2\sigma_{rd,i}^2\delta\frac{2\alpha\delta\sigma_n^2}{\beta\sigma_{sr,i}^2}\right)}_{:=b_i}p_{r,i}+\underbrace{\left(\frac{\delta^2\sigma_n^2}{\beta\sigma_{sr,i}^2}+\delta-\sigma_n^{-2}\sigma_{rd,i}^2\right)}_{:=c_i}=0
\end{aligned} \tag{4-63}$$

From the solution $\left(-b_i+\sqrt{b_i^2-4a_ic_i}\right)/2a_i$ we know that if $p_{r,i}$ is larger than zero, $b_i^2-4a_ic_i$ must be positive and larger than b_i^2. Since a_i and b_i are larger than zero but c_i

may not, $b_i^2 - 4a_ic_i$ is not guaranteed to be larger than b_i^2. The conditions, $c_i > 0$, indicate that the channel quality of the direct link is better than that of the relay link. In this case, we can only use the direct link to satisfy the MSE constraint. So we can use c_i as a flag to decide if we want to use the relay or not. Thus, if $c_i > 0$, the system will be degenerated to a noncooperative system and

$$p_{r,i} = 0\,, \tag{4-64}$$

If $c < 0$, we can find $p_{r,i}$ by

$$p_{r,i} = \frac{-b + \sqrt{b^2 - 4ac}}{2a}\,, \tag{4-65}$$

where

$$a = \frac{\delta^2\sigma_n^2\sigma_{rd,i}^4}{\beta\sigma_{sr,i}^2} + \delta\sigma_{rd,i}^4 + \frac{2\alpha\delta\sigma_n^2\sigma_{rd,i}^2}{\beta\sigma_{sr,i}^2} + \frac{\alpha^2\sigma_n^2}{\beta\sigma_{sr,i}^2} + \sigma_n^{-2}\sigma_{sr,i}^2\sigma_{rd,i}^4\,, \tag{4-66}$$

$$b = \frac{2\sigma_{rd,i}^2\delta^2\sigma_n^2}{\beta\sigma_{sr,i}^2} + 2\sigma_{rd,i}^2\delta\frac{2\alpha\delta\sigma_n^2}{\beta\sigma_{sr,i}^2}\,, \tag{4-67}$$

$$c = \frac{\delta^2\sigma_n^2}{\beta\sigma_{sr,i}^2} + \delta - \sigma_n^{-2}\sigma_{rd,i}^2\,. \tag{4-68}$$

The source power $p_{s,i}$ can be found by

$$p_{s,i} = \frac{\rho_i^{-1} - \sigma_s^{-2}}{\alpha_i(P_{r,i})}\,, \tag{4-69}$$

where

$$\alpha_i(P_{r,i}) = \left(\sigma_n^{-2}\frac{P_{r,i}\sigma_{sr,i}^2\sigma_{rd,i}^2}{P_{r,i}\sigma_{rd,i}^2 + 1} + \frac{1}{\left|C^{-1}{}_{(i,i)}\right|}\right). \tag{4-70}$$

As that in Special Case I, the MSE constraint ρ_i must be small than the transmitted symbol power σ_s^2.

As discussed above our purpose is to satisfy the QoS with a minimum transmission power. In our processing, we try to make the precoding matrices at the source node and the relay node diagonal, and this makes the solution suboptimal. Because the optimal precoder matrix structure may not be diagonal, we are inquisitive about the question when the constraint precoder structure becomes optimal. To answer this question and simplify the derivation, we first assume that $N_t \le N_r$, $N_t \le N_d$, $N_d \le N_r$ and ignore the transmission in the source-destination link, this make we don't need to consider upper bound problem. Then the system model of (4-1) can be rewritten as

$$\mathbf{y}_d = \underbrace{\mathbf{H}_{rd}\mathbf{F}_r\mathbf{H}_{sr}}_{:=\mathbf{H}} \mathbf{F}_s\mathbf{s} + \underbrace{\mathbf{H}_{rd}\mathbf{F}_r\mathbf{n}_r + \mathbf{n}_d}_{:=\mathbf{w}}, \tag{4-71}$$

where $\mathbf{H}=\mathbf{H}_{rd}\mathbf{F}_r\mathbf{H}_{sr}$ and $\mathbf{w}=\mathbf{H}_{rd}\mathbf{F}_r\mathbf{n}_r+\mathbf{n}_d$. With the similar approach in Section 4.2, we can obtain the optimal receive filter as

$$\mathbf{G}_{opt} = \mathbf{R}_s\mathbf{F}_s^H\mathbf{H}^H(\mathbf{H}\mathbf{F}_s\mathbf{R}_s\mathbf{F}_s^H\mathbf{H}^H + \mathbf{R}_w)^{-1}, \tag{4-72}$$

and the corresponding MMSE as

$$J_{min} = tr\left\{(\mathbf{R}_s^{-1} + \mathbf{F}_s^H\mathbf{H}^H\mathbf{R}_w^{-1}\mathbf{H}\mathbf{F}_s)^{-1}\right\}, \tag{4-73}$$

where $\mathbf{R}_s = E[\mathbf{s}\mathbf{s}^H] = \sigma_s^2\mathbf{I}_N$, $\mathbf{R}_w = \sigma_{n_d}^2\mathbf{I}_{N_d} + \sigma_{n_r}^2\mathbf{H}_{rd}\mathbf{F}_r\mathbf{F}_r^H\mathbf{H}_{rd}^H$, σ_{n_d} and σ_{n_r} are noise variances of the channels $\mathbf{H}_{sr}$ and $\mathbf{H}_{rd}$, respectively. Express the transmission power as

$$P = \sigma_s^2 tr\left\{\mathbf{F}_s\mathbf{F}_s^H\right\} + \sigma_{n_r}^2 tr\left\{\mathbf{F}_r\mathbf{F}_r^H\right\} + \sigma_s^2 tr\left\{\mathbf{F}_r\mathbf{H}_{sr}\mathbf{F}_s\mathbf{F}_s^H\mathbf{H}_{sr}^H\mathbf{F}_r^H\right\} \tag{4-74}$$

and the channel SVDs as

$$
\underbrace{\mathbf{H}_{sr}}_{N_r\times N_t} = \begin{pmatrix} \underbrace{\mathbf{U}_{sr}}_{N_r\times N_t} & \underbrace{\overline{\mathbf{U}}_{sr},}_{N_r\times(N_r-N_t)} \end{pmatrix} \begin{pmatrix} \underbrace{\mathbf{\Sigma}_{sr}}_{N_t\times N_t} \\ \underbrace{\mathbf{0}}_{(N_r-N_t)\times N_t} \end{pmatrix} \underbrace{\mathbf{V}_{sr}^H}_{N_t\times N_t} = \underbrace{\mathbf{U}_{sr}}_{N_r\times N_t} \underbrace{\mathbf{\Sigma}_{sr}}_{N_t\times N_t} \underbrace{\mathbf{V}_{sr}^H}_{N_t\times N_t} \tag{4-75}
$$

and

$$
\underbrace{\mathbf{H}_{rd}}_{N_d\times N_r} = \underbrace{\mathbf{U}_{rd}}_{N_d\times N_d} \begin{pmatrix} \underbrace{\mathbf{\Sigma}_{rd}}_{N_d\times N_d} & \underbrace{\mathbf{0}}_{N_d\times(N_r-N_d)} \end{pmatrix} \begin{pmatrix} \underbrace{\mathbf{V}_{rd}^H}_{N_d\times N_r} \\ \underbrace{\overline{\mathbf{V}}_{rd}^H}_{(N_r-N_d)\times N_r} \end{pmatrix} = \underbrace{\mathbf{U}_{rd}}_{N_d\times N_d} \underbrace{\mathbf{\Sigma}_{rd}}_{N_d\times N_d} \underbrace{\mathbf{V}_{rd}^H}_{N_d\times N_r}, \tag{4-76}
$$

where $\mathbf{\Sigma}_{sr}$ and $\mathbf{\Sigma}_{rd}$ are diagonal matrices. If we have the precoder in the source node as

$$
\mathbf{F}_s = \mathbf{V}_{sr}\mathbf{\Sigma}_s \in \mathbb{C}^{N_t\times N}, \tag{4-77}
$$

where $\mathbf{\Sigma}_s \in \mathbb{R}^{N_t\times N}$ is a diagonal matrix. Then the MMSE and the transmission power can be rewritten as

$$
\begin{aligned} J_{min}(\mathbf{F}_r) = tr\Big\{ \Big(\sigma_s^{-2}\mathbf{I}_N + \Sigma_s^H \Sigma_{sr}^H \mathbf{U}_{sr}^H \mathbf{F}_r^H \mathbf{V}_{rd} \Sigma_{rd}^H \left(\sigma_{n_d}^2 \mathbf{I}_{N_d} + \sigma_{n_r}^2 \Sigma_{rd} \mathbf{V}_{rd}^H \mathbf{F}_r \mathbf{F}_r^H \mathbf{V}_{rd} \Sigma_{rd}^H \right)^{-1} \times \\ \Sigma_{rd} \mathbf{V}_{rd}^H \mathbf{F}_r \mathbf{U}_{sr} \Sigma_{sr} \Sigma_s \Big)^{-1} \Big\} \end{aligned} \tag{4-78}
$$

and

$$
P(\mathbf{F}_r) = \sigma_s^2 tr\left\{\Sigma_s \Sigma_s^H\right\} + \sigma_{n_r}^2 tr\left\{\mathbf{F}_r\mathbf{F}_r^H\right\} + \sigma_s^2 tr\left\{\mathbf{F}_r \mathbf{U}_{sr} \Sigma_{sr} \Sigma_s \Sigma_s^H \Sigma_{sr}^H \mathbf{U}_{sr}^H \mathbf{F}_r^H\right\}. \tag{4-79}
$$

Consider a general form of $\mathbf{F}_r$ as

$$
\mathbf{F}_r = \begin{pmatrix} \mathbf{V}_{rd} & \overline{\mathbf{V}}_{rd} \end{pmatrix} \begin{pmatrix} \mathbf{\Sigma}_{r_{11}} & \mathbf{\Sigma}_{r_{12}} \\ \mathbf{\Sigma}_{r_{21}} & \mathbf{\Sigma}_{r_{22}} \end{pmatrix} \begin{pmatrix} \mathbf{U}_{sr}^H \\ \overline{\mathbf{U}}_{sr}^H \end{pmatrix} \in \mathbb{C}^{N_r\times N_r}. \tag{4-80}
$$

Through some tedious derivations, it is easy to show that if we let $\mathbf{\Sigma}_{r_{21}} = \mathbf{0}$ and $\mathbf{\Sigma}_{r_{22}} = \mathbf{0}$ there will be no impact on $J_{min}(\mathbf{F}_r)$ (while helping save power consumption). Setting $\mathbf{\Sigma}_{r_{12}} = 0$ leads to the reduction of both $J_{min}(\mathbf{F}_r)$ and $P(\mathbf{F}_r)$. Thus, we can let the precoder be

$$
\mathbf{F}_r = \mathbf{V}_{rd}\mathbf{\Sigma}_{r_{11}}\mathbf{U}_{sr}^H. \tag{4-81}
$$

The remaining work is to investigate the structure of $\mathbf{\Sigma}_{r_{11}}$. Substituting (4-82) into (4-79) and (4-80), we can obtain

$$
\begin{aligned}
J_{min}(\mathbf{F}_r) = \sigma_s^2 N_t - tr\Big\{\Big(\sigma_s^4 \tilde{\mathbf{\Sigma}}_{r_{11}} \Sigma_{sr} \Sigma_s \Sigma_s^H \Sigma_{sr}^H \tilde{\mathbf{\Sigma}}_{r_{11}}^H \times \Big(\sigma_{n_d}^2 \mathbf{I}_{N_d} + \\
\tilde{\mathbf{\Sigma}}_{r_{11}} \left(\sigma_{n_r}^2 \mathbf{I}_{N_t} + \sigma_s^2 \Sigma_{sr} \Sigma_s \Sigma_s^H \Sigma_{sr}^H\right) \tilde{\mathbf{\Sigma}}_{r_{11}}^H\Big)^{-1}\Big\}
\end{aligned}
\qquad (4\text{-}82)
$$

and

$$
\mathbf{P}(\mathbf{F}_r) = \sigma_s^2 tr\left(\Sigma_s \Sigma_s^H\right) + tr\left(\Sigma_{rd}^{-H} \Sigma_{rd}^{-1} \tilde{\mathbf{\Sigma}}_{r_{11}} \left(\sigma_{n_r}^2 I_{N_t} + \sigma_s^2 \Sigma_{sr} \Sigma_s \Sigma_s^H \Sigma_{sr}^H\right) \tilde{\mathbf{\Sigma}}_{r_{11}}^H\right). \qquad (4\text{-}83)
$$

In (4-83), we have used matrix inverse lemma in (4-5) and let $\tilde{\mathbf{\Sigma}}_{r_{11}} = \mathbf{\Sigma}_{rd} \mathbf{\Sigma}_{r_{11}}$. Consider the eigenvalue decomposition (ED) of

$$
\mathbf{D} = \tilde{\mathbf{\Sigma}}_{r_{11}} \left(\sigma_{n_r}^2 I_{N_t} + \sigma_s^2 \Sigma_{sr} \Sigma_s \Sigma_s^H \Sigma_{sr}^H\right) \tilde{\mathbf{\Sigma}}_{r_{11}}^H = \mathbf{U}_d \Sigma_d \mathbf{U}_d^H, \qquad (4\text{-}84)
$$

where the diagonal element $\{\sigma_{d,i}\}$ of Σ_d are arranged in the descending order. Note that the last (N_d - N_t) diagonal elements of Σ_d are nulls since rank($\mathbf{D}$) = N_t ($\leq$ N_d). Pre-multiplying $\tilde{\mathbf{\Sigma}}_{r_{11}}$ by $\mathbf{U}_d^H$ leads to a new precoder $\mathbf{F}_r^{'}$ with the same MSE, i.e. $J_{min}(\mathbf{F}_r) = J_{min}(\mathbf{F}_r^{'})$, and a lower power consumption, i.e.

$$
\begin{aligned}
P(\mathbf{F}_r) &= tr\left(\Sigma_{rd}^{-H} \Sigma_{rd}^{-1} D\right) + \sigma_s^2 tr\left(\Sigma_s \Sigma_s^H\right) \geq \sum_{i=1}^{N_t} \frac{\sigma_{d,i}}{\sigma_{d,i}^2} + \sigma_s^2 tr\left(\Sigma_s \Sigma_s^H\right) \\
&= tr\left(\Sigma_{rd}^{-H} \Sigma_{rd}^{-1} \Sigma_D\right) + \sigma_s^2 tr\left(\Sigma_s \Sigma_s^H\right) = P(\mathbf{F}_r^{'})
\end{aligned}
. \qquad (4\text{-}85)
$$

Here we use the fact that for any two $N \times N$ positive semi-definite matrices $\mathbf{A}$ and $\mathbf{B}$ whose eigenvalue $\lambda_i(\mathbf{A})$ and $\lambda_i(\mathbf{B})$ are arranging in the descending order, then [21]

$$
tr\{AB\} \geq \sum_{i=1}^{N} \lambda_i(\mathbf{A}) \lambda_{N-i+1}(\mathbf{B}). \qquad (4\text{-}86)
$$

Without loss of generality, we can further assume that

$$\widetilde{\boldsymbol{\Sigma}}_{r_{11}} = \begin{pmatrix} \overline{\Sigma}_d \\ 0 \end{pmatrix} \mathbf{T}^H \left(\sigma_n^2 I_{N_t} + \sigma_s^2 \Sigma_{sr} \Sigma_s \Sigma_s^H \Sigma_{sr}^H \right)^{-1/2}, \tag{4-87}$$

where $\overline{\Sigma}_d$ denotes the $N_t \times N_t$ top-left sub-matrix of Σ_d, and $\mathbf{T}$ is a $N_t \times N_t$ unitary matrix. Note that for certain $\overline{\Sigma}_d$, varying $\mathbf{T}$ impacts the MSE $J_{min}(\mathbf{F}_r)$ but not power consumption $P(\mathbf{F}_r)$. This leads to

$$J_{min}(\mathbf{F}_r) \geq \sigma_s^2 N_t - \sum_{i=1}^{N_t} \frac{\sigma_s^4 \sigma_{sr,i}^2 \Sigma_{d,i}^2}{\sigma_{rd,i}^2 + \Sigma_{d,i}^2}. \tag{4-88}$$

Here, we use the property in [21] again, i.e.,

$$tr\{\mathbf{AB}\} \leq \sum_{i=1}^{N} \lambda_i(\mathbf{A}) \lambda_i(\mathbf{B}). \tag{4-89}$$

Here, the lower bound is attained while $\mathbf{T}=\mathbf{I}_{N_t}$. Thus, for any given Σ_d, $J_{min}(\mathbf{F}_r)$ and $P(\mathbf{F}_r)$ are always minimized with

$$\boldsymbol{\Sigma}_{r_{11}} = \Sigma_{rd}^{-1} \begin{pmatrix} \overline{\Sigma}_d \\ 0 \end{pmatrix} \mathbf{T}^H \left(\sigma_n^2 I_{N_t} + \sigma_s^2 \Sigma_{sr} \Sigma_s \Sigma_s^H \Sigma_{sr}^H \right)^{-1/2}. \tag{4-90}$$

If let $\mathbf{R}_H = \mathbf{H}^H \mathbf{R}_w^{-1} \mathbf{H}$, then the $\lambda_{H,i}$'s of $\mathbf{R}_H$ for our problem is

$$\frac{\sigma_{r,i}^2 \sigma_{sr,i}^2 \sigma_{rd,i}^2}{\sigma_{r,i}^2 \sigma_{rd,i}^2 + 1}, \tag{4-91}$$

where $\sigma_{sr,i}$, $\sigma_{rd,i}$, and $\sigma_{r,i}$ are the ith diagonal elements of Σ_{sr}, Σ_{rd}, and Σ_r. From [17] we known that the suboptimal solution obtained under the diagonal constraint of the MSE matrix is the optimum if and only if

$$\lambda_{H,i} \rho_i^2 \geq \lambda_{H,i+1} \rho_{i+1}^2, \ 1 \leq i \leq L, \tag{4-92}$$

where the ρ_i 's are arranged in the decreasing order, and the $\lambda_{H,i}$'s are the largest eigenvalues of $\mathbf{R}_H$ in the increasing order. Substituting (4-92) into (4-93), we have

$$\frac{\sigma_{sr,i}^2\rho_i^2}{\sigma_{sr,i+1}^2\rho_{i+1}^2} \geq \frac{\sigma_{r,i+1}^2\sigma_{rd,i+1}^2 + \sigma_{r,i}^2\sigma_{r,i+1}^2\sigma_{rd,i}^2\sigma_{rd,i+1}^2}{\sigma_{r,i}^2\sigma_{rd,i}^2 + \sigma_{r,i}^2\sigma_{r,i+1}^2\sigma_{rd,i}^2\sigma_{rd,i+1}^2}. \tag{4-93}$$

If $\sigma_{r,i}^2\sigma_{rd,i}^2 \geq \sigma_{r,i+1}^2\sigma_{rd,i+1}^2$, then

$$\frac{\sigma_{r,i+1}^2\sigma_{rd,i+1}^2 + \sigma_{r,i}^2\sigma_{r,i+1}^2\sigma_{rd,i}^2\sigma_{rd,i+1}^2}{\sigma_{r,i}^2\sigma_{rd,i}^2 + \sigma_{r,i}^2\sigma_{r,i+1}^2\sigma_{rd,i}^2\sigma_{rd,i+1}^2} \geq \frac{\sigma_{r,i+1}^2\sigma_{rd,i+1}^2}{\sigma_{r,i}^2\sigma_{rd,i}^2}. \tag{4-94}$$

So, if

$$\sigma_{r,i}^2\sigma_{rd,i}^2 \geq \sigma_{r,i+1}^2\sigma_{rd,i+1}^2 \tag{4-95}$$

and

$$\underbrace{\sigma_{sr,i}^2\sigma_{rd,i}^2\rho_i^2}_{\lambda_i}\sigma_{r,i}^2 \geq \underbrace{\sigma_{sr,i+1}^2\sigma_{rd,i+1}^2\rho_{i+1}^2}_{\lambda_{i+1}}\sigma_{r,i+1}^2 \tag{4-96}$$

the suboptimal solution obtained under the diagonal constraint becomes the true optimum.

Chapter 5: Simulations

In this chapter, we report simulation results to evaluate the performance of the power allocation methods we have proposed. Specifically, we consider following scenarios: (1) SISO relay system (2) MIMO relay system: (a) special case I, (b) special case II, and (c) general case.

5.1 SISO Systems

We evaluate the performance of SISO relay system. In the system, the precoding problem is degenerated to a power-allocation problem. The system we consider is a typical three-node system (the source, the relay, and the destination), all channels are assumed to experience Rayleigh fading and has a same SNR which is 10 dB. Here, dB is defined as $10\log_{10}(.)$. The QoS is measured with the average BER. Here, we let the required BER be 10^{-3}, and use 50,000 symbols for each set of simulation. Figure 5-1 shows the simulated results for systems with the proposed power allocation (PA) and equal-power PA. In the equal-power PA scheme, the transmit power at the source node is equal to that at the relay node. To have a fair comparison, in each run of the simulation we first calculate the total power required for the proposed PA scheme and then use that for the equal-power PA scheme. Note that the horizontal axis in Figure 5-1 indicates the bit number mapped to a QAM symbol, and the vertical axis the BER. Since we use an upper bound to approximate the true BER, the BER yielded by the proposed algorithm will be always less than the desired BER. From the figure, we observe that our algorithm significantly better than the equal-power PA scheme. And the performance gap becomes larger when the QAM size is larger.

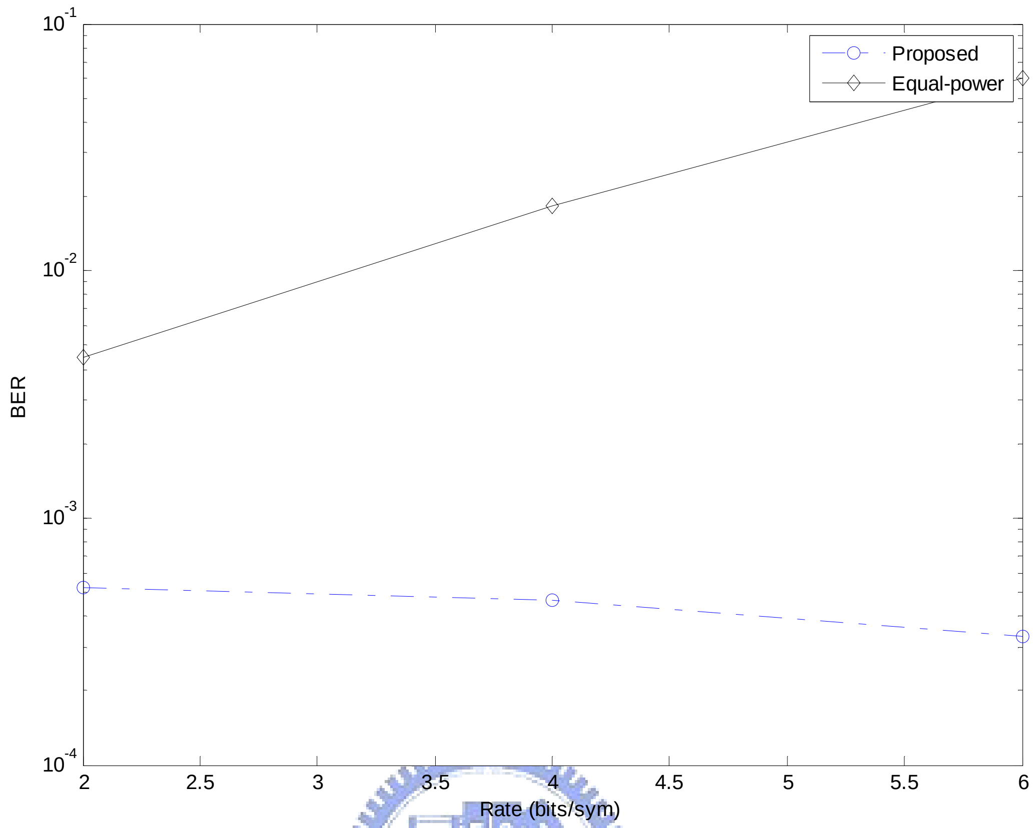

Figure 5-1: Performance comparison for equal-power and proposed PA schemes in SISO relay systems (BER constrain $=10^{-3}$).

5.2 MIMO Relay Systems

5.2.1 Special case I: Source precoding

In this subsection, we evaluate the performance of the proposed MIMO precoded relay system under the scenario that only the source precoder is considered. Each node is assumed to have four antennas. The SNRs for three channels in the system are set to be equal (SNR=10 dB), and the power used in the relay be 6.0206 dB. Also, the power is uniformly distributed in the diagonal matrix of $\mathbf{\Sigma}_r$. QoS here is measured with the MSE and its value is set as 10^{-1}. Figure 5-2 shows the MSE comparison of a conventional MIMO (non-cooperative) and the proposed MIMO precoded relay systems. For the conventional MIMO system, we can adjust the transmission power such that it can precisely satisfy the QoS constraint. However, for the proposed system, an upper bound is used in our derivation and the resultant MSE will be always less than the designated MSE. This is similar to that in the SISO system and Figure 5-2 clearly verifies the result. Figure 5-4 gives the averaged power used for both systems under various MSEs (ranging from 10^{-1} to 10^{-3}). As we can see, the required transmission power of the proposed scheme is significantly smaller than the conventional MIMO system. For the simulation setting considered here, a 7dB reduction can be obtained. Also, the reduction is almost independent of the required MSE.

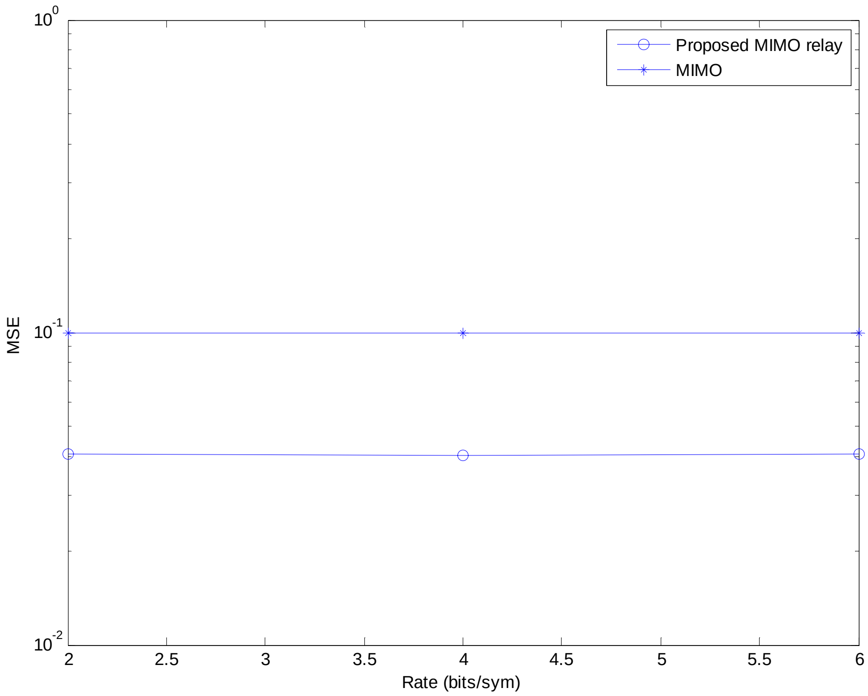

Figure 5-2: MSE comparison for conventional MIMO and proposed MIMO relay systems (MSE constrain = 10^{-1}).

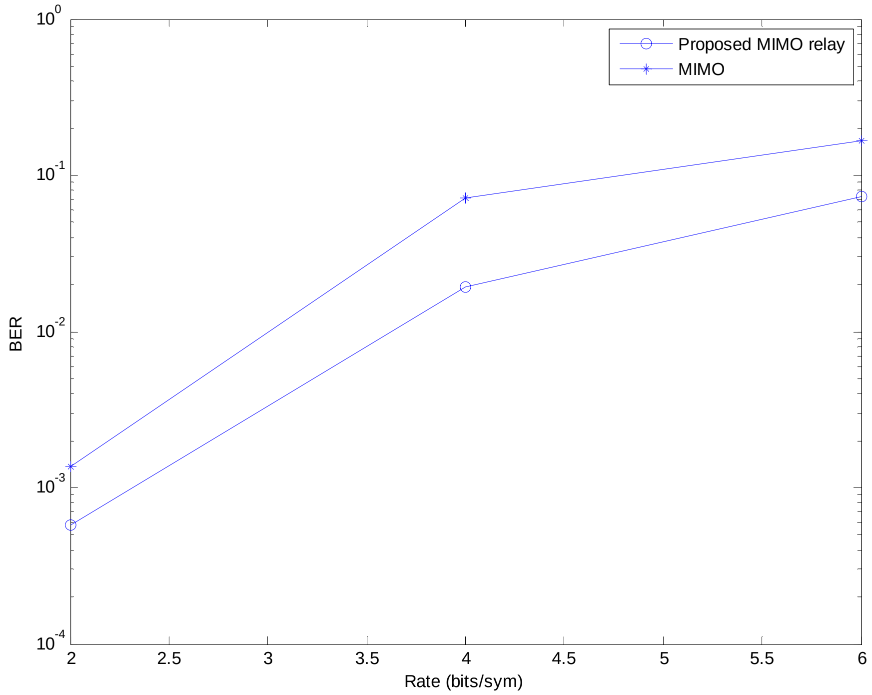

Figure 5-3: BER comparison for conventional MIMO and proposed MIMO relay systems

(MSE constrain = 10^{-1}).

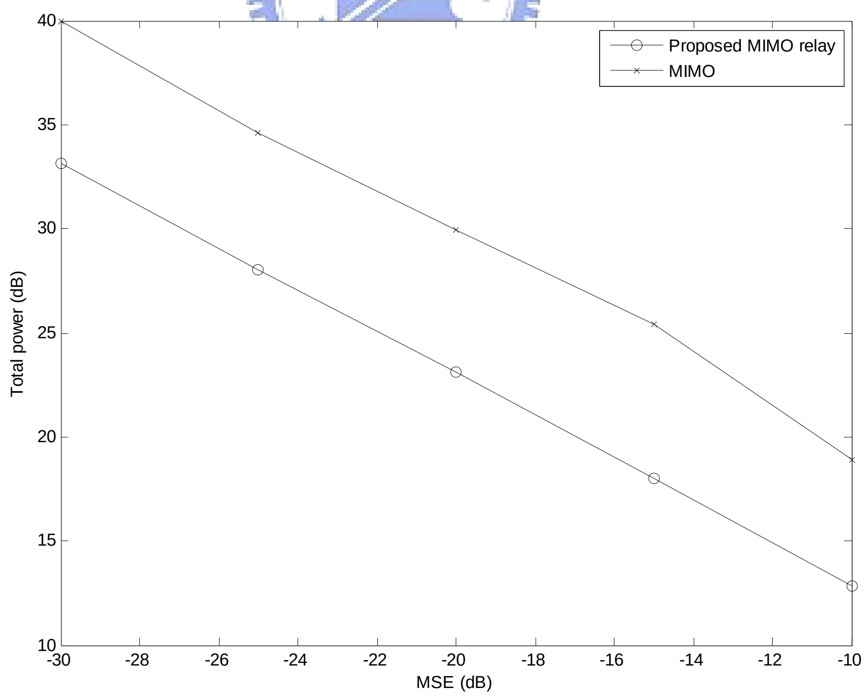

Figure 5-4: Power-consumption comparison for conventional MIMO and proposed MIMO relay systems

5.2.2 Special case II: Relay precoding

In this subsection, we evaluate the performance of the proposed MIMO precoded relay system under the scenario that only the relay precoder is considered. Each node is assumed to have four antennas. The SNR of the source-to-destination channel is set 5 dB lower than other two channels (10 dB), and the power in the source is set as 20 dB. Also, the power is uniformly distributed in the diagonal matrix of $\mathbf{\Sigma}_s$. Similar to the previous case, the QoS here is measured with the MSE and its value is set as 10^{-1}. Figure 5-5 shows the MSE comparison of the conventional MIMO and the proposed MIMO precoded relay systems. As we can see, the MSE of the proposed system is less than the designated MSE. This behavior is also similar to previous cases except that the gap seems larger. Figure 5-7 gives the averaged power used for both systems under various MSEs (ranging from 10^{-1} to 10^{-3}). As we can see, the required transmission power of the proposed scheme is still significantly smaller than the conventional MIMO system. For the simulation setting considered here, a 5dB reduction can be obtained.

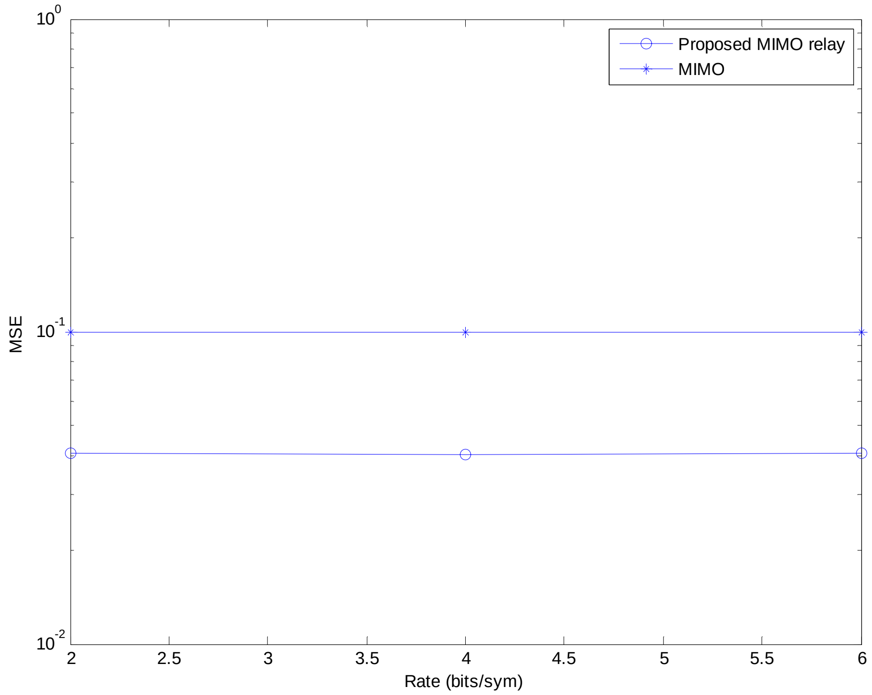

Figure 5-5: MSE comparison for conventional MIMO and proposed MIMO relay systems

(MSE constrain = 10^{-1})

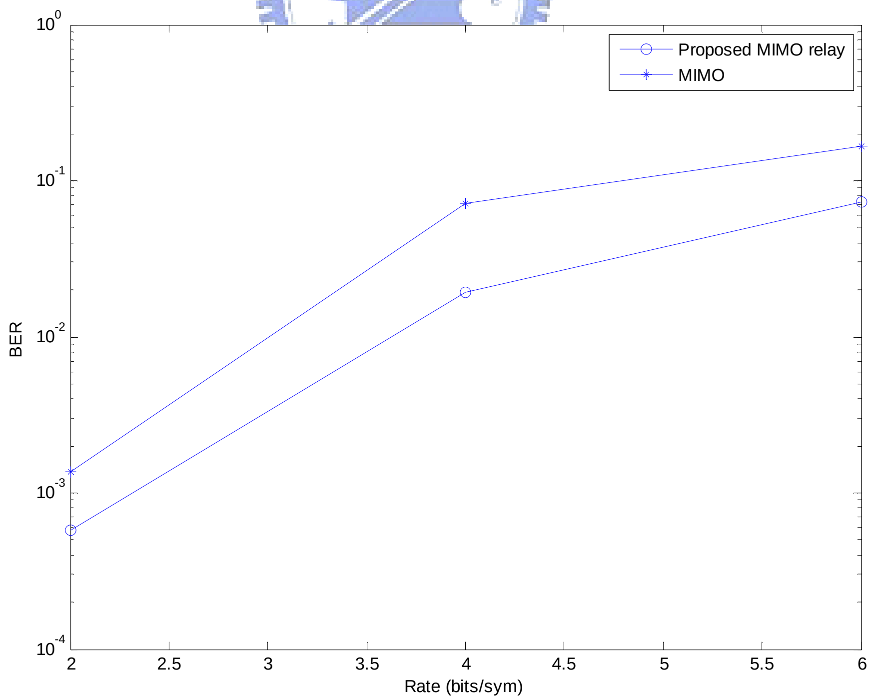

Figure 5-6: BER comparison for conventional MIMO and proposed MIMO relay systems

(MSE constrain = 10^{-1})

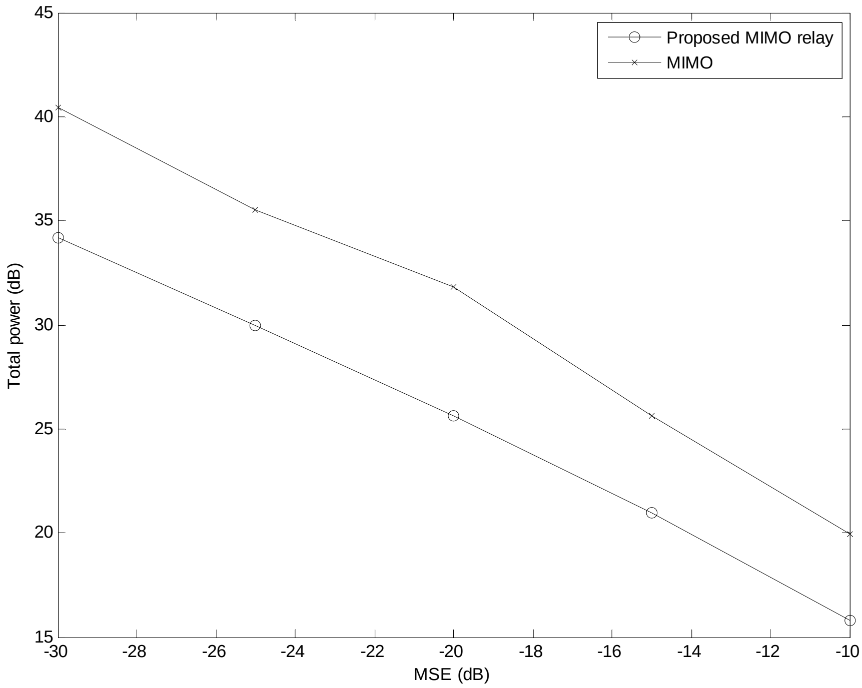

Figure 5-7: Power-consumption comparison for conventional MIMO and proposed MIMO relay systems

5.2.3 General case: Joint source/relay precoding

We now evaluate the performance of the proposed precoded MIMO relay system under the general scenario that both the source and relay precoders are considered. As previously, each node is assume to have four antennas. Here, the SNR of the source-to-destination channel is set to be lower than the other two channels by 5 dB. We still use the MSE as the measure of QoS, and let the target MSE be 10^{-1}. Figure 5-6 shows the MSE comparison of the conventional MIMO system, the proposed MIMO precoded system without the direct link, and the proposed MIMO precoded relay systems. The MSE of the proposed system with direct link is less than the designated MSE, however, its gap is smaller than the previous case. Figure 5-7 gives the averaged power used for both systems under various MSEs (ranging from 10^{-1} to 10^{-3}). As we can see, the required transmission power of the proposed scheme (with the direct link) is dramatically smaller than the conventional MIMO system. For the simulation setting considered here, a 7 dB reduction can be obtained for the proposed system without the direct link, and a 14 dB reduction can be obtained for the proposed system with the direct link.

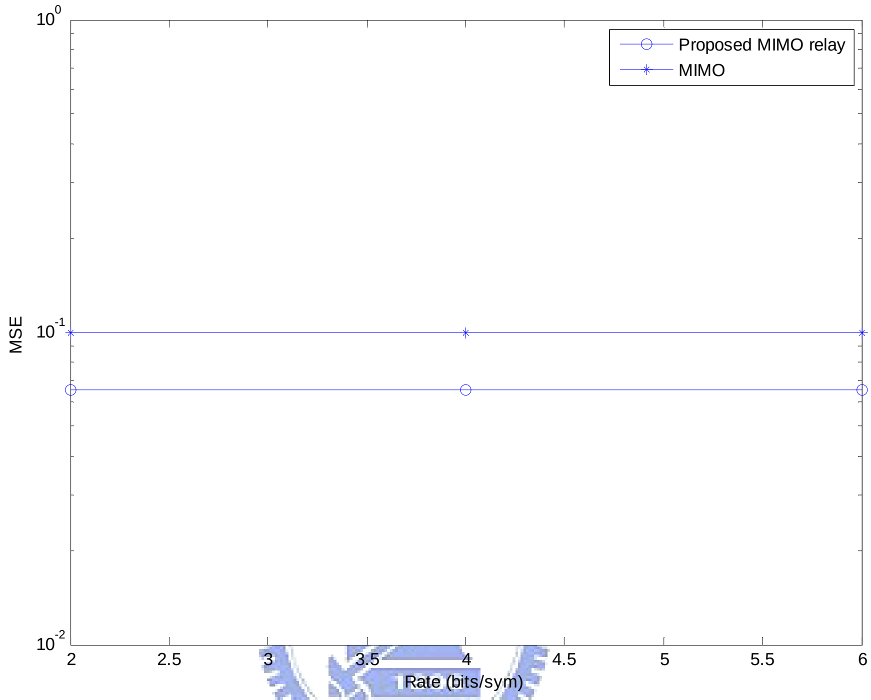

Figure 5-8: MSE comparison for conventional MIMO and proposed MIMO relay systems

(MSE constrain = 10^{-1})

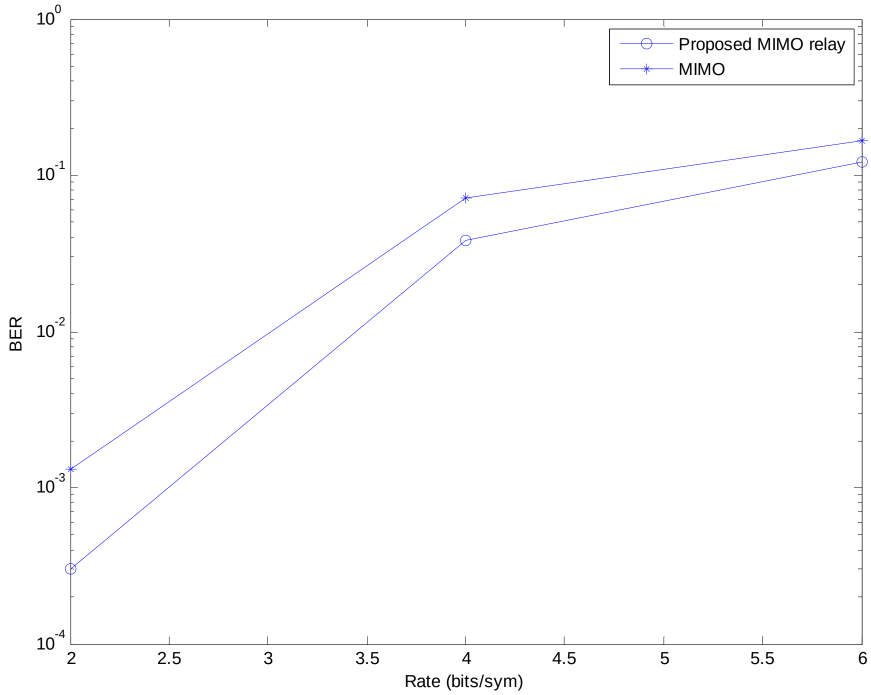

Figure 5-9: BER comparison for conventional MIMO and proposed MIMO relay systems

(MSE constrain = 10^{-1})

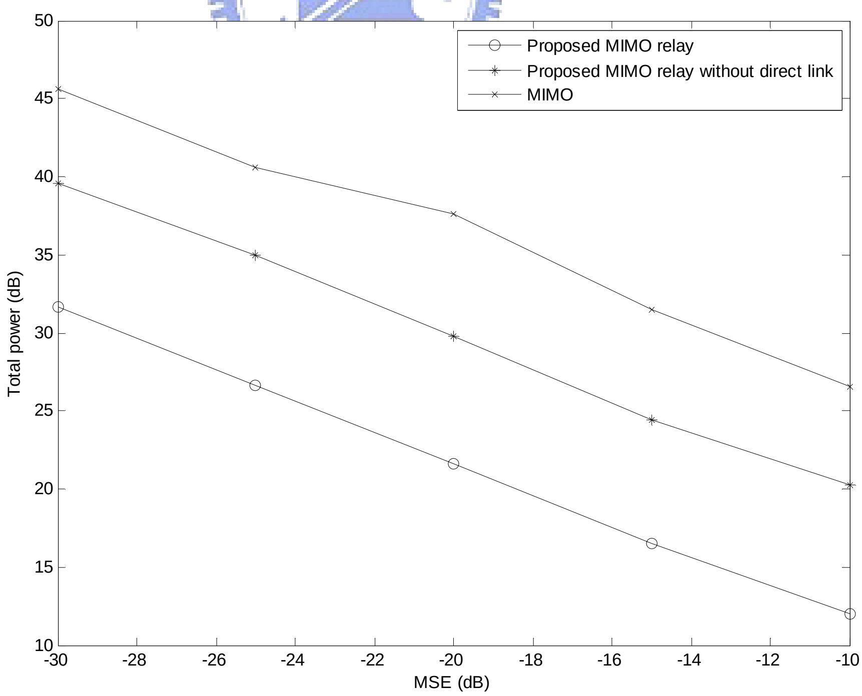

Figure 5-10: MSE comparison for conventional MIMO, the proposed MIMO relay without direct link, and the proposed MIMO systems.

From Figure 5-10, we see that even without the direct link, the proposed system requires less power than the conventional non-cooperative MIMO system. This is due to the precoders used in the source and the relay. The proposed scheme jointly designs the precoders such that the required transmission power can be minimized.

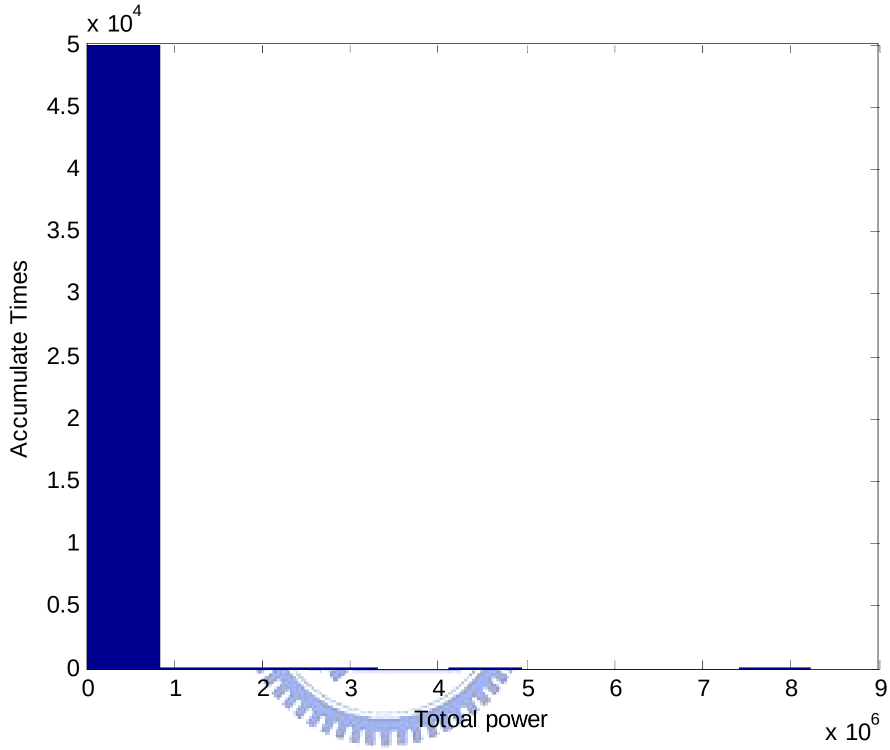

Figure 5-11: Power distribution of total power in the proposed MIMO relay system.

Figure 5.11 gives the total power distribution in the proposed system. It is observable that the distribution exhibits an impulse-like characteristic. When the required power is larger than the maximum allowable power, we say that the system is in outage. In other words, the required QoS cannot be met in the case.

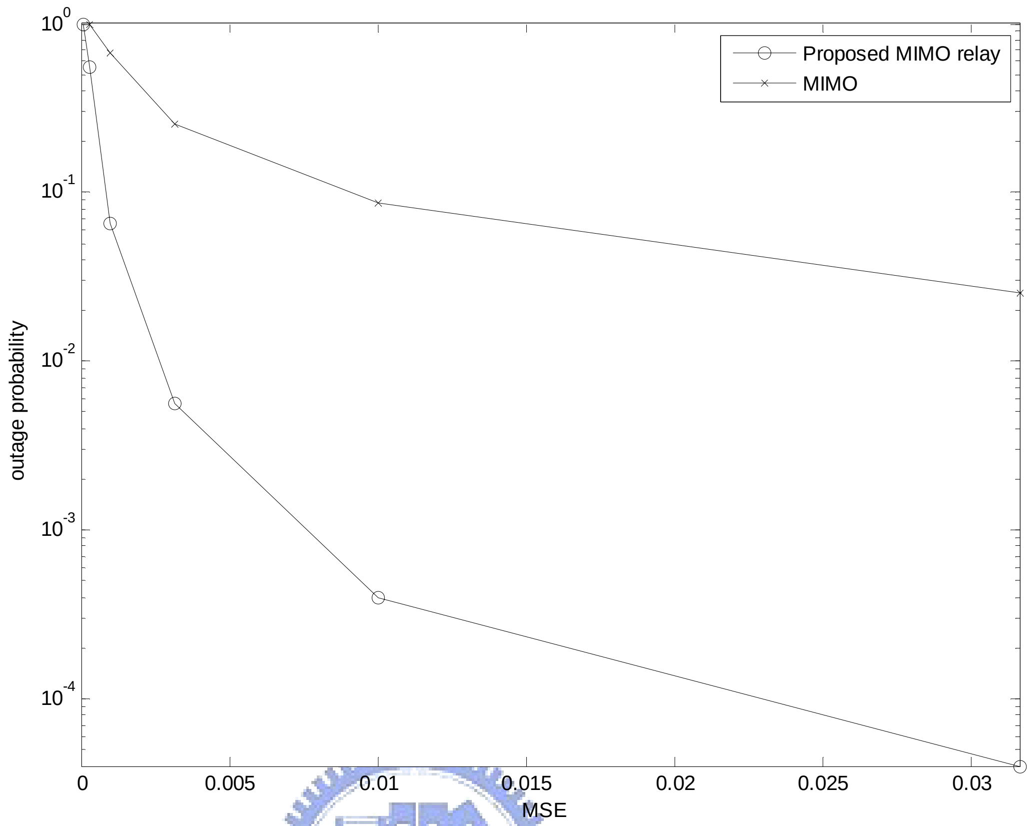

Figure 5-12: Outage probability of conventional MIMO and the proposed MIMO relay systems.

Figure 5.12 presents the outage probability for a power constraint of 40 dB. From the figure, we see that the outage probability of the proposed precoded system is much smaller than the conventional MIMO system. Also, the slope of the outage curve for the proposed system is larger than that for the MIMO system. This indicates that the proposed system has a larger diversity gain.

Chapter 6: Conclusions

In this thesis, we consider the joint source/relay precoder design in MIMO AF relay system. We assume that full channel-sate-information is available at the source, the relay, and the destination, and propose a method to minimize the total transmission power under a QoS constraint. The QoS we considered is the average MSE of the MMSE receiver. Since the cost function of the minimization problem is difficult to solve, we then propose to use an upper bound of the MSE matrix as an alternative constraint function. Using a specially designed precoder structure, we are able to convert the problem into a scalar-valued optimization problem. However, the optimal solution is still difficult to derive. We then use the primal decomposition method to translate the problem into two standard convex optimization problems. Resorting to the KKT conditions, we finally solve the optimum precoders for the MIMO relay system. Simulations show that the proposed precoded MIMO system consumes much less power than the conventional non-cooperative MIMO systems. For the simulation setting we used, the reduced power can be as large as 14 dB. In this thesis, we only consider the linear receiver, i.e, the MMSE receiver. As well known, nonlinear receivers, such as the QR successive interference cancellation (QR-SIC) receivers and the maximum-likelihood receiver, can have much better performance than linear receivers. The joint precoders design problem in the systems can serve as the topics for further research.

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