Design technique for a SISO system with uncertain nonminimum-phase plant

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Abstract: A design technique for a single-input/single-output system with nonminimum-phase plant and large parameter variations is developed. The uncertain part is transformed into a disturbance form and the problem becomes one of attenuating the external disturbance. This method gives a much simpler design procedure.

1 Introduction

Consider a single-input/single-output (SISO) linear time-invariant nonminimum-phase plant. The parameters of the plant are not known precisely; they belong to a set $\mathscr{P}$. The control problem is to design a feedback system such that the closed-loop response is within the prescribed bounds. A structure with two degrees of freedom is used (Fig. 1). F and G are to be determined to guarantee that

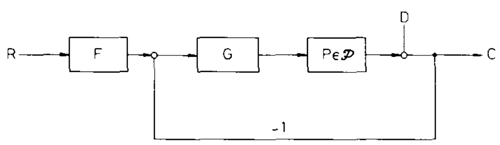

Fig. 1 *Canonical structure; D is the external disturbance*

the system satisfies the specification. Such quantitative design problems have been presented in [1–4]. These studies manipulated a plant template (a set of complex numbers due to plant parameter variation for each $s = j\omega$) on a Nichols chart to find the loop transmission bounds [5, 6]; the loop transmission functions result. This method is useful for solving an uncertain plant with large parameter variations, but the template manipulation process is somewhat tedious even for a control engineer with quantitative feedback theory (QFT) [7] design experience. However, there are now commercially available CAD packages for SISO design. East [8] has proposed a CAD method to fit an optimum loop transmission for minimum-phase plant, but the fitting work can be time-consuming.

This paper presents an equivalent disturbance attenuation method for solving a SISO quantitative design with nonminimum-phase plant without the need for plant template manipulations. We transform the plant uncertainty problem to a disturbance attenuation one, and use an algebraic equation to calculate the values of the bounds. Then the bounds can easily be found on an inverse Nichols chart, and the results are almost the same.

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2 Nonminimum-phase systems with plant uncertainties

In Fig. 1 the overall transfer function is

$$T(s) = F\frac{GP}{1 + GP} \tag{1}$$

We note that the zeros of the plant are also the zeros of $T(s)$. Hence a nonminimum phase of $P(s)$ will cause a nonminimum phase of $T(s)$, explicitly.

Note that we consider the case of strictly right-halfplane (RHP) zeros only, and exclude the case which may vary between the left and right halfplanes. If the transfer function of $P(s)$ has one or more zeros in the right halfplane, then explicitly

$$P(s) = N(-s)P_1(s) \tag{2}$$

where $N(-s) = \prod_i (1 - \tau_i s)$ and $P_1(s)$ is the minimum phase part. The $-s$ is used to emphasise the non-minimum character. Let $N_0(-s)$ be the nominal value of $N(-s)$. Then we have

$$\begin{aligned} L = PG &= \left(\frac{N_0(-s)}{N_0(s)}\right)(P_{10}(s)N_0(s)G)\left(\frac{N(-s)P_1(s)}{N_0(-s)P_{10}(s)}\right) \\ &= [A_0(s)][L_{m0}(s)][P'(s)] \\ &= L_{n0}(s)P'(s) \end{aligned} \tag{3}$$

where $P_{10}(s)$ is the nominal representation of $P_1(s)$, $L_{m0}(s)$ is the nominal transmission function with minimum phase and $L_{n0}(s)$ is the nominal transmission function with nonminimum phase, when the parameters have their nominal values. $P'(s)$ is the only uncertain part of $L(s)$, and is unity at the nominal values of the plant parameters. From eqn. 3 we note that the boundary on the loop transmission function of $L_{m0}(s)$ and $L_{n0}(s)$ is

$$L_{m0}(s) = A_0^{-1}(s)L_{n0}(s) \tag{4}$$

In eqn. 3, $P'(s)$ is the only uncertain part of L. From eqn. 1, owing to uncertainty, we note that

$$\Delta|T|_{dB} = \Delta\left|\frac{L}{1 + L}\right|_{dB} \tag{5}$$

with

$$\Delta |L|_{dB} = \Delta |P'|_{dB} \tag{6}$$

As the plant parameters range over their regions of uncertainty, $\Delta |L(j\omega)|_{dB} = \Delta |P'(j\omega)|_{dB}$ occupies a known region in the complex plane at each specified ω. This has been called the 'template' at ω. References 1 and 2 use these templates to manipulate on the Nichols chart and, according to eqn. 6, to find the acceptable boundary of L_{n0}. This template manipulation technique, which is widely used in various QFT papers, is practical but tedious. In this paper it is shown that the boundaries of the nominal loop transmission function can be chosen directly from a set of existing curves.

3 Equivalent disturbance attenuation method

3.1 Design equation derivation

From Fig. 1, P_0 and P are the nominal and perturbed plant transfer functions, respectively. $L_0 = GP_0$ is the nominal loop transmission function. T_0 and T are the nominal and perturbed system functions, respectively. We have

$$T \triangleq \frac{C}{R} = F\frac{GP}{1+GP} \tag{7a}$$

$$= F\frac{GP_0}{P_0/P + GP_0} \tag{7b}$$

$$= F\frac{L_0}{P_0/P + L_0} \tag{7c}$$

$$(P_0/P + L_0)T = FL_0 \tag{8}$$

Defining $P_v = 1 - P_0/P$, it follows that

$$(1 + L_0)T = FL_0 + P_vT \tag{9}$$

By using

$$T_0 = FL_0/(1 + L_0) \tag{10}$$

then

$$(1 + L_0)(T - T_0) = P_vT \tag{11}$$

Let $\Delta T \triangleq T - T_0$. Then

$$\Delta T = P_vT/(1 + L_0) \tag{12}$$

Eqn. 12 is implemented in Fig. 2. The plant P_0 has fixed value and the plant uncertainty becomes an equivalent

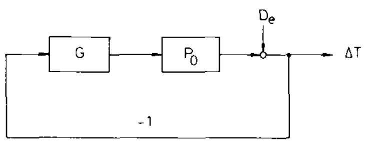

Fig. 2 *Disturbance attenuation only, $D_e \triangleq P_vT = (1 - P_0/P)T$, $\Delta T \triangleq T_0 - T$; both are treated as equivalent signals*

disturbance $D_e = P_vT$. The P is implicitly in the equivalent disturbance term P_vT; hence this is termed the equivalent disturbance attenuation (EDA) method.

From eqn. 12, define $T_v = \Delta T/T$. Then we have

$$\left|\frac{1}{1 + L_0}\right|_{dB} = |T_v|_{dB} - |P_v|_{dB} \tag{13}$$

Note that $|T_v|_{dB}$ is the acceptable system normalised tolerance (specification) and $|P_v|_{dB}$ is the variation range due to uncertainty. The point is to find the bounds of a nominal loop transmission L_0 at each frequency ω. By eqn. 13, $L_0(s)$ should be designed to satisfy

$$\left|\frac{1}{1 + L_0}\right|_{dB} \leqslant |T_v|_{dB} - |P_v|_{dB\,max} \tag{14}$$

in order to attenuate the disturbance from D_e.

If $Q = 1/L_0$, then eqn. 14 becomes

$$\left|\frac{Q}{1 + Q}\right|_{dB} \leqslant |T_v|_{dB} - |P_v|_{dB\,max} \tag{15}$$

On the Nichols chart, suppose Q is an open-loop transfer function and $Q/(1 + Q)$ is the closed-loop transfer function. Then the rectangular grid represents $|Q|_{dB}$, $\angle Q$, and the curve represents $|Q/(1 + Q)|_{dB}$, $\angle(Q/(1 + Q))$. Because $|L_0|_{dB} = -|Q|_{dB}$, $\angle L_0 = -\angle Q$. So, if we reverse the Nichols chart with the 0 dB line and rotate 180° about the $-180°$ line, then the rectangular grid and the curve represent the magnitude and phase of L_0 and $Q/(1 + Q)$, respectively. In the EDA design, we will use this relation to find the boundaries of the nominal loop transmission function.

3.2 Bounds on L_{n0} in Nichols chart

By eqn. 15 the bounds of L_{n0} can be calculated; for example, in Table 2 if $\omega = 2$ then $Q/(1 + Q) \leqslant -0.2539$. We can easily find the bound $B^n(2)$ as shown in Fig. 3.

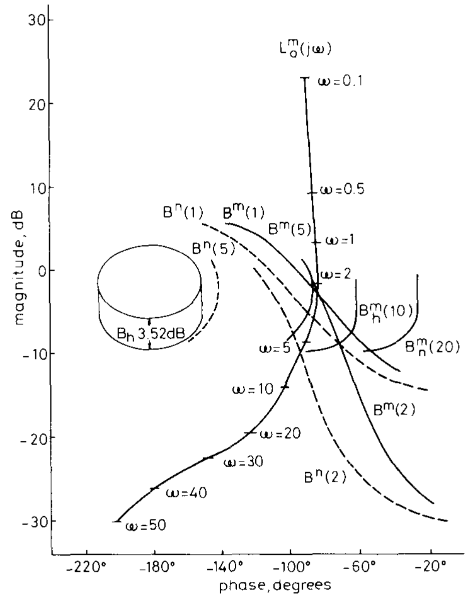

Fig. 3 *Bounds and practical shaping of loop transmission function*

We note that in the low- and intermediate-frequency ranges the sensitivity S_P^T is smaller than one. The magnitudes of the boundaries are single-valued functions of phase on the Nichols chart, and extend from 0° to $-360°$. They encircle the point 0 dB and $-180°$.

For the high-frequency range $|P_v|_{dB\,max}$ degenerates into $|(k - k_0)/k|_{dB\,max}$ for any rational function $P(s) = k\prod(s + z_i)/\prod(s + p_j)$. The acceptable normalised system bound $|T_v|_{dB}$ is larger than $|P_v|_{dB\,max}$. Hence, the bound value of $Q/(1 + Q)$ is positive. This means that the boundaries become an oval curve on the Nichols chart and shrink as the frequency increases.

3.3 Universal high-frequency bounds

In the high-frequency range, the constrained form eqn. 15 becomes less important. Therefore, we use the universal high-frequency bound (B_h) as proposed by Horowitz and Sidi [2].

At high frequency the plant $P(s)$ degenerates into ks^{-e} where e is the excess of poles of P over zeros of P. The plant variation approaches a vertical line of length $\Delta|P(j\omega)|_{dB} = |k_{max}/k_0|_{dB}$. For the example in Section 5 the length is 3.52 dB.

In the relatively high-frequency range $|S| \gg 1$ is tolerable, as far as its effects on $|T(j\omega)|$ are concerned, because the prefilter F in Fig. 1 attenuates the resulting high peak in $|L/(1+L)|$. However, the disturbance response in Fig. 1, $C/D = 1/(1+L) = S$, is then also very large, which is generally not tolerable because there is no equivalent filter available. Although the parameter ignorance problem is assumed to dominate in this paper, it is necessary to consider the disturbance response at least to the extent of adding the constraint $|T_d| = |C/D| = |1/(1+L)| \leqslant \gamma$ for all ω. In the example of Section 5, $\gamma \leqslant 6$ dB; therefore the universal high-frequency bound B_h is as shown in Fig. 3.

3.4 Nonminimum-phase conditions

For nonminimum-phase conditions, the corresponding boundary of $L_{m0}(j\omega)$ is easily derived by eqn. 4. For example, if as in Section 5 $\omega = 2$, then $L_{m0}(j2) = A_0^{-1}(j2)L_{n0}(j2)$. There is a similar boundary at each ω; the shift in angle is due to $A_0^{-1}(j\omega)$, which is small in the low-frequency range and tends to 180° as ω becomes large. The resulting boundaries for the example are shown in Fig. 3.

4 Design procedures

This section presents the design procedures of the nonminimum-phase (or transportation lag) system using the method given in Section 3.

(*a*) Choose the nominal T_0, P_{10} and N_0. T_0 is selected as the mean $|T_0| = (B_u + B_l)/2$, where B_u and B_l are the upper and lower permitted bounds, respectively. P_{10} is selected at the midpoint of the given uncertainty. N_0 is selected as the maximum lag.

(*b*) Find the bounds $B(\omega)$ of L_{n0}, then the bounds of L_{m0}. By eqn. 11, $|T_v|_{dB} = (B_u - B_l)/2$. Also $P_v = 1 - 1/P'(s)$, where $P'(s)$ is given by eqn. 3. Then we can easily find the L_{n0} bounds on the inverse Nichols chart. The L_{m0} bounds can be found by eqn. 4.

(*c*) Shape the nominal loop transmission function L_{m0} and derive both the compensator $G(s)$ and the prefilter $F(s)$. The loop shaping is achieved by selecting a rational transfer function with magnitude at or above the bounds found in step (*b*) at each specified frequency. One could use a high-order rational function to achieve an optimum loop shaping as proposed in [9]. The prefilter design is to make the $|T_0'(j\omega)| = |L_{n0}(j\omega)/(1 + L_{n0}(j\omega)|$ as close as possible to the $|T_0(j\omega)|$ selected in step (*a*)

(*d*) Modify $L_{n0}(s)$ and $F(s)$ if necessary.

5 Numerical example

For the sake of easier comparison of the EDA method with the original method, this example is taken from [1, 2]. The plant transfer function is

$$P(s) = \frac{k(1-ds)}{s(1+bs)}$$

with $k \in [1, 3]$, $b \in [0.3, 1]$, $d \in [0.05, 0.1]$. The step time response specifications and the equivalent frequency response bounds are shown in Figs. 6 and 4. The constraint for the disturbance response is $\gamma \leqslant 6$ dB.

The nominal values are chosen as $P_{10} = 2/(s(1 + 0.65s))$, $d_0 = 0.1$, $N_0(s) = 1 + 0.1s$ and

$$P'(s) = \frac{k(1+0.65s)(1-ds)}{2(1-0.1s)(1+bs)}$$

According to Section 4 step (*b*) and the acceptable tolerance in Fig. 4, we have $|T_v|_{dB}$ at each given ω as shown in Table 1. By eqn. 15, we have the bound value for the given frequency as shown in Table 2. Then the L_{n0} bounds can be easily found as in Fig. 3 (dashed line). By Section 3.2, the L_{m0} bounds are also shown in Fig. 3 (solid line). Then the nominal loop transmission function and the compensators are as follows:

$$L_{m0}(s) = \frac{1.4}{s}\frac{1+0.56s}{1+0.4s}\frac{40^2}{s^2+40s+40^2}$$

$$G(s) = \frac{1120(1+0.65s)(1+0.56s)}{(1+0.1s)(1+0.4s)(s^2+40s+1600)}$$

$$F(s) = \frac{7.2(s^2+3.8s+6.25)}{(1+0.67s)(s+1.8)(s^2+6s+25)}$$

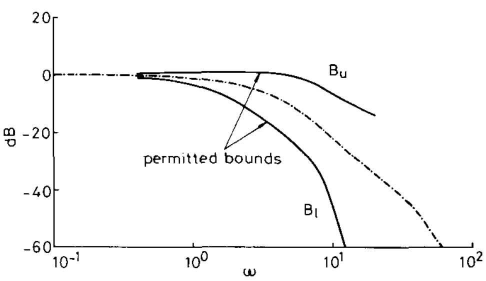

Fig. 4 *Achieved* $|T_0(j\omega)|$

Table 2: Bounds value calculation

ω	$\lvert T_v\rvert_{dB} - \lvert P_v\rvert_{dB\,max} = \lvert 1/(1+L_0)\rvert_{dB}$
0.4	0.5 − 0.8773 = −0.3733
1	1.75 − 3.1466 = −1.3966
2	5.0 − 5.2539 = −0.2539
3	7.75 − 6.2078 = 1.5172
5	11 − 7.3206 = 3.6794
8	15 − 8.6521 = 6.3479
10	20.25 − 9.4202 = 10.8298

Table 1: Normalised system tolerance

ω	0.4	1	2	3	5	8	10
B_u	0.25	0.5	1.0	0.95	0	−3.5	−6.5
B_l	−0.75	−3	−9.0	−14.5	−22	−33.5	−47
$\lvert T_v\rvert_{dB} = (B_u - B_l)/2$	0.5	1.75	5.0	7.725	11	15	20.25

Simulations for the different plant cases of Table 3 are shown in Figs. 5 and 6.

Table 3: Different plant cases

Case	k	d	b
1	1.0	0.1	1.0
2	1.0	0.1	0.3
3	3.0	0.1	1.0
4	3.0	0.1	0.3
5	1.0	0.05	1.0
6	1.0	0.05	0.3
7	3.0	0.05	1.0
8	3.0	0.05	0.3
9	2.0	0.1	0.65

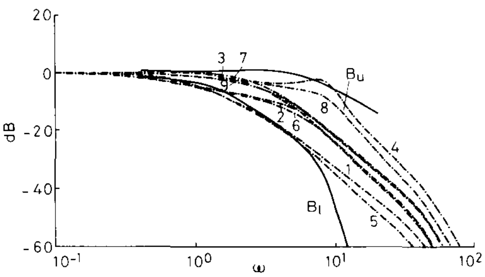

Fig. 5 *Achieved* $|T(j\omega)|$ *for different plant cases*

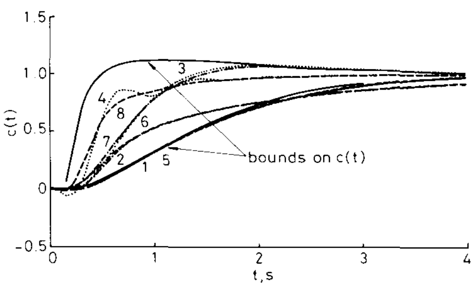

Fig. 6 *Step response for different plant cases*

6 Conclusion

This paper shows how the equivalent disturbance attenuation method is used to design a nonminimum-phase (or transportation lag) system. Theoretically, any nonminimum-phase system can be designed as well as the minimum-phase system. However, since the frequency domain is used throughout the design procedure, if the phase lag caused by the right-halfplane zero is too large, it is possible in practice that no solution exists for a nonminimum-phase problem. Some criteria for determining whether there may be no solution to the nonminimum-phase problem have been presented in [2, 3, 10].

The equivalent disturbance attenuation method is successfully applied to the nonminimum-phase plant with large parameter variations. From the numerical example, the design procedure is shown to be straightforward, especially in finding the bounds on L_0. It is interesting to compare the results of the EDA method and the template manipulation method. Fig. 7 shows compensators $G(s)$ obtained from Section 4 (solid line) and by Horowitz and Sidi (dashed line) [1, 2]. Theoretically, the EDA method from eqn. 15 should be more conservative than the template manipulation method. However, Fig. 7 shows the

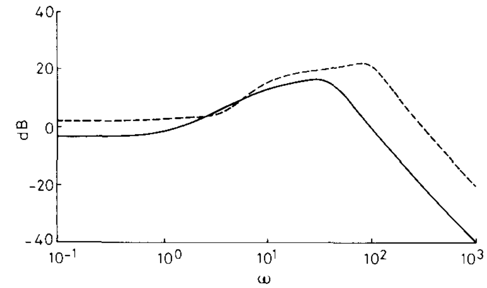

Fig. 7 *Frequency response of compensator*

——— designed by EDA method

- - - - designed in [1, 2]

opposite. This is due not to the superiority of the EDA method but to the accuracy of the loop shaping. This interesting result also shows that the criteria of the two methods are similar, and that the EDA method is more convenient for the designer. Although the EDA approach will sometimes result in a conservative design, this disadvantage can be overcome by choosing an optimum nominal plant P_0 and the method proposed in [9]. This is an interesting and challenging subject, and further research is now under way.

7 Acknowledgement

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