Chapter 1

Introduction

1.1 Motivation

Process yield is the most common criterion used in the manufacturing industry. Ng and Tsui (1992) proposed a more accurate, complete and customer-oriented measure of yield, which is referred to as quality yield Y_q. The index distinguishes the products within the specifications by increasing the penalty as the departure from the target increases. Quality yield could be expressed as the traditional yield minus the truncated expected relative loss within the specifications to quantify how well a process can reproduce product items satisfactory to the customers. However, only the sample point estimate of Y_q has been considered in the literature. The sampling distribution and sampling errors are neglected. The decision maker would be interested in a lower bound on the quality yield rather than just the sample point estimate. Johnson (1992) developed the process loss index L_e, which is defined as the ratio of the expected quadratic loss to the square of half specification width. A process is said to have a symmetric tolerance if the target value is set to be the midpoint of the specification interval. Most research in quality assurance literature has focus on cases in which the manufacturing tolerance is symmetric. Although cases with symmetric tolerances are common in practical situations, cases with asymmetric tolerances also may occur in the manufacturing industry. From the customer's point of view, asymmetric tolerances reflect that deviations from the target are less tolerable in one direction than in the other. Usually they are not related to the shape of the supplier's process distribution. Under asymmetric tolerances situation, using Y_q and L_e would be risky and probably the results obtained are misleading. This problem calls for a need to generalize the index Y_q and L_e to cover situations with asymmetric tolerances so that appropriate use of the process loss index can be continued.

1.2 Literature Review

During the last decade, numerous process capability indices, including C_p, C_{pk}, C_{pm} and C_{pmk} (see Kane (1986), Chan *et al.* (1988), Pearn *et al.* (1992)), have been proposed in the manufacturing industries to provide numerical measures on process performance. Those indices are effective tools for process capability analysis and quality assurance and convey critical information regarding whether a process is capable of reproducing items satisfying customers' requirement. In practice, a minimal capability requirement would be preset by the customers/engineers. If the prescribed minimum capability fails to be met, one would conclude that the process is incapable.

Two process characteristics including the process location in relation to its target value, and the process spread (overall process variation) are used to establish the formula of those capability indices. The closer the process output is to the target value and the smaller the process spread, the more capable is the process. That is, the larger the value of a process capability index, the more capable is the process. Because C_p and C_{pk} are independent of the target value T, they can fail to account for process loss incurred by the departure from the target. For this reason, two more-advanced indices C_{pm} and C_{pmk} were developed. Those indices have been defined explicitly as:

$$C_p = \frac{USL - LSL}{6\sigma}, \tag{1.1}$$

$$C_{pk} = \min\left\{\frac{USL - \mu}{3\sigma},\ \frac{\mu - LSL}{3\sigma}\right\}, \tag{1.2}$$

$$C_{pm} = \frac{USL - LSL}{6\sqrt{\sigma^2 + (\mu - T)^2}}, \tag{1.3}$$

$$C_{pmk} = \min\left\{\frac{USL - \mu}{3\sqrt{\sigma^2 + (\mu - T)^2}},\ \frac{\mu - LSL}{3\sqrt{\sigma^2 + (\mu - T)^2}}\right\}, \tag{1.4}$$

where μ is the process mean, σ is the process standard deviation, T is the target value, USL and LSL are the upper and the lower specification limits, respectively. We remark that the indices presented above, are designed to monitor the performance for normal and near-normal processes with symmetric tolerances. We have assumed that $T = M = (USL + LSL)/2$ (which is quite common in practical situations) in Chapters 2, 3 and 5. It is essential that process capability indices must be applied under the condition that the process is in statistical control (stable).

The index C_p considers the overall process variability relative to the manufacturing tolerance, reflecting product quality consistency. Due to simplicity of the design, C_p cannot reflect the tendency of process centering (targeting). The index C_{pk} takes the process mean into consideration but can fail to distinguish between on-target processes from off-target processes. The index C_{pm} takes the proximity of process mean from the target value into account, which is more sensitive to process departure than C_{pk}. Because C_{pm} is based on the average process loss relative to the manufacturing tolerance, it has been alternatively called the Taguchi index. The index C_{pmk} is constructed from combining the modifications to C_p that produced C_{pk} and C_{pm}, which inherits the merits of both indices.

In the literature, several authors have promoted the use of various process capability indices and examined with differing degrees of completeness. Examples include Chou and Owen (1989), Chou *et al.* (1990), Franklin and Wasserman (1992), Kushler and Hurley (1992), Kotz *et al.* (1993), Vännman and Kotz (1995), Vännman (1997), Kotz and Lovelace (1998), Hoffman (2001), Pearn and Shu (2003), and references therein. Kotz and Johnson (2002) presented a thorough review for the development of process capability indices in the past ten years and Spiring *et al.* (2003) consolidated the research papers in process capability analysis for the period 1990-2002. Applications of those indices include the manufacturing of semiconductor products (Hoskins *et al.* (1988)), head/gimbals assembly for memory storage systems (Rado (1989)), flip-chips and chip-on-board (Noguera and Nielsen (1992)), rubber edge (Pearn and Kotz (1994)), aluminum electrolytic capacitors (Pearn and Chen (1997)), couplers and wavelength division multiplexers (Wu and Pearn (2003)). Other applications include performance measures on process with toolwear problem (Spiring (1989)), supplier selections (Tseng and Wu (1991), Chou (1994)), capability measures for multiple manufacturing streams (Bothe (1999)) and many others.

Yield Index

An important measure for interpreting process capability is yield, defined as

$$Y = \int_{LSL}^{USL} dF(x), \tag{1.5}$$

where $F(x)$ is the cumulative distribution function of the measured characteristic X. The disadvantage of the yield measure is that it does not distinguish among the products that fall inside of the specification limits. Customers do notice unit-to-unit differences in these characteristics, especially if the variance is large and/or the mean is offset from the target.

Loss Function

The quadratic loss function is considered to distinguish the products fall inside the specification limits by increasing the penalty as the departure from the target increases. To provide information on the variation about the target value, several possibilities have been tried. Hsiang and Taguchi (1985) first introduced the loss function approach to quality improvement with focuses on the reduction of variation around the target value. This concept pays attention to the product designer's original intent, that is, critical values at target lead to maximum product performance. In the development of this concept, Hsiang and Taguchi noted that any value x of a particular product's critical characteristic X incurs some monetary loss, which is denoted by $L(x)$, to the customer and/or society as it moves away from the target value. This loss function is defined as

$$L(x) = k(x-T)^2, \tag{1.6}$$

where k is some positive constant. Therefore, no loss is incurred when the characteristic is 'perfect' (i.e. $x = T$) and $L(x)=0$, and increasing loses are incurred as the measured value moves away from the target. While the reasons for using a continuous loss function such as the loss function (1.6) are understood, obtaining precise estimates for the parameter k turns out to be uneasy.

Loss Index

The quadratic loss function itself does not provide comparison with the specification limits and depends on the unit of the characteristic. To address these issues, Johnson (1992) developed the relative expected loss L_e for symmetric case, to provide numerical measures on process performance for industrial applications. Tsui (1997) interpret $L_e = L_{pe} + L_{ot}$, which provide an uncontaminated separation between information concerning the potential relative expected loss (L_{pe}) and the relative off-target squared (L_{ot}). The index L_e is defined as the ratio of the expected quadratic loss and the square of half specification width as follows:

$$L_e = \int_{-\infty}^{\infty} \left[\frac{(x-T)^2}{d^2}\right] dF(x) = \left(\frac{\sigma}{d}\right)^2 + \left(\frac{\mu - T}{d}\right)^2, \tag{1.7}$$

where $d = (USL - LSL)/2$ is the half specification width. If we define $L_{pe} = (\sigma/d)^2$ and $L_{ot} = [(\mu - T)/d]^2$, then L_e can be expressed as $L_e = L_{pe} + L_{ot}$. Since L_{pe} measures the process variation relative to the specification tolerance, it has been referred to as the process relative inconsistency loss index. On the other hand, L_{ot} measures the relative process departure and has been referred to as the relative off-target loss index. We note that the mathematical relationship $L_e = (3C_{pm})^{-2}$, $L_{pe} = (3C_p)^{-2}$, and $L_{ot} = (1 - C_a)^2$ can be established, where C_{pm}, C_p and C_a (defined as $C_a = 1 - |\mu - T|/d$) are three basic process capability indices considered by Chan *et al.* (1988), Kane (1986) and Peran *et al.* (1998), respectively. The most advantage of L_e over C_{pm} is that the estimator of the former has better statistical properties than that of latter, as the former does not involve a reciprocal transformation of process mean and variance. The disadvantage of L_e index is the difficulty in setting a standard for the index since it increases from zero to infinity.

Quality Yield

To incorporate the proportion conforming measure Y with loss function based index L_e, Ng and Tsui (1992) proposed a more accurate, complete and customer-oriented measure of yield, which is referred to as quality yield Y_q. In contrast to the yield index Y, quality yield (Q-yield) emphasizes on the ability of the process clustering around the target, which therefore reflects the degree of the process targeting (centering) by considering only the relative loss within the specifications. With only taking the relative expected loss L_e within the specifications into account, Ng and Tsui (1992) defined the standardized quality as one minus the relative loss, and so the quality yield Y_q is defined as the expected value of the standardized quality within the specification, that is,

$$Y_q = \int_{LSL}^{USL} \left[1 - \frac{(x-T)^2}{d^2}\right] dF(x). \tag{1.8}$$

This quality yield index differs from the expected relative worth index defined in Johnson (1992) by truncating the deviation outside the specifications. With this truncation, the quality yield index will be between zero and one and thus provides a standardized measure. Also, by relating to the yield measure widely accepted in the manufacturing industry, it will be understood and accepted as a capability measure. Similar to the yield measure Y, an ideal value of Y_q is one, which provides the user a clear guide about the standard. Similar to the yield Y, the Q-yield Y_q requires no normality assumption. While yield is the proportion of conforming products, Q-yield can be interpreted as the average degree of products reaching "perfect" or "on target".

1.3 Research Objectives

In our investigation, we focus on obtaining lower bounds on Y_q and extending Y_q and L_e to handle processes with asymmetric tolerances. The concrete contributions of this dissertation are threefold. The first is to propose two reliable approaches for measuring Y_q by converting the estimated value into a lower confidence bound. One approach is for production processes with very low fraction of defectives under normality assumption. For arbitrary underlying distributions, we propose a bootstrap approach to obtain lower confidence bound on quality yield. The second is to generalize Y_q and L_e for asymmetric tolerances. The merit of the generalization is justified, and some statistical properties of the estimated generalization are investigated. The third is to investigate the statistical properties of these natural estimators for Y_q and L_e. The results obtained in this dissertation are useful to the practitioners in choosing good estimators and making reliable decisions on judging process capability.

1.4 Organization

In Chapter 1, we review some existing process capability indices and point out our research objectives. In Chapter 2, we propose a reliable approach for measuring Y_q by converting the estimated value into a lower confidence bound for processes with very low fraction of defectives under normality assumption. In Chapter 3, a nonparametric but computer intensive method, called bootstrap, is used to obtain lower confidence bound on quality yield for arbitrary underlying distributions. Simulation studies are conducted to examine the sampling distribution of the estimated Y_q. The lower confidence bound not only provides information regarding actual process performance which is tightly related to both the fraction of defective units and customer quality loss, but also is useful in making decisions for capability testing. In Chapter 4, we generalize the quality yield index for asymmetric tolerances. The merit of the generalization is justified, and some statistical properties of the estimated generalization are investigated. In Chapter 5, we consider these loss indices L_e, L_{pe}, L_{ot}, and investigate the statistical properties of their natural estimators. For these three indices, we obtain their UMVUEs and MLEs, and compare the reliability of the two estimators based on the relative mean square errors. In addition, we construct 90%, 95%, and 99% upper confidence limits, and the maximum values of $\hat{L}_e$ for which the process is capable 90%, 95%, and 99% of the time. In Chapter 6, we consider a generalization, which we refer to as L_e'', to deal with processes with asymmetric tolerances. The generalization is shown to be superior to the original index L_e. We investigate the statistical properties of a natural estimator of L''_e L''_{ot} and L''_{pe} when the underlying process is normally distributed. We obtained the rth moment, expected value, and the variance of the natural estimator $\hat{L}''_e$, $\hat{L}''_{ot}$, and $\hat{L}''_{pe}$. We also analyzed the bias and the mean squared error in each case. In Chapter 7, we summarized the results obtained in our investigation.