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Thickness dependence of magnetic force acting on a magnetic dipole over a type-II superconducting thin film*

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The thickness dependence of the magnetic force acting on a point magnetic dipole over a superconducting thin film in the mixed state with a single vortex line is calculated by London theory. The magnetic force is decomposed into two parts, vertical and lateral components. Both vertical and lateral force components approach to the saturated value with increasing thickness when the thickness is much larger than the London penetration depth. A single vortex created in the thin film by the field of the magnetic dipole is also considered. The thickness dependence of the critical position of the point dipole for creating the first vortex line in the thin film is exploited and derived.

1. INTRODUCTION

Since the discovery of high T_c superconductor (HTSC), the phenomenon of superconducting levitation has paid much attention. The magnetic dipole tip used to probe the vortex line structure in the superconducting thin film is highly interesting. On the theoretical side, the superconducting levitation force acting on a magnet over a semi-infinite type-II superconductor in both the Meissner and mixed states has recently been studied [1,2]. The thickness dependence of the levitation acting on the point dipole over a thin film in the Meissner state has been examined and calculated [2,3].

In this report, the thickness dependence of the magnetic force acting on a point dipole over a superconducting thin film with a single vortex line is studied by energy consideration. Also, the thickness dependence of the critical position of the point dipole is derived when the first vortex line is created.

2.THICKNESS DEPENDENCE OF MAGNETIC FORCE

Considering a single vortex embedded in the superconducting thin film with thickness d and directed perpendicularly to its surface. Set the film surface in the x-y plane and a single vortex locates at distance r_o from the origin of the x-y plane. A magnetic dipole with a moment **m** pointing along z axis is placed distance a above the film surface. From London's and Maywell's equations, the vector potential **A** can be expressed as

$$\nabla \times \nabla \times \mathbf{A} = -\mu_0 \mathbf{m} \times \nabla[\delta(\mathbf{r})\delta(z-a)], \quad z>0$$

$$\nabla \times \nabla \times \mathbf{A} + \mathbf{A}/\lambda^2 = \frac{\phi_0}{2\pi\lambda^2}\frac{1}{|\mathbf{r}-\mathbf{r}_0|}\hat{\theta}', -d<z<0$$

$$\nabla \times \nabla \times \mathbf{A} = 0 \qquad z<-d$$

where μ_o is the vacuum permeability, λ is the penetration depth, the flux quantum h/2e is ϕ_o, and $\hat{\theta}'$ is the angular unit vector while the origin is just at the center of the vortex line. The vector potential **A** is due to the magnetic dipole and the vortex line. The cylindrical coordinates (r, θ, z) with center at the origin is used. The boundary conditions for **A** (continuity and continuity of its normal derivative at the interfaces z = 0 and z - d) are imposed. The solution of A may be obtained by expanding in Bessel functions [4]. The magnetic induction **B** can be calculated by $\nabla \times \mathbf{A}$. The magnetic force acting on the magnetic point dipole can be gotten by the interaction energy through $\mathbf{F} = -\nabla U_{int}$. Here the interaction energy is

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$$U_{int} = -\frac{1}{2}\mathbf{m}.\mathbf{B}_{in}(0,0,a) - \mathbf{m}.\mathbf{B}_v(0,0,a).$$

The first term represents the self-interaction energy due to the screening current. The second term is the interaction energy between the point dipole and the vortex line. The force acting on the point dipole at $(\mathbf{r},a)$ is expressed as

$$F_z = \frac{\mu_o m^2}{4\pi\lambda^2}\int_0^\infty dk k^3 e^{-2ka}\sinh Kd/CH(k) - \frac{m\phi_o}{2\pi\lambda^2}\int_0^\infty dk(k^2/K)e^{-ka}J_o(kr_o)SH(k)/CH(k)$$

$$F_{ro} = \frac{-m\phi_o}{2\pi\lambda^2}\int_0^\infty dk(k^2/K)e^{-ka}J_1(kr_o)SH(k)/CH(k)$$

where $K = \sqrt{k^2 + 1/\lambda^2}$, SH(k)=KsinhKd+kcoshKd-k, $CH(k) = (k^2 + K^2)\sinh Kd + 2kK\cosh Kd$, J_1 is 1st-order Bessel function, F_z is the z component of the force **F**, and F_{ro} is the force **F** in the lateral direction. The repulsive m^2 term in F_z is due to point dipole interact with the superconductor in the Meissner state, and the attractive $m\phi_o$ term is contributed by the vertex line. The force due to the vertex line also contributes to the lateral force F_{ro}. $F_{ro} = 0$ when $r_o = 0$, i.e. a point dipole is just above the vertex line. In this situation, only vertical force remains. It is obvious that F_z is either attractive or repulsive depending on the dipole moment m, a and d. At $r_o = 0$, two extreme situations ($a \gg \lambda$ and $a \ll \lambda$) of F_z are given as

$$F_z = \frac{3\mu_o m^2}{32\pi a^4}[1 - \frac{4\lambda}{a}\coth(\frac{d}{\lambda})] - \frac{m\phi_o}{\pi a^3}(1 - \frac{3\lambda}{a}\frac{\cosh(\frac{d}{\lambda})+1}{\sinh(\frac{d}{\lambda})})$$

for $a \gg \lambda$

$$F_z = \frac{\mu_o m^2}{64\pi a^2\lambda^2}[1-(1+\frac{d}{a})^{-2}] - \frac{m\phi_o}{4\pi a\lambda^3}[(1-(1+\frac{d}{a})^{-1}],$$

for $a \ll \lambda$.

For $r_o \ll \lambda$, the lateral force F_{ro} in two limiting cases is given as

$$F_{ro} = \frac{-3m\phi_o r_o}{2\pi a^4}[1 - \frac{4\lambda}{a}(\frac{\cosh(\frac{d}{\lambda})+1}{\sinh(\frac{d}{\lambda})})], \text{for } a \gg \lambda$$

$$F_z = \frac{m\phi_o r_o}{8\pi a^2\lambda^2}[1-(1+\frac{d}{a})^{-2})], \text{ for } a \ll \lambda.$$

In either situations, F_z and F_{ro} approache a saturated value as $d \gg \lambda$.

3.THICKNESS DEPENDENCE OF VORTEX CREATION

The critical position a_1 of the dipole to create first vortex line in the thin film can be found by equating the free energy of the system (dipole plus thin film) before and after the creation of first vortex line. a_1 may be determined by a complicated relation

$$\int_0^\infty dt(t/T)e^{-(a_1 t/\lambda)}[S(t)/C(t)] = (d/2\alpha\lambda)K_o(\xi/\lambda) + (1/\alpha)\int_0^\infty dt(1/T^2)J_o(\xi t/\lambda)S(t)/C(t) \quad (1)$$

where $\alpha = \mu_o m/\phi_o\lambda$, ξ is the coherence length, K_o is the zero-order modified Bessel function, J_o is the zero-order Bessel function, $T = (t^2+1)^{1/2}$, $S(t) = T\sinh(Td/\lambda) + t\cosh(Td/\lambda) - t$, and $C(t) = (t^2+T^2)\sinh(Td/\lambda) + 2tT\cosh(Td/\lambda)$. From Eq.(1), we may find that a_1 decreases with increasing d for high λ/ξ.

4. CONCLUSION

The magnetic force acting on a point dipole placed above the type II superconducting thin film with a single vortex approaches to a saturated value as the thickness is much greater than the penetration depth. The creation of a single vortex caused by a point dipole is studied. The critical height (position) of a point dipole is found to decrease with increasing thickness of the film. The discussion of a single vortex in the thin film is this short report holds only for a perfect thin film.

Our formulation may be extended to the case with more vortices.

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