Coriolis Effects on Phonon Correlations in the Unclear Cranking Model

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(Received July 13, 1990)

Nuclear phonon excitations with different values of K^{π} are correlated under the influence of the coriolis interaction. This will lead to a correction term for the cranking moment of inertia. It is suggested that this phonon correlation may improve the agreements between theoretical calculations and experimental observations for nuclear ground state moments of inertia.

The theoretical calculation of the moments of inertia for nuclear ground state is an old problem since Inglis proposed the cranking model.[1] This model, with a self-consistent Hartee-Fock wave function, gives results in a rigid body value for the nuclear moments of inertia which is two to three times larger than that of the experimental observation. It was indicated by Bohr and Mottelson[2] that residual two-body forces, not inccluded in the one body self-consistent field, would lower the moment, and that the pairing correlations would be the most important. It was shown explicitly by Belyaev[3] that residual interactions of the pairing type indeed lower the moment of inertia from the rigid value through the following two effects. First, there is an increased energy denominator due to the replacement of the particle-hole excitation energy by the two-quasi-particle energy

$$E_k + E_{k'} \text{, with } E_k = [(\epsilon_k - \lambda)^2 + \Delta^2]^{12} \tag{1}$$

the quasi-particle energy, ϵ_k the Nilsson single particle energy of a state k, λ the Fermi energy and A the pairing gap parameter. Second, there is a reduction of the j_x matrix element in the numerator by a factor $(uv'-vu')$, where u and v are the coefficients of the Bololiubov transformation.[4] The BCS cranking moment of inertia then is given by

$$\Theta/(2\hbar^2) = \sum_{kk'} \frac{|\langle k\bar{k'}|j_x|\phi_0\rangle|^2}{E_k + E_{k'}} = \sum_{kk'} \frac{(u_k v_{k'} - v_k u_{k'})^2 |\langle k|j_x|k'\rangle|^2}{E_k + E_{k'}} \tag{2}$$

where $|\phi_0\rangle$ is the BCS ground state, $|k\bar{k'}\rangle$ the twoquasi-particle excited state, and $\bar{k}$ the time reversal state of k. Numerical calculations were performed by Griffin and Rich[5] and by Nilsson and Prior.[6] With the choice of the "best" parameter values they obtained a remarkable agreement with experiments for both the rare earth and the actinide nuclei. However, the theoretical values for the moments of inertia are still systematically about twenty percent too small, on the average.

Corrections due to residual interactions were derived by Migdal[7] and by Belyaev.[8] Numerical calculations by Mayer et al.[9] showed that the effects of the residual particle-hold and the residual particle-particle on the moments of inertia of all rare earth nuclei nearly cancel each other and leave the simple cranking value approximately unchanged. Calculations by Birbrair and Nikolaev[10] and by Kammuri and Kusuno[11] on the same subject did not take the rotation effects into account. Recently Luo et al.[12] used phonon ground state to calculate the moment of inertia of Eq. (2). That is, in Eq. (2), the quasi-particle ground state ϕ_0 was replaced by the RPA phonon ground state, the 2-quasi-particle excitations by the phonon excitations, and the two-quasi-particle energy denominator by the phonon energy. However, the improvement was really small and the systematic deviations of the theoretical moments of inertia from experimental values were left still not understood.

From the expression for the theoretical moment of inertia in Eq. (2), one can easily see that only elementary states with two-quasi-particles coupled to $k = 1^+$ can have contribution to the moment of inertia. This is true also for all the existing calculations for theoretical nuclear moments of inertia. In this note, it is proposed that 2-quasi-particle states with $K \neq 1^+$ could make a small contribution to the nuclear moment of inertia (may be about twenty percent in the average). This contribution to the moment of inertia can be treated in the following.

Consider a Hamiltonian H consists of a Nilsson single particle Hamiltonian with pairing correlations and residual two-body interactions. Using a Bololiubov transformation to treat the pairing force the Hamiltonian becomes

$$H = U_o + \sum_k E_k (\alpha_k^+ \alpha_k + \alpha_{\bar{k}}^+ \alpha_{\bar{k}}) + H_R \quad , \tag{3}$$

where U_o is the BCS quasi-particle ground state energy, E_k the quasi-particle energy, α_k^+ and α_k the quasi-particle creation and destrubtion operators, and H_R the residual interactions. To treat the residual interaction, we follow the standard approach of Beranger[13] for random phase approximation method (RPA) and define the phonon operator $Q_{K\alpha}^+$ as

$$Q_{K\alpha}^+ = \sum_\beta Y_\beta^{K\alpha} A_\beta^+ - \sum_\beta Z_\beta^{K\alpha} A_{\bar{\beta}} \tag{4}$$

where the A's are two-quasi-particle operators

$$A_\beta^+ = \alpha_k^+ \alpha_{k'}^+ \; , \; A_{\bar{\beta}} = \alpha_{\bar{k}'} \alpha_{\bar{k}} \quad , \tag{5}$$

with (kk') coupled to K. The inverse transformation is

$$A_\alpha^+ = \sum_\beta Y_\alpha^{K\beta} Q_{K\beta}^+ + \sum_\beta Z_\alpha^{K\beta} Q_{K\beta} \; . \tag{6}$$

In terms of phonon operators, the Hamiltonian can be written approximately as

$$H = U_o + U_Q + \sum_{K\alpha} \lambda_{K\alpha}(Q^+_{K\alpha}Q_{K\alpha} + Q^+_{\bar{K}\bar{\alpha}}Q_{\bar{K}\bar{\alpha}}) \equiv U_o + U_Q + \sum_K H_K \, , \tag{7}$$

where U_Q is the ground state phonon correlation energy and $\lambda_{K\alpha}$ the phonon excitation energy. The phonon ground state $|\Phi_o>$ and the one phonon state $|\Phi_{K\alpha}>$ are

$$Q_{K\alpha}|\Phi_o> = 0, \text{ for all K and } \alpha \, . \tag{8}$$

$$|\Phi_{K\alpha}> = Q^+_{K\alpha}|\Phi_o> \, . \tag{9}$$

The physical pictures of the ground state $|\Phi_o>$ and the excited state $|\Phi_{K\alpha}>$ are very well understood that (see for instance Ref. (13)) $|\Phi_o>$ is a superposition with 0, 4, 8, ... quasi-particle states, and $|\Phi_{K\alpha}>$ has only states with 2, 6, 10, ... quasi-particles. The commutator of the operater A with A^+ can be written as

$$[A_i, A^+_k] = \delta_{i.k} + \text{ small terms including } (a^+_\alpha a_\beta) \, . \tag{10}$$

As long as the ground state quasi-particle density is small, the small terms in Eq. (10) can be neglected at the end of the calculation, and we have obtained a good approximation solution for our problem. There is a major advantage of this approach that the difficulty of the problem of spurious states does not arise as Baranger has shown in Ref. (13).

To treat the nuclear rotation, we shall follow the approach in Ref. (14) since by using that approach many important nuclear rotational properties can be easily seen, for instance, the moment of inertia and the gapless superfluid in rotating nuclei. Introduce the semi-classical coriolis term ωj_x into the Hamiltonian, instead of dealing with the quasi-particle operators a ' and a, we treat the twoquasi-particle operators A^+ and A and consider their equations of motion. In the quasi-particle representation, this coriolis term can be written as

$$H_\omega = -\sum_{\substack{k>0 \\ k'>0}} \omega j_{kk'} [R^S_{kk'}(\alpha^+_k \alpha_{k'} - \alpha^+_{\bar{k}'}\alpha_{\bar{k}}) + R^A_{kk'}(\alpha^+_k \alpha^+_{\bar{k}'} + \alpha_{k'}\alpha_{\bar{k}})] \tag{11}$$

where

$$j_{kk'} = <k|j_x|k'>, R^S_{kk'} = (u_k v_{k'} + u_{k'} v_k), R^A_{kk} = u_k v_{k'} - u_{k'} v_k \tag{12}$$

In H_ω we have omitted terms with $k = \pm 1/2$ and $k' = \mp 1/2$ for simplicity. In actual calculations, we can add a correction term at the end. The basic commutator we need to consider is the commutator of H_ω with a two-quasi-particle operator. Let us consider the operator A ' , $= A^+_{k_1 \bar{k}_2} = \alpha^+_{k_1}\alpha^+_{\bar{k}_2}$ or A, ' $= A^+_{k_1 k_2} = \alpha^+_{k'}\alpha^+_k$ with $(k_1 \bar{k}_2)$ or (k, k_2) coupled to K. we have

$$\frac{1}{\omega}[A^{+}_{k_1\bar{k}_2}, H_\omega] = \delta_{K1} j_{k_1k_2} R^{A}_{k_1k_2} + \sum_k j_{kk_1} R^{S}_{kk_1} A^{+}_{k_1\bar{k}_2} - \sum_k j_{kk_2} R^{S}_{kk_2} A^{+}_{k_1\bar{k}} \tag{13}$$

$$\frac{1}{\omega}[A_{k_1k_2}, H_\omega] = \sum_k j_{kk_1} R^{S}_{kk_1} A^{+}_{k_1k_2} + \sum_k j_{kk_2} R^{S}_{kk_2} A^{+}_{k_1k} \tag{13a}$$

In Eqs. (13) and (13a), we have neglected terms of the quasi-particle density type as it is usually done in the RPA treatment. The first term on the right hand side of Eq. (13) vanishes if K is not 1 or $\bar{1}$. The commutator of A, with H_ω can be obtained by taking Hermitian conjugate of Eqs. (13) and (13a) then take a time reversal transformation, remembering that H_ω changes sign under time reversal operation. We now look at the summation k in Eqs. (13). Since we want to have terms where j_x matrix elements do not vanish, we must have $k=k_1\pm1$ so $k-k_2=k_1-k_2\pm1=K\pm1$; also, $k=k_2\pm1$ so $k_1-k=K\pm1$. In Eqs. (11) and (12), we assume k > 0 and k' > 0, terms with k < 0 in Eq. (13) must vanish. Therefore, a two-quasi-particle state $|\alpha,K>$ with the interaction of the coriolis potential can generate states of $|\beta,K\pm1>$. That is, states with $K\neq1$ will certainly contribute to the nuclear moment of inertia.

The equations of motion for Q^{+}_{α} and $Q_{\bar{\alpha}}$ can be worked out in a straightforward manner by using Eqs. (4), (6), (13) and (13a) as

$$[Q^{+}_{K\alpha}, H] = \sum_\beta F^{K\alpha}_{K+1\beta} Q^{+}_{K+1\beta} + \sum_\beta F^{K\alpha}_{K-1\beta} Q^{+}_{K-\beta} + \sum_\beta G^{K\alpha}_{K+1\beta} Q_{\overline{K+1\beta}}$$
$$+ \sum_\beta G^{K\alpha}_{K-1\beta} Q_{\overline{K-1\beta}} + \lambda_{K\alpha} Q_{K\alpha} + \eta_\alpha \delta_{K1} \tag{14}$$

where

$$H = H + H_\omega\ , \quad \eta_\alpha = -\omega \sum_{kk'} j_{kk'} R^{A}_{kk'} (Y^{1\alpha}_{kk'} - Z^{1\alpha}_{kk'})$$

$$F^{K\alpha}_{K+1\beta} = \sum_{(k_1k_2)} [\sum_{k_1^{(+)}} (js)_{k_1k_1^{(+)}} (Y^{K\alpha}_{k_1k_2} Y^{K+1\beta}_{k_1^{(+)}k_2} - Z^{K\alpha}_{k_1k_2} Z^{K+1\beta}_{k_1^{(+)}k_2})$$
$$+ (sn) \sum_{k_2^{(-)}} (js)_{k_2k_2^{(-)}} (Z^{K\alpha}_{k_1k_2} Z^{K+1\beta}_{k_1k_2^{(-)}} - Y^{K\alpha}_{k_1k_2} Y^{K+1\beta}_{k_1k_2^{(-)}})]$$

$$F^{K\alpha}_{K-1\beta} = \sum_{(k_1k_2)} [\sum_{k_1^{(-)}} (js)_{k_1k_1^{(-)}} (Y^{K\alpha}_{k_1k_2} Y^{K+1\beta}_{k_1^{(-)}k_2} - Z^{K\alpha}_{k_1k_2} Z^{K+1\beta}_{k_1^{(-)}k_2})]$$
$$+ (sn) \sum_{k_2^{(+)}} (js)_{k_2k_2^{(+)}} (Z^{K\alpha}_{k_1k_2} Z^{K+1\beta}_{k_1k_2^{(+)}} - Y^{K\alpha}_{k_1k_2} Y^{K+1\beta}_{k_1k_2^{(+)}}$$

$$G^{K\alpha}_{K-1\beta} = \sum_{(k_1k_2)} [\sum_{k_1^{(+)}} (js)_{k_1k_1^{(+)}} (Y^{K\alpha}_{k_1k_2} Z^{K+1\beta}_{k_1^{(+)}k_2} - Z^{K\alpha}_{k_1k_2} Y^{K+1\beta}_{k_1^{(+)}k_2})]$$
$$+ (sn) \sum_{k_2^{(-)}} (js)_{k_2k_2^{(-)}} (Z^{K\alpha}_{k_1k_2} Y^{K+1\beta}_{k_1k_2^{(-)}} - Y^{K\alpha}_{k_1k_2} Z^{K+1\beta}_{k_1k_2^{(-)}})] \tag{15}$$

$$G^{K\alpha}_{K-1\beta} = \sum_{(k_1 k_2)} [\sum_{k_1^{(-)}} (js)_{k_1 k_1^{(-)}} (Y^{K\alpha}_{k_1 k_2} Z^{K-1\beta}_{k_1^{(-)} k_2} - Z^{K\alpha}_{k_1 k_2} Y^{K+1\beta}_{k_1^{(-)} k_2})]$$

$$+ (sn) \sum_{k_2^{(+)}} (js)_{k_2 k_2^{(+)}} (Z^{K\alpha}_{k_1 k_2} Y^{K+1\beta}_{k_1 k_2^{(+)}} - Y^{K\alpha}_{k_1 k_2} Z^{K+1\beta}_{k_1 k_2^{(+)}})]$$

(k, k,) means a pair of states k_1 and k_2, $(js)_{kk'} = \langle k | j_x | k' \rangle (u_k v_{k'} + v_k u_{k'})$, $k^{(\pm)} =$ the state with the value of z-component of angular momentum $= k \pm 1$, during the summation over $(k_1 k_2)$, if the pair $(k_1 \bar{k}_2)$ is $(k_1 k_2)$, sn = +1, if $(k_1 k_2)$ is $(k_1 k_2)$, sn = -1. The equation for $Q_{\bar{\alpha}}^-$ can be obtained from equation (14) by replacing $Q^+_\beta, Q_{\bar{\beta}}$ and λ_α with $Q^+_{\bar{\beta}}, Q_\beta$ and $-\lambda_\alpha$ repectively. The equations for $Q^+_{\bar{\alpha}}$ and Q_α are the same as those for Q^+_α and $Q^-_{\bar{\alpha}}$ respectively except the η terms change signs. Equations (14) has the form

$$\frac{d}{dt} r = Ar + R_0 \quad , \quad or \quad \frac{d}{dt}(r + r_0) = A(r + r_0) \tag{16}$$

with R_0 = Ar,, or $r_0 = A^{-1} R_0$. Let the eigen vector be R = G(r + r_0), the inverse transformation is therefore

$$r = G^{-1} R - r_0 = G^{-1} R - A^{-1} R_0 \tag{17}$$

Since the rotating frequency ω is small, we are interested only in the lowest order approximation. Here R_0 is a first order quantity in ω, A is a diagonal matrix D plus a first order small matrix. In the combination of $A^{-1} R_0$ we only need D^{-1} R,. Let A = D + ωU and G = I + ϵ where I is the unit matrix and ϵ is a first order small matrix in ω. Then in the first order approximation, we have

$$\epsilon_{ij} = \omega \frac{U_{ij}}{D_i - D_j} \tag{18}$$

The inverse transformation G^{-1} is $I - \epsilon_{ij}$. The unperturbed eigen energies in Eq. (14) are $(\lambda_1, \lambda_2, \ldots, \lambda_N, -\lambda_1, -\lambda_2, \ldots, -\lambda_N)$. Using Eqs. (14), (16) and (17), we finally obtain

$$Q^+_{K\alpha} = q^+_{K\alpha} - \delta_{K1} \eta_\alpha / \lambda_\alpha - \sum_\beta \omega \left(\frac{F^{K\alpha}_{K+1\beta}}{\lambda_{K\alpha} - \lambda_{K+1\beta}} q^+_{k+1\beta} + \frac{F^{K\alpha}_{K-1\beta}}{\lambda_{k\alpha} - \lambda_{K-1\beta}} q^+_{K-1\beta} \right)$$

$$- \sum_\beta \omega \left(\frac{G^{K\alpha}_{K+1\beta}}{\lambda_{K\alpha} - \lambda_{K+1\beta}} q_{K+1\beta} + \frac{G^{K\alpha}_{K-1\beta}}{\lambda_{K\alpha} - \lambda_{K-1\beta}} q_{K-1\beta} \right) \tag{19}$$

The q^+ and q in Eq. (19) are the eigen vectors of Eq. (14). The expression for $Q^+_{\bar{\alpha}}$ can be obtained from Eq. (19) by replacing $Q^+_\alpha, q^+_\beta, q_{\bar{\beta}}^-$ and η_α with $Q^-_{\bar{\alpha}}, q^-_{\bar{\beta}}, q_\beta$ and $-\eta_\alpha$ repectively. q^+ and q are the new creation and destruction operators of the rotational phonon in the rotating system with the new rotational ground state defined as

$$q_{K\alpha} | \Phi_0(\omega) \rangle = 0, \text{ for all K and } \alpha \quad . \tag{20}$$

Using the Hamiltonian (7), the rotational energy of a system is (for ground state band)

$$E = <\Phi_0(\omega)|H|\Phi_0(\omega)> - <\Phi_0|H|\Phi_0> . \qquad (21)$$

Therefore we have

$$\Theta/(2\hbar^2) = 2 \sum_{(kk')} \sum_{\alpha} \frac{|j_{kk'}R^A_{kk'}(Y^{1\alpha}_{kk'} - Z^{1\alpha}_{kk'})|^2}{\lambda_\alpha} + 2\sum_{K\alpha\beta} [\frac{(G^{K\alpha}_{K+1\beta})^2}{\lambda_{K\alpha} + \lambda_{K+1\beta}} + \frac{(G^{K\alpha}_{K-1\beta})^2}{\lambda_{K\alpha} + \lambda_{K-1\beta}}] . \qquad (22)$$

The first term on the right hand side of Eq. (22) is just the usual cranking moment of inertia which is due to the self-energies of the K = 1 excitations, while the 2nd term is a correction coming from the off-diagonal parts of the interaction among excitations of all the K's. Note that the self-energy of K ≠ 1 excitations do not exist. Note also that the 2nd term in Eq. (22) is positive definite.

The coriolis Hamiltonian H_ω of Eq. (11) consists of a quasi-boson term H^A_ω and a fermion term H^S_ω. Let us first consider the effect of the H^A_ω on the physical Hamiltonian H of Eq. (7). In the Q-representation, we can write

$$H + H^A_\omega = U_0 + U_Q + \sum_{K\neq 1} H_K + \sum_{(K=1)\alpha} [\lambda_{1\alpha}(Q^+_{1\alpha}Q_{1\alpha} + Q^+_{\bar{1}\bar{\alpha}}Q_{\bar{1}\bar{\alpha}})$$

$$+ \eta_\alpha (Q^+_{1\alpha} + Q_{1\alpha} - Q^+_{\bar{1}\bar{\alpha}} - Q_{\bar{1}\bar{\alpha}})] \qquad (23)$$

This can be written as

$$H + H^A_\omega = U_0 + U_Q + \sum_{K\neq 1} H_K + \Sigma\lambda_{1\alpha}[\xi^+_\alpha\xi_\alpha + \xi^+_{\bar{\alpha}}\xi_{\bar{\alpha}} - 2(f_\alpha)^2/\lambda_{1\alpha}] , \qquad (24)$$

with $\xi_\alpha = Q_\alpha + \eta_\alpha/\lambda_{1\alpha}$, $\xi_{\bar{\alpha}} = Q_{\bar{\alpha}} - \eta_\alpha/\lambda_{1\alpha}$. The ground state $|\Phi_\xi>$ of the Hamiltonian (24) is defined as $\xi_\alpha|\Phi_\xi> = 0$ for all α and K = 1, and $Q_{K\alpha}|\Phi_\xi> = 0$ for all α and K ≠ 1. The boson Hamiltonian H^A_ω has no effect on H_k for K ≠ 1. If we use $|\Phi_\xi>$ to calculate $<\Phi_\xi|H|\Phi_\xi> - <\Phi_0|H|\Phi_0>$, we get exactly the moment of inertia as the first term in Eq. (22). Now the physical picture becomes clear when we add the fermion Hamiltonian H^S_ω into the Hamiltonian (24). It will introduce the effects described by Eqs. (13) and (13a). A quasi-boson state $|k_1 k_2>$ is an approximate boson state, but actually it is a fermion 2-quasi-particle state. The interaction H^S_ω can convert one of the fermions in $|k_1 k_2>$ from a state k_i into a state k'_i such that, since $k_i + k_j = K$ and $k'_i = k' \pm 1$, we have $k'_i + k_j = k_i + k_j \pm 1 = K \pm 1$. In this way, all the K-states are correlated so as to produce a correction term for the moment of inertia as described by the 2nd term in Eq. (22). However, in order to see how large this effect can contribute to the nuclear moment of inertia, numerical calculations are certainly needed. It is hoped that, in the near future, theoretical results may be available for experimental comparison.

ACKNOWLEDGMENT

Work partially supported by the National Science Council of the Republic of China.

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