



## Formation of multiple-quanta vortices at circular defects in a superconducting thin film\*

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The possibility of the formation of multiple-quanta vortices in a superconducting thin film with circular defects is discussed. It is shown that the condition for creating two-quanta vortices on a circular defect depends on the radius of defect, the separation between defects and the configuration of defects.

### 1. INTRODUCTION

The most remarkable feature of type-II superconductors is the creation of the vortex lattice in a magnetic field. The critical supercurrent in a magnetic field is determined by the strength of pinning due to the interaction between the vortices and various structural defects. The properties of ion-irradiated high-T<sub>c</sub> superconductors where the heavy ions create columnar defects [1] strongly influence the critical current of samples. The interaction between a vortex lattice and an ordered columnar defect structure has been investigated [2,3]. Such a system was discussed already in an experimental study [4]. Among other things it is interesting because a cylindrical cavity in a bulk superconductor or a hole in a superconducting film is a simple model of a pinning center.

The aim of the present work is to study the possibility of the formation of two-quanta vortices on circular defects under a magnetic field. We consider the circular defects of a radius  $b$  form an ordered lattice which may fits the triangular lattice. When the magnetic field continues to increase, more and more defects become occupied by the vortices. If the vortex core coincides with the circular defect there is no loss of superconducting condensation energy in the core and it contributes to vortex pinning. Further increase of the magnetic field may create vortices between defects. However, there exists an alternative option, the appearance of two-quanta vortices at a defect.

### 2. FREE ENERGY OF THE SYSTEM

Let us consider a thin superconducting film with defects in a perpendicular magnetic field. We choose the system zero-point free energy is in the absence of vortex. The free energy of an arbitrary lattice configuration can be expressed as

$$F = \sum_i E_{vi} + \frac{1}{2} \sum_{ij} U_{ij} + \sum_{ik} V_{ik}, \quad (1)$$

where  $E_{vi}$  is the self-energy of the  $i$ th circular defect with one quanta of the magnetic field,  $U_{ij}$  is the interaction energy between two vortices and  $V_{ik}$  is the vortex-defect interaction energy.

The energy of a circular defect with a single vortex is

$$E_v = \frac{\phi_0^2}{2\pi\mu_0\Lambda} \ln\left(\frac{\Lambda}{b}\right), \quad (2)$$

where  $\phi_0$  is the magnetic flux quantum,  $\mu_0$  is the vacuum permeability and  $\Lambda = 2\lambda^2/d$  is the effective screening length for a film of thickness  $d$ .

The interaction energy between vortices can be described by the London approximation[5],

$$U(r) = \frac{\phi_0^2}{2\mu_0\Lambda} \left[ H_0\left(\frac{r}{\Lambda}\right) - Y_0\left(\frac{r}{\Lambda}\right) \right], \quad (3)$$

where  $H_0(x)$  is the zero-order Struve function,  $Y_0(x)$  is the zero-order Bessel function and  $r$  is the distance between two vortices.

\* This work was supported by Natinal Science Council of R. O. C.

The expression of the interaction energy due to the interaction between a defect with a radius  $b$  ( $\ll \Lambda$ ) and a vortex [6] is

$$V(r) = -\frac{\phi_0^2}{2\mu_0\Lambda} \sum_{n=1}^{\infty} \frac{[H_{-n}(\frac{r}{\Lambda}) - Y_{-n}(\frac{r}{\Lambda})]^2}{[H_{-n}(\frac{b}{\Lambda}) - Y_{-n}(\frac{b}{\Lambda})]} J_{-n}(\frac{b}{\Lambda}), \quad (4)$$

where  $J_n(x)$  are the Bessel function.

### 3. THE CONDITIONS OF TWO-QUANTA VORTEX FORMATION

When the magnetic field increases and vortices start to enter into the film they occupy the defect sites. If all defects are occupied by single-quantum vortices the possibility of creation of a two-quantum vortex is formed at the point  $i = 0$ , then the change in free energy due to two-quantum vortex formation may be written as

$$\Delta F_{2v} = \frac{3\phi_0^2}{2\pi\mu_0\Lambda} \ell n\left(\frac{\Lambda}{b}\right) + \sum_{j \neq 0} U_{0j} + \sum_{k \neq 0} V_{0k}. \quad (5)$$

To perform the corresponding and analytical calculation we assume the defects form a triangular lattice with a distance  $a$  ( $\gg \Lambda$ ) between neighboring defects. Then the Eq.(5) becomes

$$\Delta F_{2v} = \frac{\phi_0^2}{2\pi\mu_0\Lambda} \left[ 3\ell n\left(\frac{\Lambda}{b}\right) + 2\sum_j \frac{\Lambda}{(P_{0j}a)} - \sum_k \frac{\Lambda^2 b^2}{(P_{0k}a)^4} \right], \quad (6)$$

where  $P_{0j}$  is the distance between reference vortex  $i=0$  and any other vortex (or defect)  $j$ .

When an additional single-quantum vortex appears at the center of a triangular unit cell  $r'_0$ . The change of the free energy

$$\Delta F_{1v} = \frac{\phi_0^2}{2\pi\mu_0\Lambda} \left[ \ell n\left(\frac{\Lambda}{\xi}\right) + 2\sum_j \frac{\Lambda}{(P'_{0j}a)} - \sum_k \frac{\Lambda^2 b^2}{(P'_{0k}a)^4} \right], \quad (7)$$

where  $P'_{0j}a = |r_j - r'_0|$  is the distance between the additional vortex and any other vortex (or defect)  $j$ .

Then under the condition  $\Delta F_{2v} < \Delta F_{1v}$ , it turns out to be energetically more favorable to create two-quantum vortices after all defects are occupied, and we have

$$\ell n\left(\frac{\xi\Lambda^2}{b^3}\right) < 2\alpha \frac{\Lambda}{a} - \beta \frac{b^2\Lambda^2}{a^4}, \quad (8)$$

where  $\alpha$  and  $\beta$  are parameters which refer to the intrinsic properties of triangular lattice, and which depend on the number of defects. If, for instance, we set the number of defects  $N=31$ , then  $\alpha \approx 2.55$  and  $\beta \approx 22$ .

### 4. CONCLUSION

We have studied the presence of circular defects in the thin superconducting film may lead to two-quantum vortices formation. The conditions of vortices to be fixed on defects depend on the symmetry of defect structure and the number of defects. Our consideration is adequate for the case  $a \gg \Lambda$ , when the defect concentration is relatively low.

### REFERENCES

1. W. Gerhauser *et al.*, Phys. Rev. Lett. 68, 879 (1992).
2. A. I. Buzdin, Phys. Rev. B 47, 11416 (1993).
3. V. N. Rud'ko, O. N. Shevtsova and S. V. Shiyanovskil, Zh. Eksp. Teor. Fiz. 109, 495 (1996) [Sov. Phys. JETP 82, 262 (1996)].
4. A. Bezryadin and B. Pannetier, J. Low Temp. Phys. 98, 251 (1994).
5. A. L. Fetter and P. C. Hohenberg, Phys. Rev. 159, 2 (1967).
6. J. L. Chen and T. Y. Tseng, Phys. Rev. B 54, 502 (1996).