國立交通大學

光電工程研究所

碩 士 論 文

非線性光子晶體波導中的
光調制不穩定性與光孤子傳播

Optical Modulation Instability and Solitons Propagation in the Nonlinear Photonic Crystal Waveguide

研 究 生：賴盈璇

指導教授：謝文峰　教授

中 華 民 國 九 十 七 年 六 月

非線性光子晶體波導中的光調制不穩定性與光孤子傳播

Optical Modulation Instability and Solitons Propagation in the Nonlinear Photonic Crystal Waveguide

研 究 生：賴盈璇　　Student：Ying-Hsuan Lai

指導教授：謝文峰　教授　　Advisor：Wen-Feng Hsieh

國 立 交 通 大 學

光電工程研究所

碩 士 論 文

A Thesis
Submitted to Institute of Electro-optical Engineering
College of Electrical Engineering and Computer Science
National Chiao Tung University
in Partial Fulfillment of the Requirements
for the Degree of
Master
in
Electro-optical Engineering
June 2007
Hsinchu, Taiwan, Republic of China

中華民國九十七年六月

非線性光子晶體波導中的光調制不穩定性與光孤子傳播

研究生：賴盈璇　　　　指導教授：謝文峰　教授

國立交通大學光電工程研究所

摘要

本論文中，我們將對一具有非線性線缺陷的光子晶體波導，研究光調制不穩定性(modulation instability)及光孤子(soliton)的傳播。藉由緊束縛理論，考慮左右三個鄰近點缺陷的耦合影響下，我們導出一不連續的非線性展開方程式，並將其視為一延伸的非線性薛丁格方程(extended NLSE)。對此方程式求解，我們可以完整地以解析解來描述光調制不穩定現象及其增益係數。在負非線性折射介質組成的光子晶體波導中，我們可以發現光調制不穩定性的增益被抑制的現象，此現象不會出現於一般光纖波導的光調制不穩定性的增益頻譜圖。另一方面，我們以四階阮奇庫塔法(4^{th} order Runge-Kutta method)數值解 extended NLSE，並與解析解相互印證。首先，我們得到模擬所得到的光調制不穩定性的增益頻譜圖與解析解相符合。利用此方法進行模擬，可在空間及時間軸上作光脈衝傳播的觀察，透過我們的模擬，可歸納出形成光孤子的條件以及光孤子傳播的特性。

Optical Modulation Instability and Solitons Propagation in the Nonlinear Photonic Crystal Waveguide

Student: Ying-Hsuan Lai   Advisor: Prof. Wen-Feng Hsieh

Institute of Electro-optical Engineering

National Chaio Tung University

Abstract

We have studied the modulation instability (MI) and solitons propagation in a photonic-crystal waveguide with nonlinear line defect. By tight-binding theory, we consider the coupling effects up to the third order nearest-neighbor defects and obtain the discrete nonlinear evolution equations as a new extended nonlinear Schrödinger equation (NLSE). Solving this equation, MI and MI gain can be analytically determined. We can find the phenomenon of gain suppression when nonlinear coefficient $\gamma < 0$ and this is a big different from the gain profile in general. On the other hand, we do the simulation by the 4^{th} order Runge-Kutta method to consistent our analytic solutions and we can observe MI and soliton propagation in the spatial and time domains. The results of analytic solution and simulation

are the same. Besides, we also summarize the conditions of forming solitons and some particular behaviors in soliton propagation by our simulation.

誌謝

在交大的日子，時光總是匆匆，轉瞬間，已輪到我必須寫這篇謝誌。
能夠完成碩士班的修業，比我想像中還不容易，現在的我，心裡沒有太多的興奮，反倒是有些複雜，要說兩年的研究生活只有笑沒有淚是騙人的，但在這七百多個日子裡，我學到了很多，
謝謝我遇到的好事，讓我相信世界的美好;謝謝我遇到的挫折，讓我更懂得珍惜。
我想要感謝的人太多了，
謝謝有耐心，數學超好，又像爸爸一樣和藹的實驗室大頭目—謝文峰老師；
謝謝常常在研究上拉我好大一把的光子晶體組台柱—智賢學長；
謝謝像大哥哥一樣照顧我們的實驗室高雄一哥—智章學長；
謝謝又貼心，身材又好，又常常跟我聊八卦的材料組一姐—晴如學姐；
謝謝總是熱心助人又金條滿出來的實驗室新好男人&好爸爸楷模—黃董；
謝謝實驗很認真做，但很愛消遣我的同步輻射達人—維仁學長；
謝謝總是偷偷在位子上推文的卡丁車彩手級鄉民—國峰學長；
謝謝努力追求攝影最高境界的實驗室御用攝影師—小豪學長；
謝謝專研股市情報，一秒鐘幾十萬上下的好野人—松哥；
謝謝和我同樣來自中央，也同樣來自高雄這個地靈人傑的老同鄉—博濟學長；
謝謝總是帶著帥氣機車帽的有為青年—小郭學長；
謝謝總是善解人意而且臉比我小但是綽號卻叫麵包超人的可愛青春美少女—詹芸佩；
謝謝擁有舌燦蓮花特異功能，帶領實驗室邁向魔術方塊里程碑的表裡不一白面書生—陳億文；
謝謝專長是不睡覺，興趣是量子力學，幫碩二的大家把屎把尿的博士班候選人—陳厚仁；
謝謝紅遍港台，網路照片點閱率最高，股票經營有聲有色，賣香腸賺大錢的阿伯—李岳勳；
謝謝很愛裝酷，彈琴超認真的伍佰—林易慶；
謝謝實驗室最高，一年半就畢業，肚子有點大的樂團主唱—徐瑋澤；
謝謝常常陪我逛街，又超會安慰人，總是令人感到窩心的可人兒—蔡智雅；
謝謝長相斯文，身材魔鬼的溫文儒雅貴公子—黃冠智；
謝謝跟我一樣超愛看電影，對美食超有研究的美食通—林建輝；
謝謝很愛裝可愛，也真的很可愛的教授級卡丁車阿肥選手—黎延垠；
謝謝所有的老朋友、新朋友，願意和我一起乘載生活中的喜怒哀樂，你們是我生活中最大的支柱；
最後謝謝給我最最最最最多關心與溫暖的家人，無論在何時何地我都想跟你們說，我真的超愛你們的♥

2008.07.22 盈璇 於交大雷射診斷實驗室

Contents

Abstract (in Chinese)……………………………………………………………………………i

Abstract (in English)……………………………………………………………………………ii

Acknowledgement………………………………………………………………………….iv

Contents…………………………………………………………………………………......v

List of figures……………………………………………………………………………... vii

Chapter 1 Introduction

1.1 Background……………………………………………………………………………... 1

1.2 Motivation………………………………………………………………………………3

1.3 Organization of the thesis…………………………………………………………….....4

Chapter 2 Theory and Calculation Method

2.1 Soliton– nonlinear Schrödinger equation………………………………………………… 5

2.2 Modulation instability…………………………………………………………….... 12

2.3 Dispersion relation by PWEM………………………………………………………… 15

2.4 Discrete soliton– extended nonlinear Schrödinger equation………………………….... 17

2.5 Dispersion relation by extended nonlinear Schrödinger equation.................................... 21

2.6 Modulation instability– extended nonlinear Schrödinger equation……………………… 22

2.7 Fourth-order Runge-Kutta method for solving extended NLSE…………………………24

Chapter 3 Simulation and discussion

3.1 Results of analytic solution……………………………………………………….... 31

3.2 Results of simulation………………………………………………………......43

3.3 Soliton propagation………………………………………………………………49

3.4 Summary…………………………………………………………………………57

Chapter 4 Conclusion and Perspectives

4.1 Conclusion……………………………………………………………………...58

4.2 Future works…………………………………………………………....................59

References……………………………………………………………………….... 60

List of Figures

Fig. 2.1 Gain spectrum g(Q) of modulation instability for u_0=1 and 2……………………..**14**

Fig. 2.2 The bandgap of perfect photonic structure………………………………………...**15**

Fig. 2.3 The allowed frequencies in nonlinear line defect PCW and their dispersion relation……………………………………………………………………..**16**

Fig. 2.4 Photonic crystal structure with nonlinear defects…………………………………**17**

Fig. 2.5 Square-array PCW……………………………………………………………**19**

Fig. 2.6 Fitting to the dispersion relation……………………………………………..........**21**

Fig. 2.7 Euler's method for integrating an ordinary differential eq. (ODE)……….............**28**

Fig. 2.8 Midpoint method for integrating ODE……………………………………………**28**

Fig. 2.9 Fourth-order Runge-Kutta method for solving ODE…………………………...... **30**

Fig. 3.1 (A+B) versus q for square-array PCW considering coupling up to the third NN...**31**

Fig. 3.2 MI gain profile (γ = - 0.005< 0) in the square-array PCW……..............................**32**

Fig. 3.3 MI gain profile (γ = 0.005> 0) in the square-array PCW……............................... **33**

Fig. 3.4 MI gain profile for γ <0 in square-array PCW only considering coupling of the NN defects………………………………………..................................**33**

Fig. 3.5 MI gain profile for γ > 0 in square-array PCW only considering coupling of the NN defects………………………………………………………………...**34**

Fig. 3.6 The MI gain, variation of (A+B), and (A+B+4F) versus q with p=0.49π…………**36**

Fig. 3.7 MI gain profile for γ < 0 in square-array PCW considering coupling up to the third NN defects………………………………………………………….........**38**

Fig. 3.8 Triangular-array PCW…………………………………………………………...**39**

Fig. 3.9 MI gain profile (γ = 0.005> 0) in the triangular-array PCW………………………....... **39**

Fig. 3.10 MI gain profile (γ = - 0.005< 0) in the triangular -array PCW…………………...**40**

Fig. 3.11 MI gain profile for $\gamma > 0$ in triangular-array PCW only considering coupling of the NN defects………………………………………………………….......... **41**

Fig. 3.12 MI gain profile for $\gamma < 0$ in triangular-array PCW only considering coupling of the NN defects………………………………………………………….......... **41**

Fig. 3.13 The cases of our simulation…………………………………………………… **43**

Fig. 3.14 The initial intensity of electric fields in three cases…………………………... **44**

Fig. 3.15 The intensity of electric fields in three cases after evolution.............................. **44**

Fig. 3.16 Invariable spatial domain and variable time domain……………………………..**45**

Fig. 3.17 The intensity of electric field in time domain……………………………………**46**

Fig. 3.18 Taking logarithmic scale from Figure 3.17………………………………………**46**

Fig. 3.19 Comparison of analytic solution and simulation with different p……………...... **47**

Fig. 3.20 Case Ⅰ (p = 0.4π, q = 0.1π): the variation in space and spectrum……………...... **48**

Fig. 3.21 Case Ⅱ (p=0.5π, q=0.2π): the variation in space and spectrum…………………...... **49**

Fig. 3.22 Case Ⅱ (p=0.5π, q=0.9π): the variation in space and spectrum…………………...... **49**

Fig. 3.23 Case Ⅲ (p=0.6π, q=0.3π): the variation in space and spectrum………………… **50**

Fig. 3.24 the profiles of V_g, β_2, and β_3…………………………………………………..**52**

Fig. 3.25 The soliton propagation in space and corresponding spectra for various centralspatial frequency of 0.22π, 0.32π, and 0.5π... **54**

Fig. 3.26 The soliton propagation in space and corresponding spectra for various intensity P_0 for p= 0.22π, 0.32π, and 0.5π...**55**