Chapter 4

Quality Yield Measure with Asymmetric Tolerances

A process is said to have a symmetric tolerance if the target value T is set to be the midpoint of the specification interval [LSL, USL], i.e. $T = M = (USL + LSL)/2$. Most research in quality assurance literature has focus on cases in which the manufacturing tolerance is symmetric. Examples include Kane (1986), Chan *et al.* (1988), Choi and Owen (1990), Boyles (1991), Pearn *et al.* (1992), Vännman (1995), Vännman and Kotz (1995), and Spiring (1997). Although cases with symmetric tolerances are common in practical situations, cases with asymmetric tolerances often occur in the manufacturing industry. In general, asymmetric tolerances simply reflect that deviations from the target value are less tolerable in one direction than the other (see Boyles (1994), Vännman (1997), and Wu and Tang (1998)). Asymmetric tolerances can also arise from a situation where the tolerances are symmetric to begin with, but the process follows a non-normal distribution and the data is transformed to achieve approximate normality, as shown by Chou *et al.* (1998) who have used Johnson curves to transform non-normal process data. Unfortunately, there has been comparatively little research published on cases with asymmetric tolerances. Exceptions are Boyles (1994), Vännman (1997), Chen (1998), Pearn *et al.* (1998, 1999), and Chen *et al.* (1999).

In this chapter, we consider the quality yield index for processes with asymmetric tolerances. We consider the asymmetric loss function, and the corresponding truncated worth function to generalize the quality yield index. Comparisons among the yield, the quality yield, and some popular process capability indices are examined. Distributional properties of the estimated Y_q are also investigated. A confidence interval of Y_q is constructed to estimate the manufacturing capability. Finally, an application example using the index Y_q to the manufacturing capability of the Light Emitting Diodes (LEDs) is presented to illustrate the applicability of the proposed approach.

4.1 Quality Yield with Asymmetric Tolerances

Yield is currently defined as the percentage of processed units that pass inspection. Therefore, the yield index Y can be defined mathematically as the expected value of the worth $W(X)$ where $W(x) = 1$ for $LSL < x < USL$ and $W(x) = 0$ for $x \le LSL$ or $x \ge USL$, that is, $Y = E[W(X)]$. The disadvantage of yield measure is that it does not distinguish the worth of the products that fall inside of the specification limits, i.e., they are equally good. A product has the maximal worth W_T as the corresponding characteristic X having the target value T (Johnson (1992)). Using the loss function as Equation (1.6), the worth of the product with characteristic X is

$$W(x) = W_T - k(x - T)^2 . \tag{4.1}$$

Therefore, as deviation of X from T increases, the worth becomes less eventually becoming 0 and then negative.

Asymmetric Loss Function

Now, for process with manufacturing specification (LSL, T, USL), we can redefine $W(x) = 0$ for $x \le LSL$ or $x \ge USL$ and $W(x) = W_T - k(x-T)^2$ for $LSL < x < USL$. Using $W(LSL) = 0$, we obtain $k = W_T/(d_l)^2$, where $d_l = T - LSL$. On the other hand, using $W(USL) = 0$, we obtain $k = W_T/(d_u)^2$, where $d_u = USL - T$. For case of symmetry, both the two values of k reduce to $W_T/(d)^2$, where $d = (USL - LSL)/2$. Without loss of generality, we can set $W_T = 1$. Therefore, for process with manufacturing specification (LSL, T, USL), we can define a general truncated loss function of x as:

$$L(x) = \begin{cases} [(T-x)/d_l]^2, & LSL < x \le T, \\ [(x-T)/d_u]^2, & T \le x < USL, \\ 1, & \text{otherwise.} \end{cases} \tag{4.2}$$

Hence, the corresponding general truncated worth function of x becomes

$$W(x) = \begin{cases} 1-[(T-x)/d_l]^2, & LSL < x \le T, \\ 1-[(x-T)/d_u]^2, & T \le x < USL, \\ 0, & \text{otherwise.} \end{cases} \tag{4.3}$$

Then, the expected loss L_e, defined as $E[L(X)]$, can be expressed as:

$$\begin{aligned} L_e &= \int_{-\infty}^{\infty} L(x)dF_X(x) \\ &= 1 + F_X(LSL) - F_X(USL) + (d_l)^{-2}E[(T-X)^2 \mid LSL < X \le T] \\ &+ (d_u)^{-2}E[(X-T)^2 \mid T \le X < USL]. \end{aligned} \tag{4.4}$$

Figure 26 displays the plot of $L(x)$ for a process with a process with asymmetric manufacturing specification (LSL, T, USL) = (10, 40, 50). Figure 27 displays the plot of $W(x)$ for a process with asymmetric tolerance (LSL, T, USL) = (10, 40, 50). Now, using the worth function, we can distinguish the worth of the products that fall inside of the specification limits. We consider two items x_1 and x_2 with $x_1 > T$ and $x_2 < T$, satisfying the relationship $(x_1 - T)/d_u = (T - x_2)/d_l$, equal departure ratio in this case, the worth values given to items x_1 and x_2 are the same. For example, we consider that for the midpoint of the left-hand side tolerance, $x_1 = (T + LSL)/2$ and the midpoint of the right-hand side tolerance, $x_2 = (T + USL)/2$, the corresponding worth can be calculated as:

$$W(x_1) = 1 - [(T - x_1)/d_l]^2 = 1 - \{[T - (T + LSL)/2]/(T - LSL)\}^2 = 3/4 ,$$

$$W(x_2) = 1 - [(x_2 - T)/d_u]^2 = 1 - \{[((T + USL)/2) - T]/(USL - T)\}^2 = 3/4 .$$

Obviously, the two points x_1 and x_2 have the same departure ratio (relative departure) $k = (T - x_1)/d_l = (x_2 - T)/d_u = 1/2$. Checking process loss at x_1 and x_2, we have $L(x_1) = L(x_2) = 1/4$ and equal worth value 3/4. In fact, $0 < W(x) < 1$ for $LSL < x < USL$ and $W(T) = 1$. On the other hand, $W(x) = 0$ while x falls outside of the specification limits.

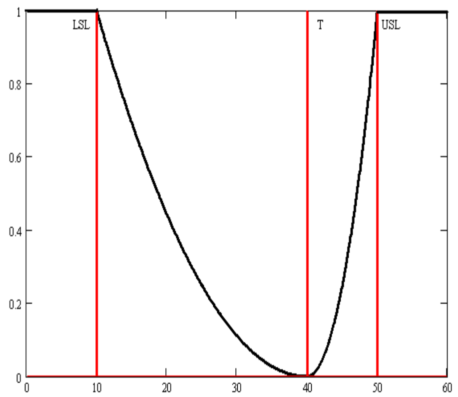

Figure 26. The plot of $L(x)$, loss function for an asymmetric specification $(LSL,\ T,\ USL) = (10, 40, 50)$.

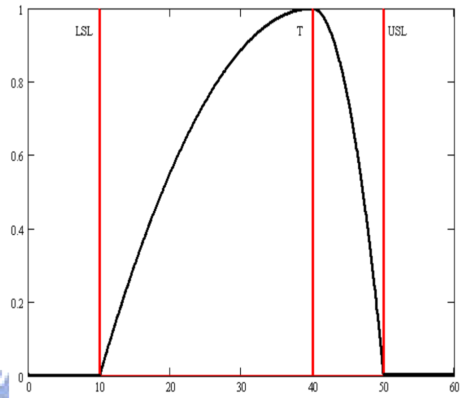

Figure 27. The plot of $W(x)$, worth function for an asymmetric specification $(LSL,\ T,\ USL) = (10, 40, 50)$.

Quality Yield with Asymmetric Tolerances

Suppose a process characteristic X follows a distribution with the cumulative distribution function $F_X(x)$ and the probability density function $f_X(x)$. $F_W(w)$, the cumulative distribution function of $W(X)$, can be expressed as (see Appendix B):

$$F_W(w) = 1 + F_X(T - d_l\sqrt{1-w}) - F_X(T + d_u\sqrt{1-w}),\ 0 \le w \le 1. \quad (4.5)$$

Particularly, the fraction of nonconforming, the probability of an item falling outside specified tolerance limits, can be calculated as:

$$F_W(0) = 1 + F_X(LSL) - F_X(USL). \quad (4.6)$$

Hence $f_W(w)$, the probability density function of $W(X)$, can be expressed as:

$$f_W(w) = \frac{1}{2\sqrt{1-w}}\left\{d_l f_X(T - d_l\sqrt{1-w}) + d_u f_X(T + d_u\sqrt{1-w})\right\},\ 0 < w < 1. \quad (4.7)$$

The mean value and variance of $W(X)$ can be calculated as:

$$E[W(X)] = \int_0^1 w dF_W(w)$$

$$= \int_0^1 \frac{w}{2\sqrt{1-w}} \left\{ d_l f_X(T - d_l\sqrt{1-w}) + d_u f_X(T + d_u\sqrt{1-w}) \right\} dw \,, \quad (4.8)$$

$$E[W(X)]^2 = \int_0^1 w^2 dF_W(w)$$

$$= \int_0^1 \frac{w^2}{2\sqrt{1-w}} \left\{ d_l f_X(T - d_l\sqrt{1-w}) + d_u f_X(T + d_u\sqrt{1-w}) \right\} dw \,, \quad (4.9)$$

$$Var[W(X)] = E[W(X)]^2 - E^2[W(X)] \,. \quad (4.10)$$

Now we can rewrite Q-yield as $E[W(X)]$, the expected value of the worth $W(X)$. The Q-yield Y_q will be between zero and one, can be used as an index to access the ability of a process under the consideration of process yield and process loss. The Q-yield index Y_q can be interpreted as the proportion of the "perfect" items while the yield index Y is the proportion of conforming items. As the existed process capability indices (PCIs), Q-yield index Y_q also has the large-the-better property.

This quality yield index differs from the expected relative worth index defined in Johnson (1992) by truncating the deviation outside the specifications. With this truncation, the quality yield index will be between zero and one and, thus provides a standardized measure. Also, by relating to the yield measure, which is widely accepted in the manufacturing industry, it will be better understood and accepted as a capability measure. The advantage of the Y_q index over the L_e index is the value of the former goes from zero to one. Similar to the yield index Y, an ideal value of Y_q is one, which provides the user a clear guide about the standard. Similar to the yield Y, the yield index Y_q requires no normality assumption.

Table 7. Normal distribution with
$\mu = T$ versus $Y_q = 0.5(0.1)0.9$.

Y_q %	Case 1	
	μ	σ
50	T	3.558213
60	T	2.782604
70	T	2.176123
80	T	1.6512655
90	T	1.12161

To illustrate some basic behavior of quality yield Y_q versus normal distribution with various application cases, we consider the following parameter settings as listed in Tables 7-10. For a process with asymmetric tolerance (LSL, T, USL) = (−3, 0, 4.5), five levels of Y_q, 0.5(0.1)0.9 are selected in each case.

And cases studied are arranged in the following manner. In Case 1, we set $\mu = T$ to calculated the corresponding σ in each Y_q level. In Cases 2-4, μ is shifted from T to USL by $d_u/6$, $d_u/4$, and $d_u/3$, respectively. We then solve for σ in each setting. In Cases 5-7, μ is shifted from T to LSL by $d_l/6$, $d_l/4$, and $d_l/3$, respectively. We then solve for σ in each cases. Finally, in Cases 8-10, σ are fixed in three levels, 1/3, 1/2, and 1. The corresponding values of μ in each setting have again, been computed.

Table 8. Normal distribution with μ shifted from T to USL by $d_u/6$, $d_u/4$, and $d_u/3$, respectively, versus $Y_q = 0.5(0.1)0.9$.

Y_q %	Case 2		Case 3		Case 4	
	μ	σ	μ	σ	μ	σ
50	$T+d_u/6$	3.593474	$T+d_u/4$	3.551352	$T+d_u/3$	3.4652255
60	$T+d_u/6$	2.8240045	$T+d_u/4$	2.767893	$T+d_u/3$	2.651555
70	$T+d_u/6$	2.221167	$T+d_u/4$	2.1443699	$T+d_u/3$	1.9813995
80	$T+d_u/6$	1.6909245	$T+d_u/4$	1.5751335	$T+d_u/3$	1.316363
90	$T+d_u/6$	1.1111475	$T+d_u/4$	0.852496	$T+d_u/3$	---

Table 9. Normal distribution with μ shifted from T to LSL by $d_l/6$, $d_l/4$, and $d_l/3$, respectively, versus $Y_q = 0.5(0.1)0.9$.

Y_q %	Case 5		Case 6		Case 7	
	μ	σ	μ	σ	μ	σ
50	$T-d_l/6$	3.440189	$T-d_l/4$	3.345944	$T-d_l/3$	3.221025
60	$T-d_l/6$	2.6308625	$T-d_l/4$	2.5039585	$T-d_l/3$	2.3262755
70	$T-d_l/6$	1.985113	$T-d_l/4$	1.8183015	$T-d_l/3$	1.576054
80	$T-d_l/6$	1.4197015	$T-d_l/4$	1.216756	$T-d_l/3$	0.930123
90	$T-d_l/6$	0.85078	$T-d_l/4$	0.5874915	$T-d_l/3$	---

Table 10. Normal distribution with σ fixed in three levels, 1/3, 1/2, and 1, respectively, versus $Y_q = 0.5(0.1)0.9$.

Y_q %	Case 8		Case 9		Case 10	
	μ	σ	μ	σ	μ	σ
50	3.1644764	1/3	3.1432	0.5	3.076668	1
60	2.82646245	1/3	2.8018575	0.5	2.689652	1
70	2.4421075	1/3	2.4135035	0.5	2.2613755	1
80	1.9846635	1/3	1.9493365	0.5	1.744542	1
90	1.383308	1/3	1.33217	0.5	0.960625	1

Figures 28-31 display four selected normally distributed processes, which are $N(\mu = T, \sigma)$, $N(\mu = T + d_u/4, \sigma)$, $N(\mu = T - d_l/4, \sigma)$ and $N(\mu, \sigma = 1/2)$ respectively, with the quadratic loss function and five levels of quality yield (see cases 1, 3, 6, 9). The relationship between the squared loss function and some normal probability distributions can be easily examined. Quality yield could be treated as traditional yield minus truncated expected relative loss within the specifications to quantify how well a process can reproduce product items satisfactory to the customers. While yield is the proportion of conforming products, Q-yield can be interpreted as the average degree of products reaching "perfect" or "on target".

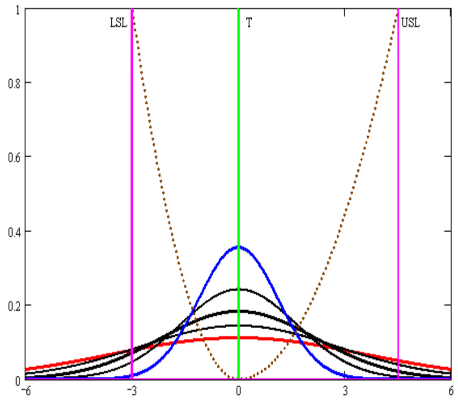

Figure 28. Distribution plots of normal distribution $N(\mu = T, \sigma)$ with the loss function for various σ.

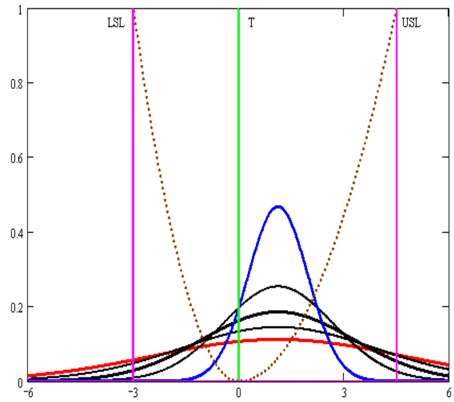

Figure 29. Distribution plots of normal distribution $N(\mu = T + d_u / 4, \sigma)$ with the loss function for various σ.

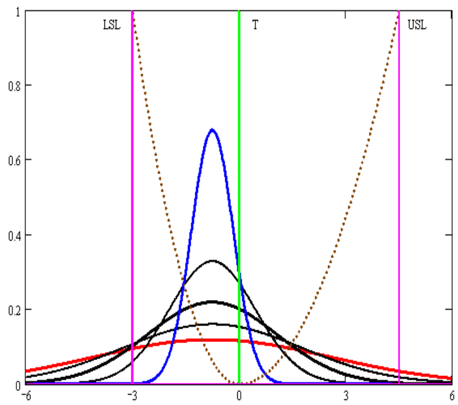

Figure 30. Distribution plots of normal distribution $N(\mu = T - d_l / 4, \sigma)$ with the loss function for various σ.

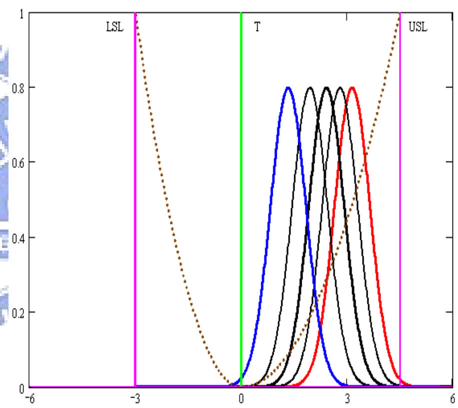

Figure 31. Distribution plots of normal distribution $N(\mu, \sigma = 1/2)$ with the loss function for various μ.

4.2 Comparison of Yield, Q-Yield, and PCIs with Asymmetric Tolerances

To illustrate the basic differences among yield Y, quality yield Y_q, and the four well-known process capability indices C_p, C_{pk}, C_{pm} and C_{pmk}, we compare the measured values based on the yield Y, quality yield Y_q, and the four indices on some processes.

Comparison of Q-yield and Yield

Both the Q-yield index Y_q and the conventional yield index Y can be applied to processes with any distribution. The conventional yield Y, however, does not distinguish the products falling inside the specification tolerance. For example, if X follows the uniform distribution $U(LSL, USL)$ with target T where $LSL < T < USL$, then yield $Y = 1.00$ and Q-yield $Y_q = 0.667$, respectively.

From manufacturing perspective all the produced units are good products, but certainly the consumer would consider the process low quality even although the yield Y = 1.00. To further demonstrate the difference between yield Y and Q-yield Y_q, we consider a set of triangular distributed processes with $a < x < b$ and mode c. Table 11 displays the quality yield measure of those triangular distributed processes with modes c = 11(1)49, $(a, b) = (LSL, USL)$ = (10, 50), and target value T = 30(5)45. For those processes, the conventional yield values given to all processes are Y = 1.00. On the other hand, Q-yield Y_q obtains its maximum 0.833 not at $\mu = T$ but at the mode $c = T$ for those triangular distributed processes. The plots of Y_q versus mode c = 11(1)49 with T = 30(5)45 are displayed in Figure 32. The figure shows that Y_q always obtaining the maximum 0.833 at the mode, as the target value moves from 30 to 45 by 5.

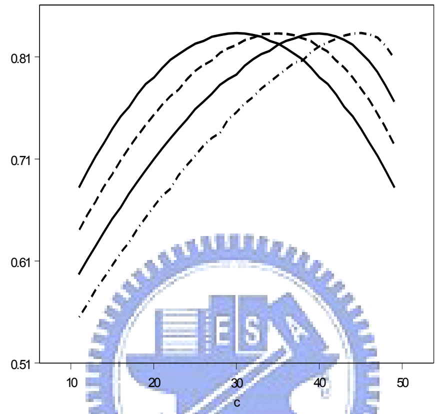

Figure 32. Plots of Y_q for processes with c = 11(1)49, and T = 30(5)45 (left to right).

For a normal distributed process with mean μ and standard deviation σ, we denote $X \sim N(\mu, \sigma^2)$. By Equation (4.5), the corresponding cumulative distribution function of $W(X)$ is

$$F_W(w) = 1 + \Phi((T - \mu - d_l\sqrt{1-w})/\sigma) - \Phi((T - \mu + d_u\sqrt{1-w})/\sigma), \quad 0 \le w \le 1, \tag{4.11}$$

where Φ is the cumulative distribution function of the standard normal distribution. The corresponding probability density function of $W(X)$ is

$$f_W(w) = \frac{1}{2\sigma\sqrt{1-w}}\left\{d_l\phi((T - \mu - d_l\sqrt{1-w})/\sigma) + d_u\phi((T - \mu + d_u\sqrt{1-w})/\sigma)\right\}, \tag{4.12}$$

where $0 < w < 1$, and ϕ is the probability density function of the standard normal distribution. The corresponding Q-yield Y_q, which is defined as the expect function of $W(X)$, therefore, can be expressed as:

$$Y_q = \int_0^1 \frac{w}{2\sigma\sqrt{1-w}} \left\{ d_l \phi((T-\mu-d_l\sqrt{1-w})/\sigma) + d_u \phi((T-\mu+d_u\sqrt{1-w})/\sigma) \right\} dw. \tag{4.13}$$

Table 12 displays the comparisons of Q-yield Y_q for normal distributed processes with $\mu = 10(1)50$, $\sigma = 10/3$ and $20/3$ respectively, where (LSL, USL) = (10, 50) and T = 30(5)45. For symmetric cases (T = M), the maximal Y_q occurs at $\mu = T$. But for asymmetric cases ($T \neq M$), the maximal Y_q occurs not at $\mu = T$, but at μ which is between the target value T and M (the center of the specification interval). This is reasonable, because that the on-target process ($\mu = T$) has larger proportion low-quality products than the process with maximal Y_q value. For example, let's compare two processes A and B with $\mu_A = 40$, $\mu_B = 45$, $\sigma_A = \sigma_B = 10/3$, and ($LSL$, T, USL) = (10, 45, 50). By Table 12, we have Y_q = 0.961 for process A and Y_q = 0.823 for process B, the result corresponds that on-target process B has larger proportion low-quality products than process A.

As we mentioned earlier, two items satisfying equal departure ratio have equal worth. But for two processes A and B satisfying equal departure ratio $(\mu_A - T)/d_u = (T - \mu_B)/d_l$ and $\sigma_A = \sigma_B$, there are not equal average worth for two samples come from the two process A and B. For example, normal distributed processes A and B with $\mu_A = USL$, $\mu_B = LSL$ and $\sigma_A = \sigma_B$ have equal yield Y, proportions of conforming items 50%, but Y_q values given to processes A and B are different for asymmetric cases. In fact, Table 12 also displays that Y_q value given to process B are less than that given to process A, since that average quality of products coming from process A is better than that coming from process B for cases of $T > M$.

Comparison of Q-yield and PCIs

Most of the investigations for the existed PCIs, C_p, C_{pk}, C_{pm} and C_{pmk}, depend heavily on the assumption of normal variability. If the underlying distributions are non-normal, then the capability calculations are highly unreliable since the conventional estimate estimator S^2 of σ^2 is sensitive to departures from normality, and estimators of those indices are calculated using S^2 (see Somerville and Montgomery (1997)). Table 13 displays the comparisons among the six indices yield Y, Q-yield Y_q, C_p, C_{pk}, C_{pm} and C_{pmk} using in normal processes for various values of μ with fixed σ = 20/3, and (LSL, T, USL) = (10, 30, 50). For the symmetric case, all the six indices obtain their maximum at $\mu = T$.

Figure 33 and Figure 34 display the plots of μ versus Y_q, Y and four PCIs C_p, C_{pk}, C_{pm} and C_{pmk}, respectively. With fixed σ = 20/3, and (LSL, T, USL) = (10, 30, 50), μ is varies from 10 to 50 by 1 to examine the sensitivity of these indices with respect to μ. For the symmetric case, all the six indices obtain their maximum at $\mu = T = 30$ as one can easily check in the plots. And as μ departs from T, except C_p, the other five indices all decrease, as one may expect.

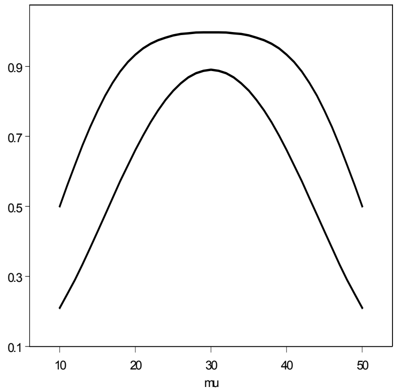

Figure 33. Plots of Y and Y_q (top to bottom) versus $\mu = 10(1)50$ for processes with fixed $\sigma = 20/3$, and $(LSL, T, USL) = (10, 30, 50)$.

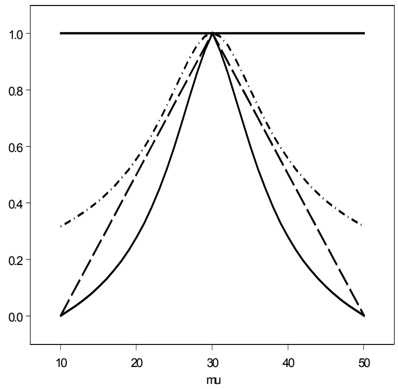

Figure 34. Plots of C_p, C_{pk}, C_{pm} and C_{pmk} (top to bottom) versus $\mu = 10(1)50$ for processes with fixed $\sigma = 20/3$, $(LSL, T, USL) = (10, 30, 50)$.

Table 14 and Table 15 demonstrate the comparisons among the six indices using in normal processes for various values of μ with fixed $\sigma = 10/3$, $20/3$ respectively, and $(LSL, T, USL) = (10, 40, 50)$. For the asymmetric case, with fixed $\sigma = 10/3$, and $(LSL, T, USL) = (10, 40, 50)$, Figure 35 and Figure 36 display the plots of μ versus Y_q, Y and four PCIs C_p, C_{pk}, C_{pm} and C_{pmk}, respectively. Similarly, Figure 37 and Figure 38 display the plots of μ versus Y_q, Y and four PCIs C_p, C_{pk}, C_{pm} and C_{pmk}, respectively. Specification limits are set to $(LSL, T, USL) = (10, 40, 50)$ and $\sigma = 20/3$ is fixed. In this setting, μ is varies from 10 to 50 by 1 to examine the sensitivity of these indices with respect to μ, for processes with asymmetric tolerances.

For the asymmetric cases, none expect Y_q in the six indices reflects process performance accurately. In fact, the index Y only reflects the quantity, not quality of conforming items, C_p cannot reflect the shift of the process mean, C_{pk} being a yield-based index cannot reflect the departure of process mean μ from the target value T. The index C_{pm} obtains its maximum at $\mu = T$, but the corresponding on-target process is not the process with the best average quality (proportion of perfect items, under the consideration of both process yield and loss) for asymmetric cases as we pointed out earlier. The index C_{pmk} cannot distinguish accurately the average quality of productions from different processes. For example, give the same zero C_{pmk} value to two processes A and B with $\mu_A = USL$, $\mu_B = LSL$ and $\sigma_A = \sigma_B$, but as we mentioned earlier the average quality of productions coming from process A is better than that coming from process B for cases of $T > M$.

4.3 Distributional Properties of the Estimated Y_q

In the following, some distributional properties of the estimated Y_q are investigated. A confidence interval of Y_q is constructed. For factory applications purpose, an approximate process performance testing is also investigated.

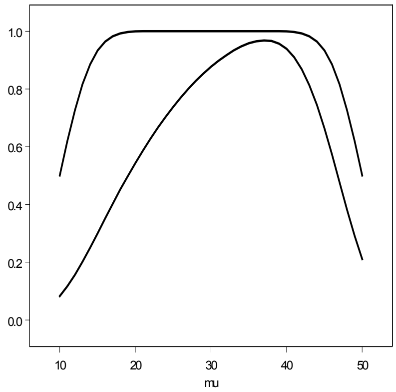

Figure 35. Plots of Y (top) and Y_q (bottom) versus $\mu = 10(1)50$, for processes with $\sigma = 10/3$, $(LSL, T, USL) = (10, 40, 50)$.

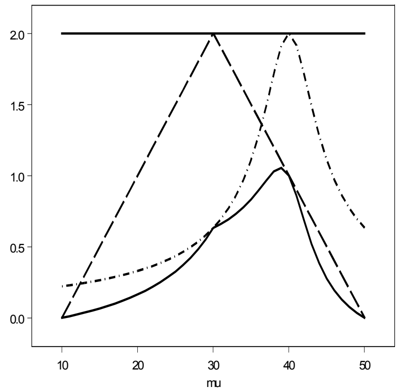

Figure 36. Plots of C_p (top), C_{pk} (left), C_{pm} (right) and C_{pmk} (bottom) versus $\mu = 10(1)50$, for processes with $\sigma = 10/3$, $(LSL, T, USL) = (10, 40, 50)$.

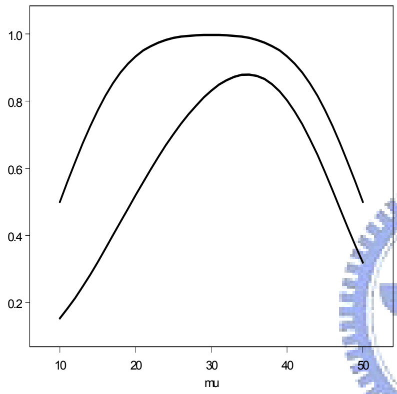

Figure 37. Plots of Y (top) and Y_q (bottom) versus $\mu = 10(1)50$, for processes with $\sigma = 20/3$, and $(LSL, T, USL) = (10, 40, 50)$.

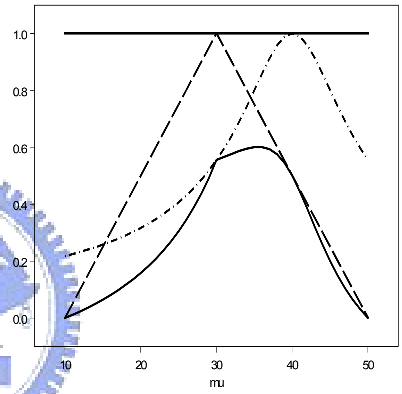

Figure 38. Plots of C_p (top), C_{pk} (left), C_{pm} (right) and C_{pmk} (bottom) versus $\mu = 10(1)50$, $\sigma = 20/3$, $(LSL, T, USL) = (10, 40, 50)$.

Estimation of Q-yield

If the process parameters μ and σ are unknown, then Y_q must be estimated from a sample. Let X_1, $X_2, \ldots,$ X_n be a random sample taken from the process, and W_1, $W_2, \ldots, W_n$ be the corresponding worth. To estimate the Q-yield Y_q, we can consider the following estimator:

$$\hat{Y}_q = \sum_{i=1}^{n} \frac{W_i}{n} = \overline{W} . \tag{4.14}$$

It is easy to verify that $E(\hat{Y}_q) = Y_q$. Therefore, $\hat{Y}_q$ is an unbiased estimator of Y_q with $Var(\hat{Y}_q) = n^{-1} Var(W_1)$. Using the unbiased estimator $\hat{Y}_q$ does not require any knowledge of process distribution. But, if the distribution of the characteristic X is given with cumulative distribution function F_X, then the cumulative distribution function of the corresponding worth F_W can be calculated, and the cumulative distribution function of $\hat{Y}_q$ can be expressed as the n-fold convolution of F_W:

$$F_{\hat{Y}_q}(y) = P(\hat{Y}_q \le y) = P(\sum_{i=1}^{n} W_i \le ny) = G(ny), \tag{4.15}$$

where G is the n-fold convolution of F_W. The complexity of the cumulative distribution function of $\hat{Y}_q$ comes from the truncation property of the worth function. There is no analytic closed form for $F_{\hat{Y}_q}(y)$. But, for large sample size n, we can show that:

$$\frac{\sqrt{n}(\hat{Y}_q - Y_q)}{S} \to N(0,1), \tag{4.16}$$

where sample variance $S^2 = \sum_{i=1}^{n}(W_i - \overline{W})^2/(n-1)$. Consequently, a $(1-\alpha)100\%$ confidence interval of Y_q can be established as:

$$(\hat{Y}_q - z_{1-\alpha/2}S/\sqrt{n},\ \hat{Y}_q + z_{1-\alpha/2}S/\sqrt{n}), \tag{4.17}$$

where $z_{1-\alpha/2}$ is the $(1-\alpha/2)$ quantile value of the standard normal distribution $N(0,1)$. We note that a lower $(1-\alpha)100\%$ confidence limit can be obtained from the lower (one-sided) confidence interval. If the calculated lower confidence limit is greater than the predetermined index value, then we would judge that the process is capable. Otherwise, the process is considered to be incapable, and some quality improvement activities must be initiated.

Distribution Plot of the Q-yield Estimator

Some Monte Carlo simulations are conducted to investigate the behavior of the sampling distribution of the estimated Y_q, for several selected cases, where the underlying process distributions are normal, skewed, or heavy tailed. True value of quality yield $Y_q = 0.6$ is picked, with underlying process distributions set to

(1) Normal distribution $N(\mu, \sigma^2)$ with probability density function

$$f(x) = (\sqrt{2\pi}\sigma)^{-1}\exp[-(\mathrm{x}-\mu)^2/2\sigma^2], \tag{4.18}$$

with mean μ and variance σ^2, for $-\infty < x < \infty$.

(2) Lognormal distribution $LN(\mu, \sigma^2)$ with probability density function

$$f(x) = (x\sqrt{2\pi}\sigma)^{-1}\exp[-(\ln x - \mu)^2/2\sigma^2], \tag{4.19}$$

with mean $\exp(\mu + \sigma^2/2)$ and variance $\exp(2\mu + 2\sigma^2) - \exp(2\mu + \sigma^2)$, for $x > 0$.

(3) Student's t distribution t_k with degree of freedom k, where probability density function is

$$f(x) = [\Gamma((k+1)/2)/\Gamma(k/2)](\sqrt{k\pi})^{-1}(1 + x^2/k)^{-(k+1)/2}, \quad -\infty < x < \infty, \tag{4.20}$$

with mean $\mu = 0$, for $k > 1$ and variance $\sigma^2 = k/(k-2)$, for $k > 2$.

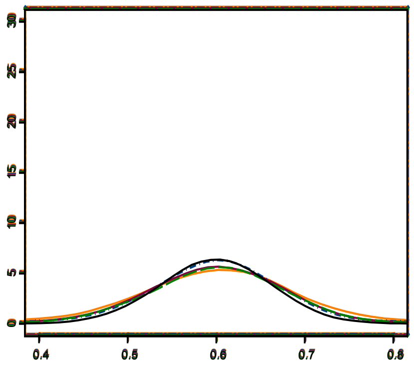

Figure 39. Distribution plots of $\hat{Y}_q$ for $N(\mu,\sigma^2)$, χ_k^2, t_k, $LN(\mu,\sigma^2)$, $W(\alpha,\beta)$ (bottom to top) with $n = 25$.

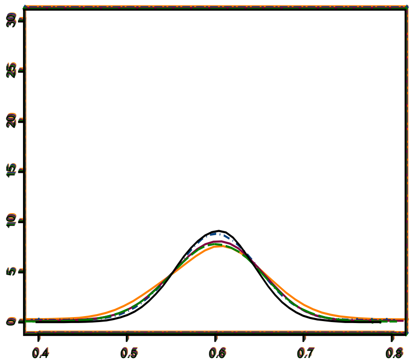

Figure 40. Distribution plots of $\hat{Y}_q$ for $N(\mu,\sigma^2)$, χ_k^2, t_k, $LN(\mu,\sigma^2)$, $W(\alpha,\beta)$ (bottom to top) with $n = 50$.

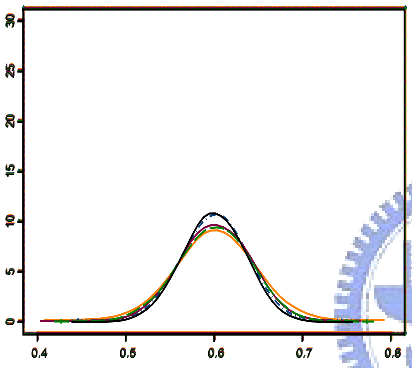

Figure 41. Distribution plots of $\hat{Y}_q$ for $N(\mu,\sigma^2)$, χ_k^2, t_k, $LN(\mu,\sigma^2)$, $W(\alpha,\beta)$ (bottom to top) with $n = 75$.

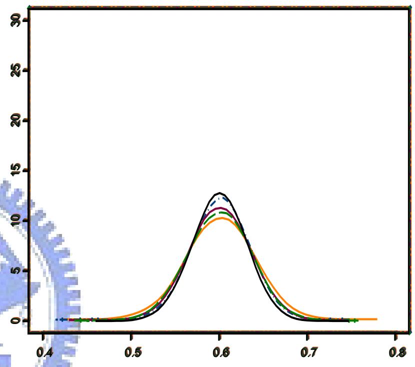

Figure 42. Distribution plots of $\hat{Y}_q$ for $N(\mu,\sigma^2)$, χ_k^2, t_k, $LN(\mu,\sigma^2)$, $W(\alpha,\beta)$ (bottom to top) with $n = 100$.

(4) Chi-square distribution χ_k^2 with degree of freedom k, where probability density function of is

$$f(x) = [1/\Gamma(k/2)](1/2)^{k/2} x^{k/2-1} e^{-x/2},\ x > 0, \tag{4.21}$$

with mean $\mu = k$ and variance $\sigma^2 = 2k$, $k = 1, 2, \ldots$.

(5) Weibull distribution $W(\alpha,\beta)$ with probability density function

$$f(x) = \alpha\beta x^{\beta-1}\exp(-\alpha x^{\beta}), \tag{4.22}$$

with mean $\mu = \alpha^{-1/\beta}\Gamma(1+\beta^{-1})$ and variance $\sigma^2 = \alpha^{-2/\beta}[\Gamma(1+2\beta^{-1}) - \Gamma^2(1+\beta^{-1})]$, for $x > 0$.

We randomly generate $N = 10{,}000$ samples of sizes $n = 25$, 50, 75, 100, 150, 200, 250, 300 for each distribution, then calculate the estimate value of Y_q for each sample. Figures 39-46 plot the distribution of $\hat{Y}_q$ for the eight levels of sample size with $Y_q = 0.6$, respectively. In each figure, five underlying process distributions including normal, lognormal, Student's t, chi-square, and Weibull are drawn with fixed sample size in order to investigate how the sample size

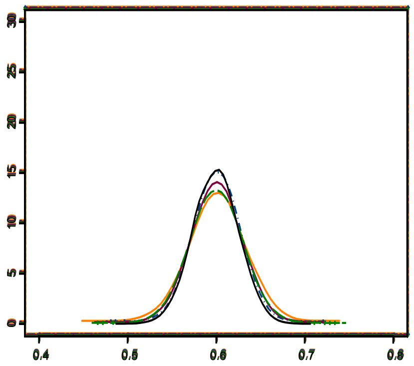

Figure 43. Distribution plots of $\hat{Y}_q$ for $N(\mu,\sigma^2)$, χ_k^2, t_k, $LN(\mu,\sigma^2)$, $W(\alpha,\beta)$ (bottom to top) with $n = 150$.

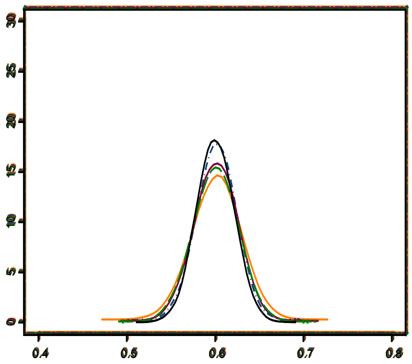

Figure 44. Distribution plots of $\hat{Y}_q$ for $N(\mu,\sigma^2)$, χ_k^2, t_k, $LN(\mu,\sigma^2)$, $W(\alpha,\beta)$ (bottom to top) with $n = 200$.

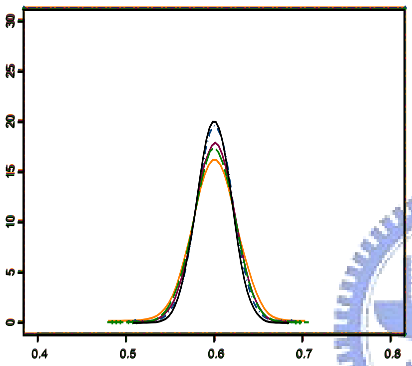

Figure 45. Distribution plots of $\hat{Y}_q$ for $N(\mu,\sigma^2)$, χ_k^2, t_k, $LN(\mu,\sigma^2)$, $W(\alpha,\beta)$ (bottom to top) with $n = 250$.

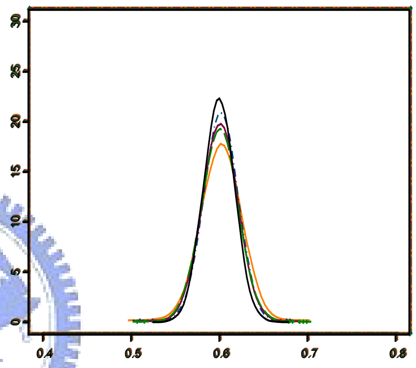

Figure 46. Distribution plots of $\hat{Y}_q$ for $N(\mu,\sigma^2)$, χ_k^2, t_k, $LN(\mu,\sigma^2)$, $W(\alpha,\beta)$ (bottom to top) with $n = 300$.

affects the distribution of $\hat{Y}_q$. From those plots, one may observe that for moderate sample size n (about 100) the distributions of the estimated Q-yield index all appear quite close to normal. Therefore, for practical purpose, normal approximations approach may be used for capability testing of Y_q.

4.4 An Application Example

We consider a case study for illustration purpose. Application of LEDs is expanding rapidly since high intensity LEDs of wide range of colors have been recently developed and become available, which enabled application of LEDs in a wide variety of areas such as instrument cluster lighting, color displays, traffic signals, roadway signs (barricade lights), airport signaling and lighting, automotive backlighting in dashboards and switches, telecommunication indicator and backlighting in telephone and fax backlighting for audio and video equipment, backlighting in office equipment, indoor and outdoor message boards, flat backlight for LCDs, switches and symbols, illumination purposes, alternative to incandescent lamps, etc.

LEDs are peculiar light sources much different from lamps in terms of physical size, flux level, spectrum, and spatial intensity distribution. LED technology provides a number of benefits over incandescent bulbs. Some benefits of LEDs for instrument cluster lighting are: (1) LEDs have lower power consumption: the LED instrument cluster uses approximately 1/5 of the electrical current of the incandescent instrument cluster. (2) LEDs have less heat generation: interior thermal measurements within the instrument cluster case indicate that the LED design operates 10-15°C cooler than the incandescent design. Interior thermal measurements within the telltale cavity airspace indicate that the LED design operates 25-50°C cooler than the incandescent design. (3) LEDs provide equivalent or better lighting: some comparative performances are that red LEDs are 3X brighter and amber LEDs are 2X brighter. (4) LEDs provide better reliability: LEDs are capable of withstanding high degrees of mechanical shock and vibration without failure. LEDs are capable of withstanding over 1000 temperature cycles –40/100°C, non-operating, without failure. (5) LEDs allow for smaller telltales: since LEDs are available in sizes less than 1/8" in diameter, LED telltales can be placed on spacing of 0.25-0.30 inches, if desired. (6) LEDs are dimmable with potentiometer: LEDs are normally wired in series with a current limiting resistor. In general, LEDs can be dimmed with a single potentiometer, as long as all series strings use the same number of LEDs. LEDs can also be dimmed through pulse width modulation. In this case, the number of lamps in each series string is not critical. (7) LEDs provide direct cost savings: potentially, LEDs allow for less expensive drive circuits. LEDs operate at lower currents (20 mA instead of 255 mA). Also, LEDs do not have a high inrush current when first turned on. In general, LEDs outperformed the incandescent bulbs for all gauge colors.

With a focus on the critical characteristic, the luminous intensity of LED sources, we examine a particular LED product model, with the upper and the lower specification limits of luminous intensity are set to $USL = 90$ mcd, $LSL = 40$ mcd, and the target value is set to $T = 60$ mcd. We note that it's an asymmetric tolerances case. The LED is said to be defective if the characteristic data does not fall within the specification limits (LSL, USL). For the purpose of making use of the methodology more convenient and accelerate the computation, an integrated S-PLUS computer program is developed (available from the authors) to calculate the lower confidence bounds. We only need to input the manufacturing specification limits, USL, LSL, target value T, and the collected sample data of size n. Then the estimated values $\hat{Y}$, $\hat{Y}_q$ and the lower confidence bounds of $\hat{Y}_q$ may be obtained easily. Thus, whether or not the process is capable may be determined.

A total of 150 observations were collected from a stable process in the factory, which are displayed in Table 16. Figure 47 displays the histogram of the sample data. From Figure 47, it is evident from the density line that the underlying process distribution is away from normality. Refer to the distribution plots of the Q-yield estimator, a random sample of size $n = 150$ seems to be large enough to apply the normal approximation approach for capability testing of Y_q.

Proceeding with the calculations by running the integrated S-PLUS program with 95% of confidence, we obtain the values of the sample estimators $\hat{Y}_q$ = 0.8082 and the corresponding lower confidence bound (LCB) L_{Y_q} as 0.7768. We note that the estimated $\hat{Y}_q$ index value is about 0.81. In fact, all 150 observations fall within the specification interval (LSL, USL) resulting that sample estimators of yield $\hat{Y} = 1$. From the producer's point of view, the proportion of conforming products is 100%. However, to quantify how well a process can meet the customer requirement, the lower confidence bound of $\hat{Y}_q$ is approximately equal to 0.78 can be interpreted as the satisfaction degree of the products, on the average, is 78% approximately, with 95% of confidence. From the corresponding lower confidence bound on Y_q, 0.7768, an example of capability testing is that if the Q-yield requirement preprint on the contract Y_q is set to 0.78, we may only conclude that the process is marginally capable, with 95% of confidence.

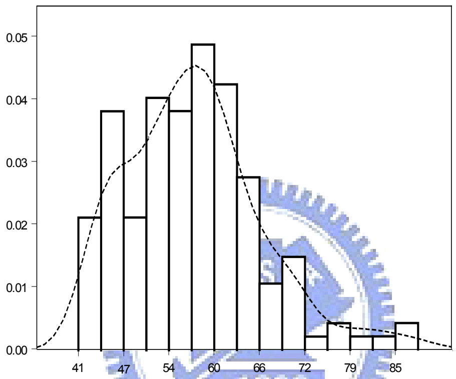

Figure 47. Histogram plot of the sample data of size n = 150.