

# 運用 $QFT/H_\infty$ 方法設計多輸入多輸出之懸吊系統

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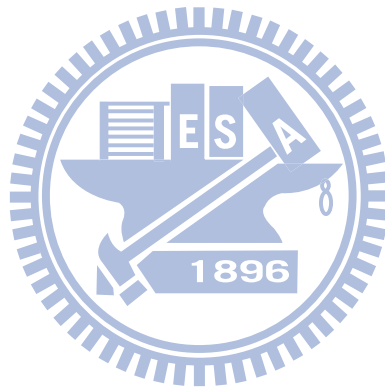
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## 摘 要

本論文將  $QFT/H_\infty$  之設計方法用於四分之一車子 (Quarter Car) 的懸吊系統 (Suspension System) 上，再配合 MIMO 控制理論將控制器設計出來。此四分之一車子的懸吊系統的控制器是以混合靈敏度標準  $H_\infty$  架構 (Mixed Sensitivity Standard  $H_\infty$  Regular Structure) 計算求得的。此控制器之主要架構亦包含三個權重函數 (Weight Function)，此三個權重函數分別為靈敏度權重函數 (Sensitivity Weighting Function,  $W_s$ )、控制權重函數 (Control Effort Weighting Function,  $W_{in}$ ) 以及互補靈敏度權重函數 (Complementary Sensitivity Weighting Function,  $W_r$ )。靈敏度權重函數屬低頻加權，是用來調整靈敏度函數  $S$ 。選取夠小的靈敏度權重函數使得在给定的控制頻寬內，以降低干擾，如性能 (Performance)，以及選取夠小的互補靈敏度權重函數，使得在希望的控制頻寬以外，能幫助確保較好的穩定度邊界，如穩健度 (Robustness)。控制權重函數目的是用來限制系統在高頻時之增益，抑制感測器雜訊干擾，以及調整系統在高頻時增益衰減的速度。

控制方法係結合  $QFT/H_\infty$  技術解決二維自由度 (2-Degree-of-Freedom) 之系統。本論文所提出的方法包含兩種型式，第一種是系統有未確定 (Uncertainty) 變數之穩健控制

(Robust Control)方法。在此控制方法中，控制器是處於一維自由度(1-Degree-of-Freedom)的架構經由  $H_{\infty}$  方法所設計的。第二種的設計方法是  $H_{\infty}$  結合  $QFT$  的方法，在迴路外加上一前置濾波器(Prefilter)，此架構為二維自由度之系統。此一架構可藉由傳統控制理論來完成軌跡追蹤(Tracking)的結果，並將  $H_{\infty}$  結合  $QFT$  的設計程序應用到四分之一車的懸吊系統上，使得 MIMO 懸吊系統之位移在給定的邊界內。



# *QFT / H<sub>∞</sub>* Controller Design of an MIMO Suspension System

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## ABSTRACT

This dissertation uses a combined *QFT / H<sub>∞</sub>* design method for the quarter-model vehicle suspension system to design a controller for the multi-input multi-out (MIMO) control algorithm. This research calculates the controller parameters of the quarter-model suspension system by the mixed-sensitivity standard *H<sub>∞</sub>* regular structure. The controller of the main structure containing three weighting functions, sensitivity weighting function  $W_s$ , control effort weighting function  $W_{un}$ , and complementary weighting function  $W_T$ . The sensitivity weighting function, weighted for low frequency, shapes sensitivity function  $S$ . Choosing  $W_s$  to be small inside the desired control bandwidth to achieve good disturbance attenuation (i.e., performance), and choosing  $W_T$  to be small outside the control bandwidth, help to ensure a good stability margin (i.e., robustness). A control effort weighting function confines the system gain at high frequency, to suppress sensor noise disturbance, and adjusts the gain decay speed at high frequency. The *QFT / H<sub>∞</sub>* combined

techniques copy a 2-degree-of-freedom (2DOF) system. This dissertation proposes two-types of design methodologies. The first design method describes the robust control method, by which the 1-degree-of-freedom (1DOF) structure designs the controller. The second design method adds a pre-filter into the 1DOF, or the 2DOF structure. The current study uses this structure to outcome tracking by the classical control theory, and applies the combined  $QFT/H_\infty$  design procedures in the vehicle quarter-model suspension system, such that displacement of the MIMO suspension system lies in the given boundaries.

**Keywords:**  $QFT/H_\infty$  robustness, Weighting function, MIMO system



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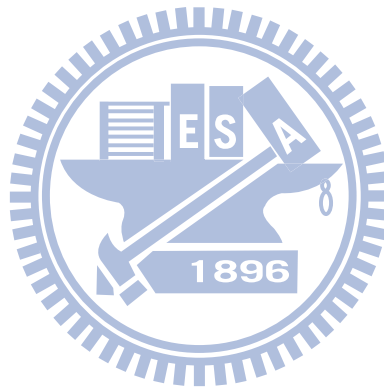
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## NOMENCLATURE

$f_1$	: The driving force
$f_2$	: The driving force
$y_1$	: The displacement
$y_2$	: The displacement
$M_1$	: The mass
$M_2$	: The mass
$B_1$	: The damper
$B_2$	: The damper
$K_1$	: The spring constant
$K_2$	: The spring constant
$W_s$	: The weighting function of sensitivity function
$W_{un}$	: The weighting function of control effort function
$W_T$	: The weighting function of complementary sensitivity function
$F$	: The pre-filter
$d$	: The disturbance
$n$	: The sensor noise
$G$	: The controller
$P$	: The plant
$r$	: The input command
$z$	: The error signal
$w$	: The external disturbance
$\omega_n$	: Undamped natural
$\zeta$	: Damping ratio
$M_p$	: Maximum overshoot
$t_s$	: Settling time