Analytic models of queuing delays for single machine with setup time and dynamic job arrivals

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Abstract

For a single finite-capacity machine that can process several product types of jobs, uncertainties in job arrival time and product type can make the calculation of required setup time and the setting of output target very complicated. Setup activities may cause wastage in machine capacity and extend job lead time. An analytical expression for the expected waiting time by queuing models constitutes a major part of the manufacturing lead time, including the setup time. Models in the literatures assumed batch arrivals and computed the expectation and variance of waiting time of batches with setup time by applying the queuing theory; however, they are unable to respond completely to the impact of setup time on waiting time. Therefore, this research consists of two parts: one is to establish analytical model of queuing delays with setup time of single machine for estimating the lead time for each product type by First-In First-Out (FIFO) rule in order to facilitate the performance evaluation from the customer's perspective, and the other is to efficiently evaluate the effects on capacity-saving by the family-based scheduling rule (FSR) because FSR can reduce setup frequency and amount of setup time. In this research, an individual job arrival with different types of products is assumed as a Poisson process. A setup process is necessary before the machine can switch from the current setting to a different one, and this cannot be regarded as a part of job processing time. The performances of FIFO and FSR analytic models are evaluated in the experimental design, and simulation models under FIFO and FSR are built for accuracy comparisons with the analytic models.

Keywords: lead time, waiting time, setup time, first-in first-out rule, family-based scheduling rule, single machine

考量整備時間與工件動態到達之單機延遲時間分析模型

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摘要

單一機台在產能的限制之下處理多種產品類別，工件到達系統時間與產品類別的不確定性會使得整備時間以及生產目標的設定變得複雜。整備時間的作業會引起機台產能浪費並且延長前置時間。考量整備時間，前置時間的估計主要是以等候模型所建立的期望等候時間分析模型分析之。文獻中多數模型假設在批次到達的條件下，以等後理論標準公式計算等候時間（包含整備時間）的期望值與變異數。然而，直接套用標準公式的作法並未完全考量整備時間對於等候時間的影響。因此，本研究由兩個部分組成：第一，考量整備時間並且透過先進先出派工法則，建立單機延遲時間的分析模型，以此估計每一產品類別的前置時間；其次，透過同族派工法降低換線頻率與整備時間的總量並且建立分析模型以評估其效益。在本研究中，假設不同產品類別個別工件到達系統為一Poisson過程，同時機台整備作業是必要的當進行不同產品類別加工作業轉換時而且此時間不包含在加工時間之內。最後，建立先進先出和同族派工法則的模擬模型用以評估兩者分析模型的效益。

關鍵字：前置時間、等候時間、整備時間、先進先出派工法則、同族派工法、單機

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1. Parameters

j	Serial number for product type
J	Number of product type
k	Serial number for waiting job in queue
n_j	Number of arrived jobs of product type j in time interval (0, RT]
n	Number of jobs in the system
r	Serial number for product type
pt_j	Processing time of product type j
RT	Time interval
s_{rj}	Setup time for product type j job after product type r job
α	Significance level
λ_j	Arrival rate of product type j
λ_r	Arrival rate of product type r
λ^c	Sum of all arrival rates except product type j
λ	Total arrival rate

2. Variables

C_v^2	Squared coefficient of variation of the service time
$DS_{ij,FIFO\text{-}FSR}$	Difference of setup time for the i^{th} job of type j by comparing FIFO and FSR
$\overline{DS}_{FIFO-FSR}$	Mean of the difference of setup time for the i^{th} job of type j by comparing FIFO and FSR
$ETST_{n\text{-}1,ij,FIFO}$	First moment of total service time of $(n-1)$ waiting jobs in queue ahead of the i^{th} job of product type j if the i^{th} job of product type j arrives at the system in the interval (0, RT] and there are n ($n \geqq 2$) jobs present in the system
$ESTST_{n\text{-}1,ij,FIFO}$	Second moment of total service time of $(n-1)$ waiting jobs in queue ahead of the i^{th} job of product type j if the i^{th} job of product type j arrives at the system in the interval (0, RT] and there are n ($n \geqq 2$) jobs present in the system

NS_{FIFO}	Number of setups for all jobs under FIFO
$NS_{j,FIFO}$	Number of setups for product type j under FIFO
$NS_{ij,FIFO}$	Number of requiring setups for the i^{th} job of product type j under FIFO
NS_{FSR}	Number of setups for all jobs under FSR
$NS_{j,FSR}$	Number of setups for product type j under FSR
$NS_{ij,FSR}$	Number of requiring setups for the i^{th} job of product type j under FSR
$LT_{j,FIFO}$	Lead time of the product type j job under FIFO
$p_{0,FIFO}$	Probability that there are no jobs present in the system under FIFO
$p_{n,FIFO}$	Probability that there are n ($n \geqq 1$) jobs present in the system under FIFO
$P_{s,j,FIFO}$	Probability that a setup is required given a product type j job arrived at the system in time interval (0, RT], at where jobs are dispatched by FIFO
$P_{s,ij,FIFO}$	Probability for the i^{th} job of product type j arriving at the system in time interval (0, RT] with a setup under FIFO
$P_{ns,ij,FIFO}$	Probability that a setup does not need for the i^{th} job of product type j under FIFO
$P_{setups,FIFO}^{n=0}$	Probability of an arriving job of the product type j requiring a setup under FIFO with no jobs in the system
$P_{setups,FIFO}^{n\geq 1}$	Probability of an arriving job of the product type j requiring a setups under FIFO with $n \geqq 1$ jobs in the system
$p_{0,FSR}$	Probability that there are no jobs present in the system under FSR
$p_{n,FSR}$	Probability that there are n ($n \geqq 1$) jobs present in the system under FSR
$P_{s,j,FSR}$	Probability that a setup is required given a product type j job arrived at the system in time interval (0, RT], at where jobs are dispatched by FSR
$P_{s,ij,FSR}$	Probability for the i^{th} job of product type j arriving at the system in time interval (0, RT] with a setup under FSR
$P_{ns,ij,FSR}$	Probability that a setup does not need for the i^{th} job of product type j under FSR
$P_{setups,FSR}^{n=0}$	Probability of an arriving job of the product type j requiring a

	setup under FSR with no jobs in the system
$P^{n\geq 1}_{setups,FSR}$	Probability of an arriving job of the product type j needs a setups under FSR with $n \geqq 1$ jobs in the system
PT_j	Variable of processing time of the product type j job
$R_{ijn,FIFO}$	Residual service time of the i^{th} job of product type j if there are n ($n \geqq 1$) jobs in the system during the arrival time of the i^{th} job of product type j
$R_{ij,FIFO}$	Time of the i^{th} arrived job of product type j has to wait until one unfinished job completed its processing on the machine
$R^2_{ijn,FIFO}$	Square of residual service time of the i^{th} job of product type j if there are n ($n \geqq 1$) jobs in the system during the arrival time of the i^{th} job of product type j
$R^2_{ij,FIFO}$	Square of time of the i^{th} arrived job of product type j has to wait until one unfinished job completed its processing on the machine
$S_{ij,FIFO}$	Setup time of the i^{th} job of product type j under FIFO
$S_{j,FIFO}$	Setup time of the product type j job under FIFO
S_{FIFO}	Overall mean setup time of jobs under FIFO
$S_{ij,FSR}$	Setup time of the i^{th} job of product type j under FSR
$S_{j,FSR}$	Setup time of the product type j job under FSR
S_{FSR}	Overall mean setup time of jobs under FSR
$ST_{ij,FIFO}$	Service time of the i^{th} job of product type j under FIFO
$ST_{j,FIFO}$	Service time of the product type j job under FIFO
ST_{FIFO}	Overall mean service time of jobs under FIFO
$ST^2_{ij,FIFO}$	Square of service time of the i^{th} job of product type j under FIFO
$ST^2_{j,FIFO}$	Square of service time of the product type j under FIFO
$ST_{ij,FSR}$	Service time of the i^{th} job of product type j under FSR
$ST_{j,FSR}$	Service time of the product type j job under FSR
ST_{FSR}	Overall mean service time of jobs under FSR
T_{ij}	Arrival time of the i^{th} arrived job of product type j
$TST_{ij,FIFO}$	Total service time of all waiting jobs in queue ahead of the i^{th} arrived job of product type j
$TST^2_{ij,FIFO}$	Square of total service time of all waiting jobs in queue ahead of the i^{th} arrived job of product type j
$W_{q,ij,FIFO}$	Waiting time of the i^{th} arrived job of product type j under FIFO
$W_{q,j,FIFO}$	Waiting time of the product type j job under FIFO

$W^2_{q,j,FIFO}$	Square of waiting time of the product type j job under FIFO
w_j	Mean of all probabilities of product type j jobs arriving at the system in the time interval (0, RT]
ρ_{FIFO}	Utilization rate of the machine under FIFO