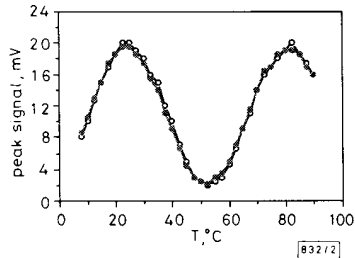


**Experimentation:** An experimental system was set up to evaluate the system as shown in Fig. 1 with a length of 50 m high-birefringence fibre (York HB800). A semiconductor laser (TXCK1203) was used as the low coherence light source. Like other multimode laser diodes, in the coherence domain it has several side-band peaks. In the experiment, the diode was driven below its threshold so that all the coherence side peaks associated with the required signal peaks were suppressed to below the noise floor.<sup>4</sup> In the preliminary experiment, the spatial scan was simulated by a manual translation stage and a mirror oscillator which, driven by a saw-tooth signal, provided linear spatial scan locally up to several wavelengths peak to peak. The AC interference signal produced was displayed on a spectrum analyser. The sensors were located by maximising the fundamental signal through adjusting the position of the mirror M1 in Fig. 1.

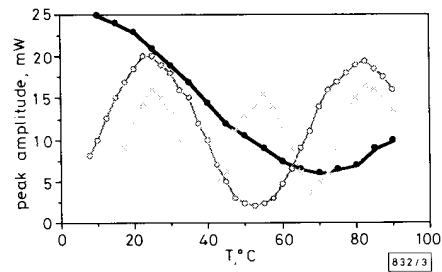
Without any loss of generality, temperature was used as the measurand. Fig. 2 shows the signal amplitude variation as a function of temperature with the two squeezers 2.5 cm apart.



**Fig. 2** Peak signal amplitude variation of one sensor under temperature cycle

Squeezer separation: 2.5 cm  
 ○ — heat up  
 ● — cool down

An additional advantage of this novel quasi-distributed interferometric sensor system is that the sensitivity of each sensor can be changed through arranging the separations of their two squeezers within the coherence range of the source. The modulation depth generally declines with the separation



**Fig. 3** Peak signal amplitude variation of three sensors with different squeezer separations under same temperature cycle

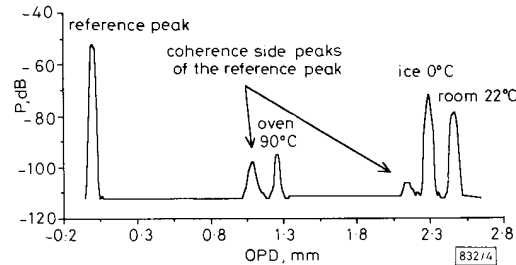
● —  $L = 1$  cm  
 ○ —  $L = 2.5$  cm  
 × —  $L = 5$  cm

increasing as shown in Fig. 3, where  $L$  is the squeezer separation.

The full system performance was assessed by multiplexing three sensors which were placed in an ice-water mixture, room environment and an oven. The squeezer separation of all three sensors were kept the same at 1 cm. The result was satisfying as shown in Fig. 4 with all the cross-term and side-band spurious peaks depressed below the noise floor. Only the coherence side peaks related to the giant reference peak at the origin emerge as spurious signals. They will not degrade the system performance because they are close to the origin and have fixed positions.

**Discussion:** A quasi-distributed interferometric sensor system has been demonstrated using the optical coherence domain

polarimetry technique. It is capable of detecting both the positions and phases of a large number of local or 'on-spot' interferometers along a single piece of high-birefringence fibre. It is superior to the previously reported systems since the phase of each local interferometer can be independently sensed. It inherited all the advantages of the former systems such as high multiplexing capacity, high signal-noise ratio and high resolution. The cross-term spurious signal problem is overcome by keeping the coupling ratio of each sensor very small. A very low cross-talking level was experimentally achieved.



**Fig. 4** System interferogramme with three sensors distributed along fibre

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#### References

- FRANKS, R. B., TORRUELLAS, W., and YOUNGQUIST, R. C.: SPIE Proceedings, Vol. 586, Fibre Optic Sensors, 1985, p. 84
- GUSMEROLI, V., VASSORI, P., and MARTINELLI, M.: 'A coherence multiplexed quasi distributed polarimetric sensor suitable for structural monitoring', Springer proceedings in physics, 'Optical fibre sensors', 1989, **44**, pp. 513
- GUSMEROLI, V., MARTINELLI, M., and VAVASSORI, P.: *Opt. Lett.*, 1989, **14**, (23), p. 1330
- YOUNGQUIST, R. C., CARR, S., and DAVIES, D. E. N.: *Opt. Lett.*, 1987, **12**, p. 158

#### PROGRAMMABLE FRACTIONAL SAMPLE DELAY FILTER WITH LAGRANGE INTERPOLATION

Indexing terms: Filters, Delay, Interpolation

A new linear time-invariant FIR filter which can be programmed to synthesise any fractional sample delay with Lagrange interpolation is presented. An analytic closed-form expression for the coefficients of such an FIR filter is derived. Computer verification is provided for the  $N = 4$  case.

**Introduction:** To transfer digital samples from one system to another system with different clock rates requires a phase shifter (or time delay) to compensate for the delay between the two clocks even if they are clocked at the same rate.<sup>1,2</sup> It may be desirable to delay a signal by a fractional multiples of the sample period. For this application, the signal must be interpolated to obtain new samples at noninteger sample instants. We propose a new programmable fractional sample delay

filter based on a FIR filter whose coefficients are functions of the delay time,  $\alpha T$ . A nonlinear interpolation technique using the conventional Lagrange interpolation formula is adopted. A general formula for the  $N$ th-order FIR programmable delay filter is derived.

**Design method:** The basic idea behind the Lagrange method is first to find a polynomial with value equal to unity at some particular sample points and zero at all the other sample points.<sup>3</sup> The function

$$L_j(t) = \frac{(t-t_1)(t-t_2)\dots(t-t_{j-1})(t-t_{j+1})\dots(t-t_{N+1})}{(t_j-t_1)(t_j-t_2)\dots(t_j-t_{j-1})(t_j-t_{j+1})\dots(t_j-t_{N+1})} \frac{\prod_{i=1, i \neq j}^{N+1} (t-t_i)}{\prod_{i=1, i \neq j}^{N+1} (t_j-t_i)} \quad (1)$$

is such a polynomial of degree  $N$ . Note that

$$L_j(t) = \begin{cases} 1 & t = t_j \\ 0 & t \neq t_j \end{cases} \quad j = 1, 2, 3, \dots, N+1 \quad (2)$$

Thus

$$L_j(t)x_j = \begin{cases} x_j & t = t_j \\ 0 & t \neq t_j \end{cases} \quad j = 1, 2, 3, \dots, N+1 \quad (3)$$

It is easy to see that

$$x(t) = \sum_{j=1}^{N+1} L_j(t)x_j \quad (4)$$

is a polynomial of degree  $N$  passing through the  $N+1$  points  $(t_i, x_i)$ . The Lagrange interpolation formula  $x(t)$  is not very practical because of its computational complexity. For the sake of convenience, some modifications for  $x(t)$  must be executed. Consider the special case when all  $x_j = 1$ , then  $x(t) = 1$  for all  $t$

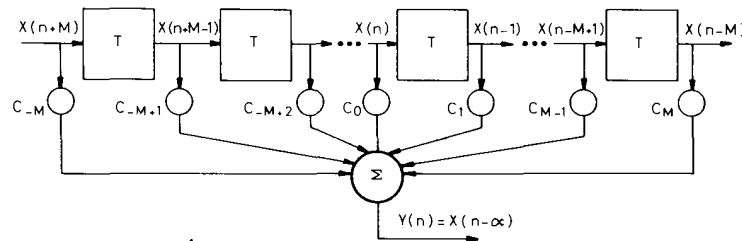
$$1 = \sum_{j=1}^{N+1} L_j(t) \quad (2)$$

Dividing eqn. 4 by eqn. 5

$$x(t) = \frac{\sum_{j=1}^{N+1} L_j(t)x_j}{\sum_{j=1}^{N+1} L_j(t)} \quad (6)$$

Furthermore, dividing the numerator and the denominator of eqn. 6 by

$$\prod_{j=1}^{N+1} (t-t_j) \quad (7)$$



$$C_j = \frac{1}{\sum_{i=-M}^M (-1)^{j-i} \frac{(M+j)!(M-j)!(j-\alpha)}{(M+j)!(M-i)!(i-\alpha)}} \quad j = -M, \dots, M$$

Fig. 1 Programmable fractional sample delay filter with Lagrange interpolation

we then have

$$x(t) = \frac{\sum_{j=1}^{N+1} \frac{x_j}{(t-t_j) \prod_{i=1, i \neq j}^{N+1} (t_j-t_i)}}{\sum_{j=1}^{N+1} \frac{1}{(t-t_j) \prod_{i=1, i \neq j}^{N+1} (t_j-t_i)}} = \frac{\sum_{j=1}^{N+1} A_j x_j(t-t_j)}{\sum_{j=1}^{N+1} A_j(t-t_j)} \quad (8)$$

where

$$A_j = \frac{1}{\prod_{i=1, i \neq j}^{N+1} (t_j-t_i)} \quad (9)$$

For an equally spaced data sequence  $\{x(n+M), x(n+M-1), \dots, x(n+1), x(n), x(n-1), \dots, x(n-M+1)$  and  $x(n-M)\}$ , as shown in Fig. 1, define

$$t_j = nT - jT \quad j = -M, -M+1, \dots, M-1, M \quad (10)$$

$$x_j = x(n-j)$$

If each  $T$ -interval is divided into  $LT'$  subintervals, where  $T' = T/L$ , then from eqns. 8-10 the interpolated sample at  $t = (nT - mT') = (nT - mT/L) = (n - \alpha)T$  can be described as

$$x(n - \alpha) = x(t) \Big|_{t=(n-\alpha)T} = \frac{\sum_{j=-M}^M A_j x(n-j) / [(n-\alpha)T - (n-j)T]}{\sum_{j=-M}^M A_j [(n-\alpha)T - (n-j)T]} \quad (11)$$

where

$$A_j = \frac{1}{\prod_{i=-M, i \neq j}^M (t_j-t_i)} = \frac{1}{\prod_{i=-M, i \neq j}^M (i-j)T} = \frac{1}{(-1)^{M+j} (M+j)!(M-j)! T^{2M}} \quad j = -M, -M+1, \dots, M-1, M \quad (12)$$

By rearranging eqn. 11, we obtain

$$x(n-\alpha) = \sum_{j=-M}^M \left\{ \frac{A_j(j-\alpha)}{\sum_{i=-M}^M A_i(i-\alpha)} \right\} x(n-j) = \sum_{j=-M}^M C_j x(n-j) \quad (13)$$

where

$$C_j = \frac{A_j(j-\alpha)}{\sum_{i=-M}^M A_i(i-\alpha)} \quad (14)$$

Combining eqns. 12 and 14, the coefficients  $C_j$ , which are functions of  $\alpha$ , can be expressed as

$$C_j = \frac{\frac{1}{(j-\alpha)(-1)^{M+j}(M+j)!(M-j)!T^{2M}}}{\sum_{i=-M}^M \frac{1}{(i-\alpha)(-1)^{M+i}(M+i)!(M-i)!T^{2M}}} = \frac{1}{\sum_{i=-M}^M \left[ \frac{(-1)^{j-i} (M+j)!(M-j)!j-\alpha}{(M+i)!(M-i)!i-\alpha} \right]} \quad j = -M, -M+1, \dots, M-1, M \quad (15)$$

The interpolated sample at noninteger multiples of the sampling period,  $x(n-\alpha)$ , can thus be expressed as a linear combination of the samples at integer multiples of sampling period. It can be implemented as a programmable FIR digital filter to synthesise the fractional sample delay  $\alpha T$ . Note that  $\alpha = m/L$ , where  $m$  and  $L$  are integers, is the fractional sample delay factor and  $x(n-\alpha)$  represents the delayed replica of  $x(n)$ .

Fig. 1 shows a general FIR delay filter using the coefficients  $C_j$  given by eqn. 15. The frequency responses for various  $\alpha$  parameters of a fourth-order delay filter ( $N=2, M=4$ ) are shown in Fig. 2. It provides nearly constant delay and nearly flat magnitude response over the passband. The range of constant delay and flat magnitude increases with the order ( $N$ ) of the FIR filter.

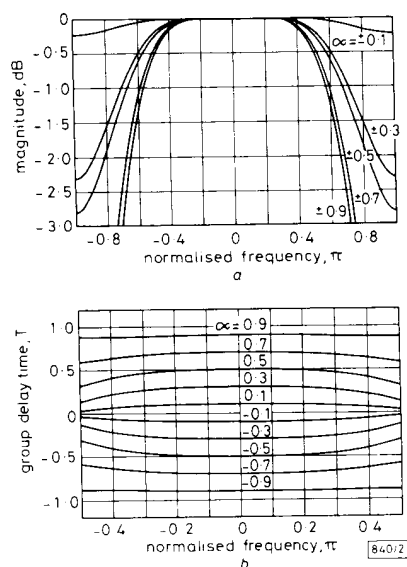


Fig. 2 Frequency responses  
 $N=4$   
 a Magnitude response  
 b Group delay response

**Conclusion:** A new technique is presented for designing and implementing a programmable digital phase shifter (or time delay) which is capable of synthesising any noninteger delays. Derivation of the analytic closed-form has been described. It provides nearly constant delay and nearly flat magnitude response over the passband.

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### References

- 1 FARROW, C. W.: 'A continuously variable digital delay element'. Proc. IEEE Int. Symp. on Circuits and Systems, Espoo, Finland, June 7-9, 1988, pp. 2461-2465
- 2 CROCIENE, R. E., RABINER, L. R., and SHIVELY, R. R.: 'A novel implementation of digital phase shifters', *Bell Syst. Tech. J.*, 1975, **54**, pp. 1497-1502
- 3 HAMMING, R. W.: 'Numerical methods for scientists and engineers' (McGraw-Hill, 1962)

### VERY LINEAR CMOS FLOATING RESISTOR

*Indexing terms:* Circuit theory and design, Active filters, CMOS devices

A very linear CMOS floating resistor is introduced. The proposed topology takes advantage of the MOS transistor characteristics biased in the linear region. It is claimed that the resistor linearity can be improved by reducing the AC voltage swing in the transistor terminals, the drain and the source, and using the linear behaviour between the gate voltage and the drain current. Simulated results, even in the presence of large transistors mismatches, have shown that the total harmonic distortion (THD) is lower than 0.1% for applied voltages up to 2V peak to peak,  $V_{PTP}$ . Resistance values of 500 $\Omega$  and a frequency response up to 10 MHz have been simulated in a typical 3 $\mu\text{m}$  CMOS process. The supply voltages was only  $\pm 2.5$  V.

**Introduction:** Several CMOS fully compatible continuous time systems have been proposed.<sup>1-2</sup> These systems overcome the lack of additional filtering of switched capacitor circuits and offer the possibility of operation in the megaHertz frequency range. It is very important to reduce the harmonic distortion of the building blocks, e.g., OTAs and resistors because these circuits work with large input signals.

Several OTA linearisation techniques have been proposed, but only a few for active resistors. One reason is because the resistors can be simulated by OTAs with the input connected to the output. However, for narrow band applications the time constant of the loop which determines the bandwidth should be much larger than the time constant of the loop which determines the resonant frequency. For these kind of applications the use of active resistors is very attractive.<sup>1,2</sup> CMOS resistors have been designed using saturated transistors<sup>3</sup> or using triode biased transistors.<sup>1,2,4,5</sup> The frequency response of the former type of resistor is quite good but the linear range is limited, especially when the supply voltages are scaled down from  $\pm 5$  V to  $\pm 2.5$  V. The latter resistor implementation uses CMOS transistors biased in the linear region. In this region the output current is approximately<sup>4</sup>

$$i_d = \beta_1 \left\{ \begin{aligned} & (V_g - V_B - V_{FB} - 2\phi_F)(V_d - V_s) \\ & - \frac{1}{2}[(V_d - V_B)^2 - (V_s - V_B)^2] \\ & - \frac{2}{3}7[(-V_d + V_B - 2\phi_F)^{3/2} \\ & - (-V_s + V_B - 2\phi_F)^{3/2}] \end{aligned} \right\} \quad (1)$$