

國立交通大學

光電工程研究所

碩 士 論 文

非線性光子晶體波導中的  
光調制不穩定性與光孤子傳播



Optical Modulation Instability and Solitons  
Propagation in the Nonlinear Photonic Crystal  
Waveguide

研 究 生：賴盈璇

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# 非線性光子晶體波導中的光調制不穩定性與光孤子 傳播

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## 摘要

本論文中，我們將對一具有非線性線缺陷的光子晶體波導，研究光調制不穩定性(modulation instability)及光孤子(soliton)的傳播。藉由緊束縛理論，考慮左右三個鄰近點缺陷的耦合影響下，我們導出一不連續的非線性展開方程式，並將其視為一延伸的非線性薛丁格方程(extended NLSE)。對此方程式求解，我們可以完整地以解析解來描述光調制不穩定現象及其增益係數。在負非線性折射介質組成的光子晶體波導中，我們可以發現光調制不穩定性的增益被抑制的現象，此現象不會出現於一般光纖波導的光調制不穩定性的增益頻譜圖。另一方面，我們以四階阮奇庫塔法(4<sup>th</sup> order Runge-Kutta method)數值解 extended NLSE，並與解析解相互印證。首先，我們得到模擬所得到的光調制不穩定性的增益頻譜圖與解析解相符合。利用此方法進行模擬，可在空間及時間軸上作光脈衝傳播的觀察，透過我們的模擬，可歸納出形成光孤子的條件以及光孤子傳播的特性。

# **Optical Modulation Instability and Solitons Propagation in the Nonlinear Photonic Crystal Waveguide**

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## **Abstract**

We have studied the modulation instability (MI) and solitons propagation in a photonic-crystal waveguide with nonlinear line defect. By tight-binding theory, we consider the coupling effects up to the third order nearest-neighbor defects and obtain the discrete nonlinear evolution equations as a new extended nonlinear Schrödinger equation (NLSE). Solving this equation, MI and MI gain can be analytically determined. We can find the phenomenon of gain suppression when nonlinear coefficient  $\gamma < 0$  and this is a big different from the gain profile in general. On the other hand, we do the simulation by the 4<sup>th</sup> order Runge-Kutta method to consistent our analytic solutions and we can observe MI and soliton propagation in the spatial and time domains. The results of analytic solution and simulation

are the same. Besides, we also summarize the conditions of forming solitons and some particular behaviors in soliton propagation by our simulation.



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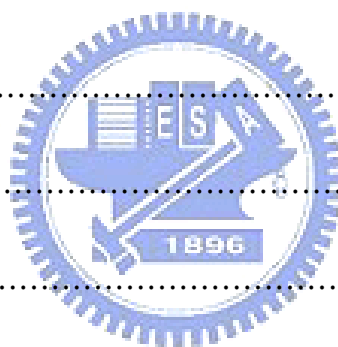
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# Contents

|                            |     |
|----------------------------|-----|
| Abstract (in Chinese)..... | i   |
| Abstract (in English)..... | ii  |
| Acknowledgement.....       | iv  |
| Contents.....              | v   |
| List of figures.....       | vii |

## Chapter 1 Introduction

|                                     |   |
|-------------------------------------|---|
| 1.1 Background.....                 | 1 |
| 1.2 Motivation.....                 | 3 |
| 1.3 Organization of the thesis..... | 4 |



## Chapter 2 Theory and Calculation Method

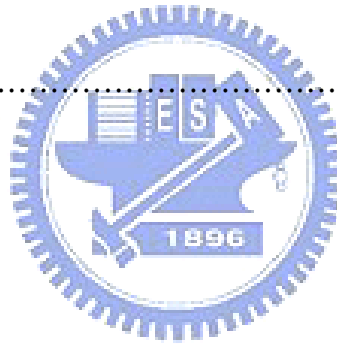
|                                                                          |    |
|--------------------------------------------------------------------------|----|
| 2.1 Soliton— nonlinear Schrödinger equation.....                         | 5  |
| 2.2 Modulation instability.....                                          | 12 |
| 2.3 Dispersion relation by PWEM.....                                     | 15 |
| 2.4 Discrete soliton— extended nonlinear Schrödinger equation.....       | 17 |
| 2.5 Dispersion relation by extended nonlinear Schrödinger equation.....  | 21 |
| 2.6 Modulation instability— extended nonlinear Schrödinger equation..... | 22 |
| 2.7 Fourth-order Runge-Kutta method for solving extended NLSE.....       | 24 |

## **Chapter 3 Simulation and discussion**

|                                       |    |
|---------------------------------------|----|
| 3.1 Results of analytic solution..... | 31 |
| 3.2 Results of simulation.....        | 43 |
| 3.3 Soliton propagation.....          | 49 |
| 3.4 Summary.....                      | 57 |

## **Chapter 4 Conclusion and Perspectives**

|                        |           |
|------------------------|-----------|
| 4.1 Conclusion.....    | 58        |
| 4.2 Future works.....  | 59        |
| <b>References.....</b> | <b>60</b> |



## List of Figures

|                 |                                                                                                           |           |
|-----------------|-----------------------------------------------------------------------------------------------------------|-----------|
| <b>Fig. 2.1</b> | Gain spectrum $g(Q)$ of modulation instability for $u_0=1$ and 2.....                                     | <b>14</b> |
| <b>Fig. 2.2</b> | The bandgap of perfect photonic structure.....                                                            | <b>15</b> |
| <b>Fig. 2.3</b> | The allowed frequencies in nonlinear line defect PCW and their dispersion relation.....                   | <b>16</b> |
| <b>Fig. 2.4</b> | Photonic crystal structure with nonlinear defects.....                                                    | <b>17</b> |
| <b>Fig. 2.5</b> | Square-array PCW.....                                                                                     | <b>19</b> |
| <b>Fig. 2.6</b> | Fitting to the dispersion relation.....                                                                   | <b>21</b> |
| <b>Fig. 2.7</b> | Euler's method for integrating an ordinary differential eq. (ODE).....                                    | <b>28</b> |
| <b>Fig. 2.8</b> | Midpoint method for integrating ODE.....                                                                  | <b>28</b> |
| <b>Fig. 2.9</b> | Fourth-order Runge-Kutta method for solving ODE.....                                                      | <b>30</b> |
| <b>Fig. 3.1</b> | $(A+B)$ versus $q$ for square-array PCW considering coupling up to the third NN.....                      | <b>31</b> |
| <b>Fig. 3.2</b> | MI gain profile ( $\gamma = -0.005 < 0$ ) in the square-array PCW.....                                    | <b>32</b> |
| <b>Fig. 3.3</b> | MI gain profile ( $\gamma = 0.005 > 0$ ) in the square-array PCW.....                                     | <b>33</b> |
| <b>Fig. 3.4</b> | MI gain profile for $\gamma < 0$ in square-array PCW only considering coupling of the NN defects.....     | <b>33</b> |
| <b>Fig. 3.5</b> | MI gain profile for $\gamma > 0$ in square-array PCW only considering coupling of the NN defects.....     | <b>34</b> |
| <b>Fig. 3.6</b> | The MI gain, variation of $(A+B)$ , and $(A+B+4F)$ versus $q$ with $p=0.49\pi$ .....                      | <b>36</b> |
| <b>Fig. 3.7</b> | MI gain profile for $\gamma < 0$ in square-array PCW considering coupling up to the third NN defects..... | <b>38</b> |
| <b>Fig. 3.8</b> | Triangular-array PCW.....                                                                                 | <b>39</b> |

|                  |                                                                                                                                                  |    |
|------------------|--------------------------------------------------------------------------------------------------------------------------------------------------|----|
| <b>Fig. 3.9</b>  | MI gain profile ( $\gamma = 0.005 > 0$ ) in the triangular-array PCW.....                                                                        | 39 |
| <b>Fig. 3.10</b> | MI gain profile ( $\gamma = -0.005 < 0$ ) in the triangular -array PCW.....                                                                      | 40 |
| <b>Fig. 3.11</b> | MI gain profile for $\gamma > 0$ in triangular-array PCW only considering coupling<br>of the NN defects.....                                     | 41 |
| <b>Fig. 3.12</b> | MI gain profile for $\gamma < 0$ in triangular-array PCW only considering coupling<br>of the NN defects.....                                     | 41 |
| <b>Fig. 3.13</b> | The cases of our simulation.....                                                                                                                 | 43 |
| <b>Fig. 3.14</b> | The initial intensity of electric fields in three cases.....                                                                                     | 44 |
| <b>Fig. 3.15</b> | The intensity of electric fields in three cases after evolution.....                                                                             | 44 |
| <b>Fig. 3.16</b> | Invariable spatial domain and variable time domain.....                                                                                          | 45 |
| <b>Fig. 3.17</b> | The intensity of electric field in time domain.....                                                                                              | 46 |
| <b>Fig. 3.18</b> | Taking logarithmic scale from Figure 3.17.....                                                                                                   | 46 |
| <b>Fig. 3.19</b> | Comparison of analytic solution and simulation with different p.....                                                                             | 47 |
| <b>Fig. 3.20</b> | Case I ( $p = 0.4\pi$ , $q = 0.1\pi$ ): the variation in space and spectrum.....                                                                 | 48 |
| <b>Fig. 3.21</b> | Case II ( $p=0.5\pi$ , $q=0.2\pi$ ): the variation in space and spectrum.....                                                                    | 49 |
| <b>Fig. 3.22</b> | Case II ( $p=0.5\pi$ , $q=0.9\pi$ ): the variation in space and spectrum.....                                                                    | 49 |
| <b>Fig. 3.23</b> | Case III ( $p=0.6\pi$ , $q=0.3\pi$ ): the variation in space and spectrum.....                                                                   | 50 |
| <b>Fig. 3.24</b> | the profiles of $V_g$ , $\beta_2$ , and $\beta_3$ .....                                                                                          | 52 |
| <b>Fig. 3.25</b> | The soliton propagation in space and corresponding spectra for various<br>centralspatial frequency of $0.22\pi$ , $0.32\pi$ , and $0.5\pi$ ..... | 54 |
| <b>Fig. 3.26</b> | The soliton propagation in space and corresponding spectra for various intensity<br>$P_0$ for $p= 0.22\pi$ , $0.32\pi$ , and $0.5\pi$ .....      | 55 |