國立交通大學

財務金融研究所

碩 士 論 文

Modified Ritchken and Trevor tree於GARCH Option

評價之應用

Value the GARCH Option applying the Modified Ritchken and Trevor tree

研 究 生：黃靖謙

指導教授：王克陸 教授

中 華 民 國 九 十 七 年 七 月

Modified Ritchken and Trevor Tree於GARCH Option評價之應用

Value the GARCH Option applying Modified Ritchken and Trevor tree

研 究 生：黃靖謙　　Student：Ching-Chien Huang

指導教授：王克陸 博士　　Advisor：Dr. Keh-Luh Wang

國 立 交 通 大 學

財務金融研究所　碩士班

碩 士 論 文

A Thesis

Submitted to Graduate Institute of Finance

College of Management

National Chiao Tung University

in partial Fulfillment of the Requirements

for the Degree of

Master of Science

in

Finance

June 2008

Hsinchu, Taiwan, Republic of China

中華民國九十七年七月

Modified Ritchken and Trevor Tree 於 GARCH Option

評價之應用

學生：黃靖謙　　　　　　　　　　　　指導教授：王克陸 教授

國立交通大學財務金融研究所 碩士班

摘 要

一般而言，用來評價選擇權的方式大部分為Black-Scholes Model與數值分析方法，其中數值分析方法又分為多種不同的模型。例如: 蒙地卡羅法、二項式法等等。雖然Black-Scholes Model在早期被各界廣泛採用，但它的缺點是有太多的假設，隨著今日日新月異的多種選擇權的發明，Black-Scholes Model在實證分析時出現了一些不合理的問題；我們可以知道Black-Scholes Model面對這些新奇選擇權的評價時並不適用。

Duan (1995)發表了 GARCH 選擇權定價模型，論文中指出根本資產之價格動態過程，在服從 GARCH 模型的行程下，引入經濟學上均衡概念的主張，經過適當的風險測度轉換之後，可以導出歐式選擇

權的價格。但是在此條件狀態下的選擇權訂價理論，其數值分析方法，仍不夠完備，以致於實務上未能完全地擷取而加以運用。其問題的主要癥結在於 GARCH 模型，其本質上必然會產生路徑相依(path dependence)的問題，導致運算與處理上的困難程度增加。而所謂的路徑相依，是指在選擇權存續期間，其價格會受到標的資產價格本身波動性的影響。反之，路徑獨立(path independence)是指選擇權價格只受到標的資產其到期日時之價格影響。GARCH 模型的路徑相依的性質，會使得欲用樹狀圖來刻劃價格的波動過程中，各時點的可能狀態個數會因時間的往前推移，而呈現指數的遞增情形，而使得樹狀圖陣列非常的龐大，使得 GARCH 選擇權定價模型在實務上的應用並不理想。而 Ritchken 和 Trevor (1999)針對在非連續時間的 GARCH 模型，對歐式選擇權和美式選擇權的訂價，建構一個所謂的樹狀演算法。且說明此一樹狀演算法可以進一步擴展到標的資產服從一般化 GARCH 模型之下，建立出有效的運算方法，此一具體運算方法，不僅適用於 GARCH 模型之下選擇權的訂價，而且，也可以用來處理很多雙變數的擴散模型。*RT* 演算法的優點在於可以捕捉各個時點的條件變異數，可以解決 GARCH 模型路徑相依的問題，使得評價能更有效率。

於1999，S. Figlewski與B. Gao提出了適應性網狀模型(Adaptive Mesh Model, AMM), 同時解決了分配誤差(distribution errors)與非線

性誤差(non-linearity errors)，並且提升了評價模型運算的效率。由於AMM在評價上表現出不錯的彈性以及效率，後來，有不少研究將AMM應用於權證的評價上。

本篇論文將 AMM 中的概念應用在 *RT* 模型上，我們稱之為 *AMM-RT* 模型。由於非線性誤差大部分出現在執行價格附近，因此AMM執行價格增加網格節點的密度來提升估計的精確度和減少非線性誤差。我們將這種想法應用於 *RT* 模型的到期日前一天，在到期日的前一日與到期日之間，我們仍然使用 *RT* 模型的演算法，將這段期間切割的較細。這樣的方式可以達到跟 AMM 一樣的效用，同時也可以如同之前的 *RT* 模型一樣具有捕捉條件變異數的能力。波動性(volatility)對於任何一種金融商品而言，都有相當顯著的關係存在，因此我們選擇 *RT* 模型搭配 GARCH 模型來預測選擇權價格，然而，我們又希望增加其精確度與減少其誤差，故到期日前一天增加切割期數以期能達到我們想要的效果。本論文將嘗試分別以傳統的 BS 模型(在不同的 volatility 下)與 *RT* 模型以及 AMM-RT 模型再搭配 GARCH (1,1)模型去模擬並比較股票選擇權價格。

Value the GARCH Option applying the Modified Ritchken and Trevor tree

Student：Ching-Chien Huang Advisors：Dr. Keh-Luh Wang

Institute of Finance, National Chiao Tung University

ABSTRACT

Evaluating stock option price with traditional predictive techniques have proven to be difficult. GARCH option pricing model proposed by Duan has been proven to be more suitable for the task. BS model have so many assumptions that it cannot be suitable in some exotic option. GARCH option pricing model solve the problem which may occur while using the BS model.

This thesis focuses on the stock option price estimating based on GARCH (1, 1) model, which have been surveyed by earlier researcher as well as the comparison between each model is discussed. Derived from the first GARCH option price model proposed by Duan (1995), the Ritchken-Trevor Model offers more accurate pricing than CRR model and traditional trinomial tree model. AMM proposed by S. Figlewski and B. Gao adds the mesh point density partially to modify the inefficiency and calculating error of the *CRR* and trinomial lattice model, which

addresses the problems of distribution errors and non-linearity errors as well as upgrade the efficiency of the pricing model. We apply the idea of AMM in the date T (i.e. the day before the maturity day). Rather than the fine mesh structure like AMM, we develop another fine mesh by the same approach of *RT* model. We just increase the number of time step by changing parameter m (Here m is the segmental level of the last trading day; m=2, 3, 5 will be discussed) in the last date T. We call this justified model "*Modified RT* Model (*AMM-RT*)" in this thesis. The same as AMM, the *AMM-RT* model solve the nonlinearity error around the strike price while evaluating exotic price like, barrier option. By this modified *RT* model, we also solve the nonlinearity error as well as increase the accuracy. In this thesis, we demonstrate a comparison of accuracy between BS model (with different volatility), *RT* model and *AMM-RT* model.

With their ability to discover patterns in nonlinear and chaotic financial systems, the GARCH option pricing model with *AMM-RT* algorithm not only offer the ability to predict market directions more accurately than current techniques bur also reduce the complexity of computing of the original *RT* model. Numerical analysis via above methods are discussed and compared with performance. Finally, future directions for applying the *AMM-RT* model to the financial markets are also disserted.

Acknowledgement

That I exist is a perpetual surprise which is life
- Rabindranath Tagore

我的碩士求學過程—現在回想起來感覺像是一場冒險旅程。旅程中充滿各種的關卡。每個關卡都需要一把鑰匙，而通過之後是絕無僅有的寶物與更多的抉擇。冒險中的鑰匙是我的老師、朋友、以及家人、甚至於一些從未見過的朋友。寶物是我從錯誤中學習的人生經驗。而抉擇是我充滿未知的人生。

在此，我想感謝王克陸老師。在過去近兩年中，王老師扮演著亦師亦友的角色。王老師幫助我的不僅是學業，更多的是在我碰到難關時給我極大的幫助。從老師身上看到的是學者與家庭的典範。王老師不僅在研究上提供我很好的舞台，也讓我在國內的學習環境多了國際視野與各種難得的經驗。同時謝謝所長鍾惠民老師，鐘老師每次遇到我時總會不斷的給我鼓勵，提供我源源不決的信心跟肯定，我很想感謝鐘老師與我一起經歷求學的酸甜苦辣。

另外，我非常感謝博士班學長劉炳麟。炳麟學長除了給予我研究的方向之外，也花了極大的心力與我一起討論研究方法以及協助我處理程式方面的問題，很感謝炳麟學長的傾囊相授。我也從學長身上我學到積極的研究精神以及謙虛的處事態度。我也必須要感謝電子所的吳東佑學弟，謝謝他能夠在兩年期間課業上助我一臂之力。也要感謝財金所的秋男，感謝他在資料蒐集上的幫助，謝謝他在初稿完成前最後幾天，毫無怨言的幫我整理繁雜的資料。最後我想謝謝我的父母、女友泰琳和從電研所博一認識到現在的鄭淳護博士多年的支持與幫助，學長很像是我旅途中一起度難關的伙伴。也謝謝財金所的同學文誠、以文、詩政、建佑、妤芳、佳彣等。並謝謝財金所所有夥伴們。同時在此感謝所有口試委員們的寶貴意見。

To my parents, brother and friends.

Content

CHAPTER 1

Introduction

1.1 Overview

The prevailing notion in society is that wealth brings comfort and luxury, so it is not surprising that there has been so much work done on ways to predict the markets. Various technical, fundamental, and statistical indicators have been proposed and used with varying results. However, no one technique or combination of techniques has been successful enough to consistently "beat the market". With the development of GARCH option pricing model, researchers and investors can wish that the market mysteries can be unraveled. This thesis is an investigation of GARCH option pricing model combining different lattice model with an emphasis on stock price volatility prediction.

Because it is often important to obtain price fast, the efficient numerical algorithms play a vital role in derivatives pricing when prices changes quickly in stock market. In financial econometrics, General Autoregressive Conditionally Heteroskedastic (GARCH) processes are wildly used to model the returns at regular intervals on stocks, currency and other assets.

Specifically, the GARCH process typically represents the increments, $\ln S_{t+1} - \ln S_t$, of the logarithms of the asset price at date 1, 2, 3…. These models capture many of so-called stylized features of such data, e.g. tail heaviness, volatility clustering and dependence without correlation.

Many financial time series data suffer from the stochastic change in volatility over time. For most financial commodities, return innovation will influence future volatilities. This issue has become an important and imperative empirical fact. Mandelbrot (1963) showed that large absolute returns are more likely to follow the large absolute return innovation, which is called volatility clustering. The volatility will be influenced by the extrinsic environment changes. If the news is bad, the volatility will be larger. Black (1976) called this phenomenon "leverage effect". This implies that there is a negative correlation between asset return innovation and volatility innovation. (Bollerslev, Chou, and Kroner, 1992)

Using interaction effect between returns and volatility is very important in the option price model. In 1973, Black and Scholes use the history volatility to calculate the option value. On the assumption of setting the volatility as constant, they ignore the issue about the volatility itself changes with the time. Although BS Model is wildly used, many empirical analysis showed that BS model will bring the issues of pricing error, for example: underestimate the value of out-of-money-option and volatility smile. Duan (1995) was the first to propose a GARCH option pricing model. He indicates that option can be priced when the dynamics of the price of the underlying asset comply with the GARCH process. Unfortunately, plenty of the path dependence of the pricing models prefer to use Mnote Carlo simulation over trees which would increase the calculating difficulty. Thus the analytical solutions to prices of options are not generally available and hence numerical approaches to prices have to be invoked. Ritchken and Trevor (1999) propose trinomial lattice tree to address these problems. They provide an efficient numerical procedure (a lattice approach) for pricing European and American options under discrete-time GARCH processes. Furthermore, in order to handling American option, Duan and Simonato (2000) proposed another numerical algorithm “a Markov chain approach” almost at the same time.

Because the Monte Carlo estimate is probabilistic and the American options can be accurately priced only with simulation schemes that employ advanced techniques, a numerical approach that processed the American option more efficiently than previous Monte Carlo simulation is the binomial tree. Although the binomial approach works well under constant volatility, there will be a formidable challenge to apply this method in stochastic volatility. Rithken and Trevor (1999) construct a tailored lattice approximation algorithm for the GARCH model by restricting the storage of conditional variance to the minimum and maximum values at each node of the discretized underlying asset price under the forward building process.

S. Figlewski and B. Gao (1999) propose the Adaptive Mesh Model (AMM) which adds the mesh point density partially to modify the inefficiency and calculating error of the *CRR* and trinomial lattice model. In this thesis, we apply the idea of AMM in the date T (i.e. the day before the maturity day). Rather than the fine mesh structure like AMM, we develop another fine mesh by the same approach of *RT* model. We just increase the number of time step by changing parameter m in the last date T (Here, we call the segmental level of the last trading day *m*). We call this justified model "*Modified RT* Model (*AMM-RT*)" in this thesis. The emphasis of this thesis is to completely investigate the stock option price estimation under Duan's GARCH model in combination with different algorithms. *BS* model with different volatility, *RT* model and *AMM-RT* model (modified *RT* model) will be discussed. Moreover, using this modification of the later *RT* model also makes it possible to apply Duan's GARCH option pricing model to a broader domain of exchange traded option contracts.

The thesis organized as follows. In section 2 we will review the basic GARCH option pricing Model proposed by Duan (1995), the lattice algorithm of Ritchken-Trevor (1999), and Adaptive Mesh Model. Section 3 describes the empirical procedure of our work using *AMM-RT* to evaluate the target commodity price volatility. The crux of the work, in Section 4, details the numerical illustrations of BS model, *RT* model and our *AMM-RT* model in concert with GARCH option pricing model. This thesis also concludes with comments on possible future work in the area and some conclusions.

1.2 Research Motivation

There are several motivations for trying to predict stock market prices. The most basic of these is financial gain. Any system that can consistently pick winners and losers in the dynamic market place would make the owner of the system very wealthy. Thus, many individuals including researchers, investment professionals, and average investors are continually looking for this superior system which will yield them high returns. There is a second motivation in the research and financial communities. It has been proposed in the Efficient Market Hypothesis (EMH) that markets are efficient in that opportunities for profit are discovered so quickly that they cease to be opportunities. The EMH effectively states that no system can continually beat the market because if this system becomes public, everyone will use it, thus negating its potential gain.

Doing stock option price predictions have never been easy even for professional investors. Stock market experts are continuously researching and devising methods that could aid them and others in foreseeing an accurate stock market outcome. Stock market commodities prediction is continuously being attempted. But unfortunately until now, there isn't a 100% accurate technique created to do it yet.

Stock market is the term given to the act of trading company shares, options, stocks, and other securities and its derivatives. The stock option has a number of players, which could be range from an individual stockholder to a very large corporate trader. These players can be anybody coming from any part of the world. Trading in the stock option can be done privately with an attorney or with a professional stock exchange dealer who have the power to execute the order.

For the most part, stock option price is very volatile in nature so that the price is very tough to predict. That's the reason why volatility is studied in this thesis. In the past, people almost widely used the regression method, time series methods, and the neural network methods to predict stock price. Due to persistent studies, the changes in the stock market can now be calculated in a relatively acceptable precision.

In this thesis, we use a different kind of approach to predict the option price. The performance there are the various efforts carried out by stock market experts to predict the market's movements. I depict the empirical procedure in Section 3.2 and the applicability of *Modified RT (AMM-RT)* model is also discussed.

CHAPTER 2

Literatures Review

2.1 Stock Option Pricing

Traditionally, the approach of pricing the option divides three major sections. Section One: Formula solution (Closed solution): Black and Scholes option pricing model. Section Two: Numerical Analysis solution: Using numerical approach, like computer simulation, to calculate option price. For example, tree algorithm, Monte-Carlo simulation and finite differential approach. Section Three: Analytic approximate model: This approach combines the above two methods. For example, Barone-Adesi and Whaley (1987) deduce the analytic formula solution of American option.

Most researchers use risk free arbitrage to deduce closed form solution and find a partial differential equation and its solution. However, the derivation process is more complicated and difficult since we couldn't find its closed-solution in many situations, especially the path-dependency option. Harrison and Kreps (1979) develop another kind of method to solve the pricing issue of the derivative commodity which is so-called "martingale pricing method”. This method, comparing with solving the partial differential equation, is easier to solve and involve with fewer mathematical techniques. Thus, recently the martingale pricing method is used repetitiously.

Although closed form formula is simple and computing fast, not all the pricing of options exist the closed form solution. Besides, it is usually applicable to the pricing of European option but not to American option and other exotic option. Moreover, we should adopt the numerical approach to handle the option pricing under disconnected time. If we know the path of our target asset price, we can use the Monte Carlo approach to simulate target asset price’s possible path repeatedly. Thus, we can get the price of plain vanilla type option. Yet, this approach would cost a lot of processed time and suffer from poor computing efficiency.

Cox, Ross, and Rubinstein (1979) develop binominal tree model (CRR model), which breakthrough the original BS model’s assumptions and applicative range. CRR model describe the target asset price’s behavior in discrete time status. It also deduces the risk nature pricing model except the arbitrage opportunity. It should be noted that CRR model assume target asset return’s volatility is constant when it is built. Besides, the binominal tree model can add the segmental time steps on tree diagram to increase pricing accuracy, which also solve the issue of consuming a lot of time of Monte Carlo method. Yet, when the path-dependency issue exists, the nodes of the tree diagram will increase exponentially due to the increase of segmental time steps. Thus we can conclude that the traditional binominal tree model hardly to handle as soon as the option becomes more complicated. Boyle (1986) extends the binominal tree to trinomial tree. Further, for binomial tree and trinomial tree, Tian (1993) proposes different kind of estimation methods of parameter. Tian also verifies and compares the pricing efficiency of the two models. Even though the trinomial tree's diffusive ended nodes are more than the binomial tree's, the segmental time step will be smaller. Thus, the trinomial tree model can capture more complete price probabilistic distribution function. Base on this advantage, we can find relatively accurate option price and we can also verdict that the pricing efficiency of the trinomial tree model is better than which of the binomial tree model.

2.2 GARCH Option Pricing Model

Duan (1995) proposes the GARCH option model in 1995. He develops the option pricing model when stock option follows the discrete time GARCH (*p*, *q*) process (proposed by Bollerslev, 1986). Following, we will describe the GARCH option pricing model using the standard discrete time GARCH (1, 1) specification. Because the simple GARCH (1, 1) with normal distribution assumption is the most commonly used, we will use GARCH (1, 1) as our estimating model. Based on the LRNVR of Duan (1995), the estimation of variance will not change with the Measure situation, thus, we only need to apply simply GARCH (1, 1) to estimate the variance out of sample. We let S_t be the target asset price at data t, h_t be the conditional variance at interval $\left[t, t+1\right]$ that is one day. The behavior of target asset price can be expressed as below:

$$\ln(\frac{S_{t+1}}{S_t}) = r_f - \lambda\sqrt{h_t} - \frac{1}{2}h_t + \varepsilon_t,\ \varepsilon \,|\, \phi_{t-1} \sim N(0, h_t) \qquad (1)$$

$$h_{t+1} = \omega + \alpha\varepsilon_t^2 + \beta h_t \qquad (2)$$

The other corresponding definitions of symbol are illustrated as follows:

S_t: the target asset price at period (day) t

h_t: the conditional variance of the target asset price at period t

ϕ: the collection of all information before period t-1

ε_t: the standard normal random variable at period t, that is, $\varepsilon \mid \phi_{t-1} \sim N(0, h_t)$

r_f: risk free rate

λ: unit risk premium

The above GARCH (1,1) system follows the standard GARCH parameter restriction. And this model follows the restriction as:

$\omega > 0,\ 0 < \alpha\ ,\ \beta < 1,\ (\alpha + \beta) < 1$

Based on Duan's model (1995), the asset price process under locally risk-neutralized pricing measure Q can be rewritten as:

$$\ln\left(\frac{S_{t+1}}{S_t}\right) = r_f - \frac{1}{2}h_t + \varepsilon_t,\ \varepsilon_t \mid \phi_{t-1} \sim N(0, h_t) \tag{3}$$

$$h_{t+1} = \omega + \alpha(\varepsilon_t - \lambda\sqrt{h_t})^2 + \beta h_t \tag{4}$$

Among these equations, ε_t is the standard normal random variable in the corresponding risk-neutralized pricing measure Q. Under this modified model, the parameters waiting for estimating are ω, α, β, respectively. Furthermore, the risk premium λ amputates from the equation (3) under measure Q. In other word, the equation (3) is independent of the parameter λ. This property indicates that ones can assume that the investors are risk-neutral. In the risk-neutral world, the present value of any cash flow can be obtained by discounting its expected value at risk-free rate as well as the expected return of any stock commodity is the risk-free interest rate.

2.3 Ritchken and Trevor Tree

Ritchken and Trevor (1999) develop the trinomial tree algorithm based on trinomial tree model to capture the path of price and volatility and advanced to evaluate the European option and American option. Since the stock option volatility is incompletely standard distribution, Ritchken and Trevor assume that stock option volatility follows the GARCH model and the stochastic process. They develop the tree lattice of the variances of stock yield rate, probability, and stock price, etc.

A. Construct the tree lattice of the variance and probabilities:

The key to an efficient implementation is to design an algorithm that avoids an exponentially exploding number of state, Toward this goal, we begin by approximating the sequence of single period lognormal random variables in equation (3) by a sequence of discrete random variables. In particular, we set $\ln(S_t) = y_t$. Based on GARCH (1, 1) model under Q measure, the model in equation (3) ~ (4) can be rewritten as :

$$y_{t+1} = y_t + r_f - \frac{1}{2}h_t + \varepsilon_t \quad , \ \varepsilon_t \mid \phi_{t-1} \sim N(0, h_t) \tag{5}$$

$$h_{t+1} = \omega + \alpha(\varepsilon_t - \lambda\sqrt{h_t}\,)^2 + \beta h_t \tag{6}$$

Thus,

$$E(y_{t+1} \mid \phi_t) = y_t + r_f - \frac{1}{2}h_t$$

$$Var(y_{t+1} \mid \phi_t) = h_t$$

The GARCH (1,1) process can be approximated by the following lattice model, and the superscript of each parameter a denoting “approximation”:

$$y^a_{t+1} = y^a_t + j\eta\gamma_n, \ \ j = \pm 0, \pm 1, \pm 2, \dots, \pm n \tag{7}$$

$$h_{t+1}^{a} = \omega + \beta h_{t}^{a} + \alpha(\varepsilon_{t}^{a} - \lambda\sqrt{h_{t}^{a}})^{2} \quad (8)$$

Symbol n determine the segmental number in each period (day). If n=2, then we segment two subinterval in each period. The symbol n also decides the branch of the tree diagram that takes on 2n+1 value. When n=2, we have five state variables of each period (day) such that there are two values smaller the current price, one value unchanged, and another two value larger than the current price. The symbol η indicates the jump parameter which allows the variance and mean of the next period's logarithmic price to match the true moments as well as ensuring that the (2n+1) probability values are valid in the interval [0, 1]. By *RT*'s theorem, η is chosen such that

$$(\eta + 1) < \frac{\sqrt{h_{t}^{a}}}{\gamma} \leq \eta \quad (9)$$

The gap between two neighbor logarithmic prices is decided by the spacing parameter γ_n. By *RT*'s theorem, γ is a fixed constant which follows the relationship:

$$\gamma_n = \frac{\gamma}{\sqrt{n}} \quad (10)$$

It would be noted that the "path dependence" issue will occur when we use GARCH model. In Figure 1, we should note that the number of variance may not be 1. Observe that different states may pick different states η. Take node (2,-1) for example, there will be three possible paths achieving node (2,-1), which means there would be three possible variances on node (2,-1). We only reserve the maximum and the minimum variance on each node. Thus, the two variances on node (2,-1) will leads to two possible η. For the smaller variance, we just need set η=1; however, we should set η=2 to satisfy equation (10). Consequently, if we don't put a limitation to these variances, we will have 5 paths deriving from node (2,-1).

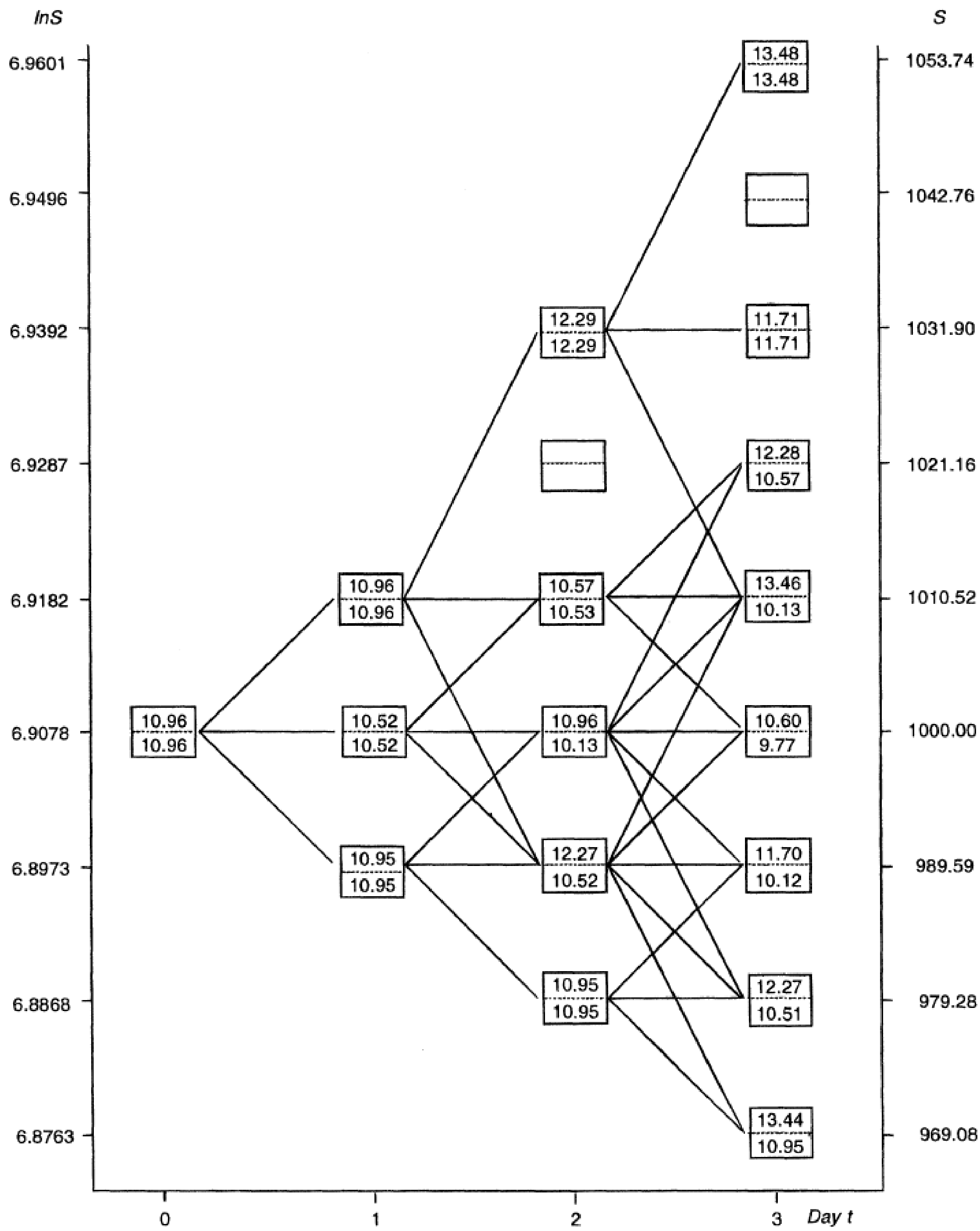

Fig.1 The tree lattice of variance based on GARCH model (t=3, n=1,). We assume K=2, r_f = 0, λ=0, β_0=6.575×10^{-6}, β_1=0.9, β_1=0.04, c=0, S_0=1000, h_0=0.0001096 (h_0 is the initial variance). The top (bottom)

number is the maximum (minimum) variance (multiplied by 10^5).

Due to the path dependence issue, the variance of next time will be influenced by the variance of the previous time. With the time increase, the number of variance of each node will not be only one. In order to overcome this problem, we must let the number of variance of each node fixed as well as express the all possible variance one node may possessed. We utilize interpolation method to get the other K-2 variances using the maximum and minimum variance, thus we can keep the number of variance of each node fixed.

$h_t^{\max}$ and $h_t^{\min}$ represent the possible maximum and the minimum conditional variance which come from all paths of the lattice. *RT* model divide interval between $h_t^{\max}$ and $h_t^{\min}$ into K parts. Let $h_t^a(i,k)$ be the k^{th} conditional variance of node (t, i):

$$h_t^a(i,k) = h_t^{\min} + \frac{h_t^{\max}(i) - h_t^{\min}(i)}{K-1}.(k-1),\ k = 1,2,...,K \tag{11}$$

Thus, we finish the frame of *RT* lattice model.

When we construct the variance of the *RT* lattice model, meanwhile, we can obtain the following probabilities:

$$p_u = \frac{h_t^a}{2\eta^2\gamma_n^2} + \frac{(r_f - h_t^a/2)\sqrt{1/n}}{2\eta\gamma_n} \tag{12}$$

$$p_m = 1 - \frac{h_t^a}{\eta^2\gamma_n^2} \tag{13}$$

$$p_d = \frac{h_t^a}{2\eta^2\gamma_n^2} - \frac{(r_f - h_t^a/2)\sqrt{1/n}}{2\eta\gamma_n} \tag{14}$$

h_t^a: the variance of target stock's rate of return (unit: year)

η: jump parameter

γ_n: stock price step between before and after date, $\gamma_n = \frac{\gamma}{\sqrt{n}}$ and $\gamma = \sqrt{h_0^a}$

r_f: risk neutral interest rate

n: decide the segmental number of each period (1 day). If n=2, then we segment n subintervals of one day.

B. *Construct the tree lattice of stock price*

After setting the variances and probabilities of the *RT* lattice model, we next begin to construct the stock price of the tree.

Let $y_t = \ln S_t$ and $y_t = y_t + j\eta\gamma_n, j = 0, \pm 1, \pm 2, \ldots, \pm n$.

Following we define the node (t, i) represents the t^{th} date and the i^{th} price, thus, $y_t(i) = y(0) + i\gamma, i = -M_d(t), \ldots 0, \ldots M_u(t)$, $M_d(t)$ and $M_u(t)$ stand for the maximum and minimum units of price-ascending and price-descending, respective. For example, $y_t(1) = y(0) + 2\cdot\gamma$ or $y_t(1) = y(0) + 3\cdot\gamma$.

C. *Pricing the stock option price*

In order to establish the lattice model, in the above discussion, *RT* model set the logarithmic price at date t is y_t^a and the conditional variance is

h_t^a. In the Step C, after the underlying asset price and volatility lattice are setting, stock option price (not variance) can be evaluated on the tree using the backward induction. Let $C_t^a(i,k)$ is option price of node (t, i) related to the k^{th} conditional variance. At this time, the corresponding stock price is $S_t(i)=e^{y_t(i)}$ and k=1, 2,…, K. Regardless of the corresponding variance, when the option strikes, the return should be

$$C_T(i,1)=C_T(i,2)=...=C_T(i,K)=Max\{0,S_T(i)-X\} \tag{15}$$

where X is the strike price and $S_T(i)$ is the stock price at date T. We will show the corresponding relation between variance and stock price at maturity day (t=T) in Figure 2.

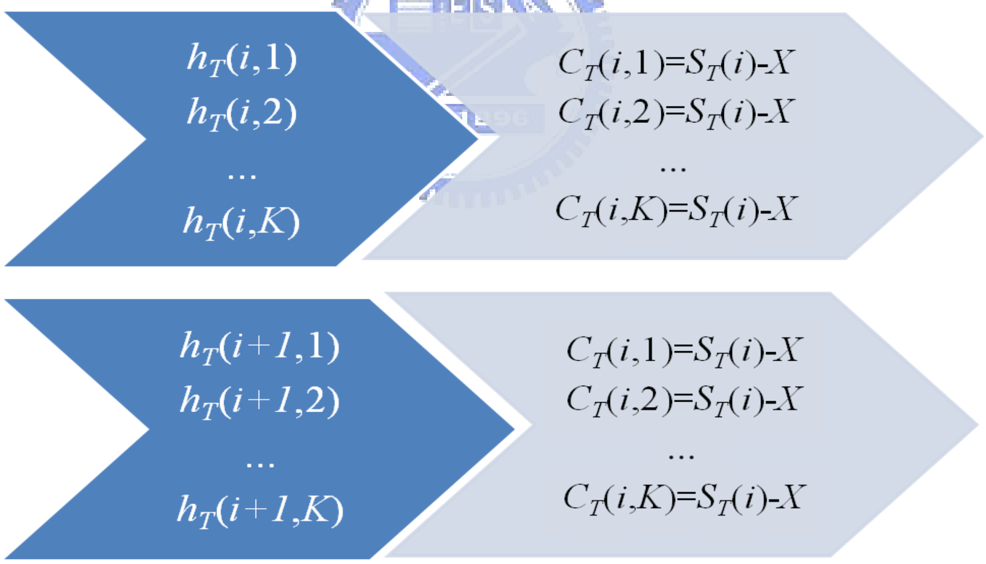

Fig. 2 The corresponding relation between variance and stock price at maturity day (t=T).

If the k^{th} conditional variance of node (t, i) is $h_t(i,k)$, we can calculate the "*true*" conditional variance of node (t+1, i+$j\eta$) at next period:

$$h_t^{next}(j) = \omega + \beta h_t + \alpha h_t \left[\frac{(j\eta\gamma_n - (r_f - \frac{1}{2}h_t^a(i,k))}{\sqrt{h_t^a(i,k)}} - \lambda\right]^2 \quad (16)$$

where $j = 0, \pm 1, \pm 2, \ldots, \pm n$

However, at node (t+1, i+$j\eta$), we have stored options for only K different variance level. When (2n+1) is larger than K, there may not be a variance entry corresponding exactly to $h_t^{next}(j)$. Following we use "interpolated method" to decide the stock option price which corresponds to $h_t^{next}(j)$. We assume $h_t^{next}(j)$ locates between the L^{th} conditional variance and $(L+1)^{th}$ conditional variance of node $(t+1, i+j\eta)$. (one node can possess K variance) Thus, the corresponding stock option price of $h_t^{next}(j)$ is:

$$C^{\text{interp}}(j) = q(j)C_{t+1}^a(i+j\eta, L) + (1-q(j))C_{t+1}^a(i+j\eta, L+1) \quad (17)$$

where,

$$q(j) = \frac{h_{t+1}^a(i+j\eta, L+1) - h^{next}(j)}{h_{t+1}^a(i+j\eta, L+1) - h_{t+1}^a(i+j\eta, L)} \quad (18)$$

We show the relation of $h_t^{next}(j)$ in Figure 3. In this way, either node (t+1, i+$j\eta$) contains a variance entry (or, option price) that matches $h_t^{next}(j)$, or the relevant information is interpolated from the closet two entries, will have its corresponding option price.

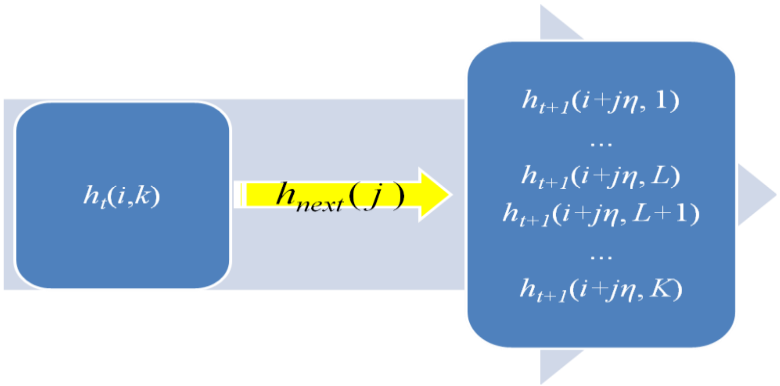

Fig. 3 The illustruction of the location of $h_t^{next}(j)$

As a result, we can get the stock option price corresponding to k^{th} conditional variance at node (t, i):

$$C_t^a(i,k) = e^{-r_f} \sum_{j=-n}^{n} P(j) C^{\text{interp}}(j), \quad j = 0, \pm 1, \pm 2, \dots, \pm n \tag{19}$$

We can use equation (16) ~ (19) with the backward recursion to get the option price at day 0. We also describe equation (19) using following Figure 4.

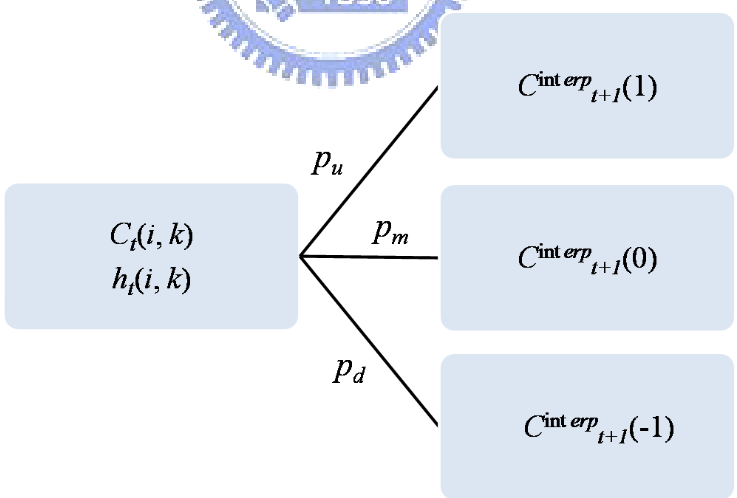

Fig. 4 The illustration of evaluating stock option price $C_t^a(i,k)$

The above mentioned probability distribution (12)-(14) can be expressed as

$$p(j) = \sum_{j_u, j_m, j_d} \binom{n}{j_u \; j_m \; j_d} p_u^{j_u} p_m^{j_m} p_d^{j_d}, \text{ with } j_u, j_m, j_d \geq 0 \text{ and } j_u + j_m + j_d = n.$$

The variance number at each node in this tree lattice is different; indeed, one node maybe has more than one variance (i.e. K variances). The pricing result shows that the convergence velocity will be influenced by the lattice branches and the number of variance at each node. When the number of each node is fixed as well as the multinomial tree's branch extends, then, the convergence velocity increases. On the other hand, when the multinomial tree's branch is fixed as well as the number of each node increases, then, the price would converge closely to its true price.

Cakici and Topyan (2000) modify the *RT* model, which is so-called *RTCT* model. In their point of view, *RT*'s model is not meaningful due to the next date's possible variance produced by interpolation method. This step could be reserved until the option price is calculated in the "backward" induction. In the "forward" step which constructs the tree lattice, we only need to reserve the maximum and minimum variances and calculate these two factors' influence on next period. Thus, Cakici and Topyan find that the model's volatility would be more close to true asset's volatility when the difference of the variance in each node is equal. Furthermore, when the accuracy increases, the calculating time would decrease, convergence velocity be better, and the price be more unbiased. Even the GARCH model's parameters change; the result will still be the same.

2.4 Adaptive Mesh Model

When we use the lattice model to price stock option, there are essentially two kinds of approximation errors in any pricing techniques of lattice framework, which are distribution error and nonlinearity error, respectively.

1. Distribution Error:

When we use lattice model to price stock option, we use a finite set of nodes with probabilities (i.e. binominal or trinomial) to approximates the true asset price distribution with continuous lognormal density. Although the mean and variance of the discrete distribution of lattice model are matched by the continuous distribution of lattice model, the discrepancy between them still leads to distribution error in stock option value. If we increase the time step number of the lattice model (i.e. increase *n*), the discrete distribution of lattice model will approach to the continuous distribution of lattice model. With the time step number of lattice model increasing, the distribution error decreases.

2. Nonlinearity Error:

If the option payoff function is highly nonlinear, pricing this nonlinear region with only one or several nodes (i.e. binominal or trinomial) would give a poor approximation to the average value over the whole interval. For example, when stock price pass through these regions: around the strike price, the stock option price meets the crossroad, and barrier option approaches to the barrier price, then the stock price's bitty perturbation will lead to the large change of the stock option price(i.e. jumps or meets the crossroad). The above situations (the stock option price jumps or meets the crossroad) are called nonlinearity error. The nonlinearity error can be reduced by increasing the time step number of the lattice model. Even though the nonlinearity error can be reduced, it also will occur while the time step number increases to some threshold number. Thus, we apply Adaptive Mesh model in pricing stock option to minimized nonlinearity error with only slight computation increase.

Originating from trinomial lattice model, S. Figlewski and B. Gao (1999) propose the Adaptive Mesh Model (AMM) which adds the mesh point density partially to modify the inefficiency and calculating error of the trinomial lattice model. AMM is a kind of trinomial tree lattices that applying higher resolution fine mesh to where nonlinearity error occurs. AMM model use $\ln(S)$ to substitute the original target asset price S as the variable for calculating node's price. This is also the major difference between AMM and the original trinomial tree model proposed by Boyle (1986). The AMM adopts the characteristic of numerical analysis method. It can adjust its setting and limitation of its parameters based on different warrant contract.

In the following section, we discuss AMM applying in European option and American option, respectively.

(I) European option:

AMM follows the serious assumption of BS model. In the risk neutral pricing environment, the target asset S obeys the generalized Wiener process and satisfies the logarithm normal distribution. The target asset in AMM can be expressed as:

$$\frac{dS}{S} = \alpha dt + \sigma dz \tag{20}$$

We assume $X^* \equiv \ln(S)$, then $dX^* = \alpha dt + \sigma dz$. Among this equation, $\alpha = r - q - \frac{\sigma^2}{2}$ (α is the expected rate of return; q is the instantaneously dividend payment rate; σ is the instantaneously volatility rate; dz expresses the Wiener process). Thus we can rewrite equation (9) as:

$$d\ln(S)-(r-q-\frac{\sigma^2}{2})dt=\sigma dz \tag{21}$$

The advantage of this transform is letting the all asset price to change regularly at a fixed quantity *dt* and *dz*. Therefore, we can use the idea of the *finite difference* to handle the price frontier. Thus, we go on to increase the density of mesh point on the price frontier locally and ensure the price can converge rapidly to increase the calculating accuracy.

Trinomial tree model assumes the target asset price will have three kinds of changes until the next period comes: ascending, unchanging, and descending. The AMM also retains this characteristic and hypothesize the occurring probabilities are P_u, P_m and P_d, respectively. If the probability and the range that price changes are symmetrical, the range of price change *h* (*h* is so-called price step) should satisfies $\sigma dz \sim N(0,\sigma^2 dt)$ under the Geometric Brownian Motion. For this reason, in the continuous diffusion process, the hypothesis of model is composed of the summation of occurring probabilities, first degree partial differential equation (1st PDE), second degree partial differential equation (2nd PDE), and forth degree partial differential equation (4th PDE)to form the following simultaneous equation:

$$1=P_u+P_m+P_d \tag{22}$$

$$E[X(t+\Delta t)-X(t)]=0=P_u\times h+P_m\times 0+P_d\times(-h) \tag{23}$$

$$E[(X(t+\Delta t)-X(t))^2]=\sigma^2\Delta t=P_u\times h^2+P_m\times 0+P_d\times(-h)^2 \tag{24}$$

$$E[(X(t+\Delta t)-X(t))^4]=3\sigma^4\Delta t^2=P_u\times h^4+P_m\times 0+P_d\times(-h)^4 \tag{25}$$

With some algebraic efforts, we obtain the following equation:

$$P_u = 1/6,\ P_m = 2/3,\ P_d = 1/6,\ h = \sigma\sqrt{3\Delta t} \tag{26}$$

The above deduction is the trinomial process for appropriating the target asset price distribution, in other words:

$$X^*_{t+\Delta t} - X^*_t = \begin{cases} h, & \text{with } \mathrm{P}_u = 1/6 \\ 0, & \text{with } \mathrm{P}_m = 2/3 \\ -h, & \text{with } \mathrm{P}_d = 1/6 \end{cases}$$

According to the above derivation, S. Figlewski and B. Gao (1999) use AMM to find the single node's price at time t without considering the fine mesh structure. As the above-mentioned, in the logarithm normal distribution, if the step between each node of the tree is constant (h and Δt), we can make use of the "*explicit finite difference*" to develop the fine mesh structure. Thus the approach would decrease the linearity error.

Because the contract of the European option is succinct, the model with symmetry will increase the convergence speed while computing. Figlewski and Gao (1999) suggest to replace the original logarithmic asset price X^* by the average mean-adjusted logarithmic asset price ($X \equiv X^*(t) - \alpha t$). In other words, the mean of X^* will be zero at any time. This also implies the early process for the original X is:

$$X_{t+\Delta t} - X_t = \begin{cases} \alpha\Delta t + h, & \text{with } \mathrm{P}_u = 1/6 \\ \alpha\Delta t, & \text{with } \mathrm{P}_m = 2/3 \\ \alpha\Delta t - h, & \text{with } \mathrm{P}_d = 1/6 \end{cases} \tag{27}$$

Therefore, in the condition that the asset price x and maturity date T, the general formula of stock option price can be written as:

$$C(X,t)=e^{-r\Delta t}(P_u(h,\Delta t)\cdot C(X+h,t+\Delta t)+P_m(h,\Delta t)\cdot C(X,t+\Delta t)$$
$$+P_d(h,\Delta t)\cdot C(X-h,t+\Delta t))$$
$$C(X,T)=(e^X-X)^+,\forall X \tag{28}$$

In the above equations, the boundary condition " $(\cdot)^+$ " of the maturity day means the value in the bracket is positive or zero, which is the same with the situation of general lattice model. Note that Eq. (28) allows the probabilities (i.e. P_u, P_m, P_d) would vary with h and Δt, whereas they are fixed in the current case of Eq. (27).

Following, we will describe the application of AMM to European Option (i.e. Plain Vanilla Option). We use Figure 5 to illustrate the fine mesh structure of one-level AMM around strike price at maturity day. We will construct the one-level fine mesh between date T and date T-Δt. In Figure 5, the coarse lattice is the original trinomial tree with price and time steps h and Δt, is denoted by heavy lines. The light lines represent the fine mesh with price step size h/4 and time step size Δt/4. The fine mesh covers all the node of the coarse mesh at time state T-Δt. The starting nodes of the fine mesh include A_2, A_3, A_4, and A_5. In the fine mesh branching from node A_2, X^+ is the highest out-of-the-money node while X^- is the lowest *in-the-money* node. Since all branches starting from nodes below A_1 all end up *in-the-money* and all branches starting from nodes above A_6 are all expired *out-of-the-money*. So there is no need to fine the mesh below node A_1 and above node A_6.

When the lattice model used to evaluate stock option, the nonlinearity error would occur in the date closing to the maturity day. Thus, in the Wiener process, price step h is directly related to the variation duration $\sqrt{\Delta t}$ (i.e.

$h = \varepsilon\sqrt{\Delta t}$). For that reason, while we apply the one-level fine mesh structure for pricing our target asset, the price volatility h and duration length Δt would convert to $h/2$ and $\Delta t/4$, respectively. Besides, the one-level fine mesh will construct between time T and time T-Δt. For two-level mesh, the price volatility h and duration length Δt would convert to $h/4$ and $\Delta t/16$, respectively and it will be construct between time T and time T-$\Delta t/4$. Consequently, if we take M-level fine mesh structure, the corresponding parameters will change to $h_M = h/2^M$ and $\Delta t_M = \Delta t/4^M$, respectively. And it will be constructed between time T and time $T - \frac{\Delta t}{4^{(M-1)}}$. If we increase one level to the lattice, the number of node will increase 52. Even though the *CRR* model and trinomial tree model could achieve to convergence by increasing the segmental number of period n comprehensively, these approaches are not effective enough lime AMM.

For *CRR* model or trinomial tree model, there are $(N+1)^2$ nodes of price computation in total, where N is the number of price step. Therefore, while cutting the price step in half to reduce the nonlinear error, it would lead N become quadrupled (h is directly related to Δt) which implies 16 times computation amount than before. On the other hand, we compare them with AMM. For example, we see the 1-level AMM in Figure 5 and find that we only need to add 40 nodes of price computation to the critical region. (The total number of node of 1-level AMM: 52; The coarse mesh region of 1-level AMM: 12; the fine mesh region of 1-level AMM including the overlap region, then we only need to increase 52-12=40 nodes) On the other hand, 2-level AMM with only 25 time steps, which is much more accurate than a standard trinomial tree with 250 time steps, and only a little less accurate than a 1000 time steps binomial tree which require 250 times greater execution time. Although the binomial tree runs distinctly faster, it is only about half as accurate as the standard trinomial tree and much less accurate than the AMM. Furthermore, the 1-level AMM is about four times as accurate as the standard trinomial tree. The 2-level AMM, with "finer mesh", is even about four times as accurate as the1-level AMM. These descriptions also indicate AMM can reduce the nonlinearity error without sacrificing its efficiency. If we increase more level number M, we will obtain more accuracy. When we increase one level to the lattice, the number of node will only increase 52. It won't add too much computing time to the whole model.

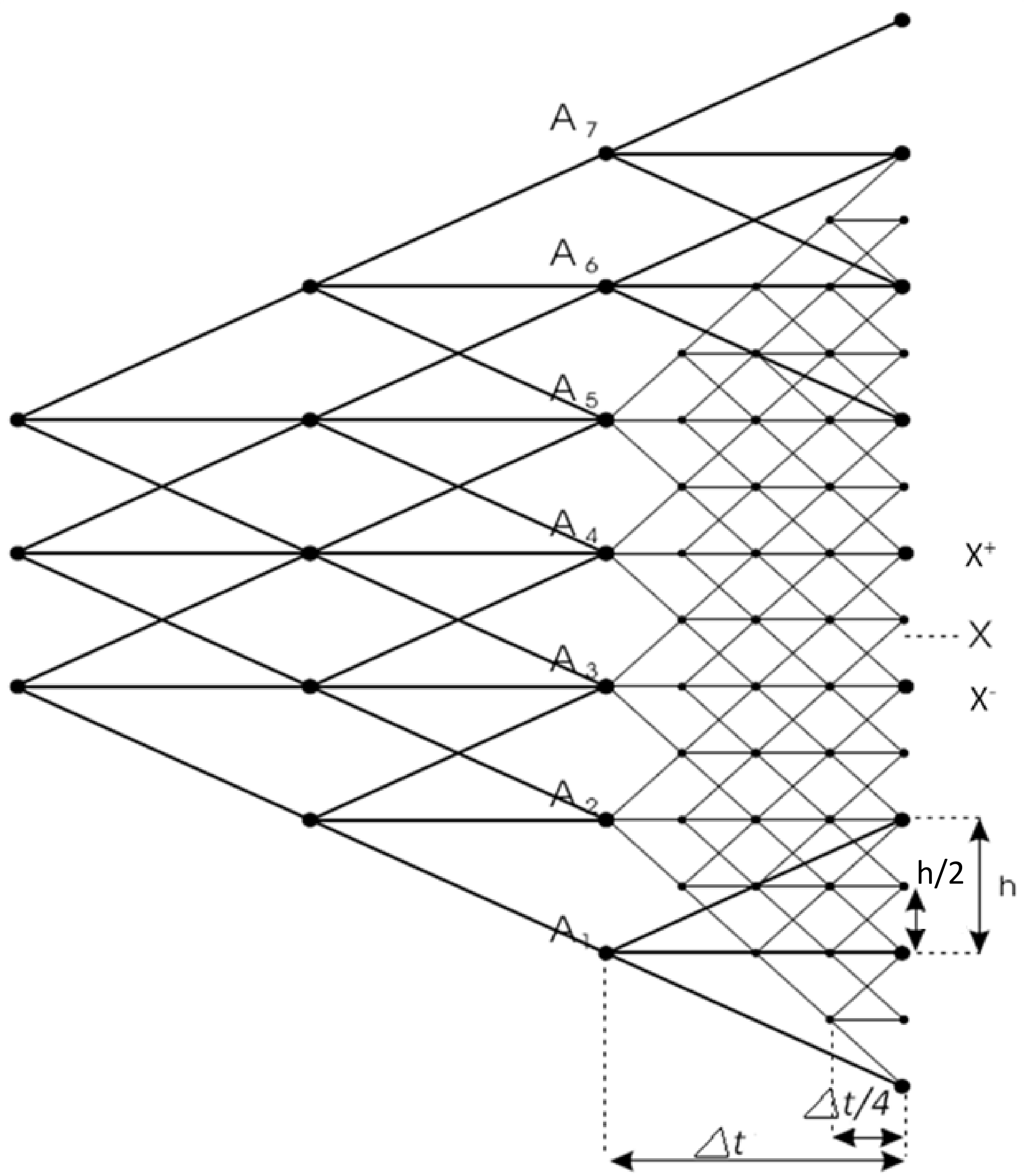

Fig. 5 A one-level AMM for a put option of Plain Vanilla Option around strike price at maturity day.

(II) American option:

For American option, the nonlinearity error is also largely accounted for the error in the last time step. Besides, there is also an approximation error with regard to where the early exercise occurs.

While we use AMM to evaluate American option, we should set up the fine mesh structure around the last several periods' executing prices, using the calculating method of the previous AMM for European option.

We use the AMM lattice in Figure 6 to illustrate. In the coarse mesh, we set the strike price X as the *"center point"* and select the two neighbor asset price X^-(node A_{11}) and X^+(node A_{12}) as the *"critical region"* of the fine mesh structure. In order to achieve the accurate result, Figlewski and Gao (1999) believe that the calculating path of the fine mesh structure should covers the region of *in-the-money* and *out-of-the-money*. Hence the calculating range of the coarse mesh node which connects the fine mesh should extend from (X^-, X^+) to $(X^+ - 2h, X^- + 2h)$. In other word, in the maturity day, we extend the original critical region from (A_{11}, A_{12}) to (A_{10}, A_{13}). For the date *T-Δ t*, the nodes A_2 and A_5 have the same asset price with nodes A_{10} and A_{13} at maturity date. From (A_2, A_5), we also spread their calculating range to $(X_i - 2h, X_i + 2h)$, that is, (A_8, A_{15}). Thus, the whole fine mesh structure is surrounded by the *trapezoid* composed of nodes A_2, A_5, A_{15} and A_8.

When we calculate the American option, we must handle the fine mesh structure first. Its process is similar to the general lattice model. Take subscription to warrant for example, the fine mesh node B_2 in Figure 6, whose warrant price is formed by A_{15}, B_1, and A_{14}:

$$f_{B_2} = e^{-r\Delta t/4}(P_u(h/2, \Delta t/4) \cdot C_{A_{15}} + P_m(h/2, \Delta t/4) \cdot C_{B_1} + P_u(h/2, \Delta t/4) \cdot C_{A_{15}} + P_d(h/2, \Delta t/4) \cdot C_{A_{14}}) \tag{29}$$

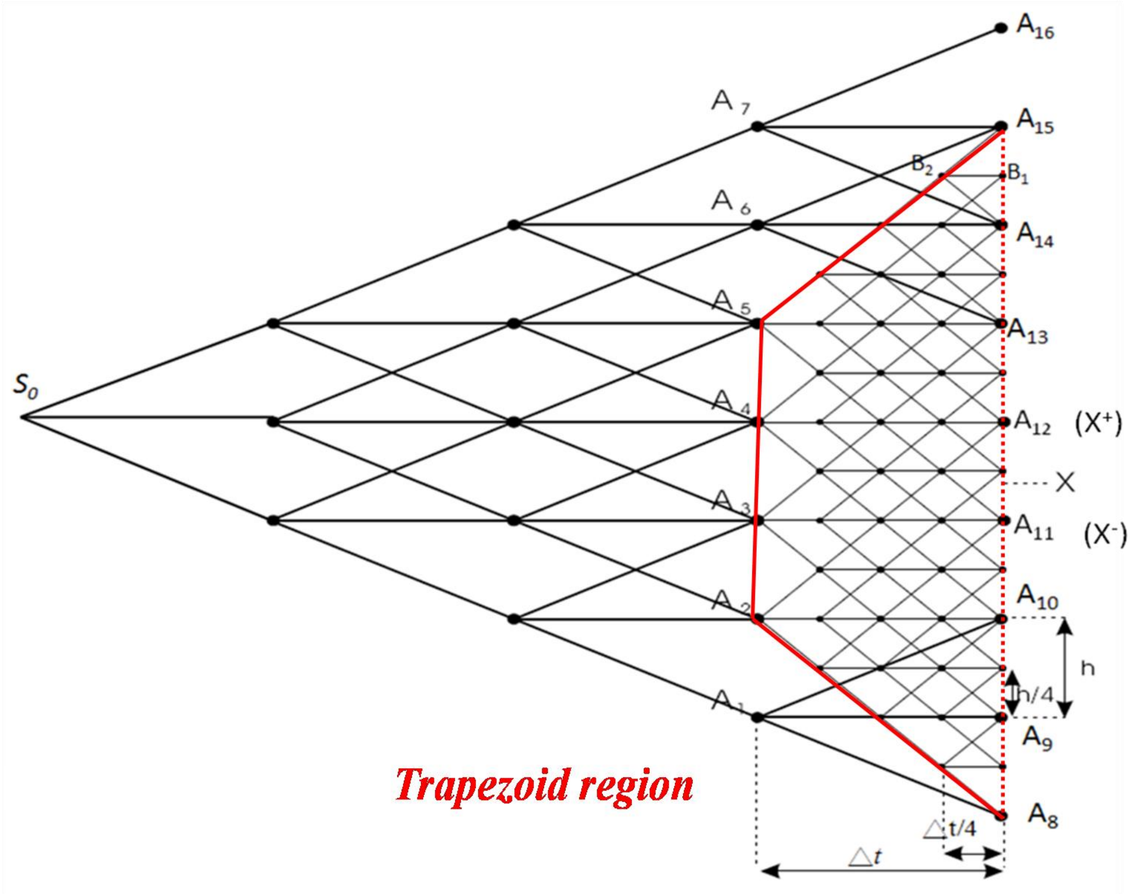

Fig. 6 AMM mesh structure chart.

CHAPTER 3

Methodology

3.1 Empirical Procedure

In this section, we discuss the corresponding assumption, limitation and the operation method of *Modified RT* model (*AMM-RT*). Originated form Adaptive Mesh Model proposed in 1999 by S. Figlewski and B. Gao., we also apply fine mesh structure during the period of (T-1, T). T is the maturity day here. In the *RT* model, the time step during (T-1, T) is 1, that is n=1. The lattice structure of *AMM-RT* is not only based on *RT* model, but also with the idea of AMM. We cut the period of (T-1, T) into m subinterval (we call the segmental level of the last trading day m, i.e. we use m=2, 3, 5 in the thesis). The approach in (T-1, T) possesses the essence of AMM. In the following mentions, we introduce the empirical procedure of *Modified RT* model under the stock option price prediction.

A. Using Original RT tree lattice before period T-1

In this section, we simplify GARCH (1, 1) model and do parameter estimation in the first. We use the assumption in *RT* model and *K*=3 (three variance in each node). The risk free rate r_f is 2.5%. We use the target stock's rate of return and GARCH model with ***out of sample estimation*** to estimate the parameters of our estimating period ω, α, β, λ and the initial variance h_0. For each day of our evaluating period, each day will has itself GARCH parameters (ω, α, β, λ, h_0). Then, for each evaluating day, we set ω, α, β and the variance of rate of returns of asset h_0 as the beginning value to construct *RT* model. The pricing empirical procedure before date T-Δt is shown step by step as follows:

Step1: Let n=1 to construct the trinomial tree and j=1, 0, -1.

Step2: Calculate $\gamma = \sqrt{h_0}$ and $\gamma_n = \frac{\gamma}{\sqrt{n}}$. Since n=1, thus $\gamma = \gamma_n$.

Step3: Using the inequality $\eta - 1 < \frac{\sqrt{h_t}}{\gamma} \leq \eta$ to find the value η.

Step4: Substitute the variance h_t (the initial value of h_t is h_0 and h_0 is known) of this period (day) into formula of h_{t+1} (the variance of the next period under past t period variances have known)

$$h_{t+1} = \omega + \alpha(\varepsilon_t - \lambda\sqrt{h_t})^2 + \beta h_t$$

to find the variance of next period (i.e. h_{t+1}).

Step5: Due to the path dependence issue, the variance of next day will be influenced by the variance of the previous day. With the time increase, the number of path arriving at each node will increase. Thus, the number of variance of each node will increase too. Then, there will be more than one variance in each node. Thus, when we proceed to the next date, we should compare the value of variance of each node in the date. Next, we reserve the maximum and the minimum variance to calculate the η of next date. (That is, we substitute h_t into $\eta - 1 < \frac{\sqrt{h_t}}{\gamma} \leq \eta$ to calculate the η of next date).

We ***REPEAT*** Step4 to Step5 until day ***T*-1**.

Step6: After Step5, we already construct the tree lattice of variance before date ***T*-1**. At the same time, we also construct the tree lattice of probability .

$$p_u = \frac{h_t}{2\eta^2\gamma_n^2} + \frac{(r_f - h_t/2)}{2\eta\gamma_n}$$

$$p_m = 1 - \frac{h_t}{\eta^2\gamma_n^2}$$

$$p_d = \frac{h_t}{2\eta^2\gamma_n^2} - \frac{(r_f - h_t/2)}{2\eta\gamma_n}$$

B. Using AMM-RT Model in the last (T-1, T) period

After using the *RT* lattice structure before *T*-1 period (this is so-called coarse mesh structure in our model), we will apply our modified method to construct the fine mesh structure during last (*T*-1, *T*) period. In the original *RT* model, *n*=1 is used during last (*T*-1, *T*) period. In the following mention, we will use different value of *m* (*m* is the segmental level in the last trading day) in the last period. The modified *RT (AMM-RT)* model with *m=2, 3, 5* will be discussed and compared. For convenience to describe, we show Figure 7 to explain the lattice in the last period. A day is cut into *m* = 3 periods, and the jump size turn to $\eta \cdot \gamma_m = \eta \cdot \frac{\gamma}{\sqrt{3}}$. For one intermediate node at day T-1, $2m + 1$ states at day T follow each state at day T-1.

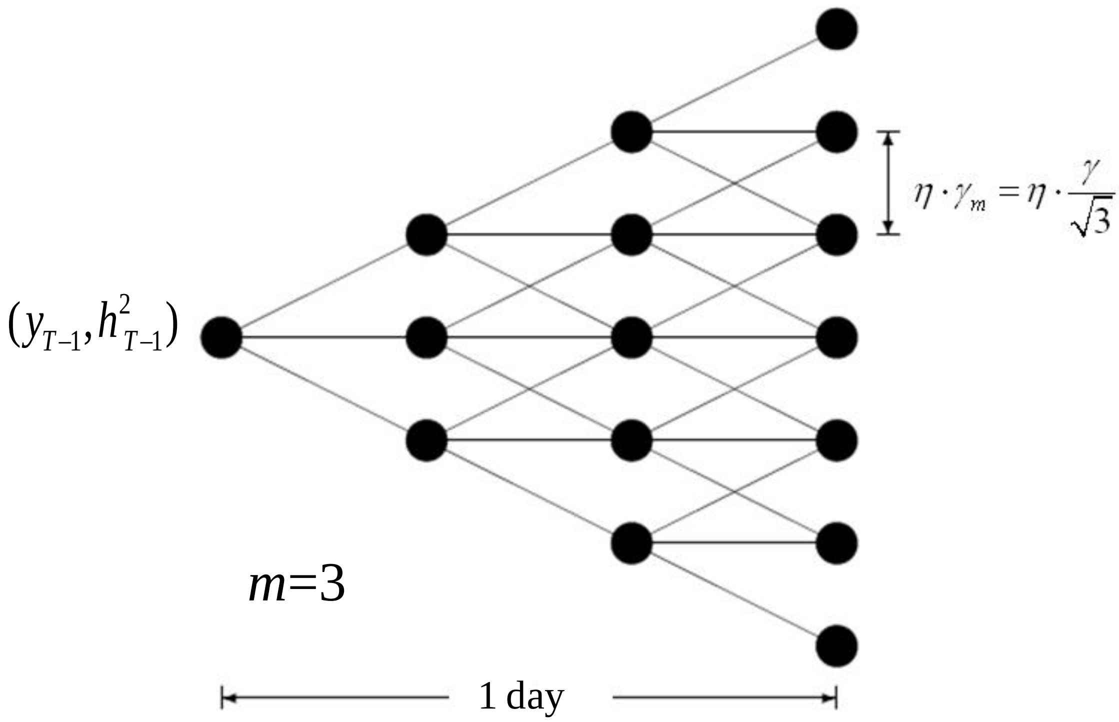

Fig. 7 For node at period T-1, when a period (day) is cut into m = 3 periods, and the jump size is $\eta \cdot \gamma_m = \eta \cdot \frac{\gamma}{\sqrt{3}}$

In our *AMM-RT* model, we only need to add moderate node in the last during last (T-1, T) period. This will not cost much computing amount as well as increase pricing accuracy. Although AMM also increase its node in the last time step (for *1-level* AMM, we increase the mesh density in the last (T-1, T) period), AMM can't capture more complete price probabilistic distribution function and the conditional variance. We have introduced AMM algorithm in Chapter 2. The probability distribution of AMM tree lattice is fixed and the price step and time step are also fixed too. Even though the fine mesh in the last time step increase the accuracy of pricing, it seems not to be enough efficiently. For our *AMM-RT* model, the probabilistic functions are not only non-fixed but also the variance of each time step update with time. This will capture more information of target asset than AMM and achieve more accuracy at the same time. The *AMM-RT* model cut the last period (day) into m subintervals (i.e. increase m), and thus, the discrete distribution of lattice model will more approach to the continuous distribution of lattice model. With m increasing, the distribution error decreases. Besides, the nonlinearity error will occur when *RT* model applying in some exotic option. For example, when barrier option approaches to the barrier price, the nonlinearity error occurs. I suggest our *AMM-RT* model with the same essence as AMM will be able to price this type option.

The procedure in the (T-1, T) will be shown as following Step7~Step14. We assume m=2, 3, 5 in the (T-1, T).

Step7: In the Step 7, we increase m to add the mesh density.

Let m=2, 3, 5, respectively to construct the trinomial tree and $j=0,\ \pm 1,\ \pm 2,\ldots,\ \pm m$

Step8: Calculate $\gamma=\sqrt{h_0}$ and $\gamma_m=\dfrac{\gamma}{\sqrt{m}}$. m=2, 3, 5.

Step9: Using the inequality $\eta-1<\dfrac{\sqrt{h_{T-1}}}{\gamma}\le\eta$ to find the value η of last period (day) T.

Step10: Substitute the variance h_{T-1} into formula of h_{t+1}

$$h_{t+1}=\omega+\alpha(\varepsilon_t-\lambda\sqrt{h_t})^2+\beta h_t$$

to find the variance of next period (i.e. h_T).

Step11 After Step 10, we already construct the variance tree lattice. At the same time, we also construct the tree lattice of probability of last period (day).

$$p_u = \frac{h_t}{2\eta^2\gamma_n^2} + \frac{(r_f - h_t/2)\sqrt{1/m}}{2\eta\gamma_n}$$

$$p_m = 1 - \frac{h_t}{\eta^2\gamma_n^2}$$

$$p_d = \frac{h_t}{2\eta^2\gamma_n^2} - \frac{(r_f - h_t/2)\sqrt{1/m}}{2\eta\gamma_n}$$

Step12: We construct the tree lattice of stock price.

For period (0, T-1):

$$y_{t+1} = y_t + j\eta\gamma, \quad j = 0, \pm 1, \pm 2, \ldots, \pm n$$

For period (T-1, T):

$$y_T = y_{T-1} + j\eta\gamma_m, \quad j = 0, \pm 1, \pm 2, \ldots, \pm m$$

Step13: Calculate the stock option price; this price at every node should be the same.

$$C_T^{\max}(i) = C_T^{\min}(i) = Max\{0,\ S_T(i)\text{-}X\}$$

Step14: After the Step13, we apply the backward recursion and discount, and then we can get the stock option price at day *t=0*. Using the equations recursively as follow:

$$h_t^{next}(j) = \omega + \beta h_t + \alpha h_t\ [\frac{(j\eta\gamma_n - (r_f - \frac{1}{2}h_t^a(i,k))}{\sqrt{h_t^a(i,k)}} - \lambda]^2 \quad ; j = 0, \pm 1, \pm 2, \ldots, \pm n \text{ (or } m)$$

$$C^{\text{interp}}(j) = q(j)C_{t+1}^a(i + j\eta, L) + (1 - q(j))C_{t+1}^a(i + j\eta, L+1)$$

$$C_t^a(i,k) = e^{-r_f}\sum_{j=-n}^{n} P(j)C^{\text{interp}}(j), \quad j = 0, \pm 1, \pm 2, \ldots, \pm n \text{ (or } m)$$

If we increase m, the distribution error will decrease. Furthermore, when the lattice model used to evaluate stock option, the nonlinearity error would occur in the date closing to the maturity day. Thus we only cut the time step in the last period to track the asset price and reduce the nonlinearity. If the segmental level m is larger in the period (T-1, T), we can obtain more accuracy. Besides, it won't add too much computing time to the whole model.

CHAPTER 4

Numerical Illustration

4.1 Data Analysis

To examine the empirical performance of the GARCH option pricing model, we applied the model to daily closing prices of the Taiwan Stock Exchange Capitalization Weighted Stock Index (TAIEX) and its corresponding TAIEX options. For simplicity, we will just consider the call options here. We use the index and its corresponding options based on the following consideration. The first reason is that the index and the option data are freely available on the websites. Furthermore, the TAIEX index option is the most actively traded European-style option in Taiwan. Thus, the TAIEX option market is chosen to test the empirical performance of the Black-Scholes model, *RT* model and *AMM-RT* model. In next section, we will focus to estimate the call option price in September 2007 (2007/9/3~ 2007/9/31, 18 trading days). We use the TAIEX index with the sample period from September to December 2007 (past 5 years) to establish the GARCH volatility dynamic. There are 1239 observations.

4.2 Numerical Analysis

In this thesis, we will apply our *AMM-RT* trinomial lattice model to price the stock option price. First, we should estimate the parameters of GARCH model under *P* measure. We use TAIEX as our approximating samples. Here, we choose TAIEX index with the sample period from September 2, 2002 to August 31, 2007 as estimative period of GARCH model. For example, the call option price of 2007/9/3 will be estimated under the estimative period “5 years” prior to this day (i.e. 2002/9/2~2007/8/31). Following, we use rolling sample method to estimate the subsequent parameters of GARCH model. Fig. 8 shows the daily observations of TAIEX during 2002/9/2 – 2007/12/31.

Based on Bakshi, Cao, and Chen (1997), Duan and Zhang (2001), we define a call option is said to be at-the-money if the moneyness is between (1.00, 1.03), in-the-money if the moneyness is between (1.03, 1.06), out-of-the-money if the moneyness is between (0.94, 0.97) and deep in-the-money if the moneyness is greater than1.06 and deep out-of-the-money if the moneyness is less than 0.94. We amputate the data whose moneyness greater than 1.1 or smaller than 0.9, because the volume if trade of them are small. Table 1 provides the average and standard deviation of call option prices reported for each moneyness category, and also shows the numbers of observations in these categories for the period from September 1, 2007 to September 31, 2007 in Figure 8.

Table 1 Summary Statistics for TAIEX Call Options (September)*

	Moneyness (S/K)					
	DOTM	OTM	ATM1	ATM2	ITM	DITM
	<0.94	0.94–0.97	0.97-1	1-1.03	1.03-1.06	>1.06
Average	37.343	101.557	197.963	336.559	507.471	725.149
Std. Dev.	33.386	57.311	67.933	79.198	70.121	87.551
Number	96	61	54	59	51	67
Sum			388			

*The summary statistics of TAIEX call option near closing prices are reported for each moneyness category. Moneyness is defined as *S/K*, where *S* denotes the closing value of the TAIEX and *K* denotes the exercise price of the option. The sample period is from September 1, 2007 to September 31, 2007 with a total of 559 call options.

For the selection of option data, we amputate the trading days which are less than 7 days (since the volatility is large) and more than 40 days (since the volume of trade is small) away from the estimated trading day.

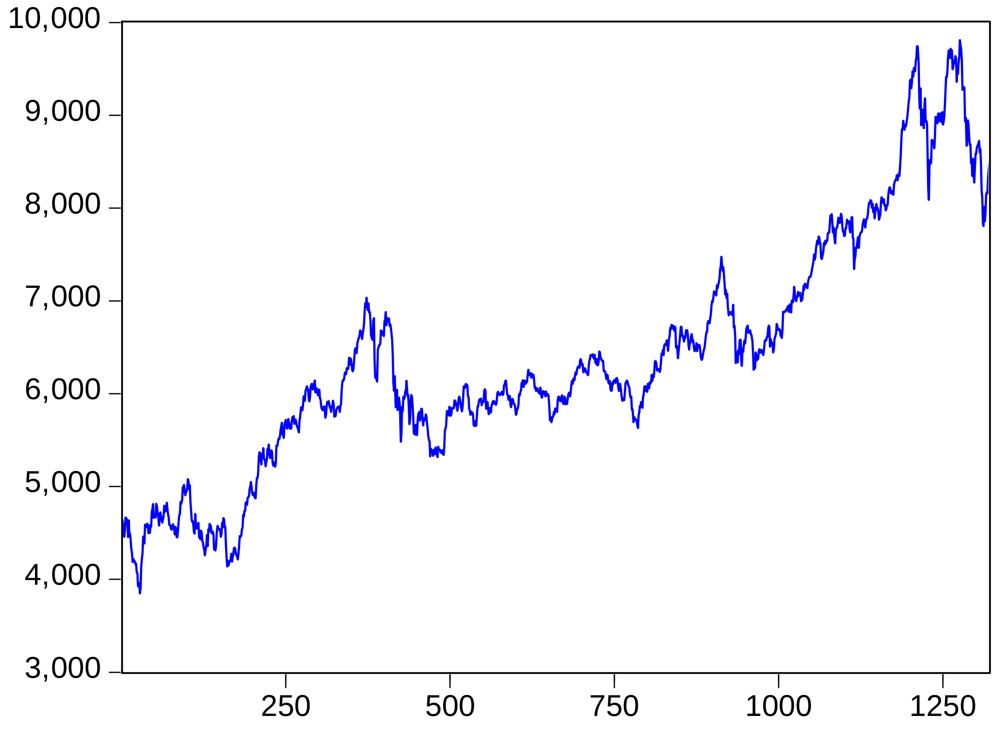

Figure 8. TAIEX during 2002/9/2–2007/12/31, 1321 daily observations.

From Figure 8, we find that the TAIEX trend appears buoyancy during 2002~2007. The index rises from 4644.58 (2002/9/2) to 8982.16 (2007/8/31) and subsequently has intense vibration. The index is 8506.28 at 2007/12/31. We also show the rate of return (log return) of TAIEX during 2002/9/2 ~ 2007/12/31 in Figure 9.

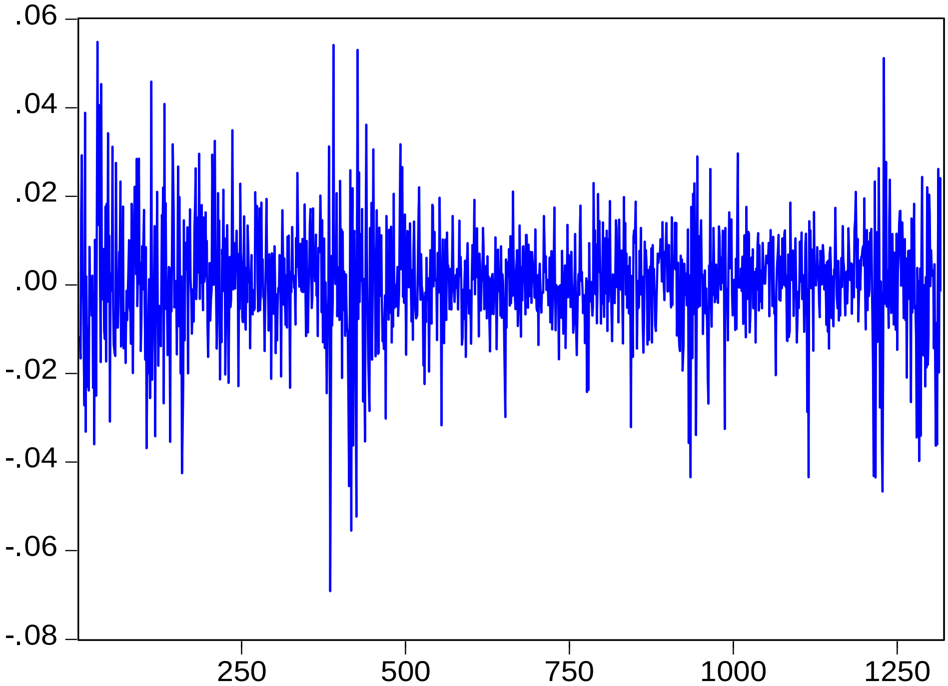

Fig. 9 Rate of return (log return) of TAIEX during 2002/9/2–2007/12/31 with 1320 daily observations. It is noted that the observation will lessen 1 after selecting the log return.

Volatility clustering is also observed in the Figure 9, a large value tends to follow by another large value. This is known as the conditional heteroscedasticity. Thus this data is suitable to be analyzed by GARCH option pricing model.

We also show the relative statistics of TAIEX in Table 2.

Table 2 The elemental statistic of TAIEX during 2002/9/2–2007/12/31 with 1320 daily observations.

Statistics	
Mean	0.000458
Median	0.000467
Maximum	0.054845
Minimum	-0.06912
Std. Dev.	0.012714
Skewness	-0.29432
Kurtosis	5.91097

From Table 2, the average rate of return is positive, which also indicates the trend between 2002/9/2–2007/12/31 appears buoyancy. The rate of return appears to shift to left (the skewness is negative) and possesses "fat tail", which also accords with the characteristic of the rate of return of Index.

We also show the estimated parameters of the GARCH model under *P measure* in Table 3.

Table 3 the estimation of the GARCH model under *P Measure* (2002/9/2 – 2007/8/31, 1239 observations)

$$\ln\left(\frac{S_{t+1}}{S_t}\right) = r_f - \lambda\sqrt{h_t} - \frac{1}{2}h_t + \varepsilon_t,\ \ \varepsilon_t \mid \phi_{t-1} \sim N(0, h_t)$$

$$h_{t+1} = \omega + \alpha\varepsilon_t^2 + \beta h_t$$

Estimated parameter	
λ	-0.076 (-2.724)
ω	2.53×10^{-6} (3.166)
α	0.071 (8.293)
β	0.914 (80.194)

*The value in the bracket is the "*t*" value, which used to evaluate the option price of 2007/9/3. We still need to estimate the parameters of the GARCH model again using rolling sample method when we evaluate the forthcoming days' option price (2007/9/4~2007/9/31).

Figure 10 shows the implied volatility of fitted GARCH model (not implied volatility for option). By comparing Figure 10 (the estimating data by GARCH model) with Figure 9 (the true data), the result indicates GARCH model can capture the characteristic of time-varying of volatility.

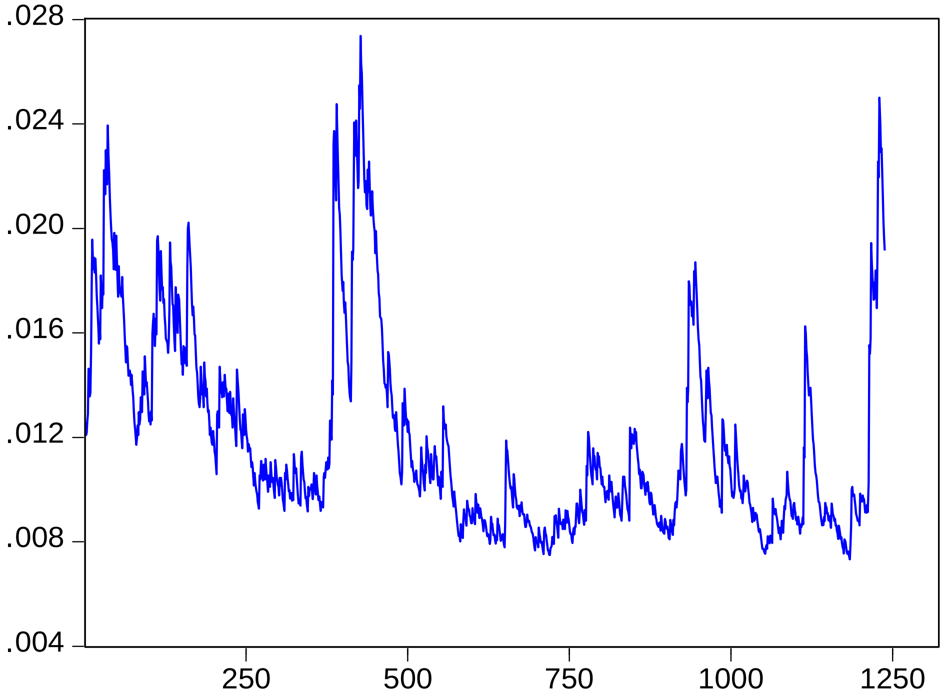

Fig. 10 The implied volatility of fitted GARCH model (2002/9/2 – 2007/8/31, 1239 observations).

Even though Figure 10 shows the implied volatility of fitted GARCH model, the situation of volatility is *in the sample*. However, this thesis focuses on the viewpoint of *out of sample*. We are interested in the implied volatility of the fitted GARCH model out of sample. In other words, we stand on the viewpoint of future estimating to check the applicability of GARCH model.

It is noted that the when we estimate the variance of rate of return *out of sample*, we should estimate the volatility of each trading day (we want to estimate) until maturity day. Based on the LRNVR of Duan (1995), the estimation of variance won't change with the measure situation, thus, we only need to apply simply GARCH (1, 1) to estimate the variance out of sample. About our out of sample estimation, we use the estimated data from 2002/9/2 to 2007/8/31 to appraise the parameters during 2007/9/3~2007/9/31.

We discuss out of sample estimation as follows. The 1st period out of sample estimation for the variance of GARCH (1, 1) is:

$$h^f_{t,t+1} = E(h_{t+1} \mid \phi_t) = \hat{\omega} + \hat{\alpha}\varepsilon_t^2 + \hat{\beta}h_t$$

The 2nd period out of sample estimation is:

$$\begin{aligned} h^f_{t,t+2} &= E(h_{t+2} \mid \phi_t) = \hat{\omega} + \hat{\alpha}E(\varepsilon_{t+1}^2 \mid \phi_t) + \hat{\beta}E(h_{t+1} \mid \phi_t) \\ &= \hat{\omega} + \hat{\alpha}E(h_{t+1} \mid \phi_t) + \hat{\beta}E(h_{t+1} \mid \phi_t) \\ &= \hat{\omega} + (\hat{\alpha} + \hat{\beta})\hat{\omega} + (\hat{\alpha} + \hat{\beta})^2 h_t \end{aligned}$$

The general formula of k^{th} period out of sample estimation can be rewritten as:

$$h^f_{t,t+k} = E(h_{t+k} \mid \phi_t) = \frac{\hat{\omega}(1-(\hat{\alpha}+\hat{\beta})^k)}{1-(\hat{\alpha}+\hat{\beta})} + (\hat{\alpha}+\hat{\beta})^k h_t$$

, where $h^f_{t,t+k}$ denotes estimation of the $t+k$ period conditional variance when we have known the preceding t periods' information.

Base on the approximated GARCH model of Table 3, the volatility route out of sample is shown in Figure 11. Because we only discuss the option which is at most 40 days away from its maturity day (according to the trading day), we only show 40-period volatilities in the Figure 11.

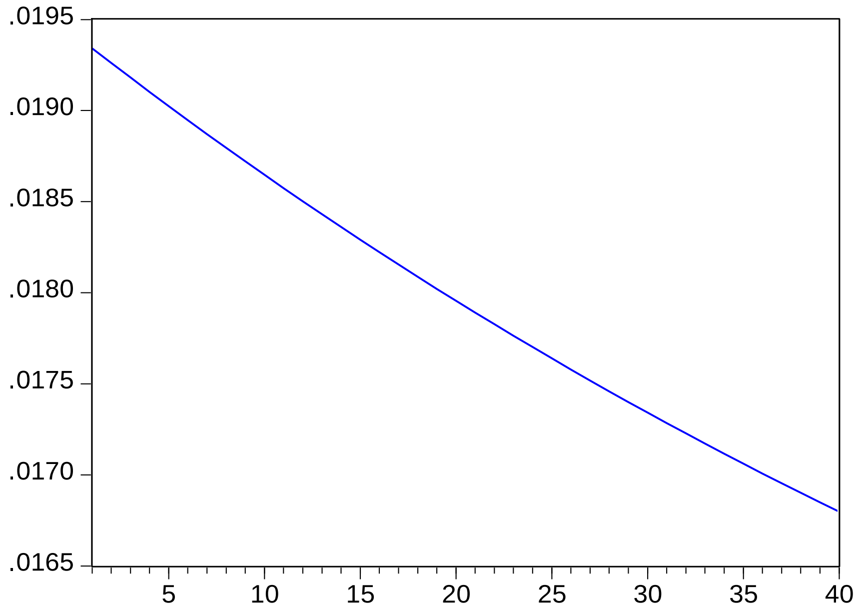

Fig. 11 Out of sample GARCH volatility (2007/9/3 – 2007/11/1, 40 observations)

And then, we use" *rolling sample*" method to estimate the parameters of the GARCH model. In other words, we can use TAIEX index with the sample during 2002/9/3~ 2007/9/3 (past 5 years) to estimate the option price of 2007/9/4. The number of "*rolling sample*" during the estimative period (2007/9/3 ~ 2007/9/31) will be fixed under this frame. For our case, we will estimate until September 31, 2007.

Finally, we can rewrite the above mentioned model under *Q Measure* as

$$\ln\left(\frac{S_{t+1}}{S_t}\right) = r_f - \frac{1}{2}h_t + \varepsilon_t,\ \varepsilon \mid \phi_{t-1} \sim N(0, h_t)$$

$$h_{t+1} = \omega + \alpha(\varepsilon_t - \lambda\sqrt{h_t})^2 + \beta h_t$$

Then we estimate option price during 2007/9/3~2007/9/31 under *Q Measure.*

In this thesis, BS model is one of the chosen models comparing with our *AMM-RT* model. We use the historical volatility as BS model's volatility. We allow different volatilities for different lengths of time to maturity: past one month (22 days), past half year (122 days) and past one year (243 days) data to calculate BS model's historical volatility (Using rolling sample method). We plot the historical volatility in Figure 12. It is noted that, the volatility in Figure 12 ignores the fact that volatility will change with time.

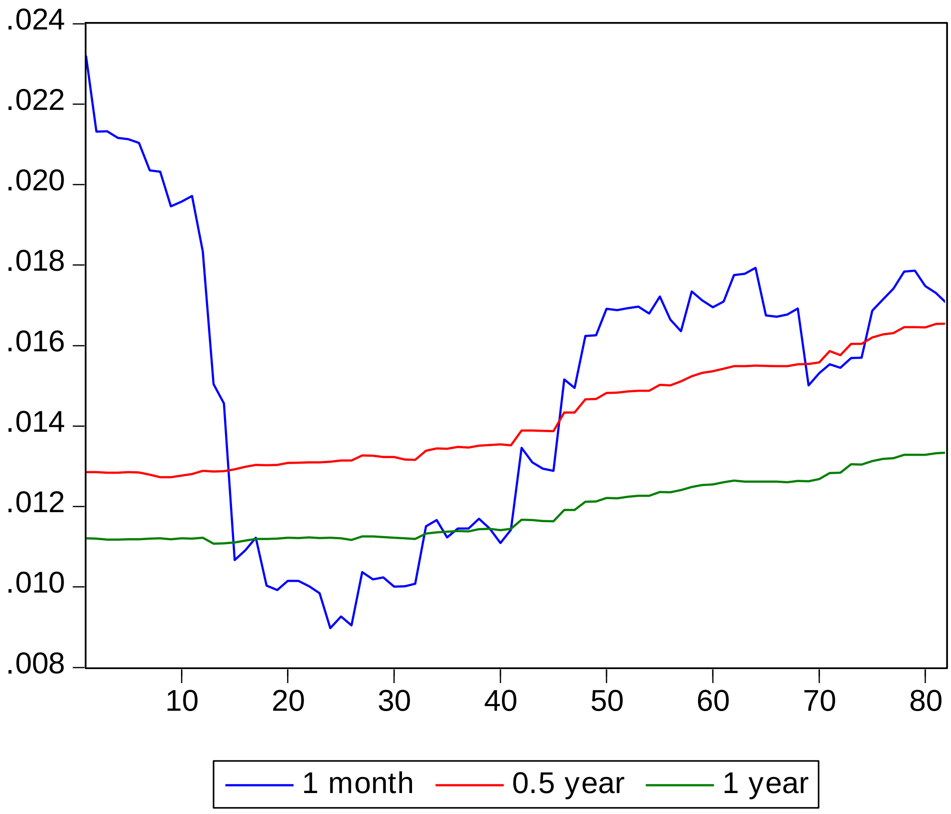

Fig. 12 Historical volatility (2007/9/3 – 2007/12/31, 81 observations). Because we only evaluate option price at September 2007, this plot needs to be modified. (We only choose first 18 days in the plot)

In order to further examine *AMM-RT* model's performance, we conduct a numerical simulation and then empirically examine its performance on the pricing of the call warrants in Taiwan Stock Exchange. To affirm our *AMM-RT* model's performance and practicability, we also compare our model with the well-behaved *TBS* model (Trinomial Black-Scholes model). Chou and Wang proposes the Trinomial Black-Scholes （*TBS*）GARCH option pricing algorithm in 2007, which graft Black and Scholes model on RT trinomial lattice algorithm. *TBS* model use Ritchken and Trevor's algorithm in the n-1 periods whiling utilizing the BS model in the last period.

Table 4 The TXO estimative performance comparisons between Ritchken-Trevor GARCH option model (*RT*), Modified Ritchken-Trevor GARCH option mode (*AMM-RT*), different type Black-Scholes option model (*BS*) and well-behaved Trinomial Black-Schole model (*TBS*).

$$RMSE = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(\frac{\hat{C}_i - C_i}{C_i}\right)^2}$$

	Moneyness (S/K)						
	DOTM <0.94	OTM 0.94–0.97	ATM1 0.97-1	ATM2 1-1.03	ITM 1.03-1.06	DITM >1.06	Overall
BS(σ_1)	0.528	0.457	0.895	2.153	3.954	6.203	3.131
BS(σ_2)	0.624	0.509	1.309	2.642	4.417	6.754	3.446
BS(σ_3)	0.742	0.565	1.434	2.793	4.581	6.930	3.555
RT(n=1)	0.447	0.459	0.564	0.779	1.556	1.856	1.167
RT(n=2)	0.421	0.432	0.509	0.722	1.333	1.622	1.089
AMM-RT (n=1, m=2)	0.427	0.446	0.528	0.744	1.389	1.711	1.153
AMM-RT (n=1, m=3)	0.417	0.431	0.514	0.720	1.344	1.631	1.108
AMM-RT (n=1, m=5)	**0.408**	**0.423**	**0.485**	**0.711**	**1.287**	**1.492**	**1.033**
TBS (n=1)	**0.406**	**0.422**	**0.485**	**0.710**	**1.252**	**1.335**	**1.022**

In Table 4, we demonstrate the *Modified RT* model (*AMM-RT*) applying for the prediction of TXO and make a completed comparison of *AMM-RT* model with original *RT* model and BS model with different σ both in efficiency and accuracy. The numerical evaluating results indicate the *AMM-RT* model is generally suitable to price other exotic options.

In the following, we use root-mean-square relative error (RMSE) to measure the accuracy. The *RMSE* error is defined as

$$RMSE = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(\frac{\hat{C}_i - C_i}{C_i})^2} \tag{30}$$

where C_i is the mean of true option price used as our benchmark, $\hat{C}_i$ is the evaluating price applying the different models and N is the option number of similar contracts except for the parameters or variable are changed. First, we use traditional BS model with different volatility to evaluate option price. We choose three different kind of volatilities (one month, half year and one year prior to maturity day) and substitute to BS model. It is because volatility can be observed directly, thus, we should choose some substitutive amount to express the volatility of one period. For the *RT* model, we choose different segmental level (i.e. n=1 and n=2) in one trading day. In the *AMM-RT* model, except for the last day, the segmental period of other trading day is 1 and the segmental periods of the last trading day are 2, 3, 5, respectively (i.e. m=2, m=3, m=5). From Table 4, we know the *AMM-RT* and *RT* model are both better than BS model. Although *RT* model with n=2 is bitty better than *AMM-RT* model with m=2, the complexity of computing for *AMM-RT* model (m=2) is effectively reduced. *AMM-RT* model with m=5 is significantly better than *RT* model with n=2, while use less amount of computation. These results suggest *AMM-RT* with m=5 behave best accuracy as well as efficiency, compared with traditional *RT* model. For further investigation, we compare *AMM-RT* model with the *TBS* model. Our *AMM-RT* model with m=5 can achieve comparable accuracy, comparing with well-behaved *TBS* model without scarifying much efficiency. These discussions will be shown in the following.

To furthermore investigate the performance for literature models, we use $\ln(\frac{MSE_{\text{model-B}}}{MSE_{\text{model-A}}})$ to express the relative accuracy between two different models. We show the comparing results in Table 5~Table 12. If the value of $\ln(\frac{MSE_{\text{model-B}}}{MSE_{\text{model-A}}})$ is negative, then this result indicates model A shows better accuracy than model B. Besides, its absolute value of this negative value is larger, model A have better accuracy than model B. From the point of view, we can easily see the superiority of model A, which we intend to demonstrate.

Table 5 *AMM-RT* (*m*=2) and BS model (with σ_1, σ_2, σ_3)

	Moneyness (S/K)					
	DOTM	OTM	ATM1	ATM2	ITM	DITM
	<0.94	0.94–0.97	0.97-1	1-1.03	1.03-1.06	>1.06
$\ln(\frac{MSE_{AMM-RT-2}}{MSE_{BS-1}})$	-0.212	-0.024	-0.561	-1.063	-1.046	-1.288
$\ln(\frac{MSE_{AMM-RT-2}}{MSE_{BS-2}})$	-0.379	-0.132	-0.908	-1.267	-1.569	-1.373
$\ln(\frac{MSE_{AMM-RT-2}}{MSE_{BS-3}})$	-0.553	-0.237	-0.999	-1.323	-1.193	-1.399

Table 6 *AMM-RT* (*m*=3) and BS model (with σ_1, σ_2, σ_3)

	Moneyness (S/K)					
	DOTM	OTM	ATM1	ATM2	ITM	DITM
	<0.94	0.94–0.97	0.97-1	1-1.03	1.03-1.06	>1.06
$\ln(\frac{MSE_{AMM-RT-3}}{MSE_{BS-1}})$	-0.236	-0.056	-0.554	-1.095	-1.079	-1.517
$\ln(\frac{MSE_{AMM-RT-3}}{MSE_{BS-2}})$	-0.403	-0.164	-0.935	-1.312	-1.191	-1.421
$\ln(\frac{MSE_{AMM-RT-3}}{MSE_{BS-3}})$	-0.577	-0.268	-1.026	-1.356	-1.226	-1.447

Table 7 *AMM-RT* (*m*=5) and BS model (with σ_1, σ_2, σ_3)

	Moneyness (S/K)					
	DOTM	OTM	ATM1	ATM2	ITM	DITM
	<0.94	0.94–0.97	0.97-1	1-1.03	1.03-1.06	>1.06
$\ln(\frac{MSE_{AMM-RT-5}}{MSE_{BS-1}})$	-0.258	-0.077	-0.613	-1.108	-1.123	-1.425
$\ln(\frac{MSE_{AMM-RT5}}{MSE_{BS-2}})$	-0.425	-0.185	-0.993	-1.313	-1.233	-1.511
$\ln(\frac{MSE_{AMM-RT-5}}{MSE_{BS-3}})$	-0.598	-0.289	-1.084	-1.368	-1.271	-1.536

Table 8 *AMM-RT* (*m*=2) and *RT* (*n*=1 and *n*=2)

	Moneyness (S/K)					
	DOTM	OTM	ATM1	ATM2	ITM	DITM
	<0.94	0.94–0.97	0.97-1	1-1.03	1.03-1.06	>1.06
$\ln(\frac{MSE_{AMM-RT-2}}{MSE_{RT-1}})$	-0.046	-0.029	-0.066	-0.046	-0.114	-0.082
$\ln(\frac{MSE_{AMM-RT-2}}{MSE_{RT-2}})$	0.014	0.031	0.036	0.030	0.041	0.053

Table 9 *AMM-RT* (*m*=3) and *RT* (*n*=1 and *n*=2)

	Moneyness (S/K)					
	DOTM	OTM	ATM1	ATM2	ITM	DITM
	<0.94	0.94–0.97	0.97-1	1-1.03	1.03-1.06	>1.06
$\ln(\frac{MSE_{AMM-RT-2}}{MSE_{RT-1}})$	-0.069	-0.061	-0.093	-0.079	-0.146	-0.129
$\ln(\frac{MSE_{AMM-RT-2}}{MSE_{RT-2}})$	-0.0096	-0.0023	0.0097	-0.0027	0.0082	0.0053

Table 10 *AMM-RT* (*m*=5) and *RT* (*n*=1 and *n*=2)

	Moneyness (S/K)					
	DOTM	OTM	ATM1	ATM2	ITM	DITM
	<0.94	0.94–0.97	0.97-1	1-1.03	1.03-1.06	>1.06
$\ln(\frac{MSE_{AMM-RT-2}}{MSE_{RT-1}})$	-0.092	-0.083	-0.151	-0.092	-0.190	-0.218
$\ln(\frac{MSE_{AMM-RT-2}}{MSE_{RT-2}})$	-0.032	-0.021	-0.048	-0.0154	-0.035	-0.084

Table 11 *AMM-RT* (m=2, 3, 5) and *TBS*

	Moneyness (S/K)					
	DOTM	OTM	ATM1	ATM2	ITM	DITM
	<0.94	0.94–0.97	0.97-1	1-1.03	1.03-1.06	>1.06
$\ln(\frac{MSE_{AMM-RT-2}}{TBS})$	0.05	0.054	0.084	0.046	0.103	0.248
$\ln(\frac{MSE_{AMM-RT-3}}{TBS})$	0.026	0.021	0.058	0.013	0.071	0.185
$\ln(\frac{MSE_{AMM-RT-5}}{TBS})$	0.0049	0.0023	0	0.0014	0.027	-0.11

From Table 5~Table 7, we can find our *AMM-RT* model behave markedly accuracy than *BS* model, whose variances are obtained by "out of sample estimation". It is very important to note that *BS* model can behave best accuracy while its variances are determined; however, in this thesis, the variances using in *BS* model are estimated. Thus, the non-determined variances would lead to very poor accuracy, while they are used in *BS* model.

From Table 8~Table 10, we also can see *AMM-RT* model could achieve better accuracy and efficiency than traditional *RT* model, which attest the theory and facts discussed in the former part.

Finally, in Table 11, although our *AMM-RT* model cannot achieve better accuracy than the well-behaved *TBS* model, the *AMM-RT* model also behaves the very close accuracy in comparison with TBS model. It indicates *AMM-RT* model is also an attracting and promising candidate for GARCH option pricing.

CHAPTER 5

Conclusion

Rithcken and Trevor (1999) develop the *RT* trinomial tree that demonstrates the stock option price can be computed when the underlying stock price is driven using GARCH process.

The stock markets provide a framework for investors to allocate their funds into stocks, and try to make profits by buying "under valued" stocks and selling "over valued" stocks. Stock markets are one of the most complex and rewarding systems to economics model accurately. Since their incorporation into the latter part of the 20th century, there have been a vast number of different techniques to predict their future behavior.

This thesis extend *RT* model and modify its last time step to obtain higher accuracy and efficiency. It is noted that many exotic option will confront nonlinear error around the maturity day, close to the barrier price, ect. We well know the AMM approach can solve this problem by applying higher resolution fine mesh to where nonlinearity error occurs. We utilize this idea of AMM in concert with the lattice algorithm of *RT* model. We apply fine mesh structure during the period of (T-1, T) in the original *RT* model. T is the maturity day here. In the RT model, the time step during (T-1, T) is 1, that is m=1. In this thesis, we divide the period of (T-1, T) into more time step (we call the segmental level of the last trading day m, i.e. m=2, 3, 5). On other hand, if we increase the segmental number of the last period (day) of the lattice model (i.e. increase m), the discrete distribution of lattice model will approach to the continuous distribution of lattice model. With m increasing, the distribution error decreases. By this modified RT model or *AMM-RT* model, the complexity of computing will be obviously reduced and we also decrease the distribution error and nonlinearity error as well as increase the accuracy. However, I only apply the *AMM-RT* model in European option to test its feasibility. For the future direction, I suggest *AMM-RT* model can be applied in American option, barrier option and other exotic options. I think this model will also work well in the upcoming novel financial commodities. Furthermore, we also can compare this model with M-level AMM for the future work.

Bibliography

[1] 巫春州 (2002), 「認購權證價格行爲之實證研究」, 管理學報, 第十九卷第四期, 頁759-779.

[2] 巫春州 (2002), "The GARCH Option Pricing Model Modification, Application and Empirical Study", *National Central University, Ph. D. Dissertation*.

[3] 李怡宗, 劉玉珍, 李健璋 (1999), 「Black-Scholes評價模型在台灣認購權證市場之實證」, 管理評論, 第十八卷第三期, 頁83-104.

[4] 李存修 and Ching Yang (2001), "An Empirical Analysis of the Market Structure and the Price Behavior of Warrants: The Case of Taiwan", 台灣財務金融季刊第一輯第二期, 頁89-101.

[5] 吳佳純 (2001), 「GARCH選擇權評價模型之探討」，國立中正大學應用數學研究所, *Master Thesis*.

[6] 周恆志, 王功亮 (2007), 「TRINOMIAL BLACK-SCHOLES 選擇權演算法之模擬與實證」, 商管科技期刊, 第八卷第一期, 頁91-115.

[7] 徐守德, 官顯庭, 黃玉娟 (1998), 「台股認購權證訂價之研究」，管理評論，第十七卷第二期, 頁759-779.

[8] 游達錦 (2004), 「GARCH選擇權樹狀圖演算法在台指選擇權的應用」, 銘傳大學, *Master Thesis*.

[9] 黃巧婷 (2000), 「GARCH選擇權評價模型理論與應用」, 國立台灣大學財務金融研究所,*Master Thesis*.

[10] Amin, K., and V. Ng (1993), "ARCH Progresses and Option Valuation", *Working Paper, University of Michigan*.

[11] Black, F., and M. Scholes (1973), "The Pricing of Options and Corporate Liabilities", *Journal of Political Economics*, 81, pp. 637-659.

[12] Black, F. (1975), "Fact and Fantasy in the Use of Options", *Journal of Financial Analysis*, 31, 36-41, 61-72.

[13] Engle, R. (1982), "Autoregressive Conditional Heteroskedasticity, with Estimates of the Variance of U. K. Inflation", *Econometrica*, 50, pp. 987-1008.

[14]Bollerslev, T. (1986), "Generalized Autoregressive Conditional Heteroskedasticity", Journal of Econometrics, 31, pp. 307-327.

[15] Barone-Adesi, G., and R. E. Whaley (1987), "Efficient Analytical Approximation of American Option Values", *Journal of Finance*, 42. pp. 301-320.

[16] Bate, D. S. (1996), "Testing Option Pricing Models", *Unpublished manuscript*, The Wharton School of the University of Pennsylvania.

[17] Bakshi, G., Cao, C., and Chen, Z, (1997), "Empirical Performance of alternative option pricing models", *Journal of Finance*, Dec., pp. 2003-2049.

[18] Cox, J., (1975), "Notes on Option Pricing I, Constant Elasticity of Variance Diffusions", *Class Notes, Stanford University*.

[19] Cox, J., S. Ross, and M. Rubinstein (1979), "Option Pricing: A Simplified Approach ", *Journal of Financial Economics*, 7, pp. 229-264.

[20] Cakici, N. and K. Topyan (2002), "The GARCH Option Pricing Model: a Lattice Approach", *Journal of Computational Finance*, 3, pp. 71-85.

[21] Duan, J. (1996), "Cracking the Smile", *Risk*, 9, pp. 55-59.

[22] Duan, J. (1995), "The GARCH Option Pricing Model", *Mathematical Finance*, 5, pp. 13-32.

[23] Duan, J., and Simonato (2001), "American Option Pricing Under GARCH by A Markov Financial Crisis-A GARCH Approach", *Journal of Banking and Finance*, pp. 1989-2014.

[24] Engle, R., and V. Ng (1993), "Measuring and Testing the Impact of News in Volatility", Journal of Finance, 43, pp. 1749-1778.

[25] Harrison, J. M., and D. M. Kreps (1979), "Martingales and Arbitrage in Multiperiod Securities Markets", Journal of Economic Theory, 20, pp. 381-408.

[26] Hull, J., and A. White (1987), "The Pricing of Options on Assets with Stochastic Volatilities", *Journal of Finance*, 42, pp. 281-300.

[27] Merton, R. (1976), "Option Pricing When Underlying Stock Returns are Discontinuous", *Journal of Financial Economics*, 3, pp. 125-144.

[28] MacBeth, and Merville (1979), "An Empirical of the Black-Scholes Call Option Pricing Model", *Journal of Finance*, 34, pp. 1173-1186.

[29] Nelson, D. (1991), "Conditional Heteroskedasticity in Asset Returns: A New Approach", *Econometrica*, 45, pp. 7-38.

[30] Ritchken, P., and R. Trevor (1999), "Pricing Option Under Generalized GARCH and Stochastic Volatility Process", *Journal of Finance*, 54, pp. 377-402.

[31] Schwartz, R. A., and Cohen, L. M. (1993), "Reshaping the Equity Markets: A Guide for the 1990s", *Business One Irwin*, pp. 126-128.

[32] Stephen Figlewski and Bin Gao (1998), " The Adaptive Mesh Model: A New Approach to Efficient Option Pricing", *Leonard N. Stern School of Business- in its series New York University, Leonard N. Stern School Finance Department Working Paper Seires*, 98-032, Mar.

[33] Tian, Y. S. (1993), "A Modified Lattice Approach to Option Pricing", *Journal of*

Futures Markets, 13, 5, pp. 563-577.

[34] Whaley, R. (1982), "Valuation of American Call Options on Dividends-Paying Stocks", Journal of Financial Economics, 10, pp. 29-55.

[35] Yuh-Dauh Lyuu and Chi-Ning Wu (2003), "Complexity of the Ritchken-Trevor-Cakici- Topyan GARCH Option Pricing Algorithm" (401) *Financial Engineering and Applications*.

[36] Yuh-Dauh Lyuu and Chi-Ning Wu (2005), "On accurate and provably efficient GARCH option pricing algorithms", *Quantitative Finance*, Vol. 5, No. 2, April, 181–198.

Ching-Chien, Carson Huang

Personal Information

- Master Student, Institute of Finance, National Chiao-Tung University
- 1001 Ta-Hsueh Road, HsinChu 30010, Taiwan.
- Tel: +886-(0)968-671-972
- E-mail: aoao1018@hotmail.com
  Backup: carrson.huang@hotmail.com

Education

- Master Candidate, Institute of Finance, National Chiao-Tung University, HsinChu, Taiwan, 2006-Present.
  – Master Thesis: Value the GARCH Option Applying the Modified Ritchken and Trevor Tree