國 立 交 通 大 學

經營管理研究所

碩 士 論 文

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The Evaluation of the Short ETFs

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中 華 民 國 九十七 年 六 月

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經營管理研究所
碩士論文

A Thesis
Submitted to Institute of Business and Management
College of Management
National Chiao Tung University
in Partial Fulfillment of the Requirements
for the Degree of
Master of Business Administration

June 2008
Taipei, Taiwan, Republic of China
中華民國 九十七 年 六 月

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摘　　要

本篇文章使用一般化自我相關條件異質變異模型(Generalized Autoregressive Conditional Heteroscedasticity ; GARCH)以及動態條件相關係數模型(Dynamic Conditional Correlation；DCC)來對放空型指數股票型基金(Exchanged-Traded Fund；ETF)的追蹤誤差和避險績效進行評價。本文發現對於同一指數的放空和雙倍放空 ETF 的追蹤誤差而言，道瓊工業平均指數以及標準普爾中型企業 400 的放空型 ETF 比起雙倍放空 ETF 有較小的追蹤誤差，相反的，標準普爾的雙倍放空 ETF 比放空型 ETF 有較佳的追蹤能力。而在不同指數間的比較上，那斯達克 100 的放空型 ETF 與標準普爾中型企業 400 的雙倍放空型 ETF 有最大的追蹤誤差。本文也證實了指數與 ETF 報酬間的不完全相關會產生 ETF 的追蹤誤差。在產生追蹤誤差的因素上，我們發現由於 ProShares 在操作雙倍放空 ETF 時使用了較多的指數期貨，因此如同預期，實證結果也顯示出標準普爾以及標準普爾中型企業 400 此兩種指數的雙倍放空 ETF 的追蹤誤差比起放空 ETF 的追蹤誤差更容易受到指數期貨的波動所影響，此外，本文也觀察到放空和雙倍放空 ETF 的追蹤誤差會隨著交易量的增加而上升。最後，我們比較了放空以及雙倍放空型 ETF 的避險績效，對於道瓊工業平均指數以及標準普爾中型企業 400 而言，放空型 ETF 比起雙倍放空 ETF 有較佳的避險績效，而標準普爾的雙倍放空 ETF 比起放空型 ETF 有較好的避險績效。在跨指數的比較中，標準普爾中型企業400的放空型ETF與標準普爾的雙倍放空ETF 擁有最好的避險績效。這些結果可以作為投資人在投機交易以及避險上的一個參考依據。

關鍵字：指數型股票基金，追蹤誤差，避險績效，一般化自我相關條件異質變異模型，動態條件相關係數模型

The Evaluation of the Short ETFs

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ABSTRACT

Based on the Generalized Autoregressive Conditional Heteroscedasticity (GARCH) of Bollerslev (1986) and the Dynamic Conditional Correlation (DCC) Model of Engle (2002), we investigate the tracking errors and the hedging effectiveness of each short ETF. We find that when it comes to tracking errors of Short/UltraShort ETFs related to the same benchmark, the Short ETFs of DJIA and S&P400 MidCap outperform the UltraShort ETFs of these two indices. On the contrary, the UltraShort ETF of S&P500 has the better tracking ability than the Short ETF of the S&P500. As for the cross indices comparison, the Short ETF of NASDAQ100 is the worst on tracking performance in the group of Short ETFs while the MZZ has the worst tracking ability in the group of UltraShort ETFs. Furthermore, we also examine the relationship between tracking errors and volatilities of their related index futures as well as that between tracking errors and trading volumes. We conclude that the tracking errors of DOG and DXD are affected almost equally by the volatilities of DJIA index futures while the volatilities of S&P500 (S&P400 MidCap) index futures have more influences on the tracking errors of SDS (MZZ) than on those of SH (MYY). These results coincide with the facts that the ProShares uses more index futures on UltraShort ETFs than on Short ETFs. We also find that over-trading on the shot ETFs may lead to larger tracking errors, and this effect is quite obvious regarding MYY and MZZ. Finally, we research the hedging performance of each short ETFs. We find that Short ETFs outperform UltraShort ETF when DJIA and S&P400 MidCap are concerned while the UltraShort (SDS) ETF of S&P500 has the better hedging performance than SH. Besides, the MYY has the best hedging performance among the Short ETFs when SDS has the best hedging effectiveness among the UltraShort ETFs.

Keywords: ETF, Tracking Errors, Hedging Performance, GARCH Model, DCC Model

誌　謝

兩年的研究生生活，現在回想起來，真的是過得很快，人生的一個階段完成後，下個階段已經在眼前了，首先，感謝經營管理研究所提供的教學資源以及所有教授熱心的指導，讓我可以在每一門專業課程中學到了很多寶貴的知識和經驗，使我在這兩年獲益匪淺。

本論文能夠順利完成，最重要的，要感謝我的指導教授周雨田教授，周老師讓我們選擇自己有興趣的題目，使我們得以自由發揮，並且於研究期間適時提供寶貴的意見與細心的指導，將我們導引到正確的寫作方向同時也提供了許多的靈感，讓本論文充實許多。還要感謝劉炳麟學長幾次參與我們的研究討論，給予我們在操作計量模型與軟體上相當大的助益。除此之外，感謝口試委員胡均立教授、張元晨教授與周恆志教授於口試時所提出的建議以及更正，以及書審委員丁承教授於書審時給予的許多寶貴意見，使本研究內容更趨於完整。

我也非常感謝同門的伙伴們：致宏、念青與維苡在學業的幫助；動作總是比我們快的志豪、志勤、美娟、安婷以及士漢學長的鼓勵，特別是安婷在我們口試時給予了非常大的幫助，讓我們在緊張之餘不至於慌張；也感謝守正、繼良、宣琪、珮婷、柔均、彥良還有瓊文總是陪我吃飯玩樂，使我的研究生生活更加的有趣。

感謝我的大學好朋友們：大家三不五時的聚餐總是讓我可以充充電、轉換心情、提振一下精神。也謝謝我的國中死黨們，雖然大家都很忙，但總是能保持聯絡甚至挪出時間一同出遊，因為有你們的友情支柱，讓我總是能面對接下來的挑戰！

最後也最感謝的就是我的家人，特別是爸爸、媽媽，因為有你們在我這二十多年的求學生涯中，不斷地對我付出、關懷，讓我可以無後顧之憂地、恣意地享受自己所選擇的生活。

林文元 謹誌

2008 年 6 月

Contents

List of Tables

List of Figures

Ⅰ. Introduction

Exchange-traded funds (ETFs) are a rapidly growing class of financial instruments and they are now widely used investment vehicles. Although the Exchange-traded funds became more and more popular in the last few years, they yet received much attention in the academic literatures comparing to the mutual funds (Kostovetsky 2003). Each ETF is designed to track a specific index. They provide the availability to a wide range of investment styles, asset classes, and individual sectors. The idea of trading a portfolio in a single transaction did not come from the Toronto Stock Exchange Index Participation (TIPS) or Standard & Poor's Depositary Receipts (SPDRS) that are the earliest examples of the modern portfolio-traded-as-a-share structure. It originated in what has come to be known as program trading. In the late 1970s and early 1980s, program trading was the revolutionary ability to trade a whole portfolio. The progress in electronic order entry technology and the availability of large order desks in the investment banking industry made early portfolio trades attainable (2001).

For the retail and institutional investor, buying and selling ETFs is the essence of simplicity. The trading rules are the same as those of the stock market. Instead of being purchased from a fund and resold to a fund, the ETFs are purchased and sold in the secondary market, like stocks or closed-end funds. ETFs are traded like stocks, so they can be bought or sold any time during the trading day, not just at 4:00 p.m. when net asset values (NAV) of funds are determined. Though the opportunities for intraday trading may not be important to everyone who trades ETFs, they doubtless have appeal to many investors whenever it comes to one's ability to get out of a position before the market close when the market is volatile.

In the years ahead, the objections to more extensive use of the ETF will be overcome.

We would expect almost all index funds to have an ETF share in time.

For the first time in July 2007, eight short ETFs launched into the market. Unlike the traditional ETFs which use the creation-redemption process to operate the products. ProShares uses derivatives to operate the Short ETFs and UltraShort ETF to gain profits that reverse the performance of the broad market indices or to gain the effects that double reverse the performance of tracking benchmark indices. These derivatives include index futures and swap, which are contracts between two parties to exchange an income stream. The index futures are sold, or sold short. The ETF uses swaps with a negative correlation to the index which essentially means shorting the swaps as well.

The swaps exchange the income streams depending on the direction of the index. The short side of the swap receives an interest payment all the time for allowing the long side of the swap to get the fund's potential upside. However, if the price of the fund falls, the short side of the swap receives the interest as well as the downside returns. Besides, index futures are margined tools which give leverage. For an ETF returning the reverse return of an index, the ETF needs to put only 10% of its money into the futures. If the ETF needs a 200% negative return of an index, it puts 20% of its cash into the futures.

The short ETFs allow investors to bet against a market without having to sell stocks short or sell the related exchange-traded funds short. This makes short ETFs a much easier, cheaper instrument to taking a bearish position on a sector or market compared to short sales.

In this article, we would like to evaluate the short ETFs in two ways. Specifically, we will investigate the tracking errors of the short ETFs, as well as their effectiveness of hedging the broad market indices. The prior studies focused on the hedging efficiency of index futures due to that these investment instruments make the investment and risk management strategies more flexible. The index futures greatly enhance one's ability to hedge their stock portfolios (see Figlewski, 1984). However, when the short ETFs hit the market, they provide a cheaper and easier way to hedge the broad market indices. This is because that one has no need to pay the margin calls to short broad market indices. As to the tracking error, it can be represented as the volatility of return differences between the tracking portfolio and their benchmark (Ammann and Zimmermann, 2001), and it actually means that the fund exposes to great risk. For the passively-managed portfolios such as ETFs particularly, a small tracking error is generally considered desirable due to these funds seek to replicate index returns. From the point view of fund managers, if the creation and redemption mechanism for ETFs can't allow arbitrage chances to be exploited profitably whenever the ETFs' prices deviate from the NAV of the underlying portfolio, the ETFs fail to achieve the goal. Moreover, if the premiums (discounts) are large and persistent, the ETFs will lose their characteristic and become worthless.

There are numerous studies examining the efficiency of hedging stock indices with index-linked instruments such as index futures. Since the short ETFs hit the market, they provide investors a new choice for hedging stock indices. In order to estimate the minimum-variance hedge ratio, we need to estimate the correlation between individual assets first. Engle (2002) developed the Dynamic Conditional Correlation (DCC) model which provides a very good approximation to a variety of time varying correlation processes, so we will estimate the hedge ratio based on DCC model. For the comparison between the hedging performances of each ETF, we build the portfolios implied by the calculated hedge ratios each day and compute the variance of the returns of these portfolios. There are also abundant literatures discussing the tracking error between assets which are the same in essence. In this study, we follow Trynor and Black (1973) to define the tracking error of an ETF to be the volatility of returns of a portfolio relative to that of its benchmark index. However, we make some modifications that we use the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model presented by Bollerslev (1986) to estimate the volatility. Furthermore, we will discuss the relationship between tracking error and trading volumes of short ETF as well as the relationship between tracking error and the volatilities of their related index futures.

This article evaluates the shot ETFs concerning the tracking errors and the hedging effectiveness of each short ETF. As for the tracking errors, there is no clear conclusion whether Short or UltraShort ETFs have the better tracking ability and we show that the unperfect correlation between shot ETFs and their benchmarks will lead to tracking erros. Furthermore, we examine the relationship between tracking errors and volatilities of their related index futures as well as that between tracking errors and trading volumes. We find that volatilities of S&P500 and S&P400 MidCap index futures have more influences on tracking errors of UltraShort ETFs than on those of short ETFs. These results coincide with the facts that the ProShares uses more index futures on UltraShort ETFs than on Short ETFs. We also find that over-trading on the shot ETFs may lead to larger tracking errors.

Finally, we research the hedging performance of each short ETFs. We find that Short ETFs outperform UltraShort ETF when DJIA and S&P400 MidCap are concerned while the UltraShort (SDS) ETF of S&P500 has the better hedging performance than SH. Besides, the MYY has the best hedging performance among the Short ETFs when SDS has the best hedging effectiveness among the UltraShort ETFs.

The rest of this article is organized as follows: In the next section, we introduce literature related to ETF as well as the measurement of tracking error in the first part, and we review the literature concerning cross-market hedge with index-linked products in the second part. In the third part, we introduce the Autoregressive Conditional Heteroskedasticity (ARCH) family and review the literature regarding the development of the DCC model. Section Ⅲ presents the method employed in this article. Section Ⅳ shows the data used in this study together with their descriptive statistics and discuss the empirical results. The conclusions are given in section Ⅴ.

Ⅱ. Literature Review

There are plenty research on the tracking error of ETFs as well as hedging effectiveness. In this section, we provide a review of literature related to this article for the further empirical discussions.

2.1 Exchange-Traded Fund (ETF) and the Tracking Error

Chen and Stockum (1986) as well as Lee and Rahnian (1990) find that there is a limited number of fund managers have the selectivity and market-timing skills required to outperform the market, analysis by Malkiel (1995) and Bogle (1998) has shown that without prior knowledge of the few superior fund managers, investors would do best to stay in index funds. Furthermore, the reason individual investors might be persuaded to pay out 2% of assets annually, plus 20% of profits, is that it's hard for them to hedge on their own.

ProShares has launched 29 ETFs that short the broad market and its subsectors. Clash (2007) suggests that with a short ETF, one's risk is limited to his initial investment as well as there is no margin calls. However, with a stock the risk can be infinite. Therefore, with the short ETFs, one can create his own hedge fund, at a lower cost. Besides, Tax rules favor ProShares (see Poterba and Shoven, 2002; Gastineau, 2002 chapter 4; Bergstresser and Poterba, 2002), at least if one makes money on them.

There are numerous literatures examining comovements of prices of substantially the same assets in different markets. This leads to the measurement of correlation of these assets which are the same in essence. Closed-end mutual funds and futures markets are the two widely researched examples of essentially the same asset trading in different forms. In this article, we focus on the comovements of the returns of ETFs and the returns of their benchmark indices.

Ackert and Tian (2000) find that Standard & Poor's Depository Receipts or SPDR or Spiders do not trade at economically significant discount because the SPDRs redemption feature facilitates arbitrage so that the traders can eliminate mispricing. However, they report an economically significant discount for MidCap SPDRs due to higher arbitrage costs. The arbitrage costs come from higher fundamental risk, higher transaction cost, and lower dividend yields.

Elton, Gruber, Comer, and Li (2002) also examine the characteristics and performance of Spider. They suggest that the differences in return based on the price of the Spider and its net asset value (NAV) is less than 1.8 basis points per year on average and that almost all of the difference disappear within one day. Furthermore, they find that the NAV of the Spider, measured before management fees and dividends on the underlying securities, keeps close to market price by the ability to create and delete the Spider by in-kind transactions. They report that the Spiders (NAV) underperform the S&P Index by 28.4 basis points. The two principal causes of the tracking errors are the management fee of 18.45 basis points and the loss of return from dividend reinvestment of 9.95 basis points.

Engle and Sarkar (2006) examine the magnitude of premiums and discounts for a wide range of Exchange Traded Funds. Because of both the price and NAV may be measured with errors, they develop a statistical approach to measuring the true premium by correcting some of the measurement errors in net asset value. They take futures prices and the futures returns from 4:00 PM to 4:15 PM into account to generate a model calling dynamic model. Due to this, they reduce further the observed standard deviation. They also examine how the standard deviation moves over time. The resulting standard deviation of the premium is averages 14.7

bps for the domestic funds and 77.7 bps for the international funds. And they also show that for the international ETFs the premiums (discounts) are much larger and more persistent than the domestic ETFs. This is probably the higher cost of creation and redemption for the international products.

Hehn (2005) suggests that index-linked ETFs are subject to 'tracking error' risks. Factor such as imperfect correlation between ETFs and their underlying index may cause the ETFs' performance to diverge from that of their benchmark index. Although there are small divergences in performance between an ETF and its benchmark index, the optimised replication of the tracked index means that ETF performance is usually close to that of its benchmark index, regardless of the trading volume. This is because the liquidity of ETFs is mostly caused by the liquidity of the underlying shares instead of demand for the ETF itself. She also mentions that ETFs are flexible investments that allow investors to quickly react to what they needs; besides, ETFs can be used for hedging purposes. They can be sold short to hedge a portfolio of stocks, and allow an investor to protect a portfolio from overall market losses. In other words, ETFs can be used in a same way to index futures, but they have more flexibility. Based on the reasons above, she concludes that ETFs can match the main advantage of index futures, the advantage which enables investors to trade both long and short; moreover, ETFs have several advantages over index futures.

Ammann and Zimmermann (2001) research the relationship between statistical measures of tracking error and asset allocation restrictions expressed as acceptable weight ranges. Particularly, they investigate how the size of admissible deviations from the benchmark weights relates to the tracking error. The authors use two different methods to measure tracking error. The first way is to use the standard deviation of the difference in the portfolio and benchmark returns. Alternatively, they follow Treynor and Black (1973) to define the tracking error of a portfolio as the residual volatility of the tracking portfolio with respect to the benchmark. Specifically, the tracking error of a tracking portfolio can be computed as the standard deviation of the residuals of a linear regression between the tracking portfolio's returns and those of the benchmark portfolio. They conclude that imposing rather large tactical asset allocation ranges leads to surprising small tracking errors.

2.2 Hedging With Index-Linked Products

In early 1982, trading in futures contracts based on stock indices began at three different exchanges. Stock index futures were a success, and led to the spread of new futures and options markets tied to many different indices.

Figlewski (1984) was the first one who analyzed the hedging effectiveness of stock index futures. He suggested that the reason for this success was that index futures enlarged the range of investment and risk management strategies available to investors. In considering the potential applications of index futures, it is clear that almost in every case a cross-hedge is involved. He mentioned that return and risk for an index futures hedge will depend upon the behavior of the difference between the futures price and the cash price. Hedging a position in stock will inevitably expose it to some risk that the change in the futures price over time will not track exactly the value of the cash position. Furthermore, he argued that there are two primary risks of hedging indices with index futures. The first risk is that returns of the index portfolio include dividends, while the index futures only track the capital value of the portfolio. This may not be a terrible shortcoming because dividends are low and stable. The more important risk is that the futures price is not undeviatingly tied to the underlying index, expect for the settlement price on the expiration date. Just as the tracking error risks between index-linked ETFs and the indices can be traded away by the creation-redemption process; the magnitude of risk that the futures price is not undeviatingly tied to the underlying index is limited by the feasibility of arbitrage between cash and futures markets. For stock index futures, however, a perfect arbitrage appears to be impossible.

Still, he investigated hedging performance for three stock index futures and concluded that a more effective hedge may be reachable with a more specialized investment tools, such as an industry group index option or futures. He also observed that, different from what has been suggested in other literatures, the risk minimizing hedge ratio was smaller than the beta of the portfolio being hedged. Finally, he found that about 70 percent of a discrepancy between the actual futures price and the spot index is eliminated in one day. Overall, he argued that the stock index futures market is now rather efficient and the efficiency is getting better and better.

Junkus and Lee (1985) examined the hedging effectiveness of USA stock index futures contracts across the three exchanges (Kansas City Board of Trade, New York Futures Exchange, and Chicago Mercantile Exchange) due to differences in these stock index contract specifications. This article also used four hedging strategies as well as different maturities of contract (a short, intermediate, and long maturity) to evaluate the hedging performance. They found the minimum-variance hedge ratio was the most effective method at decreasing the risk of a portfolio comprising the index underlying the index futures contract.

Graham and Jennings (1987) were first to evaluate hedging effectiveness for cash portfolios not matching a broad market index. They used random sampling methods to form portfolios of common stocks, so that the portfolios exposed to different systematic risk. Then, they added short position of the S&P 500 Stock Index futures to each portfolio and used three hedge methods (naïve, beta and minimum-variance) to calculate the hedge ratio. They conclude that the minimum-variance hedge strategy was considerably better than the other two strategies. Besides, this study indicated that hedging these non-index portfolios with short position of the index futures was less than half as effective as hedging broad market indices.

Butterworth and Holmes (2001) provide the first evaluation of hedging performance of the FTSE-Mid250 (Mid250) stock index futures contract. In contrast to previous researches, the cash portfolio to be hedged is an actual diversified portfolio in the form of investment trust companies (ITCs, an ITC is similar to a mutual fund), rather than a broad market index. Their results show that despite relatively thin trading, the Mid250 contract plays an important additional hedging role. Surprisingly, when it comes to hedge the actual cash portfolios in the form of ITCs, the results distinctly demonstrate the average standard deviation of returns is lower when the portfolios is hedged with Mid250 as compared to be hedged with FTSE-100 contract. Furthermore, they also show that previous studies of hedging effectiveness of UK stock index futures have overstated the risk reduction which can be obtained in that they use the broad market index as the portfolio to be hedged.

Laws and Thompson (2005) used a variety of strategies to estimate the optimal hedge ratio. The hedged portfolios in this article were assets of seventeen investment companies as well as two portfolios which were designed to match the corresponding cash index. They used FTSE100 and FTSE250 to hedge those portfolios described above. They concluded that the Exponential Weighted Moving Average method was superior to other methods used in this article in estimating the hedge ratios and the FTSE250 index provided a better hedging effectiveness than the FTSE100 index. Furthermore, the risk reduction afforded by hedging was quite small for the investment companies' portfolios than the two composite portfolios which were designed to match the corresponding cash index.

Merrick (1988) mentions that the presence of the mispricing return of stock index futures has implications for hedge ratio and hedging effectiveness. The article argues that some adjustments should be made for the hedge ratios to eliminate the variance of stock market return.

2.3 The Development of the Dynamic Conditional Correlation Model (DCC)

The autoregressive conditional heteroskedasticity (ARCH) model, introduced by Engle(1982), has been widely use to formulate time-varying conditional volatility in time series data. It proves to be an effective tool in modeling temporal behavior and the volatility clustering phenomenon of many economic variable, especially financial market data. The traditional econometrics models assume the one period forecast variance to be constant, however the ARCH model free this assumption and assumes that variance of residuals to be time-varying and conditional on past samples.

Bollerslev (1986) extends the ARCH model to Generalized ARCH, or GARCH which brings the previous volatility term into the ARCH model. The GARCH model provides a more flexible framework to capture various dynamic structures of conditional variance. In particular, Bollerslev (1987) as well as Lamoureux and Lastrapes (1990) mention that the GARCH(1,1) model has been especially popular in econometric modeling since it has been shown to be a parsimonious representation of conditional variance that adequately fits many economic time series. Bollerslev, Chou, and Kroner (1992) also suggest that such small numbers of parameters appear to modeling the variance dynamics sufficiently over a very long run sample period.

Moreover, some studies strive to estimate the covariance and correlation matrices of multiple variables, especially large sets of asset prices. Bollerslev, Engle, and Wooldridge (1988) proposed the VECH model which provided a general framework for the multivariate volatility models. Bollerslev (1990) presented the constant conditional correlation (CCC) model, where univariate GARCH models are estimated for each asset and then the correlation matrix is estimated using MLE correlation estimator using transformed residuals. The strong assumption of constant correlation makes the estimation process simple, but this assumption imposes restrictive constraints, which the dynamic structure of covariance is completely determined by individual volatilities.

The BEKK (Baba-Engle-Kraft-Kroner) model of Engle and Kroner (1995) model developed a general quadratic form for the conditional covariance equation. The large number of parameters needing to be estimated for the BEKK model makes the estimation difficult. The VECH and the BEKK models are more flexible comparing to the CCC model because they allow time-varying correlations.

Engle (2002) proposed the Dynamic Conditional Correlation (DCC) model which have the flexibility of univariate GARCH but not the complexity of multivariate GARCH. The DCC model, which parameterizes the conditional correlations directly, are naturally estimated in two steps – the first is a series of univariate GARCH estimates and the second the correlation estimate.

The comparison of DCC with simple multivariate GARCH and several other estimators shows that the DCC is often the most accurate. With all the advantages of DCC model, I will use this model to perform the further analyses.

Ⅲ. Methods

3.1 Tracking Error and volatility Measures

As we can see in the last section, tracking errors can be captured by a variety of statistical measures. Treynor and Black (1973), Ammann and Zimmermann (2001) define the tracking error of a portfolio to be the residual volatilities of the tracking portfolio with respect to the benchmark. In particular, they mention that the tracking error (TE) can be calculated as the standard deviation of the residuals of a linear regression between the returns of the tracking portfolio and those of their benchmark portfolio:

$$TE = \sigma(\varepsilon_P) = \sigma(R_P)\sqrt{1-\rho_{PB}^2}\ , \qquad (1)$$

where $\sigma(R_P)$ is the volatility of the tracking portfolio and ρ_{PB} represents the correlation of the returns of the portfolio with the returns of their benchmark portfolio.

In this article, we use Generalized Autoregressive Conditional Heteroscedasticity (GARCH) model to compute the standard deviation of the discrepancy between returns of the portfolio and returns of its benchmark instead of linear regression method.

In this way, we can define the tracking error of Short ETFs as:

$$TE = \sqrt{Var(r_s + r_b)}\ , \qquad (2)$$

where r_s is the return of each Short ETF, and r_b is the return of benchmark index of each ETF. Because the goal of Short ETF is to seek daily investment results that are equivalent to the inverse of daily performance of the corresponding benchmark index, we use the sum of the return of Short ETF and the return of its benchmark here.

For UltraShort ETF, we modify the above equation of tracking error to:

$$TE = \sqrt{Var((\frac{r_u}{2}) + r_b)}\ , \qquad (3)$$

where r_u is the return of each UltraShort ETF. Because the goal of UltraShort ETF is to seek investment results that correspond to twice the opposite of daily performance of the corresponding benchmark index, we divide r_u by 2.

The GARCH volatility structure can be illustrated as below:

$$y_t = \varepsilon_t \quad , \quad \varepsilon_t | I_{t-1} \sim \mathrm{N}(0, h_t), \tag{4}$$

$$h_t = \omega + \alpha\varepsilon_{t-1}^2 + \beta h_{t-1}, \tag{5}$$

where the first equation is the conditional mean equation and the second equation is the conditional variance equation. I_{t-1} is the information set at time t-1, y_t is the difference of return between short ETF and the benchmark index, and $N(0, h_t)$ represents the normal density with zero mean and variance h_t. The advantage of a GARCH model is that it captures the tendency in financial data for volatility clustering. For a GARCH structure to be well-defined and stationary, it is necessary for the coefficients (ω, α, β) are all non-negative and $\alpha + \beta$ <1 (see Bollerslev, Chou, and Kroner, 1992).

We also use the univariate GARCH (1,1) models to measure the volatilities of Index Futures. We simply adjust the model to:

$$r_t = \varepsilon_t \quad , \quad \varepsilon_t | I_{t-1} \sim \mathrm{N}(0, h_t), \tag{6}$$

$$h_t = \omega + \alpha\varepsilon_{t-1}^2 + \beta h_{t-1}, \tag{7}$$

where only r_t is changed to denote the returns of index futures.

3.2 The Dynamic Conditional Correlation (DCC) Model

The DCC model remains the flexibility of the univariate Generalized Autoregressive Conditional Heteroscedasticity (GARCH) model of individual assets' volatilities with a simple GARCH-like time varying correlation.

Traditionally, we can define the conditional covariance and correlation between two random variables $r_{1,t}$ and $r_{2,t}$ with zero mean as:

$$COV_{12,t} = E_{t-1}(r_{1,t}r_{2,t}), \tag{8}$$

$$\rho_{12,t} = \frac{E_{t-1}(r_{1,t}r_{2,t})}{\sqrt{E_{t-1}(r_{1,t}^2)E_{t-1}(r_{2,t}^2)}}, \tag{9}$$

In the definition above, we can see that the conditional covariance and correlation are determined by previous information. However, this method has two shortcomings: the first is that we give previous information equal weight so that it will cause uncoupling estimation, and the other is that we might use too premature data.

Bollerslev (1990) presents the Constant Correlation Coefficient (CCC) model which can be shown as:

$$H_t = D_t R_t D_t, \tag{10}$$

where R is the correlation matrix and $D_t = diag\{\sqrt{h_{i,t}}\}$. As to the $\sqrt{h_{i.t}}$, it's the square root of the estimated variance for the i^{th} return series. The assumption of a constant correlation makes estimating a large model achievable. However, the constant conditional correlation could be too restrictive since that the correlation tends to be time varying in real application.

Engle (2002) extends the CCC to DCC which can be viewed as a generalization of CCC. The DCC model differs from CCC model only in that the DCC allows the correlation matrix, R, to be time varying. The DCC model can be written as:

$H_t = D_t R_t D_t$, $H_t \equiv E_{t-1}(r_t r_t^{'})$ is the conditional covariance matrix of returns.

$R_t = diag\{Q_t\}^{-1/2} Q_t diag\{Q_t\}^{-1/2}$ is the time-varying correlation matrix,

where $D_t = diag\{\sqrt{h_{i,t}}\}$. As to the $\sqrt{h_{i.t}}$, it's the square root of the estimated variance for the i^{th} return series. Q_t is the conditional standardized residuals(z_t) covariance matrix, in a bivariate case specifically,

$$\begin{bmatrix} q_{11,t} & q_{12,t} \\ q_{21,t} & q_{22,t} \end{bmatrix} = (1-a-b)\begin{bmatrix} 1 & \bar{q}_{12,t} \\ \bar{q}_{12,t} & 1 \end{bmatrix} + a\begin{bmatrix} z_{1,t-1}^2 & z_{1,t-1}z_{2,t-1} \\ z_{2,t-1}z_{1,t-1} & z_{2,t-1}^2 \end{bmatrix} + b\begin{bmatrix} q_{11,t-1} & q_{12,t-1} \\ q_{21,t-1} & q_{22,t-1} \end{bmatrix} \quad (11)$$

and $\bar{q}_{12,t} = E(z_{1,t} z_{2,t})$, then the typical element of R_t can be obtained in the form of

$$\rho_{ijt} = q_{ijt} / \sqrt{q_{ii} q_{jj}} \quad (12)$$

The DCC model is built to permit for two-stage estimation of the conditional covariance matrix H_t. In the first step, we utilize an univariate volatility model fitted by the returns of each asset and the estimates of $h_{i,t}$, are obtained.

The univariate volatility model we use here is GARCH, and the GARCH model can be illustrated as:

$$r_{i,t} = \varepsilon_{i,t} \quad \varepsilon_{i,t} | \mathrm{I}_{t-1} \sim \mathrm{N}(0, h_{i,t}) \ , i=1,2 \quad (13)$$

$$h_{i,t} = \omega_i + \alpha_i \varepsilon_{i,t-1}^2 + \beta_i h_{i,t-1} \quad (14)$$

$$z_{i,t} = r_{i,t} / \sqrt{h_{i,t}} \quad (15)$$

In the second step, the asset returns transformed by their estimated standard deviations and then we can use the standardized residuals (z_t) and $\bar{q}_{12,t} = E(z_{1,t} z_{2,t})$ to obtain the conditional correlations.

$$\begin{bmatrix} q_{11,t} & q_{12,t} \\ q_{21,t} & q_{22,t} \end{bmatrix} = (1-a-b)\begin{bmatrix} 1 & \bar{q}_{12,t} \\ \bar{q}_{12,t} & 1 \end{bmatrix} + a\begin{bmatrix} z_{1,t-1}^{2} & z_{1,t-1}z_{2,t-1} \\ z_{2,t-1}z_{1,t-1} & z_{2,t-1}^{2} \end{bmatrix} + b\begin{bmatrix} q_{11,t-1} & q_{12,t-1} \\ q_{21,t-1} & q_{22,t-1} \end{bmatrix} \quad (16)$$

The conditional correlation matrix is given by $q_{12,t} / \sqrt{q_{11,t}q_{12,t}}$. (17)

For its log-likelihood function, we can express it as:

$$\begin{aligned} L &= -\frac{1}{2}\sum_{t}(k\log(2\pi) + \log|H_t| + r_t^{'}H_t^{-1}r_t) \\ &= -\frac{1}{2}\sum_{t}(k\log(2\pi) + \log|D_t R_t D_t| + r^{'}D_t^{-1}R_t^{-1}D_t^{-1}r_t) \\ &= -\frac{1}{2}\sum_{t}(k\log(2\pi) + 2\log|D_t| + \log|R_t| + \varepsilon_t^{'}R_t^{-1}\varepsilon_t) \end{aligned} \quad (18)$$

Let the parameters in D_t be denoted by θ_1 and the other parameters in R_t to be denoted as θ_2. The log-likelihood can be rewritten as the sum of a volatility part $L_v(\theta_1)$ and a correlation part $L_c(\theta_1,\theta_2)$. The two step approach is to maximize the log-likelihood and find $\hat{\theta}_1 = \max\{L_v(\theta_1)\}$ and then take this value into the second step: $\max\{L_c(\theta_1,\theta_2)\}$ to obtain $\hat{\theta}_2$.

3.3 The minimum-variance hedge ratio model and the hedging performance

After performing the DCC model, we use the covariance and the variance collecting from the model to calculate the minimum-variance (MV) hedge ratios.

At time t, we set R_t^b to be the return of the broad market index and R_t^e to be the return of its corresponding short ETF. We assume that the investor has a hedged portfolio that includes both *h* units of the short ETFs and a stock portfolio that represents the broad market index. Then, the return of the hedged portfolio can be written as $r_t = R_t^b + h_{t-1}R_t^e$. (19)

The conditional variance of r at time $t-1$ is

$$Var_{t-1}(r_t) = Var_{t-1}(R_t^b) + 2h_{t-1}Cov_{t-1}(R_t^b, R_t^e) + h_{t-1}^2 Var_{t-1}(R_t^e) \tag{20}$$

Based on the partial difference equation, we can acquire the minimum-variance (MV) hedge ratio $h_{t-1} = -\dfrac{Cov_{t-1}(R_t^b, R_t^e)}{Var_{t-1}(R_t^e)}$ (21)

For the comparison between the hedging performances of each ETF, we build the portfolios implied by the calculated hedge ratios each day and compute the variance of the returns of these portfolios. In particular, we evaluate

$Var(R^b + h^* R^e)$, where h^* is the computed hedge ratios.

After calculating the variance of the returns of these portfolios, we use the equation $HE_{MV} = \dfrac{\sigma_u^2 - \sigma_h^2}{\sigma_u^2}$ to compute the hedging effectiveness (HE),

where σ_h^2 is the variance of return of hedged portfolio and σ_u^2 is the variance of return of unhedged portfolio.

This equation measures the variance reduction from hedge, and the more the variance reduction, the better the hedging effectiveness. Because of this reason, the increase of HE value represents the increasing performance of hedge.

Ⅳ. Empirical results and Discussions

4.1 Data

The data employed in this study consist of four U.S. stock market indices (Dow Jones Industrial Average Index, S&P500 Index, S&P400 MidCap Index, and Nasdaq100 index) as well as their corresponding Short/UltraShort ETFs and index futures spanning from 07/13/2006 to 03/18/2008, which comprises 423 daily observations for each asset. We acquire the indices and ETFs data from Yahoo's database (www.yahoo.com/finance), and we gain the return data of index futures from DataStream.

< Table 1 is inserted about here >

Panel A in Table 1 presents the Short/UltraShort Exchange-traded funds (ETFs) we used in this article. We can see clearly the relationship between Short/UltraShort ETFs and their benchmark index. For the terms 'Short' of the product names mean that the ETFs seek daily investment results, before fees and expenses, that correspond to inverse (100%) the daily performance of the corresponding benchmark indices. Moreover, the terms 'UltraShort' of the product names indicate that the ETFs pursue daily investment results, before fees and expenses, that are equivalent to twice (200%) the daily performance of their benchmark indices. We remove the UltraShort ETF (QID) of Nsadaq100 index because when we apply GARCH (1,1) model to this asset, the coefficients are not all non-negative. As we mention in the section of literature review, the management fees and dividends on the underlying securities will also cause the tracking errors. In this article, we use the adjusted closing prices of all broad market indices and ETF products to avoid the problem of dividends. However, we omit the management fees due to their essential stability. For simplicity, we will use the ticker to represent each ETF product in the following analyses.

Panel B in Table 1 shows the index futures products of each U.S. stock market index. We remove the index futures of Nsadaq100 index because when we apply GARCH (1,1) model to this asset, the coefficients are not all non-negative. The Dow Jones index futures is the product of The Chicago Board of Trade (CBOT), whereas others are products of Chicago Mercantile Exchange (CME). We will also use the ticker to represent each index futures for the reason of simplicity.

4.2 Descriptive Statistics

< Figure 1 is inserted about here >

< Figure 2 is inserted about here >

< Figure 3 is inserted about here >

< Figure 4 is inserted about here >

Figure 1-4 show the graphs for the daily returns of four U.S. stock market indices (Dow Jones Industrial Average Index, S&P500 Index, S&P400 MidCap Index, and Nasdaq100 index) as well as their corresponding Short/UltraShort ETFs and index futures spanning from 07/13/2006 to 03/18/2008. The returns of all stock market indices, ETFs, and index futures are defined as $r_t = 100(\log(p_t^{close} / p_{t-1}^{close}))$. As we can see, the shape of figure of stock index and index futures are very similar, and so are the Short and UltraShort ETF. Also, the returns of Short (UltraShort) ETFs are (twice) opposite to those of their benchmark. This reveals the nature of Short and UltraShort ETFs.

< Table 2 is inserted about here >

The descriptive statistics for the returns of these univariate series spinning from 07/13/2006 to 03/18/2008 are given in Table 2. The table shows that the mean returns of the four U.S. stock indices and their index futures are positive, and those of the short ETFs are all negative. This result is agreeing with the feature of short ETFs. The mean returns of Short/UltraShort ETFs are not exactly the value that inverse and twice the inverse of mean returns of their benchmarks, while the mean returns of index futures are very close to the returns of their corresponding U.S. stock indices. The standard deviations in this table also reveal that the Nasdaq100 index and its ETF product are more volatile than other indices and their ETF products. Besides, the standard deviations of Short ETFs and index futures are close to those of their benchmarks whereas the standard deviations of UltraShort ETFs are close to twice the values of their benchmarks. As for the higher moments of the return data, each of them has the excess kurtosis. This indicates that these data exhibit fat-tail distributions. Furthermore, return data of the four stock indices and their related index futures have negative skewness while return data of each short ETF has positive skewness. As to all the data series, the Jarque-Bera statistics strongly reject the null hypothesis of a normal distribution.

4.3 Empirical Analysis

< Table 3 is inserted about here >

< Table 4 is inserted about here >

< Table 5 is inserted about here >

< Table 6 is inserted about here >

Table 3-6 present the empirical results of the estimation with the DCC model over the sample period from 07/13/2006 to 03/18/2008. Because of the procedure for parameters which are estimated under the setting of standard DCC mode, we divide these tables into two parts consistent with the two steps in the DCC estimation. In Panel A of each table, we apply the GARCH model to individual assets to obtain the standardized residuals. Then, these standardized residuals series are brought into the second stage for dynamic conditional correlation estimating, and we show the estimated parameters of DCC model in Panel B of each table.

Furthermore, in panel A of Table 3-6, we can find that most of the coefficients estimated in the univariate GARCH (1,1) models are significant under 5% level excluding some coefficients of constant parameters in the conditional variance equations. The results reveal that very strong time-varying conditional heteroskedasticity is shown by the large t-statistics of the coefficients of the lagged squared residuals (α) and the lagged conditional variance terms (β). Besides, the sums of $\alpha+\beta$ for all series are near to one, and this is the evidence that there exists strong persistence in the conditional variances.

Finally, in Panel B of Table 3-6, the results show that almost all of the estimated coefficients (b) are significant at 5% level. These outcomes indicate that the correlations are significantly dynamic, and we can conclude that current dynamic conditional correlations are significantly affected by previous dynamic conditional correlations.

Based on the results above, we will focus on the tracking errors of each ETF in this section. First, the comparison of the tracking error between Short ETF and UltraShort ETF related to the same benchmark will be delivered. Furthermore, we try to observe whether the less perfect conditional correlation leads to the larger tracking error. Although the conditional correlations between returns of stock market indices and returns of their short ETF products are negative, we modify these numbers to positive for intuitive understanding. Then, we will also make the comparison of tracking error across the different stock market indices. Besides, we will investigate the relationship between tracking error and trading volume of each short ETF as well as the relationship between tracking error of ETF and the return volatilities of its corresponding index futures.

< Table 7 is inserted about here >

< Table 8 is inserted about here >

Table 7 show that all of the coefficients estimated in the univariate GARCH (1,1) models are significant under 5% level. The results reveal very strong time-varying conditional heteroskedasticity. The sums of $\alpha+\beta$ for are near to one, and this is the evidence that there exists strong persistence in the conditional variances. Table 8 presents the descriptive statistics of the tracking error (TE) between the Dow Jones Industrial Average index (DJIA) and its corresponding ETFs. As we can see, the TE also presents the fat-tail distributions and is found to reject the null hypothesis of a normal distribution. This indicates that the TE also has the same characteristics like most of the financial data. Also, in Table 8, the mean of TE between DJIA and its related Short ETF (DOG) is smaller than that between DJIA and UltraShort product (DXD). This means, on average, the DXD has the larger TE than that of DOG while the standard deviation between DOG and DXD does not have large differences. Based on these results, a conclusion can be made that the TE of DOG is smaller than TE of DXD.

< Figure 5 is inserted about here >

Figure 5 shows the comparison between the dynamic conditional correlation and the tracking error of short ETFs related to the DJIA. Although we can't observe the perfect relationship between these two series, the less perfect conditional correlation seems to produce the larger tracking error. This phenomenon exists in both DOG and DXD. The unconditional correlation between the TE and the dynamic conditional correlation for DOG is -0.557 while that for DXD is -0.377, which is much lower.

< Table 9 is inserted about here >

< Table 10 is inserted about here >

Table 9 reveals that all of the coefficients estimated in the univariate GARCH (1,1) models are significant under 5% level except one coefficients of constant parameter. Table 10 shows the descriptive statistics of the tracking error (TE) between the S&P500 index and its corresponding Short (SH)/UltraShort (SDS) EFTs. The TE series of the SH presents the fat-tail distributions, whereas that of SDS is not. Both series are found to reject the null hypothesis of a normal distribution. Moreover, as shown in Table 10, both the mean and the standard deviation of TE between S&P500 and its related UltraShort ETF (SDS) outperform that between S&P500 and its Short product (SH). As a result, we can conclude that the SDS is better on the tracking ability than SH.

< Figure 6 is inserted about here >

Figure 6 shows the comparison between the dynamic conditional correlation and the tracking error of short ETFs related to the S&P500. In this figure, the relationship between these two series is not obvious. However, we still find the positive unconditional correlations between these two series. The value for SH is -0.529, and the value is only -0.112. Consequently, the less perfect conditional correlation will cause the TE to be larger.

< Table 11 is inserted about here >

< Table 12 is inserted about here >

Table 11 reveals that all of the coefficients estimated in the univariate GARCH (1,1) models are significant under 5% level. Table 12 shows the descriptive statistics of the tracking error (TE) between the S&P400 MidCap index and its corresponding Short (MYY)/UltraShort (MZZ) EFTs. The TE series of the MYY and MZZ present the fat-tail distributions, and both series are found to reject the null hypothesis of a normal distribution. Furthermore, Table 12 also exhibit that both the mean and the standard deviation of TE between S&P400 MidCap and its related Short ETF (MYY) outperform that between S&P400 MidCap and its UltraShort product (MZZ). Based on these results, we can conclude that the MYY is better on the tracking performance than MZZ.

< Figure 7 is inserted about here >

Figure 7 reveals the comparison between the dynamic conditional correlation and the tracking error of short ETFs related to the S&P400 MidCap. We can see the strong positive relationship between these two series. The unconditional correlation between these two series of Short ETF (MYY) is -0.726, and that of UltraShort (MZZ) is -0.598. The less perfect conditional correlation between S&P400 MidCap and its corresponding short ETFs also leads to larger TE for these two ETFs.

< Table 13 is inserted about here >

< Table 14 is inserted about here >

Table 13 reveals that all of the coefficients estimated in the univariate GARCH (1,1) models are significant under 5% level. Table 14 shows the descriptive statistics of the tracking error (TE) between the NASDAQ100 index and its corresponding Short (PSQ) EFTs. The TE series of the PSQ shows the fat-tail distributions, and are found to reject the null hypothesis of a normal distribution. Furthermore, Table 12 exhibits the mean of TE is 0.317, and the standard deviation is 0.247.

< Figure 8 is inserted about here >

Figure 8 reveals the comparison between the dynamic conditional correlation and the tracking error of PSQ. We can see the comovement between the two series is very obvious. The unconditional correlation between the two series of PSQ is -0.893.

After we investigate the TEs of each Short/UltraShort ETF related to the same stock market index, here we try to compare the TE of Short/UltraShort ETF related to the different stock market index. Specifically, we try to show which Short/UltraShort ETF, across the different stock market indices, has the smallest TE.

< Table 15 is inserted about here >

Table 15 shows the statistics of tracking error of Short/UltraShort ETFs across different market indices. As we can see in Table 15, the PSQ is the worst on tracking performance in the group of short ETFs because it has the largest mean and standard deviation of tracking error while in the group of UltraShort ETFs, the MZZ is the worst on tracking performance due to the same reason.

In this section, we will investigate the relationship between tracking error and trading volumes of each ETF.

< Figure 9 is inserted about here >

The relationsip between tracking error and trading volumes of the “Short”(DOG)/ “UltraShort”(DXD) ETF of Dow Jones Industrial Average index seems vague. The unconditional correlation of these two series is 0.538 for DOG and 0.621 for DXD. This result shows that the larger the volumes, the larger the tracking error.

< Figure 10 is inserted about here >

In figure 10, we can't not see clearly the relatioship between tracking error and trading volumes of the "Short"(SH)/ "UltraShort"(SDS) ETF of S&P500. We report that the unconditional correlation of these two series is 0.526 for SH and 0.857 for SDS. This result also shows that the larger the volumes, the larger the tracking error.

< Figure 11 is inserted about here >

In figure 11, the relatioship between tracking error and trading volumes of the "UltraShort"(MZZ) ETF is easier to observe than that of "UltraShort"(MZZ) ETF . The unconditional correlation of these two series is 0.439 for MZZ, and the value is mush smaller for MYY (0.007). This result reveals that the larger the volumes, the larger the tracking error.

< Figure 12 is inserted about here >

In figure 12, it's hard for us to tell whether there is any relation between tracking error and trading volumes of the "Short"(PSQ) ETF. The unconditional correlation of these two series is only 0.191. This result also reveals that there is weak positive relation between the tracking error and trading volumes.

< Table 16 is inserted about here >

We utilize Table 16 to discuss the positive relationship between tracking error and trading volumes of each ETF. As we can see in Table 16, the higher correlation between tracking errors and trading volumes accompanies higher trading volumes. One possible reason for this phenomenon is that when the trading volumes go too large, the large trading volumes themselves generate the large tracking errors. This implies that when investors throng to market to buy these products, the over-trading will produce tracking errors. We can see the trading volumes of MYY and MZZ.

When the trading volumes are six times more for MZZ than MYY, the unconditional correlation goes from 0.007 for MYY to o.439 for MZZ

< Table 17 is inserted about here >

We use Table 17 to confirm our conjecture. In Table 17, we can find that the more the trading volumes, the bigger the tracking errors of each short ETF except MYY. This table also shows that the tracking errors come from the quarter which contains the larger trading volumes, are larger than the average tracking errors. Because of this reason, we conclude that the over-trading will lead to larger tracking error.

We will investigate the relationship between tracking error and volatilities of index futures in this section. ProShares uses index futures to rebalance its UltraShort ETFs daily to keep leverage consistent with each ETF's daily investment objective so ProShares uses more index futures on UltraShort ETFs than on Short ETFs. When the volatilities of index futures go up, it may cause ProShares to miss the target prices and lead to tracking errors of ETFs. Because of this reason, we presume that volatilities of index futures have more influences on tracking errors of the UltraShort ETF than on those of Short ETF.

< Table 18 is inserted about here >

Table 18 reveals that all of the coefficients estimated in the univariate GARCH (1,1) models are significant under 5% level except one coefficients of constant parameter. The results show very strong time-varying conditional heteroskedasticity. The sums of $\alpha+\beta$ for are near to one, and this is the evidence that there exists strong persistence in the conditional variances.

After the the estimations of GARCH (1,1) for each index futures are made, we can difine the conditional standard deviations form GARCH (1,1) as the volatilities of index futures. Now we can use these results to discuss the relationship between the volatilities of index futures and the tracking errors.

< Figure 13 is inserted about here >

Figure 13 shows that the relationsip between the tracking error of DOG/DXD and the volatilities of DJIA index futures seems to correlate positively. The unconditional correlation of these two series is 0.688 for DOG and 0.680 for DXD. This result shows that the tracking errors of “Short” (DOG)/”UltraShort” (DXD) ETF of DJIA are affected almost equally by the volatilities of DJIA index futures.

< Figure 14 is inserted about here >

The relationsip between the tracking error of SH/SDS and the volatilities of S&P500 index futures shown in Figure 14 also seems to correlate positively. The unconditional correlation of these two series is 0.676 for SH and 0.882 for SDS. This result shows that the volatilities of S&P500 index futures have more influences on the tracking errors of the UltraShort (SDS) ETF than on those of Short (SH) ETF.

< Figure 15 is inserted about here >

The relationsip between the tracking error of MYY/MZZ and the volatilities of S&P400 MidCap index futures shown in Figure 15 is obscure. The unconditional correlation of these two series is 0.388 for MYY and 0.556 for MZZ. This result shows that the volatilities of S&P400 MidCap index futures have more influences on tracking errors of the UltraShort (MZZ) ETF than on those of Short (MYY) ETF.

According to the daily holdings of short ETFs revealed by ProShares, this company uses more index futures on the UltreaShort ETFs than on the Short ETFs. Coinciding with this fact, our results show that the volatilities of S&P500 and S&P400 MidCap index futures have more influences on tracking errors of the UltraShort ETFs than on those of Short ETFs.

However, the tracking errors of “Short” (DOG)/”UltraShort” (DXD) ETF of DJIA are affected almost equally by the volatilities of DJIA index futures.

Finally, the hedge performance of short ETFs will be shown, and the comparison will be made. Based on the results of performing DCC, we can use the conditional covariance and variance to calculate the hedge ratios.

< Figure 16 is inserted about here >

< Figure 17 is inserted about here >

Figures 16 and 17 show that the minimum-variance hedge ratios (MVHRs) for stock market indices using their related Short ETF are all close to 1 and the values are close to 0.5 using their related UltraShort ETF. We can conclude that the basic functions of the Short ETF and the UltraShort ETF exist.

< Table 19 is inserted about here >

Table 19 reports that there is no certain answer that which knid of ETF outperform the other kind when it comes to hedging performance. For ETFs relate to DJIA, the Short (DOG) ETF has the better hedging performance than the UltraShort (DXD) ETF. The Short ETF (MYY) of S&P400 MidCap also outperform MZZ in hedging performance while the UltraShort (SDS) ETF of S&P500 outperform SH in hedging performance. Futhermore, for the comparison across different market indices, the MYY has the best hedging performance among the Short ETFs. SDS has the best hedging effectiveness among the UltraShort ETFs.

Ⅴ. Conclusion

We investigate the tracking errors and the hedging effectiveness of each short ETF. This article shows that when it comes to tracking errors of Short/UltraShort ETFs related to the same benchmark, the Short ETFs of DJIA and S&P400 MidCap outperform the UltraShort ETFs of these two indices. On the contrary, the UltraShort ETF of S&P500 has the better tracking ability than the Short ETF of the S&P500. As for the cross indices comparison, the Short ETF of NASDAQ100 is the worst on tracking performance in the group of Short ETFs while the MZZ has the worst tracking ability in the group of UltraShort ETFs. Still, after the time-varying correlations between ETFs and their benchmark are estimated from the DCC model, we report the negtive unconditional correlation between tracking errors and these time-varying correlations. This result corroborates that the unperfect correlation between ETFs and their benchmarks will lead to tracking errors as mentioned by Hehn (2005).

Furthermore, we also examine the relationship between tracking errors and volatilities of their related index futures as well as that between tracking errors and trading volumes. We conclude that the tracking errors of DOG and DXD are affected almost equally by the volatilities of DJIA index futures while the volatilities of S&P500 (S&P400 MidCap) index futures have more influences on tracking errors of SDS (MZZ) than on those of SH (MYY). The results, except for the short ETFs of DJIA, coincide with the facts that ProShares uses more index futures on UltraShort ETFs than on Short ETFs. ProShares uses index futures to rebalance its UltraShort ETFs daily to keep leverage consistent with each ETF’s daily investment objective. When the volatilities of index futures go up, it may cause ProShares to miss the target prices and lead to tracking errors of ETFs. We also find that over-trading on the shot ETFs may lead to larger tracking errors, and this effect is quite obvious regarding MYY and MZZ.

Finally, we research the hedging performance of each short ETFs. We find that Short

ETFs outperform UltraShort ETF when DJIA and S&P400 MidCap are concerned while the UltraShort (SDS) ETF of S&P500 has the better hedging performance than SH. Besides, the MYY has the best hedging performance among the Short ETFs when SDS has the best hedging effectiveness among the UltraShort ETFs.

As to the further research, one may try to evaluate the hedging performance using short ETFs to hedge the portfolio not matching a broad market index (as Graham and Jennings (1987), Butterworth and Holmes (2001)). After all, not everyone has the ability to create such a portfolio so much like a benchmark unless one buys another ETF. More recently, ProShares also launches Short international ETFs, and therefore one can research if there are more tracking errors for these Short international ETFs than Short domestic ETFs as suggested by Engle and Sarkar (2006) as far as ordinary ETFs are concerned.

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Table 1

The Short/UltraShort ETFs and the Index Futures

Panel A is the Short/UltraShort Exchange-traded funds (ETFs) we used in this article. For the terms ”Short” and ”UltraShort” of the product names mean the ETFs seek daily investment results, before fees and expenses, that correspond to inverse (100%) and twice (200%) the inverse of the daily performance of the corresponding benchmark index. Panel B is the list of index futures used in this article.

Panel A: short ETFs

Fund	**Ticker**	**Benchmark Index**
Short Dow30	DOG	Dow Jones Industrial Average
Short S&P500	SH	S&P 500 Index
Short MidCap400	MYY	S&P400 MidCap Index
Short QQQ	PSQ	NASDAQ-100 Index
UltraShort Dow30	DXD	Dow Jones Industrial Average
UltraShort S&P500	SDS	S&P 500 Index
UltraShort MidCap400	MZZ	S&P400 MidCap Index

Panel B :Index Futures

Stock index futures	**Ticker**	**Benchmark Index**
CBT BIG DOW DJIA	DD	Dow Jones Industrial Average
CME E-Mini S&P 500index	ES	S&P 500 Index
CME E-Mini S&P MidCap400	EMD	S&P400 MidCap Index

Table 2

Descriptive Statistics for Return of U.S. Broad Market Indices, ETFs, and Index Futures

This table provides the descriptive statistics for the data used in this paper. This table summary statistics for the daily return data on Dow Jones Industrial Average Index, S&P500 Index, S&P400 MidCap Index, and Nasdaq100 index as well as their corresponding Short/UltraShort ETFs and index futures. The columns in the table are arranged by stock market index, Short ETF, UltraShort ETF, and index futures accordingly. The returns are computed by $r_t = 100(\log(p_t^{close} / p_{t-1}^{close}))$, and the Jarque-Bera statistic is used to test the null hypothesis of whether the return data are normally distributed. The Std. Dev. denotes standard deviation. The sample period ranges from 07/13/2006 to 03/18/2008.

Return of broad indices	DJIA_RET	S&P500_RET	S&P400 MIDCAP_RET	NASDAQ_RET
Mean	0.032	0.016	0.013	0.041
Median	0.068	0.084	0.107	0.156
Maximum	3.487	4.153	3.946	4.285
Minimum	-3.349	-3.534	-3.129	-4.396
Std. Dev.	0.912	0.993	1.073	1.205
Skewness	-0.364	-0.307	-0.274	-0.238
Kurtosis	5.072	5.226	3.921	4.177
Jarque-Bera	84.759	93.729	20.180	28.323
Observations	423	423	423	423

Table 2

(Continued)

Return of Short ETFs	DOG_RET	SH_RET	MYY_RET	PSQ_RET
Mean	-0.015	0.001	0.005	-0.022
Median	-0.033	-0.066	-0.070	-0.134
Maximum	3.375	3.816	3.434	4.499
Minimum	-3.696	-4.319	-3.849	-4.317
Std. Dev.	0.904	0.984	1.080	1.194
Skewness	0.092	0.102	0.147	0.199
Kurtosis	5.118	5.352	3.864	4.221
Jarque-Bera	79.470	97.997	14.632	29.006
Observations	423	423	423	423

Return of UltraShort ETFs	DXD_RET	SDS_RET	MZZ_RET
Mean	-0.054	-0.023	-0.020
Median	-0.115	-0.140	-0.162
Maximum	7.199	7.505	6.752
Minimum	-7.391	-8.625	-8.892
Std. Dev.	1.717	1.942	2.195
Skewness	0.140	0.056	0.075
Kurtosis	5.490	5.392	4.020
Jarque-Bera	110.362	111.842	18.700
Observations	423	423	423

Return of index futures	DD_ret	ES_ret	EMD_ret
Mean	0.031	0.016	0.013
Median	0.088	0.061	0.106
Maximum	3.429	4.171	4.216
Minimum	-3.818	-4.025	-3.737
Std. Dev.	0.875	0.979	1.100
Skewness	-0.291	-0.252	-0.214
Kurtosis	5.313	5.427	4.024
Jarque-Bera	100.003	107.996	21.682
Observations	423	423	423

Table 3

Estimation of Bivariate Return-based DCC Model Using Daily Dow Jones Industrial Average Index and Its corresponding ETFs

Step 1 of DCC estimation: $h_{i,t} = \omega_i + \alpha_i \varepsilon_{i,t-1}^2 + \beta_i h_{i,t-1}, \varepsilon_{i,t} | \mathrm{I}_{t-1} \sim \mathrm{N}(0, h_{i,t})$, i=1,2

Step 2 of DCC estimation:

$$\begin{bmatrix} q_{11,t} & q_{12,t} \\ q_{21,t} & q_{22,t} \end{bmatrix} = (1-a-b)\begin{bmatrix} 1 & \bar{q}_{12,t} \\ \bar{q}_{12,t} & 1 \end{bmatrix} + a\begin{bmatrix} z_{1,t-1}^2 & z_{1,t-1}z_{2,t-1} \\ z_{2,t-1}z_{1,t-1} & z_{2,t-1}^2 \end{bmatrix} + b\begin{bmatrix} q_{11,t-1} & q_{12,t-1} \\ q_{21,t-1} & q_{22,t-1} \end{bmatrix}$$

This table reports the estimations for the bivariate return-based DCC model using daily Dow Jones Industrial Average index and its corresponding "Short"(DOG) and "UltraShort"(DXD) products. The two formulas above two steps estimation are GARCH and the conditional correlation equation respectively of the standard DCC model with mean reversion. In the first step, we use the GARCH model to estimate the volatilities ($\hat{h}_t$) for each asset and compute the standardized residuals (z_t). In the second steps, we bring the standardized residuals series and $\bar{q}_{12,t} = E(z_{1,t}z_{2,t})$ into the dynamic conditional correlation estimating. The conditional correlation matrix is given by $q_{12,t}/\sqrt{q_{11,t}q_{12,t}}$, and the conditional covariance can be expressed using the product of conditional correlation between these two variables and their individual conditional standard deviations. This table shows estimations of the two models using the MLE method. Numbers in the parentheses are t-values.

Panel A: Step 1 of DCC estimation			
	DJIA_ret	DOG_ret	DXD_ret
	GARCH	GARCH	GARCH
$\hat{\omega}$	0.010(3.057)	0.011(3.170)	0.053(2.637)
$\hat{\alpha}$	0.065(3.547)	0.069(3.664)	0.065(2.955)
$\hat{\beta}$	0.927(50.426)	0.921(47.890)	0.921(36.740)

Panel B: Step 2 of DCC estimation		
	DJIA versus DOG	DJIA versus DXD
	Return-based DCC	Return-based DCC
$\hat{a}$	0.109(6.277)	0.127(4.858)
$\hat{b}$	0.861(34.709)	0.357(2.232)

Table 4

Estimation of Bivariate Return-based DCC Model Using Daily S&P 500 Index and Its corresponding ETFs

Step 1 of DCC estimation: $h_{i,t} = \omega_i + \alpha_i \varepsilon_{i,t-1}^2 + \beta_i h_{i,t-1}, \varepsilon_{i,t} \left| \mathrm{I}_{t-1} \sim \mathrm{N}\left(0, h_{i,t}\right) \right.$, i=1,2

Step 2 of DCC estimation:

$$\begin{bmatrix} q_{11,t} & q_{12,t} \\ q_{21,t} & q_{22,t} \end{bmatrix} = (1-a-b)\begin{bmatrix} 1 & \bar{q}_{12,t} \\ \bar{q}_{12,t} & 1 \end{bmatrix} + a\begin{bmatrix} z_{1,t-1}^2 & z_{1,t-1}z_{2,t-1} \\ z_{2,t-1}z_{1,t-1} & z_{2,t-1}^2 \end{bmatrix} + b\begin{bmatrix} q_{11,t-1} & q_{12,t-1} \\ q_{21,t-1} & q_{22,t-1} \end{bmatrix}$$

This table reports the estimations for the bivariate return-based DCC model using daily S&P500 index and its corresponding “Short”(SH) and “UltraShort”(SDS) products. The two formulas above two steps estimation are GARCH and the conditional correlation equation respectively of the standard DCC model with mean reversion. In the first step, we use the GARCH model to estimate the volatilities ($\hat{h}_t$) for each asset and compute the standardized residuals (z_t). In the second steps, we bring the standardized residuals series and $\bar{q}_{12,t} = E(z_{1,t} z_{2,t})$ into the dynamic conditional correlation estimating. The conditional correlation matrix is given by $q_{12,t} / \sqrt{q_{11,t} q_{12,t}}$, and the conditional covariance can be expressed using the product of conditional correlation between these two variables and their individual conditional standard deviations. This table shows estimations of the two models using the MLE method. Numbers in the parentheses are t-values.

Panel A: Step 1 of DCC estimation			
	S&P500_ret	SH_ret	SDS_ret
	GARCH	GARCH	GARCH
$\hat{\omega}$	0.008(2.281)	0.013(2.510)	0.042(2.164)
$\hat{\alpha}$	0.052(3.333)	0.059(3.342)	0.054(3.078)
$\hat{\beta}$	0.944(54.843)	0.933(47.217)	0.938(45.372)

Panel B: Step 2 of DCC estimation		
	S&P500 versus SH	S&P500 versus SDS
	Return-based DCC	Return-based DCC
$\hat{a}$	0.021(2.210)	0.020(1.282)
$\hat{b}$	0.957(38.743)	0.683(1.377)

Table 5

Estimation of Bivariate Return-based DCC Model Using Daily S&P400 MidCap Index and Its corresponding ETFs

Step 1 of DCC estimation: $h_{i,t} = \omega_i + \alpha_i \varepsilon^2_{i,t-1} + \beta_i h_{i,t-1}, \varepsilon_{i,t} | \mathrm{I}_{t-1} \sim \mathrm{N}(0, h_{i,t})$, i=1,2

Step 2 of DCC estimation:

$$\begin{bmatrix} q_{11,t} & q_{12,t} \\ q_{21,t} & q_{22,t} \end{bmatrix} = (1-a-b)\begin{bmatrix} 1 & \bar{q}_{12,t} \\ \bar{q}_{12,t} & 1 \end{bmatrix} + a\begin{bmatrix} z^2_{1,t-1} & z_{1,t-1}z_{2,t-1} \\ z_{2,t-1}z_{1,t-1} & z^2_{2,t-1} \end{bmatrix} + b\begin{bmatrix} q_{11,t-1} & q_{12,t-1} \\ q_{21,t-1} & q_{22,t-1} \end{bmatrix}$$

This table reports the estimations for the bivariate return-based DCC model using daily S&P400 MidCap index and its corresponding “Short”(MYY) and “UltraShort”(MZZ) products. The two formulas above two steps estimation are GARCH and the conditional correlation equation respectively of the standard DCC model with mean reversion. In the first step, we use the GARCH model to estimate the volatilities ($\hat{h}_t$) for each asset and compute the standardized residuals (z_t). In the second steps, we bring the standardized residuals series and $\bar{q}_{12,t} = E(z_{1,t}z_{2,t})$ into the dynamic conditional correlation estimating. The conditional correlation matrix is given by $q_{12,t} / \sqrt{q_{11,t}q_{12,t}}$, and the conditional covariance can be expressed using the product of conditional correlation between these two variables and their individual conditional standard deviations. This table shows estimations of the two models using the MLE method. Numbers in the parentheses are t-values.

Panel A: Step 1 of DCC estimation			
	S&P400 MidCap_ret	MYY_ret	MZZ_ret
	GARCH	GARCH	GARCH
$\hat{\omega}$	0.014(1.486)	0.016(1.393)	0.062(1.686)
$\hat{\alpha}$	0.055(2.776)	0.050(2.573)	0.051(2.991)
$\hat{\beta}$	0.935(36.900)	0.938(35.091)	0.938(46.612)

Panel B: Step 2 of DCC estimation		
	S&P MidCap400 versus MYY	S&P400 MidCap versus MZZ
	Return-based DCC	Return-based DCC
$\hat{a}$	0.104(5.152)	0.059(5.535)
$\hat{b}$	0.655(8.833)	0.925(65.742)

Table 6

Estimation of Bivariate Return-based DCC Model Using Daily NASDAQ100 Index and Its corresponding ETFs

Step 1 of DCC estimation: $h_{i,t} = \omega_i + \alpha_i \varepsilon_{i,t-1}^2 + \beta_i h_{i,t-1}, \varepsilon_{i,t} \left| \mathrm{I}_{t-1} \sim \mathrm{N}\left(0, h_{i,t}\right)\right.$, i=1,2

Step 2 of DCC estimation:

$$\begin{bmatrix} q_{11,t} & q_{12,t} \\ q_{21,t} & q_{22,t} \end{bmatrix} = (1-a-b)\begin{bmatrix} 1 & \bar{q}_{12,t} \\ \bar{q}_{12,t} & 1 \end{bmatrix} + a\begin{bmatrix} z_{1,t-1}^2 & z_{1,t-1}z_{2,t-1} \\ z_{2,t-1}z_{1,t-1} & z_{2,t-1}^2 \end{bmatrix} + b\begin{bmatrix} q_{11,t-1} & q_{12,t-1} \\ q_{21,t-1} & q_{22,t-1} \end{bmatrix}$$

This table reports the estimations for the bivariate return-based DCC model using daily NASDAQ100 index and its corresponding "Short"(PSQ) products. The two formulas above two steps estimation are GARCH and the conditional correlation equation respectively of the standard DCC model with mean reversion. In the first step, we use the GARCH model to estimate the volatilities ($\hat{h}_t$) for each asset and compute the standardized residuals (z_t). In the second steps, we bring the standardized residuals series and $\bar{q}_{12,t} = E(z_{1,t}z_{2,t})$ into the dynamic conditional correlation estimating. The conditional correlation matrix is given by $q_{12,t} / \sqrt{q_{11,t}q_{12,t}}$, and the conditional covariance can be expressed using the product of conditional correlation between these two variables and their individual conditional standard deviations. This table shows estimations of the two models using the MLE method. Numbers in the parentheses are t-values.

Panel A: Step 1 of DCC estimation		
	NASDAQ100_ret	PSQ_ret
	GARCH	GARCH
$\hat{\omega}$	0.003(0.394)	0.010(0.821)
$\hat{\alpha}$	0.029(2.489)	0.034(2.472)
$\hat{\beta}$	0.972(61.445)	0.962(47.254)

Panel B: Step 2 of DCC estimation	
	NASDAQ100 versus PSQ
	Return-based DCC
$\hat{a}$	0.049(9.570)
$\hat{b}$	0.950(169.275)

Table 7

Estimation of Univariate GARCH(1,1) Models for Discrepancy between Returns of Daily Dow Jones Industrial Average Index and The Daily Returns of Its Corresponding ETFs

$y_t = \varepsilon_t$, $\varepsilon_t | I_{t-1} \sim \mathrm{N}(0, h_t)$,

$h_t = \omega + \alpha\varepsilon_{t-1}^2 + \beta h_{t-1}$,

This table reports the estimations for the Univariate GARCH(1,1) models using daily Dow Jones Industrial Average (DJIA) index and its corresponding 'Short'(DOG) and 'UltraShort'(DXD) products. The two formulas above estimation are Univariate GARCH(1,1) models where the first equation is the conditional mean equation and the second equation is the conditional variance equation. I_{t-1} is the information set at time t-1, y_t is the difference of return between short ETF and the benchmark index. For Short ETF, we define the difference is $r_s + r_b$ while the difference is $\frac{r_u}{2} + r_b$ for UltraShort ETF. r_s is the return of DOG, r_b is the return of DJIA, and r_u is the return of DXD. $N(0, h_t)$ represents the normal density with zero mean and variance h_t. Numbers in the parentheses are t-values.

	Difference of Returns Between DOG and DJIA	Difference of Returns Between DXD and DJIA
$\hat{\omega}$	0.001(3.023)	0.002(2.511)
$\hat{\alpha}$	0.222(5.643)	0.278(5.409)
$\hat{\beta}$	0.766(20.851)	0.715(16.786)

Table 8

Descriptive Statistics of the Tracking Error between the Dow Jones Industrial Average Index and Its Corresponding ETFs

This table shows the descriptive statistics of tracking error (TE) between the Dow Jones Industrial Average index and its corresponding ‘Short’(DOG)/’UltraShort’(DXD) products. After the GARCH (1,1) estimation in table 7, we can obtain the conditional variance of $r_s + r_b$ for Short ETF, and $\frac{r_u}{2} + r_b$ for UltraShort ETF. In order to get the tracking error ($TE = \sqrt{Var(r_s + r_b)}$) of Short ETF as well as that ($TE = \sqrt{Var((\frac{r_u}{2}) + r_b)}$) of Ultrashort ETF, we simply calculate the square root of the conditional variance of $r_s + r_b$ for Short ETF and $\frac{r_u}{2} + r_b$ for UltraShort ETF. r_s is the return of DOG, r_b is the return of DJIA, and r_u is the return of DXD.

	TE Between DJIA and DOG	TE Between DJIA and DXD
Mean	0.176	0.192
Median	0.132	0.159
Maximum	0.525	0.615
Minimum	0.075	0.085
Std. Dev.	0.102	0.095
Skewness	1.396	1.683
Kurtosis	4.169	6.613
Jarque-Bera	161.173	428.693

Table 9

Estimation of Univariate GARCH(1,1) Models for Discrepancy between Returns of Daily S&P500 Index and The Daily Returns of Its Corresponding ETFs

$y_t = \varepsilon_t$, $\varepsilon_t | I_{t-1} \sim \mathrm{N}(0, h_t)$,

$h_t = \omega + \alpha\varepsilon_{t-1}^2 + \beta h_{t-1}$,

This table reports the estimations for the Univariate GARCH(1,1) models using daily S&P 500 index and its corresponding 'Short'(SH) and 'UltraShort'(SDS) products

. The two formulas above estimation are Univariate GARCH(1,1) models where the first equation is the conditional mean equation and the second equation is the conditional variance equation. I_{t-1} is the information set at time t-1, y_t is the difference of return between short ETF and the benchmark index. For Short ETF, we define the difference is $r_s + r_b$ while the difference is $\frac{r_u}{2} + r_b$ for UltraShort ETF. r_s is the return of SH, r_b is the return of S&P500, and r_u is the return of SDS. $N(0, h_t)$ represents the normal density with zero mean and variance h_t. Numbers in the parentheses are t-values.

	Difference of Returns Between SH and S&P500	Difference of Returns Between SDS and S&P500
$\hat{\omega}$	0.002(3.770)	0.000(0.255)
$\hat{\alpha}$	0.256(5.993)	0.032(2.965)
$\hat{\beta}$	0.723(21.422)	0.973(72.36735)

Table 10

Descriptive Statistics of the Tracking Error between the S&P500 Index and Its Corresponding ETFs

This table shows the descriptive statistics of tracking error (TE) between the S&P500 index and its corresponding 'Short'(SH)/'UltraShort'(SDS) products. After the GARCH (1,1) estimation in table 7, we can obtain the conditional variance of $r_s + r_b$ for Short ETF, and $\frac{r_u}{2} + r_b$ for UltraShort ETF. In order to get the tracking error ($TE = \sqrt{Var(r_s + r_b)}$) of Short ETF as well as that ($TE = \sqrt{Var((\frac{r_u}{2}) + r_b)}$) of Ultrashort ETF, we simply calculate the square root of the conditional variance of $r_s + r_b$ for Short ETF and $\frac{r_u}{2} + r_b$ for UltraShort ETF. r_s is the return of SH, r_b is the return of S&P500 index, and r_u is the return of SDS.

	TE Between S&P500 and SH	TE Between S&P500 and SDS
Mean	0.196	0.195
Median	0.157	0.156
Maximum	0.840	0.372
Minimum	0.093	0.114
Std. Dev.	0.113	0.071
Skewness	2.086	0.748
Kurtosis	8.220	2.342
Jarque-Bera	785.269	46.918

Table 11

Estimation of Univariate GARCH(1,1) Models for Discrepancy between Returns of Daily S&P400 MidCap Index and The Daily Returns of Its Corresponding ETFs

$$y_t = \varepsilon_t \quad , \quad \varepsilon_t | I_{t-1} \sim \mathrm{N}(0, h_t),$$

$$h_t = \omega + \alpha \varepsilon_{t-1}^2 + \beta h_{t-1},$$

This table reports the estimations for the Univariate GARCH(1,1) models using daily S&P400 MidCap index and its corresponding 'Short'(MYY) and 'UltraShort'(MZZ) products. The two formulas above estimation are Univariate GARCH(1,1) models where the first equation is the conditional mean equation and the second equation is the conditional variance equation. I_{t-1} is the information set at time t-1, y_t is the difference of return between short ETF and the benchmark index. For Short ETF, we define the difference is $r_s + r_b$ while the difference is $\frac{r_u}{2} + r_b$ for UltraShort ETF. r_s is the return of MYY, r_b is the return of S&P400 MidCap, and r_u is the return of MZZ. $N(0, h_t)$ represents the normal density with zero mean and variance h_t. Numbers in the parentheses are t-values.

	Difference of Returns Between MYY and S&P400 MidCap	Difference of Returns Between MZZ and S&P400 MidCap
$\hat{\omega}$	0.007(3.564)	0.002(2.954)
$\hat{\alpha}$	0.302(5.043)	0.227(5.742)
$\hat{\beta}$	0.527(5.980)	0.772(30.966)

Table 12

Descriptive Statistics of the Tracking Error between the S&P400 MidCap Index and Its Corresponding ETFs

This table shows the descriptive statistics of tracking error (TE) between the S&P400 MidCap index and its corresponding ‘Short’(MYY)/’UltraShort’(MZZ) products. After the GARCH (1,1) estimation in table 7, we can obtain the conditional variance of $r_s + r_b$ for Short ETF, and $\frac{r_u}{2} + r_b$ for UltraShort ETF. In order to get the tracking error ($TE = \sqrt{Var(r_s + r_b)}$) of Short ETF as well as that ($TE = \sqrt{Var((\frac{r_u}{2}) + r_b)}$) of Ultrashort ETF, we simply calculate the square root of the conditional variance of $r_s + r_b$ for Short ETF and $\frac{r_u}{2} + r_b$ for UltraShort ETF. r_s is the return of MYY, r_b is the return of S&P400 MidCap index, and r_u is the return of MZZ.

	TE Between S&P400 MidCap and MYY	TE Between S&P400 MidCap and MZZ
Mean	0.185	0.220
Median	0.157	0.201
Maximum	0.766	0.619
Minimum	0.122	0.103
Std. Dev.	0.080	0.094
Skewness	3.277	1.304
Kurtosis	17.630	4.930
Jarque-Bera	4518.771	185.027

Table 13

Estimation of Univariate GARCH(1,1) Models for Discrepancy between Returns of Daily NASDAQ100 Index and The Daily Returns of Its Corresponding ETFs

$y_t = \varepsilon_t$, $\varepsilon_t | I_{t-1} \sim \mathrm{N}(0, h_t)$,

$h_t = \omega + \alpha\varepsilon_{t-1}^2 + \beta h_{t-1}$,

This table reports the estimations for the Univariate GARCH(1,1) models using daily S&P 500 index and its corresponding 'Short'(PSQ) products. The two formulas above estimation are Univariate GARCH(1,1) models where the first equation is the conditional mean equation and the second equation is the conditional variance equation. I_{t-1} is the information set at time t-1, y_t is the difference of return between short ETF and the benchmark index. For Short ETF, we define the difference is $r_s + r_b$. r_s is the return of PSQ, and r_b is the return of NASDAQ100. $N(0, h_t)$ represents the normal density with zero mean and variance h_t. Numbers in the parentheses are t-values.

	Difference of Returns Between PSQ and NASDAQ100
$\hat{\omega}$	0.001(2.189)
$\hat{\alpha}$	0.132(9.072)
$\hat{\beta}$	0.883(68.340)

Table 14

Descriptive Statistics of the Tracking Error between the NASDAQ100 Index and Its Corresponding ETF

This table shows the descriptive statistics of tracking error (TE) between the S&P400 MidCap index and its corresponding ‘Short’(PSQ) product. After the GARCH (1,1) estimation in table 7, we can obtain the conditional variance of $r_s + r_b$ for Short ETF, and $\frac{r_u}{2} + r_b$ for UltraShort ETF. In order to get the tracking error ($TE = \sqrt{Var(r_s + r_b)}$) of Short ETF, we simply calculate the square root of the conditional variance of $r_s + r_b$ for Short ETF. r_s is the return of PSQ, r_b is the return of NASDAQ100 index.

	TE Between NASDAQ100 and PSQ
Mean	0.317
Median	0.210
Maximum	1.266
Minimum	0.118
Std. Dev.	0.247
Skewness	1.977
Kurtosis	463.068
Jarque-Bera	4518.771

Table 15

Comparison of Tracking Errors of Short/UltraShort ETF Related to Different Stock Market Indices

This table summarizes the descriptive statistics of tracking errors of each Short/UltraShort ETF. The ETF products are divided into two groups: one is Short ETFs and the other is UltraShort ETFs. The Short ETFs are DOG, SH, MYY, and PSQ which are corresponding to Dow Jones Industrial Average index (DJIA), S&P500 index, S&P400MidCap index, and NASDAQ100 index accordingly. The UltraShort ETFs are DXD, SDS, and MZZ which are corresponding to Dow Jones Industrial Average index (DJIA), S&P500 index, and S&P400MidCap index accordingly.

Stock market index	Statistics of the Tracking Error	Short ETF	UltraShort ETF
DJIA		DOG	DXD
	Mean	0.176	0.192
	Standard Deviation	0.102	0.095
S&P500		SH	SDS
	Mean	0.196	0.195
	Standard Deviation	0.113	0.071
S&P400 MidCap		MYY	MZZ
	Mean	0.185	0.220
	Standard Deviation	0.080	0.094
NASDAQ100		PSQ	
	Mean	0.317	
	Standard Deviation	0.247	

Table 16

The Average Trading Volumes of Each ETF and the Unconditional Correlation between Tracking Errors and Trading Volumes

This table summarizes the average trading volumes of each ETF as well as the unconditional correlation between tracking errors and trading volumes. The ETF products are divided into two groups: one is Short ETFs and the other is UltraShort ETFs. The Short ETFs are DOG, SH, MYY, and PSQ which are corresponding to Dow Jones Industrial Average index (DJIA), S&P500 index, S&P400MidCap index, and NASDAQ100 index accordingly. The UltraShort ETF are DXD, SDS, and MZZ which are corresponding to Dow Jones Industrial Average index (DJIA), S&P500 index, and S&P400MidCap index accordingly.

Stock market index		Short ETF	UltraShort ETF
DJIA		DOG	DXD
	Correlation	0.538	0.621
	Trading volumes	152,290.995	2,081,689.810
S&P500		SH	SDS
	Correlation	0.526	0.857
	Trading volumes	164,284.360	9,022,643.365
S&P400 MidCap		MYY	MZZ
	Correlation	0.007	0.439
	Trading volumes	68,777.251	454,596.919
NASDAQ100		PSQ	
	Correlation	0.191	
	Trading volumes	102,922.749	

Table17

The Trading Volumes and the Tracking Errors of Each Short ETF

This table divides the data of trading volumes and tracking errors (TE) of each ETF into quarters and the averages of each part of data are calculated. The averages are arranged in order of the trading volumes. The Short ETFs are DOG, SH, MYY, and PSQ which are corresponding to Dow Jones Industrial Average index (DJIA), S&P500 index, S&P400MidCap index, and NASDAQ100 index accordingly. The UltraShort ETF are DXD, SDS, and MZZ which are corresponding to Dow Jones Industrial Average index (DJIA), S&P500 index, and S&P400MidCap index accordingly.

	DOG		DXD	
	Volume	TE	Volume	TE
First quarter	344,575	0.261	5,695,470	0.277
Second quarter	144371	0.189	2,028,303	0.216
Third quarter	77,549	0.134	458,894	0.147
Forth quarter	41,317	0.119	116,978	0.127
Overall	152,291	0.176	2,081,690	0.192
	SH		SDS	
	Volume	TE	Volume	TE
First quarter	406,859	0.299	25,937,670	0.228
Second quarter	145,585	0.194	8,437,621	0.216
Third quarter	70,914	0.151	1,338,925	0.141
Forth quarter	33,181	0.140	240,068	0.135
Overall	164,284	0.196	9,022,643	0.195
	MYY		MZZ	
	Volume	TE	Volume	TE
First quarter	163,819	0.184	1,130,121	0.301
Second quarter	60,835	0.192	398,035	0.237
Third quarter	33,563	0.184	190,949	0.176
Forth quarter	16,255	0.178	94,384	0.162
Overall	68,777	0.185	454,597	0.220
	PSQ			
	Volume	TE		
First quarter	216,323	0.429		
Second quarter	100,063	0.339		
Third quarter	61,345	0.254		
Forth quarter	33,220	0.246		
Overall	102,923	0.317		

Table 18

Estimation of Univariate GARCH(1,1) Models for Index Futures

$$r_t = \varepsilon_t \quad , \quad \varepsilon_t \middle| I_{t-1} \sim \mathrm{N}(0, h_t),$$

$$h_t = \omega + \alpha \varepsilon_{t-1}^2 + \beta h_{t-1},$$

This table reports the estimations for the Univariate GARCH(1,1) models using daily return of Dow Jones Industrial Average index futures (DD), S&P 500 index futures (ES), and S&P400 MidCap index futures (EMD). The two formulas above estimation are Univariate GARCH(1,1) models where the first equation is the conditional mean equation and the second equation is the conditional variance equation. I_{t-1} is the information set at time t-1, r_t is the return of each index futures. Numbers in the parentheses are t-values.

	Return of DD	Return of ES	Return of EMD
$\hat{\omega}$	0.014(2.738)	0.009(2.174)	0.023(1.835)
$\hat{\alpha}$	0.059(3.116)	0.047(2.931)	0.059(2.808)
$\hat{\beta}$	0.925(40.669)	0.948(51.486)	0.925(32.849)

Table 19

The hedging effectiveness of each short ETF

This table summarizes the hedging effectiveness of each ETF. After performing the DCC model, we use the covariance and the variance collecting from the model to calculate the minimum-variance (MV) hedge ratios. The minimum-variance (MV) hedge ratio is calculated by $h_{t-1} = -\frac{Cov_{t-1}(R_t^b, R_t^e)}{Var_{t-1}(R_t^e)}$. For the comparison between the hedging performances of each ETF, we build the portfolios implied by the calculated hedge ratios each day and compute the variance of the returns of these portfolios. In particular, we evaluate $Var(R^b + h^* R^e)$, where h^* are the computed hedge ratios.

After calculating the variance of the returns of these portfolios, we use the equation $HE_{MV} = \frac{\sigma_u^2 - \sigma_h^2}{\sigma_u^2}$ to compute the hedging effectiveness (HE), where σ_h^2 is the variance of return of hedged portfolio and σ_u^2 is the variance of return of unhedged portfolio.

Stock market index	Short ETF	UltraShort ETF
DJIA	DOG	DXD
Hedging Effectiveness	0.951	0.948
S&P500	SH	SDS
Hedging Effectiveness	0.949	0.956
S&P400 MidCap	MYY	MZZ
Hedging Effectiveness	0.999	0.955
NASDAQ100	PSQ	
Hedging Effectiveness	0.893	

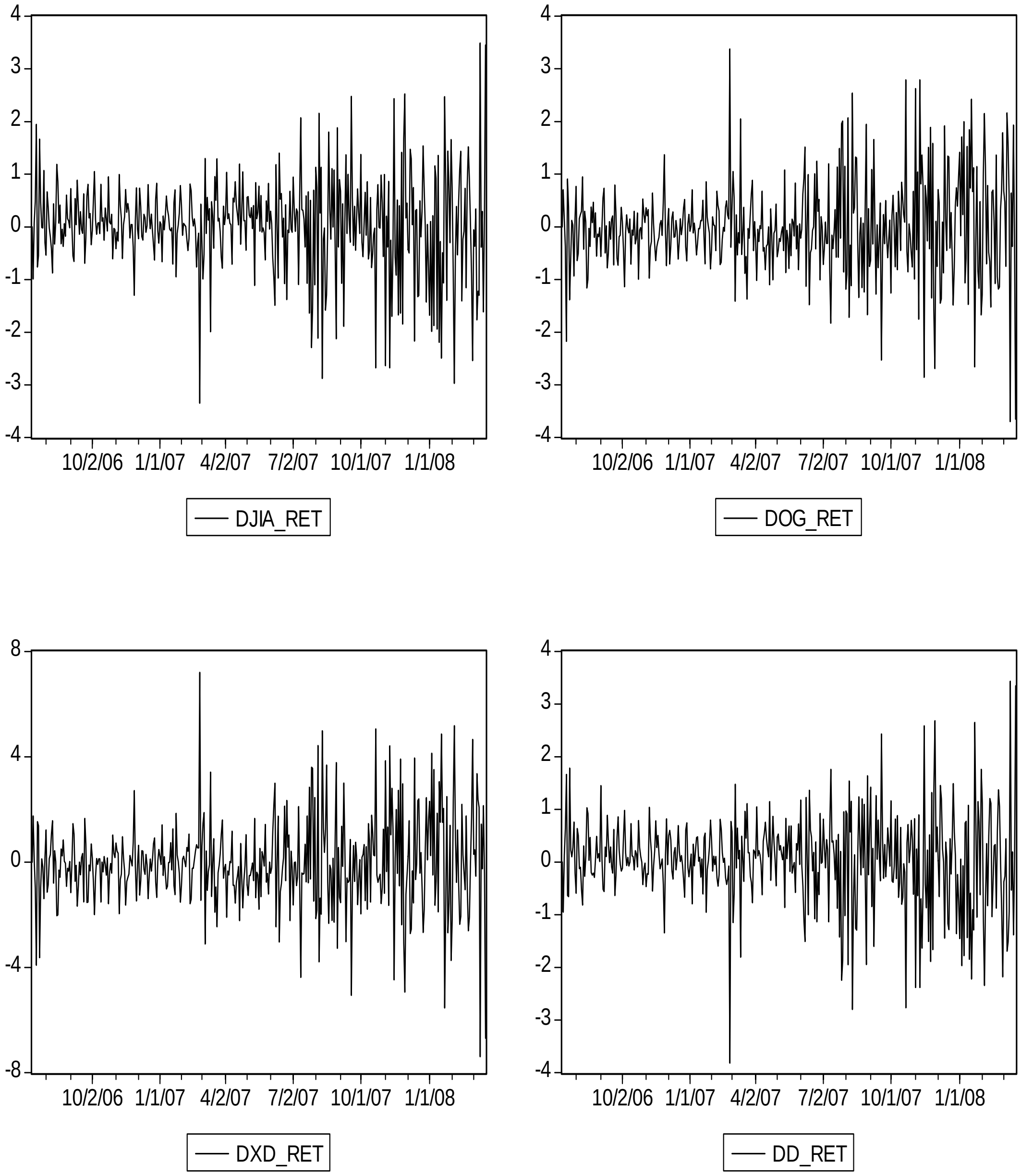


Figure 1: Returns of DJIA, DOG, DXD, and DD

The figure shows the daily returns of the Dow Jones Industrial Average index as well as the daily returns of its corresponding ‘Short’(DOG), ‘UltraShort’(DXD), and index futures (DD) products over the sample period. The returns are defined as $r_t = 100(\log(p_t^{close} / p_{t-1}^{close}))$.

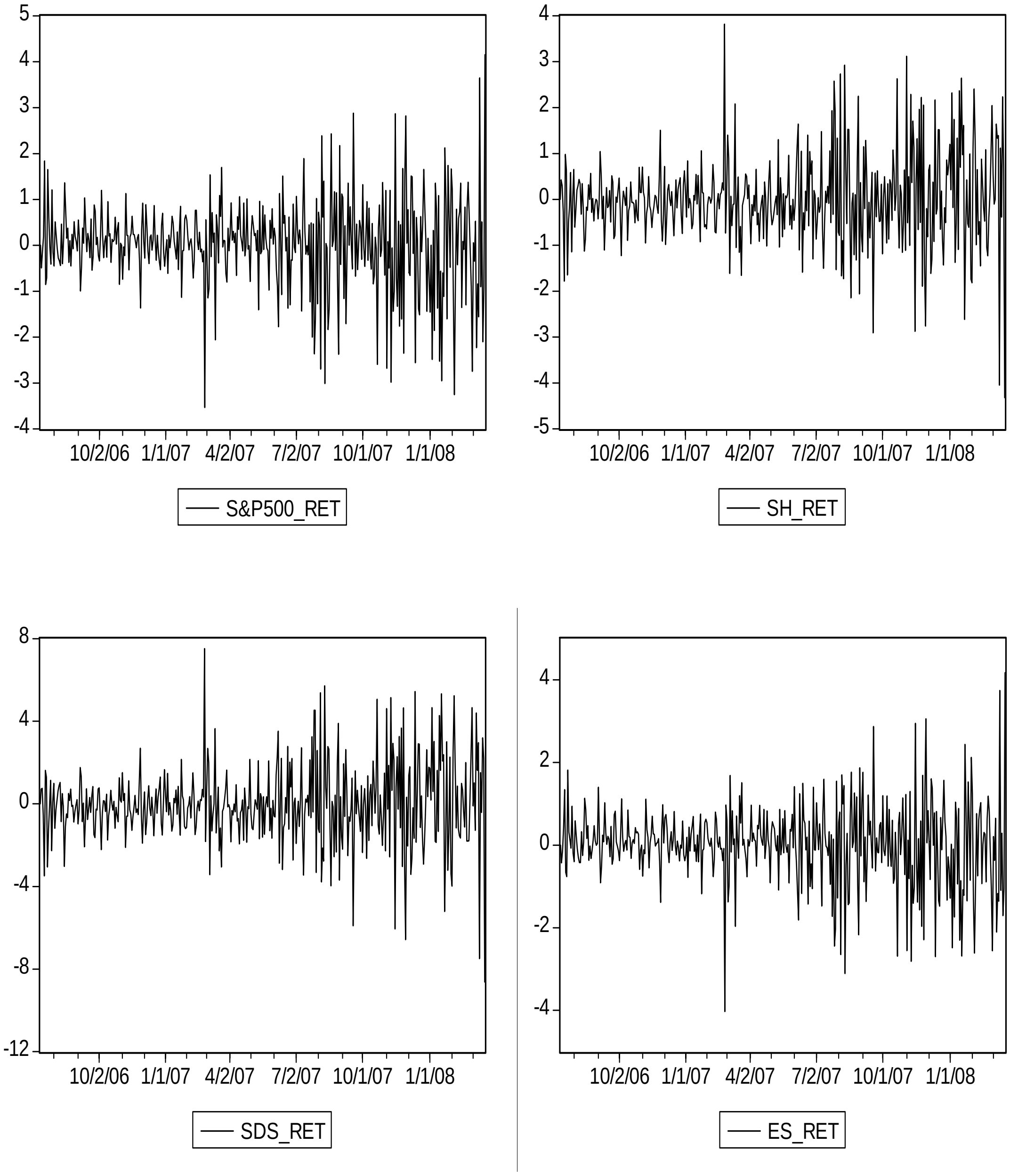


Figure 2: Returns of S&P500, SH, SDS, and ES

The figure shows the daily returns of the S&P500 index as well as the daily returns of its corresponding ‘Short’(SH), ‘UltraShort’(SDS), and index futures (ES) products over the sample period. The returns are defined as $r_t = 100(\log(p_t^{close} / p_{t-1}^{close}))$.

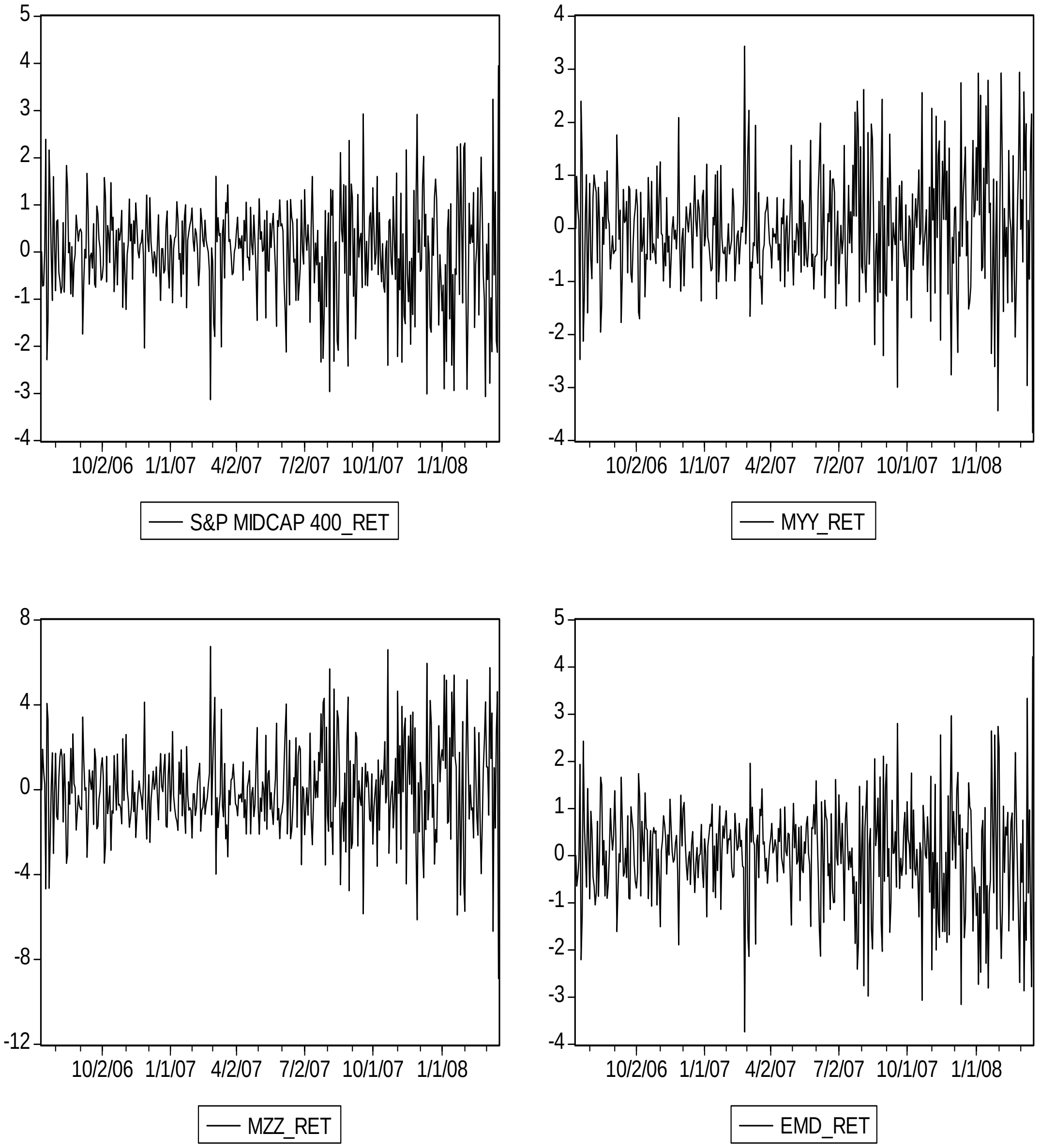


Figure 3: Returns of S&P400 MidCap, MYY, MZZ, and EMD

The figure shows the daily returns of the S&P400 MidCap index as well as the daily returns of its corresponding 'Short'(MYY), 'UltraShort'(MZZ), and index futures (EMD) products over the sample period. The returns are defined as $r_t = 100(\log(p_t^{close} / p_{t-1}^{close}))$.

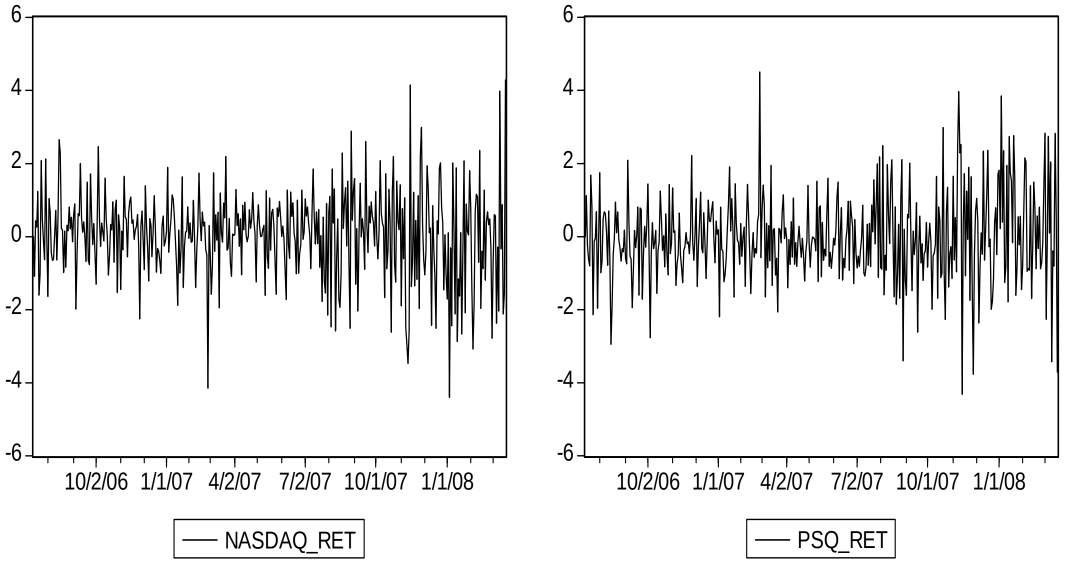
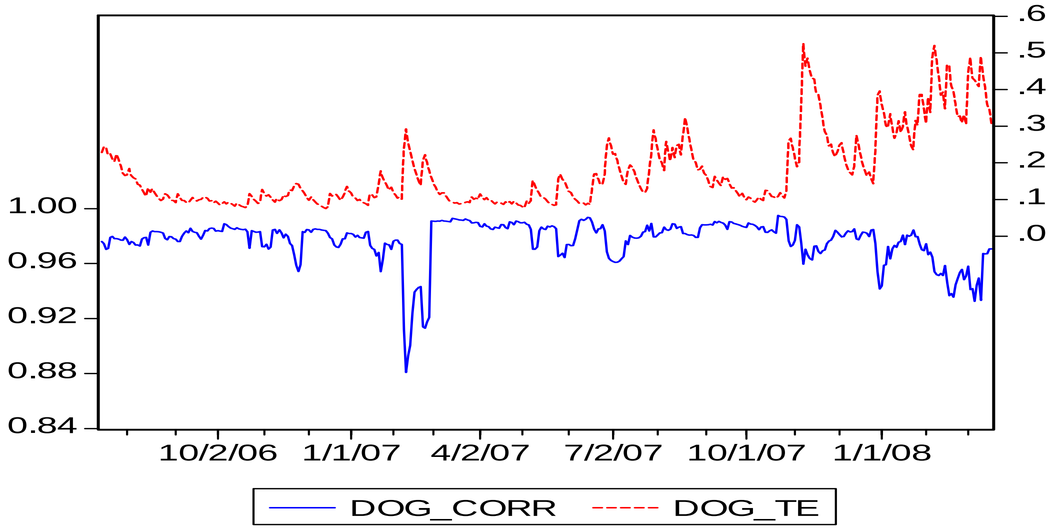


Figure 4: Returns of NASDAQ100, and PSQ

The figure shows the daily returns of the NASDAQ100 index and its corresponding 'Short'(PSQ) over the sample period. The returns are defined as $r_t = 100(\log(p_t^{close} / p_{t-1}^{close}))$.



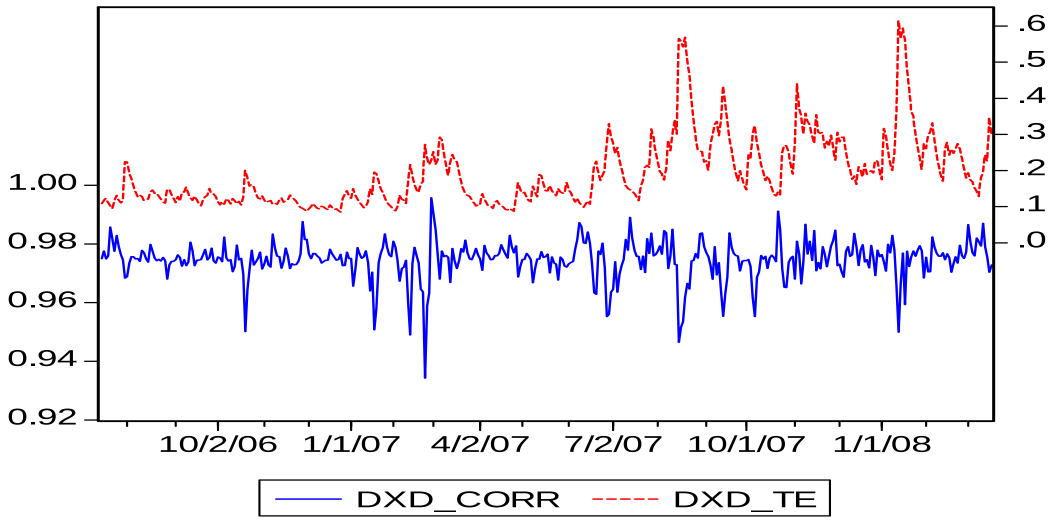
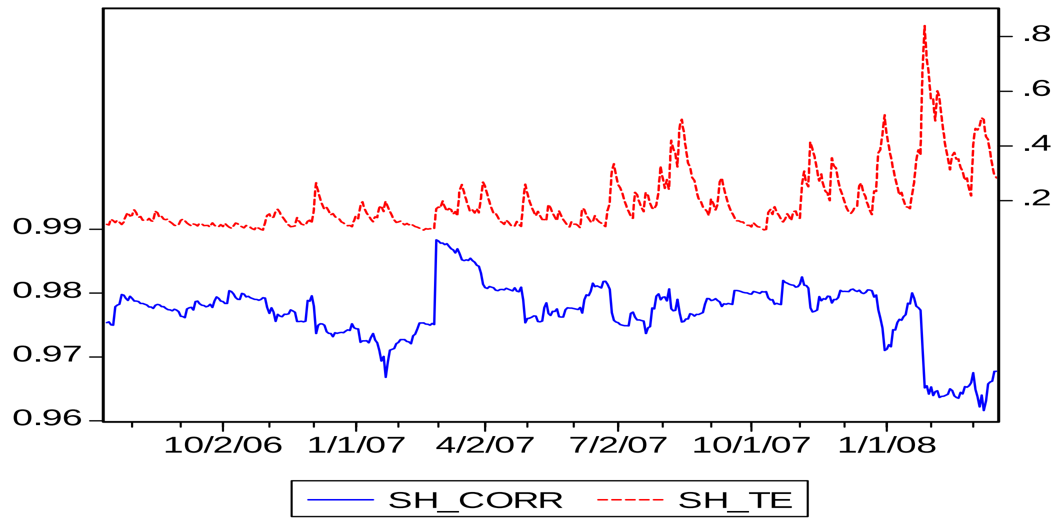


Figure 5: Tracking Error and Dynamic Conditional Correlation Between the Dow Jones Industrial Average Index and Its Corresponding 'Short'(DOG)/ 'UltraShort'(DXD) ETF

The figure shows the comparison between the dynamic conditional correlation and the tracking error of short ETFs related to the Dow Jones Industrial Average index (DJIA) over the sample period. The Short ETF for DJIA is DOG and the UltraShort ETF for DJIA is DXD. CORR means the dynamic conditional correlation and TE means the tracking error.



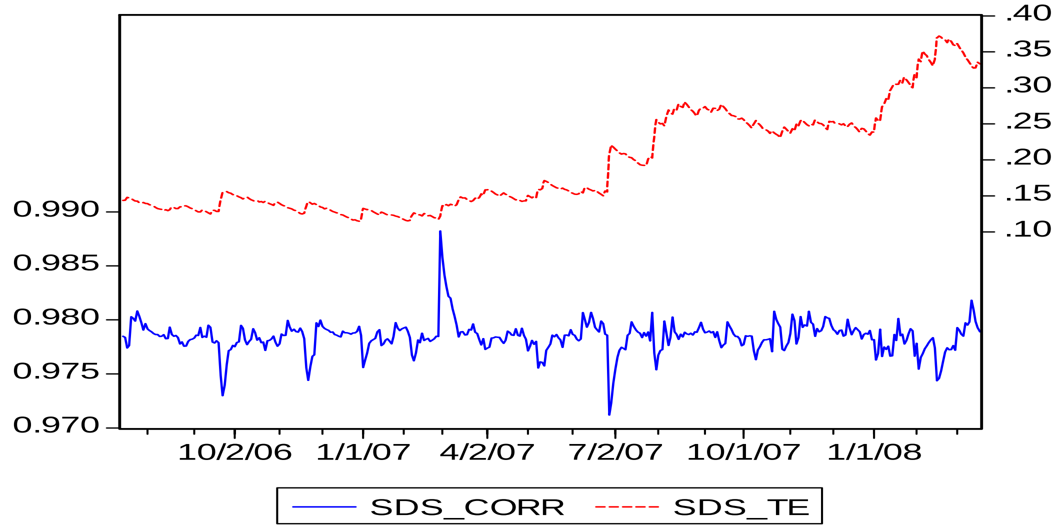


Figure 6: Tracking Error and Dynamic Conditional Correlation Between the S&P500 Index and Its Corresponding 'Short'(SH)/ 'UltraShort'(SDS) ETF

The figure shows the comparison between the dynamic conditional correlation the and tracking error of short ETFs related to the S&P500 index over the sample period. The Short ETF for S&P500 index is SH and the UltraShort ETF for S&P500 index is SDS. CORR means the dynamic conditional correlation and TE means the tracking error.

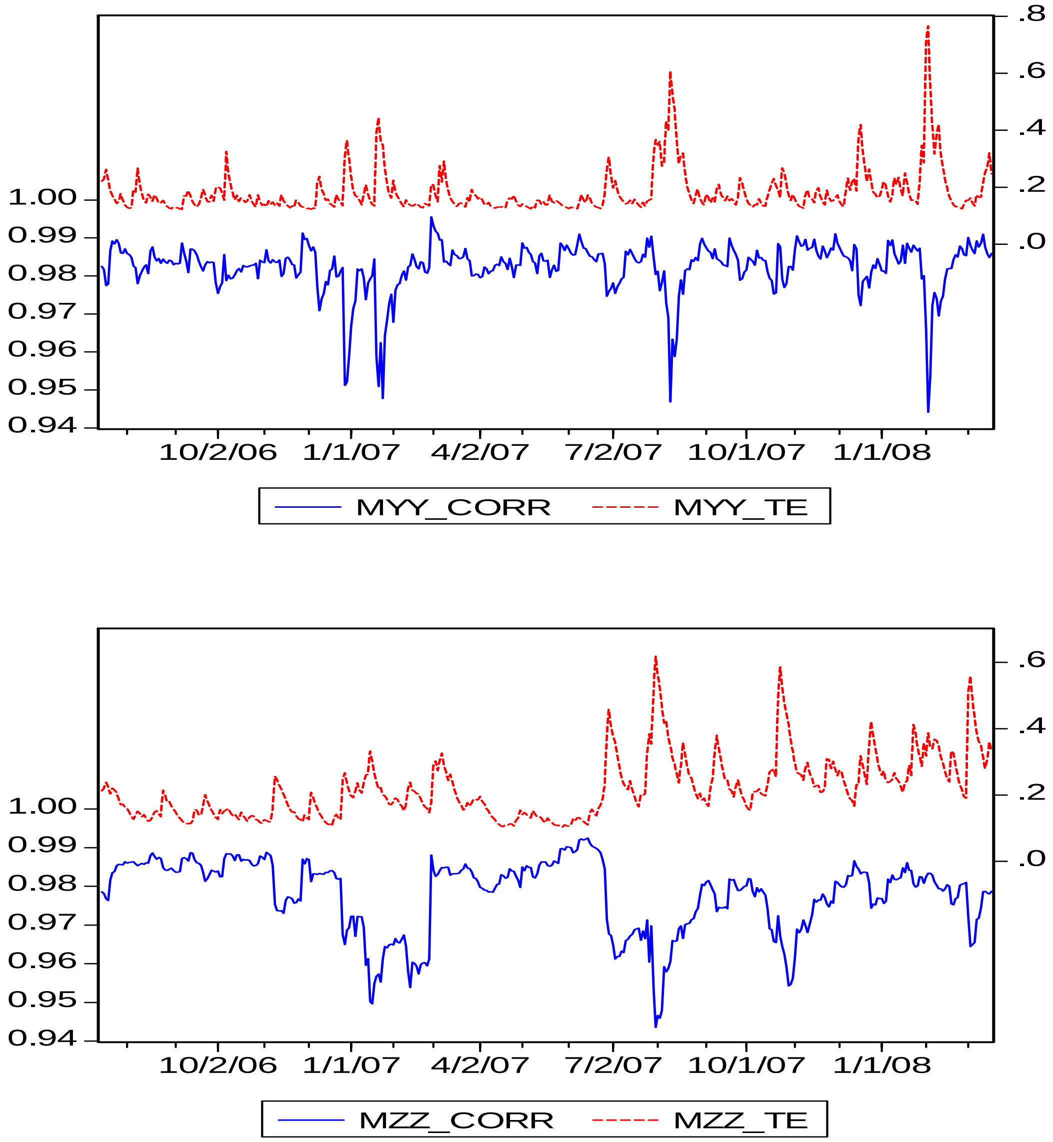


Figure 7: Tracking Error and Dynamic Conditional Correlation Between the S&P400 MidCap Index and Its Corresponding 'Short'(MYY)/ 'UltraShort'(MZZ) ETF

The figure shows the comparison between the dynamic conditional correlation and the tracking error of short ETFs related to the S&P400 MidCap index over the sample period. The Short ETF for S&P400 MidCap index is MYY and the UltraShort ETF for S&P500 index is MZZ. CORR means the dynamic conditional correlation and TE means the tracking error.

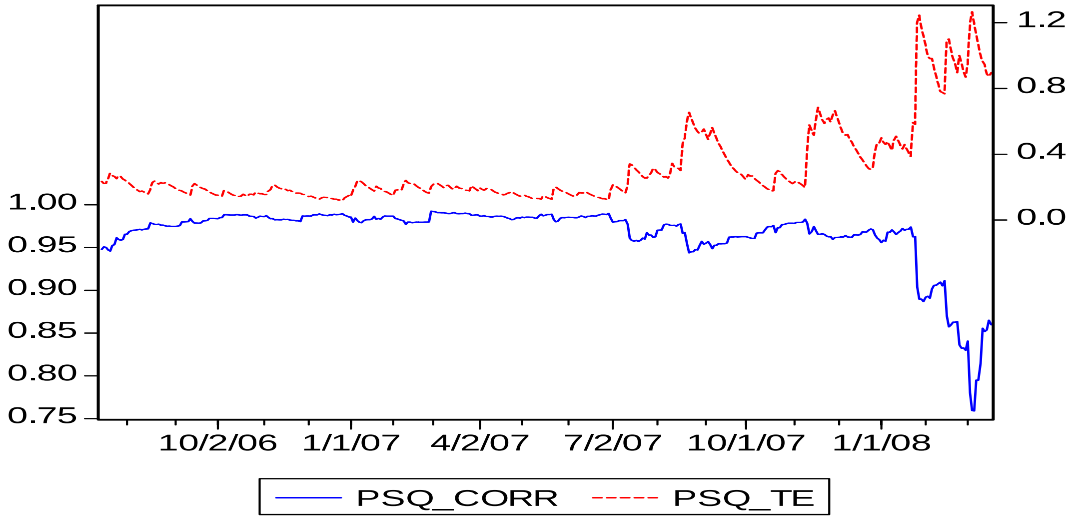
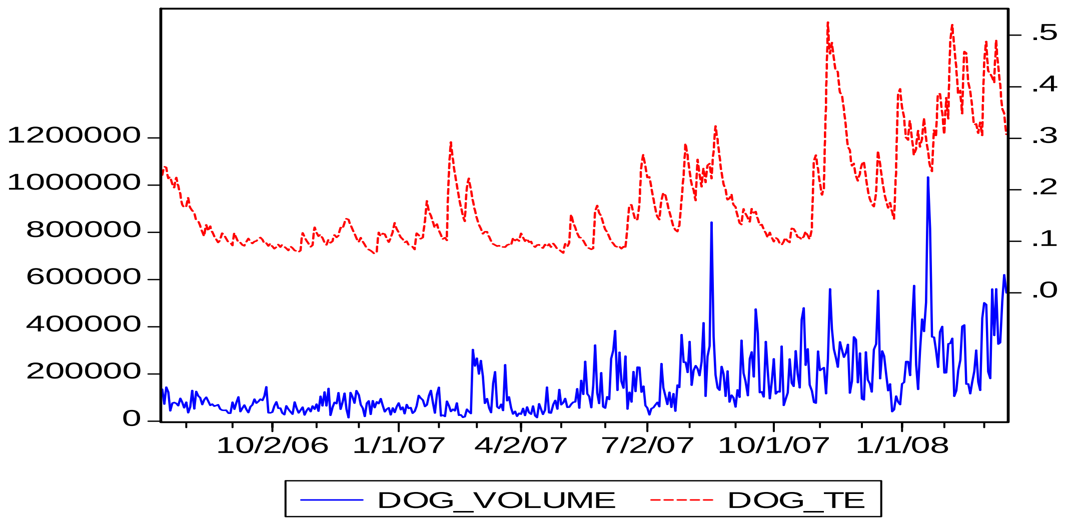


Figure 8: Tracking Error and Dynamic Conditional Correlation Between the NASDAQ100 Index and Its Corresponding 'Short'(PSQ) ETF

The figure shows the comparison between the dynamic conditional correlation and the tracking error of short ETFs related to the NASDAQ100 index over the sample period. The Short ETF for NASDAQ100 index is PSQ. CORR means the dynamic conditional correlation and TE means the tracking error.



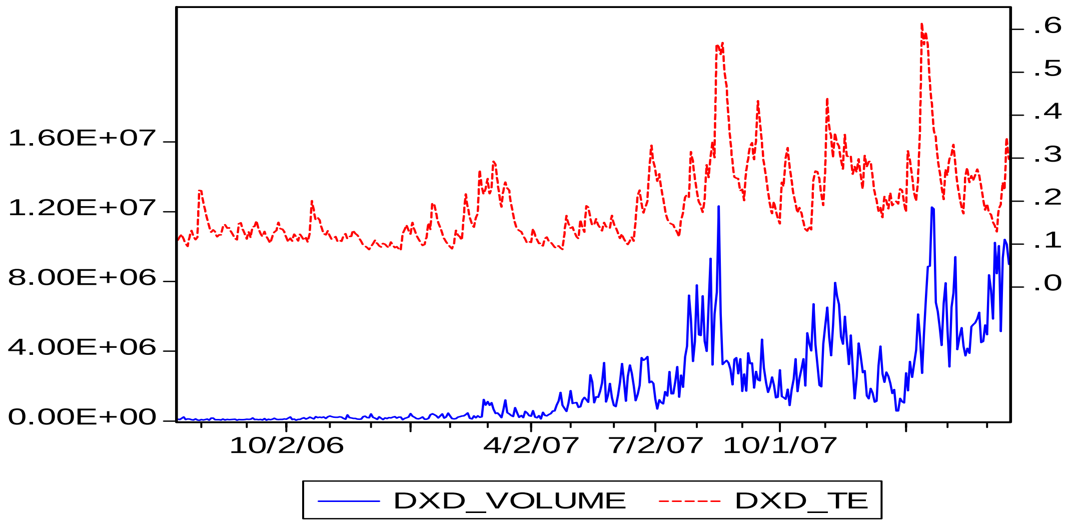


Figure 9: Tracking Error and Trading Volumes of the 'Short'(DOG)/ 'UltraShort'(DXD) ETF of Dow Jones Industrial Average Index

The figure shows the comparison between the tracking error and trading volumes of short ETFs related to the Dow Jones Industrial Average index (DJIA) over the sample period. The Short ETF for DJIA is DOG and the UltraShort ETF for DJIA is DXD. TE means the tracking error.

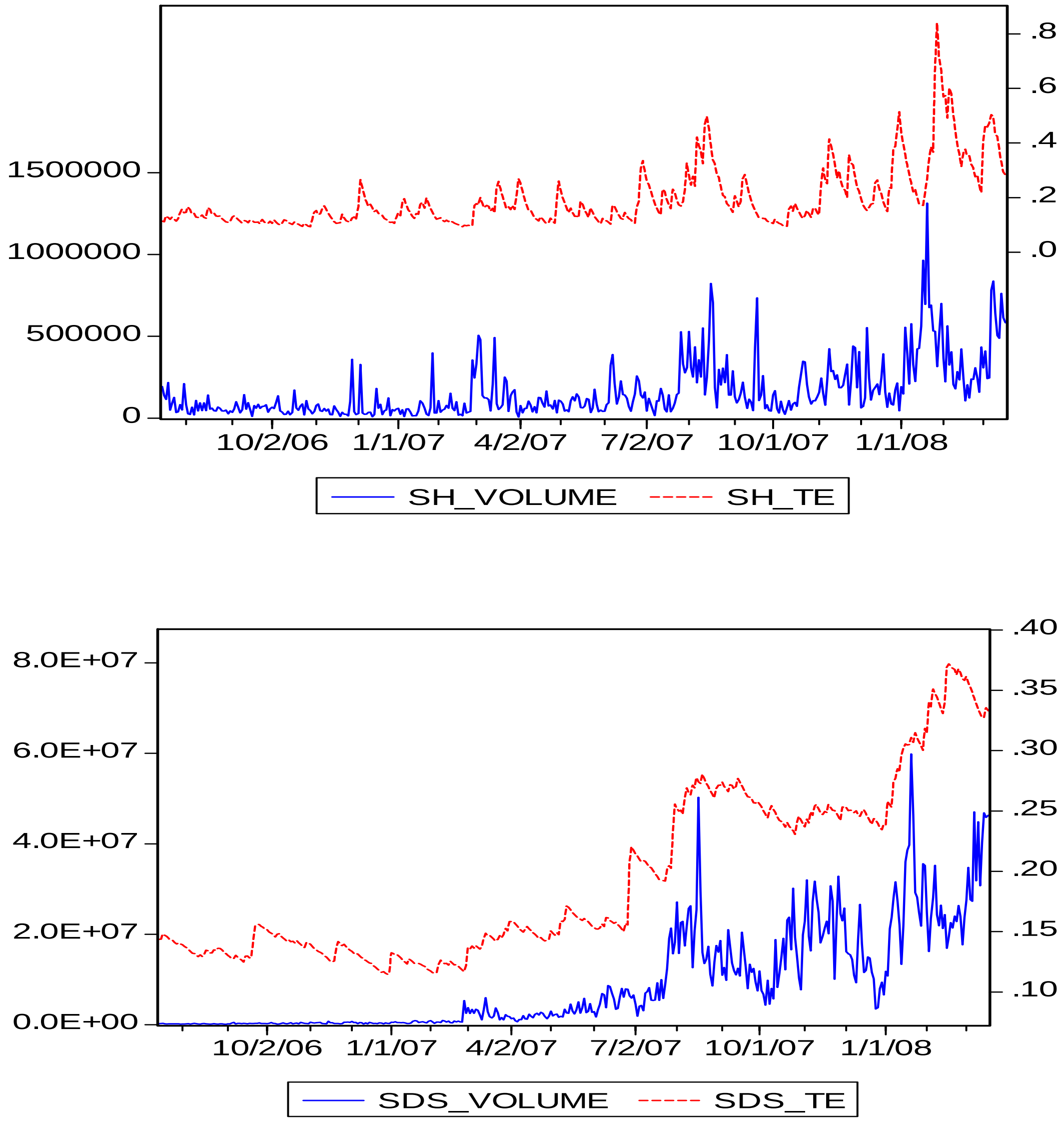


Figure 10: Tracking Error and Trading Volumes of the 'Short'(SH)/ 'UltraShort'(SDS) ETF of S&P500 Index

The figure shows the comparison between the tracking error and trading volumes of short ETFs related to the S&P500 index over the sample period. The Short ETF for S&P500 is SH and the UltraShort ETF for S&P500 is SDS. TE means the tracking error.

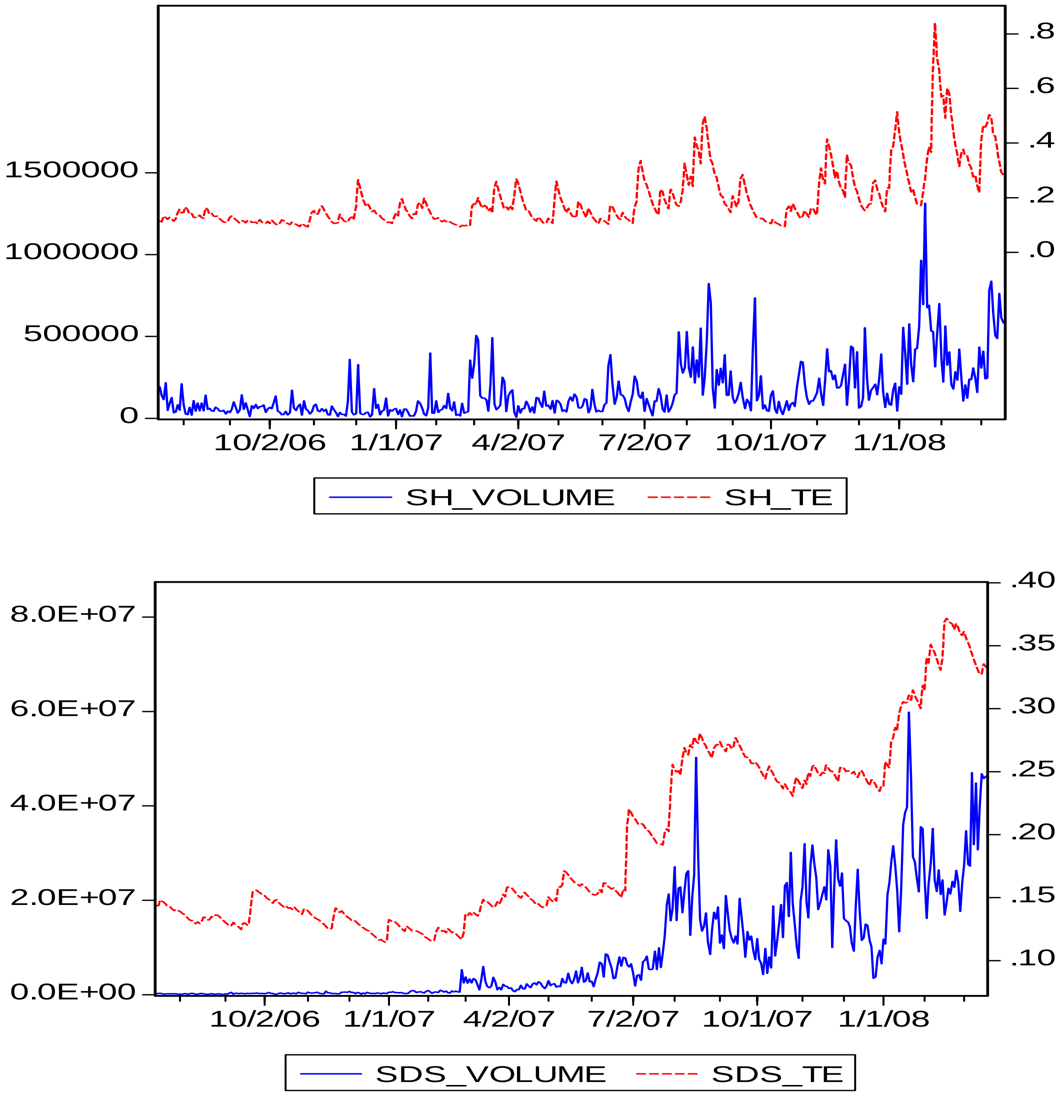


Figure 11: Tracking Error and Trading Volumes of the 'Short'(MYY)/ 'UltraShort'(MZZ) ETF of S&P400 MidCap Index

The figure shows the comparison between the tracking error and trading volumes of short ETFs related to the S&P400 MidCap index over the sample period. The Short ETF for S&P400 MidCap is MYY and the UltraShort ETF for S&P400 MidCap is MZZ. TE means the tracking error.

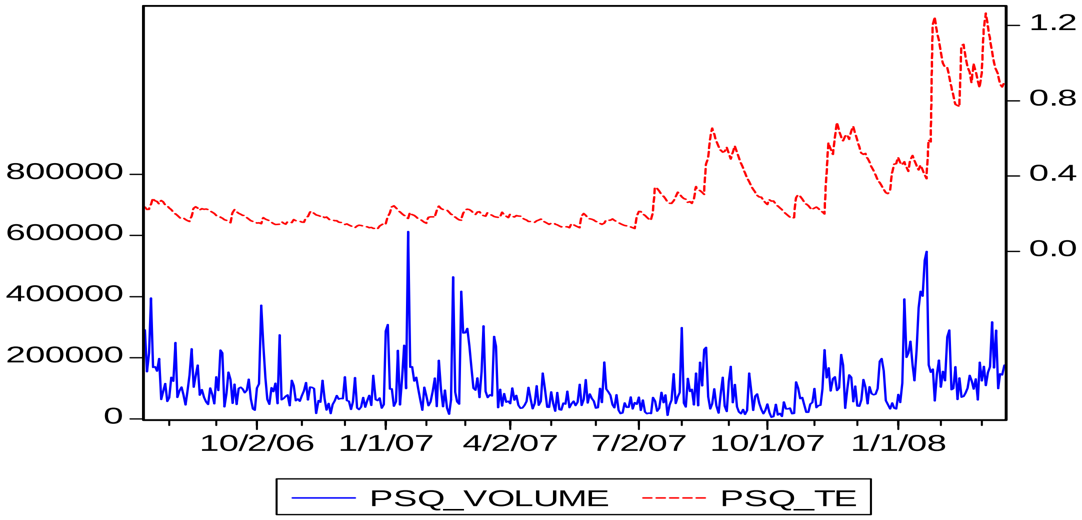
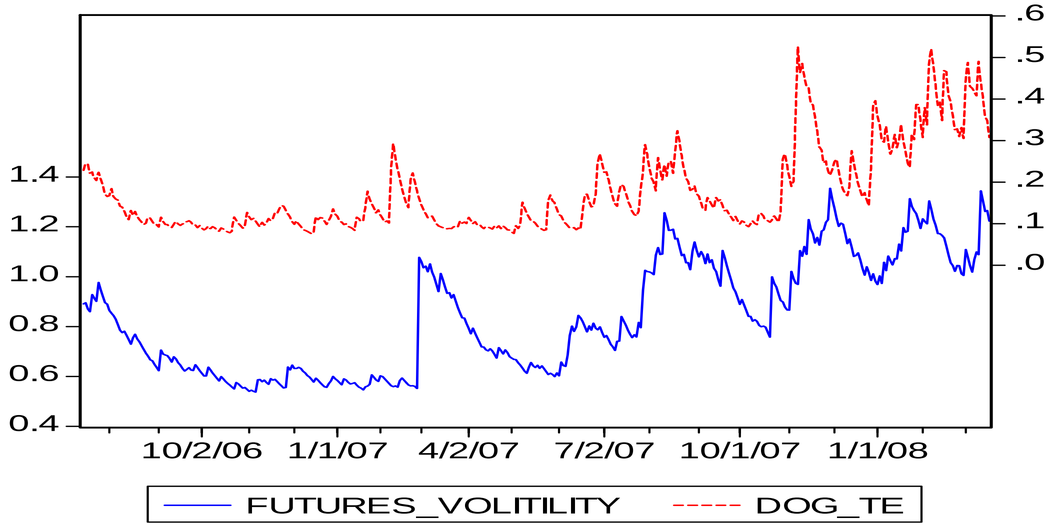


Figure 12: Tracking Error and Trading Volumes of the 'Short'(PSQ) ETF of NASDAQ100 Index

The figure shows the comparison between the tracking error and trading volumes of short ETFs related to the NASDAQ100 index over the sample period. The Short ETF for NASDAQ100 is PSQ. TE means the tracking error.



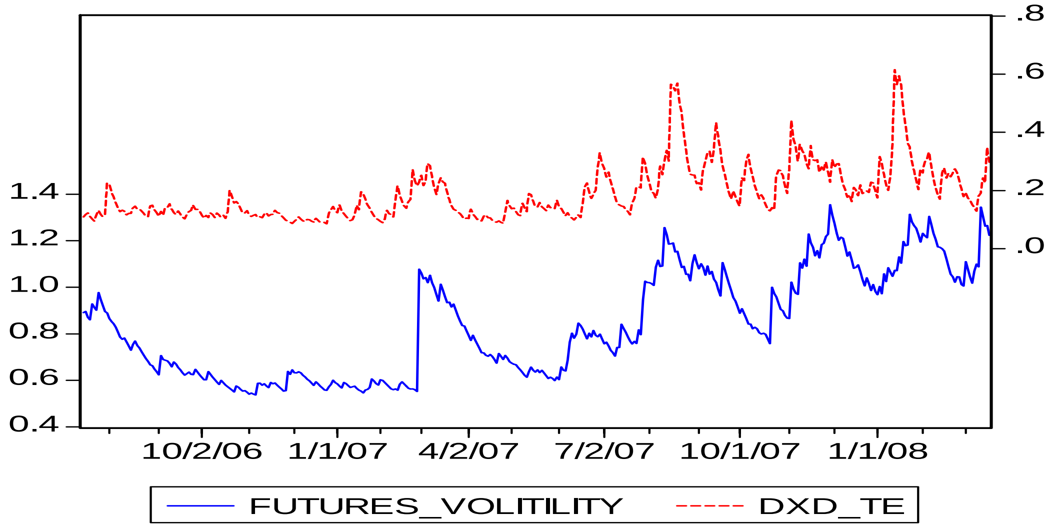


Figure 13: Tracking Error of DOG/DXD and volatility of DJIA Index Futures

The figure shows the comparison between the tracking error and volatilities of index futures over the sample period. The Short ETF of DJIA is DOG and the UltraShort ETF of DJIA is DXD. The DJIA index futures we use here is CBT BIG DOW DJIA.

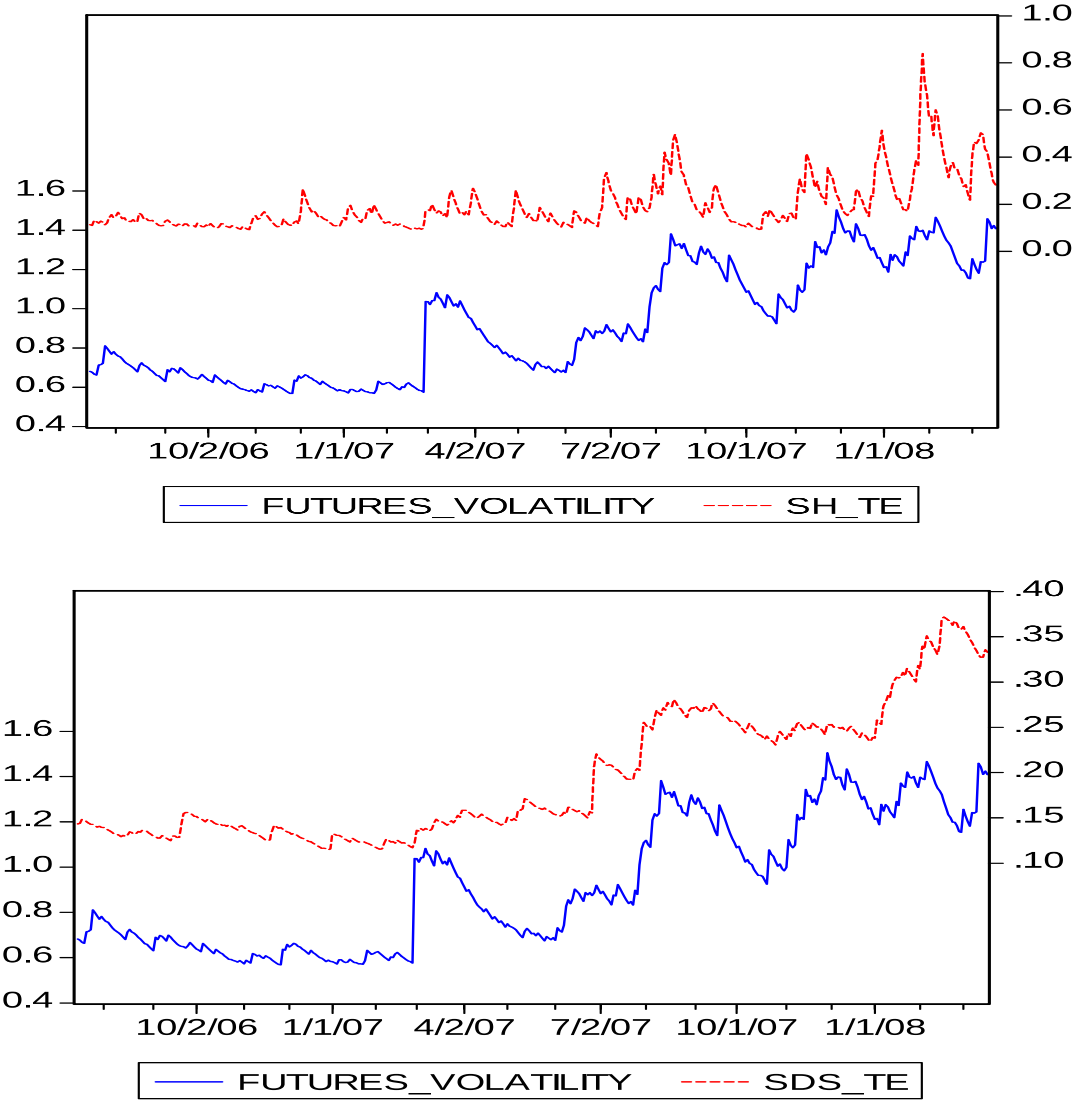
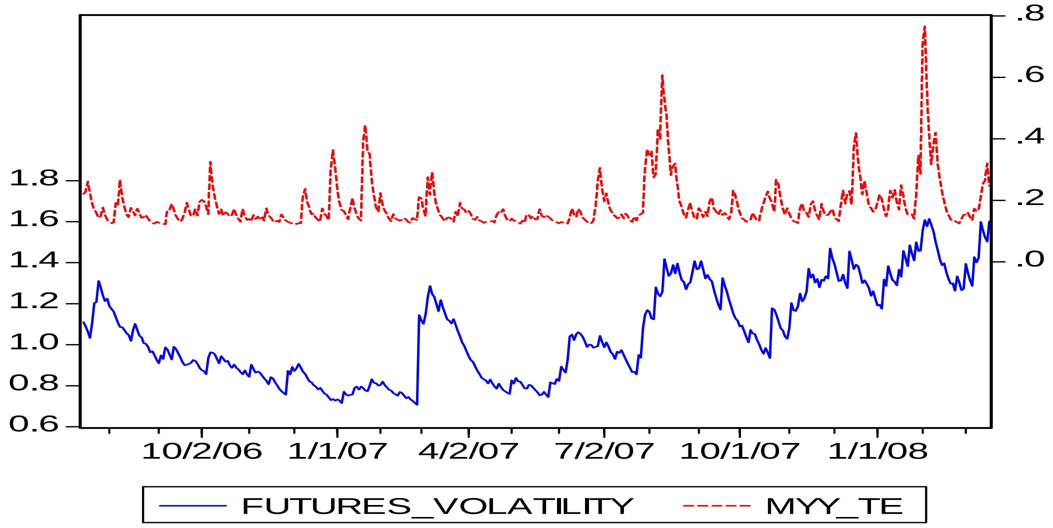


Figure 14: Tracking Error of DOG/DXD and volatility of S&P500 Index Futures

The figure shows the comparison between the tracking error and volatilities of index futures over the sample period. The Short ETF of S&P500 is SH and the UltraShort ETF of S&P500 is SDS. The S&P500 index futures we use here is CME E-Mini S&P 500 index.



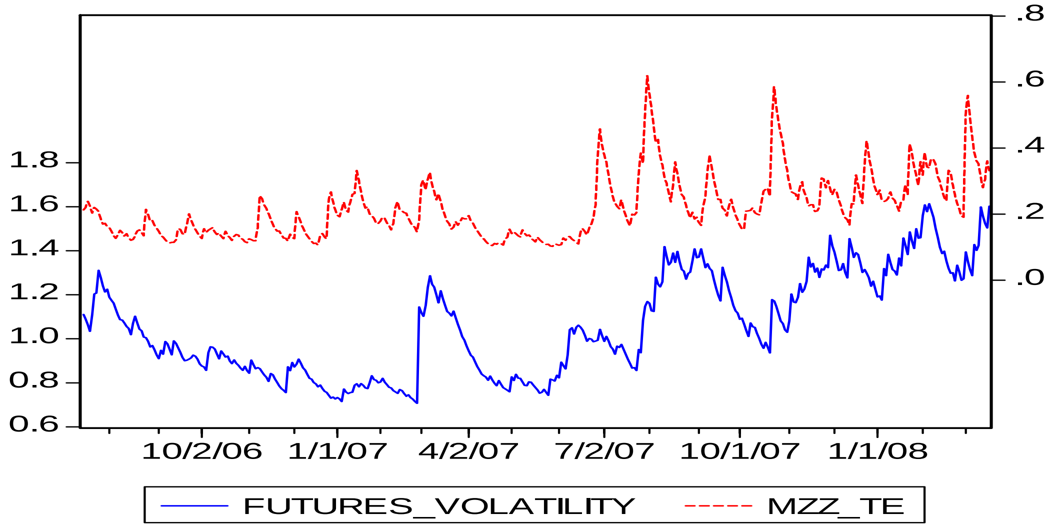
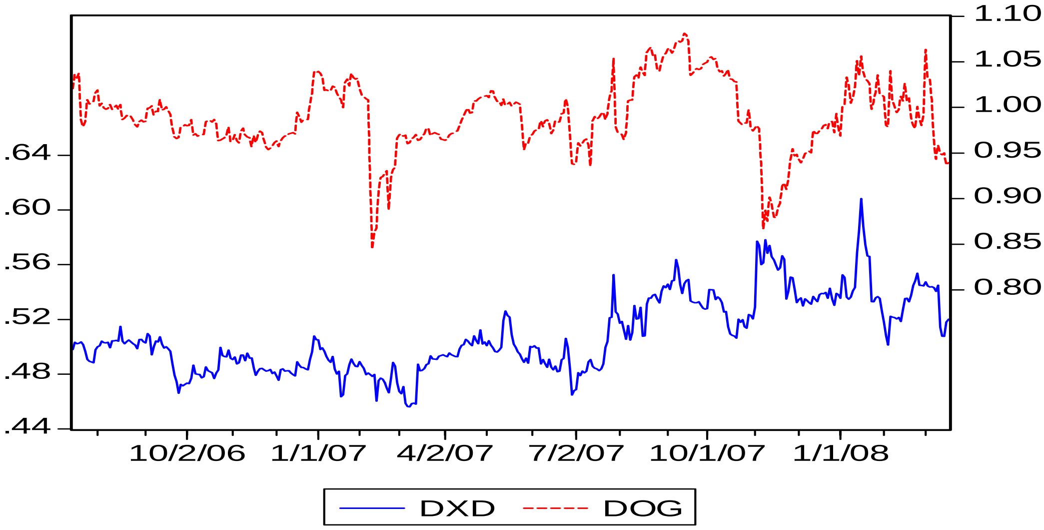


Figure 15: Tracking Error of MYY/MZZ and volatility of S&P400 MidCap Index Futures

The figure shows the comparison between the tracking error and volatilities of index futures over the sample period. The Short ETF of S&P400 MidCap is MYY and the UltraShort ETF of S&P400 MidCap is MZZ. The S&P400 MidCap index futures we use here is CME E-Mini S&P MidCap400.



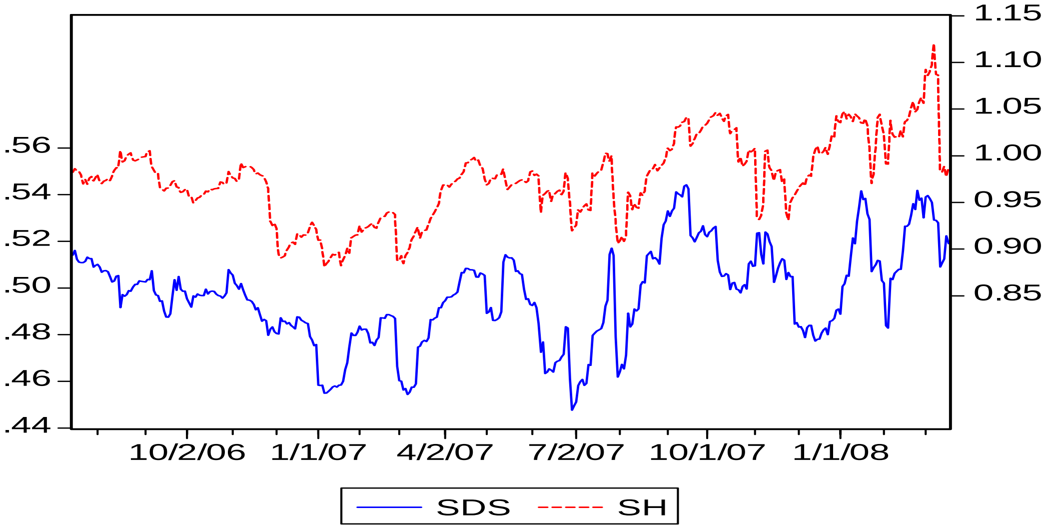
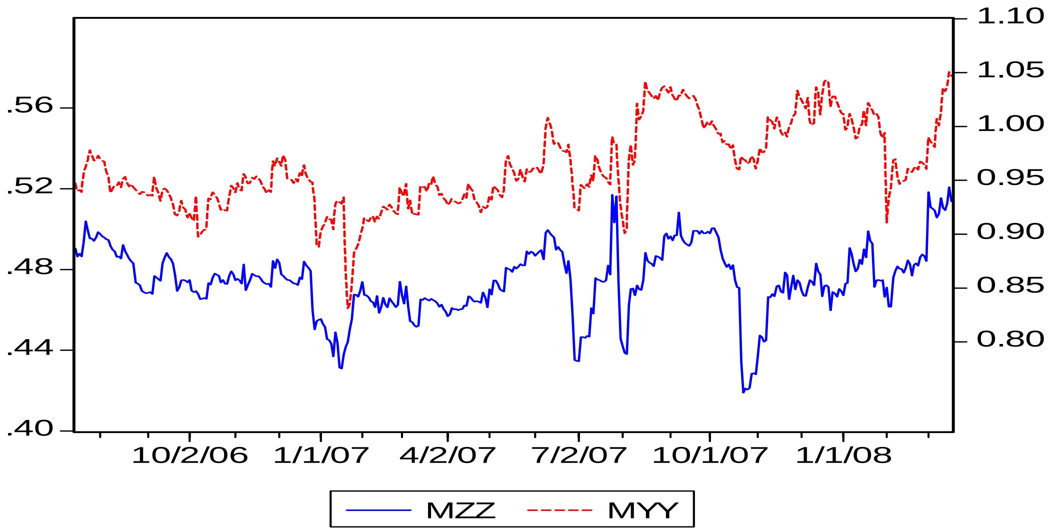


Figure 16: MVHRs for Dow Jones Industrial Average Index and S&P500

This figure shows the minimum-variance hedge ratios (MVHRs) for DJIA and S&P500 using their corresponding Short/UltraShort ETFs. The Short ETF of DJIA (S&P500) is DOG (SH) and the UltraShort ETF of DJIA (S&P500) is DXD (SDS).



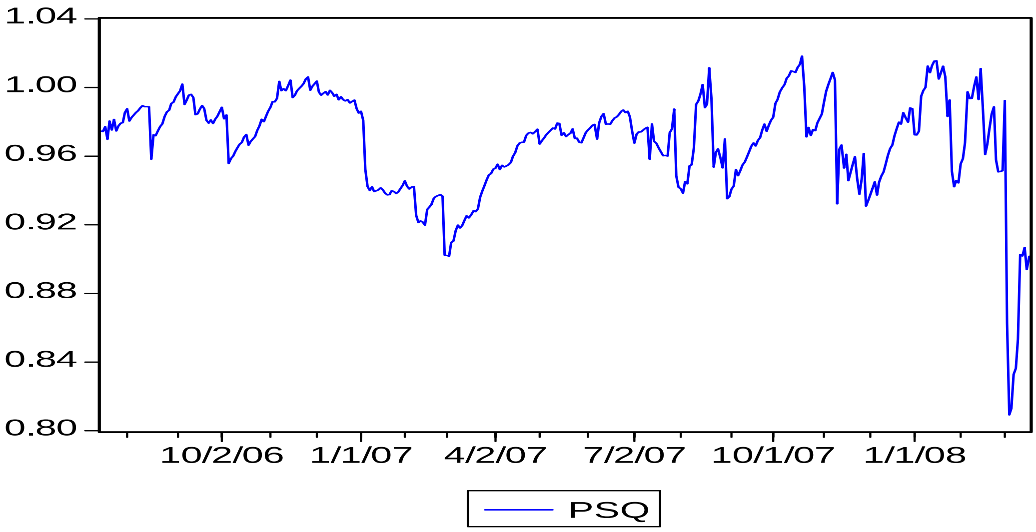


Figure 17: MVHRs for S&P400 MidCap and NASDAQ100

This figure shows the minimum-variance hedge ratios (MVHRs) for S&P400 MidCap using their corresponding Short/UltraShort ETFs and that for NASDAQ100 using Short ETF. The Short ETF of S&P400 MidCap (NASDAQ100) is MYY (PSQ) and the UltraShort ETF of S&P400 MidCap is MZZ.