CHAPTER 5

IMPLEMENTATION OF DYNAMICAL DOUBLE EWMA CONTROLLER

In this chapter, we first present how to determine the discount factor, and then we implement the proposed tuning method by using the software of Matlab/Simulink version 4.1. After that, we will make a comparison between the fixed weight trade-off solution and the proposed time-varying tuning method by running the Monte Carlo simulations.

5.1 Implement Time-Varying Weights Tuning Strategy

For the control performance characterization, the normalized mean square error (MSE/σ_ε^2) is used as the performance measure. The prediction MSE is defined as follows:

$$\hat{MSE} = \frac{\sum_{t=1}^{m} y_t^2}{m}$$

Therefore, the normalized mean square error is the measure of inflation for the controlled process produced against the natural disturbance (ε_t). To implement the proposed tuning method, the first task is to determine the discount factor. The objective of the chosen discount factor is to minimize the normalized mean square error.

Consider a drifting process model in Equation (2.13) with $\alpha = 0$, $T = 0$, $\xi = \beta / b = 1$, $\delta = 1$, $\sigma_\varepsilon^2 = 1$ and $m = 200$. By solving Equation (2.33), we obtain the all-variance solution weight $(\lambda_{1,v}, \lambda_{2,v}) = (0.0247, 0.1486)$. Therefore, our proposed tuning method can be expressed as follows:

$$\lambda_1^* = \max\{0.0247, 0.1486\} \tag{5.1}$$

$$\lambda_2^*(t) = \min\{0.0247, 0.1486\} + (f)^t \tag{5.2}$$

A plot of the normalized mean square errors of the controlled process output, obtained from a series of trial values of the discount factor, is shown in Figure 5.1. The minimum value of $\hat{MSE}/\sigma_\varepsilon^2$ appears at $f^* = 0.92$. Table 5.1 shows a similar process for determining the discount factor under other drifting speeds. We can see that whatever the drifting speed is, the optimal discount factor always appears at $f^* = 0.92$. Thus, it shows the robustness for determining the discount factor under our proposed tuning method. Substituting $f^* = 0.92$ into Equation (5.2), Figure 5.2 shows the controlled process output, and it shows that observations wander around the target value (*T=0*) with $\hat{MSE}/\sigma_\varepsilon^2 = 1.2481$. In addition, if we use the fixed trade-off solution weights to control the process, then $\hat{MSE}/\sigma_\varepsilon^2 = 1.6183$. Therefore, our proposed time-varying tuning method is 22.88% better than the fixed weights control scheme. The following section will make a more detailed comparison between the trade-off solution, and the proposed tuning method, by using the Monte Carlo simulations.

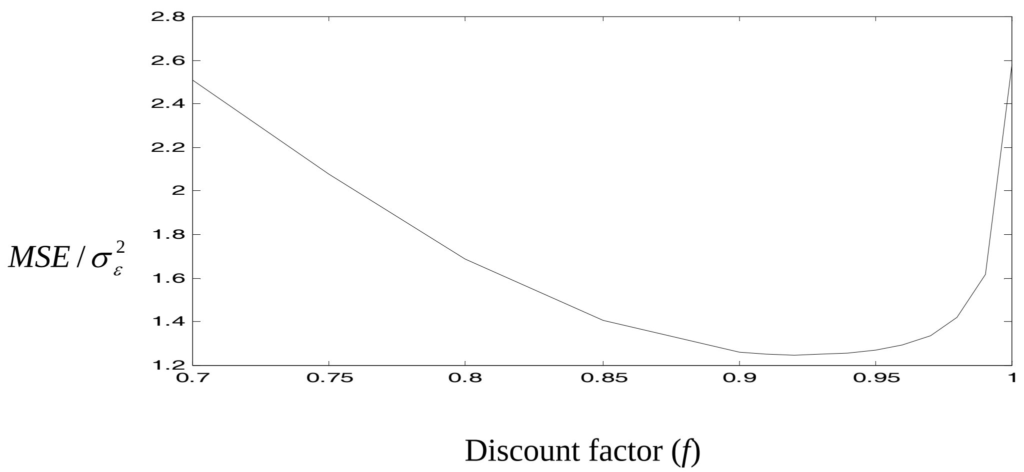

Figure 5.1 MSE/σ_ε^2 versus discount factor

Table 5.1 Optimal discount factors under various drifting speeds

δ	$\lambda_{1,v}$	$\lambda_{2,v}$	Optimal Discount factor
0.1	0.0067	0.0485	0.92
0.5	0.0173	0.1067	0.92
1.0	0.0247	0.1486	0.92
1.5	0.0303	0.1799	0.92
2.0	0.0351	0.2057	0.92
2.5	0.0393	0.2281	0.92
3.0	0.0432	0.2480	0.92

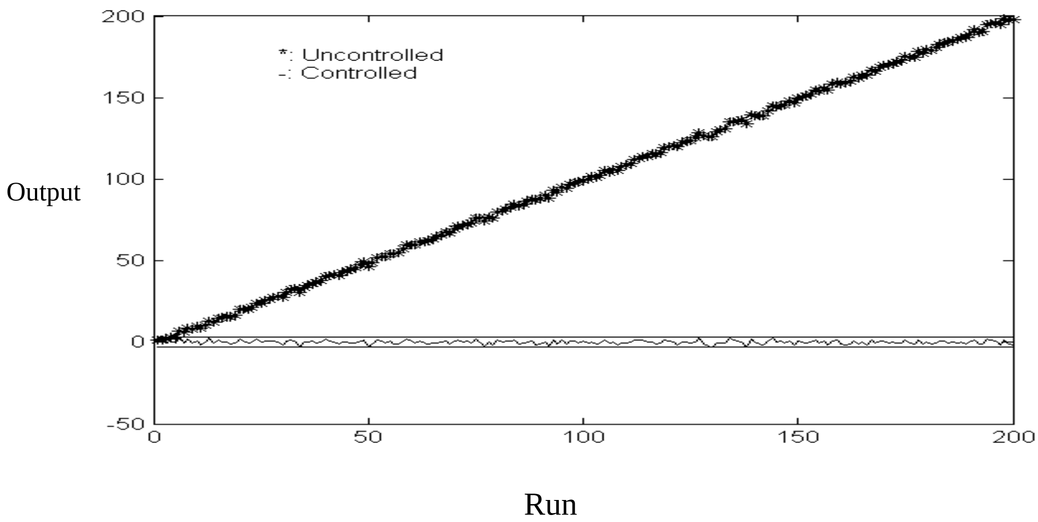

Figure 5.2 Controlled process output under the proposed time-varying tuning method

5.2 Comparison Results

In order to validate the effectiveness of the proposed time-varying weight tuning method, Monte Carlo simulations are performed under various random seeds. In these Monte Carlo simulations, we assume the target value $T = 0$, $\alpha = 0$, $\xi = \beta / b = 1$ and $\varepsilon_t \sim N(0,1)$. At each simulation, the performance index, $\hat{MSE}/\sigma_\varepsilon^2$, is calculated based on the simulation results of 200 runs (m) and 200 simulation cycles (initial seed from 0 to 199).

Table 5.2 shows the comparison results between the fixed trade-off solution weights, and the proposed time-varying tuning methods under various drifting speeds. The estimated standard deviation errors are shown in the parentheses. We can see that the performance between the two control schemes is not significantly different with the slow drifting speed (say $\delta = 0.1$). But, when the drifting speed is moderate to large (say $\delta \geq 0.5$), then the proposed time-varying tuning method is much better than the fixed trade-off control scheme. The last column of Table 5.2 shows the percent improvement of the proposed time-varying tuning method over the fixed trade-off control scheme. It shows that the larger the drifting speed, the more improved the performance. Thus, it is recommended that when a drifting disturbance model exists in the process, using the proposed method to tune the double EWMA controller will produce a satisfactory performance.

Table 5.2 Comparison results

δ	Trade-Off	Proposed (MGC-2)	Improvement (%) Over Trade-Off
0.1	1.1309 (0.1124)	1.1322 (0.1153)	---
0.5	1.3315 (0.1303)	1.1458 (0.1167)	13.95%
1.0	1.5096 (0.1447)	1.1955 (0.1214)	20.81%
1.5	1.6581 (0.1564)	1.2454 (0.1254)	24.89%
2.0	1.7939 (0.1667)	1.2957 (0.1292)	27.77%
2.5	1.9222 (0.1763)	1.3469 (0.1329)	29.93%
3.0	2.0456 (0.1855)	1.3995 (0.1366)	31.58%

5.3 Implement Dynamical Double EWMA Controller

Consider a simple dynamic system model follows $y_t = x_{t-1}$ (*T=0*). Assume the drifting disturbance parameter (δ) changed from 0.1 to 0.3 at run 200 ($t' = 200$) with $\varepsilon_t \sim N(0,1)$. Figure 5.3 shows the uncontrolled drifting process output over 400 runs.

Design criteria, $q = 0.2$ and $L = 2.962$ are selected for the EWMA chart. Figure 5.4 shows the EWMA chart applied to the controlled process output. The chart shows that a shift is detected at the 208th run, that is $k = 208$. Next, the estimated drifting rate is shown in Figure 5.5, it shows $\hat{\delta} \to 0.3$ after the 208th run. From Table 3.1, we know that the ARL for detection a drifting process with $\delta = 0.3$ is $ARL_1 \approx 8$. By solving Equation (2.33), we can obtain $(\lambda_{1,v}, \lambda_{2,v}) = (0.0203, 0.1241)$.

So, the Dynamic Tuning Loop module becomes:

$$\lambda_1(t) = 0.1241$$
$$\lambda_2(t) = 0.0203 + (0.92)^{8+(t-208)}$$

Figure 5.6 shows the control cycle of $\lambda_2(t)$. It shows the control cycle is triggered at the 208th run and then converges to the all variance solution weight at the 319th run. The all variance weights will be held until the EWMA chart triggers a new signal. Finally, the performance ($\hat{MSE}/\sigma_{\varepsilon}^{2}$) of the controlled process output is only 1.196, which is close to the MMSE controlled process output (i.e. $MSE/\sigma_{\varepsilon}^{2}=1$).

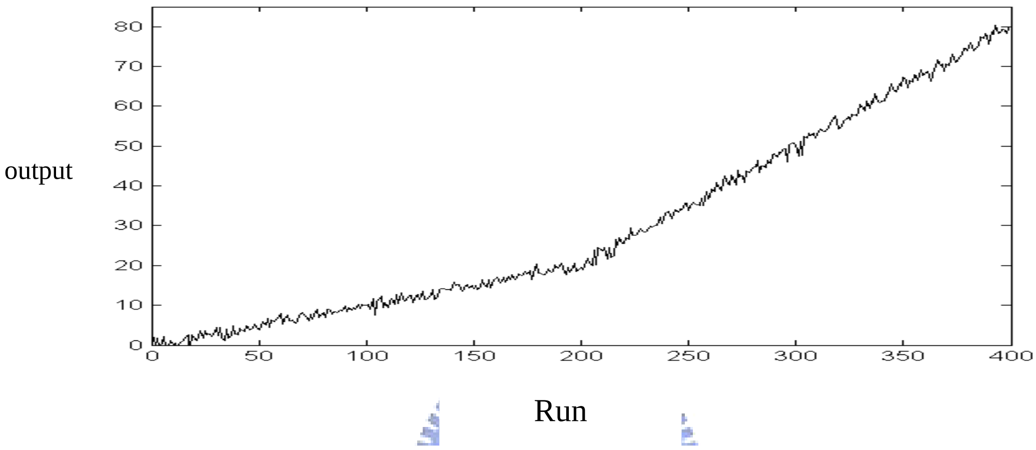

Figure 5.3 Uncontrolled drifting process output

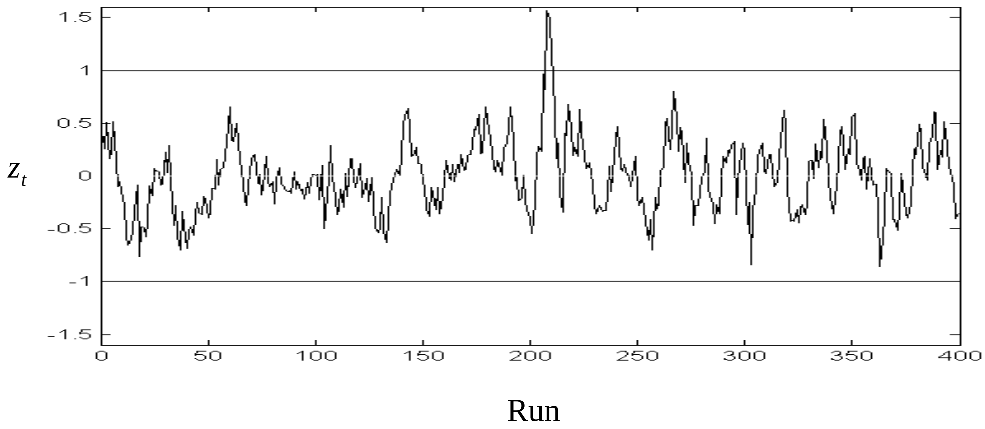

Figure 5.4 EWMA chart on the controlled process output

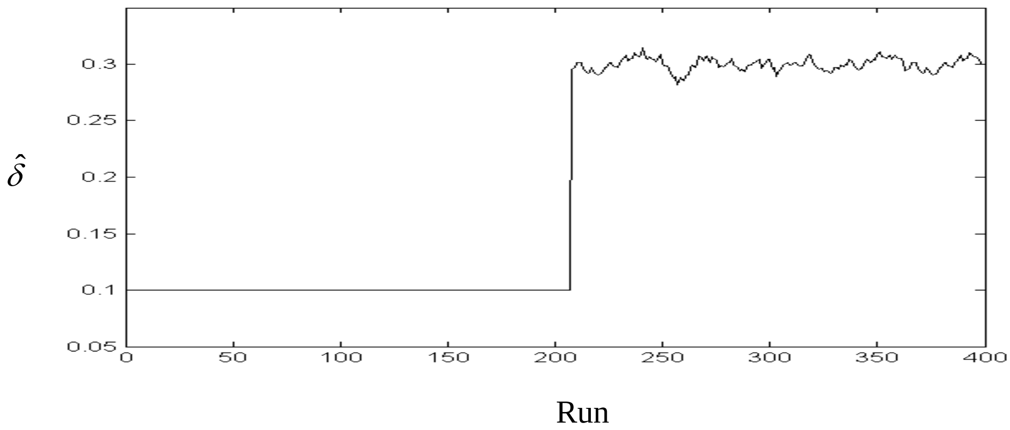

Figure 5.5 Estimated drifting rates

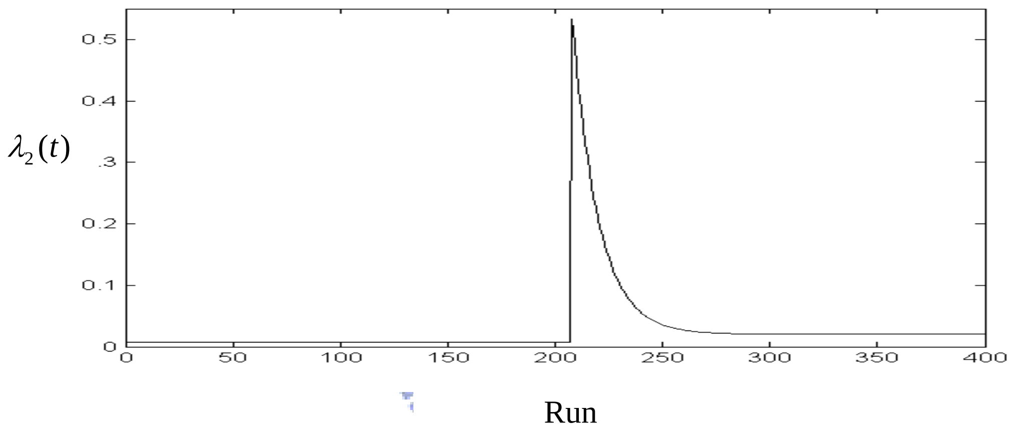

Figure 5.6 Control cycle of $\lambda_2(t)$

5.4 Discussion and Concluding Remarks

In this chapter, we have shown that the proposed tuning strategy possesses a significant improvement over the fixed trade-off solution weights control scheme, especially for processes with a moderate to large drifting rate. In addition we also proposed a dynamic tuning double EWMA controller. In the proposed controller, the EWMA control chart was used to trigger the Dynamic Tuning Loop Module, in order to adjust the control parameters. We have shown that the proposed controller is effective in responding to the disturbance changes.