Appendix B

Suppose a process characteristic X follows a distribution with the cumulative distribution function $F_X(x)$ and the probability density function $f_X(x)$. The fraction of nonconforming, the probability of an item falling outside specified tolerance limits, can be derived as:

$$
\begin{aligned}
F_W(0) &= P[W(X)=0] \\
&= P[X \le LSL] + P[X \ge USL] \\
&= F_X(LSL) + 1 - F_X(USL). \qquad \text{(B.1)}
\end{aligned}
$$

For the case with $w > 0$, the cumulative distribution function of $W(X)$, can be obtained as

$$
\begin{aligned}
F_W(w) &= P[W(X) \le w] \\
&= P[W(X)=0] + P[0 < W(X) \le w \mid LSL < X \le T] \\
&+ P[0 < W(X) \le w \mid T \le X < USL] \\
&= P[W(X)=0] + P[0 < 1 - [(T-X)/d_l]^2 \le w \mid LSL < X \le T] \\
&+ P[0 < 1 - [(X-T)/d_u]^2 \le w \mid T \le X < USL] \\
&= P[W(X)=0] + P[0 < d_l^2 - (T-X)^2 \le d_l^2 w \mid LSL < X \le T] \\
&+ P[0 < d_u{}^2 - (X-T)^2 \le d_u{}^2 w \le w \mid T \le X < USL] \\
&= P[W(X)=0] + P[d_l^2(1-w) < (T-X)^2 \le d_l^2 \mid LSL < X \le T] \\
&+ P[d_u{}^2(1-w) \le (X-T)^2 < d_u{}^2 \mid T \le X < USL] \\
&= P[W(X)=0] + P[d_l\sqrt{1-w} \le (T-X) < d_l \mid LSL < X \le T] \\
&+ P[d_u\sqrt{1-w} \le (X-T) < d_u \mid T \le X < USL] \\
&= P[W(X)=0] + P[LSL < X \le T - d_l\sqrt{1-w}] \\
&+ P[T + d_u\sqrt{1-w} \le X < USL] \\
&= [F_X(LSL) + 1 - F_X(USL)] + [F_X(T - d_l\sqrt{1-w}) - F_X(LSL)] \\
&+ [F_X(USL) - F_X(T + d_u\sqrt{1-w})] \\
&= 1 + F_X(T - d_l\sqrt{1-w}) - F_X(T + d_u\sqrt{1-w}), \quad 0 \le w \le 1. \qquad \text{(B.2)}
\end{aligned}
$$