Analysis of a Two-Dimensional Radome of Arbitrarily Curved Surface

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Abstract—**The transmission effect of a dielectric shell on electromagnetic radiation is analyzed by modal cylindrical-wave spectrum technique. This method takes into consideration the curvature effect, which is generally ignored in classical approaches such as the ray method and the plane-wave spectrum analysis.**

I. Introduction

THE EFFECT OF a dielectric layer on the penetration of electromagnetic waves is always an interesting subject which finds many applications such as in the study of the performance of a radar antenna enclosed by radome. A radome is a dielectric shell used to protect the radar from water, sun, wind, etc. The presence of the radome, however, inevitably distorts the radiation pattern of the radar antenna, which causes a shift in the radar's beampointing angle. In addition, part of the radiation energy is lost as a consequence of scattering of the wave from the radome surface. The difference between the apparent and the distorted beam directions and the loss of peak gain are called the boresight error and the peak-gain attenuation, respectively, which are of greatest concern in the traditional radome design.

A precise analysis of radome performance is difficult, and nearly impossible in practice, because the general shape of a radome layer does not fit into the frame suitable for exact analysis. One must therefore resort to some approximation methods. The basic principle of approximation is to find a canonical configuration to approximate the surface of the dielectric layer locally, which can be solved rigorously by analytic means. The accuracy of the approximation depends on how closely the canonical problem resembles the original one. Traditional approaches to the problem, notably the geometric optics and the plane-wave spectrum (PWS) analysis [1], are based on the assumption that the shape of the layer can be approximated locally by a flat dielectric slab. One can thus apply the well-known Fresnel coefficients to characterize the wave transmission through the layer. By doing so the effect due to the curvature of the layer surface is completely ignored. With the modern advance in radar technology, however, this factor is likely to be critical in many applications, and therefore requires further investigation, especially in the case of layer surface with continuously varying surface curvature.

To account properly for the curvature effect, the rigorous analysis of a two-dimensional circular dielectric layer has been performed [2], which results in a ray-tracing scheme with curvature correction. In this investigation we introduce an alternative approach as an extension to treat layers of arbitrarily curved surface. In contrast with the plane-wave spectrum analysis, we make use of the modal cylindrical-wave spectrum. Here we choose to approximate the local layer surface as circular and resolve the excitation field into a series of modal cylindrical waves.

In Section II we introduce the basic concept of the method and the formulation of the problem. The technique is then applied in Section III to the analysis of radomes of elliptic shape. Numerical results are presented in Section IV. Discussion and conclusion follow in the last section.

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II. Formulation

The basic concept underlying in the analysis is to represent arbitrary primary wave fields as an expansion of a series of cylindrical wave constituents, and analyze the effect of transmission through the dielectric layer for each wave constituent separately. At the same time the dielectric layer of interest is approximated locally as a circular one. By doing this we inherently eliminate the need for curvature compensation since the transmission coefficient deduced therein will be cylindrical in nature instead of planar, as was always assumed in the early analysis.

A. Field Representation by Modal Cylindrical Waves

Consider an infinite z-directed current distribution $\mathbf{J}(\boldsymbol{\rho})$ in a cylindrical coordinate system. A harmonic time-dependence $e^{-i\omega t}$ is assumed and deleted throughout. The problem is two-dimensional and the resultant fields are invariant with respect to z.

The electric field due to this current distribution is given as

$$\mathbf{E}^i(\boldsymbol{\rho}) = \hat{\mathbf{z}}\frac{-\omega\mu}{4}\int_{S'} H_0^{(1)}(k_0|\boldsymbol{\rho}-\boldsymbol{\rho}'|)\mathbf{J}(\boldsymbol{\rho}')\,dS' \quad (1)$$

where $\boldsymbol{\rho} = (\rho, \phi)$, k_0 denotes the wavenumber in free space and $H_0^{(1)}(z)$ the Hankel function of first kind of order zero. $\hat{\mathbf{z}}$ is the unit vector along z.

A modal representation of the excitation field, which is useful for our purpose, can be derived by making use of the

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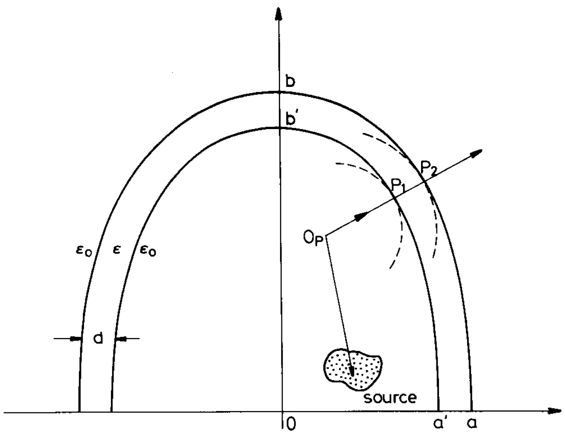

Fig. 1. Geometry of the problem.

addition theorem of the Bessel function [3]. As a result

$$\mathbf{E}^i(\rho) = \hat{\mathbf{z}}\frac{-\omega\mu}{4}\sum_{n=0}^{\infty}\epsilon_n\int_{S'} J_n(k_0\rho_<)H_n^{(1)}(k_0\rho_>) \cdot \cos n(\phi-\phi')\mathbf{J}(\rho')\, dS' \quad (2)$$

where $\epsilon_0 = 1$, $\epsilon_n = 2$ for $n \neq 0$, $J_n(x)$ is the Bessel function of order n, and $\rho_>$, $\rho_<$ denote respectively the greater and the lesser of ρ and ρ'. Physically, this representation expresses the excitation fields at a point P as superposition of modal cylindrical waves of various orders, emanated from some arbitrary reference point O.

This expression will be employed in the following to represent the excitation field due to the source of radiation enclosed by a dielectric shell.

B. Transmission Effect due to the Dielectric Layer

Fig. 1 depicts the situation where the antenna represented by a current distribution is enclosed by a dielectric shell. The shell has dielectric constant ϵ and uniform thickness d. The surface of the shell is sufficiently smooth such that at each point P_1 on the surface we are able to locate the center of the curvature O_P, as shown in the figure. We can therefore approximate the dielectric shell in the vicinity of the point of interest P_1 by a local circular layer.

Next we represent the primary field as a series of modal cylindrical waves expanded at the center of curvature O_P corresponding to the local circular layer at P_1, as given by (2). Notice that for $\rho < \rho'$ we have modal excitation fields of standing-wave type represented by $J_n(k\rho)$. These, however, can be decomposed into traveling wave components by means of the relation $J_n(k_0\rho) = \frac{1}{2}[H_n^{(1)}(k_0\rho) + H_n^{(2)}(k_0\rho)]$, where according to our convention $H_n^{(1)}(k_0\rho)$ represents the outgoing wave whereas $H_n^{(2)}(K_0\rho)$ represents the wave traveling toward the center of curvature.

To evaluate the fields affected by the shell, we take into account the transmission and reflection characteristics for each cylindrical wave constituent. To this end let

$T_n^{(1)}(\rho_1, \rho_2)$ the transmission coefficient at P_1 for outgoing wave $H_n^{(1)}(k\rho)$

$R_n^{(1)}(\rho_1, \rho_2)$ the reflection coefficient at P_2 for outgoing wave $H_n^{(1)}(k\rho)$

$T_n^{(2)}(\rho_1, \rho_2)$ the transmission coefficient at P_1 for incoming wave $H_n^{(2)}(k\rho)$

$R_n^{(2)}(\rho_1, \rho_2)$ the reflection coefficient at P_2 for incoming wave $H_n^{(2)}(k\rho)$.

By incorporating the relevant transmission and reflection coefficients into (2) we obtain the field at P_2 on the outer surface of the shell

$$E_z = \frac{-\omega\mu}{4}\sum_{n=0}^{\infty}\epsilon_n\int_{S'} T_n^{(1)}J_n(k_0\rho')H_n^{(1)}(k_0\rho_2) \cdot \cos n(\phi-\phi')J(\rho')\, dS'$$

$$H_\phi = \frac{-ik_0}{4}\sum_{n=0}^{\infty}\epsilon_n\int_{S'} T_n^{(1)}J_n(k_0\rho')H_n^{(1)'}(k_0\rho_2) \cdot \cos n(\phi-\phi')J(\rho')\, dS', \qquad \rho' < \rho_2. \quad (3a)$$

$$E_z = \frac{-\omega\mu}{8}\sum_{n=0}^{\infty}\epsilon_n\int_{S'}\left[T_n^{(1)}H_n^{(1)}(k_0\rho_2) + H_n^{(2)}(k_0\rho_2) + R_n^{(2)}H_n^{(1)}(k_0\rho_2)\right] \times H_n^{(1)}(k_0\rho')\cos n(\phi-\phi')J(\rho')\, dS',$$

$$H_\phi = \frac{-ik_0}{8}\sum_{n=0}^{\infty}\epsilon_n\int_{S'}\left[T_n^{(1)}H_n^{(1)'}(k_0\rho_2) + H_n^{(2)'}(k_0\rho_2) + R_n^{(2)}H_n^{(1)'}(k_0\rho_2)\right] \times H_n^{(1)}(k_0\rho_2)\cos n(\phi-\phi')J(\rho')\, dS', \quad \rho' > \rho_2. \quad (3b)$$

The transmission and reflection coefficients are derived from the rigorous analysis of a circular layer, as given in [2].

C. Evaluation of the Radiation Fields

From (3a) and (3b) the equivalent electric and magnetic currents at P_2 on the surface are given, respectively, as $\mathbf{J}_s = \hat{\mathbf{n}} \times \mathbf{H}$ and $\mathbf{M}_s = \mathbf{E} \times \hat{\mathbf{n}}$, where $\hat{\mathbf{n}}$ is the outward normal from the surface. Once these currents are known, then by equivalence principle the radiation fields transmitting through the dielectric shell can be derived by means of surface integration [4].

The integration is taken over a closed surface which encloses the radome. In practice, only the major part of the surface illuminated by the antenna suffices.

III. Numerical Results for Radome Application

Major quantities which are most concerned in radome analysis are the boresight error and the peak-gain attenuation, defined, respectively, as the deviation of the antenna main beam from its apparent look angle and the power lost in the direction of the main beam. In this investigation we consider specifically the radome wall of elliptical shape which has continuously varying curvature along the surface. The geometry is shown in Fig. 2, where the outer surface is chosen to be elliptical, with long axis b and short axis a. Fig. 3 shows

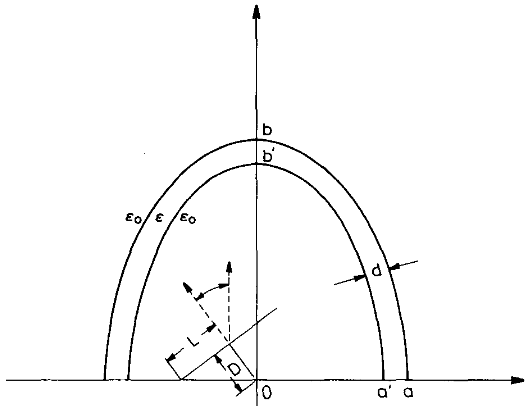

Fig. 2. Geometry of elliptic radome.

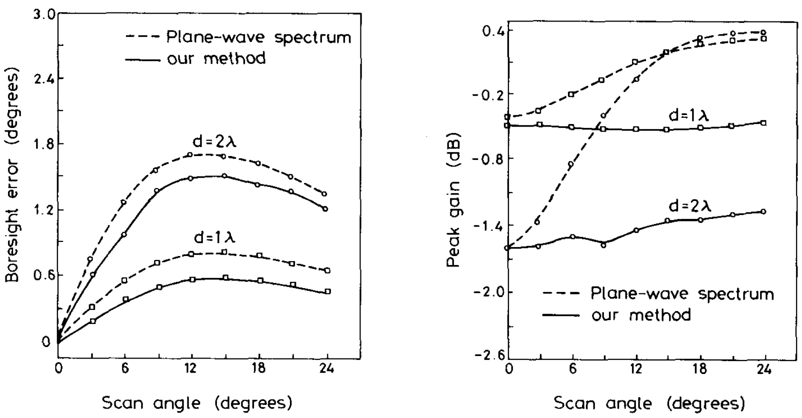

Fig. 3. Comparison of boresight error and peak-gain attenuation of elliptic radome derived by plane-wave spectrum and model cylindrical-wave spectrum analysis.

the boresight error and peak-gain attenuation calculated for various thickness d of the layer with $L = 5\ \lambda_0$, $D = (10/3)\ \lambda_0$, $a = 10\ \lambda_0$ and $b = 20\ \lambda_0$. The current distribution is assumed to be uniform. The boresight error, as well as the peak-gain attenuation, increase as d. As d increases further, the method loses its property of locality and it might become inapplicable.

The numerical program has been carefully checked to ensure its validity before we proceed to assess the radome performance. As examples we calculate the boresight errors of radomes for different radome parameters and source distribution, but with the dielectric constants set to that of free space. In such cases the radome should have no effect on the radiation fields. As shown in our calculation this is indeed the case; both the boresight error and the peak-gain attenuation are well within the range of error of tolerance incurred in the numerical computations.

A. Comparison with the Results by Plane-Wave Spectrum Analysis

As a comparison we include in Fig. 3 the results obtained by plane-wave spectrum analysis [5]. Notice that the boresight error deviates appreciably though it follows that obtained by PWS analysis. Apparent discrepancy exists, however, between the two as regards the peak-gain attenuation calculation.

IV. Conclusion

The shape and the thickness of the radome layer are two major factors that influence the radome performance, such as the boresight error and the peak-gain attenuation. Since few experimental results have appeared in the open literature, it is not easy to conclude how the curvature effect influences these quantities. As compared to the results obtained for an elliptic radome by plane-wave spectrum analysis, there is significant deviation between the two methods, which reflects the fact that the radome surface in the elliptic case varies in a continuous, more sophisticated manner. It is generally believed that the curvature effect should become an important factor in assessing the radome performance, and needs to be seriously considered in case the random surface has large curvature variation.

The method of cylindrical-wave spectrum should provide more accurate results than those obtained by conventional techniques because this method includes essentially the curvature influence inherently. It is only in the aspect of the efficiency of numerical computation that it compares unfavorably with previous ones. For radomes with a sufficiently flat surface the curvature effect may not be significant, and PWS, for instance, should offer reasonable good answer. As a compromise between the accuracy and computation efficiency, a hybrid scheme can be adopted which employs PWS in regions of small curvature and our approach otherwise.

In this paper we introduced the method of modal cylindrical-wave spectrum and applied it to the analysis of a two-dimensional elliptic radome. The method is essentially an extension of the plane-wave spectrum technique and provides an analytic means for studying phenomena of wave transmission through arbitrarily curved surfaces of dielectric layers.

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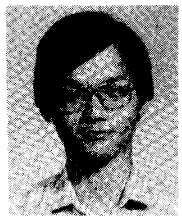

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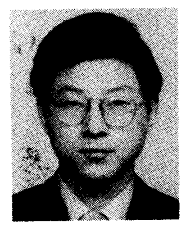

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