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多變量變幅波動模型的理論與應用

Three Essays of Range-based Multivariate Volatility Models

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摘要

本文提出多變量的動態變幅（range）波動模型，並探討其在財務相關議題的應用，內容主要分成三個部分。第一部份提出以變幅為基礎的動態條件相關係數（dynamic conditional correlation，DCC）模型，簡稱為 range-based DCC 模型，其結合 DCC 模型以及條件變幅波動（conditional autoregressive range，CARR）模型在波動性預測方面的優勢，藉此改善共變異數矩陣估計的準確性，並且以 S&P 500 股價指數和 10 年期的債券期貨做為樣本，進行樣本內和樣本外共變異數預測能力的比較，實證結果指出，在所建立的已實現變異數（realized covariance）指標下，range-based DCC 模型表現優於文獻上常見之報酬為基礎的波動模型（包含 MA100、EWMA、CCC、BEKK 和 DCC 模型）。第二部分則是基於平均數-共變異數的架構下，結合效用函數，驗證 range-based DCC 模型之波動擇時的經濟價值，實證結果支持此模型具有顯著的經濟價值，並且更勝於以報酬為基礎的 DCC 模型。第三部分則是以 range-based CCC 和 range-based DCC 模型計算最小變異數的避險比例（minimum variance hedge ratio），並且應用在商品期貨的避險，透過所選用 15 種商品（包含股價指數、匯率、金屬、農產品、軟性商品和能源市場）的驗證，得知變幅為基礎的波動模型可以有效改善避險績效，並且明顯優於其他以報酬為基礎的波動模型（包含 OLS、rollover OLS、CCC 和 DCC 模型）。

關鍵字：DCC 模型、CARR 模型、變幅、動態波動性、經濟價值、波動擇時、避險比例和最小變異數避險

Three Essays of Range-based Multivariate Volatility Models

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Abstract

This dissertation is intended as an investigation of dynamic range volatility models. There are three main parts in this study. In the first part, we propose a range-based DCC model combined by the return-based DCC model and the CARR model. The substantial gain in efficiency of volatility estimation can boost the accuracy for estimating time-varying covariances. As to the empirical study, we use the S&P 500 stock index and the 10-year treasury bond futures to examine both in-sample and out-of-sample results for six models, including MA100, EWMA, CCC, BEKK, return-based DCC, and range-based DCC. In the second part, the range-based volatility model is used to examine the economic value of volatility timing in a mean-variance framework. We compare its performance with a return-based dynamic volatility model in both in-sample and out-of-sample volatility timing strategies. For a risk-averse investor, we examine whether the predictable ability captured by the range-based volatility models is economically significant or not. In the last part, we use ranges to estimate the minimum variance hedge ratios within the framework of the CCC model and the DCC model. Other alternative methods used for comparison include the static OLS model, the week-by-week rollover OLS model, the return-based CCC model, and the return-based DCC model. While the spot price risk is hedged by their corresponding futures, we compare the out-of-sample performances of the hedging strategies for the selected commodities, including stock index, currency, metal, grain, soft, and energy markets. Overall, the range-based volatility models perform better than the other selected volatility models in the empirical studies.

Keywords: DCC model, CARR model, Range, Dynamic volatility, Economic value, Volatility timing, Hedge ratio, Minimum variance hedge.

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Chapter 1. Introduction

With the continual development of new financial instruments, there is a growing demand for theoretical and empirical knowledge of the financial volatility. It is well-known that financial volatility has played such a central role in derivative pricing, asset allocation, and risk management.

Many studies show that financial time series exhibit volatility clustering or autocorrelation. In incorporating the characteristics into the dynamic process, the generalized autoregressive conditional heteroskedasticity (GARCH) family of models proposed by Engle (1982) and Bollerslev (1986) are popular and useful alternatives for estimating and modeling time-varying financial volatility. However, as pointed by Alizadeh, Brandt, and Diebold (2002), Brandt and Diebold (2006), Chou (2005) and other authors, GARCH models are inaccurate and inefficient, because they are based on the closing prices, of the reference period, failing to use the information contents inside the reference. In other words, the path of the price inside the reference period is totally ignored when volatility is estimated by these models. Especially in turbulent days with drops and recoveries of the markets, the traditional close-to-close volatility indicates a low level while the daily price range shows correctly that the volatility is high.

The price range, defined as the difference between the highest and lowest market prices over a fixed sampling interval, has been known for a long time and recently experienced renewed interest as an estimator of the latent volatility. This information is widely used in Japanese candlestick charting techniques and other technical indicators (Nisson, 1991). Early application of range in the field of finance can be traced to Mandelbrot (1971), and the academic work on the range-based volatility estimator started from the early 1980s. Several authors, back to Parkinson (1980), developed from it several volatility measures far more efficient than the classical return-based volatility estimators.

Building on the earlier results of Parkinson (1980), many studies[1] show that one can use the price range information to improve volatility estimation. Cox and Rubinstein (1985) stated the puzzle that despite the elegant theory and the support of simulation results, the range-based volatility estimator has performed poorly in empirical studies. Chou (2005) argued that the failure of all the range-based models in the literature is caused by their ignorance of the temporal movements of price range. Using a proper dynamic structure for the conditional expectation of range, the conditional autoregressive range (CARR) model, proposed by Chou (2005), successfully resolves this puzzle and retains its superiority in empirical forecasting abilities.

There are three parts in this essay. They present three independent papers, respectively. In the first part of this dissertation, we extend the CARR model to a multivariate context using the dynamic conditional correlation (DCC) model proposed by Engle (2002a). In the empirical studied, we use the S&P 500 stock index and the 10-year treasury bond futures to examine both in-sample and out-of-sample results for six models, including MA100, EWMA, CCC, BEKK, return-based DCC, and range-based DCC. Of all the models considered, the range-based DCC model is largely supported in estimating and forecasting the covariance matrices.

In the second part, we calculate the economic value gained by the range-based DCC model. Moreover, we also compare its performance with the return-based DCC model in both in-sample and out-of-sample volatility timing strategies. For a risk-averse investor, it is shown that the predictable ability captured by the dynamic

[1] See Garman and Klass (1980), Wiggins (1991), Rogers and Satchell (1991), Kunitomo (1992), Yang and Zhang (2000), Alizadeh, Brandt and Diebold (2002), Brandt and Diebold (2006), Brandt and Jones (2006), Chou (2005, 2006), Martens and van Dijk (2007), Chou, Wu and Liu (2007).

volatility models is economically significant, and that the range-based volatility model performs better than the return-based one.

In the last part, we use the range-based volatility models to calculate the hedge ratio and compare their hedging performance with other methods, including the static OLS model, the week-by-week rollover OLS model, the return-based CCC model, and the return-based DCC model. Based on minimum-variance hedging criterion, the out-of-sample comparisons show that the range-based volatility models perform better than the other hedging models for most of the selected commodities, including the stock index, the currency, the metal, the grain, the soft, and the energy markets. Compared with the static OLS model, on average, the range-based DCC model has about 30 percent efficiency gain. Furthermore, with the same setting of dynamic structure of the return-based hedging strategies, the range-based ones can get about 10 percent additional efficiency gain.

Chapter 2. Forecasting Time-varying Covariance with a Range-Based Dynamic Conditional Correlation Model

2.1 Introduction

It is of primary importance in the practice of portfolio management, asset allocation and risk management to have an accurate estimate of the covariance matrices for asset returns. Meanwhile, a useful approach for estimating volatilities and covariances in valuing derivatives is necessary. Surveying from a bundle of past related literature, the univariate ARCH/GARCH family of models have provided effective tools in estimating the volatility of individual asset. Tailored to the needs of different asset classes, these various models have achieved remarkable success (see Bollerslev, Chou, and Kroner (1992), and Engle (2004), for a comprehensive review). However, estimating the covariance and correlation matrices of multiple variables, especially large sets of asset prices, is still an active research issue. Early attempts include the VECH model[2] of Bollerslev, Engle, and Wooldridge (1988), the BEKK (Baba-Engle-Kraft-Kroner) model[3] of Engle and Kroner (1995), and the constant conditional correlation (CCC) model of Bollerslev (1990), among others. To our knowledge, the constant correlation model is too restrictive in that it imposes stringent constraints whereby the dynamic structure of the covariance is completely determined by individual volatilities. VECH and BEKK are, however, more flexible in that they allow time-varying correlations. While the BEKK parameterization for a bivariate model involves 11 parameters, for higher-dimensional systems, the additional parameters in BEKK make estimation very difficult.

---

[2] The k-dimensional VECH model is written as vech(H_t)=A+B vech($\xi_{t-1}\xi_{t-1}'$)+C vech(H_{t-1}), where H_t is the conditional covariance matrix at time t and vech(H_t) is the vector that stacks all the elements of the covariance matrix.

[3] It is a general parameterization that involves the minimum number of parameters while imposing no cross equation restrictions and ensuring positive definiteness for any parameter value.

In a series of related papers, Engle and Sheppard (2001), Engle (2002a), and Cappiello, Engle, and Sheppard (2006) provide another viewpoint to this problem by using a model referred to the dynamic conditional correlation (DCC) multivariate GARCH[4]. Intuitively, the conditional covariance estimation for two variables is simplified by estimating univariate GARCH models for each asset's variance process. Then, the estimation of the time-varying conditional correlation is performed by using the transformed standardized residuals. A meaningful and excellent performance of this model is demonstrated in these studies.

The objective of this article is to propose an alternative to the return-based DCC approach. In this paper, we consider a refinement of the return-based DCC model by utilizing the high/low range data of asset prices during a fixed time interval. In estimating the volatility of asset prices, there is a growing recognition of the fact that the range data of asset prices can provide sharper estimates and forecasts than the return data based on close-to-close prices. Many insightful studies have provided powerful evidence including Parkinson (1980), Garman and Klass (1980), Wiggins (1991), Rogers and Satchell (1991), Kunitomo (1992) and, more recently, Gallant, Hsu, and Tauchen (1999), Yang and Zhang (2000), Alizadeh, Brandt, and Diebold (2002), Brandt and Jones (2006), Chou (2005, 2006), and Martens and van Dijk (2007). Above all, Chou (2005) proposes the conditional autoregressive range (CARR) model which can capture the dynamic volatility process and has obtained some insightful evidence in terms of real trading data. In other words, a range-based volatility model can serve as a useful substitution for the return-based volatility model in describing the process of volatility.

Range data intuitively have more information than return data for estimating

[4] Other econometric methods for estimating the time-varying correlation are proposed by Tsay (2002) and by Tse and Tsui (2002).

volatility. Again, they are easy to obtain for many financial markets. The previous studies have proved that range is an efficient volatility estimator[5]. Moreover, Chou (2005) puts the range into the dynamic process, and verifies that the range model can also fit time-varying volatility well. In light of the success of the range-based univariate volatility models, it is natural to inquire whether the efficiency of the range structure can be extended and incorporated into a multivariate framework[6] for constructing covariance process.

The remainder of this chapter is laid out in the following manner. Section 2.2 reviews the bivariate models for estimating the covariance process. Section 2.3 introduces the range-based volatility model and the DCC model. Section 2.4 describes the properties of data used and discusses the empirical results. Finally, the conclusion is showed in section 2.5.

2.2 Covariance Estimation

This section provides an overview of methods for describing the current level of covariance. Conventionally, the conditional covariance estimation between two return series is defined as:

$$COV_{12,t} = E_{t-1}[(r_{1,t} - \mu_1)(r_{2,t} - \mu_2)], \tag{2.1}$$

where $\mu_i = E(r_{i,t})$. In most applications, asset returns are assumed to have zero means. This common viewpoint is adopted in our study. Thus, equation (2.1) can be expressed as $COV_{12,t} = E_{t-1}(r_{1,t} r_{2,t})$.

It is useful to estimate time-varying covariance parameters between asset returns

---

5 Shu and Zhang (2006) provide relative performance of different range-based volatility estimators, and find that the range estimators all perform very well when an asset price follows a continuous geometric Brownian motion.

6 Fernandes, Mota, and Rocha (2005) utilize the formula *Cov(X,Y)=[V(X+Y)-V(X)-V(Y)] /2* to propose a kind of multivariate CARR model. However, this method limits the multivariate CARR model to a bivariate case only.

in many financial applications. For example, they can be used to deal with the hedging ratio for futures, the optimal weights for the portfolio allocation, the time-varying beta for the market model, and so on. The information of the conditional covariance is derived from previous trading data. One commonly used method is to compute the historical covariance. For capturing the time-varying property of covariances, however, one approach we use works with a moving average with a 100-week window, namely MA100, which is rich enough to be relevant and yet simple enough to permit a streamlined exposition:

$$COV_{12,t}^{MA100} = \frac{1}{100}\sum_{s=t-100}^{t-1} r_{1,s} r_{2,s} \ . \tag{2.2}$$

Intuitively, it is reasonable to attach more weight to recent data. Going by this, we introduce an exponentially weighted moving average (EWMA) model where the weights decrease exponentially as we move back through time. Exponential smoothing is used to model the unobservable variables for volatility in JP Morgan's RiskMetrics, too. EWMA has an attractive feature in that relatively little data need to be stored. Exponential averages arrange the most weight to the most recent observations, with weights declining exponentially as observations go back in time. It turns out that EWMA for covariance estimation can briefly be illustrated as follows.

$$COV_{12,t}^{EWMA} = (1-\lambda)\sum_{s=1}^{\infty} \lambda^{s-1} r_{1,t-s} r_{2,t-s} \ , \tag{2.3}$$

where the smoothing parameter λ lies between zero and unity. The value of λ governs how sensitive the estimate of the current variable is to percent changes in the most recent period. The popular RiskMetrics approach adopts exponential moving averages[7] to estimate future volatility because it believes the method responds rapidly

[7] The RiskMetrics database uses the exponentially-weighted moving average model with λ=0.94 for updating daily volatility estimates. J.P. Morgan found that, across variant market variables, this value

to market shocks.

The conditional variance-covariance matrix can build a multivariate ARCH model. This approach has been extracted by Engle and Kroner (1995), who proposed the so-called BEKK model. The parameters, however, easily diverge from the acceptable scope when the type of the full-rank BEKK model is adopted. In the related literature, the diagonal BEKK (DBEKK) model is adopted more frequently due to its property of convergence of parameters used in general empirical research. Considering the bivariate case for DBEKK, its covariance matrix $H_t^{DBEKK} = [h_{ij,t}]$ is shown as below:

$$H_t^{DBEKK} = \begin{bmatrix} c_{11} & 0 \\ c_{12} & c_{22} \end{bmatrix} \begin{bmatrix} c_{11} & c_{12} \\ 0 & c_{22} \end{bmatrix} + \begin{bmatrix} a_{11} & 0 \\ 0 & a_{22} \end{bmatrix} \begin{bmatrix} \varepsilon_{1,t-1}^2 & \varepsilon_{1,t-1}\varepsilon_{2,t-1} \\ \varepsilon_{1,t-1}\varepsilon_{2,t-1} & \varepsilon_{t-1}^2 \end{bmatrix} \begin{bmatrix} a_{11} & 0 \\ 0 & a_{22} \end{bmatrix}$$

$$+ \begin{bmatrix} b_{11} & 0 \\ 0 & b_{22} \end{bmatrix} \begin{bmatrix} h_{1,t-1} & h_{12,t-1} \\ h_{21,t-1} & h_{22,t-1} \end{bmatrix} \begin{bmatrix} b_{11} & 0 \\ 0 & b_{22} \end{bmatrix}, \quad (2.4)$$

where a_{ij}, b_{ij}, c_{ij} are estimated parameters. ε_t represents the innovation term of the mean equation under the assumption $\varepsilon_t \mid I_{t-1} \sim (0, H_t)$.

2.3 The Range-based Volatility Model and the DCC Model

The asset high/low range, $\Re_t$, is defined as the difference between the daily high and low prices in a logarithm type over a fixed time period. It is readily available for some assets and can be written as:

$$\Re_t = \ln(H_t) - \ln(L_t), \quad (2.5)$$

where H_t and L_t are the highest and lowest intraday price over a fixed period such as daily, weekly, or monthly. For weekly data, the highest price of a week is its intraday highest price that we can observe over the trading time in the week. Unlike the intraday

of λ results in forecasts of the volatility that come closest to the realized volatility. Following J.P. Morgan's suggestion, the variable λ equals 0.94 for the time being in the later empirical discussion.

realized volatility, the range therefore does not have a time-aggregation problem.

The previous studies indicated that range has relative efficient, but did not empirical support. Chou (2005) argues that its poor performance is due to the poor dynamic fitting, and further, proposes the CARR model to capture its dynamic structure. The CARR can be expressed as:

$$\begin{aligned} &\Re_t = \lambda_t u_t,\ u_t \mid I_{t-1} \sim \exp(1;\cdot) \\ &\lambda_t = \omega + \alpha \Re_{t-1} + \beta \lambda_{t-1} \end{aligned}, \tag{2.6}$$

where $\Re_t$ and λ_t is the high/low range and the conditional mean of the range during the time interval *t*, respectively. u_t is the innovation assumed to follow the exponential distribution with a unit mean.

The CARR model is a special case of the multiplicative error model (MEM) of Engle (2002b)[8]. The specification of the exponential distribution for the disturbance term provides a consistent estimator of the parameters. For specific discussions, see Chou (2005) for a review. This paper extends this range model to a multivariate case by the DCC model.

Bollerslev (1990) proposed the CCC model with a constant correlation matrix, where univariate GARCH models are estimated for each asset and then the corresponding correlation matrix is constructed. An illustration of CCC is shown below. The covariance matrix H_t^{CCC} for a vector of k asset returns can be decomposed as follows:

$$H_t^{CCC} = D_t R D_t, \tag{2.7}$$

where *R* is the correlation matrix and D_t is the $k \times k$ diagonal matrix of time-varying standard deviations from univariate GARCH models with $\sqrt{h_{i,t}}$ on the i^{th} diagonal. As

[8] The MEM model is designed to fit a non-negative series, like duration or realized volatility.

for the $\sqrt{h_{i,t}}$, it is the square root of the estimated variance for the i^{th} return series. The assumption of a constant correlation makes estimating a large model feasible and ensures that the estimator is positive definite, simply requiring each univariate conditional variance to be non-zero and the correlation matrix to be of full rank. Under such a situation, the estimate of the conditional covariance can be obtained, based on information regarding the fixed correlation and the product of the two conditional standard deviations.

Although CCC is meaningful, the setting of constant conditional correlations could sometimes be too restrictive and the estimators in the constant correlation setting, as proposed, do not offer a rule to construct consistent standard errors, using the multi-stage estimation process. Another shortcoming for the constant correlation model is that the correlation coefficient tends to change over time in real applications. Engle (2002a) extended CCC to the more comprehensive DCC type. DCC retains the parsimony of the univariate GARCH model of individual assets' volatilities with a simple GARCH-like time varying correlation. Meanwhile, DCC differs from CCC mainly in that it allows the correlation matrix to be changed over time. Accordingly, we can write DCC as:

$$H_t^{DCC} = D_t R_t D_t , \tag{2.8}$$

$$R_t = diag\{Q_t\}^{-1/2} Q_t diag\{Q_t\}^{-1/2} , \tag{2.9}$$

$$Q_t = S \circ (\iota\iota' - A - B) + A \circ Z_{t-1} Z_{t-1}' + B \circ Q_{t-1} , \tag{2.10}$$

where D_t is defined as in equation (2.7) and R_t is the possibly time-varying correlation matrix. $Q_t = [q_{ij,t}]$ denotes the conditional covariance matrix of the standardized residuals.

In equation (2.10), A and B are parameter matrices and $\circ$ denotes the Hadamard matrix product operator, i.e. element-wise multiplication. The symbol ι denotes a vector of ones and S denotes the unconditional covariance matrix of the standardized residuals. Finally, $Z_t = [z_{i,t}]$ is the standardized but correlated residual vector, and its conditional correlation matrix is given by variable R_t. If A and B are zeros, then the DCC model can revert to the structure of CCC. Related literature shows that if A, B, and $(\iota\iota' - A - B)$ are positive semi-definite, then Q_t will also be positive semi-definite. If any one of the matrices is positive definite, then Q_t will also be so. For the ij^{th} element of R_t, the conditional correlation matrix is given by $q_{ij,t}/\sqrt{q_{ii,t}q_{jj,t}}$. In our study, we focus on the comparison of forecasting covariances for two assets and equation (2.10) has the following structure in a bivariate case,

$$\begin{bmatrix} q_{11,t} & q_{12,t} \\ q_{12,t} & q_{22,t} \end{bmatrix} = (1-a-b)\begin{bmatrix} 1 & \bar{q}_{12} \\ \bar{q}_{12} & 1 \end{bmatrix} + a\begin{bmatrix} z_{1,t-1}^2 & z_{1,t-1}z_{2,t-1} \\ z_{1,t-1}z_{2,t-1} & z_{2,t-1}^2 \end{bmatrix} + b\begin{bmatrix} q_{11,t-1} & q_{12,t-1} \\ q_{12,t-1} & q_{22,t-1} \end{bmatrix}, \quad (2.11)$$

where a and b are parameters. In most cases, they can substitute for complicated matrices A and B. $\bar{q}_{12}$ is the unconditional covariance of the two standardized residuals.

The DCC model is constructed to permit for two-stage estimation of the conditional covariance matrix H_t. Briefly speaking, during the first step, a univariate volatility model is fitted for each of the assets and the estimates of $h_{i,t}$ are obtained. In the second step, the asset returns transformed by their estimated standard deviations are used to estimate the parameters of the conditional correlation.

The log-likelihood of this estimator is straightforward. One simply maximizes the log-likelihood:

$$L = -\frac{1}{2}\sum_t \left(k\log(2\pi) + \log|H_t| + r_t' H_t^{-1} r_t\right)$$

$$= -\frac{1}{2}\sum_t \left(k\log(2\pi) + \log|D_t R_t D_t| + r_t' D_t^{-1} R_t^{-1} D_t^{-1} r_t\right) \qquad (2.12)$$

$$= -\frac{1}{2}\sum_t \left(k\log(2\pi) + 2\log|D_t| + \log|R_t| + Z_t' R_t^{-1} Z_t\right).$$

Following Engle (2002a)'s argument, one can perform the estimation by means of qusi-maximum likelihood estimation (QMLE) to yield consistent parameter estimates. The advantages of QMLE are its simplicity and consistency. However, its disadvantages are that the estimates are inefficient, even asymptotically, and more importantly, its small-sample properties are suspect. (also see Hafner and Franses (2003) for a review.) Let the parameters in D_t be denoted by θ_1 and the additional parameters in R_t be denoted by θ_2. According to Engle (2002a), one can divide the log-likelihood function into two parts:

$$L(\theta_1, \theta_2) = L_{Vol}(\theta_1) + L_{Corr}(\theta_1, \theta_2). \qquad (2.13)$$

The former term in the right hand side of equation (2.13) represents the volatility part:

$$L_{Vol}(\theta_1) = -\frac{1}{2}\sum_t \left(k\log(2\pi) + \log|D_t|^2 + r_t' D_t^{-2} r_t\right), \qquad (2.14)$$

and the latter term can be viewed as the correlation component:

$$L_{Corr}(\theta_1, \theta_2) = -\frac{1}{2}\sum_t \left(\log|R_t| + Z_t' R_t^{-1} Z_t - Z_t' Z_t\right). \qquad (2.15)$$

Following the recipe for the first stage, we can pick up a suitable θ_1 easily, which satisfies equation (2.14) and is maximized after the estimate of $\hat{\theta}_1$ is computed. Subsequently, in the second stage, the correlation part in equation (2.15) can be maximized with respect to the optimized θ_1 and θ_2 simultaneously. Consequently, the formidable task of maximizing equation (2.13) is attainable. Estimates for $\hat{\theta}_1$ and

$\hat{\theta}_2$ are useful in subsequent analysis.

It is interesting and important to recognize that although the dynamics of the D_t matrix has usually been structured as a standard GARCH model, it can be easily extended to many other types of models. For instance, one could adopt the EGARCH or GJR-GARCH model to replace the simple GARCH model for describing the asymmetric phenomenon in the actual volatility process or use the FIGARCH model to allow for the long memory volatility processes. In this paper, the CARR model of Chou (2005) will be used as an alternative to verify if the specification selected adequately fit the DCC model.

When the specific GARCH model is fitted, the term of volatility in the likelihood function can be demonstrated as below:

$$L_{Vol}^{GARCH}(\theta) = -\frac{1}{2}\sum_{t}\sum_{i=1}^{k}\left(\log(2\pi) + \log(h_{i,t}) + \frac{r_{i,t}^2}{h_{i,t}}\right). \tag{2.16}$$

By the same token, if D_t is determined by a CARR specification, then the likelihood function of the volatility term will be modified as:

$$L_{Vol}^{CARR}(\theta) = -\frac{1}{2}\sum_{t}\sum_{i=1}^{k}\left(\log(2\pi) + 2\log(\lambda_{i,t}^{*}) + \frac{r_{i,t}^2}{\lambda_{i,t}^{*2}}\right), \tag{2.17}$$

where $\lambda_{i,t}^{*}$ denotes the conditional standard deviation as computed from a scaled expected range, using the CARR model.

The second part of the likelihood function will be used to estimate the parameters for correlations. As the squared residuals are not dependent on these parameters, they will not appear in the first-order conditions and can be neglected. A simple transformation of the two-stage framework to maximize the likelihood function is achieved. Apparently, $\hat{\theta}_1 = \arg\max\{L_{Vol}(\theta_1)\}$ and then we extract this value $\hat{\theta}_1$ as given, into the second step, $\max_{\theta_2}\left\{L_{Corr}\left(\hat{\theta}_1,\theta_2\right)\right\}$. It is shown in Engle and Sheppard (2001) that under some regularity conditions, the condition for consistency will be satisfied. Maximization of equation (2.15) will be a function of the parameter estimates from equation (2.14). These conditions are similar to those given in White (1994), where the asymptotic normality and the consistency of the two-step QMLE estimator are established.

The following GARCH and CARR structures can be performed in the first step of the DCC estimation. As to the GARCH volatility structure, the function form can be illustrated as below:

$$r_{i,t}=\varepsilon_{i,t} \qquad \varepsilon_{i,t}\mid I_{t-1}\sim N(0,h_{i,t}),\ i\text{=1,2.}$$

$$h_{i,t}=\omega_i+\alpha_i\varepsilon^2_{i,t-1}+\beta_i h_{i,t-1}, \tag{2.18}$$

$$z^{GARCH}_{i,t}=r_{i,t}/\sqrt{h_{i,t}}\ .$$

In addition to the original GARCH model embedded in DCC, one can replace it with the CARR framework. CARR is powerful in capturing the volatility process. It is intuitive to put CARR into the first stage, which is particularly convenient for complex dynamic systems in operation. It means the new standardized residuals can be obtained from the CARR model, that is $z^{CARR}_{i,t}=r_{i,t}/\lambda^*_{i,t}$, where $\lambda^*_{i,t}=adj_i\times\lambda_{i,t}$ and $adj_i=\dfrac{\overline{\sigma}_i}{\overline{\hat{\lambda}}_i}$.

The rescaled expected range $\lambda^*_{i,t}$ is used to replace the conditional standard deviation. It is computed by a product of $\lambda_{i,t}$ and the adjusted coefficient adj_i which is the ratio of unconditional standard deviations $\overline{\sigma}_i$ for the return series to the sample mean $\overline{\hat{\lambda}}_i$ of the estimated conditional range.

In performing a comparison of the in-sample data during subsequent empirical analysis of the covariance matrices, several related and conventional models are included - MA100, EWMA[9] with $\lambda = 0.94$, CCC, and DBEKK models.

For robustness of inference, we also perform out-of-sample forecast comparisons. The out-of-sample forecast of the DCC model for correlations can be obtained using the standard forward iterative approach; given T as the sample size, the $T+1^{th}$ observation will be obtained.

At time T, the out-of-sample forecast for conditional correlation in the period ($T+1$) is presented by:

$$\begin{bmatrix} q_{11,T+1} & q_{12,T+1} \\ q_{12,T+1} & q_{22,T+1} \end{bmatrix} = (1-a-b)\begin{bmatrix} 1 & \bar{q}_{12} \\ \bar{q}_{12} & 1 \end{bmatrix} + a\begin{bmatrix} z_{1,T}^2 & z_{1,T}z_{2,T} \\ z_{1,T}z_{2,T} & z_{2,T}^2 \end{bmatrix} + b\begin{bmatrix} q_{11,T} & q_{12,T} \\ q_{12,T} & q_{22,T} \end{bmatrix}, \quad (2.19)$$

The estimated correlation at time $T+1$ can be calculated as $\rho_{T+1} = q_{12,T+1} / \sqrt{q_{11,T+1}q_{22,T+1}}$. The out-of-sample prediction for correlation for the period ($T+p$), where $p \geq 2$, can be expressed as shown below:

$$\begin{bmatrix} q_{11,T+p} & q_{12,T+p} \\ q_{12,T+p} & q_{22,T+p} \end{bmatrix} = (1-a-b)\begin{bmatrix} 1 & \bar{q}_{12} \\ \bar{q}_{12} & 1 \end{bmatrix} + (a+b)\begin{bmatrix} q_{11,T+p-1} & q_{12,T+p-1} \\ q_{12,T+p-1} & q_{22,T+p-1} \end{bmatrix} \quad (2.20)$$

In addition to range-based and return-based DCC, MA100, EWMA, CCC and DBEKK are introduced for an out-of-sample predictive comparison[10]. For distinguishing the forecasting abilities of these models, as in Taylor (2004), we still use root mean square error (RMSE) and mean absolute error (MAE) as two criterions for comparison.

2.4 Comparison of Various Methods for Conditional Covariance Forecasts

The data employed for our empirical study comprise 782 weekly observations on the S&P 500 stock index (S&P 500) futures, and the 10-year treasury bond (Tbond) futures

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[9] The estimate of λ is 0.94 approximately for the returns that we adopted in this study.
[10] It is also intuitively clear that the out-of-sample forecasts for the covariance are all constant in the EWMA model.

spanning the period from January 6, 1992 to December 29, 2006 (15 years). We retrieve range and return data for the entire period from Datastream.

< Figure 2.1 is inserted about here >

Figure 2.1 shows the graphs for close prices (Panel A), returns (Panel B) and ranges (Panel C) of S&P 500 and Tbond futures over the sample period. The descriptive statistics for the returns and ranges of the series are given in Table 2.1. For the weekly returns and ranges of the S&P 500 and Tbond futures, they are computed by $100\times\log(p_t^{High}/p_t^{Low})$ and $100\times\log(p_t^{close}/p_{t-1}^{close})$, respectively. Table 2.1 shows that the means of two futures returns are positive. Both the standard deviations and the means of the ranges indicate that S&P 500 is more volatile than Tbond. For higher moments of the return data, each of them has negative skewness and excess kurtosis. As to the range data, they also have excess kurtosis values, but positive skewness coefficients. These largely contribute to the rejection for the null hypothesis of a normal distribution with the Jarque-Bera statistic.

< Table 2.1 is inserted about here >

2.4.1 Measured Covariances

Like the specific property of volatilities, the covariance matrices are also unobservable. In this work, we use daily data to construct the proxies for the weekly covariances. The purpose behind doing this is to extract the values of the measured covariances (MCOVs), as one kind of benchmark for determining the relative performance of return-based DCC and range-based DCC, for the time being.

Daily data are used to build four proxies for covariances, including implied return-based DCC, implied range-based DCC, implied DBEKK, and realized covariances. Initially, the sample period for daily data from 1/6/1992 to 12/29/2006 is extracted. In total, we collect 3779 daily data for model fitting with return-based DCC, range-based DCC and DBEKK, respectively. Meanwhile, the implied daily covariances are calculated in this stage. Sequentially, it is easy to get the implied weekly estimates for covariance series, followed by the computation below:

$$MCOV_t^{implied} = \sum_j \mathrm{cov}_t^j , \tag{2.21}$$

where cov_t^j denotes implied daily covariance on the j^{th} trading day during the corresponding week t. Ferland and Lalancette (2006) also use this idea to build the weekly covariance and correlation.

As to the realized volatility, its concept has been used productively by French, Schwert, and Stambaugh (1987) and Andersen et al. (2001). The realized covariance can be expressed as:

$$MCOV_t^{realized} = \sum_j (r_{1t}^j \times r_{2t}^j) , \tag{2.22}$$

where r_{it}^j denotes return for the asset i on the j^{th} trading day during the corresponding week t.

< Figure 2.2 is inserted about here >

Checking Figure 2.2, we depict the different covariance patterns between S&P 500 and Tbond series for return-based DCC, range-based DCC, DBEKK and the realized pattern, respectively. Some useful insights can be obtained from these figures. It seems to reflect strong interactions around these MCOVs. Furthermore, the realized covariances are more volatile than other implied ones. This shows that the realized pattern is not easy to be fitted. The empirical result also demonstrates this conjecture.

2.4.2 In-sample Forecast Comparison

In this section, we present the empirical results for the in-sample forecast comparison of covariances. Mainly, we exhibit the in-sample forecasting ability of return-based DCC, ranged-based DCC and some related models for the purpose of performance comparison. As for the parameters fitted for DCC, we estimate and arrange them in Table 2.2. Due to the procedure for parameters estimated under the DCC setting, we have to cope with two inherent stages. In the first stage, one can utilize GARCH fitted by returns, or CARR fitted by ranges, with individual assets, for obtaining standardized residuals. Afterwards, we bring these standardized residuals series into the second stage for dynamic conditional correlation estimating.

< Table 2.2 is inserted about here >

Table 2.3 illustrates some brief results of covariances estimated for in-sample prediction, based on different econometrical models that we have mentioned previously. We draw clear inference from Table 2.3 to the effect that they all appeared to be more accurate in range-based DCC than in the other five models, regardless of what criterion is adopted. This appears to be consistent not only in RMSE but also in MAE. The worst performance in predicting the covariance under the in-sample analysis is the MA100.

< Table 2.3 is inserted about here >

Generally speaking, there are no significant differences in covariance forecasting performance between return-based DCC and DBEKK under the in-sample context. In addition, predicting results of CCC perform even worse than EWMA. One reasonable conjecture is that the simple correlation between S&P 500 and Tbond is just an average and rough value. In contrast to the dynamic correlation process generated by other models, the correlations are very volatile in this sample period. For example, see Figure 2.3 for an illustration. Looking at the forecasted covariances (FCOVs) generated by return-based DCC and CCC, the only difference between them is the estimated correlation process. However, we can find that their covariance process have salient difference. Accordingly, it seems inappropriate to assume that the correlation parameter between different assets is constant over time.

< Figure 2.3 is inserted about here >

2.4.3 Out-of-sample Forecast Comparison

For completeness, we assess the out-of-sample forecasting performance for different models by using RMSE and MAE, discussed in the previous in-sample comparison. Given that the data set contains a total of 782 usable observations, it is possible to use a holdback period of observations. This way, there are 521 observations (10 years) in each estimated model and 258 out-of-sample forecasting values for comparison. Here, the rolling sample approach for out-of-sample measurement is adopted and the first forecasted value for one period ahead forecast respectively occurs on the week of January 4, 2002. Table 2.4 reports one, two, and four periods ahead of out-of-sample forecasting results for covariance.

< Table 2.4 is inserted about here >

We obtain a consistent inference for covariance prediction’s performance based on different competitive models. All of the inferences demonstrate an overwhelming phenomenon, namely, that the range-based DCC approach dominates other methods in accuracy from out-of-sample forecasting. Various forecasting results for covariance with different periods ahead are presented in Table 2.4. Except for MA100 in the forecasting models, the results in Table 2.4 appear to show a trend that the forecasting errors are proportionate to the forecasted periods. One period ahead out-of-sample forecasting covariances of all compared models are given in Figure 2.4.

< Figure 2.4 is inserted about here >

Exploring other characteristics of out-of-sample forecasting, CCC, among these competitive models is the worst one, even inferior than MA100. One possible explanation for this is that the relationship between S&P 500 and Tbond in the post-sample has structural change. Unlike previous results in the in-sample comparison, however, return-based DCC performs significantly better than DBEKK. With the exception of range-based DCC, it is surprising that EWMA, holding constant post-sample covariance, even has outstanding performance compared to those of the other models.

Moreover, we can take another look at the out-of-sample forecast comparison. Table 2.5 shows the simple correlations between MCOVs and FCOVs for one, two, and four periods ahead covariance forecasts. The results show a clear and strong relationship between the FCOV built by range-based DCC and MCOVs. The correlation coefficients in the CCC case are negative and all are lower than -0.4. It is clear that the assumption of the constant correlation may cause the serious influence. In general, the correlations show a declining trend along with forecasting horizons.

< Table 2.5 is inserted about here >

In view of in-sample and out-of-sample empirical results, we can not clearly put all forecasting models in a proper order. However, it is undoubted that the range-based DCC model possesses the optimal forecasting power in covariance.

2.5 Conclusion

In this paper, we propose a new estimator of the time-varying covariance matrices, utilizing the range data that combines the CARR model with the framework of the DCC model. The advantage of this range-based DCC model, in terms of its forecasting ability to outperform the standard return-based DCC model, hinges on the relative efficiency of the range data over the return data in estimating volatilities. Using weekly futures data of S&P 500 and Tbond, we find a consistent result that the range-based DCC model outperforms the return-based models in estimating and forecasting covariance matrices for both in-sample and out-of-sample analysis.

In addition to using conventional realized covariance for the purpose of comparison, we introduce the viewpoint of implied covariance, which is derived from return-based DCC, range-based DCC and DBEKK for benchmarking robustness. Nonetheless, no matter what realized covariance or implied covariances are adopted for comparison, we obtain a consistent conclusion that the range-based DCC approach is the best one for predicting covariance process.

Although we only applied this estimator to the bivariate systems, it can also be applied to larger systems in a manner which is similar to the application of the DCC model structures, having already been demonstrated in Engle and Sheppard (2001). It will be surely useful to utilize more diagnostic statistics or to test based on value-at-risk calculations as proposed by Engle and Manganelli (2004) in future research. Other applications such as estimating the optimal portfolio weighting matrices and calculating the dynamic hedge ratio in the futures market will also bear fruit.

Table 2.1: Summary Statistics for the Weekly Returns and Ranges, 1992-2006.

This table reports the summary statistics for the weekly return and range data on S&P 500 and Tbond futures in our empirical study. There are 782 weekly sample observations ranging from January 6, 1992 to Dec 29, 2006. All data are extracted from Datastream. The returns and ranges are computed by $100\times\log(p_t^{close}/p_{t-1}^{close})$ and $100\times\log(p^{high}/p^{low})$, respectively. Jarque-Bera is the statistic for normality. All of them reject the null hypothesis of a normal distribution.

	S&P 500		Tbond	
	Return	Range	Return	Range
Mean	0.158	3.134	0.016	1.306
Median	0.224	2.607	0.033	1.194
Maximum	8.124	13.556	2.462	4.552
Minimum	-12.395	0.690	-4.050	0.301
Std. Dev.	2.112	1.809	0.855	0.560
Skewness	-0.503	1.756	-0.498	1.390
Kurtosis	6.455	7.232	4.217	6.462
Jarque-Bera	421.317	985.454	80.441	642.367

Table 2.2: Estimation of Bivariate Return-based and Range-based DCC Model Using Weekly S&P 500 and Tbond Futures, 1992-2006.

Step 1 of DCC estimation:

$h_{i,t} = \omega_i + \alpha_i \varepsilon^2_{i,t-1} + \beta_i h_{i,t-1}$, $\varepsilon_{i,t} \mid I_{t-1} \sim N(0, h_{i,t})$

$\lambda_{i,t} = \omega_i + \alpha_i \Re_{i,t-1} + \beta_i \lambda_{i,t-1}$, $\Re_{i,t} \mid I_{t-1} \sim \exp(1;\cdot)$, $i = 1,2.$

Step 2 of DCC estimation:

$$\begin{bmatrix} q_{11,t} & q_{12,t} \\ q_{21,t} & q_{22,t} \end{bmatrix} = (1-a-b)\begin{bmatrix} 1 & \bar{q}_{12} \\ \bar{q}_{12} & 1 \end{bmatrix} + a\begin{bmatrix} z^2_{1,t-1} & z_{1,t-1}z_{2,t-1} \\ z_{2,t-1}z_{1,t-1} & z^2_{2,t-1} \end{bmatrix} + b\begin{bmatrix} q_{11,t-1} & q_{12,t-1} \\ q_{21,t-1} & q_{22,t-1} \end{bmatrix}.$$

This table provides the estimation for the bivariate return-based and range-based DCC model using weekly S&P 500 and Tbond futures. The three formulas above two steps estimation are GARCH, CARR and the conditional correlation equation respectively of the standard DCC model with mean reversion. In the first stage, we use the GARCH and CARR model to estimate their volatilities ($\hat{h}_t$ and $\hat{\lambda}_t$) for each assets and computes their standardized residuals (Z_t). Then, in the second stage, the conditional correlation process can be obtained by using their standardized residuals and $\bar{q}_{12} = E(z_{1,t} z_{2,t})$. The conditional correlation matrix is given by $q_{12,t} / \sqrt{q_{11,t} q_{22,t}}$. The conditional covariance can then be expressed using the product of conditional correlation between these two variables and their individual conditional standard deviations. The table shows estimations of the three models using the MLE method. Numbers in parentheses are t-values.

Panel A : Step 1 of DCC estimation

	S&P 500		Tbond	
	GARCH	CARR	GARCH	CARR
$\hat{\omega}$	0.018	0.103	0.027	0.075
	(1.170)	(2.923)	(1.533)	(2.809)
$\hat{\alpha}$	0.048	0.248	0.059	0.157
	(3.744)	(9.090)	(2.046)	(5.208)
$\hat{\beta}$	0.949	0.719	0.903	0.785
	(76.443)	(23.167)	(18.994)	(18.041)

Panel B: Step 2 of DCC estimation

	S&P 500 Versus Tbond	
	Return-based DCC	Range-based DCC
$\hat{a}$	0.034	0.041
	(4.323)	(4.624)
$\hat{b}$	0.960	0.954
	(96.873)	(86.943)

Table 2.3: In-sample Forecast Errors for Covariances between the S&P 500 and Tbond Futures, 1992-2006.

$$RMSE = \sqrt{\frac{1}{T}\sum_{t=1}^{T}(FCOV_t - MCOV_t)^2} \quad , \quad MAE = \frac{1}{T}\sum_{t=1}^{T}| FCOV_t - MCOV_t | .$$

This table reports the in-sample forecast errors for covariances between S&P 500 and Tbond Futures. RMSE and MAE are the error functions. MCOV represents the covariance proxy derived from the base model. FCOV is the forecast covariance for the forecasting model and is used to fit each MCOV. Daily data are used to compute the weekly implied MCOVs (Return DCC. Range DCC, and DBEKK), and the realized MCOV (Realized). MA100, EWMA, return-based DCC, range-based DCC, CCC and DBEKK, are estimated from the weekly data to build FCOVs.

Forecast Errors		Forecasting Model					
	Base Model	MA100	EWMA	Return DCC	Range DCC	CCC	DBEKK
RMSE	Return DCC	0.741	0.420	0.392	0.296	0.693	0.395
	Range DCC	0.861	0.467	0.469	0.344	0.807	0.475
	DBEKK	0.780	0.490	0.469	0.377	0.732	0.457
	Realized	1.515	1.302	1.301	1.261	1.426	1.300
MAE	Return DCC	0.543	0.305	0.274	0.219	0.502	0.261
	Range DCC	0.638	0.324	0.316	0.240	0.588	0.302
	DBEKK	0.566	0.350	0.322	0.270	0.529	0.298
	Realized	0.897	0.789	0.764	0.753	0.842	0.765

Table 2.4: One, Two, and Four Periods Ahead Out-of-sample Forecast Errors for Covariances between the S&P 500 and Tbond Futures, 1992-2006.

$$RMSE = \sqrt{\frac{1}{n}\sum_{t=T+1}^{T+n}(FCOV_t - MCOV_t)^2}\ ,\quad MAE = \frac{1}{n}\sum_{t=T+1}^{T+n}|FCOV_t - MCOV_t|$$

This table reports the one, two and four periods ahead out-of-sample forecast errors for covariances between S&P 500 and Tbond futures. RMSE and MAE are the error functions. MCOV represents the covariance proxy derived from the base model. FCOV is the forecast covariance for the forecasting model and is used to fit each MCOV. Daily data are used to compute the weekly implied MCOVs (Return DCC. Range DCC, and DBEKK), and the realized MCOV (Realized). MA100, EWMA, Return DCC, Range DCC, CCC and DBEKK, are estimated from the weekly data to build FCOVs. There are 521 observations (10 years) in each of the estimated models. Additionally, the rolling sample method provides 258 forecasting values (n in the criteria above) for every out-of-sample comparison. The first forecasted values for one, two, and four periods ahead forecasts respectively occur the week of January 4, 11, and 25 in 2002.

		Forecasting Model					
	Base Model	MA100	EWMA	Return DCC	Range DCC	CCC	DBEKK
Panel A: One period ahead forecast errors							
RMSE	Return DCC	0.823	0.439	0.439	0.283	0.883	0.596
	Range DCC	0.935	0.469	0.495	0.301	0.994	0.684
	DBEKK	0.875	0.519	0.528	0.354	0.935	0.655
	Realized	1.508	1.254	1.285	1.183	1.556	1.366
MAE	Return DCC	0.495	0.331	0.320	0.219	0.557	0.344
	Range DCC	0.562	0.323	0.322	0.223	0.622	0.395
	DBEKK	0.523	0.389	0.384	0.279	0.596	0.385
	Realized	0.877	0.807	0.800	0.756	0.923	0.805
Panel B: Two periods ahead forecast errors							
RMSE	Return DCC	0.823	0.454	0.456	0.312	0.885	0.638
	Range DCC	0.935	0.481	0.511	0.344	0.996	0.729
	DBEKK	0.875	0.537	0.548	0.387	0.938	0.701
	Realized	1.507	1.263	1.294	1.214	1.557	1.403
MAE	Return DCC	0.495	0.342	0.336	0.237	0.558	0.366
	Range DCC	0.561	0.336	0.336	0.242	0.622	0.415
	DBEKK	0.523	0.403	0.403	0.302	0.598	0.408
	Realized	0.875	0.816	0.813	0.771	0.921	0.843
Panel C: Four periods ahead forecast errors							
RMSE	Return DCC	0.823	0.482	0.487	0.380	0.889	0.656
	Range DCC	0.935	0.514	0.546	0.432	0.999	0.748
	DBEKK	0.875	0.567	0.581	0.461	0.942	0.725
	Realized	1.506	1.281	1.312	1.252	1.558	1.411
MAE	Return DCC	0.494	0.359	0.357	0.285	0.560	0.392
	Range DCC	0.559	0.360	0.357	0.291	0.623	0.434
	DBEKK	0.523	0.425	0.428	0.351	0.601	0.441
	Realized	0.872	0.826	0.822	0.792	0.916	0.846

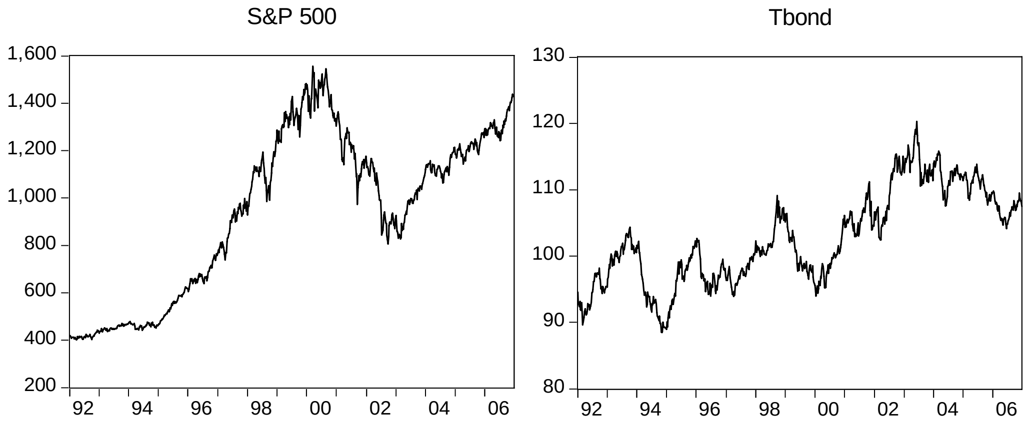
In the first column, MA100 has minor changes in comparing one, two, and four periods ahead forecast errors.

Table 2.5: Simple Correlations between MCOVs and FCOVs for One, Two, and Four Periods Ahead Out-of-sample Covariance Forecasts, 1992-2006.

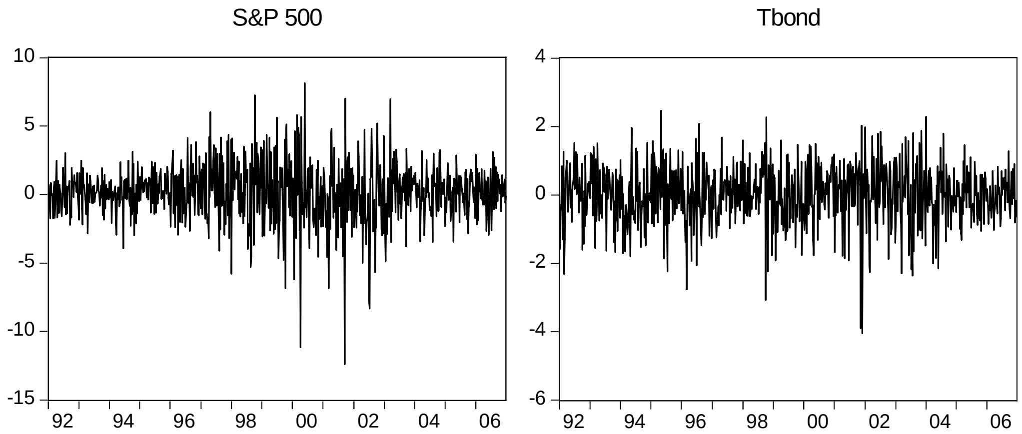
This table reports the simple correlations between MCOVs and FCOVs for one, two and four periods ahead out-of-sample covariance forecasts. MCOV represents the covariance proxy derived from the base model. FCOV is the forecast covariance for the forecasting model. Daily data are used to compute the weekly implied MCOVs (Return DCC. Range DCC, and DBEKK), and the realized MCOV (Realized). MA100, EWMA, Return DCC, Range DCC, CCC and DBEKK, are estimated from the weekly data to build FCOVs. There are 521 observations (10 years) in each of the estimated models. Additionally, the rolling sample method provides 258 forecasting values for every out-of-sample comparison. The first forecasted values for one, two, and four periods ahead forecasts respectively occur the week of January 4, 11, and 25 in 2002.

		FCOVs					
		MA100	EWMA	Return DCC	Range DCC	CCC	DBEKK
Panel A: Correlations for one period ahead forecast							
MCOVs	Return DCC	0.646	0.836	0.815	0.941	-0.543	0.579
	Range DCC	0.660	0.833	0.815	0.940	-0.537	0.578
	DBEKK	0.600	0.803	0.778	0.922	-0.514	0.568
	Realized	0.340	0.471	0.428	0.557	-0.408	0.310
Panel B: Correlations for two periods ahead forecast							
MCOVs	Return DCC	0.635	0.821	0.794	0.922	-0.571	0.504
	Range DCC	0.651	0.822	0.799	0.916	-0.563	0.502
	DBEKK	0.588	0.783	0.751	0.898	-0.545	0.485
	Realized	0.334	0.458	0.411	0.520	-0.430	0.236
Panel C: Correlations for four periods ahead forecast							
MCOVs	Return DCC	0.616	0.793	0.754	0.867	-0.624	0.469
	Range DCC	0.631	0.791	0.757	0.855	-0.612	0.474
	DBEKK	0.567	0.749	0.703	0.836	-0.604	0.436
	Realized	0.321	0.433	0.380	0.470	-0.455	0.220

Panel A: Close Prices



Panel B: Returns



Panel C: Ranges

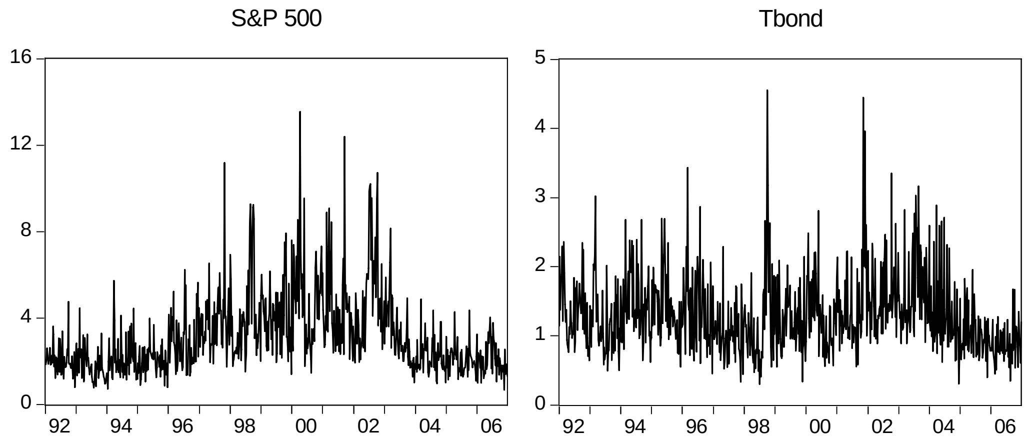


Figure 2.1: S&P 500 and Tbond Futures Weekly Closing Prices, Returns and Ranges, 1992-2006. This figure shows the weekly close prices, returns, and ranges of S&P 500 and Tbond Futures over the sample period.

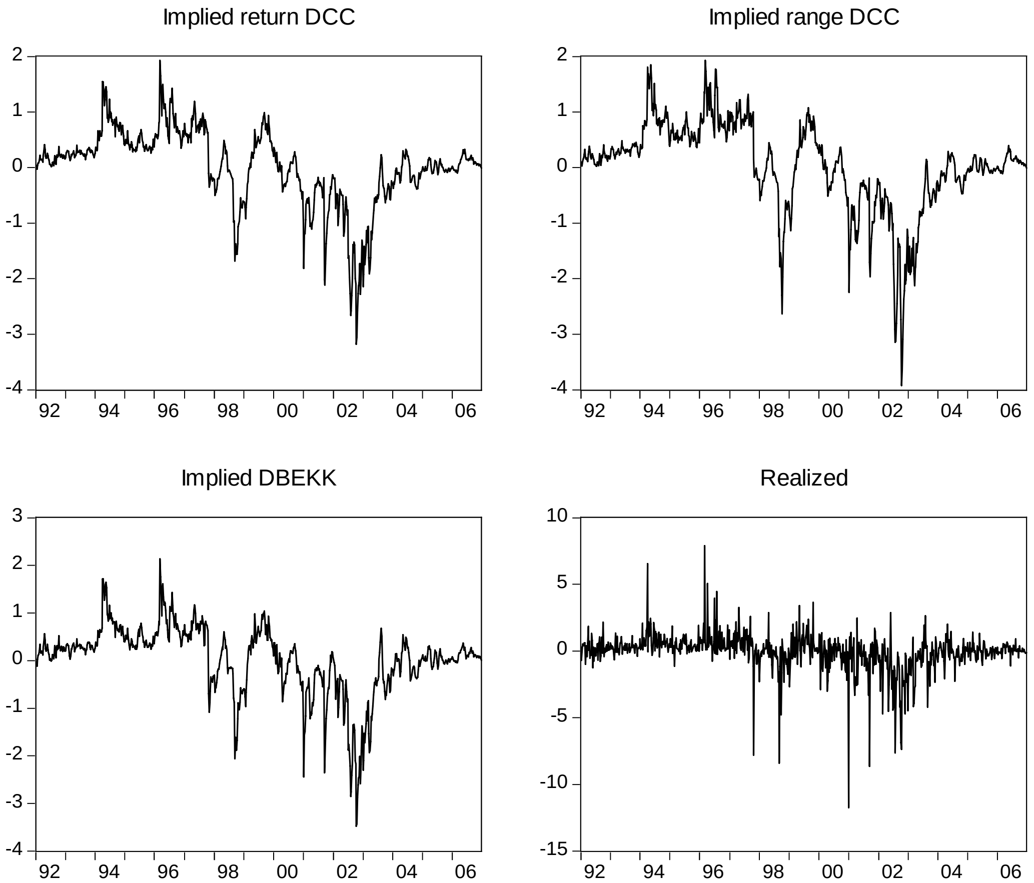


Figure 2.2: Four Measured Covariances between S&P 500 and Tbond Futures, 1992-2006. This figure plots the four measured weekly covariances between S&P 500 and Tbond futures. The measured weekly covariances are built from the daily data and are used to be the weekly covariance proxies in our empirical comparison. For getting the implied and realized weekly covariance series, we sum their daily covariances on the trading days of the corresponding week.

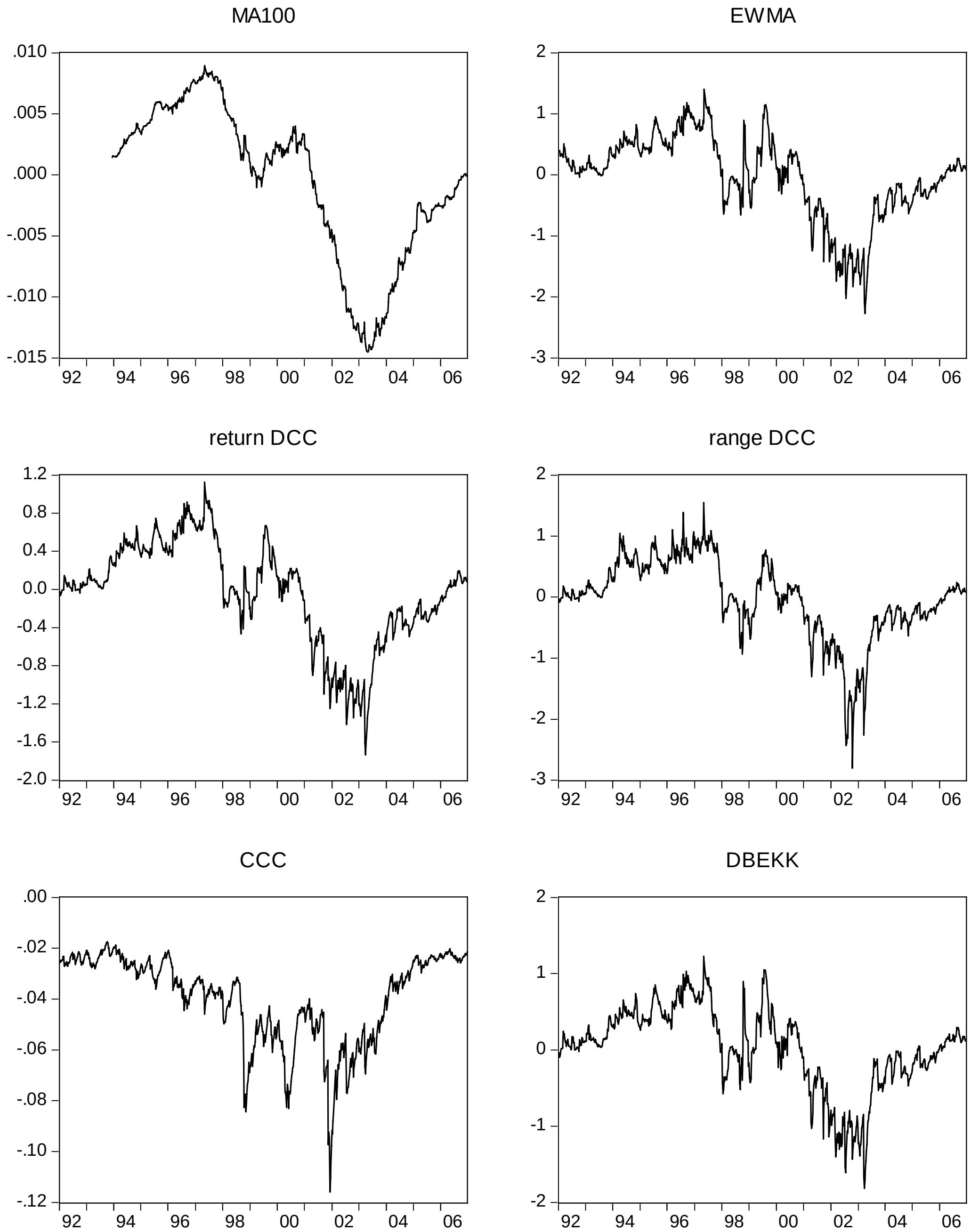


Figure 2.3: In-sample Forecasting Covariances between S&P 500 and Tbond Futures for Six Models, 1992-2006. This figure provides the fitted covariances between S&P 500 and Tbond futures for six different models. We lost some former values in MA100. This is because the first estimated value must be derived by the former 100 observation. The covariances of CCC are all negative and quite smaller than ones of DCC. The reasonable explanation is its negative and small constant correlation (-0.0229).

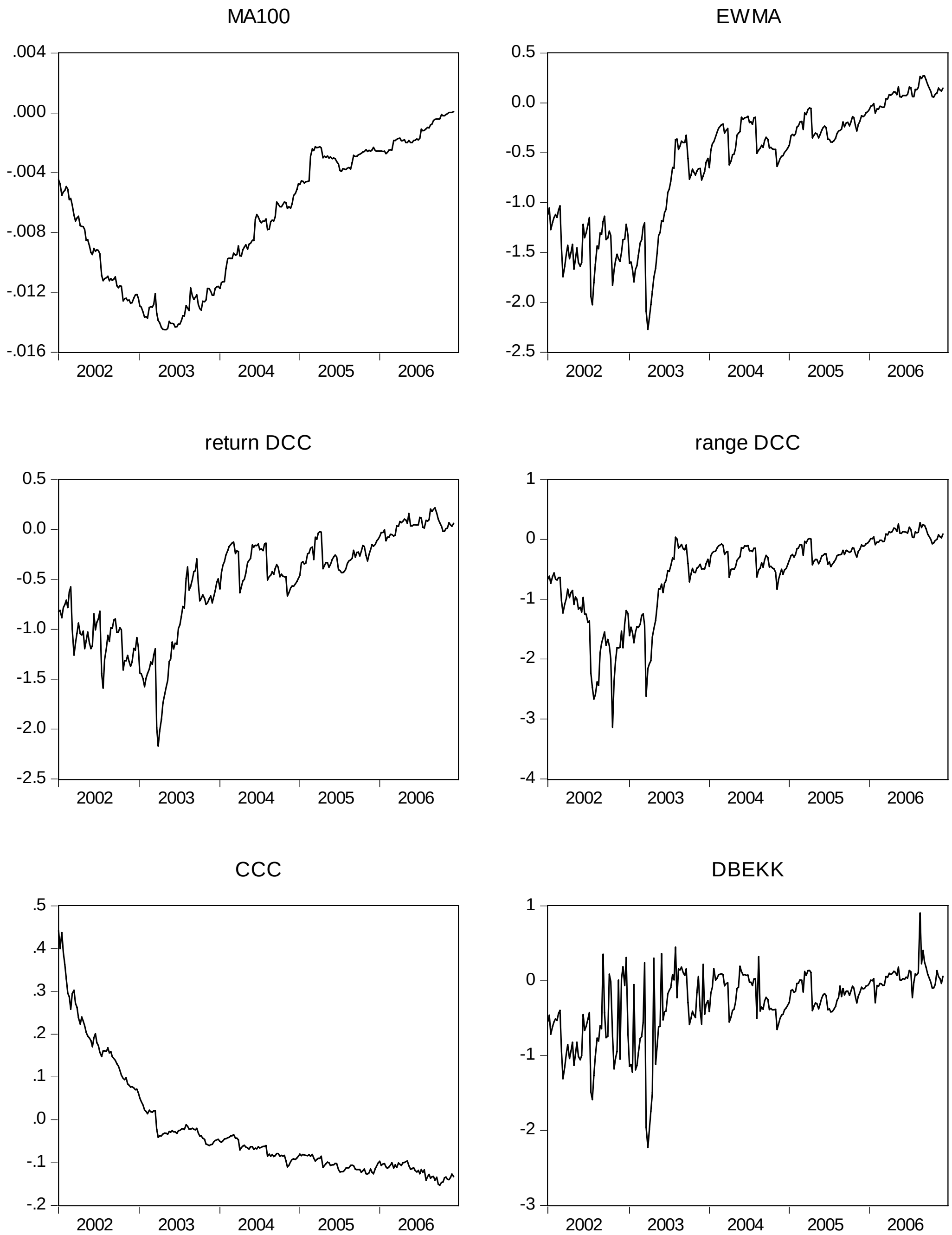


Figure 2.4: One Period Ahead Out-of-sample Forecasting Covariances between S&P 500 and Tbond Futures for Six Models, 1992-2006. This figure shows one period ahead out-of-sample forecasting result of six different models. The rolling sample approach is adopted for each model with 521 observations (10 years). The first forecasted value for one period ahead forecast respectively occurs the week of January 4, 2002. In all, we have 258 out-of-sample forecasting covariances.

Chapter 3. The Economic Value of Volatility Timing Using a Range-based Volatility Model

3.1 Introduction

In recent years, there has been considerable interest in volatility. The extensive development of volatility modeling has been motivated by the related applications in risk management, portfolio allocation, assets pricing, and futures hedging. In discussions of econometric methodologies in estimating the volatility of individual assets, ARCH [see Engle (1982)] and GARCH [see Bollerslev (1986)]have been emphasized most. Various applications in finance and economics are provided as a review in Bollerslev, Chou, and Kroner (1992), Bollerslev, Engle, and Nelson (1994), and Engle (2004).

Several studies, having noted that the range data based on the difference of high and low prices in a fixed interval, can offer a sharper estimate of volatility than the return data. A number of studies have investigated this issue started with Parkinson's (1980) research, and more recently, Brandt and Jones (2006), Chou (2005, 2006), and Martens and van Dijk (2007)[11]. Especially, Chou (2005) proposes a conditional autoregressive range (CARR) model which can easily capture the dynamic volatility structure and has obtained some insightful empirical evidences.

However, the literature above just focuses on volatility forecast of a univariate asset. It should be noted that there have been some attempts to establish a relationship between multiple assets, such as VECH [see Bollerslev, Engle, and Wooldridge (1988)], BEKK [see Engle and Kroner (1995)], and a constant conditional correlation model (CCC) [see Bollerslev (1990)], among others. VECH and BEKK allowing time-varying

[11] See also Garman and Klass (1980), Wiggins (1991), Rogers and Satchell (1991), Kunitomo (1992), Yang and Zhang (2000), and Alizadeh, Brandt, and Diebold (2002).

covariance process are too flexible to estimate, and CCC with a constant correlation is too restrictive to apply on general applications. Seminal work on solving the puzzle is carried out by Engle (2002a). A dynamic conditional correlation[12] (DCC) model proposed by Engle (2002a) provides another viewpoint to this problem. The estimation of DCC can be divided into two stages. The first step is to estimate univariate GARCH, and the second is to utilize the transformed standardized residuals to estimate time-varying correlations [see Engle and Sheppard (2001), Cappiello, Engle, and Sheppard (2006)].

A new multivariate volatility, recently proposed by Chou, Wu, and Liu (2007), combines the range data of asset prices with the framework of DCC, namely range-based DCC[13]. They conclude that the range-based DCC model performs better than other return-based models (MA100, EWMA, CCC, return-based DCC, and diagonal BEKK) through the statistical measures, RMSE and MAE based on four benchmarks of implied and realized covariance[14].

Because the empirical results in many studies show that the forecast models only can explain little part of variations in time-varying volatilities, some studies are concentrated on whether volatility timing has economic value [see Busse (1999), Fleming, Kirby, and Osdiek (2001, 2003), Marquering and Verbeek (2004), Thorp and Milunovich (2007)]. The question we focus on is whether the economic value of volatility timing for range-based volatility model still exists and to test whether investors are willing to switch from a return-based DCC to a range-based DCC model.

For comparing the economic value of the return-based and range-based models, it is helpful to use a suitable measure to capture the trade-off between risk and return.

[12] See Tsay (2002) and Tse and Tsui (2002) for other related methods for estimating the time-varying correlations.
[13] See also footnote 6.
[14] Daily data are used to build four proxies for weekly covariances, i.e. implied return-based DCC, implied range-based DCC, implied DBEKK, and realized covariances.

Most literatures evaluate volatility models through error statistics and related applications, but neglect the influence of asset expected returns. A more precise measurement should consider both of them, but only few studies have so far been made at this point. However, a utility function can easily connect them and build a comparable standard. Before entering into a detailed discussion for the economic value of volatility timing, it is necessary to clarify its definition in this paper. In short, the economic value of volatility timing is the gain compared with a static strategy. For an investor with a mean variance utility, our concern is to estimate his will to pay for a new volatility model rather than a static one.

In light of the success of the range-based volatility model, the purpose of this paper is to examine its economic value of volatility timing by using conditional mean-variance framework developed by Fleming, Kirby, and Ostdiek (2001). We consider an investor with different risk-averse levels uses conditional volatility analysis to allocate three assets: stock, bond, and cash. Fleming, Kirby, and Ostdiek (2001) extend West, Edison, and Cho (1993) utility criterion to test the economic value of volatility timing for the short-horizon investors with different risk tolerance levels[15]. In addition to the short-horizon forecast of selected models, we also examine the economic value for longer horizon forecasts and an asymmetric range-based volatility model in our empirical study. This study may lead to a better understanding of range volatility.

The reminder is laid out as follows. Section 3.2 introduces the asset allocation methodology and economic value measurement. Section 3.3 describes the properties of data used and evaluates the performance of the different strategies. Finally, the conclusion is showed in section 3.4.

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[15] They find that volatility-timing strategy based on one-step ahead estimates of the conditional covariance matrix [see Foster and Nelson (1996)] significantly outperformed the unconditional efficient static portfolios.

3.2. Methodologies

The method to carry out this study is to use a framework of a minimum variance strategy, which is conductive to determine the accuracy of the time-varying covariances. For a risk-averse investor, we want to find the optimal dynamic weights of the selected assets and the implied economic value compared with a static strategy. Before applying the volatility timing strategies, we need to build a time-varying covariance matrix. The Details of the methodology are as the following.

3.2.1 Optimal Portfolio Weights in a Minimum Variance Framework

Initially, we consider a minimization problem for the portfolio variance subjected to a target return constraint. To derive our strategy, we let $\mathbf{R}_t$ is the $k \times 1$ vector of spot returns at time t[16]. Its conditional expected return $\boldsymbol{\mu}_t$ and conditional covariance matrix $\boldsymbol{\Sigma}_t$ are calculated by $E[\mathbf{R}_t \mid \Omega_{t-1}]$ and $E[(\mathbf{R}_t - \boldsymbol{\mu}_t)(\mathbf{R}_t - \boldsymbol{\mu}_t)' \mid \Omega_{t-1}]$, respectively. Here Ω_t is assumed as the information set at time t. To minimize portfolio volatility subject to a required target return μ_{target}, it can be formulated as

$$\min_{w_t} \ \mathbf{w}_t' \boldsymbol{\Sigma}_t \mathbf{w}_t ,$$

$$s.t \ \ \mathbf{w}_t' \boldsymbol{\mu}_t + \left(1 - \mathbf{w}_t' \mathbf{1}\right) R_f = \mu_{target} , \tag{3.1}$$

where $\mathbf{w}_t$ is a $k \times 1$ vector of portfolio weights for time t. R_f is the return for the risk-free asset. The optimal solution to the quadratic form (3.1) is:

$$\mathbf{w}_t = \frac{(\mu_{target} - R_f)\boldsymbol{\Sigma}_t^{-1}(\boldsymbol{\mu}_t - R_f \mathbf{1})}{(\boldsymbol{\mu}_t - R_f \mathbf{1})' \boldsymbol{\Sigma}_t^{-1} (\boldsymbol{\mu}_t - R_f \mathbf{1})} . \tag{3.2}$$

A bivariate case ($k = 2$) can be expressed as:

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[16] Through out this paper, we use blackened letters to denote vectors or matrices.

$$w_{1,t} = \frac{\dot{\mu}_{target}(\dot{\mu}_{1,t}\sigma_{2,t}^2 - \dot{\mu}_{2,t}\sigma_{12,t})}{\dot{\mu}_{1,t}^2\sigma_{2,t}^2 + \dot{\mu}_{2,t}^2\sigma_{1,t}^2 - 2\dot{\mu}_{1,t}\dot{\mu}_{2,t}\sigma_{12,t}},$$

$$w_{2,t} = \frac{\dot{\mu}_{target}(\dot{\mu}_{2,t}\sigma_{1,t}^2 - \dot{\mu}_{1,t}\sigma_{12,t})}{\dot{\mu}_{1,t}^2\sigma_{2,t}^2 + \dot{\mu}_{2,t}^2\sigma_{1,t}^2 - 2\dot{\mu}_{1,t}\dot{\mu}_{2,t}\sigma_{12,t}}, \tag{3.3}$$

where $\dot{\mu}_{target} = \mu_{target} - R_f$, $\dot{\mu}_{1,t} = \mu_{1,t} - R_f$, and $\dot{\mu}_{2,t} = \mu_{2,t} - R_f$ are the excess target returns and the excess spot returns of S&P 500 index (S&P 500) and 10-year Treasury bond (T-bond) in our empirical study. Under the cost of carry model, we can regard the excess returns as the futures returns by applying regular no-arbitrage arguments[17]. It is clear that the covariance matrix $\mathbf{\Sigma}_t$ of the spot returns is the same as that of the excess returns.

The above analysis points that the optimal portfolio weights are time-varying. Here we assume the conditional mean $\boldsymbol{\mu}_t$ is constant[18]. Therefore, the dynamics of weights only depend on the conditional covariance $\mathbf{\Sigma}_t$. In this study, the optimal strategy is obtained based on a minimum variance framework subject to a given return. We use return-based and range-based DCC models to estimate the covariance matrix of multiple asset returns.

3.2.2 Economic Value of Volatility Timing

Fleming, Kirby, and Ostdiek (2001) use a generalization of the West, Edison, and Cho (1993) criterion which builds the relation between a mean-variance framework and a quadratic utility to capture the trade-off between risk and return for ranking the performance of forecasting models. According to their work, the investor's utility can be defined as:

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[17] There is no cost for futures investment. It means the futures return equals the spot return minus the risk-free rate.

[18] The changes in expected returns are not easy to be detected. Merton (1980) points out that the volatility process is more predictable than return series.

$$U(W_t) = W_t R_{p,t} - \frac{\alpha W_t^2}{2} R_{p,t}^2, \tag{3.4}$$

where W_t is the investor's wealth at time t, α is his absolute risk aversion, and the portfolio return at period t is $R_{p,t} = \mathbf{w}_t' \mathbf{R}_t$.

For comparisons across portfolios, we assume that the investor has a constant relative risk aversion (CRRA), $\gamma_t = \alpha W_t / (1 - \alpha W_t) = \gamma$. This implies αW_t is a constant. With this assumption, the average realized utility $\overline{U}(\cdot)$ can be used in estimating the expected utility with a given initial wealth W_0.

$$\overline{U}(\cdot) = W_0 \sum_{t=1}^{T} \left[R_{p,t} - \frac{\gamma}{2(1+\gamma)} R_{p,t}^2 \right], \tag{3.5}$$

where W_0 is the initial wealth.

Therefore, the value of volatility timing by equating the average utilities for two alternative portfolios is expressed as:

$$\sum_{t=1}^{T} \left[(R_{b,t} - \Delta) - \frac{\gamma}{2(1+\gamma)} (R_{b,t} - \Delta)^2 \right] = \sum_{t=1}^{T} \left[R_{a,t} - \frac{\gamma}{2(1+\gamma)} R_{a,t}^2 \right], \tag{3.6}$$

where Δ is the maximum expense that an investor would be willing to pay to switch from the strategy a to the strategy b. $R_{a,t}$ and $R_{b,t}$ here are the returns of the portfolios from the strategy a and b[19]. If the expense Δ is a positive value, it means the strategy b is more valuable than the strategy a. In our empirical study, we report Δ as an annualized expense with three risk aversion levels of γ=1, 5, and 10.

3.3. Empirical Results

The empirical data employed in this paper consist of the stock index futures, bond

[19] In our setting, we let the strategy pair (*a,b*) be (OLS, return-based DCC), (OLS, range-based DCC), and (return-based DCC, range-based DCC), respectively. Because the rolling sample method is adopted in the out-of-sample comparison, this type of OLS is named by rollover OLS.

futures and the risk-free rate. As to the above-mentioned method, we apply the futures data to examine the economic value of volatility timing for return-based and range-based DCC. Under the cost of carry model, the result in this case can be extended to underlying spot assets [see Fleming, Kirby, and Ostdiek (2001)]. In addition to avoiding the short sale constraints, this procedure will reduce the complexity of model setting. To address this issue, we use the S&P 500 futures (traded at CME), and the T-bond futures (traded at CBOT) as the empirical samples. According to Chou et al. (2007), the futures data are taken from Datastream, sampling from January 6, 1992 to December 29, 2006 (15 years, 782 weekly observations). Datastream provides the nearest contract and rolls over to the second nearby contract when the nearby contract approaches maturity. We also use the 3-month Treasury bill rate to substitute for the risk-free rate. The Treasury bill rate is available in the Federal Reserve Board.

< Figure 3.1 is inserted about here >

Figure 3.1 shows the graphs for close prices (Panel A), returns (Panel B) and ranges (Panel C) of the S&P 500 and T-bond futures over the sample period. Table 3.1 shows summary statistics for the return and range data on the S&P 500 and T-bond futures. The return is computed as the difference of logarithm close prices on two continuous weeks. The range is defined by the difference of the high and low prices in a logarithm type. The annualized mean and standard deviation in percentage, (8.210, 15.232) of the stock futures returns are both larger than those (0.853, 6.168) of the bond futures returns. The fact indicates that the more volatile market may have higher risk premium. Both futures returns have negative skewness and excess kurtosis, indicating violation of the normal distribution. The range mean (3.134) of the stock futures prices is larger than that (1.306) of the bond futures prices. It is reasonable because the range is a proxy of volatility. The Jarque-Bera statistic is used to test the null of whether the

return and range data are normally distributed. Undoubtedly, both of return and range data reject the null hypothesis. The simple correlation between stock and bond returns is small[20] (-0.023), but it does not imply that their relation is very weak. In our latter analysis, we show that the dynamic relationship of stock and bond will be more realistically revealed by the conditional correlations analysis.

< Table 3.1 is inserted about here >

3.3.1 In-sample Comparison

For obtaining an optimal portfolio, we use the dynamic volatility models to estimate the covariance matrices. As for the parameters fitted for return-based and range-based DCC, they are both estimated and arranged in Table 3.2. We divide the table into two parts corresponding to the two steps in the DCC estimation. In Panel A of Table 3.2, one can use GARCH (fitted by return) or CARR (fitted by range) with individual assets to obtain the standardized residuals. Figure 3.2 provides the volatility estimated of the S&P 500 futures and the T-bond futures based on GARCH and CARR. Then, these standardized residuals series can be brought into the second stage for dynamic conditional correlation estimating. Panel B of Table 3.2 shows the estimated parameters of DCC under the quasi-maximum likelihood estimation (QMLE).

< Table 3.2 is inserted about here >

< Figure 3.2 is inserted about here >

The correlation and covariance estimates for return-based and range-based DCC are shown in Figure 3.3. It seems that the correlation becomes more negative at the end of 1997. A deeper investigation is given in Connolly , Stivers, and Sun. (2005).

< Figure 3.3 is inserted about here >

[20] The result is different from the positive correlation value (sample period 1983-1997) in Fleming, Kirby, and Ostdiek (2001). About after 1997, the relationship between S&P 500 and T-bond presents a reverse condition.

Following the model estimation, we construct the static portfolio (built by OLS) using the unconditional mean and covariance matrices for getting the economic value of dynamic models. Under the minimum variance framework, the weights of the portfolio are computed by the given expected return and the conditional covariance matrices estimated by return-based and range-based DCC. Then, we want to compare the performance of the volatility models on 11 different target annualized returns (5% - 15%, 1% in an interval) .

< Table 3.3 is inserted about here >

Table 3.3 shows how the performance comparisons vary with the target returns and the risk aversions. Panel A of Table 3.3 shows the annualized means (μ) and volatilities (σ) of the portfolios estimated from three methods, return-based DCC, range-based DCC, and OLS. For a quick look, the annualized Sharpe ratios[21] calculated from return-based DCC (0.680) and range-based DCC (0.699) are higher than the static model (0.560). Panel B of Table 3.3 shows the average switching fees (Δ_r) from one strategy to another. The value settings of CRRA γ are 1, 5, and 10. As for the performance fees with different relative risk aversions, in general, an investor with a higher risk aversion would be willing to pay more to switch from the static portfolio to the dynamic ones. With higher target returns, the performance fees are increasing steadily. In addition, Panel B of Table 3.3 also reports the performance fees switching from return-based DCC to range-based DCC. Positive values for all cases show that the range-based volatility model can give significant economic value in forecasting covariance matrices than the return-based ones. Figure 3.4 plots the weights of in-sample minimum volatility portfolio derived from two dynamic models. In the meanwhile, OLS has constant weights for cash, stock, and bond, i.e. -0.1934, 0.7079,

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[21] The Sharpe ratio is constant with different target multipliers. For the further details, see Engle and Colacito (2006).

and 0.4855.

< Figure 3.4 is inserted about here >

3.3.2 Out-of-sample Comparisons

For robust inference, a similar approach is utilized to estimate the value of volatility timing in the out-of-sample analysis. Here the rolling sample approach is adopted for all out-of-sample estimations. It means that the rollover OLS method replaces the conventional OLS method used in the in-sample analysis. Each forecasting value is estimated by 521 observations, about 10 years. Then, the rolling sample method provides 261 forecasting values for the one period ahead comparison. The first forecasted value occurs the week of January 4, 2002.

< Table 3.4 is inserted about here >

Table 3.4 reports how the performance comparisons vary with the target returns and the risk aversions for one period ahead out-of-sample forecast. We obtain a consistent conclusion with Table 3.3. The estimated Sharpe ratios calculated from return-based DCC, range-based DCC, and rollover OLS are 0.540, 0.586, and 0.326, respectively. The performance fees switching from rollover OLS to DCC are all positive. In total, the out-of-sample comparison supports the former inference. Figure 3.5 plots the weights that minimize conditional volatility while setting the expected annualized return equal to 10%.

< Figure 3.5 is inserted about here >

In addition to examining the performance of short-horizon investors, we further report the results of the long-horizon asset allocations. Table 3.5 reports one to thirteen periods ahead out-of-sample performance for three methods. Here the rolling sample approach provides 249 forecasting values for each out-of-sample comparison. The portfolio weights for all strategies are obtained from the weekly estimates of the out-of-sample conditional covariance matrices with a fixed target return (10%). In general, the Sharpe ratios got from range-based DCC is the largest, and return-based DCC is the next. For each strategy, however, we can not find an obvious trend in the Sharpe ratios with forecasting periods ahead. As for the result of the performance fees, it seems reasonable to conclude that an investor is still willing to pay to switch from rollover OLS to DCC. Moreover, the economic value seems to appear a decreasing trend with forecasting periods ahead. For longer forecasting horizon (12-13 weeks), however, the results of estimated switching fees are mixed. As to the will switching from return-based DCC to range-based DCC, it always keeps positive.

< Table 3.5 is inserted about here >

Thorp and Milunovich (2007) show that a risk-averse investor holding selected international equity indices, with γ = 2, 5, and 10, just want to pay little from symmetric to asymmetric forecasts. In some cases, the switching fees are even negative. In order to further understand this argument, we examine it based on the range-based volatility model. Chou (2005) provides an asymmetric range model namely CARRX: $\lambda_t = \omega + \alpha\Re_{t-1} + \beta\lambda_{t-1} + \phi ret_{t-1}$. The lagged return in the conditional range equation is used to capture the leverage effect. For building an asymmetric range-based volatility model, CARR in the first step of range-based DCC can be replaced by CARRX. Cappiello, Engle, and Sheppard (2006) introduce asymmetric DCC: $\mathbf{Q}_t = (1-a-b)\bar{\mathbf{Q}} - c\bar{\mathbf{N}} + a\mathbf{Z}_{t-1}\mathbf{Z}'_{t-1} + b\mathbf{Q}_{t-1} + c\mathbf{n}_{t-1}\mathbf{n}'_{t-1}$. $\mathbf{n}_t$ is the $k \times 1$ vector calculated by $\mathrm{I}(\mathbf{Z}_t < 0) \circ \mathbf{Z}_t$ to allow correlation to increase more in both falling returns than in both rising returns and $\bar{\mathbf{N}} = E(\mathbf{n}_t\mathbf{n}'_t)$, where $\circ$ denotes the Hadamard matrix product operator, i.e. element-wise multiplication. Table 3.6 shows the one period ahead performance of the volatility timing values for asymmetric range-based DCC compared with rollover OLS. The switching fees from rollover OLS to asymmetric range DCC seem to be smaller than the fees from rollover OLS to symmetric range DCC in Table 3.4. One of the reasons may result from the poor performance of the bond data. In this case, it is not valuable to switch the symmetric strategy to the asymmetric one.

< Table 3.6 is inserted about here >

3.4. Conclusion

In this paper, we examine the economic value of volatility timing for the range-based volatility model in utilizing the range data which combines CARR with a DCC structure. Applying S&P 500 and T-bond futures to a mean-variance framework with a no-arbitrage setting, the result can be extended to spot asset analysis. By means of the utility of portfolio, the economic value of dynamic models can be obtained from comparing with OLS. Both of in-sample and out-of-sample results show that a risk-averse investor is willing to switch from OLS to DCC. Moreover, the switching fees from return-based DCC to range-based DCC are always positive. We can conclude that the range-based volatility model has more significant economic value compared to the return-based one. The results give robust inferences for supporting the range-based volatility model in forecasting volatility.

Table 3.1: Summary Statistics for Weekly S&P 500 and T-bond Futures Return and Range Data, 1992-2006

The table provides summary statistics for the weekly return and range data on S&P 500 stock index futures and T-bond Futures. The returns and ranges are computed by $100 \times \log(p_t^{close} / p_{t-1}^{close})$ and $100 \times \log(p^{high} / p^{low})$, respectively. The *Jarque-Bera* statistic is used to test the null of whether the return and range data are normally distributed. The values presented in parentheses are p-values. The annualized values of means (standard deviation) for S&P 500 and T-bond futures are 8.210 (15.232) and 0.853 (6.168), respectively. The simple correlation between stock and bond returns is -0.023. The sample period ranges form January 6, 1992 to December 29, 2006 (15 years, 782 observations) and all futures data are collected from Datastream.

	S&P 500 Futures		T-Bond Futures	
	Return	Range	Return	Range
Mean	0.158	3.134	0.016	1.306
Median	0.224	2.607	0.033	1.194
Maximum	8.124	13.556	2.462	4.552
Minimum	-12.395	0.690	-4.050	0.301
Std. Dev.	2.112	1.809	0.855	0.560
Skewness	-0.503	1.756	-0.498	1.390
Kurtosis	6.455	7.232	4.217	6.462
Jarque-Bera	421.317	985.454	80.441	642.367
	(0.000)	(0.000)	(0.000)	(0.000)

Table 3.2: Estimation Results of Return-based and Range-based DCC Model Using Weekly S&P500 and T-bond Futures, 1992-2006

$$r_{i,t} = c + \varepsilon_{i,t}\,,\quad h_{k,t} = \omega_k + \alpha_k \varepsilon_{k,t-i}^2 + \beta_k h_{k,t-1}\,,\quad \varepsilon_{k,t} \mid I_{t-1} \sim N(0, h_{k,t})\,,$$

$$\Re_{i,t} = u_{i,t}\,,\quad \lambda_{k,t} = \omega_k + \alpha_k \Re_{k,t-1} + \beta_k \lambda_{k,t-1}\,,\quad \Re_{k,t} \mid I_{t-1} \sim exp(1,\cdot)\,,\quad k = 1,2\,.$$

$$\mathbf{Q}_t = (1-a-b)\overline{\mathbf{Q}} + a\mathbf{Z}_{t-1}\mathbf{Z}'_{t-1} + b\mathbf{Q}_{t-1}, \text{ and then}$$

$$\rho_{12,t} = \frac{(1-a-b)\bar{q}_{12} + az_{1,t-1}z_{2,t-1} + bq_{12,t-1}}{\sqrt{[(1-a-b)\bar{q}_{11} + az_{1,t-1}^2 + bq_{11,t-1}][(1-a-b)\bar{q}_{22} + az_{2,t-1}^2 + bq_{22,t-1}]}}\,,$$

where $\Re_t$ is the range variable, $\mathbf{Z}_t$ is the standard residual vector which is standardized by GARCH or CARR volatilities. $\mathbf{Q}_t = \{q_{ij,t}\}$ and $\overline{\mathbf{Q}} = \{\bar{q}_{ij}\}$ are the conditional and unconditional covariance matrix of $\mathbf{Z}_t$. The three formulas above are GARCH, CARR and the conditional correlation equations respectively of the standard DCC model with mean reversion. The table shows estimations of the three models using the MLE method. Panel A is the first step of the DCC model estimation. The estimation results of GARCH and CARR models for two futures are presented here. $Q(12)$ is the *Ljung–Box* statistic for the autocorrelation test with 12 lags. Panel B is the second step of the DCC model estimation. The values presented in parentheses are t-ratios for the model coefficients and p-values for Q(12).

Panel A: Volatilities Estimation of GARCH and CARR models

	S&P500 Futures		T-bond Futures	
	GARCH	CARR	GARCH	CARR
c	0.188		0.008	
	(3.256)		(0.242)	
$\hat{\omega}$	0.019	0.103	0.028	0.075
	(1.149)	(2.923)	(1.533)	(2.810)
$\hat{\alpha}$	0.051	0.248	0.060	0.157
	(3.698)	(9.090)	(2.031)	(5.208)
$\hat{\beta}$	0.946	0.719	0.902	0.785
	(71.236)	(23.167)	(18.645)	(18.041)
Q(12)	26.322	5.647	15.872	23.121
	(0.010)	(0.933)	(0.197)	(0.027)

Panel B: Correlation Estimation of Return- and Range-based DCC Models

	S&P500 and T-bond	
	Return-based DCC	Range-based DCC
$\hat{a}$	0.037	0.043
	(4.444)	(4.679)
$\hat{b}$	0.955	0.951
	(85.621)	(80.411)

Table 3.3: In-sample Comparison of the Volatility Timing Values in the Minimum Volatility Strategy Using Different Target Returns, 1992-2006

The table reports the in-sample performance of the volatility timing strategies with different target returns. The target returns are from 5% to 15% (annualized). The weights for the volatility timing strategies are obtained from the weekly estimates of the conditional covariance matrix and the different target return setting. Panel A shows the annualized means (μ) and volatilities (σ) for each strategy. The estimated Sharpe ratios for the return-based DCC model, the range-based DCC model, and the OLS strategy are 0.680, 0.699, and 0.560, respectively. Panel B shows the average switching annualized fees (Δ_r) from one strategy to another. The values of the constant relative risk aversion γ are 1, 5, and 10.

Panel A: Means and Volatilities of Optimal Portfolios

Target return(%)	Return-based DCC		Range-based DCC		OLS	
	μ	σ	μ	σ	μ	σ
5	5.201	2.100	5.241	2.100	5.000	2.190
6	6.366	3.814	6.438	3.813	6.000	3.977
7	7.530	5.527	7.635	5.526	7.000	5.764
8	8.694	7.241	8.832	7.239	8.000	7.551
9	9.859	8.954	10.028	8.952	9.000	9.338
10	11.023	10.668	11.225	10.665	10.000	11.125
11	12.187	12.381	12.422	12.378	11.000	12.912
12	13.352	14.095	13.619	14.091	12.000	14.699
13	14.516	15.808	14.815	15.804	13.000	16.486
14	15.680	17.521	16.012	17.517	14.000	18.273
15	16.845	19.235	17.209	19.230	15.000	20.060

Panel B: Switching Fees with Different Relative Risk Aversions

Target return(%)	OLS to Return DCC			OLS to Range DCC			Return to Range DCC		
	Δ_1	Δ_5	Δ_{10}	Δ_1	Δ_5	Δ_{10}	Δ_1	Δ_5	Δ_{10}
5	0.303	0.376	0.393	0.343	0.417	0.434	0.040	0.041	0.041
6	0.703	0.950	1.008	0.777	1.025	1.084	0.074	0.076	0.076
7	1.244	1.771	1.897	1.353	1.883	2.009	0.109	0.112	0.112
8	1.929	2.845	3.063	2.073	2.994	3.213	0.144	0.149	0.151
9	2.761	4.173	4.507	2.940	4.360	4.696	0.180	0.189	0.191
10	3.739	5.753	6.224	3.956	5.979	6.453	0.217	0.230	0.233
11	4.866	7.578	8.206	5.121	7.846	8.477	0.255	0.273	0.277
12	6.142	9.641	10.441	6.434	9.951	10.754	0.294	0.318	0.324
13	7.565	11.932	12.914	7.897	12.283	13.270	0.334	0.365	0.373
14	9.135	14.436	15.609	9.507	14.831	16.009	0.375	0.414	0.424
15	10.851	17.142	18.509	11.262	17.580	18.952	0.418	0.466	0.479

Table 3.4: Out-of-sample Comparison for the One Period Ahead Volatility Timing Values in the Minimum Volatility Strategy with Different Target Returns, 1992-2006

The table reports the one period ahead out-of-sample performance of the volatility timing strategies with different target returns. There are 521 observations in each of the estimated models and the rolling sample approach provides 261 forecasting values for each out-of-sample comparison. The first forecasted value occurs the week of January 4, 2002. The target returns are from 5% to 15% (annualized). The weights for the volatility timing strategies are obtained from the weekly estimates of the one period ahead conditional covariance matrix and the different target return setting. Panel A shows the annualized means (μ) and volatilities (σ) for each strategy. The estimated Sharpe ratios for the return-based DCC model, the range-based DCC model, and the rollover OLS strategy are 0.540, 0.586, and 0.326, respectively. Panel B shows the average switching annualized fees (Δ_r) from one strategy to another. The values of the constant relative risk aversion are 1, 5, and 10.

Panel A: Means and Volatilities of Optimal Portfolios

Target return(%)	Return-based DCC		Range-based DCC		Rollover OLS	
	μ	σ	μ	σ	μ	σ
5	4.691	1.698	4.747	1.661	4.344	1.749
6	5.438	3.083	5.540	3.016	4.808	3.176
7	6.186	4.468	6.333	4.370	5.273	4.603
8	6.933	5.853	7.127	5.725	5.737	6.030
9	7.681	7.239	7.920	7.080	6.202	7.456
10	8.428	8.624	8.714	8.435	6.667	8.883
11	9.176	10.009	9.507	9.790	7.131	10.310
12	9.923	11.394	10.300	11.145	7.596	11.737
13	10.671	12.779	11.094	12.500	8.060	13.164
14	11.418	14.165	11.887	13.854	8.525	14.591
15	12.166	15.550	12.680	15.209	8.990	16.018

Panel B: Switching Fees with Different Relative Risk Aversions

Target return(%)	OLS to Return DCC			OLS to Range DCC			Return to Range DCC		
	Δ_1	Δ_5	Δ_{10}	Δ_1	Δ_5	Δ_{10}	Δ_1	Δ_5	Δ_{10}
5	0.393	0.425	0.433	0.481	0.537	0.550	0.089	0.112	0.118
6	0.781	0.890	0.916	0.991	1.176	1.220	0.210	0.289	0.308
7	1.232	1.463	1.518	1.606	1.998	2.090	0.377	0.545	0.585
8	1.746	2.144	2.239	2.328	3.001	3.159	0.589	0.882	0.953
9	2.323	2.935	3.079	3.156	4.185	4.425	0.848	1.303	1.413
10	2.963	3.834	4.039	4.092	5.545	5.881	1.154	1.810	1.967
11	3.667	4.842	5.116	5.133	7.077	7.522	1.509	2.402	2.617
12	4.435	5.956	6.309	6.280	8.774	9.338	1.913	3.083	3.363
13	5.267	7.174	7.614	7.531	10.629	11.321	2.366	3.851	4.206
14	6.162	8.495	9.029	8.885	12.634	13.460	2.869	4.707	5.146
15	7.121	9.914	10.548	10.340	14.781	15.746	3.422	5.651	6.181

Table 3.5: Out-of-sample Comparison for One to Thirteen Periods Ahead Volatility Timing Values in the Minimum Volatility Strategy, 1992-2006

The table reports the one to thirteen periods ahead out-of-sample performance of the volatility timing strategies with the fixed 10% (annualized) target return. The weights for the volatility timing strategies are obtained from the weekly estimates of the one to thirteen periods ahead conditional covariance matrix. There are 521 observations in each of the estimated models and the rolling sample approach provides 249 forecasting values for each out-of-sample comparison. The first forecasted mean value occurs the week of January 4, 2002. Panel A shows the annualized means (μ), volatilities (σ), and Sharpe ratios (SR) for each strategy. Panel B shows the average switching annualized fees (Δ_r) from one strategy to another. The values of the constant relative risk aversion are 1, 5, and 10.

Panel A: Means and Volatilities of Optimal Portfolios

Periods Ahead	Return-based DCC			Range-based DCC			Rollover OLS		
	μ	σ	SR	μ	σ	SR	μ	σ	SR
1	7.717	8.724	0.452	8.060	8.540	0.502	6.022	8.956	0.251
2	7.868	8.830	0.464	8.562	8.556	0.560	6.068	8.933	0.257
3	7.371	8.807	0.408	8.312	8.572	0.529	6.660	8.931	0.323
4	8.117	8.838	0.491	8.750	8.604	0.578	7.103	8.928	0.373
5	8.464	8.860	0.529	9.200	8.653	0.627	6.869	8.989	0.344
6	9.088	8.903	0.597	9.600	8.637	0.674	7.232	8.973	0.385
7	9.361	8.840	0.632	10.033	8.629	0.725	7.872	8.945	0.458
8	8.853	8.897	0.571	9.429	8.683	0.651	7.644	8.975	0.431
9	9.806	8.878	0.679	10.093	8.664	0.729	8.476	9.023	0.521
10	9.746	8.887	0.672	9.576	8.695	0.667	8.189	8.983	0.491
11	9.436	8.908	0.636	8.986	8.712	0.598	8.031	8.910	0.478
12	8.737	9.003	0.551	8.076	8.791	0.489	7.424	8.853	0.412
13	8.713	9.111	0.542	8.272	8.914	0.505	7.794	8.867	0.453

Panel B: Switching Fees with Different Relative Risk Aversions

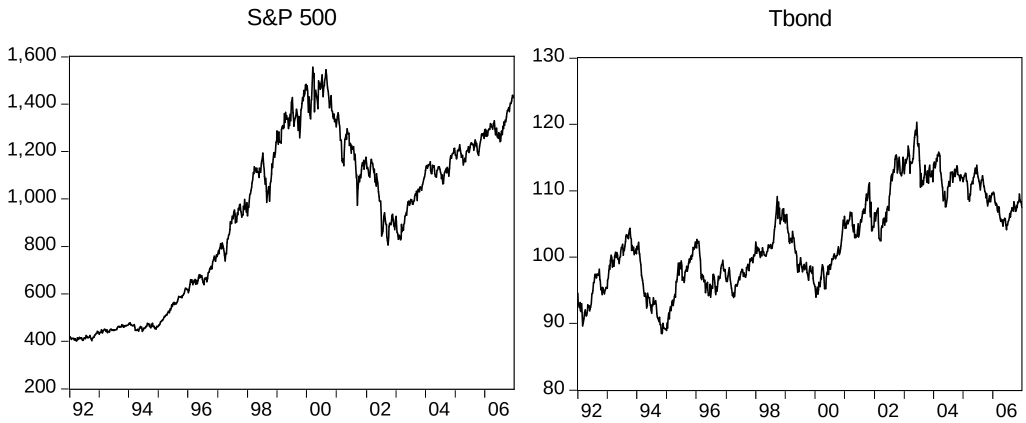
Periods Ahead	OLS to Return DCC			OLS to Range DCC			Return to Range DCC		
	Δ_1	Δ_5	Δ_{10}	Δ_1	Δ_5	Δ_{10}	Δ_1	Δ_5	Δ_{10}
1	2.772	3.546	3.727	3.944	5.289	5.599	1.196	1.831	1.983
2	2.282	2.633	2.716	4.223	5.448	5.731	1.970	2.914	3.137
3	1.293	1.721	1.823	3.308	4.495	4.772	2.029	2.830	3.019
4	1.440	1.758	1.834	3.152	4.244	4.499	1.728	2.544	2.738
5	2.210	2.665	2.773	3.900	5.032	5.297	1.712	2.446	2.622
6	2.191	2.442	2.503	3.938	5.078	5.345	1.775	2.730	2.958
7	1.993	2.373	2.464	3.647	4.740	4.997	1.674	2.440	2.625
8	1.581	1.861	1.928	3.161	4.172	4.410	1.597	2.369	2.555
9	2.028	2.556	2.683	3.319	4.578	4.875	1.313	2.103	2.295
10	2.019	2.370	2.455	2.753	3.767	4.007	0.753	1.465	1.638
11	1.416	1.424	1.426	1.891	2.591	2.758	0.489	1.209	1.383
12	0.593	0.037	-0.100	0.945	1.164	1.217	0.358	1.128	1.313
13	-0.269	-1.202	-1.436	0.251	0.078	0.035	0.518	1.243	1.417

Table 3.6: The One Period Ahead Performance of the Volatility Timing Values for the Asymmetric Range-based Volatility Model, 1992-2006

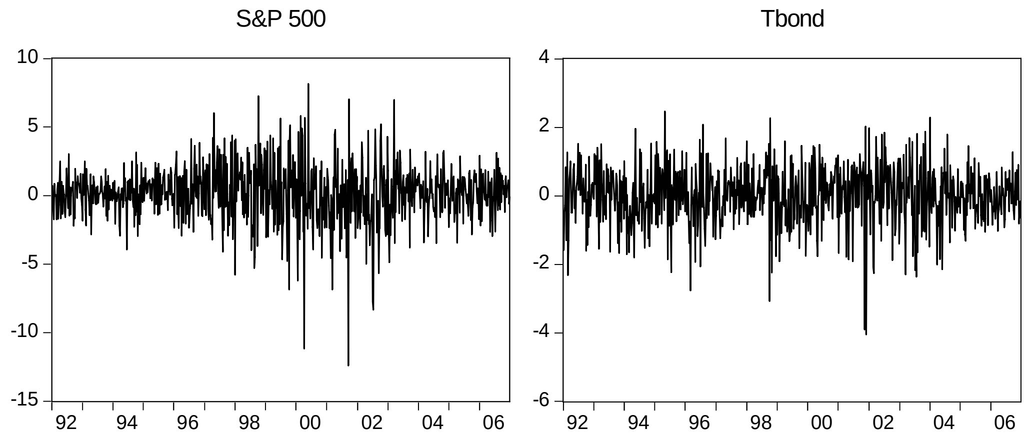
The table reports the one period ahead out-of-sample performance of the volatility timing strategies for the asymmetric range-based volatility model with different target returns. There are 521 observations in each of the estimated models and the rolling sample approach provides 261 forecasting values for each out-of-sample comparison. The first forecasted value occurs the week of January 4, 2002. The target returns are from 5% to 15% (annualized). The weights for the volatility timing strategies are obtained from the weekly estimates of the one period ahead conditional covariance matrix and the different target return setting. The annualized means (μ) and volatilities (σ) of the optimal portfolio are shown here. The estimated Sharpe ratio for the asymmetric range-based DCC model is 0.521. Δ_r is the average switching annualized fee from the rollover OLS model to the asymmetric range-based volatility model. The values of the constant relative risk aversion are set as 1, 5, and 10.

Target return(%)	Means and Volatilities of Optimal Portfolios for Asymmetric Range-based DCC		Switching Fees from Rollover OLS to Asymmetric Range-based DCC		
	μ	σ	Δ_1	Δ_5	Δ_{10}
5	4.643	1.666	0.373	0.425	0.438
6	5.352	3.025	0.787	0.962	1.003
7	6.060	4.384	1.301	1.670	1.757
8	6.769	5.744	1.915	2.550	2.699
9	7.478	7.103	2.630	3.601	3.827
10	8.187	8.462	3.445	4.818	5.136
11	8.895	9.821	4.361	6.199	6.621
12	9.604	11.180	5.377	7.738	8.274
13	10.313	12.540	6.491	9.428	10.087
14	11.022	13.899	7.703	11.262	12.050
15	11.730	15.258	9.011	13.232	14.155

Panel A: Close Prices



Panel B: Returns



Panel C: Ranges

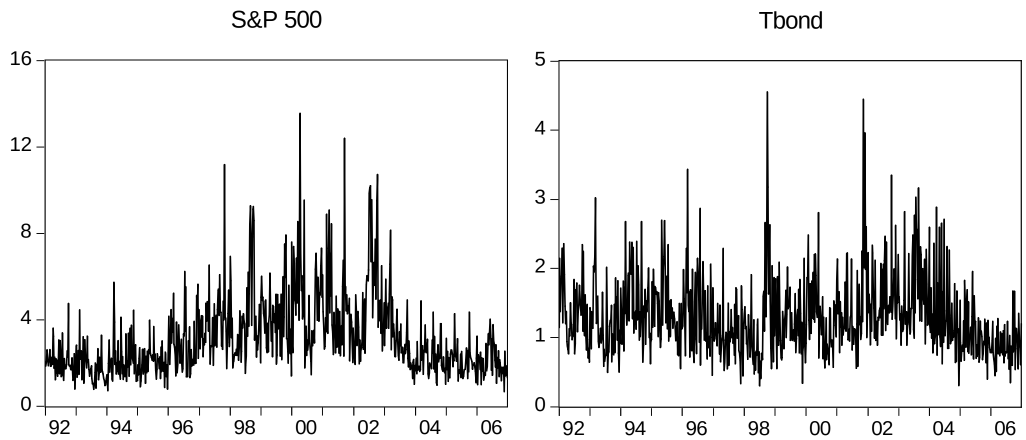
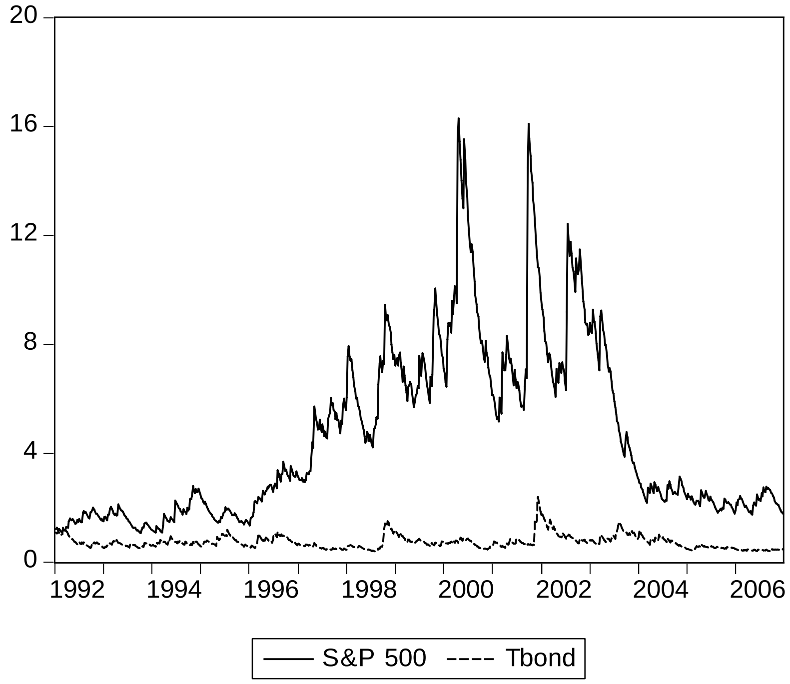


Figure 3.1: S&P 500 Index Futures and T-bond Futures Weekly Closing Prices, Returns and Ranges, 1992-2006. This figure shows the weekly close prices, returns, and ranges of S&P 500 index futures and 10-year Treasury bond (T-bond) futures over the sample period.

Panel A: Volatility Estimates for the GARCH Model



Panel B: Volatility Estimates for the CARR Model

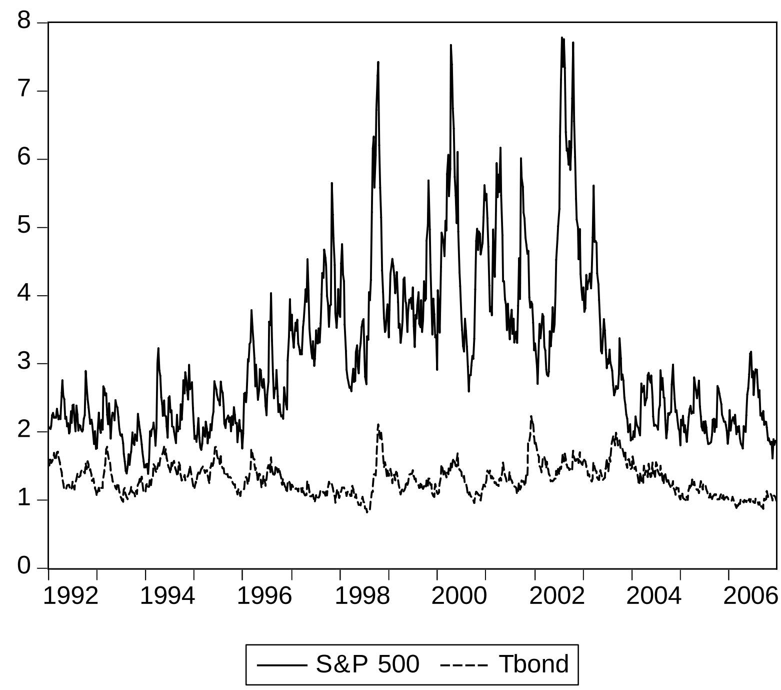
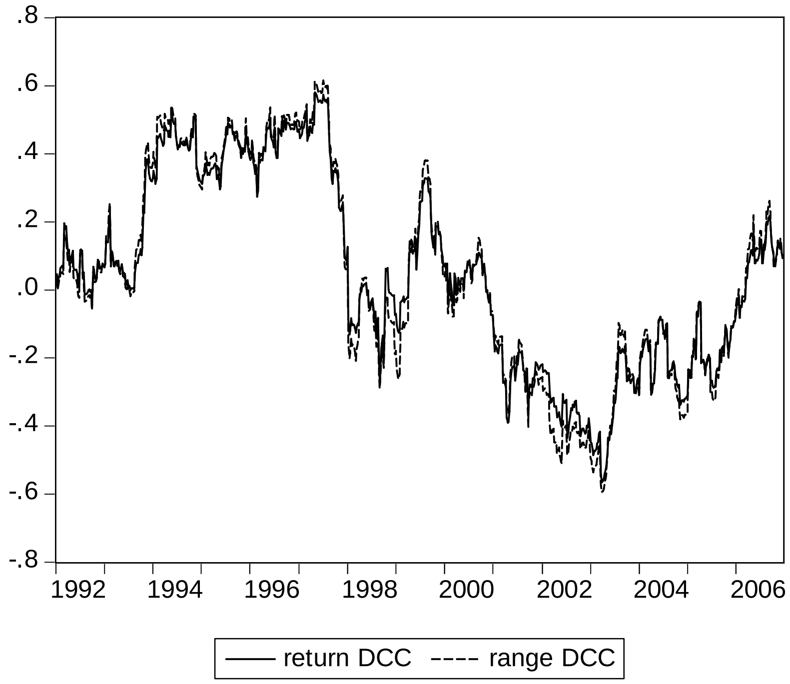


Figure 3.2: In-sample Volatility Estimates for the GARCH and CARR Model

Panel A: Correlation Estimates



Panel B: Covariance Estimates

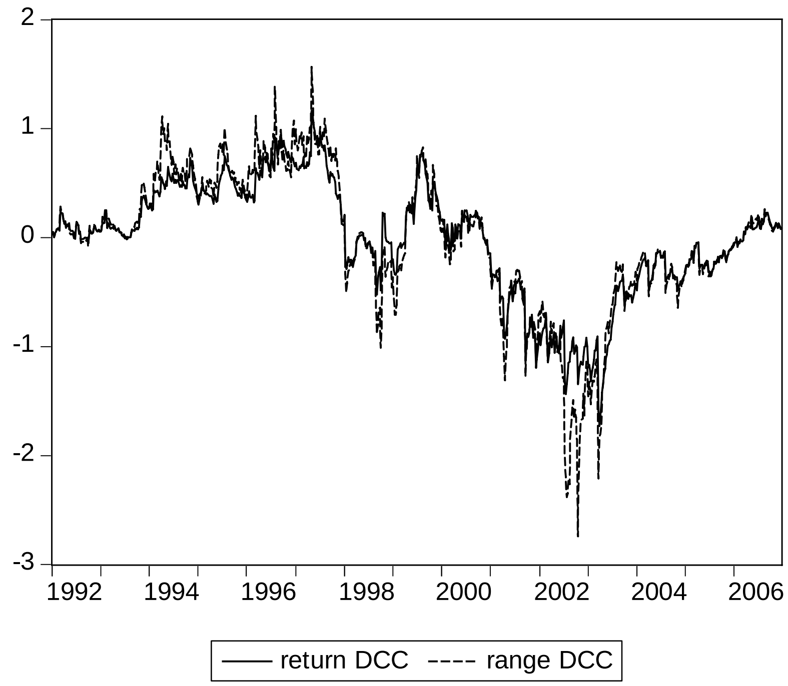
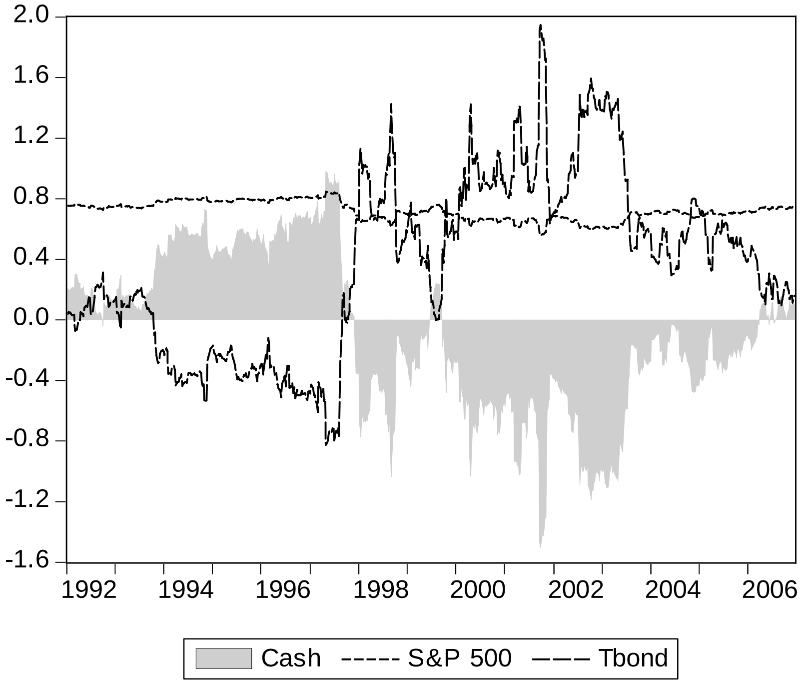


Figure 3.3: In-sample Correlation and Covariance Estimates for the Return-based and Range-based DCC Model

Panel A: In-sample Portfolio Weights Derived by the Return-based DCC Model



Panel B: In-sample Portfolio Weights Derived by the Range-based DCC Model

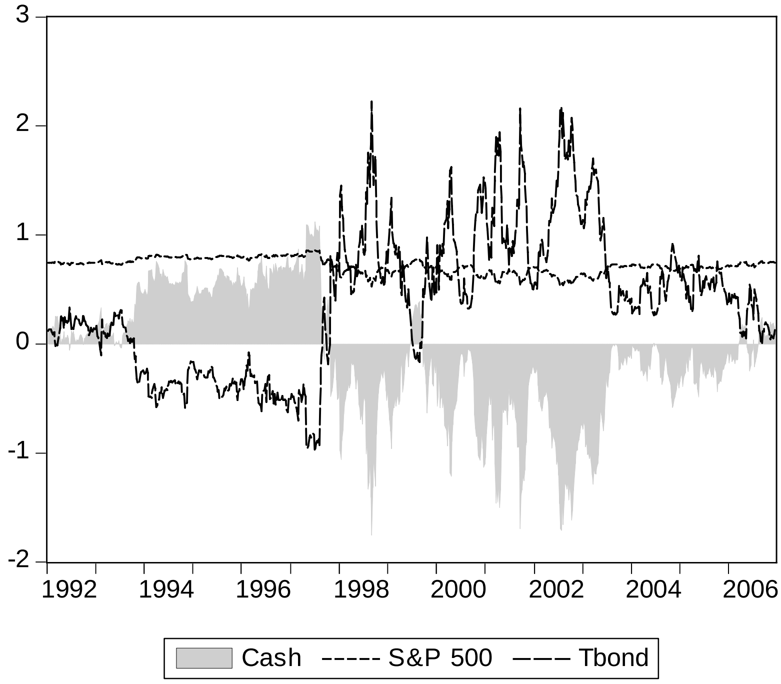
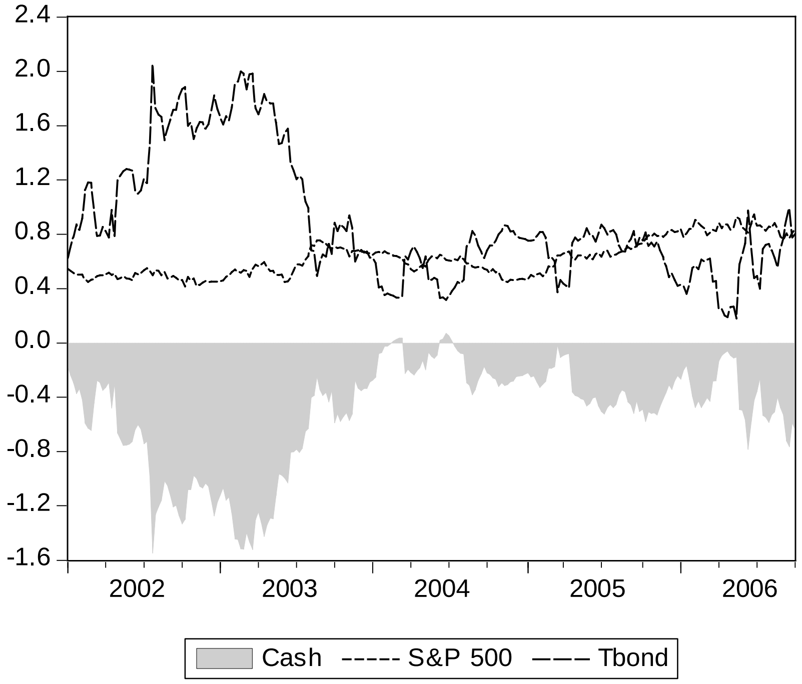
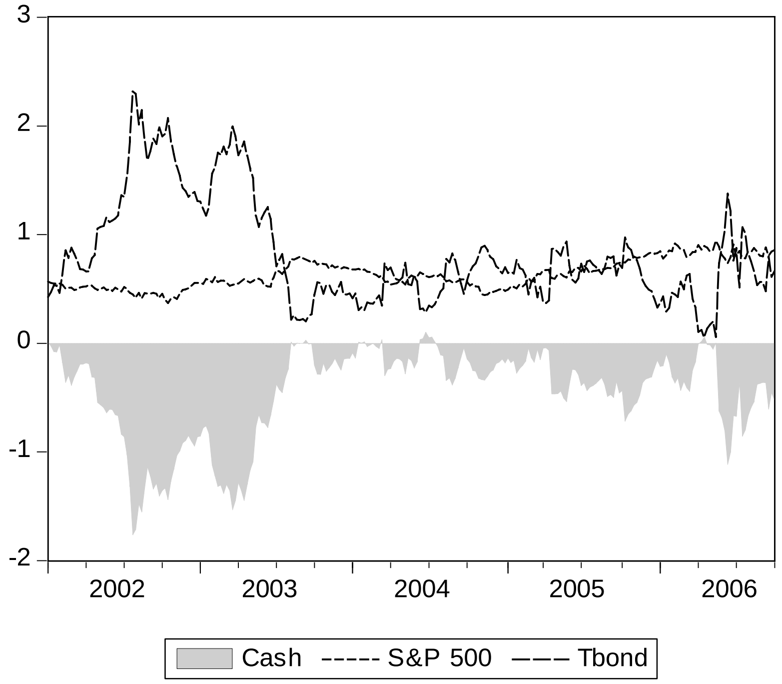


Figure 3.4: In-sample Minimum Volatility Portfolio Weight Derived by the Dynamic Volatility Model. Panels A and B show the weights that minimize conditional volatility while setting the expected annualized return equal to 10%. The OLS model has constant weights for cash, stock, and bond, i.e. -0.1934, 0.7079, and 0.4855.

Panel A: Out-of-sample Portfolio Weight Derived by the Return-based DCC Model



Panel B: Out-of-sample Portfolio Weight Derived by the Range-based DCC Model



Panel C: Out-of-sample Portfolio Weight Derived by the Rollover OLS Model

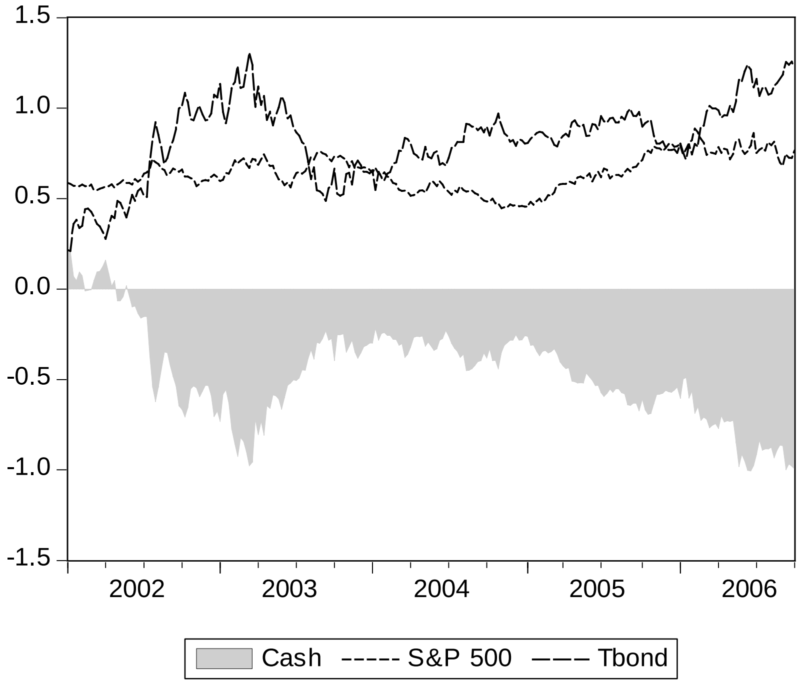


Figure 3.5: Out-of-sample Minimum Volatility Portfolio Weight Derived by the Dynamic Volatility Model for One Period Ahead Estimates. Panels A, B, and C show the one period ahead weights that minimize conditional volatility while the expected annualized return equal is set to 10%. Different from the in-sample case, the rolling sample method is used in the portfolio weights estimation. The portfolio weights in the rollover OLS model (Panel C) also vary with time. The first forecasted weights occur the week of January 4, 2002.

Chapter 4. Estimating Time-Varying Hedge Ratios with a Range-Based Multivariate Volatility Model

4.1 Introduction

Recent research has made significant contributions to theories and applications of futures hedging. In previous studies, Johnson (1960) and Stein (1961) introduced the concept of portfolio theory through hedging the spot position with futures. Here the hedging portfolio has usually been adopted as the returns of holding the spot asset on the returns together with the futures contracts. Edrington (1979) applied this concept to determine a minimum-variance hedge ratio and then proposed a measure of hedging effectiveness. From an academic perspective, an optimal hedging strategy is conventionally based on the expected-utility maximization paradigm. A simplification of this paradigm leads to the minimum-variance criterion[22]. In this case, the optimal hedge ratio can be defined as the amount of futures position for bearing one unit of spot position such that we have minimum variance hedging portfolio.

On the practical side, the research on futures hedging has benefited tremendously from recent developments in the econometrics literature. Many studies have focused on improving the estimation of the optimal hedge ratio. Moreover, some sophisticated estimation methods have been proposed after the knowledge about the statistical properties of financial time series that have been shared in the academic community. Various approaches to the optimal hedge ratios with different optimization criterions are discussed in Lien and Tse (2002), and Chen, Lee, and Shrestha (2003).

Past studies assumed the asset prices to follow a random walk with price changes

[22] Although the minimum variance hedge neglects the expected return of a hedging portfolio, it still has a consistent inference with other hedging criterions. For example, when an investor with higher risk averse or the futures prices follow martingale, the optimal hedge ratios derived from the minimum variance criterion are consistent with those from a mean-variance framework. The explanation is more fully developed in Kroner and Sultan (1993).

being identically and independently distributed. However, many commodity price changes appeared not to be independent but rather to be characterized by quiet and volatile periods as variances change over time, following Mandelbrot (1963) and Fama (1965). The unconditional distributions of commodity price changes are also found to be fat-tailed, or leptokurtic. Again, the empirical works powerfully support that volatility is time-varying in many economic and financial time series. After considering the deterministic volatility functions, some investigators adopt the framework of the GARCH model developed by Engle (1982) and Bollerslev (1986). The bivariate GARCH models, particularly, are widely adopted to explain the behavior of the spot and futures prices which produced the dynamic hedging strategy[23].

However, the results from the performance of the GARCH hedge ratios in comparing with the traditional methods are mixed. Most studies have found that the dynamic hedging strategies constructed by the GARCH methods outperform those of the static methods [Baillie and Myers (1991); Kroner and Sultan (1993)], but some ones are mostly in favor of the conventional hedging strategy. Hence, our paper intends to provide further evidence in this debate by introducing ranges in the multivariate GARCH models. Because most people are interested in knowing how well they can do in the future with a different hedging strategy and we would not change our hedging portfolio every day, this paper just highlights out-of-sample performance with weekly data.

In estimating volatility, the range data of asset prices perform better than the return data with close-to-close price [Parkinson (1980); Wiggins (1991); Alizadeh, Brandt and Diebold (2002); Chou (2005); Brandt and Jones (2006)]. Chou (2005) proposed the

[23] See Baillie and Myers (1991), Kroner and Sultan (1993), Lien, Tse, and Tsui (2002), Lien and Yang (2006) for a reference.

conditional autoregressive range (CARR) model[24] to estimate the volatility process. Compared with GARCH model, the CARR model obtained superior volatility forecast. Moreover, Chou, Wu and Liu (2007) extend it to a multivariate context using the DCC model proposed by Engle (2002a). The DCC model is a kind of two steps forecasting model which estimates univariate GARCH models for each asset and then calculates its time-varying correlation by using the transformed standardized residuals from the first step. They find that the range-based DCC model performs better than other return-based volatility models in forecasting covariances and correlations. However, there is very limited study in the practical financial applications of the range-based volatility models.

In this paper, we will test the range-based volatility model on futures hedging performance. Range data intuitively give more information than return data, and have low cost. In our empirical study, broad types of commodities are used to examine the optimal hedge ratio obtained from the new range-based volatility models. In addition to the traditional and rollover OLS[25] models, other compared strategies are all based on the frameworks of the CCC and DCC models.

This paper applies new volatility models to exercise the optimal futures hedging. The remainder of this paper is organized as follows. Section 4.2 discusses the static and dynamic hedging methodologies. Section 4.3 presents the data analysis and out-of-sample results of the optimal hedging ratios constructed from different models. The conclusion is included in the final section.

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[24] The CARR model and the autoregressive conditional duration (ACD) model of Engle and Russell (1998) are both special cases of the multiplicative error models (MEM) of Engle (2002b).

[25] The rollover approach here utilizes week-by-week updating to build the time-varying hedge ratios. It means that the rollover OLS models are viewed as dynamic hedging strategies. Lien, Tse, and Tsui (2002) used this rollover OLS approach (day-by-day updating) to build their hedge ratios and found this method performed better than the CCC model.

4.2 Hedging Methodology

Assume that the variance of the hedging portfolio ($P_H = \Delta S - h\Delta F$) using short hedge is $Var(P_H) = Var(\Delta S) - 2hCov(\Delta S, \Delta F) + h^2 Var(\Delta F)$ = $\sigma_S^2 - 2h\rho\sigma_S\sigma_F + h^2\sigma_F^2$ σ_S^2, where ΔS (r_S) and ΔF (r_F) are the difference of the logarithm types of the spot price S and the futures price F during the hedging period. σ_S and σ_F are the standard deviations of ΔS and ΔF. ρ is the correlation of ΔS and ΔF. The decision variable h is the hedge ratio. In a structure of minimum variance hedge, we can take the fist order differential to $Var(P_H)$ with h and then get the optimal hedge ratio $h^* = \rho\sigma_S / \sigma_F$. In practice, the optimal hedge ratio can be obtained from estimating the coefficient $\hat{\varphi}$ of the simple regression $r_{S,t} = \phi + \varphi r_{F,t} + \varepsilon_t$. This method has been broadly applied in the literature[26].

The classical regression method, as mentioned above, is assumed that its hedge ratio is time-invariant. In fact, the distribution of spot and futures prices may be time-varying. In the presence of an environment with changing conditional second moments, this method may not provide an effective hedge using the futures instruments. Recent studies suggest that the time-varying volatility prevails in many time series. The risk of assets changes because new information is continuously received by the markets [Bollerslev (1990); Kroner and Sultan (1993)]. Therefore, the hedge ratio should be time-varying because it depends on the conditional moments of the spot and futures returns. The conditional volatility literature has provided many models that capture the time-varying variance and covariance. Hence, the optimal hedge ratio for time *t* can be written as $h_t^* = \rho_t \sigma_{S,t} / \sigma_{F,t}$, where the conditional estimates (ρ_t, $\sigma_{S,t}$, and $\sigma_{F,t}$) are

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[26] Some studies use the error correction (EC) model proposed by Engle and Granger (1987) to calculate the optimal hedge ratio. However, there is just small difference between OLS and EC [Kroner and Sultan (1993)].

obtained from different models conditional on information set at time *t-1*.

4.3 Empirical Analysis

In this study, 887 weekly observations on the spot and futures for six classes (fifteen commodities), i.e., stock indices (FTSE 100, Nikkei 225 and S&P500 (SP)), currencies (British Pond (BP), Japanese Yen and Swiss Franc (SF)), metals (gold and silver), grains (corn, soybeans (Soy) and soybean oil (SO)), softs (coffee, cotton and sugar), and energy (crude oil (CL)), are obtained from Datastream. The detail of these data is described in Table 4.1. The time period of commodities is from January, 1, 1990 to December, 29, 2006. The futures data provided by Datastream are the nearest contract to deliver but rolled it over to the next nearest contract on the first day of the delivery month in order to avoid thin trading and expiration effects.

<Table 4.1 is inserted about here>

Table 4.2 gives summary statistics for returns and ranges of each spot and futures commodity. The returns are computed by $100 \times log(P_t / P_{t-1})$, where P_t is the close price in each week. The ranges are computed by $100 \times \log(P_t^{High} / P_t^{Low})$, where P_t^{High} and P_t^{Low} are the maximum and minimum price respectively among the daily close prices in the t^{th} week and the last trading day close price in the $t-1^{th}$ week[27]. The means of the returns are almost close to zero. As is noted by Fama (1965), this martingale behavior is often interpreted as being consistent with a weak form efficient market. Except soybeans, soybean oil and crude oil, the volatilities of all futures returns are somewhat higher than the volatilities of spot returns. The order of the magnitudes for the means of the range is roughly the same as that for the standard deviations of the

[27] Unlike financial assets, the high-low price data of most commodities in a trading day are unavailable but close price data. In this study, however, the weekly data are used to examine the hedging performance. Therefore, it is reasonable to use the measure as its proxy.

returns with the only two exceptions of corn and cotton. This reflects the fact that both range and standard deviations are measures of volatilities. Given that the range data are non-negative is present for all commodities.

< Table 4.2 is inserted about here >

In order to clarify the relative hedging performance, several models are used for comparison, including three buy-and-hold strategies (no hedge, naïve[28], and OLS) with fixed weights in the hedging period and five dynamic strategies (rollover OLS, return-based CCC, return-based DCC, range-based CCC, and range-based DCC) with time-varying weights in a framework of rolling sample.

The rolling sample approach here utilizes week-by-week updating to build the time-varying hedge ratios. There are 522 weekly observations (about ten years) in each of estimated period of the others. In addition, all cases provide 365 one period ahead out-of-sample forecasting values for comparison. The first forecasted value occurs on the week of January 3, 2000. Assume that there is one unit underlying asset in the beginning. No hedge means that the variance of its hedging portfolio is only decided by the underlying asset. Naïve here is the short hedge with selling one unit futures.

In order to formally compare the performances of each kind of hedging method, the hedging portfolios are applied by the estimated hedge ratios of each week. The variance of these portfolio returns can be written as $Var(r_{S,t} - h_t^* r_{F,t})$, where h_t^* are estimated optimal hedge ratios from different hedging methods. In this study, we focus on the out-of-sample forecasting results with one period ahead. Table 4.3 and Table 4.4 report the maximum likelihood estimations of the return-based and range-based DCC

[28] The naïve hedging strategy is the simplest way to hedge the spot price risk. This strategy suggests that an investor who has a long position in the spot market should sell a unit of futures today and buy it back when he sells the spot. If the spot and futures prices both change by the same amount at all times, this will be a perfect hedge.

models respectively. Here the first out-of-sample parameter estimates[29] are provided. In Table 4.3, Panel A and Panel B are the first step of the DCC model estimation, which are the GARCH model fittings of spot and futures returns respectively. In Table 4.4, they are for the CARR model fittings of ranges. Panel C is the second step of the DCC model estimation for both tables.

< Table 4.3 is inserted about here >

< Table 4.4 is inserted about here >

From the tables, the values of $(\hat{\alpha}+\hat{\beta})$ are close to one except for soybean oil, indicating high persistence in volatility. As for correlation persistence, however, the values of $(\hat{a}+\hat{b})$ exhibit inconsistent results. Some cases have high persistence in correlation, but the others don't. In addition, for the cases of gold and silver, the range-based DCC model shows stronger correlation persistence than the return-based one.

The comparisons of out-of-sample hedging performance are reported in Table 4.5. Panel A of Table 4.5 shows the variances of the hedging portfolios. To further gauge the hedging efficiency among various methods, Panel B also reports the efficiency gain of each alternative method compared to no hedge. Furthermore, Panel C shows the percentage variance improvement compared with the range-based DCC model.

< Table 4.5 is inserted about here >

Several observations can be made from the reading of Table 4.5. First, the portfolio variances in stock indices, currencies, and metals are much smaller than those in grains, softs, and energy. It seems that the financial market[30] has active trade and visible information to reduce the price change between spot and futures. It is obvious that trading noises lead to worse hedging performance for the agriculture and energy

29 In total, we have 365 estimations. The first parameter estimates are provided in Table 3 and Table 4.
30 In general, gold and silver are viewed as financial assets.

markets. In respect of no hedge, all portfolio variances, with the single exception of currency, are very large, especially in silver and non-financial commodities.

As for the comparison of the seven hedging methods, the naïve method generally is the worst of all. This is not surprising as the assumption of perfect correlation between the spot and the futures returns underlying this method is clearly not supported empirically. In the cases of stock indices, however, naïve performs better than the other static and rollover OLS models[31]. Next, the fact that the dynamic strategies with time-varying hedge ratios outperform the buy-and-hold ones indicates that the traditional method assuming a constant hedge ratio through the hedging period has a lot of room for improvements.

Among the dynamic hedging methods, how do the range-based methods compare with the return-based methods? The results suggest that the range-based ones are better than their corresponding opponents with the return-based ones. Specifically, the variances of the hedging portfolio derived from the range-based volatility models are smaller than the return-based volatility ones in thirteen out of fifteen commodities. The finding has its exception only in the soybean and coffee cases.

Panel B of Table 4.5 shows the hedging effectiveness of all strategies. The simple naïve hedge for all commodities can reduce over about 75% variation of spot. In addition, there are over 90% high values of hedging effectiveness for all hedging strategies in silver and two classes, stock index and currency. Again, the difference of the hedging effectiveness between the static and dynamic models for these cases is small. However, the results in the other commodities still support the superiority of the dynamic hedging strategies over the static ones. It is noteworthy that the poor

[31] Because the 10-year period might have some structural changes which would reduce the hedging efficiency of the OLS model, the 5-year OLS and rollover OLS models were considered as other comparison models. However, our empirical results indicated that the difference between two different estimated periods was very small.

effectiveness in crude oil seems to point out it is difficult to hedge by futures.

In order to more intuitively compare the performance of these hedging strategies, Panel C of Table 4.5 lists the hedging improvement ratio by the range-based DCC model. From the average percentage variance improvement reported in the last column, range-based DCC is the clear winner of all methods, with an improvement of about 30% over OLS, 27% over rollover OLS, 16% over return-based CCC, 10% over return-based DCC, and 5% over range-based CCC. It is valuable to take a more look at these values. In the same model setting, there are about 5% improvements by using the time-varying correlation strategies over the constant ones, and about 10% improvements by using the range-based strategies over the return-based ones[32]. From the hedging point of view, the range is indeed a more efficient measure of volatility than the return. Furthermore, the additional effort in modeling the time-varying pattern of the conditional correlation is with rewards.

<Figure 4.1 is inserted about here>

For illustration, Figure 4.1 plots the estimated hedge ratios using different methods for six cases, S&P 500, British Pond, gold, soybean oil, cotton, and crude oil, respectively. In addition to rollover OLS, the optimal return-based and range-based CCC or DCC models are put together for comparison. The rollover OLS model has the smoothest pattern in all cases, but still varies over time with a rolling-sample of ten years is used in the out-of-sample comparisons. To take cotton for example, there is an obvious jump in the middle of 2005. With the single exception of gold, the figures indicate that the hedge ratios from range are more volatile than those from return. To conclude, the dynamic methods provide wide variations of the hedge ratios around the

[32] In fact, we need to redo the work of Panel C of Table 5 for our target model to get the accuracy value. For simplicity, the related results are not listed in this study. Return-based (range-based) DCC has a 7.02% (4.96%) gain over return-based (range-based) CCC. Then, range-based DCC (CCC) has a 9.71% (11.76%) gain over return-based DCC (CCC).

OLS estimates. A more flexible hedge ratio seems to be necessary in order to obtain a more effective hedging strategy.

4.4 Conclusion

Range is a more efficient estimator than return in forecasting volatility. However, few researches utilize its superiority in financial applications. This paper uses range-based hedging models for calculating optimal hedge ratios in six classes of commodity futures, totally fifteen commodities, and compares with hedging performance of other models.

For a one-period forecasting horizon, empirical findings indicate that hedging performances of range-based volatility models are significantly better than the other volatility models for most commodities. Based on minimum-variance hedge criterion, the hedging portfolio variances calculated from the range-based volatility models are smaller than the return-based ones in thirteen out of fifteen commodities.

In conclusion, the results mainly indicate the following three points: (1) static hedging strategies are not suitable for most futures hedging, especially for non-financial ones; (2) assuming constant correlation generally has an approximate 5 percent loss in hedging achievement; (3) in the same dynamic structure, hedging improvement for the range data compared with the return data is about 10 percent on average.

Table 4.1: The Source of Spot and Futures Data

The table reports the related information for the fifteen futures and spots in this paper. The exchanges for futures, Datastream names for spots and their codes are included in this table.

		Futures		Spot	
Type	Name	Exchange	Code	Datastream Name	Code
Stock Index	FTSE 100	LIFFE	LSXCS00	FTSE 100 - PRICE INDEX	FTSE100
	Nikkei 225	OSX	ONACS00	NIKKEI 225 STOCK AVERAGE - PRICE INDEX	JAPDOWA
	S&P 500	CME	ISPCS00	S&P 500 COMPOSITE	S&PCOMP
Currency	British Pond	CME	IBPCS00	US $ TO UK (GTIS)	BRITPUS
	Japanese Yen	CME	IJYCS00	US $ TO JAPANESE YEN (GTIS)	JAPYNUS
	Swiss Franc	CME	ISFCS00	US $ TO SWISS FRANC (GTIS)	SWISFUS
Metal	Gold	CMX	NGCCS00	Gold, Handy & Harman Base $/Troy Oz	GOLDHAR
	Silver	CMX	NSLCS00	Silver, Handy & Harman (NY) cts/Troy OZ	SILVERH
Grain	Corn	CBOT	CC.CS00	Corn No.2 Yellow Cents/Bushel	CORNUS2
	Soybeans	CBOT	CS.CS00	Soyabeans, No.1 Yellow C/Bushel	SOYBEAN
	Soybean Oil	CBOT	CBOCS00	Soya Oil, Crude Decatur Cents/lb	SOYAOIL
Soft	Coffee	NYBOT	NKCCS00	Coffee-ICO Composite Daily ICA c/lb	COFDICA
	Cotton No. 2	NYBOT	NCTCS00	Cotton,1 1/16Str Low -Midl, Memph C/Lb	COTTONM
	Sugar No. 11	NYBOT	NSBCS00	Raw Cane Sugar, World FOB Cents/lb	SUGCNRW
Energy	Crude Oil	NYMEX	NCLCS00	Crude Oil-Brent Cur. Month FOB U$/BBL	OILBREN

Table 4.2: Summary Statistics for Returns and Ranges of Spot and Futures

The table provides summary statistics for the weekly return and range data of the spot and futures samples in this study. The returns are computed by $100 \times log(P_t / P_{t-1})$, where P_t is the close price in each week. The ranges are computed by $100 \times \log(P_t^{High} / P_t^{Low})$, where P_t^{High} and P_t^{Low} are the maximum and minimum price respectively among the daily close prices in the t^{th} week and the last trading day close price in the $t-1^{th}$ week. The sample period ranges from Jan 1, 1990 to Dec 29, 2006 (887 weekly observations).

	Spot				Futures			
	Return		Range		Return		Range	
	Mean	Std Dev	Mean	Std Dev	Mean	Std Dev	Mean	Std Dev
FTSE	0.105	2.084	2.304	1.418	0.103	2.186	2.469	1.483
Nikkei	-0.090	2.955	3.297	1.870	-0.091	3.020	3.365	1.896
SP	0.157	2.073	2.239	1.410	0.157	2.117	2.309	1.474
BP	0.020	1.290	1.373	0.824	0.022	1.322	1.405	0.847
Yen	0.022	1.565	1.612	1.030	0.023	1.603	1.660	1.056
SF	0.026	1.579	1.734	0.904	0.027	1.605	1.760	0.920
Gold	0.050	1.879	1.863	1.377	0.050	1.967	1.968	1.388
Silver	0.101	3.295	3.257	2.326	0.098	3.412	3.422	2.447
Corn	0.053	3.367	3.485	2.272	0.056	3.373	3.345	2.283
Soy	0.018	3.150	3.173	2.161	0.019	3.078	3.135	2.024
SO	0.046	3.158	3.465	1.890	0.049	3.079	3.360	1.837
Coffee	0.059	4.479	3.909	3.325	0.050	5.527	5.704	3.905
Cotton	-0.015	3.487	3.858	2.137	-0.019	3.750	3.775	2.515
Sugar	-0.011	4.166	4.569	2.744	-0.021	4.462	4.762	3.015
CL	0.107	5.229	5.395	3.640	0.110	4.930	5.281	3.540

Note: BP (British Pond), SF (Swiss Franc), Soy (soybeans), SO (soybean oil), and CL (crude oil).

Table 4.3: Return-based DCC Model Estimations

This table shows the first estimation (totally 365 estimations) of the range-based DCC models using the MLE method for the out-of-sample forecast. The estimated period here is ranging from Jan 1, 1990 to Dec 31, 1999 (522 weekly observations). Panel A and Panel B are the first step of the DCC model estimation for spot and futures returns of commodities respectively. Panel C is the second step of the DCC model estimation. LLF is the abbreviation for log likelihood function value. The values presented in parentheses are standard errors for the estimated coefficients. The return-based DCC model is shown as follows:

$$r_{k,t} = c_k + \varepsilon_{k,t}\,, \quad \varepsilon_{k,t} \mid I_{t-1} \sim N(0, h_{k,t})\,, k = 1,2\,.$$
$$h_{k,t} = \omega_k + \alpha_k \varepsilon_{k,t-i}^2 + \beta_k h_{k,t-1}\,,$$
$$\mathbf{Q}_t = (1-a-b)\overline{\mathbf{Q}} + a\mathbf{Z}_{t-1}\mathbf{Z}'_{t-1} + b\mathbf{Q}_{t-1}\,,$$

where $\mathbf{Z}_t$ is the standard residual vector which is standardized by GARCH volatilities. $\mathbf{Q}_t$ and $\overline{\mathbf{Q}}$ are the conditional and unconditional covariance matrix of $\mathbf{Z}_t$.

Panel A: Estimates of GARCH(1,1) model for spot returns

	FTSE	Nikkei	SP	BP	Yen	SF	Gold	Silver	Corn	Soy	SO	Coffee	Cotton	Sugar	CL
c_1	0.238	-0.060	0.261	0.026	0.085	-0.014	-0.074	-0.085	0.191	0.098	-0.047	-0.131	-0.067	-0.057	-0.114
	(0.086)	(0.118)	(0.068)	(0.055)	(0.0690	(0.071)	(0.048)	(0.129)	(0.121)	(0.108)	(0.118)	(0.167)	(0.133)	(0.170)	(0.208)
$\hat{\omega}_1$	0.069	1.046	0.052	0.096	0.066	0.283	0.083	0.434	0.413	0.712	5.467	2.505	1.791	0.583	1.202
	(0.050)	(0.473)	(0.036)	(0.074)	(0.056)	(0.218)	(0.042)	(0.264)	(0.224)	(0.232)	(1.627)	(1.728)	(1.197)	(0.344)	(0.472)
$\hat{\alpha}_1$	0.054	0.132	0.089	0.121	0.087	0.048	0.205	0.071	0.175	0.218	0.197	0.138	0.085	0.095	0.152
	(0.020)	(0.055)	(0.025)	(0.109)	(0.032)	(0.038)	(0.098)	(0.038)	(0.039)	(0.059)	(0.074)	(0.070)	(0.035)	(0.035)	(0.053)
$\hat{\beta}_1$	0.930	0.757	0.897	0.831	0.896	0.848	0.787	0.889	0.799	0.709	0.137	0.755	0.723	0.876	0.811
	(0.028)	(0.086)	(0.028)	(0.121)	(0.026)	(0.100)	(0.081)	(0.050)	(0.049)	(0.061)	(0.194)	(0.095)	(0.142)	(0.047)	(0.041)

Panel B: Estimates of GARCH(1,1) model for futures returns

	FTSE	Nikkei	SP	BP	Yen	SF	Gold	Silver	Corn	Soy	SO	Coffee	Cotton	Sugar	CL
c_2	0.236	-0.075	0.257	0.003	0.086	-0.014	-0.106	-0.096	0.184	0.111	-0.035	-0.173	-0.132	-0.162	-0.071
	(0.093)	(0.124)	(0.069)	(0.060)	(0.070)	(0.072)	(0.055)	(0.135)	(0.124)	(0.114)	(0.119)	(0.218)	(0.141)	(0.202)	(0.196)
$\hat{\omega}_2$	0.096	1.200	0.048	0.024	0.080	0.138	0.187	0.413	1.132	0.703	4.558	0.750	0.162	1.430	1.059
	(0.064)	(0.597)	(0.034)	(0.019)	(0.058)	(0.121)	(0.081)	(0.236)	(0.500)	(0.316)	(1.420)	(0.414)	(0.177)	(0.671)	(0.547)
$\hat{\alpha}_2$	0.054	0.120	0.084	0.055	0.092	0.044	0.274	0.066	0.204	0.186	0.200	0.151	0.105	0.119	0.111
	(0.021)	(0.054)	(0.022)	(0.034)	(0.039)	(0.028)	(0.118)	(0.031)	(0.091)	(0.074)	(0.075)	(0.050)	(0.032)	(0.044)	(0.035)
$\hat{\beta}_2$	0.925	0.761	0.903	0.932	0.887	0.908	0.696	0.900	0.687	0.740	0.233	0.851	0.897	0.816	0.847
	(0.030)	(0.093)	(0.024)	(0.032)	(0.028)	(0.059)	(0.098)	(0.041)	(0.096)	(0.088)	(0.184)	(0.036)	(0.022)	(0.061)	(0.035)

Panel C: Estimates of return-based DCC model

	FTSE	Nikkei	SP	BP	Yen	SF	Gold	Silver	Corn	Soy	SO	Coffee	Cotton	Sugar	CL
$\hat{a}$	0.021	0.054	0.024	0.043	0.019	0.096	0.119	0.039	0.215	0.247	0.328	0.046	0.218	0.388	0.151
	(0.007)	(0.008)	(0.009)	(0.005)	(0.002)	(0.014)	(0.026)	(0.016)	(0.026)	(0.024)	(0.025)	(0.010)	(0.021)	(0.026)	(0.013)
$\hat{b}$	0.976	0.942	-0.930	0.933	0.975	-0.142	-0.011	0.697	0.649	0.549	0.318	0.876	0.369	0.022	0.685
	(0.010)	(0.009)	(0.043)	(0.008)	(0.004)	(0.173)	(0.232)	(0.163)	(0.045)	(0.055)	(0.037)	(0.032)	(0.125)	(0.044)	(0.038)
LLF	809.399	914.249	850.669	818.568	898.616	901.638	395.667	522.703	353.391	577.613	535.826	194.930	186.348	375.383	376.728

Table 4.4: Range-based DCC Model Estimations

This table shows the first estimation (totally 365 estimations) of the range-based DCC models using the MLE method for the out-of-sample forecast. The estimated period here is ranging from Jan 1, 1990 to Dec 31, 1999 (522 weekly observations). Panel A and Panel B are the first step of the DCC model estimation for spot and futures ranges of commodities respectively. Panel C is the second step of the DCC model estimation. LLF is the abbreviation for log likelihood function value. The values presented in parentheses are standard errors for the estimated coefficients. The range-based DCC model is shown as follows:

$$\Re_{i,t} = u_{i,t}\,,\ \ \Re_{k,t} \mid I_{t-1} \sim f(1,\cdot)\,,\ \ k = 1,2\,.$$

$$\lambda_{k,t} = \omega_k + \alpha_k \Re_{k,t-1} + \beta_k \lambda_{k,t-1}\,,$$

$$\mathbf{Q}_t = (1-a-b)\bar{\mathbf{Q}} + a\mathbf{Z}_{t-1}\mathbf{Z}'_{t-1} + b\mathbf{Q}_{t-1}\,,$$

where $\Re_t$ is the range variable, $\mathbf{Z}_t$ is the standard residual vector which is standardized by CARR volatilities. $\mathbf{Q}_t$ and $\bar{\mathbf{Q}}$ are the conditional and unconditional covariance matrix of $\mathbf{Z}_t$.

Panel A: Estimates of CARR(1,1) model for spot ranges

	FTSE	Nikkei	SP	BP	Yen	SF	Gold	Silver	Corn	Soy	SO	Coffee	Cotton	Sugar	CL
$\hat{\omega}_1$	0.045	0.359	0.029	0.044	0.070	0.126	0.054	0.109	0.219	0.290	1.195	0.117	0.303	0.223	0.224
	(0.024)	(0.122)	(0.018)	(0.023)	(0.038)	(0.065)	(0.028)	(0.057)	(0.073)	(0.081)	(0.362)	(0.065)	(0.125)	(0.100)	(0.098)
$\hat{\alpha}_1$	0.092	0.212	0.109	0.110	0.105	0.076	0.177	0.093	0.208	0.233	0.213	0.107	0.140	0.151	0.185
	(0.022)	(0.041)	(0.024)	(0.032)	(0.024)	(0.028)	(0.045)	(0.028)	(0.037)	(0.045)	(0.051)	(0.036)	(0.032)	(0.032)	(0.030)
$\hat{\beta}_1$	0.888	0.680	0.876	0.858	0.855	0.852	0.789	0.872	0.724	0.667	0.418	0.865	0.770	0.799	0.772
	(0.027)	(0.063)	(0.027)	(0.041)	(0.037)	(0.054)	(0.054)	(0.039)	(0.050)	(0.059)	(0.137)	(0.039)	(0.059)	(0.045)	(0.037)

Panel B: Estimates of CARR(1,1) model for futures ranges

	FTSE	Nikkei	SP	BP	Yen	SF	Gold	Silver	Corn	Soy	SO	Coffee	Cotton	Sugar	CL
$\hat{\omega}_2$	0.063	0.335	0.039	0.028	0.087	0.083	0.064	0.109	0.278	0.318	0.835	0.234	0.115	0.262	0.174
	(0.031)	(0.115)	(0.021)	(0.017)	(0.045)	(0.048)	(0.031)	(0.063)	(0.106)	(0.099)	(0.261)	(0.107)	(0.084)	(0.114)	(0.084)
$\hat{\alpha}_2$	0.106	0.201	0.123	0.087	0.123	0.071	0.165	0.088	0.180	0.220	0.202	0.153	0.099	0.137	0.161
	(0.025)	(0.039)	(0.025)	(0.026)	(0.027)	(0.025)	(0.043)	(0.027)	(0.040)	(0.047)	(0.052)	(0.032)	(0.023)	(0.031)	(0.027)
$\hat{\beta}_2$	0.869	0.701	0.859	0.893	0.829	0.882	0.797	0.880	0.728	0.669	0.533	0.810	0.866	0.805	0.804
	(0.031)	(0.057)	(0.029)	(0.032)	(0.041)	(0.044)	(0.051)	(0.039)	(0.059)	(0.067)	(0.113)	(0.042)	(0.043)	(0.046)	(0.034)

Panel C: Estimates of range-based DCC model

	FTSE	Nikkei	SP	BP	Yen	SF	Gold	Silver	Corn	Soy	SO	Coffee	Cotton	Sugar	CL
$\hat{a}$	0.027	0.061	-0.026	0.051	0.005	0.092	0.046	0.032	0.243	0.289	0.337	0.110	0.241	0.439	0.142
	(0.007)	(0.009)	(0.014)	(0.005)	(0.000)	(0.013)	(0.016)	(0.011)	(0.021)	(0.023)	(0.027)	(0.009)	(0.025)	(0.029)	(0.014)
$\hat{b}$	0.978	0.933	0.818	0.930	0.996	-0.130	0.878	0.922	0.657	0.549	0.339	0.835	0.364	0.008	0.648
	(0.008)	(0.010)	(0.153)	(0.007)	(0.000)	(0.153)	(0.040)	(0.040)	(0.032)	(0.044)	(0.041)	(0.020)	(0.133)	(0.037)	(0.051)
LLF	804.909	914.491	849.783	859.567	876.946	898.055	412.295	516.787	336.278	577.734	532.714	180.451	188.785	377.413	369.952

Table 4.5: Comparisons of Out-of-Sample Hedging Performance

There are three parts in this table. Panel A shows the post-sample portfolio variances. Panel B shows the hedging effectiveness. Panel C shows hedging improvement ratio by range-based DCC for other methods. The models used for comparison include three buy-and-hold strategies (no hedging, naïve, and OLS) with fixed weights in the hedging period and five dynamic strategies (rollover OLS, return-based CCC, return-based DCC, range-based CCC, and range-based DCC) with time-varying weights in a framework of rolling sample. The rolling sample approach here utilizes week-by-week updating to build the time-varying hedge ratios. Assume that there is one unit underlying asset in the beginning. No hedging means that the variance of its hedging portfolio is only decided from the underlying asset. Naïve is the short hedge with selling one unit futures. There are 522 observations (ten years) in each of estimated period of the others. There are 365 one period ahead out-of-sample forecasting values provided for comparison. The first forecasted value occurs on the week of January 3, 2000.

Panel A: Portfolio variances for all strategies

	FTSE	Nikkei	SP	BP	Yen	SF	Gold	Silver	Corn	Soy	SO	Coffee	Cotton	Sugar	CL
no hedging	4.6695	7.8949	5.4291	1.4032	1.6720	2.1171	4.8753	12.1656	13.2761	12.4454	12.1466	15.7210	16.3349	17.7476	25.5334
naïve	0.0826	0.2366	0.1133	0.0643	0.1038	0.1293	1.1745	0.9274	2.6654	2.1024	1.2375	7.8405	5.8008	8.2354	6.5475
OLS	0.1110	0.2560	0.1262	0.0653	0.0979	0.1258	1.0777	1.0014	2.7249	2.1149	1.2713	5.8112	5.8424	6.8299	6.2694
rollover OLS	0.0901	0.2444	0.1179	0.0638	0.0971	0.1253	1.0616	0.9312	2.6168	2.0649	1.2475	5.7835	5.5008	6.8467	6.0101
return-based CCC	0.0705	0.2306	0.1252	0.0613	0.0829	0.1209	0.8342	0.7806	2.1103	1.2998	1.0443	4.3736	5.1678	6.3694	5.9494
return-based DCC	0.0704	0.2286	0.1258	0.0600	0.0824	0.1179	0.7644	0.7560	1.7623	<u>1.1903</u>	0.9536	<u>4.1437</u>	3.9894	5.0676	5.5862
range-based CCC	0.0575	0.2126	<u>0.1022</u>	0.0567	<u>0.0709</u>	0.1028	0.8654	<u>0.5659</u>	<u>1.6215</u>	1.6779	0.9557	4.7769	2.9810	5.2619	4.8775
range-based DCC	<u>0.0570</u>	<u>0.2112</u>	0.1052	<u>0.0555</u>	0.0711	<u>0.0987</u>	<u>0.7538</u>	0.5816	1.7809	1.7554	<u>0.8273</u>	4.4907	<u>2.3862</u>	<u>3.7297</u>	<u>4.5918</u>

Note: The number with an underline stands for the smallest hedging portfolio variance in each commodity column.

Panel B: Hedging effectiveness ($1 - Var_{Hedgie} / Var_{no\ hedging}$)

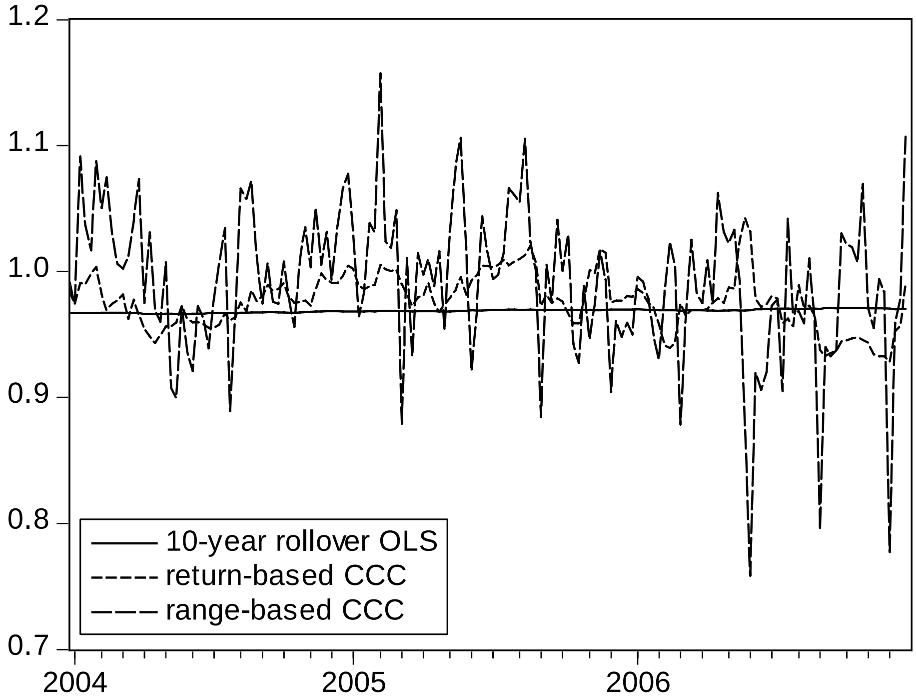
	FTSE	Nikkei	SP	BP	Yen	SF	Gold	Silver	Corn	Soy	SO	Coffee	Cotton	Sugar	CL
naive	0.9823	0.9700	0.9791	0.9542	0.9379	0.9389	0.7591	0.9238	0.7992	0.8311	0.8981	0.5013	0.6449	0.5360	0.7436
OLS	0.9762	0.9676	0.9767	0.9535	0.9414	0.9406	0.7789	0.9177	0.7948	0.8301	0.8953	0.6304	0.6423	0.6152	0.7545
rollover OLS	0.9807	0.9690	0.9783	0.9545	0.9419	0.9408	0.7822	0.9235	0.8029	0.8341	0.8973	0.6321	0.6632	0.6142	0.7646
return-based CCC	0.9849	0.9708	0.9769	0.9563	0.9504	0.9429	0.8289	0.9358	0.8410	0.8956	0.9140	0.7218	0.6836	0.6411	0.7670
return-based DCC	0.9849	0.9710	0.9768	0.9572	0.9507	0.9443	0.8432	0.9379	0.8673	0.9044	0.9215	0.7364	0.7558	0.7145	0.7812
range-based CCC	0.9877	0.9731	0.9812	0.9596	0.9576	0.9514	0.8225	0.9535	0.8779	0.8652	0.9213	0.6961	0.8175	0.7035	0.8090
range-based DCC	0.9878	0.9732	0.9806	0.9605	0.9575	0.9534	0.8454	0.9522	0.8659	0.8589	0.9319	0.7143	0.8539	0.7899	0.8202

Note: The number with an underline stands for the largest hedging effectiveness in each commodity column.

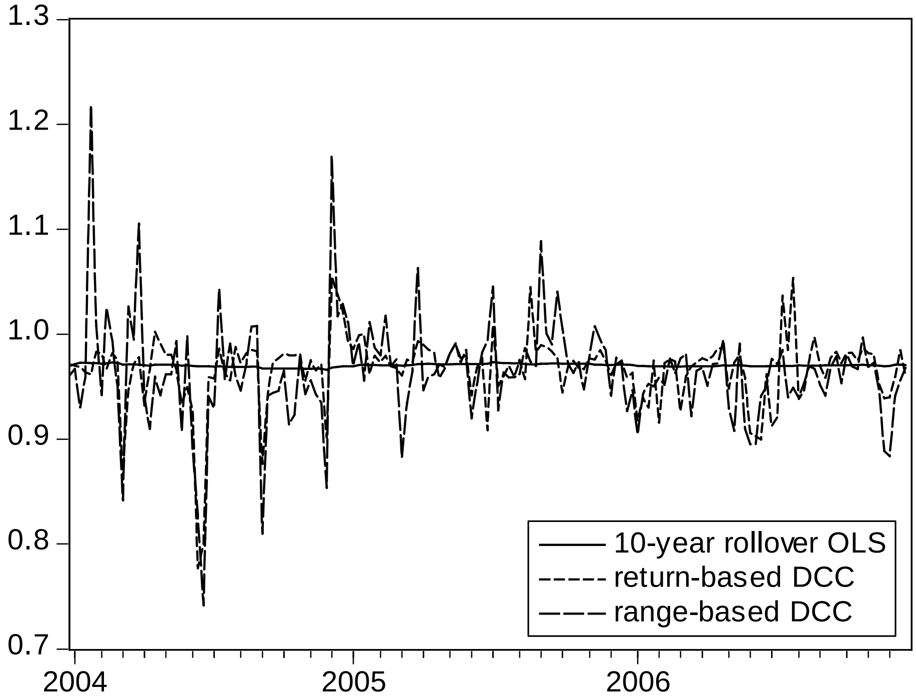
Panel C: Hedging improvement ratio by range-based DCC ($1 - Var_{range\ \mathrm{DCC}} / Var_{other\ \mathrm{model}}$)

	FTSE	Nikkei	SP	BP	Yen	SF	Gold	Silver	Corn	Soy	SO	Coffee	Cotton	Sugar	CL	Averag
naive	0.3098	0.1073	0.0715	0.1376	0.3148	0.2366	0.3582	0.3729	0.3318	0.1651	0.3315	0.4272	0.5886	0.5471	0.2987	0.3066
OLS	0.4868	0.1749	0.1664	0.1502	0.2736	0.2152	0.3006	0.4192	0.3464	0.1700	0.3493	0.2272	0.5916	0.4539	0.2676	0.3062
rollover OLS	0.3679	0.1357	0.1072	0.1307	0.2678	0.2122	0.2900	0.3754	0.3194	0.1499	0.3368	0.2235	0.5662	0.4553	0.2360	0.2783
return-based CCC	0.1916	0.0840	0.1593	0.0946	0.1417	0.1834	0.0964	0.2549	0.1561	-0.3505	0.2078	-0.0268	0.5383	0.4144	0.2282	0.1582
return-based DCC	0.1903	0.0760	0.1637	0.0754	0.1372	0.1625	0.0139	0.2307	-0.0106	-0.4748	0.1325	-0.0838	0.4019	0.2640	0.1780	0.0971
range-based CCC	0.0083	0.0067	-0.0300	0.0207	-0.0027	0.0399	0.1290	-0.0277	-0.0983	-0.0462	0.1343	0.0599	0.1995	0.2912	0.0586	0.0496

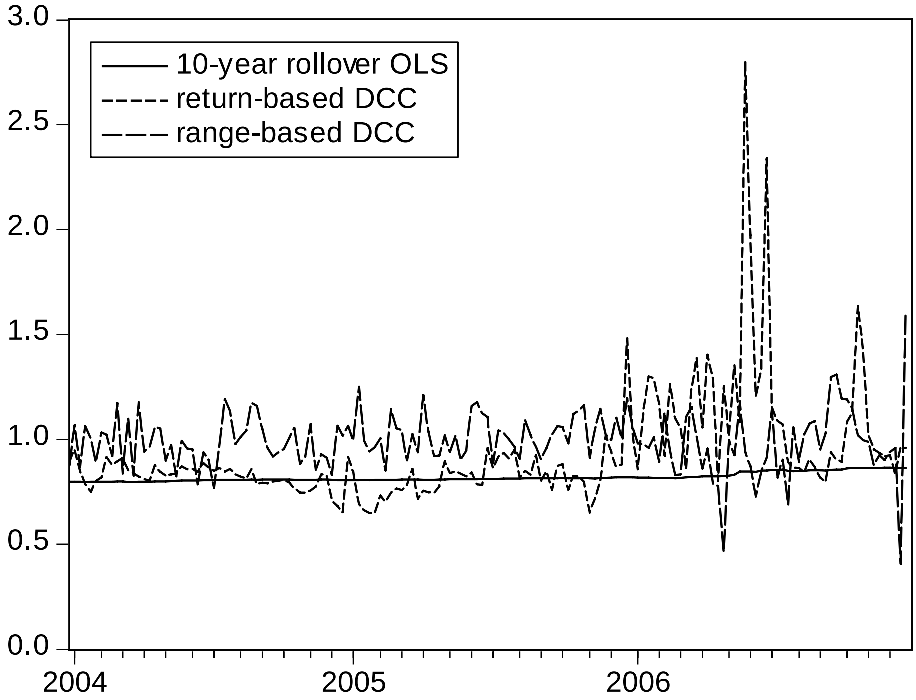
Panel A: S&P 500



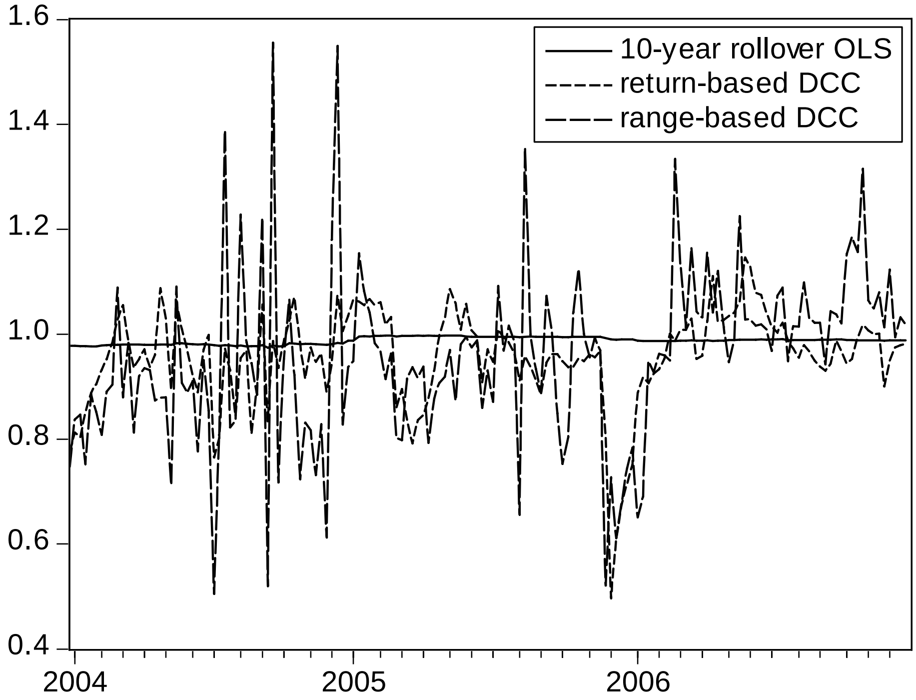
Panel B: British Pond



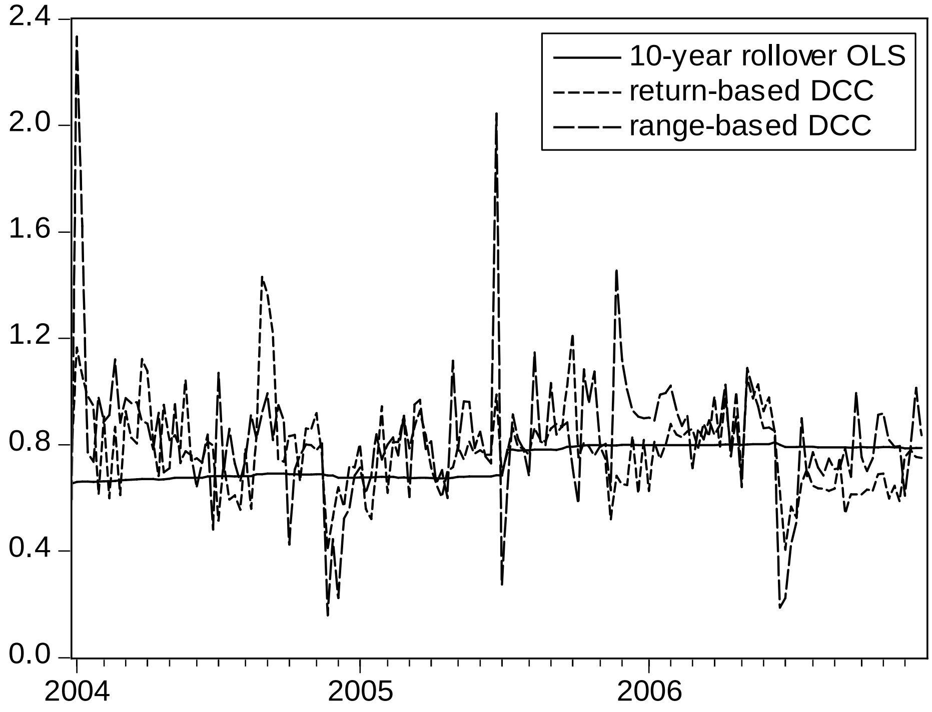
Panel C: Gold



Panel D: Soybean Oil



Panel E: Cotton



Panel F: Crude Oil

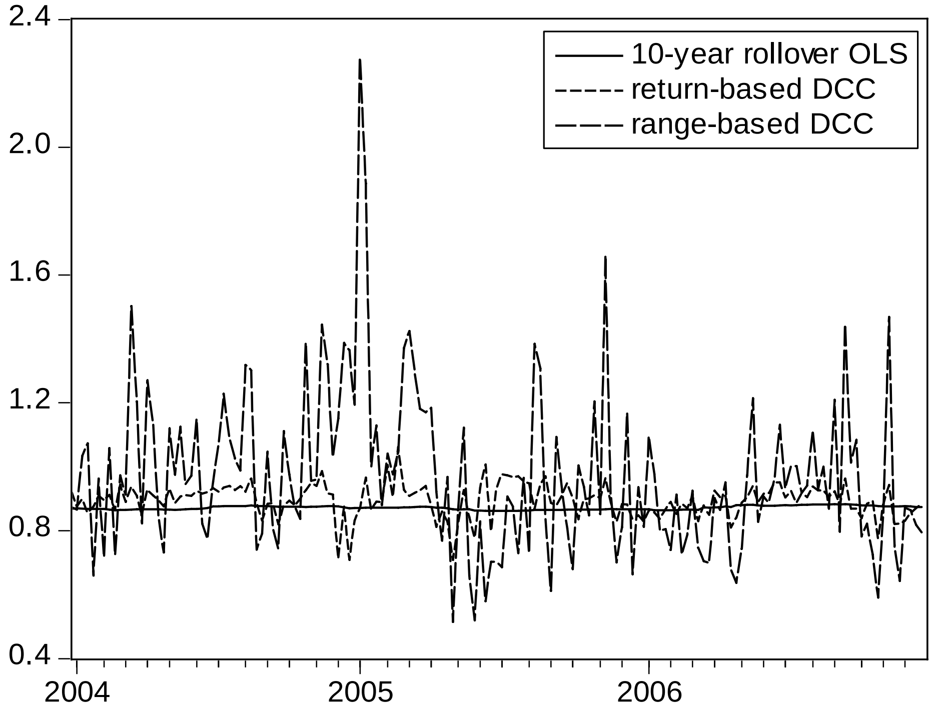


Figure 4.1: Comparison of optimal hedge ratios. There are six panels in this figure, including S&P 500, British Pond, gold, soybean oil, cotton, and crude oil. In addition to rollover OLS, we put optimal return-based and range-based CCC or DCC for comparison. For convenience in distinguishing, we reserve the last three years.

Chapter 5. Conclusions

Volatility plays a central role in many areas of finance. In view of the theoretical and practical studies, the price range provides an intuitive and efficient estimator of volatility. In this paper, we propose a new range model, which incorporates the superiority of range in forecasting volatility and of range and the elasticity of the DCC model. It contributes to the multivariate applications and can be led into broad applications in finance.

This dissertation provides three empirical methods to strengthen the suitability of the new range-based volatility model. To begin with a statistical test, the range-based DCC model performs better than other selected models for the four covariance benchmarks. Then, we test its economic value and compare its performance with the return-based DCC model. We conclude that the range-based DCC model obtains higher economic value than the return-based one. Finally, we apply the range model to calculate hedge ratios. Based on minimum-variance hedge criterion, range-based volatility models have better performance in most commodities.

Undoubtedly, the range is sensitive to outliers in statistics, and however only few researches mention this problem. It’s useful and meaningful to utilize the quantile range to replace the standard range to get a robust measure of range. Moreover, the multivariate works for range are still in its infancy. Future research is obviously required for this topic.

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